SUDOKU MADE EASY

By

Wg Cdr Gyaneshwar Singh(Retd)

BFC PUBLICATIONS

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Published by:

BFC Publications Private Limited
CP-61, Viraj Khand, Gomti Nagar,
Lucknow-226010

ISBN: 978-93-5509-002-7

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Dedication

This book is dedicated to the memory of

Late KN Ramamurthy, father of my friend

Wg Cdr K R Nagesh

Preface

This book is an attempt to simplify the game of SUDOKU. Simple techniques suggested in the book rationalize and make this game tangible and achievable. More we dwell inside; more fascinating and beautiful world opens for us. Logic and numbers had never been so thrilling and engrossing. I know there will be many ways to solve a problem but a systematic approach is always welcome.

It was during an official tour to Bangalore, I happened to stay with my Air Force Colleague, Wg Cdr K R Nagesh. There his father, late K N Ramamurthy introduced to me to this game. I found it quite interesting and soon I found this game everywhere, in all magazines and newspapers. I got inspiration from them and started solving this game. During solving these puzzles, I developed “Circle method”. This method helped me in solving comple× Sudoku puzzles. All these played a key role in inspiring me to design and write this book. Special mention of my lovely daughter, Nivedita Singh for e-typing and cover design of this book. Hopefully this book will be helpful to all, especially to the students, seniors and housewives.

Wg Cdr Gyaneshwar Singh (Retd)
05 April 2021

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Chapter 1

Introduction and Assumptions

INTRODUCTION

This game is popular all over the world and has become a hobby for people of all ages. This game can be made easy by making suitable assumptions and devising certain rules. How to do it? Here, we go…

Sudoku is a number game in which numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 are needed to be distributed in a big square divided into 9×9 small squares. So, the total number of small squares is 81. The distribution should be such that for every column, the numbers from top to bottom or from left to right in case of rows contain all numbers from 1 to 9. Further to it, there is mid-size square made up of 3 small column or three rows (indicated by blue and white colour on the figures illustrated in this book), in addition to the conditions stated above of serial appearance of numbers 1 to 9, each midsized squares formed by three columns and rows (we will use the word Block for them in subsequent paragraphs) should also contain number 1 to 9 compulsorily.

To illustrate this, let us take figure 1, we have designated different blocks/squares of the 9×9 square. The convention and formulation of rules for this game are stated as follows

Rule 1: Designating Squares

We designate squares

(a) 9×9 square as the big square, which contains all 81 small squares (or Unit Square)

(b) 3×3 squares are blocks as shown in the figure 1, which will contain 9 small squares (or Unit square). There are nine, 3×3 squares, shown alternately white and blue. These blocks may be identified as the row blocks or the column blocks as explained in subsequent paragraphs.
(c) And there are total 81 small squares or Unit squares.

Learning point

So, we have three types of squares in the game of Sudoku, 9×9 square, 3×3 squares and Small or unit Squares. We must also remember that 9×9 square has 81 unit squares, 3×3 square has 9 unit squares and the small square means unit square. So, we have big square, block square and unit square.

Rule 2: Numbering of Columns and Rows

For column, we number them 1….2....3.…up to 9 in the ascending order that is from left to right as shown in the figure 1. Similarly, we number the rows 1 to 9 from top to bottom fashion as shown in figure 1.

Learning point

Numbering of column and rows

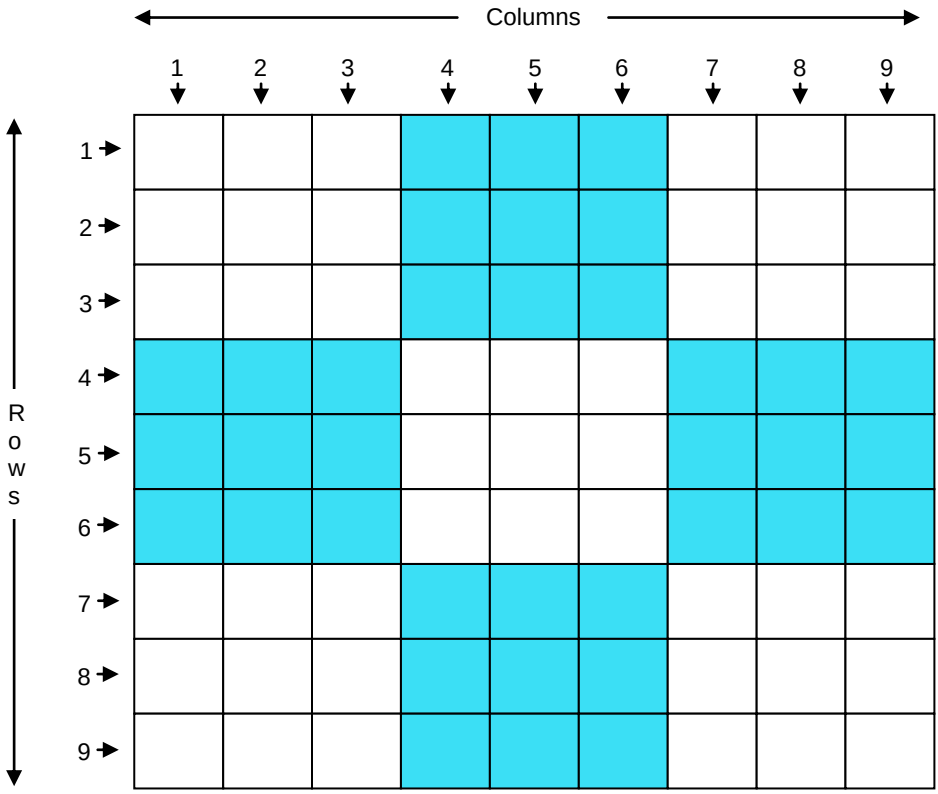


Fig 1

Rule 3: Symbols for Column and Rows

Now, we designate with symbols C or R where C stands for column and R stands for row.

So now, we can designate any of the small squares (Unit square) in terms of corresponding row and column. The concept is same as that of **two dimensional coordinate geometry systems**, where we take help of X and Y axis to describe a point on the plane of a paper. That means, we can describe the unit square in 9×9 Square (Big square) in similar fashion.

Say C_{57}. (Given by a black dot on figure 2)

C_{57} means go to column 5 and then come down to row 7. Number 7 is suffix to number 5, denotes row. So, C_{57} means column no 5 and row no 7. **It can also be described as R_{75}. Means row no.7 and column no 5, but for our convenience and simplicity, we will take reference with 'C' (means column) only.**
Similarly, in figure 2, the unit square with two black dots can be referred to as C_{84} or R48.

Learning point

Writing symbols for Unit squares.

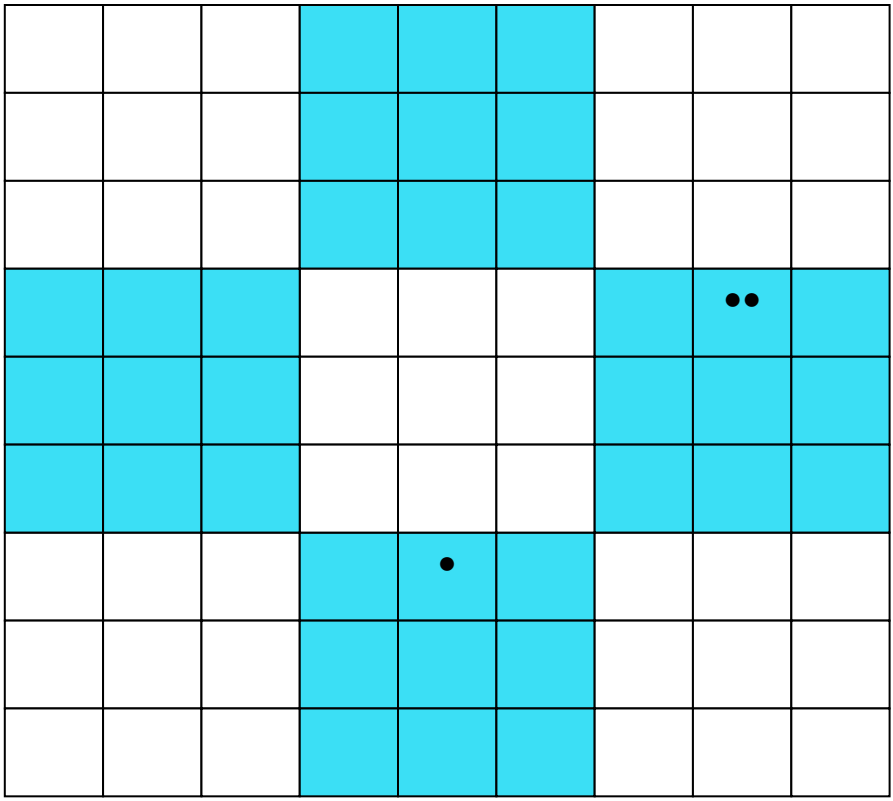


Fig 2

Exercise 1

Designate reference locations for the followings in figure 3.

(a) Square with single dot.
(b) Square with two dots.
(c) Square with single heart.
(d) Square with two hearts.
(e) Square with Single Square.
(f) Square with two squares.
(g) Square with single spade.
(h) Square with single club.
(i) Square with single triangle.

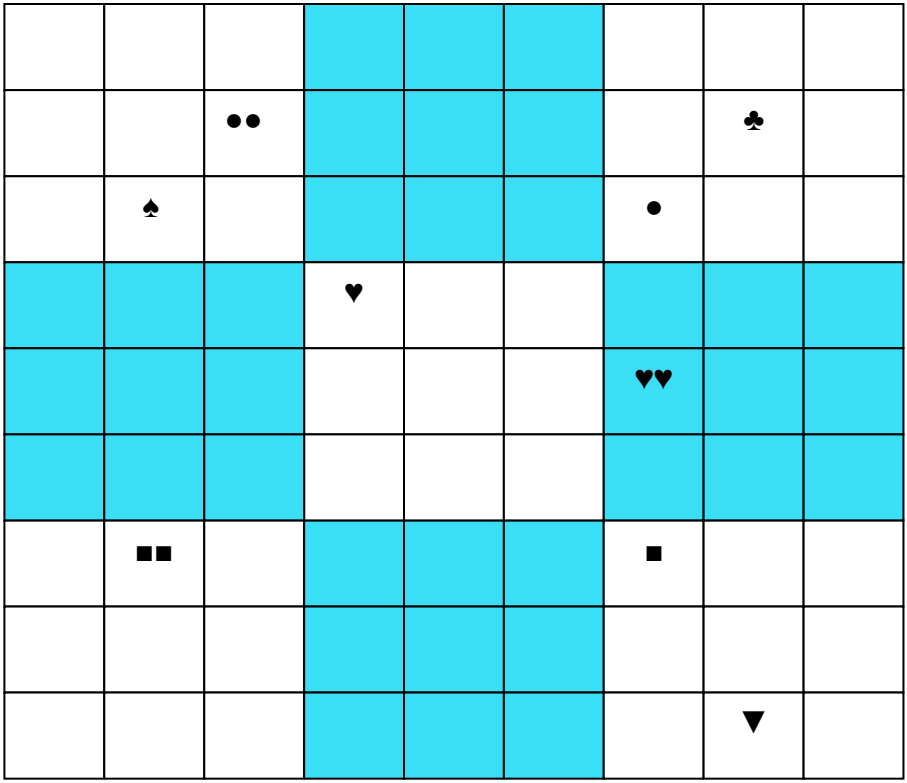


Fig 3

Rule 4 : Numbering of 3×3 square (Block square or Block) (Fig 4)

We designate row and column 3×3 square (or Block square or block), First, by block, we mean a square of 3×3 unit squares. There are total 9 blocks, we number them as follows. Vertically top to down are 1, 2 and 3 blocks. Center has 4, 5and 6 blocks. Similarly, right vertical top to down are 7, 8 and 9 blocks.

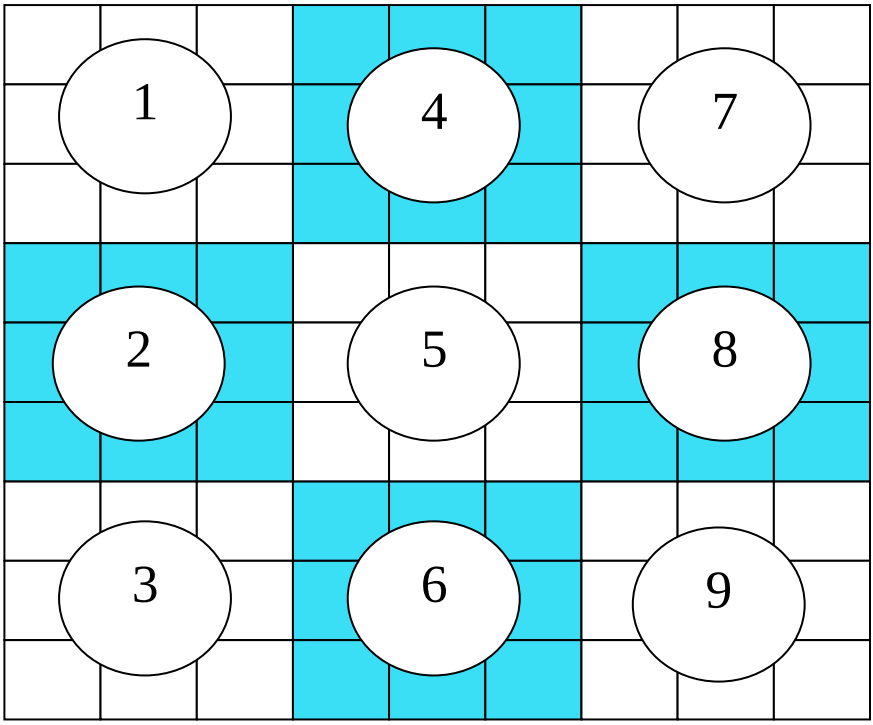


Fig 4

Learning point

Numbering of blocks or block squares.

Rule 5: Row or column block to block reference

We further designate names for each block to block for easy reference. This is done as follows - this has the same numbers as that of row 1 to 3 will be used for row block to block reference as it is easily comprehended-

That means 1st row block to block means block no 1, block no 4 and block no 7.
Second row block to block reference means block no 2, block no 5 and block no 8
Third block to block reference row means block no 3, block no 6 and block no 9.

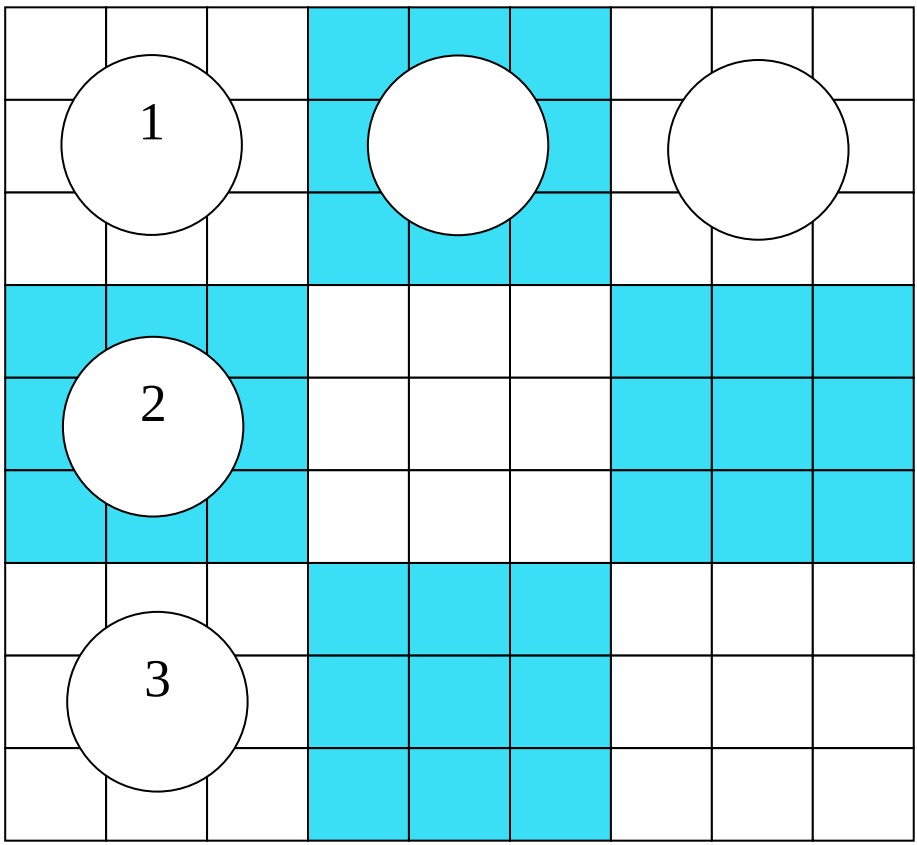
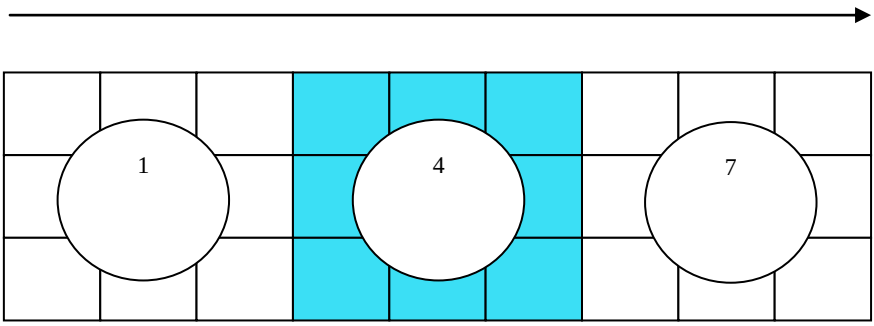
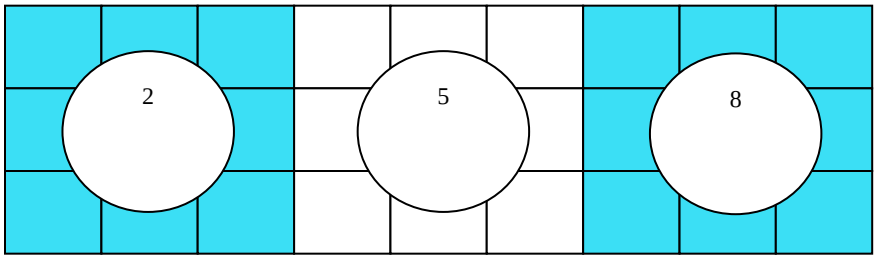


Fig 5

First row blocks (Contains block 1, 4, 7) .7and 7)



Second row blocks (contains blocks 2, 5 and 8)



Third row blocks (contains blocks 3, 6 and 9)

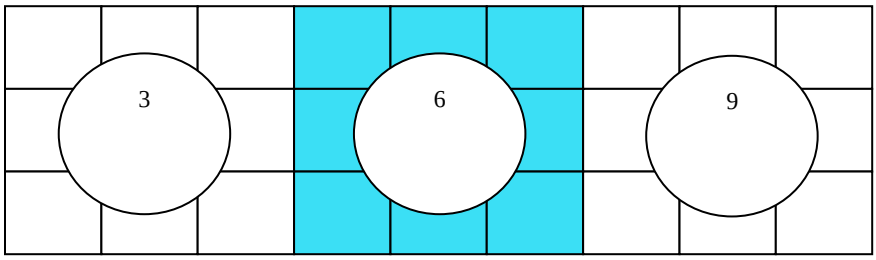


Fig 6

Learning point

Notations for row blocks

Similarly, we can designate column block to block references; only interesting part is that here block no 1 comes for both, in case of row and column block to block reference.

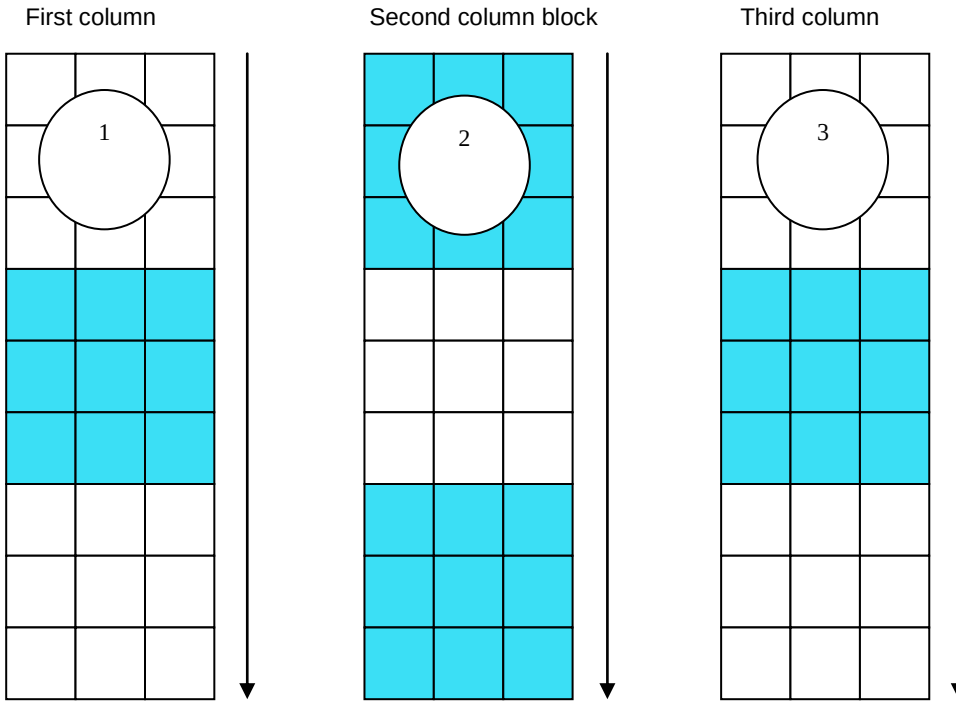
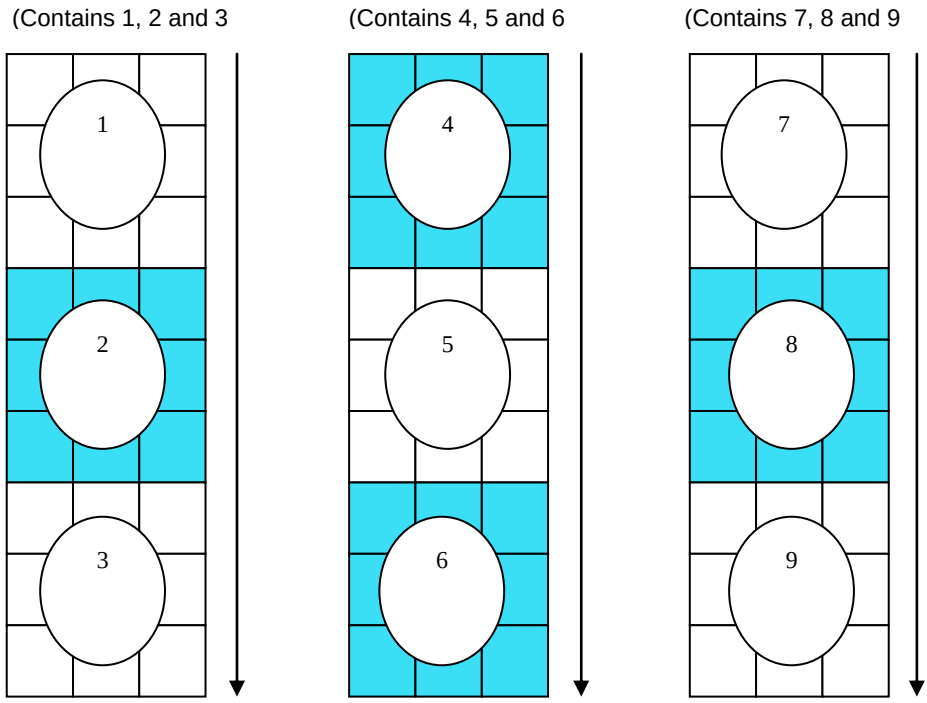


