after using the eqn (5)

$$\left\{\begin{aligned}
&\frac{1}{3}m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \\
&\quad + m_1 g l_1 C q_1 + m_2 g l_1 C q_1 = \tau_1 \\
&\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2 \ddot{q}_2}{4} + \frac{m_2 l_1 l_2 \ddot{q}_1 \cos(q_2 - q_1)}{2} \\
&\quad - \frac{m_2 l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s\, q_2 = \tau_2
\end{aligned}\right.$$

eqn. (4) is valid for any forces F_x, F_y.

we need this forces $F_x = Kx$, $F_y = Ky$

more specifically
$$\left\{\begin{aligned} F_x &= K_x(x - x_0) \\ F_y &= K_y(y - y_0) \end{aligned}\right\}$$

from (1)

$$F_x = K(l_1 C q_1 + l_2 C q_2)$$
$$F_y = K(l_1 S q_1 + l_2 S q_2)$$

from (4)

eqn. (7)
$$\left\{\begin{aligned}
&K(l_1 S q_1 + l_2 S q_2) l_2 C q_2 - K(l_1 C q_1 + l_2 C q_2) l_2 S q_2 = \tau_{2s} \\
&K(l_1 S q_1 + l_2 S q_2) l_1 C q_1 - K(l_1 C q_1 + l_2 C q_2) l_1 S q_1 = \tau_{1s}
\end{aligned}\right.$$

spring

set motor forques to be $\tau_1 + \tau_{1s}$ and $\tau_2 + \tau_{2s}$ respectively ! → Answer to T3

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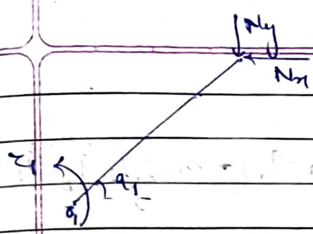

$\Sigma M_O = 0$

$N_y l_1 \cos q_1 - N_x l_1 \sin q_1 = \tau_1$

so, $N_y l_2 c q_2 - N_x l_2 s q_2 = \tau_2$

$N_y l_1 c q_1 - N_x l_1 s q_1 = \tau_1$ ④

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix}$$

③ ~~along~~ along with ④ solves T_2

for $\underline{T_3}$

Lagranges equations.

Lagrangian $\mathcal{L} = K - V$

$K \rightarrow$ kinetic energy, V - potential energy

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'} \rightarrow ⑤$$

$Q_i \rightarrow$ are generalized forces, derived using principle of virtual work.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of } l_1} + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 \underbrace{V_{c_2}^2}_{\text{velocity center of mass } l_2}$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(\cancel{l_1 s g}\ l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

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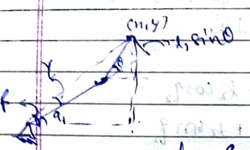
so, $\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1 l_2}\right)$



so, $q_1 = \beta - \gamma$

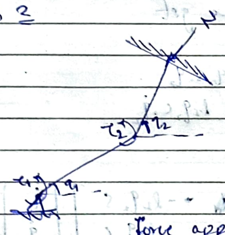
$= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right)$

and $q_2 = q_1 + \theta$

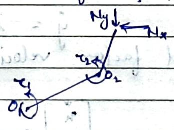
This is first level answer to I1

let denote → x_d and y_d (q_{1d} & q_{2d}) → ~~for~~ desired value

Task 2



FBD of Robot



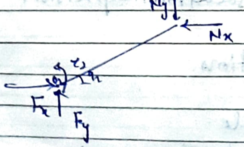
force applied by the manipulator

$F_x = -N_x$

$F_y = -N_y$

neglect gravity.

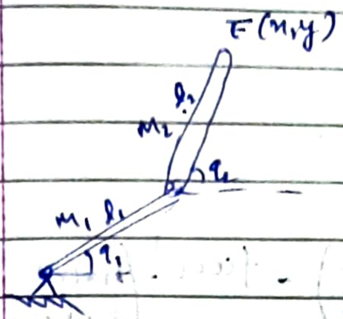
FBD of each link.



$\Sigma M_{O_2} = 0$

~~cw~~ c.c.w (+)

$+N_y l_2\cos q_2 - N_x l_2 \sin q_2 = \tau_2$



2R manipulator.

Tasks $T_1 \rightarrow$ Trajectory following.
$T_2 \rightarrow$ Apply force on wall
$T_3 \rightarrow$ act like spring

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

or using simplified notation,
cos → c   sin → s

so,
$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} (1)$$

Differentiating (1), we get

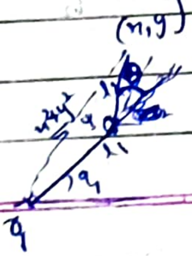
$$\dot{x} = -l_1 q_1 s \dot{q}_1 - l_2 q_2 s \cdot \dot{q}_2$$
$$\dot{y} = l_1 q_1 c \cdot \dot{q}_1 + l_2 q_2 c \cdot \dot{q}_2$$

end factor velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 q_1 s & -l_2 q_2 s \\ l_1 q_1 c & l_2 q_2 c \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

we need inverse relationship
for given x, y we need to be able to find q_1, q_2

option (1) → solve numerically
option (2) → closed form ~~solutions~~ expression
↳ Hard in general
↳ multiple solutions.



using cosine rule.
$$x^2+y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos \psi$$
$$x^2+y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(180-\theta)$$
$$x^2+y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

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