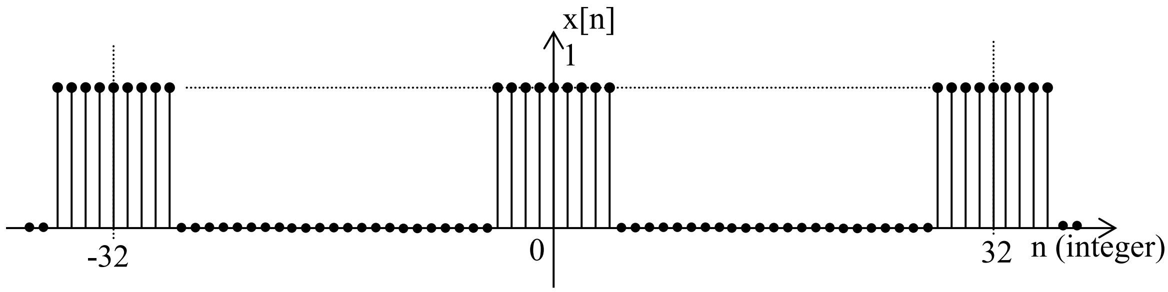
Week-10 (Fourier Analysis) –Discrete-time signals

<u>Compute and sketch the Exponential Fourier coefficients for a periodic signal:</u>

For a signal shown in figure below, find the Fourier series coefficients and sketch them.



In this case, Period $N_o = 32$ and $\Omega_o = 2\pi/32 = \pi/16$.

Fourier coefficients, D_r, can be calculated as $D_r = \frac{1}{32}\sum_{n=(32)} x[n]e^{-jr(\pi/16)n} = \frac{1}{32}\frac{\sin\left(\frac{9\pi r}{32}\right)}{\sin\left(\frac{\pi r}{32}\right)}$

Method 1: By direct computation: Following MATLAB code can be used

```
No = 32;   n = 0:No-1;   % Defining the periodicity and running integers.
xn =[ones(1,5) zeros(1,23) ones(1,4)]; % signal definition for one period.
for r=0:31
  xr(r+1) = sum(xn.*exp(-j*r*2*(pi/No)*n))/No;
end
r=n;
subplot(2,1,1); stem(r,abs(xr),'k'); ylabel('absXr'); xlabel('r');
xr=round(1000*xr)/1000;   %to omit computational error in computing angle
subplot(2,1,2); stem(r,angle(xr),'k'); ylabel('angleXr'); xlabel('r');
```

Method 2: By using *fft* command : Following MATLAB code can be used

xr1 = fft(xn)/No; % Make sure that total number of samples of xn are 2^m, where m is any integer, so that 2^m value is adjacent higher than No. This is because, the algorithm fft uses it that way.

```
subplot(2,1,1); stem(r,abs(xr1),'k'); ylabel('absXr'); xlabel('r');
subplot(2,1,2); stem(r,angle(xr1),'k'); ylabel('angleXr'); xlabel('r');
```

<u>Exercises:</u>

1) For the above example, plot Fourier series expanded signal x[n] vs. discrete-time variable “n” and sketch each (at least 5) Fourier series terms with respect to “n”, explicitly in the same window using subplots and “stem”. Observe the role of Amplitude and Phase Spectra in Wave shaping. (Use inline functions if necessary)

2) **Sketch the discrete-time signal** clearly, in the time domain, given the Fourier coefficients of the signal as shown in the following table, by direct computation (from fundamental

D_r :	0	-j0.8720	-j0.4330	+j0.3330	+j0.1440	-j0.2950
	0	+j0.2950	-j0.1440	-j0.3330	+j0.4330	+j0.8720

equation). Also **write an array of values of the signal** in the time domain for one full period. **Express the signal in the time domain as Fourier series expansion** either in the compact trigonometric form or in the exponential form.