Fig7



Fig

First column block **Second column block** **Third column block**

Learning point

Notations for column blocks

Rule 6. Naming of Individual row line and Individual column lines

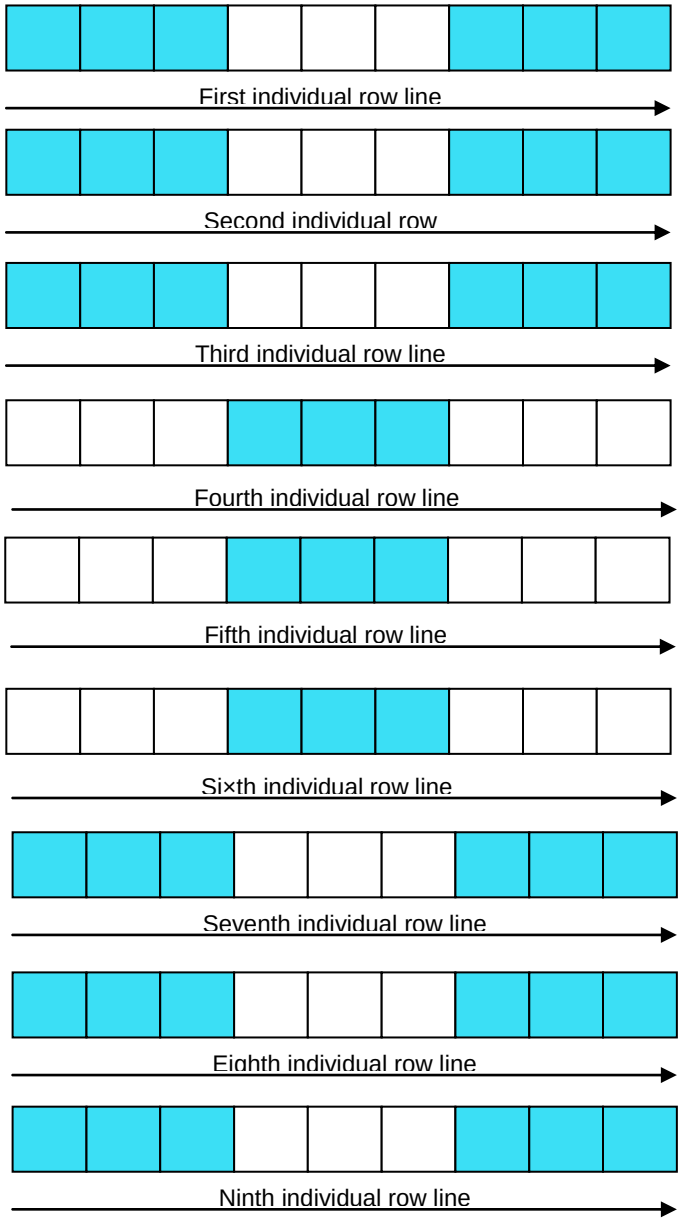


Fig 9

Individual column lines

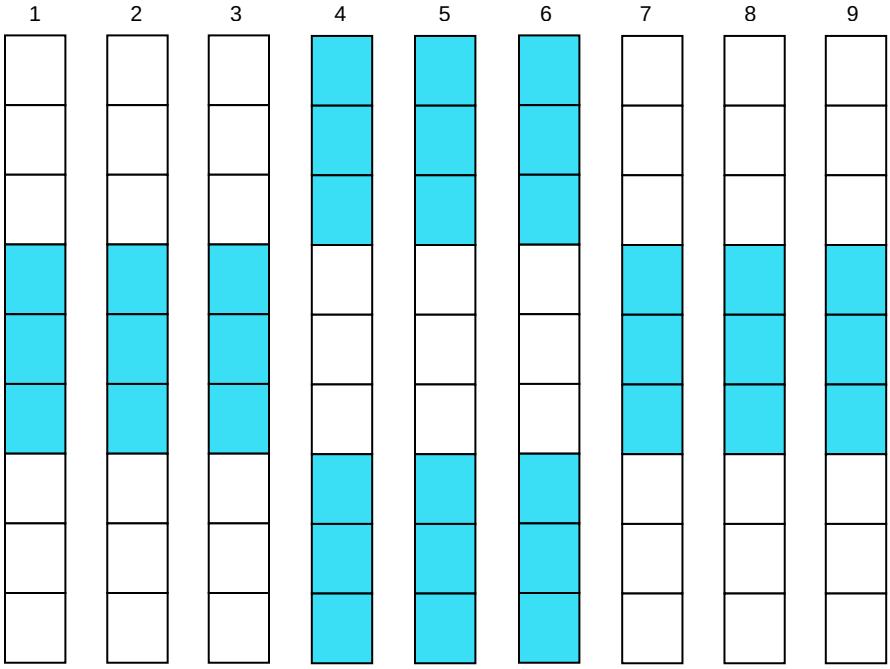


fig1

Learning point

Notations for row lines and column lines

Chapter 2

Techniques to Solve

After developing conventions for references in 9×9 squares, block to block rows, block to block column, individual row lines and individual column lines, we proceed to understand the conditions in filling the numbers in 9×9 Square.

(a) Each individual row line or the individual column line should contain 1 to 9 digits, of course with no repetition.

(b) Each block (3×3 Square) should also contain 1 to 9 digits with no repetition.

Keeping the conditions in mind, the following techniques can be adopted to fill the numbers in each blank unit square.

Technique 1: Row block to block arrangement (or Column block to block)

(a) There are three row blocks 1, 2 and 3 as described earlier. By Row block to block scanning, we fulfill the condition that digit 1 to 9 should appear in each single row in block to block scanning.

(b) The digits appearing in first row of 1^{st} block can only appear in second or third row of 4^{th} block or second or third row of 7^{th} block.

(c) The digits appearing in 2^{nd} row of 1^{st} block can only appear in 1^{st} row or 3^{rd} row of 4^{th} block or 7^{th} block.

(d) The digits appearing in 3^{rd} row of 1^{st} block can only appear in 1^{st} row or 2^{nd} row of 4^{th} block.

(e) In similar fashion, for 4^{th} and 7^{th} block, digits appearing in a particular row will find places in adjacent blocks in other two rows.

(f) Same way 2nd row block to block and 3rd row block to block scanning is performed to enter digits in individual unit squares.

(i) <u>1st row block to block scanning (Blocks 1, 4 and 7)</u>

Each block has 3 row lines and when blocks 1, 4 and 7 are combined, each row namely row 1, row 2 and row 3 is complete with 9 unit squares.

Normally, we start from first row block. This means that in this row block to block arrangements, there are three individual row lines as illustrated below. Here, three conditions need to be fulfilled - firstly each block (3×3) should contain 1 to 9 and secondly each individual row line and column line should contain 1 to 9 with no repetition.

That means any number appearing on C_{11}, C_{21}, C_{31} of 1st block can only appear on 4th block either at C_{42} or C_{52} or C_{62} or C_{43} or C_{53} or C_{63}, similarly it can only appear on block 7 at C_{72}, C_{82}, C_{92} or C_{73} or C_{83} or C_{93} etc. This will be clear with example as given figure 11. We take first row block to block arrangement-

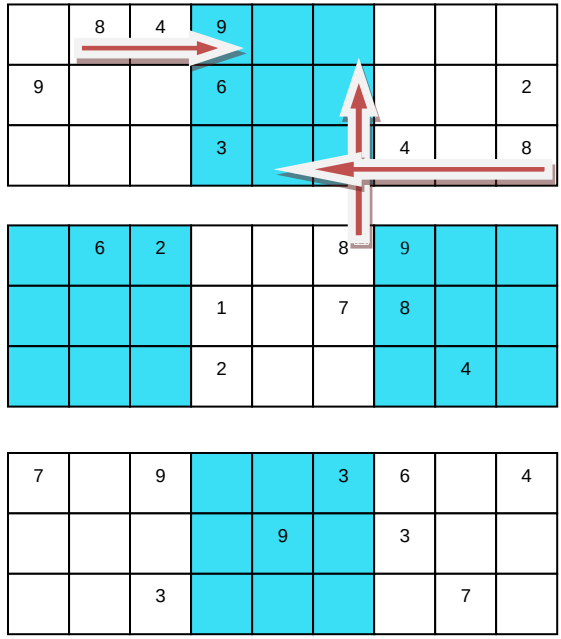


Fig 11

Number 8 is appearing in block 1 at C_{21} or R_{12} (row 1), which is also in first row line. 8 is also appearing in 3rd block at C_{93} which also happens to be 3rd line row (R3). So 8 can only appear in row 2 of 2nd block, because filling of number 8 at any other unit square say at C_{51}, C_{53}, C_{61}, and C_{63} will violate the basic condition that each individual row line should contain digits 1 to 9 only once. Hence 8 will only appear at C_{52} or C_{62}. Again 8 cannot appear at both C_{52} and C_{62} as 8 is also there at C_{64}, hence by comparing with the individual column lines C5 and C6 we find that column C_6 contains number 8 at C_{64}, so 8 can only appear at C_{52} indicated by red colour. This way we satisfy the conditions that each row line or column line will contain digit 1 to 9 only once. That means in block to block scanning we have to satisfy the conditions of row and column lines and individual blocks both together.

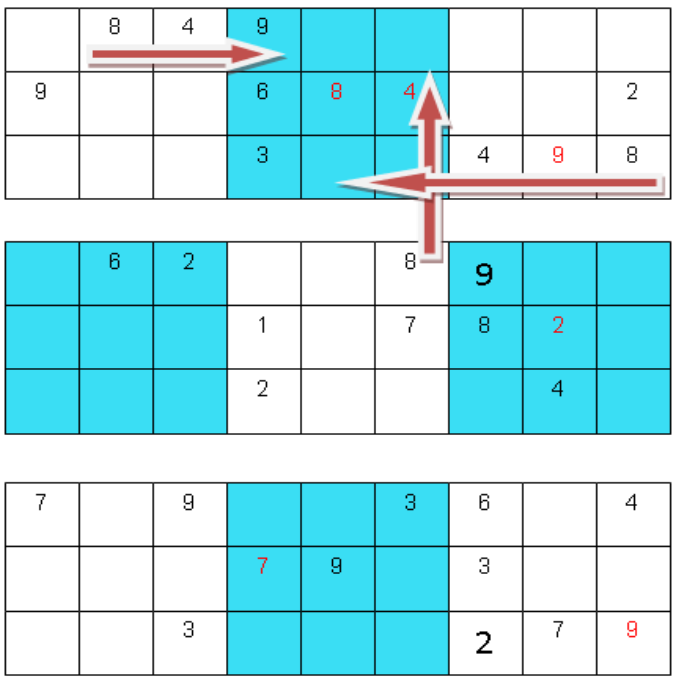


Fig 12

Similarly, digit 9 appears at C_{12} and C_{41} so by block-to-block row arrangement, we find that 9 can only be placed in 7th block at C_{83}

because that is the only vacant place and 4 is appearing in 7th block at C_{73}.

Now see that 4 is appearing at C_{31} of 1st block and by 1st row block to block scanning, we can say that 4 can appear only in C_{62} of 4th block (indicated with red) as 4 is contained in 7th block at C_{63}.

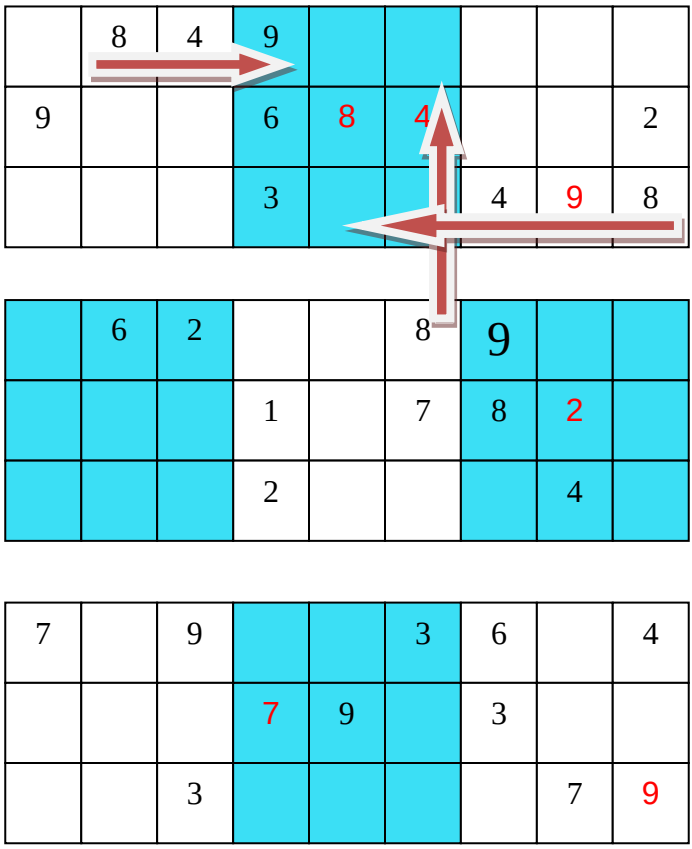


Fig 13

(ii) <u>Second Row Block to block scanning (Blocks 2, 5 and 8)</u>

Here, we have three rows, row 4, row 5 and row 6. Now, number 2 **is contained in row 4 in 2nd block and also in 5th block at C_{46} so by 2nd row block to block scanning, 2 can only appear at block 8**

at C_{85} or C_{95}, but it cannot appear in C_{95} as 2 also appears in 7th block at C_{92}. So it has to come at C_{85} only.

By this second block to block scanning, there is no clue for digit 6 so leave it as for now. Similarly, for digits 1, 7, 8, 9 and 4 there are no clues so leave them here as they are.

(iii) <u>Third row block-to-block Scanning/arrangement (Blocks 3, 6 and 9)</u>

In this case number 7 can come to C_{48} as 7 is there in C_{65} and number 9 can come at place C_{99} because 9 are contained in row 7 at C_{37}, in row 8 at C_{58} and also at C_{74}.

Similarly, block-to-block column scanning /arrangement are also done.

(a) There are three column bocks, 1, 2 and 3 as described earlier. By column block to block scanning we fulfill the condition that digit 1 to 9 should appear in each single column in block to block scanning.

(ii) The digits appearing in first column of 1st block can only appear in second or third column of 2nd block or third column of 3rd block.

(iii) The digits appearing in 2nd column of 1st block can only appear in 1st column or 3rd Column of 2nd block or 3rd block.

(iv) The digits appearing in 3rd column of 1st block can only appear in 1st column or 2nd block or 3rd block.

(V) In similar fashion, for 2nd and 3rd column block, digits appearing in a particular column will find places in adjacent blocks in other two columns in case of 2nd and 3rd blocks.

(vi) Same way, 2nd column block to block and 3rd column block to block scanning is performed to enter digits in individual unit squares.

First vertical column block-to-block Scanning/arrangement (Blocks 1, 2 and 3)

In this case block 1, 2 and 3 are involved and we have to take care of column and row lines also. The first column block to block scanning gives no clue for digit 8 at C_{21}, of first block but digit 9 is at two places, one at C_{12} in the first column line and the other at 3rd column line at C_{37}.

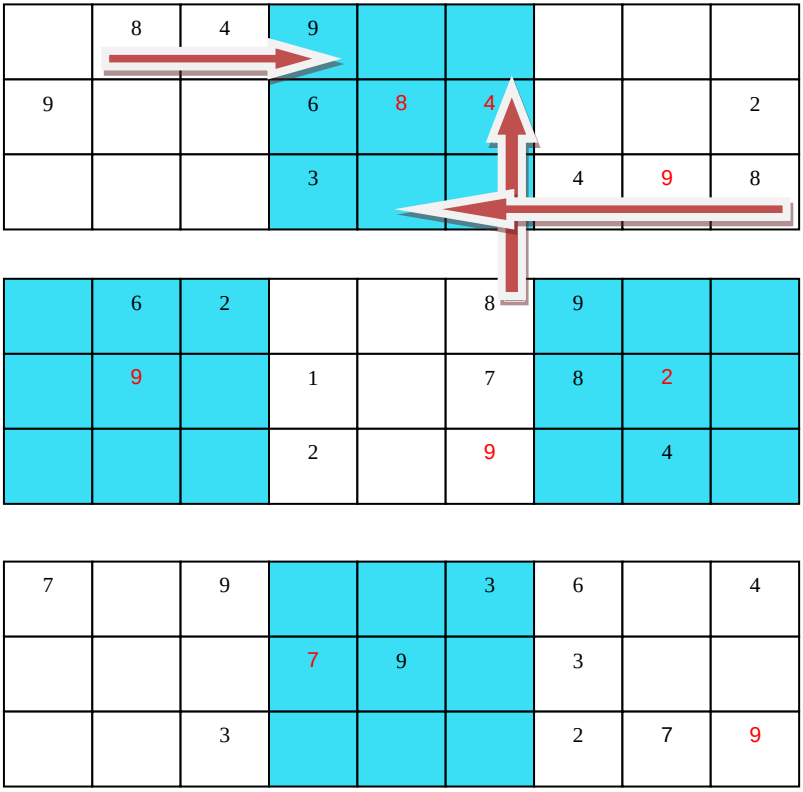


Fig 14

So, 9 have to appear in column line 2 only at C_{25} but not at C_{26} because there is digit 9 at C_{66}. This way scanning of 1st vertical column block to block gives no clue for digits 8, 4, 2, 6 and 7. So leave them as they are.

(iv) Second column block-to-block Scanning/arrangement (Blocks 4, 5 and 6)

In the second column block to block scanning, the blocks 3, 4 and 5 play the role. Here, we see that digit 9 appears in C_{41}, which is the 4th column line and at C_{58}, which is at 5th column line so now 9 can appear in 6th column only and that too in C_{66}, which already has been filled with digit 9. For digit 6, 8, 4, 3, 1 and 7 there is no clue so we leave the vacancies as they are. Digit 8 can appear in vertical line no. 4 at C_{47} and C_{49} but we cannot proceed further similarly for digit 3 there are three vacancies at C_{54} C_{55} and C_{56} so here also we cannot place digit 3. We have to wait for a condition where we can decide about its vacancy and then fill them.

(v) Third vertical column block-to-block Scanning/arrangement (Blocks 7,8 and 9)

Blocks 7, 8 and 9 play the role here and third vertical column block to block scanning gives 2 at C_{79} (Red colour). For 7, 6, 8 and 3, there is no clue.

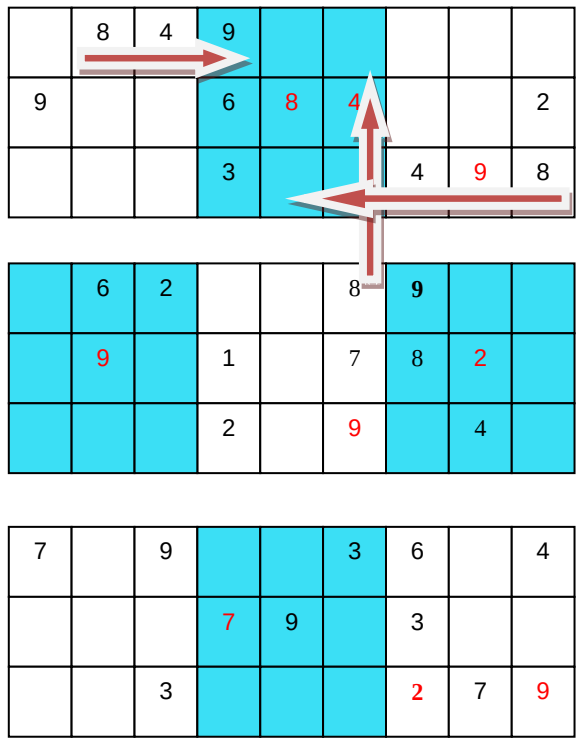


Fig 15

Technique 2 : Same block complete 1 to 9

There are total 9 blocks as designated earlier in chapter 1. Each block should contain digit 1 to 9 compulsorily. Hence, each block can be scanned for 1 to 9 digits but by not sacrificing the conditions for other/respective rows and columns lines. For example, take the case in the following block (Fig 16) - Block no. 1 contains digits 1, 2, 3, 4 and 5.

1	2	3	7		6	9	5	ऋ
4			8	1		3	ऋऋ	7
		5				1	4	6
5								
		8						
9		7						
	8	2						
	4	1						
	5	9						

Fig 16

There are two vacancies in Block 7, C_{82} and C_{91} but for block 7, digit 2 cannot happen at C_{91} because of presence of 2 at C_{21} hence 2 goes to C_{82}. You can see here that for digit 2, there is no clue in the row block to block arrangement of Block no. 1 and Block 4 but still we can fill digit 2 on the basis that each block should contain 1 to 9 and it should also satisfy other conditions of row and column lines. That is why 2 are slotted in C_{82} in 7^{th} block. Similarly, with the same logic digit 4 can happen only on C_{51} in block 4 and on C_{28} in block

3. Similarly for block 7, digit 8 will go to C_{91} to complete the block filling from 1 to 9.

1	2	3	7	4	6	9	5	8
4			8	1		3	2	7
		5				1	4	6
5								
		8						
9		7						
	8	2						
	4	1						
	5	9						

Fig 17

Technique 3: Individual Column to have 1 to 9

Each individual column and row will have digit 1 to 9 so accordingly it can be filled.

For example:

Individual row line

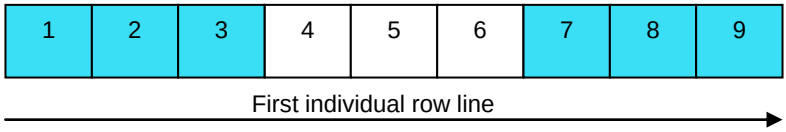


Fig 18

Individual column line

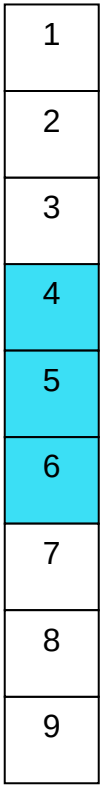


Fig 19

With this background, let us now solve the following one (Example 1, Fig 20)

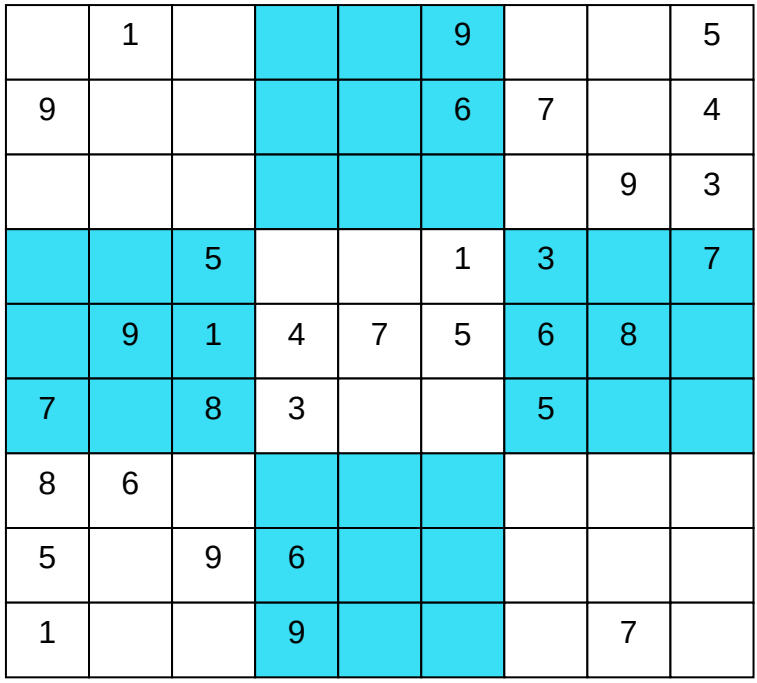


Fig 20

Chapter 3

Example

Solution to E×ample 1

Step 1 – (Row blocks scanning involving blocks1, 4 &7)

1st row block to block scanning gives nothing because there is no clue for digit 1, 6, 7, 5 and 4. Number 9 is placed evenly in 1st, 2nd and 3rd row lines and one each in the blocks 1, 4 and 7. So there is no need to do anything.

Second row block to block scanning also does not give any clue.
6_1 at C_{99} is obtained by third row to row block scanning as 6 is existing in C_{27} and C_{48} and this tells us to place 6 in 9th horizontal row line in block 9 at C_{99} but not at C_{79} as 6 is there at 7th vertical column line at C_{75}. For other digits, there is no clue so we leave them as they are till we get some leads.

