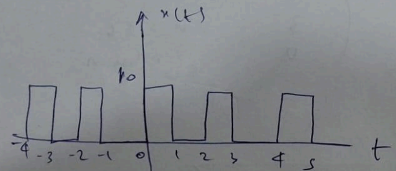
Q1.

① $(D+2)\,y(t) = x(t)$



Know,

As this is a periodic signal, we will use the Fourier Series to find frequency domain characteristics

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \qquad \text{where } T_0 = 2, \quad \omega_0 = \frac{2\pi}{T_0} = \pi$$

$$D_n = \int_{T_0} x(t) e^{-jn\omega_0 t}\,dt = \int_{-1}^{1} x(t)\, e^{-jn\omega_0 t}\,dt$$

$$D_n = \int_0^1 10\, e^{-jn\omega_0 t}\,dt = 10\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_0^1 = 10\left[\frac{e^{-jn\omega_0}}{-jn\omega_0} - \frac{1}{-jn\omega_0}\right]$$

$$D_n = \frac{10}{jn\omega_0}\left[1 - e^{-jn\omega_0}\right]$$

Substituting $\omega_0 = \pi \longrightarrow D_n = \frac{10}{jn\pi}\left(1 - e^{-jn\pi}\right)$

As for the system,

$$(D+2)\,y(t) = x(t)$$

In the frequency domain,

$$H(j\omega) = \frac{1}{j\omega + 2}$$

$$y(t) = \sum_{n=-\infty}^{\infty} D_n H(jn\omega_0) e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{10}{jn\pi}\left(1 - e^{-jn\pi}\right) \times \frac{1}{(jn\pi + 2)} \times e^{j\ldots}$$

$$y(t) = \sum_{n=-\infty}^{\infty} \frac{10}{jn\pi}\left(1 - \cos n\pi\right) \times \frac{1}{(jn\pi + 2)}\, e^{jn\pi t}$$

The Fourier series coefficients of the output $y(t)$ is

$$D_n' = \frac{10\,(1 - \cos n\pi)}{jn\pi\,(jn\pi + 2)}$$

First & Third Harmonics :

$$D_1' = \frac{20}{j\pi(j\pi+2)} \Rightarrow |D_1'| = 1.7094$$

$$D_2' = 0$$

$$D_3' = \frac{20}{3j\pi(3j\pi+2)} \Rightarrow |D_3'| = 0.22$$

Q2.

(2) Given 13 coefficients for exponential Fourier series of a discrete signal $x[n]$: D_r where $r = 1$ to 13. Assuming the signal is periodic with $N = 13$.

$$D_r = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}rn} \quad ; \quad x[n] = \sum_{r=0}^{N-1} D_r\, e^{j\frac{2\pi}{N}rn}.$$

Finding the discrete time signal (reconstructing) using MATLAB and sketching it :

In time domain, $x[n]$ can be expressed in exponential form as

$$x[n] = \sum_{k=0}^{12} D_k\, e^{j\frac{2\pi}{13}kn}, \quad n = 0, 1, \dots 12$$

where D_k for $k = 0$ to 12 are given in question.

Reconstructing x[n] from exponential fourier series coefficients using MATLAB:

```
clc; clear; close all;
N = 13;
Dr = [1, -1j*0.9430, -1j*0.3210, -1j*0.3800,+1j*0.1440, +1j*0.3950, -1j*0.1500,
+1j*0.1500, -1j*0.3950, -1j*0.1440, +1j*0.3800, +1j*0.3210, +1j*0.9430];
n = 0:N-1;
x = zeros(1, N);
% Compute x[n] using the inverse discrete Fourier series formula
for n1 = 1:N
   for r = 1:N
       x(n1) = x(n1) + Dr(r) * exp(1j*2*pi*(r-1)*(n1-1)/N);
   end
end
x = real(x);
% Display numeric values
disp('x[n] values (time-domain samples):');
disp(x);
% Plot the discrete-time signal
stem(n, x);
xlabel('n');
ylabel('x[n]');
title('Reconstructed Discrete-Time Signal from Fourier Coefficients');
grid on;
```

