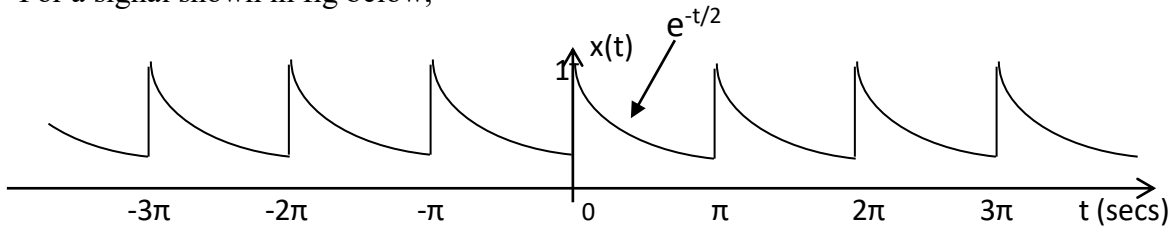


## Week-7 (Fourier Series Analysis) –Continuous-time signals

### Computation and plotting of the Fourier coefficients for a periodic signal:

For a signal shown in fig below,



**Method 1:** Write code to compute Fourier series coefficients either by creating m-function file or directly in m-script file: A sample of MATLAB code is here for your reference:

```
clc; clear; clf;
To = pi;                wo = 2*pi/To;
h = 0.001;              t = 0:h:(To-h);
y = exp(-t/2);          N = length(y);
Co = sum(y)/(N-1);
for n = 1:10             % Number of Fourier terms
a(n) = 2*sum(y.*cos(n*wo*t))/(N-1);
b(n) = 2*sum(y.*sin(n*wo*t))/(N-1);
end
Cn = sqrt(a.^2+b.^2);    thetan = atan(-b./a);
n = 0:10;
subplot(2,2,1); stem(n,[Co a], 'k'); ylabel('an'); xlabel('n');
subplot(2,2,2); stem(n,[0 a], 'k'); ylabel('bn'); xlabel('n');
subplot(2,2,3); stem(n,[Co Cn], 'k'); ylabel('cn'); xlabel('n');
subplot(2,2,4); stem(n,[0 thetan], 'k'); ylabel('\theta[rad]'); xlabel('n');
```

**Method 2:** Manually derive the expression for Fourier coefficients, ( $a_0$ ,  $a_n$ ,  $b_n$ ,  $C_n$ ,  $\theta_n$ ) and then plot using the following MATLAB code

```
clc; clear; clf;
n = 1:10; an(1) = 0.504; an(n+1) = 0.504*2./(1+16*n.^2);
bn(1) = 0; bn(n+1) = 0.504*8*n./(1+16*n.^2);
cn(1) = an(1);
cn(n+1) = sqrt(an(n+1).^2+bn(n+1).^2);
thetan(1) = 0; thetan(n+1) = atan2(-bn(n+1),an(n+1));
n=[0,n];
subplot(2,2,1); stem(n,an,'k'); ylabel('an'); xlabel('n');
subplot(2,2,2); stem(n,bn,'k'); ylabel('bn'); xlabel('n');
subplot(2,2,3); stem(n,cn,'k'); ylabel('cn'); xlabel('n');
subplot(2,2,4); stem(n,thetan,'k'); ylabel('\theta[rad]'); xlabel('n');
```

