

Week-7 (Fourier Transform) –Continuous-time signals

There is no MATLAB command which gives directly the Fourier Transform of any given continuous-time signal.

Let the energy signal be $x(t) = \begin{cases} e^{-\frac{t}{2}} & 0 \leq t \leq \pi \\ 0 & \text{otherwise} \end{cases}$ FT is $X(j\omega) = \frac{1 - 0.2079e^{-j\pi\omega}}{0.5 + j\omega}$

METHOD-1: An explicit MATLAB code can be written to compute the Fourier Transform (FT) using the “**sum**” command by appropriately modifying the code used for computing the Fourier Series. Following modifications may be attempted.

- 1) Define the time range **wide enough** to cover the entire non-zero duration of the energy signal, instead of restricting it to a single period.
- 2) Choose an appropriate step size “**h**” to accurately represent both the continuous-time energy signal and its continuous-frequency spectral response.
- 3) Define the continuous-time energy signal.
- 4) Initialize the index counter “**rk**” to zero, which will be used for indexing the array of computed spectral components of the signal.
- 5) Use a loop to compute the Fourier spectral components from first principles using the “**sum**” command, considering the appropriate lower and upper frequency limits (**w**), step size (**h**), and incrementing index counter (**rk**), as shown below:

$$xr(rk) = \text{sum}(x(t) \cdot \exp(-j \cdot w \cdot t)) \cdot h;$$

- 6) Plot the magnitude and phase of the computed spectral response with respect to frequency.

METHOD-2: Manually derive the expression for the Fourier Transform of the signal using first principles or relevant properties, and use the obtained result to sketch the spectrum.

Exercises:

1. Modify the code to reconstruct back the original signal in time-domain $x(t)$ and sketch the same with respect to time t .
2. If the above signal $x(t)$ is given as input to the system whose transfer function is $H(D) = \frac{1}{D + 2}$, then find manually the zero-state response of the system using FT approach and sketch the same. Also, get the same result through MATLAB implementation
3. Discuss/Evaluate/comprehend, how the **Fourier spectrum** modifies, when the following properties are applied to the above rectangle signal, along with any convenient another energy signals (if necessary) listed below. [Use MATLAB/penwork, wherever necessary].

a) Scalar addition	b) Scalar Multiplication	c) Conjugation
d) addition of two different time domain signals		e) Time shifting
f) Time scaling (compression, expansion, reversal		g) Frequency shifting
h) Time Convolution		i) Frequency convolution
4. Compute the Fourier transform of **unit triangle, unit half triangle, unit rectangle, signum signal, sinc signal, (Refer basic signal model notes for reference) cos(t) from 0 to $\pi/2$, sin(t) from 0 to π , and combination of these** signals and sketch the spectra.