3) Verify by plotting as well as analytically whether the following signals are periodic?

a) $3\sin\pi n + 2\sin 3n$ b) $2+5\sin 4\pi n + 4\cos 7\pi n$ c) $2\sin 3n + 7\cos\pi n$

d) $7\cos\pi n + 5\sin 2\pi n$ e) $3\cos\sqrt{2}\,\pi n + 5\cos 2\pi n$ f) $\sin 3\pi n + \cos\dfrac{15\pi n}{4}$

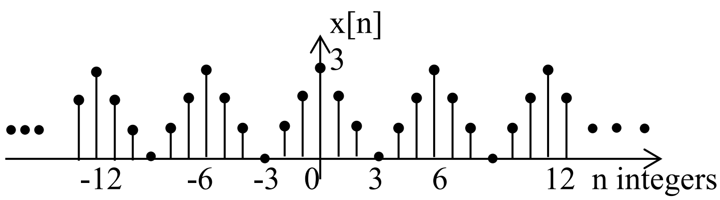
g) $\sin\dfrac{5\pi n}{2} + 3\cos\dfrac{6\pi n}{5} + 3\sin\left(\dfrac{\pi n}{7} + 30^o\right)$

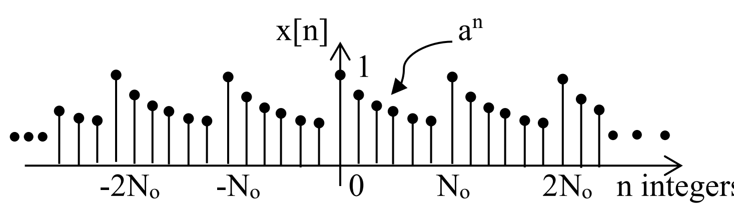
4) Find the Discrete-Time Fourier series (DTFS) analytically and sketch their spectra $|D_r|$ and $\angle D_r$ over $0 \le r \le N_o-1$ for the following periodic signal:

a) $x[n] = 4\cos 2.4\pi n + 2\sin 3.2\pi n$ b) $x[n] = \cos 2.2\pi n + \cos 3.3\pi n$

c) $x[n] = 2\cos 3.2\pi(n-3)$

d) e) $x[n] = a^n$ for $0 \le n \le No-1$. Let, $a = 0.707$ and $No = 5$.





Fourier Transform – Discrete-time signals

Let the discrete-time **energy signal** be $x(n) = \begin{cases} 0.7071^n & 0 \le n \le 4 \\ 0 & otherwise \end{cases}$.

Fourier Transform of x[n] is $X\left(e^{j\Omega}\right) = \sum_{n=-\infty}^{\infty}(0.7071)^n e^{-jn\Omega} = \dfrac{(0.7071)^5 e^{-j\Omega 5}-1}{0.7071 e^{-j\Omega}-1}$

__METHOD-1:__ **Using First Principle**. An explicit MATLAB code can be written to compute the Fourier Transform (FT) using the **"sum"** command by appropriately modifying the code used for computing the Fourier Series. Following modifications may be attempted.

1) Define the time range **wide enough** to accommodate the entire non-zero duration of the energy signal, instead of restricting it to a single period.
2) Define the discrete-time energy signal x(n) using "inline" with discrete-time "n" and step size "one".
3) Initialize the index counter **"rk"** to zero, which will be used for indexing the array of computed spectral components of the given energy signal.
4) Choose an appropriate step size **"h = 0.01"** to accurately represent the continuously varying unique phasor (spectral) components of the discrete-time energy signal.

5) Use a loop to compute the Fourier spectral components using first principles with **“sum”** command, considering the discrete frequency limits (**Ω**) from -π to π, with the defined step size (**h**), and incrementing index counter (**rk**), as shown below:

```
xr(rk) = sum(x(n).*exp(-j.*Ω.*n));
```

6) Sketch the magnitude and phase of the computed spectral response with respect to frequency using “plot” command.