Command Window

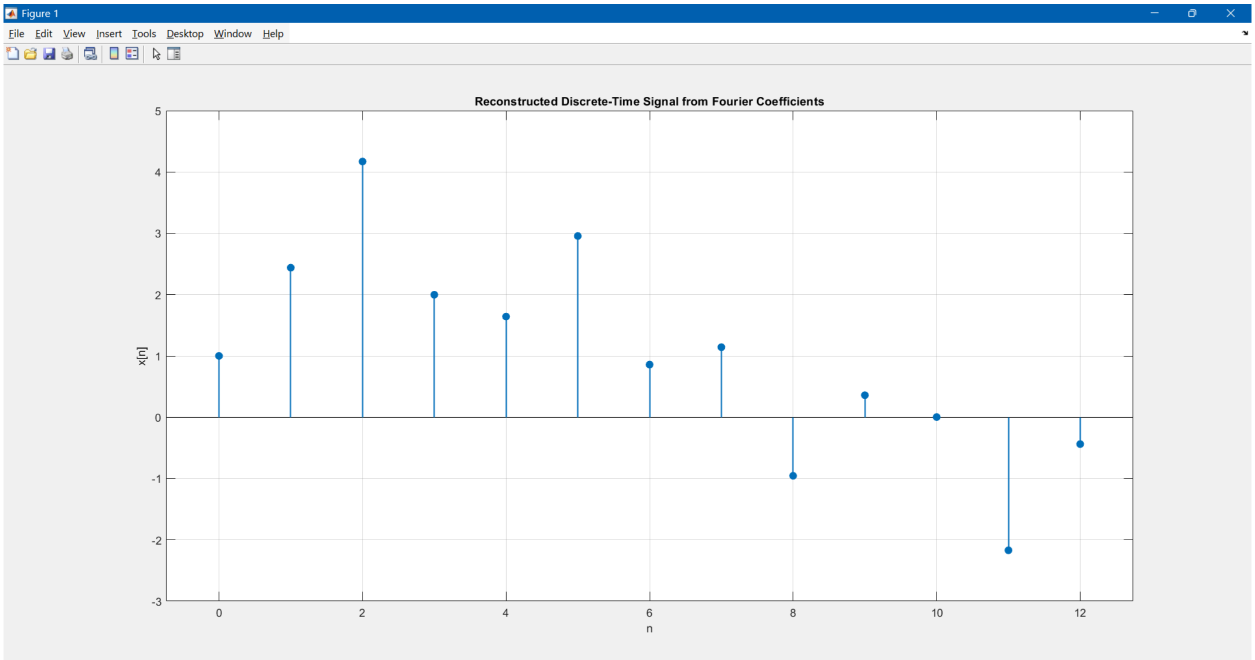
```
x[n] values (time-domain samples):
  Columns 1 through 10

    1.0000    2.4379    4.1701    1.9979    1.6408    2.9554    0.8575    1.1425   -0.9554    0.3592

  Columns 11 through 13

    0.0021   -2.1701   -0.4379

fx >>
```



Q3.

(3) From the figure.

$$x[n] = \begin{cases} 1 & , n = \pm 2, \pm 6 \\ 2 & , n = \pm 3, \pm 5 \\ 3 & , n = \pm 6 \\ 0 & , \text{other } n \end{cases}$$

we know $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$

Here, since this is an even (symmetric) function:

$$X(\Omega) = x[0] + 2\sum_{n=1}^{\infty} x[n] \cos(\Omega n)$$

$x[0] = 0$

$$X(\Omega) = 2\left[\cos 2\Omega + 2\cos 3\Omega + 3\cos 4\Omega + 2\cos 5\Omega + \cos 6\Omega\right]$$