Step 2 – (Column blocks scanning)

Now, we come to column wise block to block scanning.

1st column block to block scanning gives nothing as 1 and 9 both digits are contained in all three blocks of 1st column. Digit 5 indicates presence in block 2 & 3 but whether it will be in C_{22} or C_{23} of block 1 is not known. Similarly 3, 8 and 7 also have no clue.

	1				9		6_2	5
9					6	7		4
							9	3
		5			1	3		7
3	9	1	4	7	5	6	8	2_6
7		8	3			5		
8	6						5_3	
5		9	6				3_5	
1			9	5_4			7	6_1

Fig 20 (a)

2nd column block to block scanning also gives nothing. There are 9 and 6 in block no 4 and 6 but there is no clue how they are going to fit into block 5.Similarly the digits contained in block 5 also need to be placed in block 4 and block 6 but there exists no clue.

3rd column-to-column block scanning gives 6_2 at C_{81} because block 8 and 9 contain 6, so 6 can only be in block 7 at C_{81} also because of presence of 6 at C_{62}. Study of block 7 reveal that for digit 6

individual unit square C_{71} and C_{82} is ruled out for 6 as vertical column line 7 contains 6 at C_{75} also unit square C_{82} is ruled out for 6 as 6 is contained in unit square C_{62}.

Now, again 3rd column block-to-block scanning gives 5_3 at C_{87} and also because of 3rd row block to block scanning gives 5_3 at C_{87}.

Now, many new opportunities have developed because of filling of these 3 digits.
We can go for 5_4 at C_{59}, obtained by 3rd row block to block horizontal scanning and 6th column line scanning (because of presence of 5 at C_{65})

Again, 3_5 is obtained at C_{88} by 3rd column line scanning

2_6 at C_{95} are obtained by 5th row line scanning. With 6 new digits found many new opportunities have been opened up for finding other digits. By repetitive use of row to row, Column to column blocks scanning, scanning of individual blocks, scanning of row and lines, we can find other numbers, who can fit in to different unit squares. Let us see how it can be done.

	1				9	816	6_2	5
9					6	7	27	4
						115	9	3
		5			1	3	49	7
3	9	1	4	7	5	6	8	2_6
7	410	8	3			5	18	911
8	6					912	5_3	114
5		9	6				3_5	813
1			9	5_4			7	6_1

Fig 20 (b)

Now, for column line 8, 2_7 can happen only on C_{82} because 2_6 is on C_{95} so 2 can be either on block 7or 9. Block 9 does not have a blank space so it can happen only on C_{82}.

By 8^{th} vertical line scanning, we get 1_8 at C_{86} because of presence of 1 at C_{64} and this clears way for 4_9 at C_{84}. Alternatively, we can say for column line no 8, 2 cannot happen in C_{84} or C_{86} because of presence of 2 in block 8. This leaves with only one possibility that 2 appears in C_{82}.

4_{10} at C_{26} is obtained by 2^{nd} row block to block scanning
Study of block 8 reveals that 9_{11} can be placed at C96 because each block should contain 1 to 9 digits.

3rd column block to block scanning gives 9_{12} at C_{77}, which also conforms to 3rd block to block row scanning helped by presence of 9 at C_{49} and C_{38}.
9th column line reveals that 8 cannot happen at C_{97}, as 8 do exist at C_{17}, hence 8 for this column has to happen (8_{13}) at C_{98}.

So 1_{14} is placed at C_{96} to complete the 9th column line.
7th block scanning allows us to write 1_{15} at C_{73}, this can also be obtained by 3rd vertical column block to block scanning and similarly to complete the block 7, 8_{16} is also filled up at C_{71}.

For 6th individual row line, 6_{17} is placed atC_{56}, as 6 is present at C_{62}. Similarly, 2_{18} is also filled up to complete this row.

	1				9	8_{16}	6_{2}	5
9			1_{26}		6	7	2_{7}	4
		6_{24}				1_{15}	9	3
6_{21}	2_{22}	5	8_{20}	9_{19}	1	3	4_{9}	7
3	9	1	4	7	5	6	8	2_{6}
7	4_{10}	8	3	6_{17}	2_{18}	5	1_{8}	9_{11}
8	6					9_{12}	5_{3}	1_{14}
5		9	6	1_{25}		2_{27}	3_{5}	8_{13}
1			9	5_{4}	8_{23}	4_{28}	7	6_{1}

Fig 20 (c)

5th block scanning and presence of 9 at C_{49} allows us to write 9_{19} at C_{54}. 8_{20} complete the block 5 at C_{44}.

2nd block scanning and presence of 6 atC_{27} allows us to write 6_{21} atC_{14} and hence, 2_{22} is also filled at C_{24}.

8_{23} at C_{69} is obtained by 3rd row block to block horizontal row scanning.

6_{24} at C33 is obtained by first row block to block scanning (meaning presence of 6 at C_{81} and C_{62}) and 1st column block to block scanning. (meaning presence of 6 at c_{14} and C_{27}). Similarly 1_{25} at C_{58} is also obtained by scanning of 6th row unit line and 6th column unit line.

1_{26} at C_{42} is obtained by 2nd column to column block scanning and 1st row block to block scanning.

2_{27} is obtained by 8th individual row line study and 2 can happen only at C_{78} as in this line, 2 cannot go to C_{28} and C_{68} because of presence of 2 at C_{66} and C_{24}.

This makes 4_{28} filling at C_{79} easy as it is the only number left in block 9.

4_{47}	1	7_{36}	2_{46}	3_{38}	9	8_{16}	6_{2}	5
9	5_{44}	3_{37}	1_{26}	8_{41}	6	7	2_{7}	4
2_{48}	8_{43}	6_{24}	5_{45}	4_{39}	7_{35}	1_{15}	9	3
6_{21}	2_{22}	5	8_{20}	9_{19}	1	3	4_{9}	7
3	9	1	4	7	5	6	8	2_{6}
7	4_{10}	8	3	6_{17}	2_{18}	5	1_{8}	9_{11}
8	6	4_{31}	7_{42}	2_{40}	3_{34}	9_{12}	5_{3}	1_{14}
5	7_{32}	9	6	1_{25}	4_{33}	2_{27}	3_{5}	8_{13}
1	3_{30}	2_{29}	9	5_{4}	8_{23}	4_{28}	7	6_{1}

Fig 20 (d)

2_{29} at C_{39} because of 9th individual row line scanning and the presence of 2 at C_{24} and hence 3_{30} at C_{29} to complete the row line.

This makes it possible to fill-in 4_{31} and 7_{32} for 3rd block fillings at C_{37} and C_{28} respectively.
3rd row block-to-block scanning gives 4_{33} at C_{68} and 3_{34} at C_{67} by 6th vertical column line scanning. And by same 6th column line scanning, we get 7_{35} at C_{63}.

This helps us in filling 7_{36} at C_{31} and 3_{37} at C_{32} by 3rd vertical column line scanning.

3_{38} by 1st row block to block scanning at C_{51}. $\mathbf{4_{39}}$ by 4th block filling at C_{53}. 2_{40} by 5th vertical line scanning at C_{57} and 8_{41} at C_{52}. 7_{42} By 6th block filling at C_{47}. $\mathbf{8_{43}}$ by 2nd vertical line filling at C23 and similarly 5_{44} at C_{22}.
This gives **5_{45} at C_{43} and 2_{46} at C_{41} by 4th block filling.**

4_{47} and 2_{48} by 1st block filling at C_{11} and C_{13} respectively.
This completes the Sudoku filling in Fig 20(d).

What we have learnt from Example 1

We have used following concepts, which will lay on the basis of problem solving techniques in this book.

(a) Big square – means 9×9square- Rule 1
(b) Block square or block - means 3×3 squares – rule 1
(c) Numbering of column and row (rule 2)
(d) Symbols for column and row (rule 3)
(e) Numbering of blocks - rule 4
(f) Row block to block scanning – rule 5
(g) Column block to block scanning – rule 5
(h) Row lines 1 to 9 – rule 6
(i) Column lines 1 to 9 - rule 6

(j) Location of a unit square is indicated or denoted by (Rule 3) alphabet C (or R), C stands for columns and R stands for rows, this will have two suffix digits, indicating column and row numbers. For simplification, we have used column- C alphabet in this book for simplicity only, the first number on the suffix indicates column line and second number indicates row line. So the meaning of a number or digit, say a digit 2 is at C_{85} means digit 2 is at column 8 and row 5. Similar concept may be developed for row (R) indications also but in this book we will go with notation C only. C_{94} means the digit is at column line 9 corresponding to row line 4; intersecting unit square gives the location of the digit. This can also be denoted as R49, means the digit is at row line 4 and column line 9 and the intersecting unit square gives the position of the digit. R_{49} and C_{94} are of the same value.

(k) Digit movements- Digits with suffix numbers indicate the moves made while filling of unit squares sequentially. For an example 1_{-61} at C_{99} means it is the first move with digit 6 for filling up of the Sudoku box C_{99}. Similarly 8_{13} at C_{98} mean it the 13^{th} move in filling Sudoku boxes at C98. So and so forth.

(l) We start with row block to block scanning. First row block to block scanning. In this scanning blocks 1, 4 and 7 are involved. Then second row block to block scanning involving block 2, 5 and 8 and then we go for 3^{rd} row block to block scanning involving block 3, 6 and 9. Once row block to block scanning is over

(m) Then, we will find that many cells have been filled. After this we go for 1^{st} column block to block scanning involving block 1, 2 and 3. Then 2^{nd} column block to block scanning involving blocks 4, 5 and 6 then go for 3^{rd} column bock to block scanning involving blocks 7,8 and 9. Once this is done we will find that many empty unit square have been filled giving rise many more possibilities for filling rest of the unit square.

(n) Now go to individual blocks filling because each block should have compulsorily digits 1 to 9.

(o) Individual row and column lines filling follow after that.

(p) This sequence of block to block, block filling and lines filling continues till we fill all the empty cells.

(q) Even after doing all the above it is possible that all the cells are not filled, then we go for different techniques. One of them is Circle method developed by this author, described in later in this book. Apart from this circle method, a no of logics can also be developed to fill the remaining cells. We shall see all these in the subsequent examples.

E×ample 2

				8	7	6		
8		5			4			
7							1	8
4					9		3	
		3				9		
9	6		2					7
3	4	1						5
			8	4		1		2
		9	1	7				

Fig 21

		4_5		8	7	6		
8		5	6_4	1_3	4			
7		6_2					1	8
4					9		3	
		3				9		
9	6		2					7
3	4	1						5
			8	4		1		2
	8_1	9	1	7				

Fig 21 (a)

Solution to E×ample 2

First, second and third row block to block scanning gives nothing.
Similarly 1st, 2nd and 3rd column block to block scanning gives nothing
8_1 can happen only in C_{29} for block 3 scanning because of presence of 8 at C_{48} and C_{12}.
6_2 is slotted in C_{33} for block1 **because of presence of 6 at C_{71} and C_{26}.**
1_3 only possible in C_{52} of block 4 because of presence of 1 at C_{49} and C_{83}.
6_4 is slotted at C_{42} because of 1st row block to block scanning.

As Block 2&3 have digit 4 in their respective places hence by 1st column block to block scanning, 4_5 is slotted at C_{31}.

Now Fig 21(b)

4_8 at C_{63}, we got with 1st row block to block horizontal scanning.
4_9 at C_{45}, because of 2nd column block to block vertical scanning.

4_{10} at C_{86} because of second row block to block horizontal scanning.
4_{11} at C_{99} because of 3rd column block to block vertical scanning.

		4_7		8	7	6		
8		5	6_6	1_5	4			
7		6_4				**4_8**	1	8
4		2_{12}	7_{23}		9		3	
5_{19}		3	4_9		8_{22}	9		
9	6	8_{14}	2		1_{24}		**4_{10}**	7
3	4	1_1						5
6_{16}	5_{17}	7_{13}	8	4_2	3_{20}	1	9_{21}	2
2_{15}	8_3	9	1	7	5_{18}			4_{11}

Fig 21 (b)

2_{12} at C_{34} because of third vertical column line scanning because of presence of 2 at C_{98} and C_{46}

7_{13} at C_{38} because of third column line scanning.

8_{14} at C_{36} because of third vertical column line scanning. This completes 3rd column line.

2_{15} for block 3 only possible in C_{19} because of the presence of 2 at C_{98}.

6_{16} & 5_{17} are accordingly filled up for block 3 at C_{18} and C_{28} respectively

5_{18} at C_{69} because of third row Block to block horizontal scanning

5_{19} at C_{19} because of first column block to block vertical scanning

3_{20} only possible in C_{68} for row 8 and hence 9_{21} automatically get slotted in C_{88}.

Observe fig 21 (c)

8_{22} &7_{23} &1_{24} because of second Column block to block scanning at C_{65}, C44 and C_{66} respectively.

		4_{7}		8	7	6		
8		5	6_{6}	1_{5}	4	2_{41}		
7		6_{4}		9_{40}	2_{39}	**4_{8}**	1	8
4	1_{31}	2_{12}	7_{23}	5_{27}	9	8_{34}	3	6_{29}
5_{19}	7_{32}	3	4_{9}	6_{28}	8_{22}	9	2_{33}	1_{30}
9	6	8_{14}	2	3_{25}	1_{24}	5_{26}	**4_{10}**	7
3	4	1_{1}	9_{38}	2_{37}	6_{36}		8_{35}	5
6_{16}	5_{17}	7_{13}	8	4_{2}	3_{20}	1	9_{21}	2
2_{15}	8_{3}	9	1	7	5_{18}			4_{11}

Fig 21 (c)

3_{25} at C_{56} because block 8 cannot have 3 at column no 5 at C_{57} and Now 5_{26} at C_{76} automatically comes for row 6 filling.

5_{27} only possible at C_{54} for row 4 because of presence of 5_{26} at C_{76} and 5 at C_{97} and 5_{17} at C_{28} hence 6_{28} automatically get slotted in C_{55} for 5th column line.

6_{29} at C_{94} because of second row block to block horizontal scanning.

1_{30} at C_{95} because of 3rd column block to block vertical scanning.

1_{31} at C_{24} & 7_{32} at C_{25} & 2_{33} at C_{85} because of second Row block to block horizontal scanning

8_{34} at C_{74} is automatically gets slotted for Block 8

8_{35} at C_{87} because of 3rd column vertical block to block scanning.

6_{36} at C_{67} because of 2nd column block to block vertical scanning

2_{37} at C_{54} & 9_{38} at C_{47} because of Third row block to block horizontal scanning

2_{39} at C_{63} & 9_{40} at C_{53} because of 2nd column block to block vertical scanning.

2_{41} at C_{72} because of 3rd column block to block vertical scanning.

1_{49}	2_{48}	4_{7}	3_{47}	8	7	6	5_{44}	9_{50}
8	9_{51}	5	6_{6}	1_{5}	4	2_{41}	7_{45}	3_{52}
7	3_{53}	6_{4}	5_{46}	9_{40}	2_{39}	**4_{8}**	1	8
4	1_{31}	2_{12}	7_{23}	5_{27}	9	8_{34}	3	6_{29}
5_{19}	7_{32}	3	4_{9}	6_{28}	8_{22}	9	2_{33}	1_{30}
9	6	8_{14}	2	3_{25}	1_{24}	5_{26}	**4_{10}**	7
3	4	1_{1}	9_{38}	2_{37}	6_{36}	7_{54}	8_{35}	5
6_{16}	5_{17}	7_{13}	8	4_{2}	3_{20}	1	9_{21}	2
2_{15}	8_{3}	9	1	7	5_{18}	3_{42}	6_{43}	4_{11}

Fig 21 (d) Solved)

Now Fig 21(d)

3_{42} at C_{79} because of 3rd column block to block vertical column scanning.

6_{43} at C_{89} because of third row block to block horizontal scanning.
5_{44} at C_{81} because of 3rd column block to block vertical scanning.
7_{45} at C_{82} & 5_{46} at C_{43} & 1_{47} at C_{11}& 2_{48} at C_{21} because of first block to block horizontal scanning and hence 3_{49} at C_{41}& 9_{50} at C_{91} are automatically filled for block 4 and block 7 respectively.
9_{51} at C_{22} because of first row block to block horizontal scanning hence 3_{52} at C_{92}.
3_{53} at C_{23} for first row block to block scanning.
7_{54} at C_{77} by 3rd column block to block scanning.
This completes the Sudoku.

Example 3

Hopefully, you have enjoyed solving Sudoku with two examples given above. Before, we proceed to another Sudoku puzzle, we should summarize what we have learnt from example 1 and 2.

(a) For solving any Sudoku puzzle, we should proceed in a sequence in the sense that go for horizontal row scanning first example 1st row block to block scanning, then 2nd row block to block and then 3rd row block to block scanning. Once it is over then goes for column scanning. 1st column scanning then 2nd column scanning and then 3rd column scanning. Once that is over then, we go for line scanning. There are 18 lines, 9 row lines and 9 column lines.
(b) Then go for individual block scanning. We have 9 blocks and their identification numbers are designated as in the rule 4.
(c) Once these are done individually or in a combination thereof, new situation emerges where again row block to block or column block to block, row or column lines and individual block

scanning is needed and this process continues till the puzzle is solved.

(d) Once these techniques are exhausted and still the Sudoku is not solved then logic helps in solving the Sudoku. If the normal logics also fail to solve Sudoku then a technique developed by this author can be applied to solve Sudoku. This can be known as Gyaneshwar's circle method.

(e) This circle method has been described in details in the subsequent chapters.

2			9			7		
		5		6				4
9	4				5			
1				5			4	
5		8				3		1
	3			7				6
			2		7		1	3
7				3		2		
3	2	4			6			7

Fig 22

2			9			7		
8_{9}		5		6		1_{7}		4
9	4				5	6_{6}		
1				5			4	
5		8				3	7_{2}	1
4_{1}	3			7				6
6_{8}	5_{4}	9_{12}	2	8_{11}	7	4_{3}	1	3
7	8_{10}	1_{13}		3		2	6_{5}	
3	2	4			6			7

Fig 22 (a)

Solution to E×ample 3

First, second and third row block to block scanning gives nothing.

4_{1} at C16 is obtained by 1st column block to block scanning.

2nd column block to block scanning gives nothing

3rd column block to block scanning gives 7_{2} at C85 and 4_{3} at C77.

5_4 only possible in C_{27} for row line no 7.

6_5 at C_{88} possible for Block 9 .This makes 6_6 possible at C_{73} and 1_7 at C_{72} by 3rd column block to block scanning.

For column line no 1, 6_8 possible only in C_{17}. Further this gives 8_9 at C_{12}

Again this helps in filling with 8_{10} at C_{28} by 1st column block to block scanning.

For row line no 7, 8_{11} is possible only in C_{57} this helps in filling 9_{12} at C_{37}.
Now for block 3, only vacant place is C_{38} so it is filled with 1_{13}.

Observe fig 22(b)

Now 1st column block to block scanning give 1_{14} at C_{21}.

6_{15} by 1st row block to block scanning at C_{31}.

3_{16} at C_{33} for block 1filling and accordingly for block 1 the only vacant place is filled with 7_{17}.at C_{22}.

7_{18} at C_{34} because of 1st column block to block scanning. This gives 2_{19} at C_{36} for column line no 3.

7_{20} at C_{43} because of 2nd column block to block scanning. Now 1_{21} at C_{53} because of 1st row block to block scanning.

As block 4 has 9 at C_{41} so for row line no 2, 9_{22} possible only in C_{82}. So for row line no 2, 2_{23} is filled in C62 and this enables 3_{24} at C_{42}.

2	1_{14}	6_{15}	9	4_{26}	8_{25}	7	3_{27}	5_{28}
8_{9}	7_{17}	5	3_{24}	6	2_{23}	1_{7}	9_{22}	4
9	4	3_{16}	7_{20}	1_{21}	5	6_{6}	2_{29}	8_{30}
1		7_{18}		5			4	
5		8				3	7_{2}	1
4_{1}	3	2_{19}		7				6
6_{8}	5_{4}	9_{12}	2	8_{11}	7	4_{3}	1	3
7	8_{10}	1_{13}		3		2	6_{5}	
3	2	4			6			7

Fig 22 (b)

For block 4, 8_{25} is filled at C_{61} and accordingly only vacant unit of block 4 is filled with 4_{26} at C_{51}.

3_{27} by 3rd column block to block scanning at C_{81}so only no left 5_{28} is filled for 1st row line at C_{91}.

For 8th column line 2_{29} is possible only in C_{83} because of presence of 2 at C_{29} and C_{36}.

This allows us to fill 8_{30} at C_{93} for block 7.

Now, refer Fig 22(c)

2	1_{14}	6_{15}	9	4_{26}	8_{25}	7	3_{27}	5_{28}
8_{9}	7_{17}	5	3_{24}	6	2_{23}	1_{7}	9_{22}	4
9	4	3_{16}	7_{20}	1_{21}	5	6_{6}	2_{29}	8_{30}
1	9_{43}	7_{18}	6_{47}	5	3_{35}	8_{48}	4	2_{31}
5	6_{42}	8	4_{41}	2_{34}	9_{40}	3	7_{2}	1
4_{1}	3	2_{19}	8_{46}	7	1_{37}	9_{44}	5_{45}	6
6_{8}	5_{4}	9_{12}	2	8_{11}	7	4_{3}	1	3
7	8_{10}	1_{13}	5_{38}	3	4_{39}	2	6_{5}	9_{32}
3	2	4	1_{36}	9_{33}	6	5_{49}	8_{50}	7

Fig 22 (c)

3rd column block to block scanning gives 2_{31} at C_{94} and this allows us to fill 9_{32} at C_{98} for column line no 9.

3rd row block to block scanning gives 9_{33} at C_{59} and for column line 5 the vacant place at C_{55} is filled with 2_{34}.

2nd row block to block scanning gives 3_{35} at C_{64}

For block 6,1_{36} possible only in C_{49} this gives 1_{37} at C_{66} by second column block to block scanning. Now for block 6 again 5_{38} possible only at C_{48} and this give 4_{39} at C_{68}. This helps us in filling 9_{40} at C_{65} for column line 6.

This helps us in filling 4_{41} at C_{45} and hence 6_{42} at C_{25} for row line 5.

The vacant place of block 2 is filled with 9_{43} at C_{24},

We get 9_{44} from 2nd row block to block scanning or by 3rd column block to block scanning at C_{76}.

5_{45} is obtained at C_{86} for row line 6. Now only vacant unit at C_{66} for row line 6 is filled with 8_{46}. This leaves only vacant place of Block 5 to be filled with 6_{47}at C 44

Similarly, 8_{48}, 5_{49} and 8_{50} are filled to complete the Sudoku square

Now, we have solved three examples, so we have fair idea of what is to be done for solving Sudoku puzzles. These examples are of simple forms and we will slowly go tougher ones. But be sure, if you understood the processes above then it will not be difficult for you to attempt and solve tougher ones also. Good luck.

Chapter 4

Solved Sudoku

Solved Sudoku 1

	7			9				
		7				9		1
8	9		2		5			
			9	7	4	1		
9		1				8		7
		6	1	5				
			3		9			5
5		9				7		
			5				4	9

Fig 23

Solution: Refer fig 23 (a)

11 at C_{11} by 1st block filling and 1_2 by 1st Row block to block horizontal scanning.
7_3 at C_{83} by 3rd row line filling.
7_4 at C_{41} by 4th column line filling.
9_5 at C_{86} by 8th column line filling.
8_6 at C_{66} by 5th block filling.
7_7 at C_{69} by 6th block filling.
1_8 at C_{68} by 6th vertical line.

1_1	3	2_{17}	7_4	9		5_{13}		
6_{16}	5_{14}	7				9		1
8	9	4_{18}	2	1_2	5		7_3	
		5_{15}	9	7	4	1		
9	4_{11}	1				8	5_{12}	7
		6	1	5	8_6		9_5	
		8_{20}	3		9		1_9	5
5		9			1_8	7		
	1_{10}	3_{19}	5		7_7		4	9

Fig 23 (a)

1_{9} at C_{87} by 3rd column block to block scanning.
1_{10} at C_{29} by 1st column block to block scanning.
4_{11} at C_{25} by filling of 5th row line.
5_{12} at C_{85} by 5th row line scanning.
5_{13} at $C_{71}$by 3rd column block to block scanning.
5_{14} at C_{22} by 1st row block to block scanning
5_{15} at C_{34} by 1st column block to block vertical scanning.
6_{16} at C_{12}, 2_{17} at C_{31} and 4_{18} at C_{33} – By 1st block filling.
3_{19} at C_{39} and 8_{20} at C_{37} by 3rd column line filling.