<u>METHOD-2:</u> Use the manually derived expression for the Fourier Transform of the signal using first principles or relevant properties, and use this to sketch the spectra.

<u>Exercises:</u>

1. Modify the code to reconstruct back the original signal x(n) in discrete-time-domain and sketch the same with respect to discrete-time “n” using stem function.
2. If the above signal x(n) is given as input to the system whose transfer function is $H(E) = \frac{1}{E-0.0591}$, then find manually the zero-state response of the system using FT approach and sketch the same. Also, get the same result through MATLAB implementation.
3. Discuss/Evaluate/comprehend, how the **Fourier spectrum** modifies, when the following properties are applied to the above rectangle signal, along with any convenient another energy signals (if necessary) listed below. [Use MATLAB/penwork, wherever necessary].

a) Scalar addition   b) Scalar Multiplication   c) Conjugation
d) addition of two different time domain signals   e) Time shifting
f) Time scaling (compression, expansion, reversal   g) Frequency shifting
h) Time Convolution   i) Frequency convolution

4. Find the inverse Fourier Transform of the spectral response shown in figure P9-15, a) and b), of the discrete-time signals and sketch those discrete-time signal x(n) vs. “n”. The spectral response is shown only for one period from -π to π, as it is periodic function in Ω.
5. Compute the Fourier transform of $\gamma^n u[n]$ signal and sketch the spectra with $\gamma = -0.8$ and verify it using Inverse Fourier Transform.
6. Compute the Fourier transform of the signals, shown in figure P9-16, P9-22, P9-26, P9-27, using first principle (direct computation) and verify it using Inverse Fourier Transform. One may use properties wherever necessary.