At $\Omega = 0$

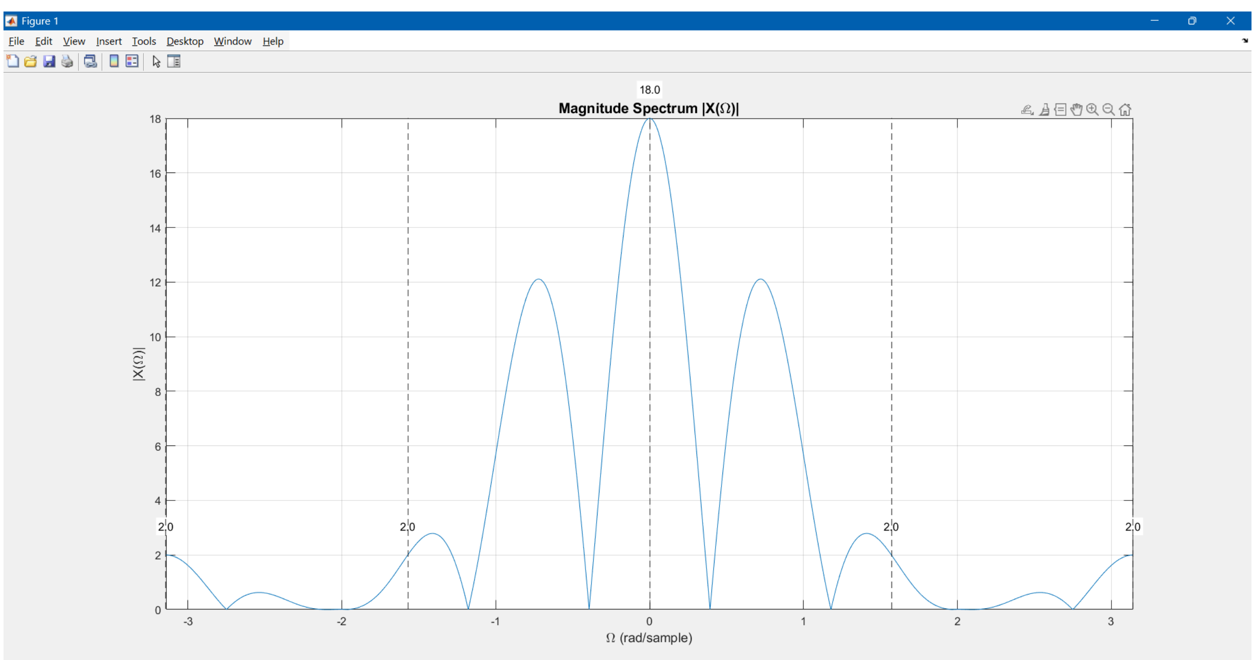
$$X(0) = 2[1+2+3+2+1] = 18$$

At $\Omega = \pm\frac{\pi}{2}$ (∵ cos is even)

$$X\left(\pm\frac{\pi}{2}\right) = 2[-1+0+3+0-1] = 2$$

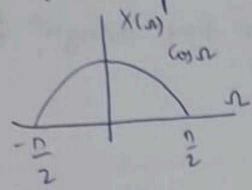
At $\Omega = \pm\pi$

$$x(\pm\pi) = 2(1-2+3-2+1) = 2$$



Q4.

(4) Given spectral response : $X(\Omega) = \begin{cases} \cos\Omega & -\frac{\pi}{2} < \Omega < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\Omega| < \pi \end{cases}$



Finding Inverse Discrete time Fourier Transform

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\, e^{j\Omega n}\, d\Omega$$

$$\Rightarrow x[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\Omega\, e^{j\Omega n}\, d\Omega \qquad \left[\cos\Omega = \frac{e^{j\Omega}+e^{-j\Omega}}{2}\right]$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{e^{j\Omega}+e^{-j\Omega}}{2}\right) e^{j\Omega n}\, d\Omega$$

$$= \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j\Omega(n+1)} + e^{j\Omega(n-1)}\, d\Omega$$

$$= \frac{1}{4\pi}\left[\frac{e^{j\Omega(n+1)}}{j(n+1)} + \frac{e^{j\Omega(n-1)}}{n-1}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{4\pi}\left(\frac{e^{j\frac{\pi}{2}(n+1)} - e^{j-\frac{\pi}{2}(n+1)}}{j(n+1)} + \frac{e^{j\frac{\pi}{2}(n-1)} - e^{j-\frac{\pi}{2}(n-1)}}{j(n-1)}\right)$$

$$\left[\frac{e^{j\theta}-e^{-j\theta}}{2j} = \sin\theta\right]$$

$$\Rightarrow x[n] = \frac{1}{4\pi}\left[\frac{2\sin\frac{\pi}{2}(n+1)}{\cancel{j}(n+1)} + \frac{2\sin\frac{\pi}{2}(n-1)}{\cancel{j}(n-1)}\right]$$

$$x[n] = \frac{1}{2\pi}\left[\frac{\cos\frac{\pi n}{2}}{n+1} - \frac{\cos\frac{\pi n}{2}}{n-1}\right]$$

$$\boxed{x[n] = -\frac{1}{\pi}\left[\frac{\cos(\frac{\pi n}{2})}{n^2-1}\right]} \quad \text{for } n \neq \pm 1$$