1_{1}	3	2_{17}	7_{4}	9	6_{30}	5_{13}	8_{31}	4_{29}
6_{16}	5_{14}	7				9		1
8	9	4_{18}	2	1_{2}	5		7_{3}	
3_{24}	8_{27}	5_{15}	9	7	4	1		
9	4_{11}	1				8	5_{12}	7
7_{23}	2_{28}	6	1	5	8_{6}	4_{33}	9_{5}	3_{34}
4_{21}	7_{22}	8_{20}	3	6_{37}	9	2_{38}	1_{9}	5
5	6_{26}	9			1_{8}	7		8_{32}
2_{25}	1_{10}	3_{19}	5	8_{35}	7_{7}	6_{36}	4	9

Fig 23 (b)

Now, refer fig 23(b)

4_{21} at C $_{17}$ by 1st column block to block scanning.

7_{22} at C $_{27}$ by 3rd row block to block scanning and 3_{24} at C_{14}, and 7_{23} at C_{16} By 1st column block to block vertical scanning. 2_{25} at C_{19} by 1st column line filling.

Empty cell of block 3 is filled with 6_{26} at C_{28}.

8_{27} at C_{24} and 2_{28} at C_{26} by 2nd column line filling.

4_{29} at C_{91} by 7th block filling.

6_{30} at C_{61} - By 4th block filling.

8_{31} by filling of 1st row line at C_{81}.

8_{32} and 433 by 3rd column block to block scanning at C_{98} and C_{76} respectively.

3_{34} at C_{96} by filling 6th row line.

8_{35} by 3rd row block to block scanning at C_{59} and 6_{36} by 9th row line filling at C_{79}.

6_{37} at C_{57} by 3rd row block to block scanning. This gives us 2_{38} at C_{77} by 7th row line filling.

Refer fig 23 (c)

1_{1}	3	2_{17}	7_{4}	9	6_{30}	5_{13}	8_{31}	4_{29}
6_{16}	5_{14}	7	8_{47}	4_{51}	3_{50}	9	2_{42}	1
8	9	4_{18}	2	1_{2}	5	3_{40}	7_{3}	6_{41}
3_{24}	8_{27}	5_{15}	9	7	4	1	6_{43}	2_{52}
9	4_{11}	1	6_{46}	3_{49}	2_{48}	8	5_{12}	7
7_{23}	2_{28}	6	1	5	8_{6}	4_{33}	9_{5}	3_{34}
4_{21}	7_{22}	8_{20}	3	6_{37}	9	2_{38}	1_{9}	5
5	6_{26}	9	4_{45}	2_{44}	1_{8}	7	3_{3}	8_{32}
2_{25}	1_{10}	3_{19}	5	8_{35}	7_{7}	6_{36}	4	9

Fig 23 (c)

3_{39} is obtained at C_{88} for block 9 filling.

3_{40} at C_{73} by 7^{th} block filling and 6_{41} at C_{93}, we get by 3^{rd} row line filling.

2_{42} at C_{82} we get by filling 8^{th} block.

Similarly,

6_{43} at C_{84}, we get by filling 8^{th} column line.

2_{44} at C_{58} and 4_{45} at C_{48} by 6^{th} block filling.

6_{46} at C_{45} and 8_{47} at C_{42} by 4^{th} column line filling.

2_{48} at C_{65} and 3_{49} at C_{55} by 7^{th} block filling.

3_{50} at C_{62} and 4_{51} at C_{52} by 4^{th} block filling

2_{52} at C_{94} by 8^{th} block filling.

SOLVED SUDOKU 2

Apart from the techniques learnt above, application of logic also helps in big way in solving Sudoku problems. Here is an example.

	3	8			5	6		7
			6					9
			1		8	2		
	5		4				9	
	4	2				1	5	
					1		6	
		5	7		9			6
4					6			
6			2			9	7	

Fig 24

Ref Fig 24 (a)

2_{8}	3	8	9_{10}	4_{6}	5	6	1_{5}	7
		4_{23}	6					9
5_{24}	6_{4}	9_{25}	1	7_{26}	8	2		
1_{3}	5	6_{1}	4			7_{15}	9	
9_{12}	4	2		6_{2}	7_{13}	1	5	
			5_{14}	9_{11}	1		6	
	2_{9}	5	7		9			6
4					6			
6			2		4_{7}	9	7	

Fig 24 (a)

First, second and third row block to block scanning gives nothing. Similarly, 1st, 2nd and 3rd column block to block scanning also gives nothing.

Scanning of 2nd block gives 6_{1} at C_{34}.

2nd column block to block scanning gives 6_{2} at C_{55}.

Scanning of 2nd row block to block gives 1_{3} at C_{14}.

1st column block to block scanning gives 6_{4} at C_{23}.

<u>Use of logic</u>

Since for block 1, digit 1 can appear at either at C_{22} or C_{32} so this helps us in filling 15 at $C_{81}$by 1st row block to block scanning.

For Row line 1, 4_6 is filled up at C_{51}. This gives 4_7 at C_{69} by 2nd column block to block to scanning. We get 2_8 by 1st row line filling. **For block 3, places C_{28} and C_{38} is reserved for 9&7**. So, if we go for 1st column block to block scanning then we can enter 2_9 at C_{27}. Further we get 9_{10} at C_{41} by 1st row line filling.

We get 9_{11} at C56 by 2nd column block to block scanning and this helps in getting 9_{12} at C_{15} by 2nd row block to block scanning

Now for row line 5 we get 7_{13} at C_{65}.

For row line 6, 5_{14} can happen only at C46 because block 3 and 8 have 5.

For block 2, digit 7 can happen only at either C_{16}, C_{26} or C_{36} so this helps us in filling 7_{15} at C_{74} by second row block to block scanning.

Now, Fig 24 (b)

2_8	3	8	9_1	4_6	5	6	1_5	7
		4_2	6			5_3	8_2	9
5_2	6_4	9_2	1	7_2	8	2	3_2	4_2
1_3	5	6_1	4			7_1	9	
9_1		2		6_2	7_1	1	5	
		3_3	5_1	9_1	1	4_2	6	2_1
	2_9	5	7	1_1	9		4_2	6
4	9_3	7_3			6		2_1	1_1
6		1_3	2		4_7	9	7	5_2

Fig 24 (b)

We get 1_{16} at C_{57} for row line 7.

C_{28} and C_{38} are reserved for 9&7 so for row line 8 we get 1_{17} at C_{98}.

This helps in getting 2_{18} at C_{88} for row line 8.

We get 2_{19} at C_{96} and 4_{20} at C_{76} easily for row line 6.

We get 4_{21} at C_{93} and 5_{22} at C_{99} for column line 9 filling.

By 1st column block to block scanning we get 4_{23} at C_{32}.

For 3rd row line we 5_{24} get at C_{13}.

1st row block to block scanning gives 9_{25} at C_{33}.

We get 7_{26} and 3_{27} at C_{53} and C_{83} by 3rd row line filling.

We get 4_{28} at C_{87} by 9th block filling.

So 8_{29} is easily obtained at C_{82} by 8th column line filling.

Hence 5_{30} at C_{72} is also obtained easily for block no 7.

We get 9_{31} and 7_{32} at C_{28} and C_{38} by 3rd row block to block scanning.

For column line 3, we get 1_{33} and 3_{34} at C_{39} and C_{36} .

Now, refer Fig 24 (c)

2_8	3	8	9_1	4_6	5	6	1_5	7
7_3	1_3	4_2	6	2_5	3_5	5_3	8_2	9
5_2	6_4	9_2	1	7_2	8	2	3_2	4_2
1_3	5	6_1	4	8_4	2_4	7_1	9	3_5
9_1		2	3_4	6_2	7_1	1	5	8_5
8_4	7_3	3_3	5_1	9_1	1	4_2	6	2_1
3_4	2_9	5	7	1_1	9	8_4	4_2	6
4	9_3	7_3	8_4	5_4	6	3_4	2_1	1_1
6	8_3	1_3	2	3_4	4_7	9	7	5_2

Fig 24 (c)

For column line 2 we fill 1_{35} at C_{22}.
This helps in filling 7_{36} at C_{12} for 1^{ST} block filling.

7_{37} and 8_{39} at C_{26} and C_{29} are filled up for column line 2.
3_{40} at C_{17} filled for block no 3 and for column line 1, we get 8_{41} at C_{16}.
We get 8_{42} at C_{77} for row line 7.
Also we get at 3_{43} C_{78} by 7^{th} column line filling.
This gives 3_{44} at C_{59} by 3^{rd} row block to block scanning.
For block 6 we get 5_{45} at C_{58} and 8_{46} at C_{48} by filling of block.

We get 3_{47} by 4^{th} column line filling at C_{45}.
Also we get 8_{48} at C_{54} and 2_{49} at C_{64} by block 5 filling.
Also we get 2_{50} at C_{52} and 3_{51} at C_{62} by block 4 filling
Also we get 3_{52} at C_{94} and 8_{53} at C_{95} by block 8 filling

Note- Sentences highlighted by green ink indicate the application of logic. This Sudoku is the example, where you can link various probable fillings based on certain logics. Many a times the application of logic increases and is interdependent and interlinked which requires stretching of memory and imagination. To simplify a method developed by this author and named **"Gyaneshwar's Circle method"** can be used to solve the Sudoku. The details of circle method have been explained in the subsequent chapters .This circle method has been used extensively in this book.

SOLVED SUDOKU 3
(GYANESHWAR'S CIRCLE METHOD)

		5	7			3	8	
		6		2	1	5		
7						1		2
				1		2	3	
6						9		8
	9	2		8		7		
3	6	7				8		5
		1	9	5		6		
	5				6	4		

Fig 25

Solution – Refer Fig 25(a)

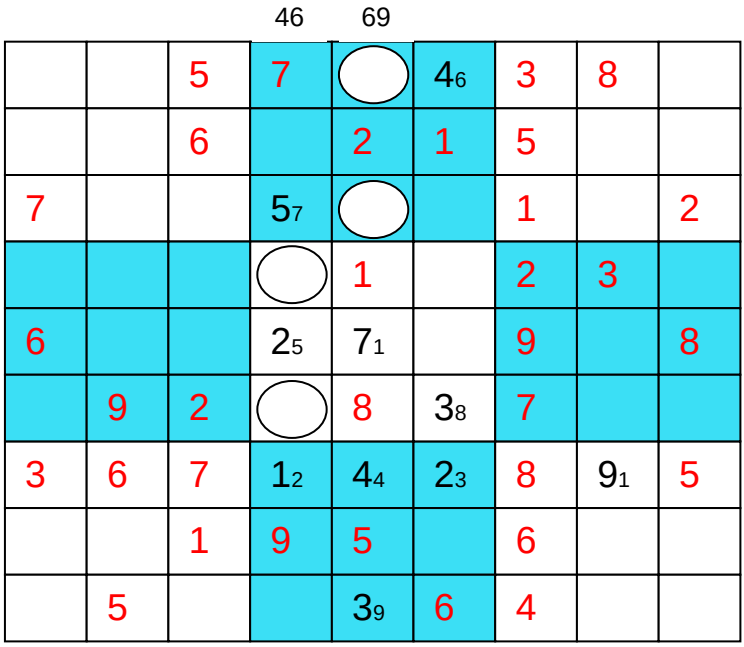


Fig 25 (a)

We get 9_1 at C_{87} because of presence of 9 in block 6 and 1_2 at C_{47} by 7^{th} row line filling.

We get 2_3 by 7^{th} row line filling at C_{67}

This gives 4_4 at C_{57} by 7^{th} row line scanning.

Second row block to block scan gives 2_5 at C_{45}.

Digit 9 and 6 cannot be placed at C_{59} and C_{55} so for column line 5 because of presence of 9 and 6 in block 6 and in row line 5 so digits 9 and 6 can only happen at C_{51} and C_{53}. We indicate this by drawing small circles on the respective unit squares (C_{51} and C_{53}) as indicated on fig 19(a).and write 69 on the top of Sudoku table for indication only. By drawing circle at C_{51} and C_{53}, we reserve these cells for digit 6and 9. So wherever we circle a cell, mean that cell is reserved for a digit and we write the digit on top of the Sudoku table or to left or right of the Sudoku table as indicated.

Now coming back to block 4, we see that C_{61} cannot have either 3, 5 or 8. These digits 3,5and 8 will occupy each either in C_{42}, C_{43} or C_{63}. So left over digit 4_6 can only be accommodated in C_{61}.

Now second column block to block scanning reserves C_{44} and C_{46} for digit 4 and 6 because of presence of 4 and 6 at C_{61} and C_{69} respectively. We again put circles on the respective place as indicated and write 46 on top for indication.

Examination of column line 4 reveals that 5_7 is possible only in C_{43}.

Examination of row line 6 reveals that 3_8 is possible in C_{66}. This 3_8 helps in filling 3_9 at C_{59} for 5th column line filling. Again for column line 5 at C_{55} we fill 7_{10}.

Now fig 25(b)

			46	69					
○	○	5	7	6_2	4_6	3	8	9_{23}	12
9_{26}	8_{27}	6	3_{13}	2	1	5	○	○	47
7			5_7	9_2	$\mathbf{8_{14}}$	1	6_{22}	2	
5_{18}	7_{17}	8_{19}	○	1	9_{11}	2	3		
6			2_5	7_{10}	5_{12}	9		8	
	9	2	○	8	3_8	7	5_{20}		
3	6	7	1_2	4_4	2_3	8	9_1	5	
		1	9	5	7_{16}	6		3_{21}	
	5		8_{15}	3_9	6	4			

Fig 25 (b)

For block 5, we can easily fill 9_{11} and 5_{12} at C_{64} and C_{65}.

Unit squares C_{11} and C_{21} are reserved for digit 1and 2 by 1st row block to block scanning. We put circles on them and write 12 to the left or right of the table indicate the circles as shown in the fig 25b.

Now for block 4, we can easily fill at 3_{13} at C_{42} and 8_{14} at C_{63}..

Now for block 6, we can easily fill 8_{15} at C_{49} and 7_{16} at C_{68}.

1st column block to block scanning gives 7_{17} at C_{24}

4th row line scanning gives 5_{18} at C_{14} and 8_{19} at C_{34}.

8th block filling gives 5_{20} at C_{86}. 9th block filling gives 3_{21} at C_{98}

8th column line filling gives 6_{22} at C_{83}.

Now c_{82} and C_{92} are reserved for 4 and 7 so we put circle and write 47 on the right side of the Sudoku square for indication. This reservation gives us 9_{23} at C_{91}. This presence of 9_{23} at C_{91}gives 6_{24} at C_{51} and hence 9_{25} at C_{53}.

2nd row line filling gives 9_{26} at C_{12} and 8_{27} at C_{22}.

Now fig 25 (c)

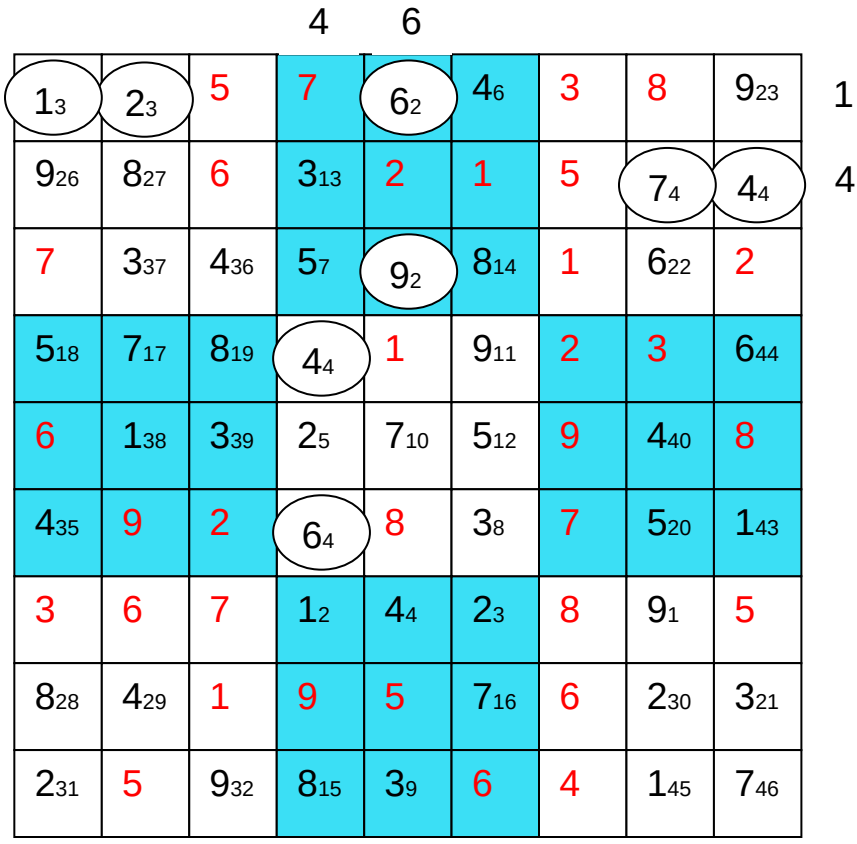


Fig 25 (c)

8th row line filling gives 8_{28} at C_{18} and 4_{29} at C_{28} and further 2_{30} at C_{88}.

3rd row block to block scanning gives 2_{31} at C_{19}. This helps in getting 9_{32} at C_{39} for block 3.

Now presence of 2_{31} at C_{19} gives 1_{33} at C_{11} (because it can either be 1 or 2) and hence 2_{34} at C_{21}.

Now 1st column line scanning gives 4_{35} at C_{16}.

1st column block to block scanning gives 4_{36} at $\mathbf{C}_{33}$.

This makes vacant place of block 1 to be filled with 3_{37} at C_{23}.

Now block 2 is filled with 1_{38} at C_{25} and with 3_{39} at C_{35} and this helps in filling at C_{85} with 4_{40} for 5th row line filling.

4_{35} at C_{16} help in filling 6_{41} at C_{46} and hence 4_{42} is filled at C_{44}.

Now block8 is filled with1_{43} at C96 and with 6_{44} at C_{94}.

3rd column block to block scanning gives 1_{45} at c_{89} and 7_{46} at C_{99}.

This helps in filling 7_{47} at C_{82} with and 4_{48} at C_{92}.

Circle method helps in memorizing various probable blank positions which helps in future moves. The number of blank cells may increase and it sometime becomes unmanageable and difficult to remember them. So if we use circles along with numbers then it becomes very easy to identify and make the next move. This definitely makes Sudoku a game of fun.

SOLVED SUDOKU 4

1		5					8	7
	4		5			1		3
				1				5
		3	6			7	1	
7	1		3		9		5	6
	6				1	3		
2				4				
4	3	1			8		7	
6	5					9		

Fig 26

Solution-

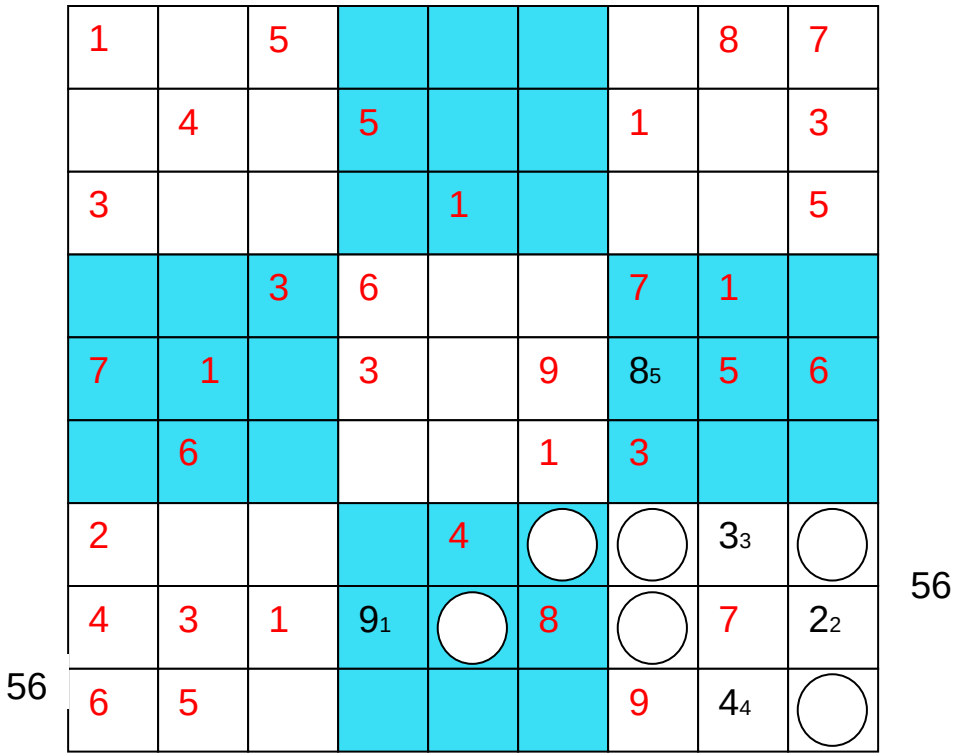


Fig 26 (a)

By logic, for row line 8, digit 5&6 can happen only on C_{58} and C_{78} because of presence of 5 &6 at C_{42} and C_{44} also at C_{93} and C_{95} so we mark them with circles and write 56 outside for indication. Also circle is drawn for C_{67} because for block 6 digit 5 and 6 can happen only on C_{58} and C_{67}. .

Further 9_1 is filled at C_{48} for row line 8. This gives 2_2 at C_{98} for 8th row line.

Now in block 6, the position of digit 3 will be either on C_{59} or C_{69}. This position of 3 at block 6 and by 3rd column block to block scanning we get 3_3 at C_{87}.

This move gives rise to a new possibility and that is for row line 7 digit 5 and 6 can only happen at C_{67} and C_{77}, we designate them by circle and write 56 outside.

Now in block 9, places C_{97} and C_{99} are reserved for digit 1 and 8 by 3rd column block to block scanning. So we draw circle for them also and write 18. This allows us to write 4_4 at C_{89}. This also helps us to write 8_5 at C_{75} by 3rd column block to block scanning.

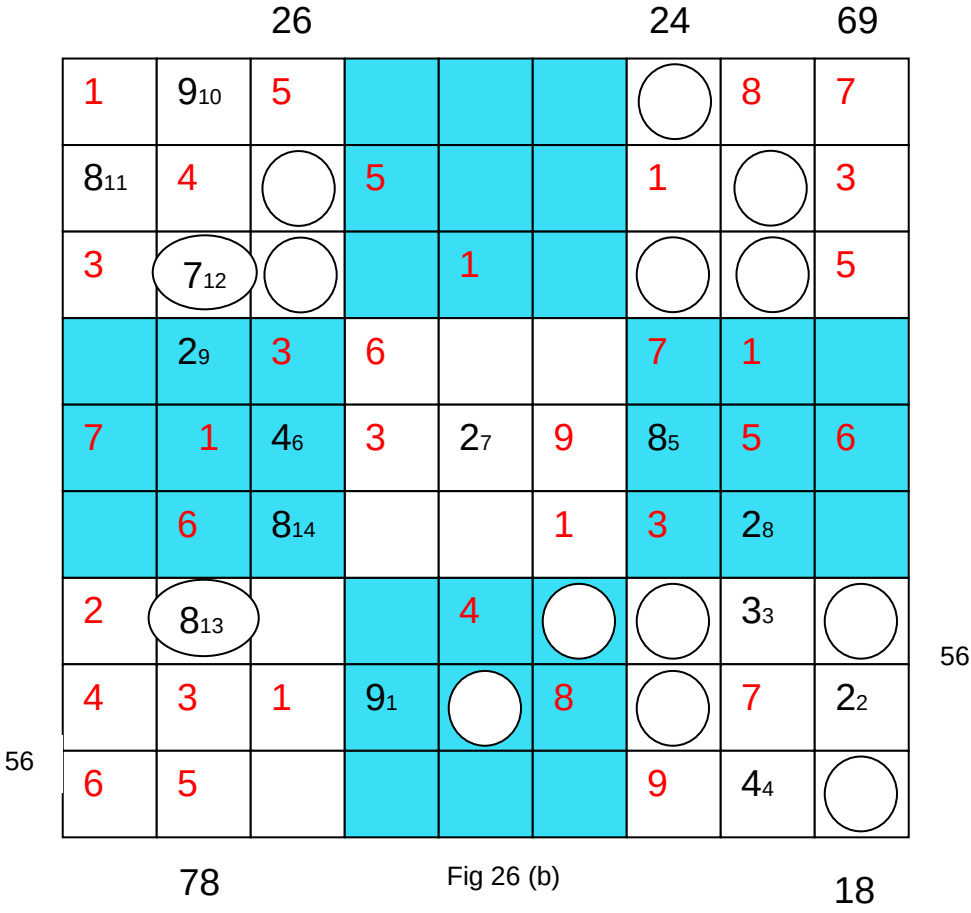


Fig 26 (b)

Now fig 26 (b)

5^{th} row line scanning gives 4_6 at C_{35}, Hence 2_7 is filled at C_{54} for 5^{th} row line.
By scanning of Block 8, we get 2_8 at C_{86}.
For block 7 again we reserve places for 24 and 69 by drawing circles at C_{71} &C_{73} and C_{82}&C_{83} respectively.
By second row block to block scanning we get 2_9 at C_{24}.
For 2^{nd} column line, digit 8and 7 cannot happen at C_{21}, it can only happen at $C_{23}$and C_{27} so they are also circled and written 78.
This allows us to enter digit 9_{10} at C_{21} for column line 2.
Again for block 1, 2 &6 can only happen at C_{32} and C_{33} so we circle them and write 26 on top.

