National Institute of Technology Karnataka, Surathkal
Department of Electrical and Electronics Engineering
EE256-Signals and Systems
Assignment -2

Please write the assignment in an unruled A4 sheet paper by indicating your Name, Roll no.
Please submit on or before 19^{th} Nov 2023.

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1. When defining the impulse function $\delta(t)$ or Dirac delta function, the specific shape of the signal used to approximate it is not crucial. The impulse function is a theoretical construct representing an infinitely narrow pulse at a specific point in time. Consider the following cases:

   (a) The triangular pulse

   $$\Lambda_\Delta(t) = \frac{1}{\Delta}\left(1 - \frac{|t|}{\Delta}\right)(u(t+\Delta) - u(t-\Delta))$$

   Carefully plot it using MATLAB, compute its area, and find its limit as $\Delta \to 0$. What do you obtain in the limit?

   (b) Consider the signal

   $$S_\Delta(t) = \frac{\sin\left(\frac{\pi t}{\Delta}\right)}{\pi t}$$

   Use the properties of the sinc signal $S = \sin(\pi t)/(\pi t)$ to express $S_\Delta(t)$ in terms of S(t). Then find its area, and the limit as $\Delta(t) \to 0$. Use symbolic MATLAB to show that for decreasing values of $\Delta(t) \to 0$, $S_\Delta(t)$ becomes like the impulse signal.

2. The Hilbert transform of a real-valued signal x(t) is defined as the convolution of x(t) and $1/\pi t$ or equivalently, the output of a Hilbert transformer (a system having impulse response $h(t) = 1/\pi t$) to the input x(t) is:

   $$y(t) = h(t) * x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} x(\tau)\frac{1}{t-\tau}\,d\tau$$

   Show that Hilbert transform has the effect of shifting the phase of positive/negative frequency components of x(t) by $-90°/+90°$, allowing the Hilbert transform a $-90°$ phase shifter.
   Hint: Analyse the system in frequency domain

3. Following is a Fourier transform pair

   $$f(t) = A(u[t+T] - u[t-T]) \Longleftrightarrow 2AT\frac{\sin(\omega T)}{\omega T}$$

   Compute the percentage of the energy of f(t) contained in the interval of $-\frac{\pi}{T} \leq \omega \leq \frac{\pi}{T}$ with respect to the total energy of the pulse f(t). Use MATLAB wherever it is required.