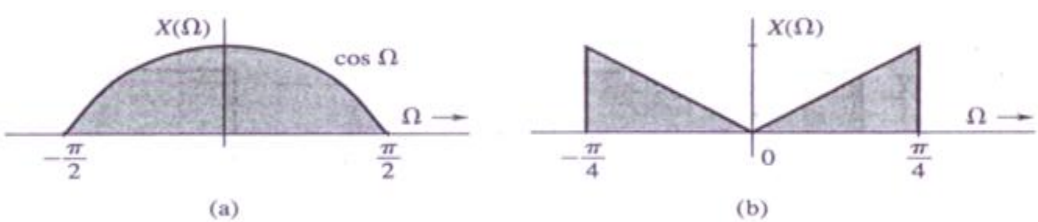


Figure P9-15

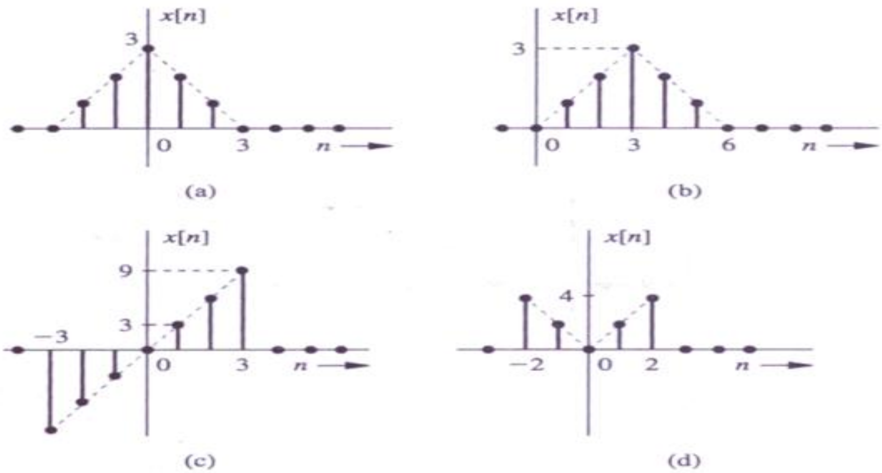


Figure P9-16

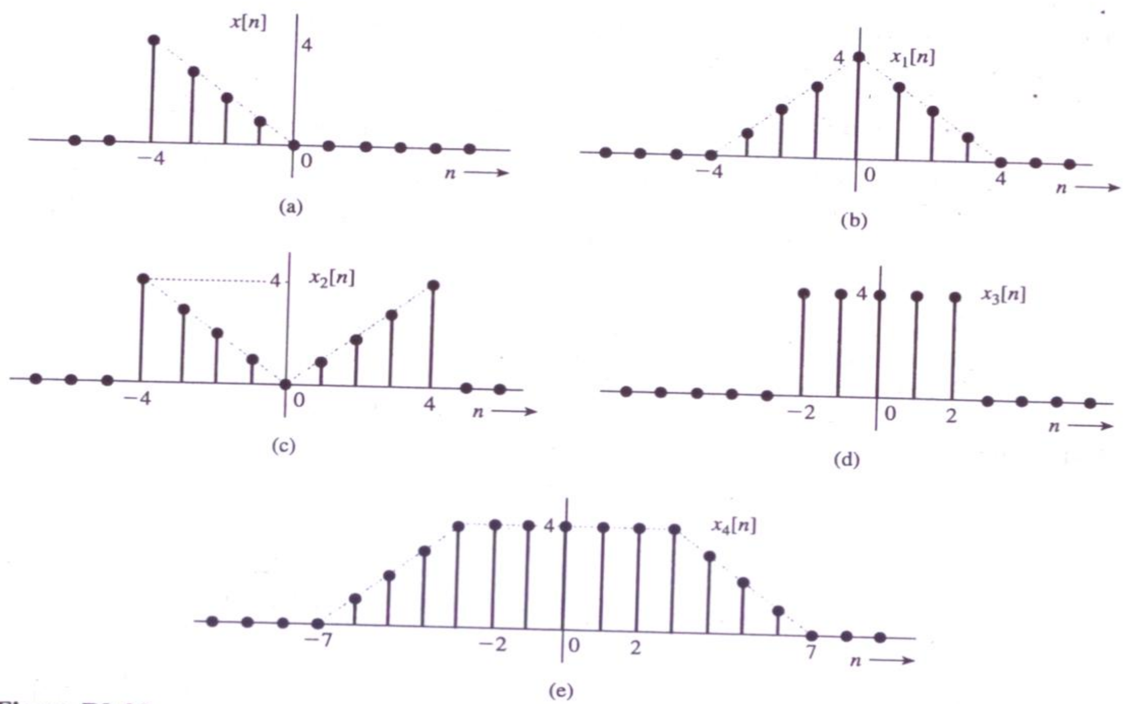


Figure P9-22

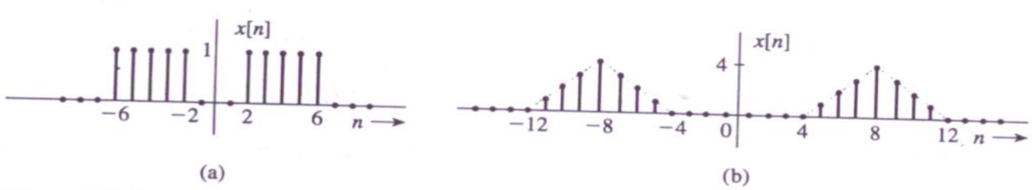


Figure P9-26

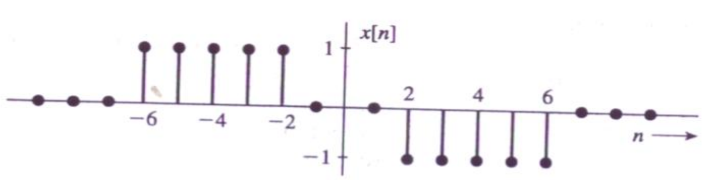


Figure P9-27