This allows us to write 8_{11} at C_{12} because of presence of 7 at C_{15}.
Again this helps us to enter digit 7_{12} at circled place C_{23}.
This also facilitates in writing 8_{13} at circled place C_{27}.
For 3^{rd} column line we can write 8_{14} at C_{36}.

Refer fig 26 (c)
This gives 8_{15} at C_{54} by 2^{nd} row block to block scanning and 8_{16} at C_{43} by 2^{nd} column block to block scanning.
Now we can write 8_{17} at C_{99} for row line 9. And this will give us 1_{18} at C_{97}.

This gives 1_{19} at C_{49} by 3^{rd} row block to block scanning.
For 7^{th} row line we can fill 9_{20} at C37 and also 7_{21} at C_{47}.

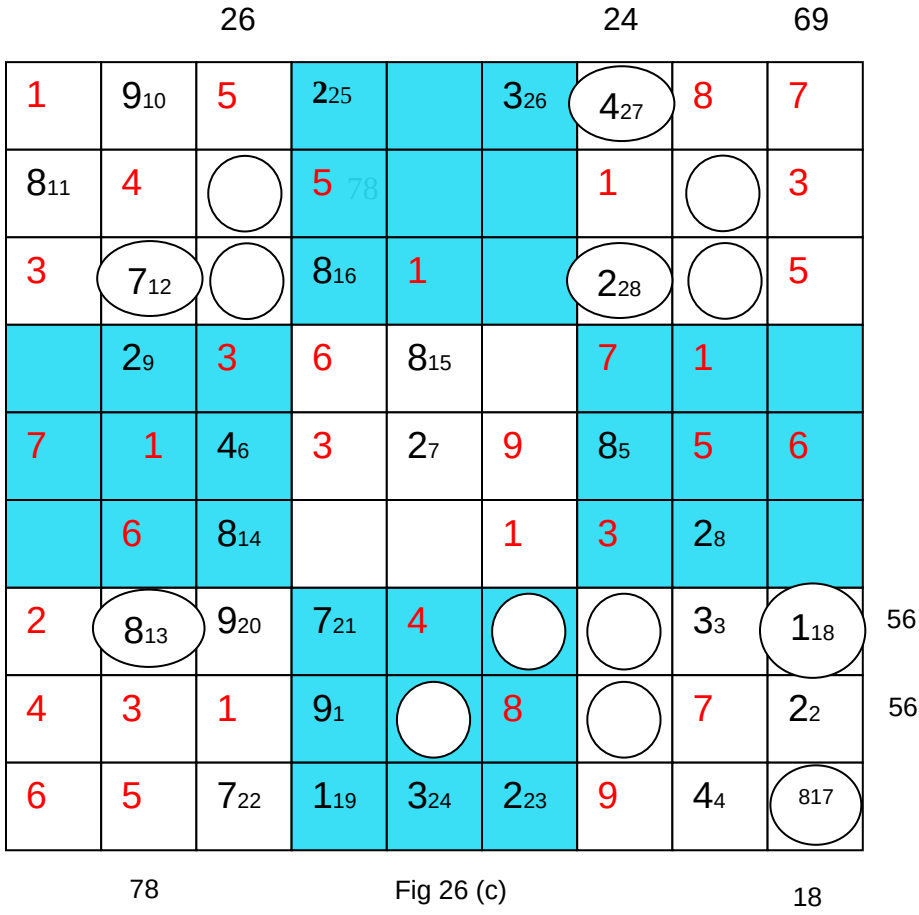


Fig 26 (c)

By block 3 filling, we can write 7_{22} at C_{39}.

For 9th row line we can write 2_{23} at C_{69}. And this allows us to fill 3_{24} at C_{59}.

2nd column block to block scanning gives 2_{25} at C_{41} and 3_{26} at C_{61}.

2_{25} at C_{41} give 4_{27} at circled C_{71} and this gives 2_{28} at C_{73}.

Now, refer fig 26 (d)

2_{28} at C_{73} give 6_{29} at circled C_{33} and this gives 2_{30} at C_{32}.

6_{29} at circled C_{33} give 9_{31} at C_{83} and this helps in getting 6_{32} at C_{82}.

For row line 3, we can write 4_{33} at C_{63}.

Similarly for row line 1, we can write 6_{34} at C_{51}.

2^{nd} column block to block scanning gives 9_{35} at C_{52}.

This helps in filling 7_{36} at C_{62} for block 4.

6_{34} at C_{51} helps in filling 5_{37} at C_{58} and hence 6_{38} at C_{78}.

This again helps in filling 5_{39} at $C_{77}$and 6_{40} at C_{67} for row line 7

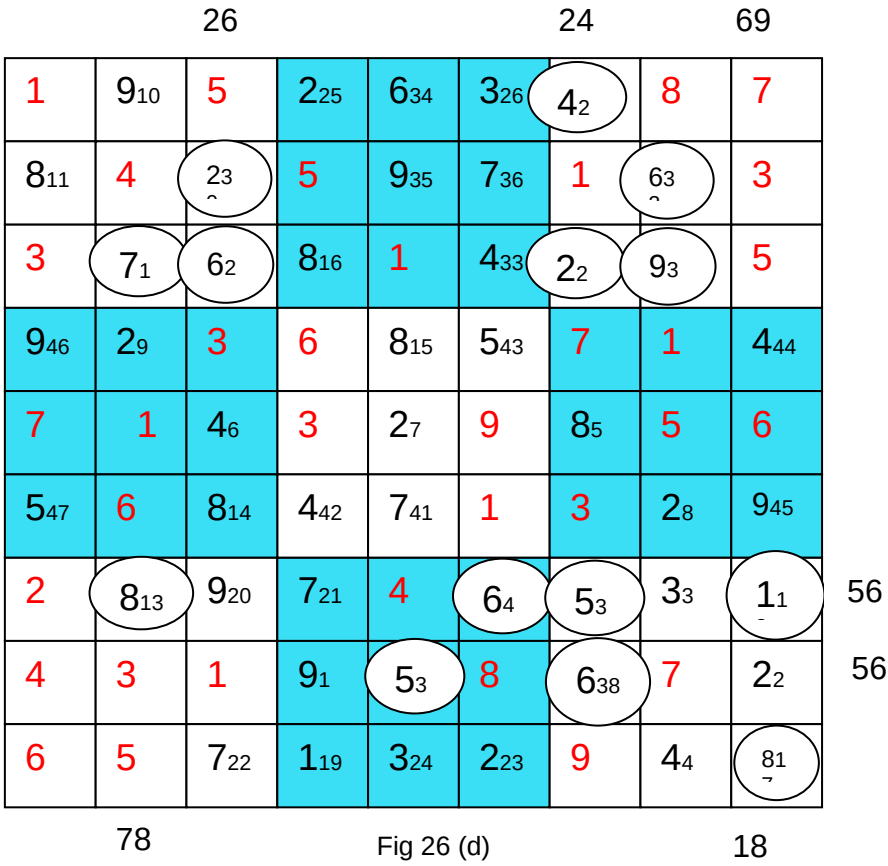


Fig 26 (d)

2^{nd} column block to block scanning give 7_{41} at C_{56}, 4_{42} at C_{46} and 5_{43} at C_{64}.

Scanning of block 8 gives 4_{44} at C_{94} and 9_{45} at C_{96}

Similarly, scanning of block 2 give 9_{46} at C_{14} and 5_{47} at C_{16}.

SOLVED SUDOKU 5

			9		7	8		6
		8					7	
6	7	1	5	2	8			3
		2		3	9			
				5				
			8	7		4		
2				9	5	3		
	6					1		
5		7			4			

Fig 27

Solution- Ref Fig 27a

By 3rd row line filling we get 4_1 at C_{83} and 9_2 at C_{73}.

Now 4 and 7 can happen only in C_{98} and C_{97} by 9th block filling because of presence of 7, 4 in C_{82} and C_{83}, also at C39 and C69. So we circle them. This gives 5_3 at C_{88} by 3rd row block to block scanning.

7_4 at C_{48} is obtained by 8th row line filling because of presence of 4 in 6th block.

This helps in filling 7_5 at C_{97} and 4_6 at C_{98} in the 9th block.

2_7 is filled at C_{68} by 8th row line filing

2_8 Is filled at C_{45} by 2nd column block to block scanning.

This helps with 3_9 at C_{49} for 6th block filling or 4th column line filling and 4_{10} for 5th block filling at C_{44} and hence 3_{11} is obtained at C_{62} by 2nd column block to block scanning.

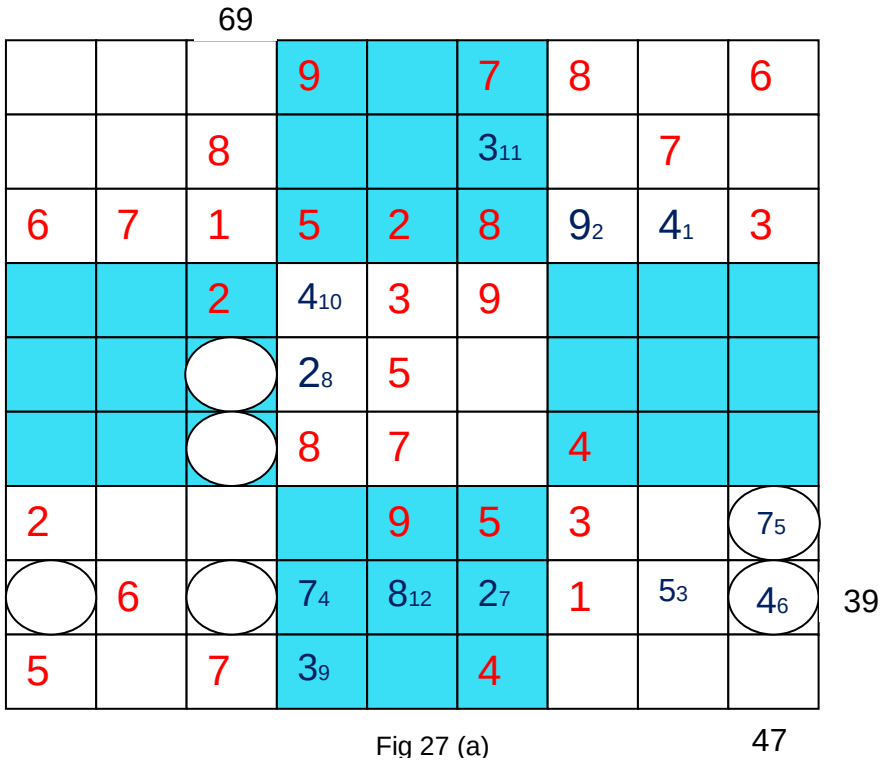


Fig 27 (a)

Now for row line 8, 3 and 9 can only happen in C_{18} and C_{38} because of 3 and 9 at C_{57} and C_{77} respectively and 9 has to happen for row line 8 so we circle them and write 39 at right as shown.

This helps us in getting 8_{12} at C_{58} for row line 8.
For third column line, 6 and 9 can only happen in C_{35} and C_{36} because of presence of 9 at C_{57} and C_{41} and also of 6 at C_{13} and C_{28}. Hence we circle them and write 69 on top of the table.

Now fig 27b
This helps in getting 5_{13} at C_{31} by third vertical column line scanning.

This automatically fills up 3_{14} at C_{38} for 3rd column line filling and 9_{15} at C_{18}.

We get 4_{16} by 3rd column line filling at C_{37}.

We get 9_{17} by 1st block scanning at C_{22}.

We get 2_{18} *at* C_{21}, 3_{19} at C_{11} and 4_{20} at C_{12} by first block filling.

Now we get 4_{21} at $C_{51}$by 1st row block to block scanning and 4_{22} at C_{25} by first column block to block scanning.

1_{23} Is filled at C_{81} for 1st row line.

Accordingly 3_{24} and 5_{25} are filled for 2nd column line at C_{26} and C_{24} respectively.

78 69

3_{19}	2_{18}	5_{13}	9	4_{21}	7	8	1_{23}	6
4_{20}	9_{17}	8			3_{11}		7	
6	7	1	5	2	8	9_2	4_1	3
	5_{25}	2	4_{10}	3	9			
	4_{22}		2_8	5				
1_{28}	3_{24}		8	7		4	2_{27}	5_{26}
2		4_{16}		9	5	3		7_5
9_{15}	6	3_{14}	7_4	8_{12}	2_7	1	5_3	4_6
5		7	3_9		4			

39

47

Fig 27 (b)

This gives 5_{26} at C_{96} by 2nd row block scanning.

Then we get 2_{27} at C_{86} by 6th row line scanning.

In block 2, 7 and 8 can only happen in C_{14} and C_{15} because of presence of 7 and 8 at C_{46} and C_{56}. So we circle them and write 78 on top as shown.

This gives 1_{28} at C_{16}.

Now refer fig 27c

Also, we get 9_{29} and 6_{30} for 6th horizontal row line scanning at C_{36} and C_{66} respectively.

Then we get 6_{31} at C_{35}, and then we get 1_{32} at C_{65} by sixth column line scanning.

We get 5_{33} at C_{72} by 3rd column block to block scanning, 2_{34} at C_{92} *by 7th block filling.*

We get 1_{35} at C_{94} and 2_{36} at C_{79} by 3rd column block to block scanning and hence 6_{37} at C_{74} and 7_{38} at C_{75} is obtained for 7th column line filling.

78 69

3_{19}	2_{18}	5_{13}	9	4_{21}	7	8	1_{23}	6
4_{20}	9_{17}	8	6_{51}	1_{50}	3_{11}	5_{33}	7	2_{34}
6	7	1	5	2	8	9_{2}	4_{1}	3
7_{4}	5_{25}	2	4_{10}	3	9	6_{37}	8_{41}	1_{35}
8_{39}	4_{22}	6_{3}	2_{8}	5	1_{32}	7_{38}	3_{42}	9_{43}
1_{28}	3_{24}	9_{2}	8	7	6_{30}	4	2_{27}	5_{26}
2	8_{47}	4_{16}	1_{52}	9	5	3	6_{45}	7_{5}
9_{15}	6	3_{14}	7_{4}	8_{12}	2_{7}	1	5_{3}	4_{6}
5	1_{48}	7	3_{9}	6_{49}	4	2_{36}	9_{44}	8_{46}

93

Fig 27 (c) 47

7_{38} at C_{75} helps in getting 8_{39} at C_{15} and hence 7_{40} at C_{14}.

This helps in filling 8_{41} at C_{84} and 3_{42} at C_{85} and 9_{43} at C_{95} by 8th block filling.

This helps in filling 9th block by 9_{44}, 6_{45}, and 8_{46} at C_{89}, C_{87} and C_{99} respectively. Further it helps in filling 8_{47} at C_{27}, 1_{48} at C_{29}, 6_{49} at C_{59} for block 8 and 1_{50} at C_{52} and 6_{51} at C_{42}. This also helps in filling 152 at C46 for 4th column line filling.

SOLVED SUDOKU 6

7	3		6		5	4		
4			9				6	
5	6							
9				5		2		
	2		8		3		9	
		5		9				1
			5				8	4
	5	4			9		2	7
		6	7		4		5	3

Fig 28

Solution (Fig 28 a)

Here, we have a condition that in both blocks 2 and 8, we see that C_{84} & C_{86} and C_{24} & C_{26} - all have possibilities of accommodating digit 4 by 1st and 3rd column block to block scanning. If this is true then by logic digit 4 will lie on the center line in block 5 at C_{55} only so we get 4_1 at C_{55}.

37

7	3		6		5	4		
4			9				6	5_6
5	6		4_2					
9			1_5	5		2		
	2		8	4_1	3	5_7	9	
		5	2_4	9				1
			5				8	4
	5	4	3_3		9		2	7
		6	7		4		5	3

37

Fig 28 (a)

This gives 4_2 at C_{43} by 2nd column block to block scanning.

For column line 4 we get 3_3 at C_{48} and 2_4 at C_{46} and 1_5 at C_{44}.

For column line 5, places of C_{52} and C_{53} are reserved for 3 &7. We circle them and write 37 on top because of 3&7 cannot happen in C_{57}, C_{58} and C_{59}, C_{57} .

In block5, digit 7 will occupy either C_{64} or C_{66} so for row line 2 , digit 7 cannot appear at C_{62} so for row line 2 , digit 3 and 7 can only appear at C_{72} and C_{52}. We circle them and write 37 on the side.

This gives 5_6 at C_{92} for 2nd row line filling.

5_7 at C_{75}.by 3rd column block to block scanning.

Now fig 28b

For column line 6 digits 1,2 and 8 can only happen at C_{62}, C_{63} and C_{67} because of presence of 1& 2 at C_{44} and C_{46}, so places C_{64} and C_{66} are reserved for digit 6 and 7 and circle them.

37 67

7	3		6		5	4			
4	1_{14}		9				6	5_6	37
5	6		4_2						
9			1_5	5		2			
1_9	2	7_8	8	4_1	3	5_7	9	6_{10}	
		5	2_4	9				1	
3_{16}	7_{11}	1_{13}	5				8	4	
8_{18}	5	4	3_3		9		2	7	
2_{17}	9_{12}	6	7		4		5	3	

Fig 28 (b) 169

7_8 at C_{35}, 1_9 at *C_{15}* and 6_{10} at C_{95} by 5th row filling

7_{11} at C_{27} by 1st column block to block scanning and 9_{12} at C_{29} by 2nd column line filling. .

This allows 1_{13} at C_{37} for block 3 and 1_{14} at C_{22} by 1st column block to block scanning.

Now refer fig 28 (c)

We get 3_{15} at C_{34} because at block 1 the places C_{31}, C_{32} and C_{33} are occupied by 289.The vacant places of block 9 will occupy 169. We circle them.

3rd row block to block scanning gives 3_{16} at C_{17}.

Similarly 2_{17} at C_{19} and 8_{18} at C_{18} is filled for block 3.

The presence of 96 in row line 9 allows us to fill 1_{19} at Circled C_{79}.

37 67

7	3		6	2_{25}	5	4	1_{28}	9_{31}
4	1_{14}	2_{29}	9		8_{27}		6	5_6
5	6		4_2		1_{26}			2_{30}
9		3_{15}	1_5	5		2		8_{32}
1_9	2	7_8	8	4_1	3	5_7	9	6_{10}
		5	2_4	9				1
3_{16}	7_{11}	1_{13}	5	6_{22}	2_{23}	9_2	8	4
8_{18}	5	4	3_3	1_{21}	9	6_2	2	7
2_{17}	9_{12}	6	7	8_{20}	4	1_1	5	3

37

169

Fig 28 (c)

8_{20} at C_{59} and 1_{21} at C_{58} are obtained by 9th column line filling.

6_{22} at C_{57} and 2_{23} at C_{67} are obtained for 6th block filling (helped by presence of 6 in column line 4 at C_{41}).

We get 9_{23} at C_{77} and 6_{24} at C_{78} by row line7 and row line 8 filling

2_{25} at C_{51} by 2nd column block to block scanning

We get 1_{26} at C_{63} and 8_{27} at C_{62} by 4th block filling.

1st row block to block scanning give 1_{28} at C_{81}.

2nd row line filling gives 2_{29} at C_{32}.

1st row block to block scanning give 2_{30} at C_{93}

9th column line scanning gives 9_{31} at C_{61} and 8_{32} at C_{94}.

3rd column block to block scanning gives 8_{33} at C_{73}.

<u>Now fig 28d</u>

37 67

7	3	8_{34}	6	2_{25}	5	4	1_{28}	9_{31}
4	1_{14}	2_{29}	9	3_4	8_{27}	7_4	6	5_6
5	6	9_{35}	4_2	7_4	1_{26}	8_{33}	3_{44}	2_{30}
9	4_{40}	3_{15}	1_5	5	6_3	2	7_{39}	8_{32}
1_9	2	7_8	8	4_1	3	5_7	9	6_{10}
6_{36}	8_{49}	5	2_4	9	7_3	3_{45}	4_{46}	1
3_{16}	7_{11}	1_{13}	5	6_{22}	2_{23}	9_2	8	4
8_{18}	5	4	3_3	1_{21}	9	6_2	2	7
2_{17}	9_{12}	6	7	8_{20}	4	1_1	5	3

37

Fig 28 (d) 169

1st row block to block scanning gives 8_{34} at C_{31}.and 9_{35} at C_{33} by 1st block filling.

We get 6_{36} at C_{16} by 2nd block filling; this gives 7_{37} at C_{66} and 6_{38} at C_{64} of 6th column line.

This gives us 7_{39} at C_{84} and 4_{40} at C_{24} by 4th row line scanning.

This allows us to have 7_{41} at C_{72} for 7th block filling and this helps in filling 3_{42} at C_{52} and 7_{43} at C_{53} for 5th column line.

This helps infilling 3_{44} at C_{83} for 7th block filling.

Now we fill 3_{45} at C_{76}, 4_{46} at C_{86} and 8_{49} at C_{26} for 6th row line filling.

SUDOKU 7

			2				7	8
		7			3	2	1	
	2		7			6	4	
	1		9	2		7		
						8		
		2		4	8		5	
	6	4			2		8	
	8	1	5		4	9		
2					6	4		

Fig 29

Solution- Refer fig 29a

2nd column block to block scanning gives 4_1 at C_{42}. This helps in getting 8_2 at C_{49} for 4th column line filling.

6th column line filling gives 7_3 at C_{65}. Now C_{61} and C_{63} are circled for 19.