For $n = 1$

$$x[1] = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\Omega\, e^{j\Omega}\, d\Omega = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(e^{2j\Omega}+1\right) d\Omega$$

$$x[1] = \frac{1}{4\pi}\left[\frac{e^{j2\Omega}}{2j} + \Omega\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$x[1] = \frac{1}{4\pi}\left(\left(\frac{e^{j\pi}}{2j} + \frac{\pi}{2}\right) - \left(\frac{e^{-j\pi}}{2j} - \frac{\pi}{2}\right)\right]$$

$$x[1] = \frac{1}{4\pi} \cdot \pi = \frac{1}{4}$$

For $n = -1$

$$x[-1] = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\Omega \, e^{-j\Omega} \, d\Omega = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 + e^{-2j\Omega} \, d\Omega$$

$$x[-1] = \frac{1}{4\pi}\left[\Omega - \frac{1}{2j}e^{-2j\Omega}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4\pi}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) - \frac{1}{2j}e^{-2j\frac{\pi}{2}} + \frac{1}{2j}e^{2j\frac{\pi}{2}}\right]$$

$$x[-1] = \frac{1}{4\pi} \times \pi = \frac{1}{4}$$

$$x[n] = \begin{cases} \dfrac{-\frac{1}{\pi}\cos\frac{\pi n}{2}}{n^2 - 1} & n \neq \pm 1 \\ \dfrac{1}{4} & n = \pm 1 \end{cases}$$

Sketch on Matlab :

SKETCH ON MATLAB:

```
n = -20:1:20 ;
f = @(n) (((-1/pi)*(cos(pi.*n./2)))./(n.^2 - 1)).*((n~=1)&(n~=-1)) +
0.25.*((n==1)|(n==-1)) ;
x = f(n);
figure;
stem(n, x);
title('Discrete-Time Signal x[n]');
xlabel('n');
ylabel('x[n]');
grid on;
```

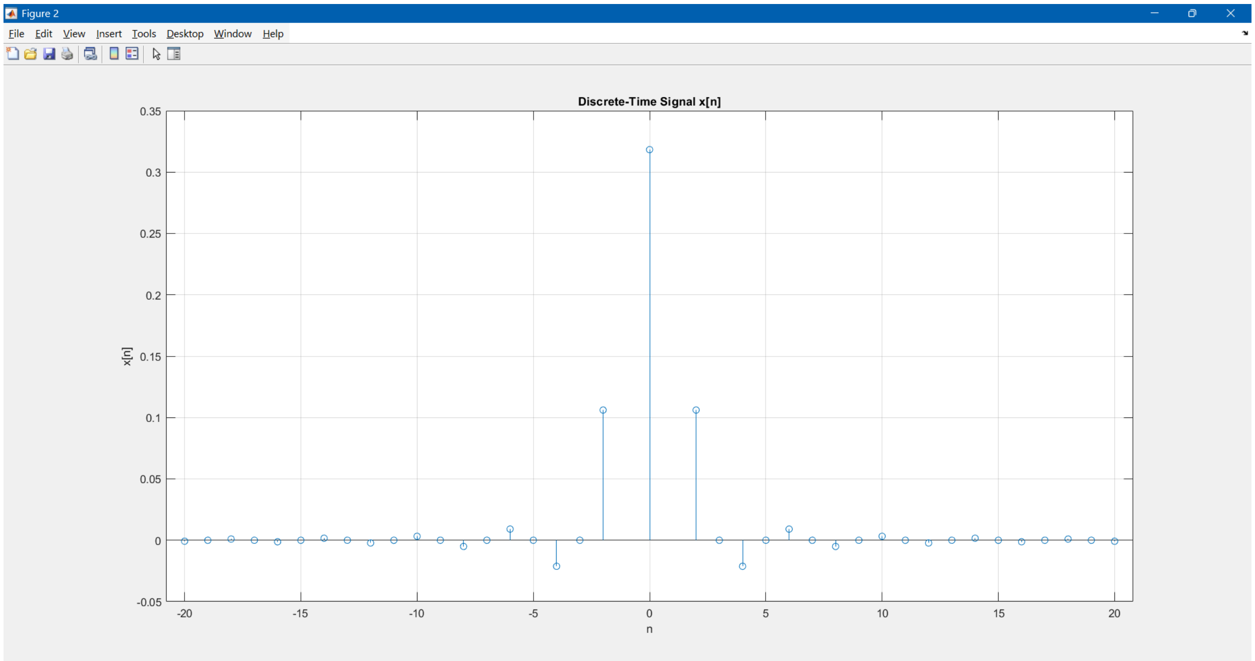


Figure 2
File Edit View Insert Tools Desktop Window Help
Discrete-Time Signal x[n]
x[n]
n
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-20
-15
-10
-5
0
5
10
15
20