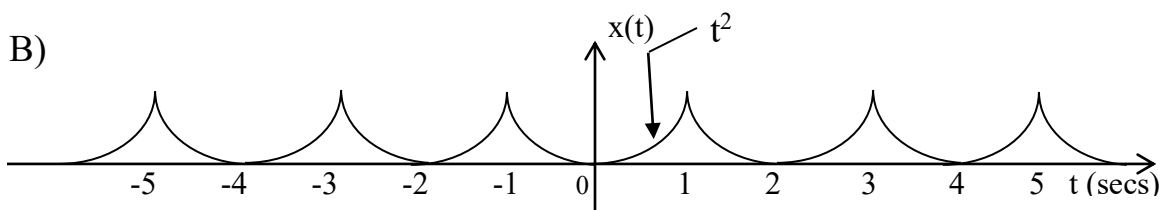
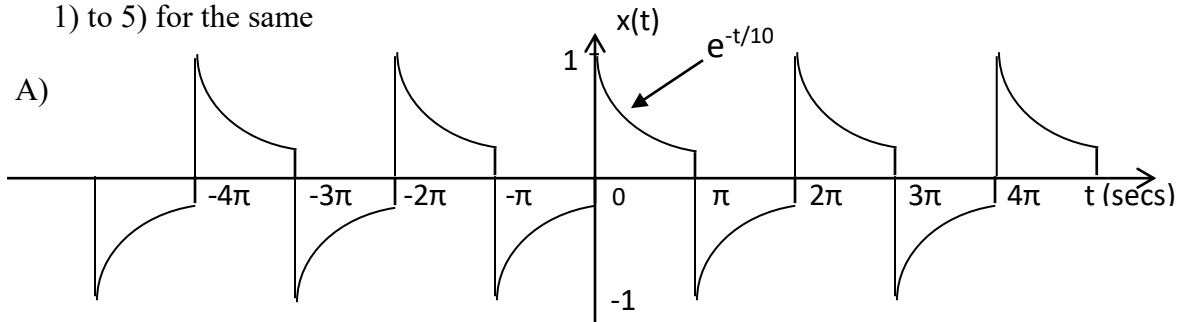
O/P ----> All the coefficients will be plotted against n

Manually derive the expression for Exponential Fourier coefficients for a periodic signal and then plot using the following MATLAB code

```
n = -10:10; dn = 0.504./(1+j*4*n);
subplot(2,1,1); stem(n,abs(dn), 'k'); ylabel('|dn|'); xlabel('n');
subplot(2,1,2); stem(n,angle(dn), 'k'); ylabel('\angle dn [rad]'); xlabel('n');
```

### Exercises:

- 1) Sketch Fourier series expanded signal  $x(t)$  vs. continuous time  $t$  considering at least 5 harmonic components of the series.
- 2) Sketch each harmonic sinusoidal signals (at least 5 Fourier series terms) w. r. t. continuous time  $t$ , explicitly in the same graph sheet, and observe the role of Amplitude and Phase Spectra in Wave shaping. (Use inline functions if necessary)
- 3) Find and sketch the Fourier series coefficients for the signal  $x(-t)$ . Compare the results with the F.S. of  $x(t)$ .
- 4) Find and sketch the Fourier series coefficients for the time compressed/expanded signal. And observe the change in the Fourier spectra. Compare the results with the F.S of  $x(t)$ .
- 5) For the above example, find and sketch the Fourier Spectra for the time delayed  $x(t-0.5)$ /time advanced  $x(t+0.5)$  signals. And observe the change in the Fourier spectra. Compare the results with the F.S of  $x(t)$ .
- 6) Identify the following signals whether periodic or aperiodic. For periodic signals, find the period and state which of the harmonics are present in the series.
  - a)  $3 \sin t + 2 \sin 3t$
  - b)  $2 + 5 \sin 4t + 4 \cos 7t$
  - c)  $2 \sin 3t + 7 \cos \pi t$
  - d)  $7 \cos \pi t + 5 \sin 2\pi t$
  - e)  $3 \cos \sqrt{2} t + 5 \cos 2t$
  - f)  $\sin \frac{5t}{2} + 3 \cos \frac{6t}{5} + 3 \sin \left( \frac{t}{7} + 30^\circ \right)$
  - g)  $\sin 3t + \cos \frac{15t}{4}$
  - h)  $(3 \sin 2t + \sin 5t)^2$
  - i)  $(5 \sin 2t)^3$
- 7) Find and sketch the Fourier Spectra for the following signals and repeat the exercises 1) to 5) for the same



C) Signal variation replaced with  $t$  instead of  $t^2$  in the above signal B)