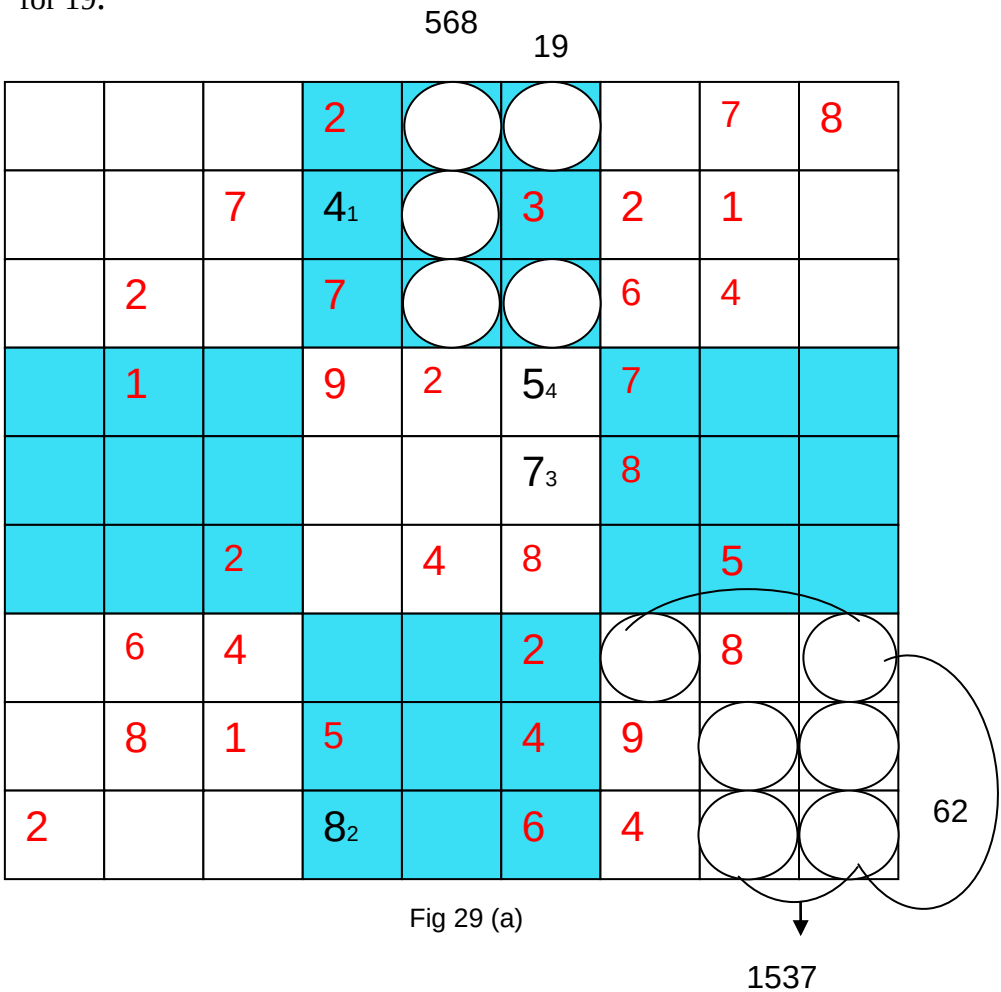


Fig 29 (a)

1, 3 &6 cannot happen at C_{64} for block 5. Now, we circle C_{41}, C_{42} and C_{43} for 5, 8 &6 and write 568 as shown.

This gives 5_4 at C_{64} for block 7 by 6th column line filling.

For block 9, place C_{88} and C_{98} is reserved for 6&2 so we circle them and write 62 as shown. That means the remaining place is occupied by 1, 5, 3&7. That also we circle and write 1537 as shown.

Now fig 29b

With these arrangements, for column line 8, 157 cannot be accommodated at C_{89} so it is filled with 3_5

By 8th column line filling we get 9_6 at C_{85} and this gives 2_7 at circled C_{88} and 6_8 at C_{84}, This also allows us to write 6_9 at C_{98}

2nd row block to block scanning gives 2_{10} at C_{95}.

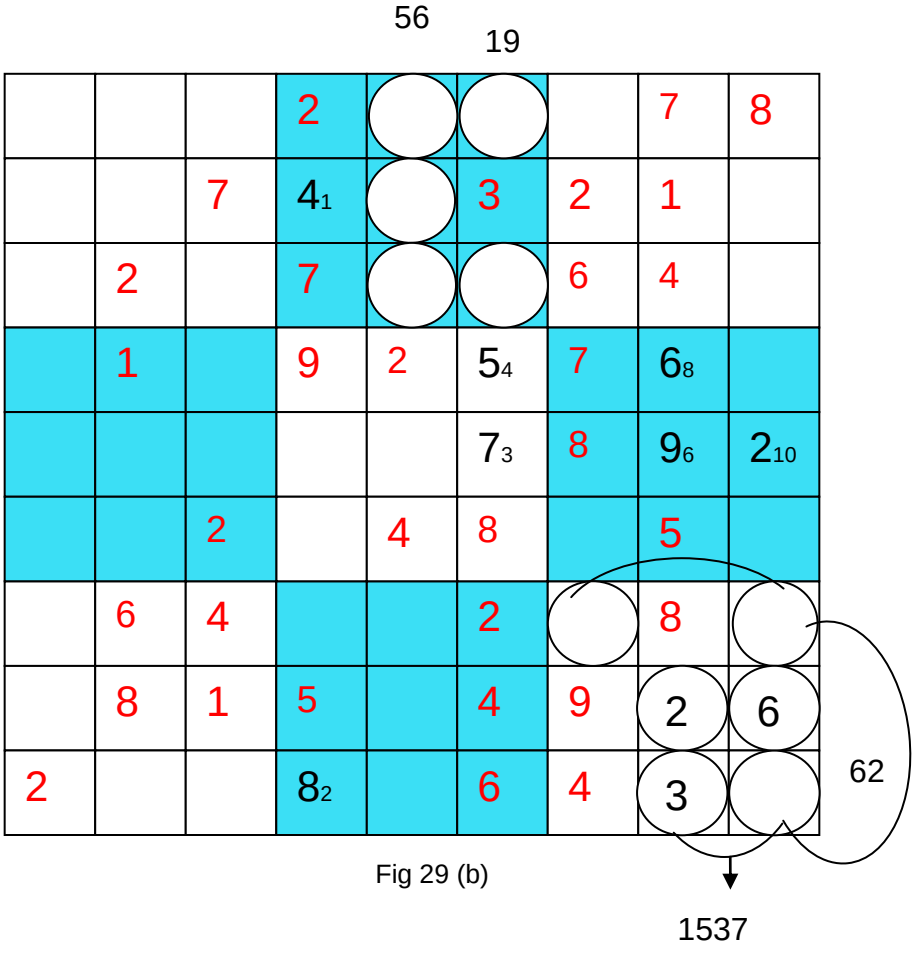


Fig 29 (b)

Now fig 29C

For 8th block filling we get 4_{11} at C_{94} this allows us to circle C_{76} and 96 for digit 1&3 and hence we circle them. This also allows us to reserve C_{16} and C_{26} for digits 9&7. We again circle them and write 97 on the right as shown.

This gives 6_{12} at C_{46} by logic for row line 6.

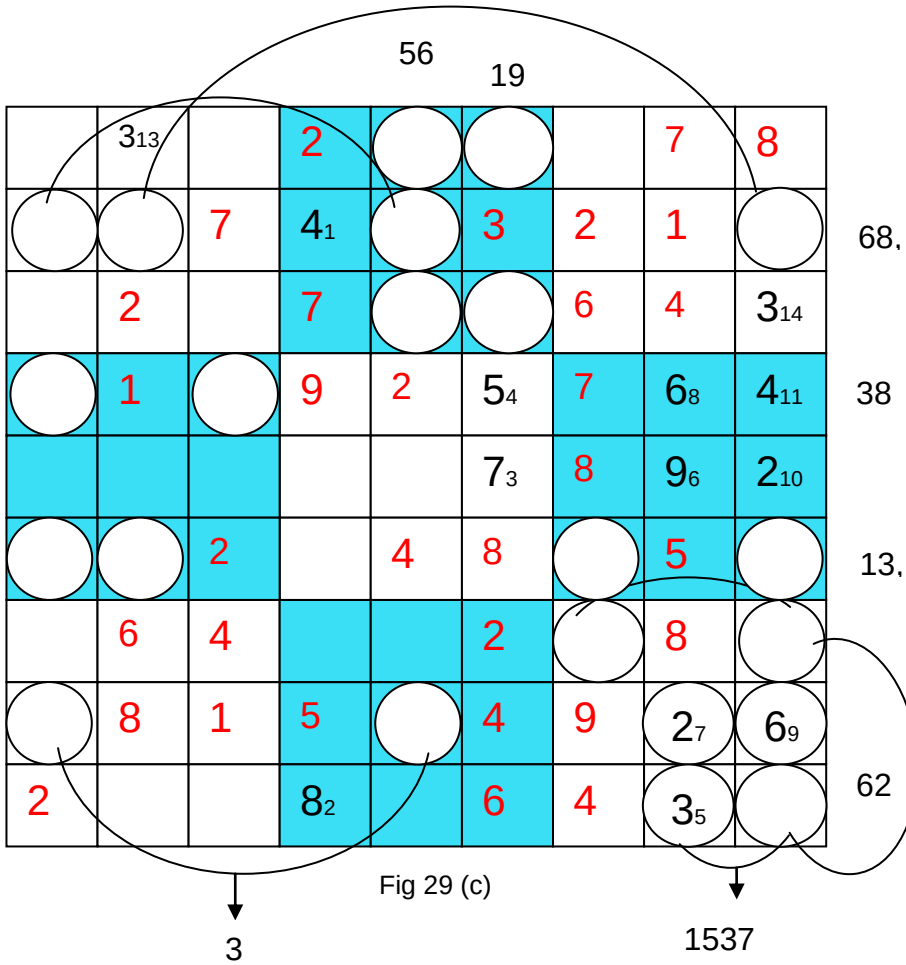


Fig 29 (c)

Now fig 29(d)

For row 2, digits 6 & 8 possible only on C_{12} and C_{52} because of the presence of 6 and 8 in block 7 and presence of 6&8 on column line 2

so we circle them and connect with an arch as shown and write 68. That means c_{22} and C_{92} are reserved for 5&9, we circle them also. Now C_{18} and C_{58} are also reserved for 3&7 for row line 8. We circle them also.C_{14} and C_{34} are also reserved for 3&8, we again circle them and write 38 on right side of the Table as shown.

Now for column line 2, we can write 3_{13} at C_{21}. This gives 3_{14} at C_{93} by 1st row bock to block scanning.

Now fig 29d

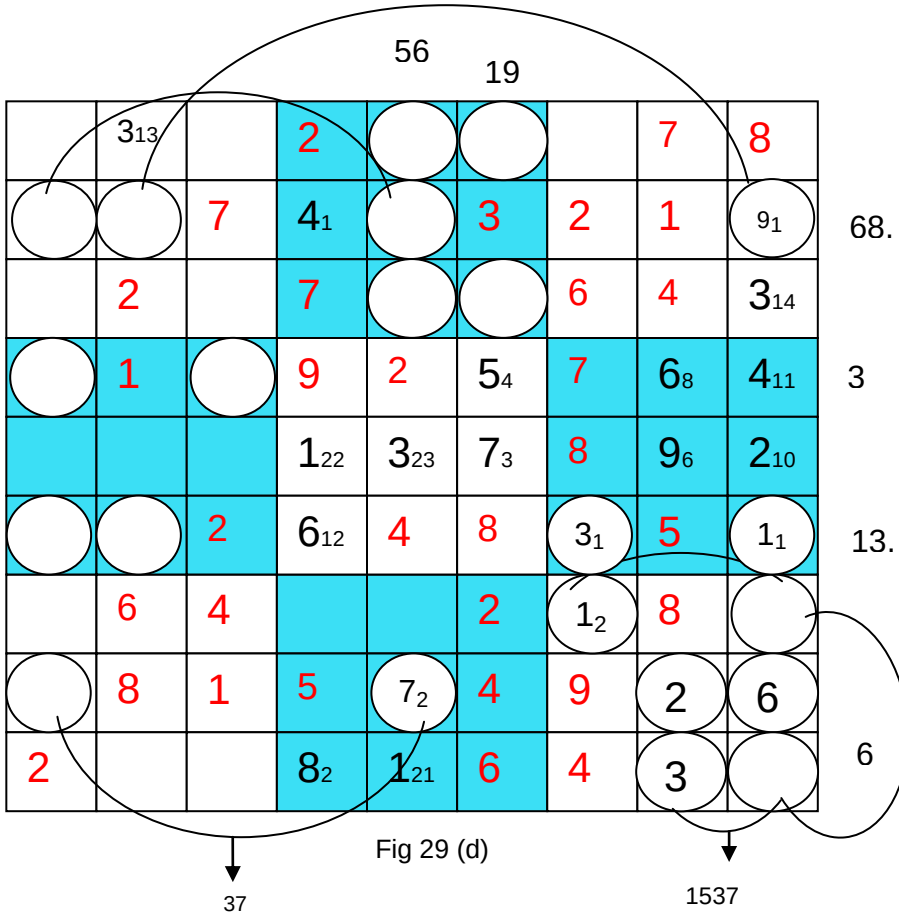


Fig 29 (d)

This give 9_{15} at C_{92} as 5&7 not possible at C_{92} and also 5_{16} at $C_{71}$for block 7 filling

This helps in getting 5_{17} at C_{22} for row line 2 filling.

This gives 3_{18} at C_{76} and 1_{19} at C_{96} for block 8 filling.

3rd column block to block scanning gives 120 at C_{77}.

We get 121 at C_{77} by 3rd block to block scanning.

3rd row block to block scanning gives 1_{21} at C_{59}.

This gives 1_{22} at C_{45} for block 5 filling and hence we get 3_{23} at C_{55}.

This gives 3_{24} at C_{47} and 9_{25} at C_{57} for block 6 filling. This also gives 7_{26} at C_{58} for block 6 filling.

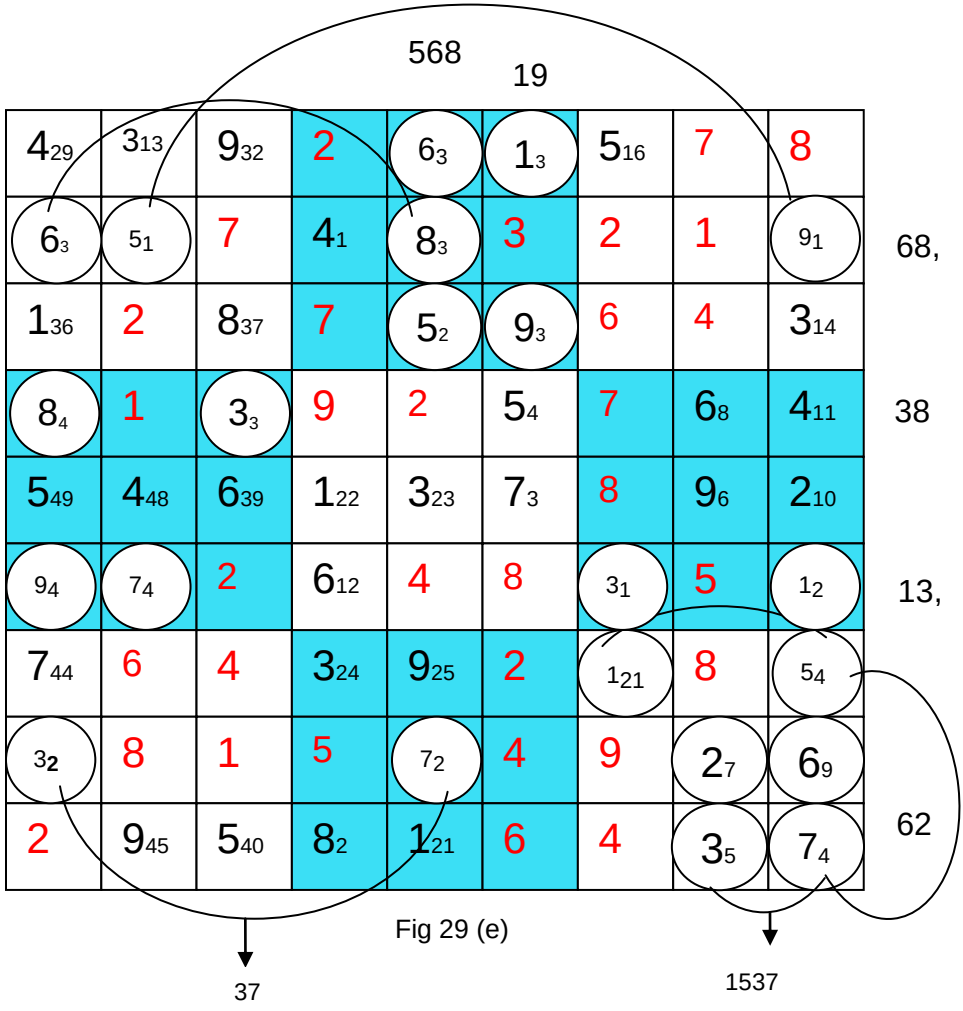


Fig 29 (e)

Now fig 29e

Now for row line 8, we get 3_{27} at C_{18}. By 3rd row line filling we get 5_{28} at C_{53}.

By 1st row line filling we get 4_{29} at C_{11}, 1_{30} at C_{61}, 6_{31} at C_{51} and 9_{32} at C_{31}.

Accordingly, we get 8_{33} at C_{52} and 9_{34} at C_{63}.

By 1st block filling we get 6_{35} at C_{12}, 1_{36} at C_{13} and 8_{37} at C_{33}.

For third column line filling we get 3_{38} at C_{34}, 6_{39} at C_{35} and 5_{40} at C_{39}.

This helps in filling 8_{41} at C_{14} and 7_{42} at C_{99}.

The vacancy at block 9 is filled with 5_{43} at C_{97}.

This gives 7_{44} at C_{17} for 7th row line filling.

This gives 9_{45} at c_{29} by 3rd block filling. For block 2, we fill 9_{46} at C_{16}, 7_{47} at C_{26}, 4_{48} at C_{25} and 5_{49} at C_{15}.

SUDOKU 8 (Self Practice)

				1	5	7		4
5			4			2	3	1
1	8							
			7			4		
		1	5		4	6		
		6			1	3		
			1				6	7
6	1	8			7			3
7		3	6	5				

Fig 30

Solved Sudoku 8

59

3_{7}	6_{6}	2_{8}	9_{17}	1	5	7	8_{2}	4	
5	9_{9}	7_{5}	4	8_{18}	6_{1}	2	3	1	68
1	8	4_{3}	3_{13}	7_{4}	2_{14}	9_{4}	5_{4}	6_{1}	59
8_{36}	3_{31}	5_{2}	7	6_{20}		4	1_{30}	2_{35}	
9_{40}	2_{34}	1	5	3_{26}	4	6	7_{38}	8_{39}	
4_{41}	7_{37}	6	8_{15}	2_{27}	1	3	9_{52}	5_{51}	
2_{33}	5_{32}	9_{2}	1	4_{23}	3_{24}	8_{11}	6	7	
6	1	8	2_{16}	9_{22}	7	5_{4}	4_{43}	3	459
7	4_{21}	3	6	5	8_{12}	1_{10}	2_{42}	9_{44}	249

Fig 30 (a)

Hints –

4_{21} is obtained from 9th row line filling. At C_{29}, 9 not possible because of presence of 9 at C_{22}. Here 2 is also not possible because of presence of 2 at C_{48}, This makes no place for 2 in the 9th block 7th row. So 2&9 has to be in 9th row and that to in 9th block. So 4_{21} is placed at C_{29}.

Now again 2&9 are in the 9th block and in 9th row so 9_{22} is placed at C58 as for 8th row only 922 at c58 only satisfy the criteria of that all digit 1 to 9 should appear in a row or column.

SUDOKU 9

7			3		2			
			8	4				
9		6	7			8		
	1		2	3	8			7
2			9		1			3
5			6	7		2	8	
		5		2	7	3		8
			5	8				
			1		3			2

Fig 31

Solved Sudoku 9

23

7	4_{19}	8_{21}	3	9_{31}	2	6_{48}	1_{47}	5_{34}	
3_{11}	5_{22}	1_{20}	8	4	6_{32}	7_{6}	2_{1}	9_{33}	
9	2_{15}	6	7	1_{17}	5_{5}	8	3_{1}	4_{18}	
6_{7}	1	4_{8}	2	3	8	9_{41}	5_{42}	7	59
2	8_{23}	7_{2}	9	5_{3}	1	4_{4}	6_{4}	3	87,46
5	9_{26}	3_{2}	6	7	4_{2}	2	8	1_{4}	93
1_{12}	6_{36}	5	4_{1}	2	7	3	9_{40}	8	
4_{10}	3_{25}	2_{16}	5	8	9_{29}	1_{39}	7_{39}	6_{35}	
8_{9}	7_{37}	9_{28}	1	6_{30}	3	5_{43}	4_{44}	2	

Fig 31 (a)

We get 4_1 at C_{47} by 4th column line filling.

This facilitates 4_2 at C_{66} by 2nd column block to block scanning and hence 5_3 at c_{55} by block 5 filling.

We get 1_4 at C_{96} by 2nd row block to block scanning.

9, 6 and 1 cannot happen in C_{63} hence only 5_5 can go there for 6th column line scanning because of presence of 9, 6 and 1 at C_{13}, C_{33} and C_{65}.

Examining block 7, we find that 2&3 can happen only in C_{82} and C_{83} because of 2&3 at C_{41}, C_{61} and C_{66} and C_{67} so we circle 2& 3 at C_{82} and C_{83}.

This gives 7_6 at C_{72} for block 7 filling.

5 and 9 cannot happen in C_{75} and C_{85} hence it can only happen in C_{74} and C_{84} hence we circle both. Also we circle C_{75} and C_{85} for 4 and 6.

This gives 6_7 at C_{14} and 4_8 at C_{34} by 4th row line scanning. We get 8_9 at C_{19} by 1st column line scanning hence 4_{10} at C_{18}, 3_{11} at C_{12} and 1_{12} at C_{17}.
This gives 3_{13} at C_{83} and 2_{14} at C_{82}, 2_{15} to C_{23} by 1st block to block scanning and 2_{16} to C_{38} by 1st column block to block scanning.

Now we get 1_{17} and 4_{18} at C_{53} and at C_{93} respectively by third row line scanning. This enables 4_{19}, 1_{20}, 8_{21} and 5_{22} by first block filling.

Hence we get 8_{23}, 7_{24} at C_{25} and at C_{35} by 1st column block to block scanning. We get 3_{25} by 3rd row block to block scanning at C_{28}.

Also, we get 9_{26} and 3_{27} by first column block to block scanning at C_{26} and C_{36}.

This gives 9_{28} at C_{39} by 3rd column line scanning and 9_{29} at C_{68} by 6th block filling,

Further, we get 6_{30} at C_{59} by 6th block filling.

This gives 9_{31} and 6_{32} by 4th block filling,

This helps us in getting 9_{33}, 5_{34} and 6_{35} at C_{92}, C_{91} and C_{98} respectively by 9th column line filling.

Then, we 6_{36} and 7_{37} by 3rd block filling.

This helps in filling 7_{38},1_{39} and 9_{40} by 9th block filling.

This way we can fill 9_{41} at C_{74} only hence we get 5_{42} at C_{84}.

This enables us to fill 5_{43} and 4_{44} for 9th block filling.

So we get 4_{45} and 6_{46} at C_{75} and C_{85} by 3rd column block to block scanning.

Now, we get 1_{47} and 6_{48} by 7th block filling. And that completes Sudoku.

SUDOKU 10 – Solved

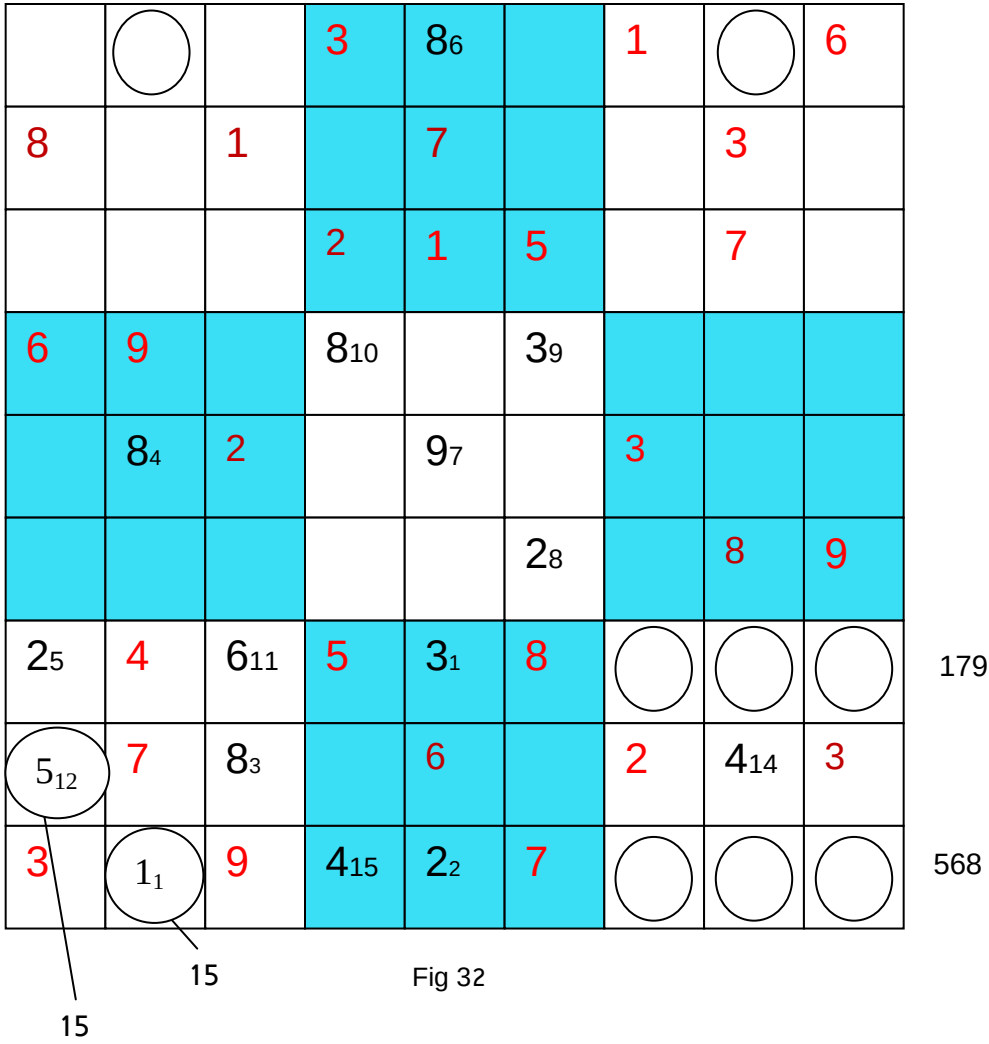


Fig 32

3_1 at C_{57} by 3rd block to block m scanning and 2_2 at C_{59} by 6th block filling.

8_3 by 3rd block filling at C_{38} or 3rd column line filling with presence of 8 in block 1 and at C_{15} or C_{25} and at C_{67}.

8_4 at C_{25} by 1st block to block vertical column scanning.

2_5 at C_{17} by 3rd block filling or 2 not possible in C_{37}, C_{77}, C_{87} and C_{97}.

86 at C_{51} by 6th block filling
9_7 by 5th vertical line filling at C_{55}. 2_8 at C_{66} by 6th row line filling.
3_9 by 5th block filling at C_{64}. 8_{10} by 5th block filling at C_{45}, 1 & 5 not possible in C_{37} because of the presence of 1 at C_{32} and 5 at C_{47} so. 6_{11} by 3rd block filling at C_{37} Now 1&5 are earmarked at C_{18} and C_{29} and they are circled. This make C_{77}, C_{87} circled for 1, 7 &9..
Row 8, block 6 contains 5 so 5_{12} at C_{18}. This gives 1_{13} at C_{29}.
1,7,9,5,6 and 8 not possible in block 6 for row 8 by 8th row scanning, this gives 5,6 and 8 in the row 9 of block 9 and hence circled and this gives at 4_{14} at C_{88} and hence 4_{15} at C_{49} by 3rd block horizontal scanning.

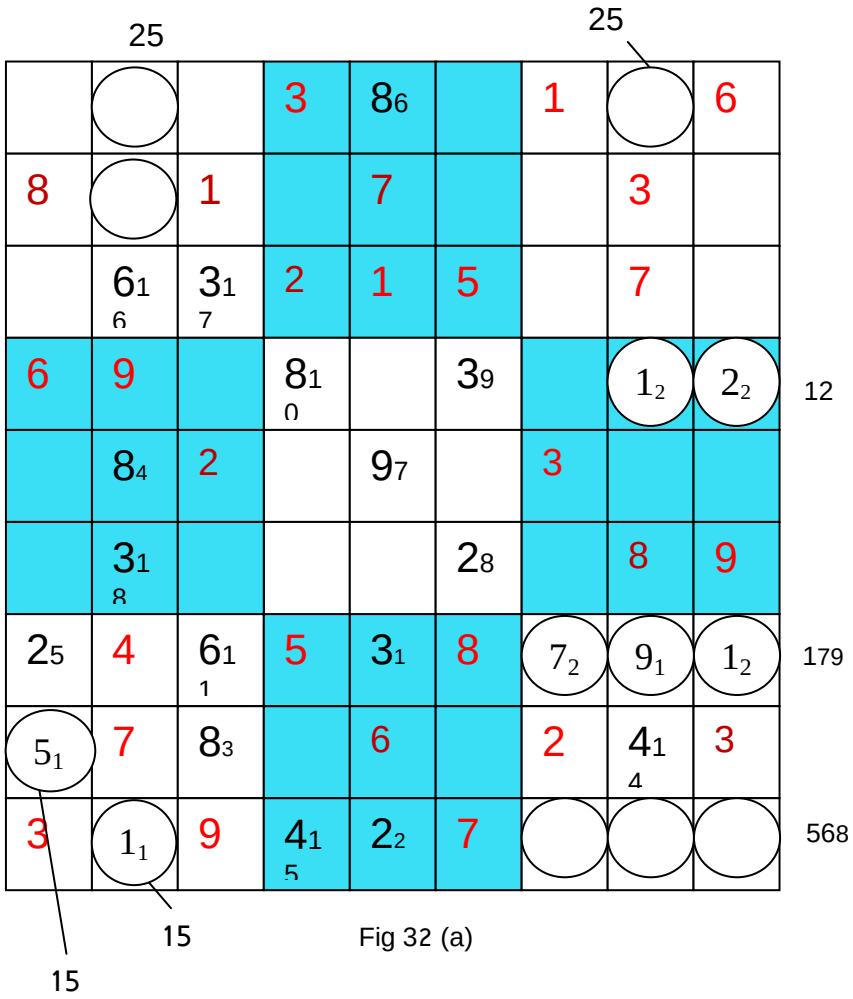


Fig 32 (a)

Now fig 32(a)

C_{21} and C_{23} is circled for 2&5.
Now since 6 is contained in block 3 row 7 and also by 1st vertical column block scanning we get 6_{16} at C_{23}.

This gives 3_{17} by 1st column block scanning and also 3_{18} by 1st vertical column block scanning.
2, 5 will occupy C_{21} and C_{22} from 2nd vertical column scanning. This gives 2 or 5 to occupy at C_{81} by first row line scanning. If that is the case then in blocks 7, 9 will occupy either at C_{72} or at C_{73}. If that is the case then 9_{19} will occupy C_{87} by 3rd column block to block scanning.
This gives 7_{21} and 1_{20} at C_{77} and C_{97} respectively.
For row 4 line 1, 2 is possible only in C_{84} and C_{94} hence circle. This gives 1_{22} and 2_{23} at C_{84} and C_{94}

Now Fig 32(b)

By third vertical block scanning, we get 224 at C81 this gives 525 and 226 at C21 and C22 respectively. From 4th row line scanning we get 727 at C34, this gives 728 at C11.by 1st column block to block scanning.
From 1st row scanning, we get 929 and 430 at C34 and C34 respectively.

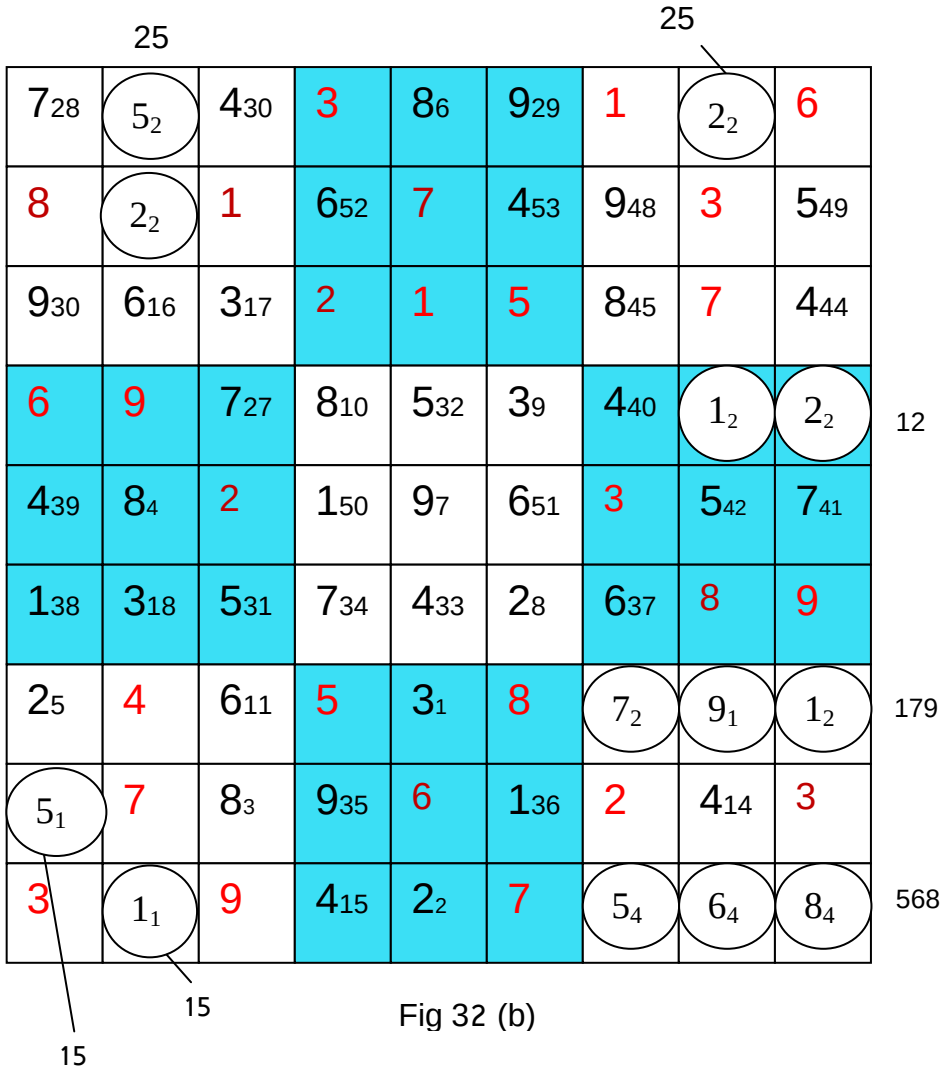


Fig 32 (b)

9_{30} at C_{34} is obtained by 1st block completion. If it is 1, 4 at C_{15} and C_{16} then by 1st vertical column line scanning the we get 5_{31} at C_{36}. This gives 5_{32} at C_{54} by 5th column line scanning and hence 4_{33} at C_{56}.

By 6th row line scanning, we get 7$_{34}$ at C$_{46}$. 9$_{35}$ possible in C$_{48}$ and hence1$_{36}$ at C$_{68}$ by 6th block filling.

We get 6$_{37}$ by 6th row line scanning at C$_{76}$ and 1$_{38}$ at C$_{16}$. Now with 2nd block filling 4$_{39}$ is obtained at C$_{15}$.

This gives 4$_{40}$ at C$_{74}$. Now by 8th block filling we get 7$_{41}$ and 5$_{42}$ at C$_{95}$ and C$_{85}$ respectively.

3rd vertical block scanning gives 6$_{43}$ at C89. 3rd row line scanning gives 4$_{44}$ at C$_{93}$ and 8$_{45}$ at C$_{73}$. This helps in filling 8$_{46}$ at C$_{99}$ and 5$_{47}$ at C$_{79}$ in block 9. 9$_{48}$ and 5$_{49}$ are obtained by 7th block filling. 1$_{50}$ and 6$_{51}$ are obtained by 5th block filling. 6$_{52}$ and 4$_{53}$ are obtained by 4th block filling.

SUDOKU 11 - Solved

31 by 2nd row block scanning and 32 by 1st vertical column block scanning at C26 and C33 respectively.
33 by 3rd and row block scanning at C47. 84 by 2nd column block scanning at C49.

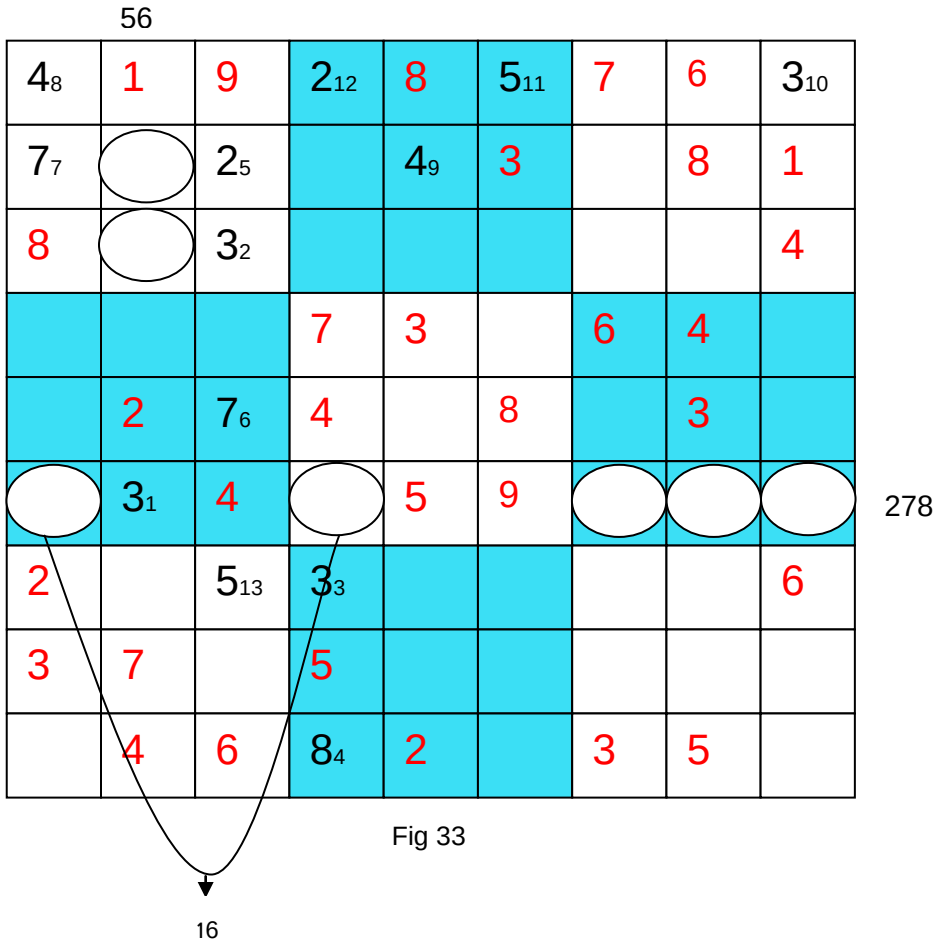


Fig 33

2_5 at C_{32} by 1st column block scanning. 7_6 by 3rd column line filling at C_{35} and this gives 7_7at C_{12}. We get 4_8 at C_{11} by 1st block filling. This gives 4_9 at C_{52} by 1st row block scanning.

3_{10} by 1st row block scanning at C_{91}. 5_{11} at C_{61} and 2_{12} at C_{41} by 1st row line scanning.

Now with this C_{22} and C_{23} are occupied by 56 hence it is circled. Now for block 3, 5_{13} can happen only in C_{36} because of presence of 5 in C_{48}, C_{89} and in column 2. C_{76}, C_{86} and C_{96} will house 278 for row line 6, so they are circled. This makes 16 to slot in C_{16} and C_{46} and hence they are circled.

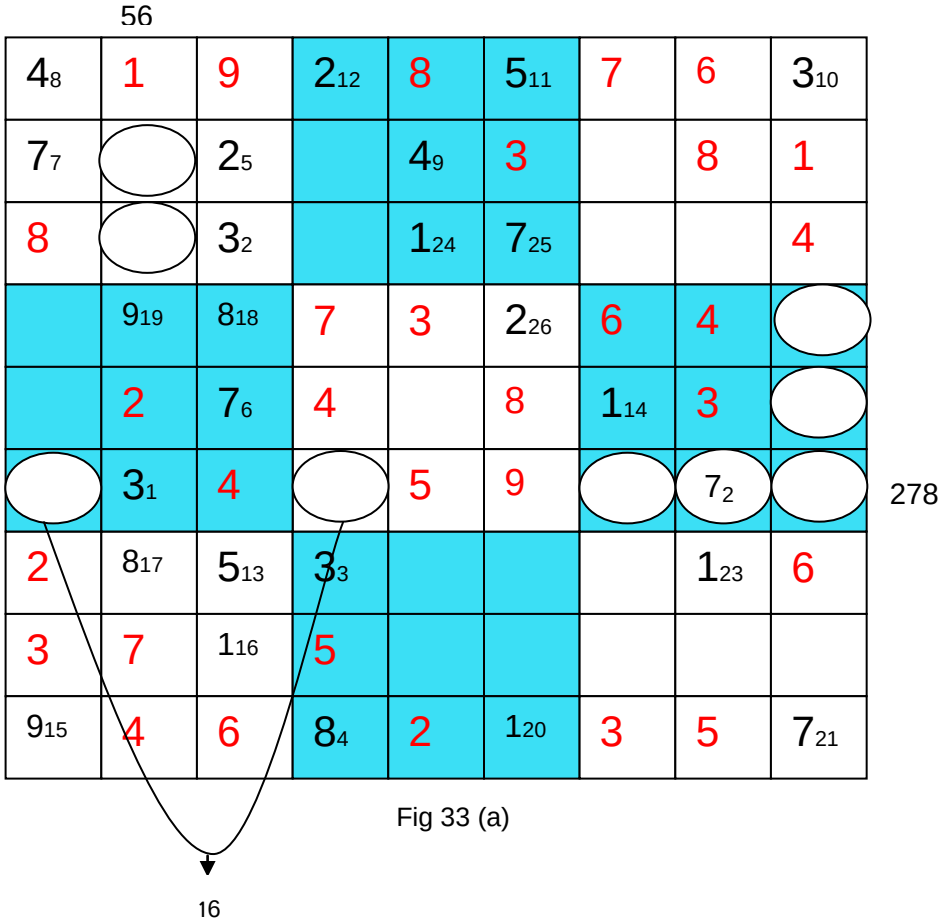


Fig 33 (a)

Now with this arrangement and with presence of 1 at C_{92}, 1_{14} can happen only in C_{75} for block 8. For row block 6, C_{94} and C_{95}will house 5&9 hence circled. For block 3, 9_{15} can happen only in C_{19}. This gives 1_{16} and 8_{17} at C_{38} and C_{27} for block 3. This gives 8_{18} at C_{34} for vertical line 3 scanning. Thus 9_{19} is obtained at C_{24} for block 2 and presence of 1 at C_{92} gives 1_{20} at C_{69} by 9th row line scanning and 7_{21} at C_{99} for row line 9. This gives 7_{22} at C_{86} by 3rd vertical block scanning. This gives 1_{23} at C_{87} by filling of block 9. 1_{24} is obtained at C_{53} by 5th vertical line scanning. 7_{25} are obtained at C_{63} by 1st row block to block scanning. 2_{26} is gotten at C_{64} by 6th vertical line scanning. This gives 1_{27} and 6_{29} for 2nd row block scanning at C_{46} and C_{55} respectively. This also gives 6_{28} at C_{16} for 6th row line scanning. Presence of 9_{19} at C_{24} gives 5_{30} and 9_{31} at C_{94} and C_{95} for column line 9 scanning. This helps in filling.

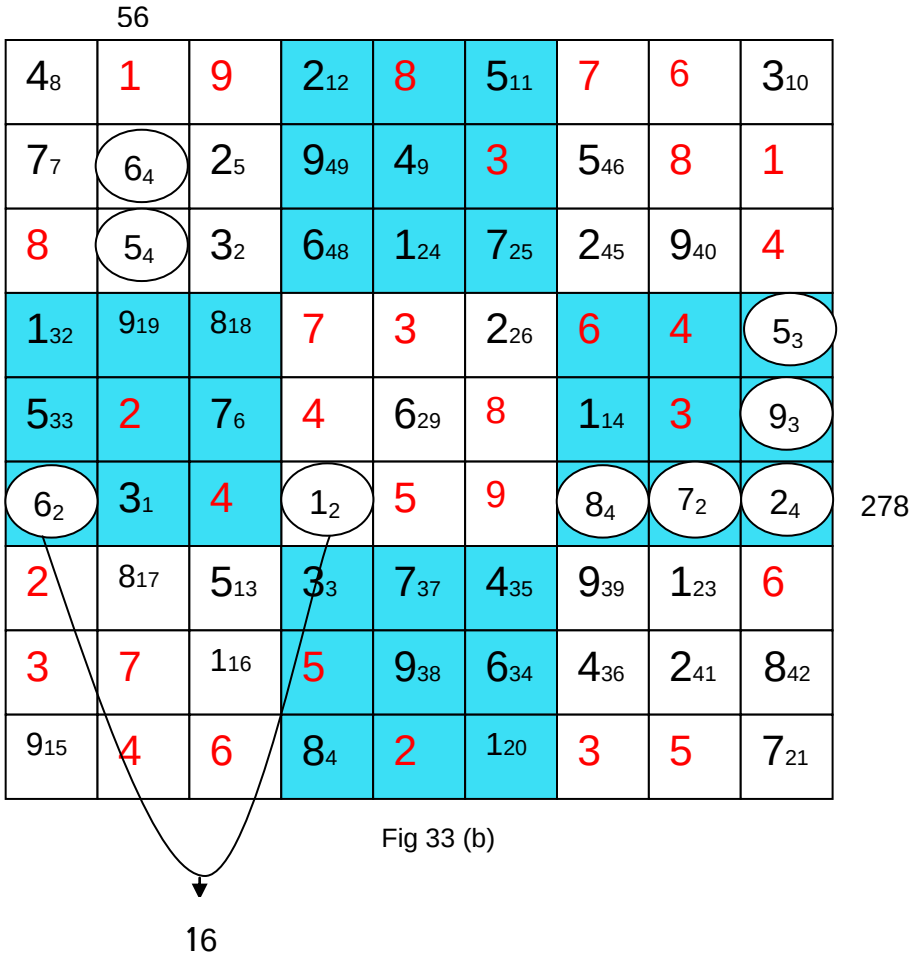


Fig 33 (b)

1_{32} and 5_{33} at C_{14} and C_{15} for 2nd block scanning. 6_{34} is obtained by 6th block filling at C_{68}. Now 4_{35} is obtained by 6th vertical line scanning at C_{67}. 4_{36} is obtained by 3rd row block to block scanning at C_{78} and 3rd row block to block filling at C_{57} and C_{58} gives 7_{37} and 9_{38} respectively. 9_{39} by 7th row line filling. 9_{40} is obtained by 3rd vertical block scanning at C_{83}. This helps in filling 2_{41} at C_{88}

by 8^{th} vertical line scanning and 8_{42} at C_{98} by 9^{th} block filling. This gives 2_{43} and 8_{44} at C_{96} and C_{76} respectively for row line 6 filling. Now it is easy to fill 2_{45},5_{46},5_{46}, 6_{47} and 5_{48}

SUDOKU 12 - Solved

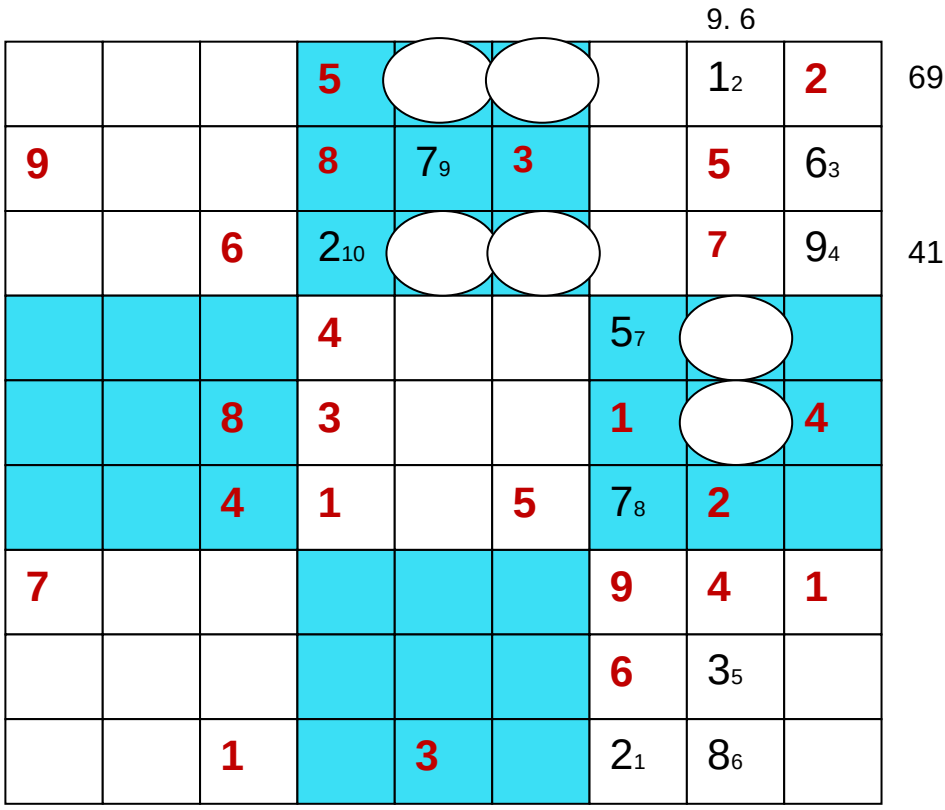


Fig 34

2_1 and 1_2 by third vertical block scanning.

$6_{3\text{ and }}9_4$ by 3rd column block to block scanning. This gives 9&6 at C_{84} and C_{85} so they are circled. So for 8th vertical line scanning, we place at 3_5 at C_{88} and 8_6 at C_{89}. 5_7 is placed at C_{74} by 7th column block scanning because of presence of 5 at 7th block and at C_{66}

7_8 is obtained at C_{73} by 7th vertical line scanning. 1st row block to block scanning gives 6&9 at C_{51} and C_{61} hence circled. This gives 7_9 at C_{52} by 4th block filling.

Now examination of block 4 gives 4, 1 at C_{53} and C_{63} hence circled. This gives 2_{10} at C_{43} by 4th block scanning.

Now refer fig 28a.

This gives 1_{11} at C_{22} by 1st row block to block scanning. Now 7and 9 not possible in C_{47} hence 7 and 9 possible in C_{48} and C_{49} and is circled. This gives 6_{12} at C_{47}. Now, we get 1_{13} at C_{14} by 1st vertical block to block scanning. This gives 2_{14} by 1st block filling. 4_{15} is obtained by 2nd row line scanning at C_{72}.

Now in vertical line 3 ,5 is occupied either at C_{37} or C_{38} so 5 cannot happen in C_{19} and C_{29} also 5 is at C_{66} , this gives 5_{16} at C_{99} for 9th row line filling. By 9th block scanning we get 7_{17} at C_{98}. This helps in getting 7_{18} at C_{49} and 9_{19} at C_{48}.

9, 6

			5				1_{2}	2	69
9	1_{11}	2_{14}	8	7_{9}	3	4_{15}	5	6_{3}	
		6	2_{10}				7	9_{4}	41
1_{13}			4			5_{7}			
		8	3			1		4	
		4	1		5	7_{8}	2		
7			6_{12}	5_{26}		9	4	1	
		5_{27}	9_{19}	1_{25}		6	3_{5}	7_{17}	
6_{24}	9_{23}	1	7_{18}	3	4_{20}	2_{1}	8_{6}	5_{16}	

Fig 34 (a)

Now 1,2,8,5 cannot happen in C_{69} in block 6 so 4_{20} is placed there. This gives 4_{21} and 1_{22} at C_{53} and C_{63} respectively.

Now in fig 28b. This gives 9_{23} and 6_{24} at C_{29} and C_{19} respectively by 9th row line scanning. 1_{25} at C_{58} by 3rd row block to block scanning then this helps in getting 5_{26} at C_{77} by 2nd vertical block to block scanning. This gives 5_{27} at C_{38} by 3rd vertical line scanning. First we get 9_{23} then 7_{24} and 3_{25} at C_{34}, C_{31} and C_{37} respectively

by 3rd vertical line scanning. 9_{23} give 9_{26} and 6_{27} at C_{85} and C_{84} respectively. 9_{28} is obtained at C_{56} by 2nd row block to block scanning. This gives 9_{29} and 6_{30} at C_{61} and C_{51} respectively. 6_{31} is obtained at C_{26} by 6th row line scanning and 6_{32} is gotten by 2nd row block to block scanning at C_{65}.

						9 6			
		7_{24}	5	6_3	9_2		1_2	2	69
9	1_{11}	2_{14}	8	7_9	3	4_{15}	5	6_3	
		6	2_{10}	4_2	1_2		7	9_4	41
1_{13}			4	8_{35}	7_{33}	5_7	6_2		
	7_{34}	8	3	2_{36}	6_{32}	1	9_2	4	
	6_{31}	4	1	9_{28}	5	7_8	2		
7			6_{12}	5_{26}		9	4	1	
		5_{27}	9_1	1_{25}		6	3_5	7_{17}	
6_{24}	9_{23}	1	7_1	3	4_{20}	2_1	8_6	5_{16}	

Fig 34 (b)

7_{33} is obtained by 6th vertical line scanning at C_{64}. 7_{34} is gotten by 2nd row block to block scanning at C_{25}. 8_{35} and 2_{36} are obtained by 5th block filling at C_{54} and C_{56} respectively.

Now, it is easy to fill rest of cells and complete the Sudoku.

SUDOKU 13

1			5		6			
2	8	1						
		7						9
5		3	2					4
		2				1		
7						6	2	3
9						8	4	
						3	9	7
			4		9			1

Fig 35

Solution (refer fig 35 (a)

169

1	4_8	9_7	5		6			
2	8	1						
6_3	5_2	7						9
5		3	2			9_4		4
4_5		2				1		
7		8_6				6	2	3
9						8	4	
1_1	2_{11}	4_{10}				3	9	7
8_9			4		9			1

45

56

237

Fig 35 (a)

1_1 by 1st vertical line scanning at C_{18}.

5_2 and 6_3 by 1st block filling at C_{23} and C_{13}. 9_4 by 3rd vertical block to block scanning at C_{74}. 4_5 at C_{18} by 1st vertical line scanning. 8_6 at C_{18} by 2nd block filling. The position for 169 is circled in Block 3 filling and te positions of 49 are circled in 1st block filling. This gives 9_7 at C_{31} and hence 4_8 at C_{21}. 8_9 at C_{19} by 1st vertical line

filling at C_{19}. For 2nd vertical line the position of 237 are circled at C_{27}, C_{28} and C_{29}.

Now 3rd row block to block scanning gives 4_{10} at C_{38}. For 8th row line 2_{11} is placed at C_{28} as 3 and 7 are not possible there. This allows positioning of 5_{68} at C_{48}, C_{58} and C_{68} respectively for 8th row line filling hence circled. Now for 6th row line, 45 not possible in C_{26} and at C_{46} so 45 is positioned at C_{56} and C_{66} and hence circled.

169

1	4_8	9_7	5		6	7_{22}		2_{13}	
2	8	1	9_{30}	4_{26}	7_{29}	5_{20}	3_{25}	6_{19}	479
6_3	5_2	7				4_{21}		9	
5	1_{33}	3	2	6_{35}	8_{36}	9_4	7_{16}	4	
4_5	6_{34}	2				1	5_{15}	8_{14}	376
7	9_{32}	8_6	1_{31}	5_{28}	4_{27}	6	2	3	45
9		6_{23}				8	4	5_{18}	
1$_1$	2_{11}	4_{10}			5_{31}	3	9	7	56
8_9		5_{24}	4		9	2_{12}	6_{17}	1	

237

Fig 35 (b)

Now study of 9th block reveals that 56 possible only in C_{97} and at C_{37} so 2_{12} is placed at C_{79} by 9th block filling. This gives 2_{13} at C_{91} by 7th block filling. This gives 8_{14} at C_{95} by 9th vertical line filling. This helps in filling 5_{15} and 7_{16} at C_{85} and C_{84} by 8th block filling. This helps in filling 6_{17} and 5_{18} at C_{89} and C_{97} respectively by 9th block filling. This gives 6_{19} at C_{92} by 9th vertical line filling. We get 5_{20} by 7th block filling at C_{72}. Now we get 4_{21} at C_{73} and 7_{22} at C_{71} by 7th block filling.

By 3rd block filling, we get 6_{23} and 5_{24} at C_{37} and at C_{39} respectively. 18 not possible in at C_{82} so 3_{25} is filled for 7th block. For vertical line 5th, 4 is possible only in C_{52} and at C_{56} but C_{56} also may contain 5, hence intuitively 4_{26} is placed at C_{52}.Also by filling 2nd row line 4_{26} is placed at $C_{52.}$ This gives 4_{27} and 5_{28} at C_{66} and C_{56} respectively. By this we get 7_{29} and 9_{30} at C_{62} and C_{42} respectively. By 6th row line filling we get 1_{31} and 9_{32} at C_{46} and C_{26} respectively. This facilitates in filling 1_{33} and 6_{34} at C_{24} and C_{25} respectively.

169

1	4_{8}	9_{7}	5	1_{50}	6	7_{22}	8_{51}	2_{13}	
2	8	1	9_{30}	4_{26}	7_{29}	5_{20}	3_{25}	6_{19}	479
6_{3}	5_{2}	7	8_{39}	3_{43}	2_{47}	4_{21}	1_{52}	9	
5	1_{33}	3	2	6_{35}	8_{36}	9_{4}	7_{16}	4	
4_{5}	6_{34}	2	7_{41}	9_{40}	3_{42}	1	5_{15}	8_{14}	376
7	9_{32}	8_{6}	1_{31}	5_{28}	4_{27}	6	2	3	45
9	7_{44}	6_{23}	3_{43}	7_{49}	1_{48}	8	4	5_{18}	
1	2_{11}	4_{10}	6_{37}	8_{38}	5_{31}	3	9	7	56
8_{9}	3_{45}	5_{24}	4	2_{46}	9	2_{12}	6_{17}	1	

237

Fig 35 (c)

This helps in filling 6_{35} and 8_{36} at C_{54} and C_{64} respectively by 4th row line filling. This helps in filling 6_{37} and 8_{38} at C_{48} and C_{58} respectively. Now by 2nd vertical block to block scanning gives 8_{39} at C_{43}. 9_{40} is obtained at C_{55} by 5th vertical line filling. We get 7_{41} and 3_{42} by 5th row line filling at C_{45} and C_{65} respectively. 3_{43} by 4th vertical line filling at C_{47}. This helps in getting 7_{44} and 3_{45} at C_{27} and C_{29} in block 3. This way 2_{46}, 2_{47},1_{48},7_{49},1_{50}, 8_{51} and 1_{52} are filled to complete the Sudoku.

SUDOKU 14 – Solved

3	4	5		8_{14}	6	2		
7		9_4	4	2			8	5
8_3		2	5		9_{17}			4
5_9	3_{13}	6			7		4	2
2_5	7_2			5_{18}	4_{15}			
4	9		2		8_{16}	5		
6	5_{11}				2	1		8_1
1_{10}	8				5		2_7	9
9_8	2_6	7	8			4	5_{12}	6

Fig 36

8_1 by 3rd row block to block scanning at C_{97}. 7_2 by 2nd vertical line scanning at C_{25}.

8_3 by 1st block filling at C_{13}. 2_5 by 2nd block filling at C_{15}. 2_6 by 1st vertical block to block scanning at C_{29}. 2_7 at C_{78} by 3rd row block to block scanning. 9_8 at C_{19} by 1st vertical block to block scanning.5_9 and 1_{10} by 1st vertical line filling at C_{14} and C_{18} respectively. 5_{11} and 5_{12} by 3rd row line block to block scanning at C_{27} and C_{89} respectively. For block 3, 3 has to be either at C_{37} or at C_{38}, this helps in filling 3_{13} at C_{24} also C_{37} and C_{38} is circled for

34, 8_{14} at C_{51} by 4th block filling. 4_{15}, 8_{16} and 9_{17} by 6th vertical line filling at C_{65}, C_{66} and C_{63} respectively. 5_{18} by 2nd row block to block scanning at C_{55}.

Now fig 36(a

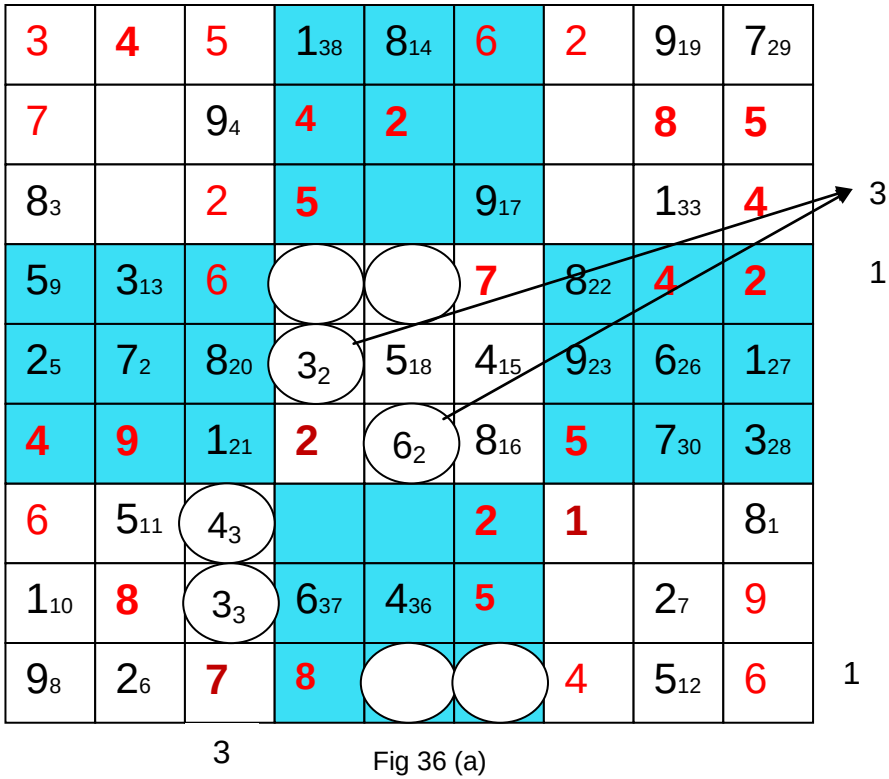


Fig 36 (a)

9_{19} by 7th block filling at C_{81}. 8_{20} and 1_{21} by 3rd block filling at C_{35} and C_{36} because of presence of 8_{16} at C_{66}. 8_{22} and 9_{23} by 7th vertical line filling at C_{74} and C_{75} respectively. 5th bock is circled for 19 in 4th row line and 36 at C_{52} and C_{52} respectively. 6th block also circled for 13 at C_{59} and C_{69}.Now for 4th vertical line, 3_{24} only possible at C_{45} because of presence of 3 at C_{11} and in block 6.

This gives 6_{25} and 6_{26} by 2nd row block to block scanning at C_{56} and C_{85}. 1_{27} by 5th row line filling at C_{95}. 3_{28} and 7_{29} by 9th vertical line scanning at C_{52} and C_{52} respectively. This helps in getting 7_{30} and 7_{31} by 3rd vertical block to block scanning at C_{86} and C_{78} respectively.

Now Fig 36(b)

We get 3_{32} and 1_{33} by 8th vertical line filling at C_{87} and C_{83} respectively. Thus, 4_{34} and 3_{35} are obtained at C_{37} and C_{38}. Scanning of 8th row line gives 4_{36} and 6_{37} at C_{68} and C_{58}. 1_{38} by 1st row line scanning at C_{41}.

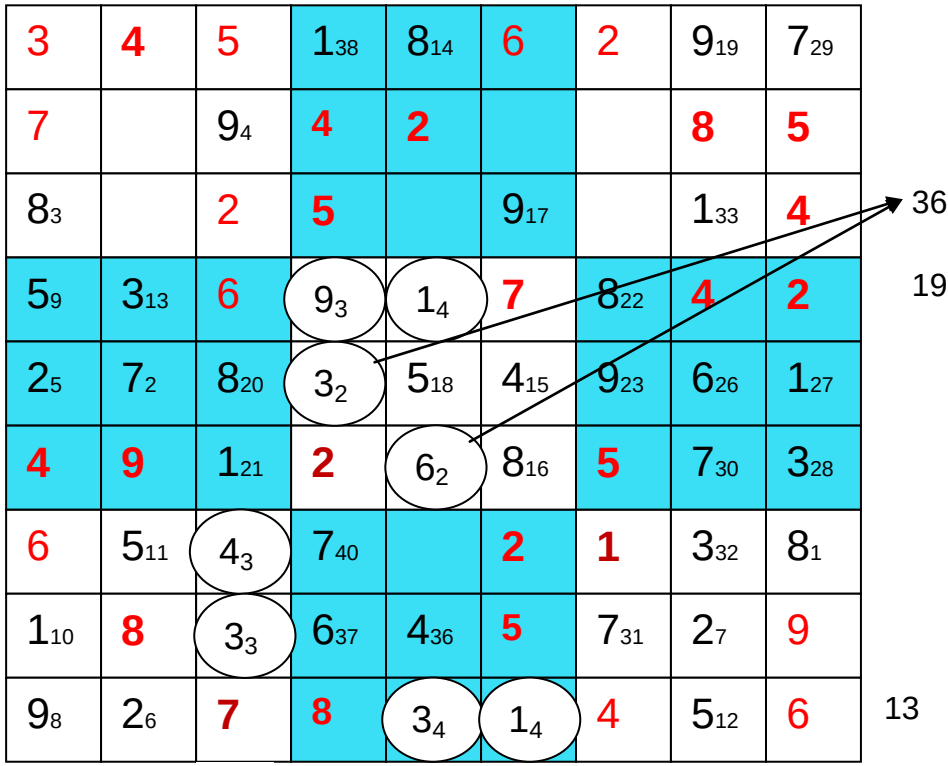


Fig 36 (b)

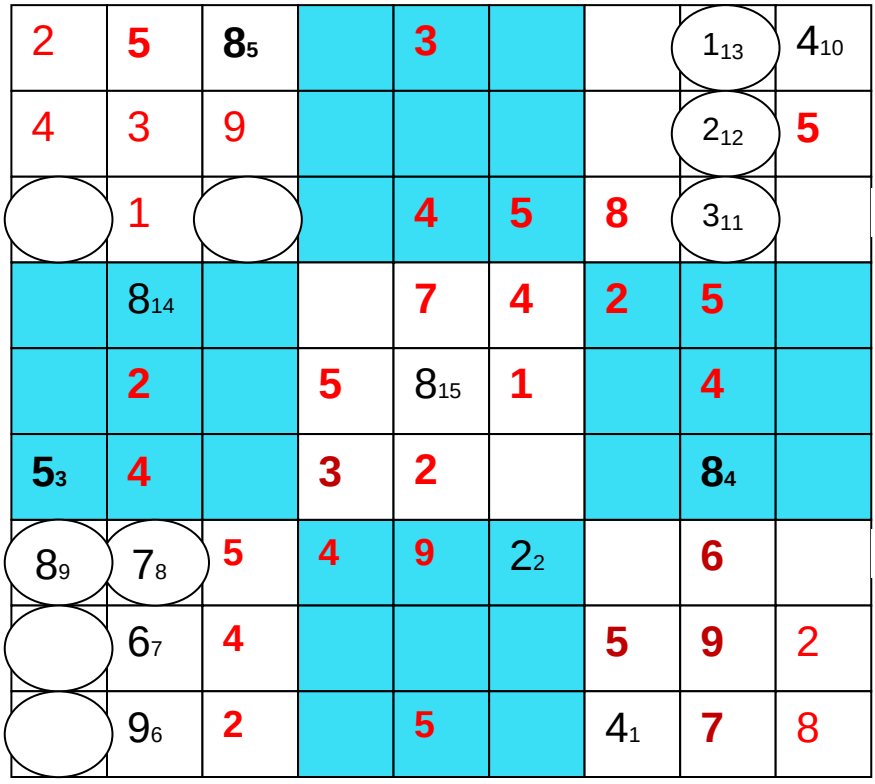
This gives 9_{39} and 7_{40} by 4th vertical line scanning at C_{44} and C_{47}. 1_{41} is obtained by 4th row line scanning at C_{54}. Now it is easy to complete the SUDOKU.

SUDOKU 15

2	5			3				
4	3	9						5
	1			4	5	8		
				7	4	2	5	
	2		5		1		4	
	4		3	2				
		5	4	9			6	
		4				5	9	2
		2		5			7	8

Fig 37

123



13

Fig 37 (a)

Solution - refer Fig 37 (a)

4_1 and 2_2 by 3rd row block to block scanning at C_{79} and C_{67} respectively. $\mathbf{5_3}$ **by** 1st vertical block to block scanning and $\mathbf{8_4}$ by 3rd vertical block to block scanning at C_{16} and C_{86} respectively. $\mathbf{8_5}$ **by** 1st block filling at C_{31} because of presence of 8 at C73. By inspection of 3rd block, we see that C_{17} and C_{27} are reserved for 78 by 3rd row block to block scanning so it is circled.

For 3rd block, we find that C_{18} and C_{19} contains 13 because of 1st column line filing and hence circled so by 3rd row block to block scanning we place 9_6 at C_{29} and hence this allows us to place 6_7 at C_{28}. Scanning of 2nd vertical line yields 7_8 at C_{27} and hence 8_9 at C_{17}. 4_{10} by 1st row block to block scanning at C_{91}. Also we get 3_{11}, 2_{12} and 1_{13} at C_{83}, C_{82} and C_{81} respectively by 1st row block to block scanning. 8_{14} and 8_{15} by 1st vertical block to block scanning at C_{24} and by 2nd row block to block scanning at C_{55}

123

2	5	**8**$_5$	7$_{28}$	3	9$_{29}$	6$_{22}$	1$_{13}$	4$_{10}$
4	3	9	1$_{20}$	6$_{18}$	8$_{21}$	7$_{23}$	2$_{12}$	5
	1		2$_{16}$	4	5	8	3$_{11}$	9$_{17}$
6$_{41}$	8$_{14}$	3$_{39}$	9$_{30}$	7	4	2	5	1$_{40}$
9$_{34}$	2		5	8$_{15}$	1	3$_{37}$	4	
5$_3$	4		3	2	6$_{31}$	9$_{35}$	**8**$_4$	
8$_9$	7$_8$	5	4	9	2$_2$	1$_{36}$	6	3$_{38}$
3$_3$	6$_7$	4	8$_{24}$	1$_{19}$	7$_{25}$	5	9	2
1$_3$	9$_6$	2	6$_{27}$	5	3$_{26}$	4$_1$	7	8

67

78

13

Fig 37 (b)

Now, fig 37 (b)

2_{16} and 9_{17} by 3rd row line scanning at C_{43} and C_{93}. 6_{18} and 1_{19} by 5th vertical line scanning at C_{52} and C_{58}. 1_{21} and 8_{22} by 4th block filling at C_{52} and C_{62} respectively. This gives 6_{22} and 7_{23} at C_{71} and C_{72} respectively. $8_{24}, 7_{25}, 3_{26}$ and

6_{27} by 6th block filling at C_{48}, C_{68}, C_{69} and C_{49} respectively. 7_{28} and 9_{29} by 1st row line filling at C_{41} and C_{61}. 9_{30} and 6_{31} by 5th block filling at C_{44} and C_{66}. 1_{32} and 3_{33} by 3rd block filling at C_{18} and C_{19}. Now we get, 9_{34} by 2nd block filling at C_{15} and 9_{35} at C_{76} by 2nd row block to block scanning. 1_{36} and 3_{37} by 7th vertical line filling at C_{77} and C_{75}. 3_{38} by 9th block filling at C_{97}. $3_{39}, 1_{40}$ and 6_{41} by 4th row line filling at C_{34}, C_{94} and C_{14} respectively. Similarly rest of unit squares can be filled to complete the SUDUKO.

SUDOKU 16

4		8	7	1				
7		2		9		8		
5			2	8		1		
	7					4	6	
	5						8	
	8	3					1	9
				3				1
				7		9		6
				6	2			8

Fig 38

Solution (ref Fig 38 a)

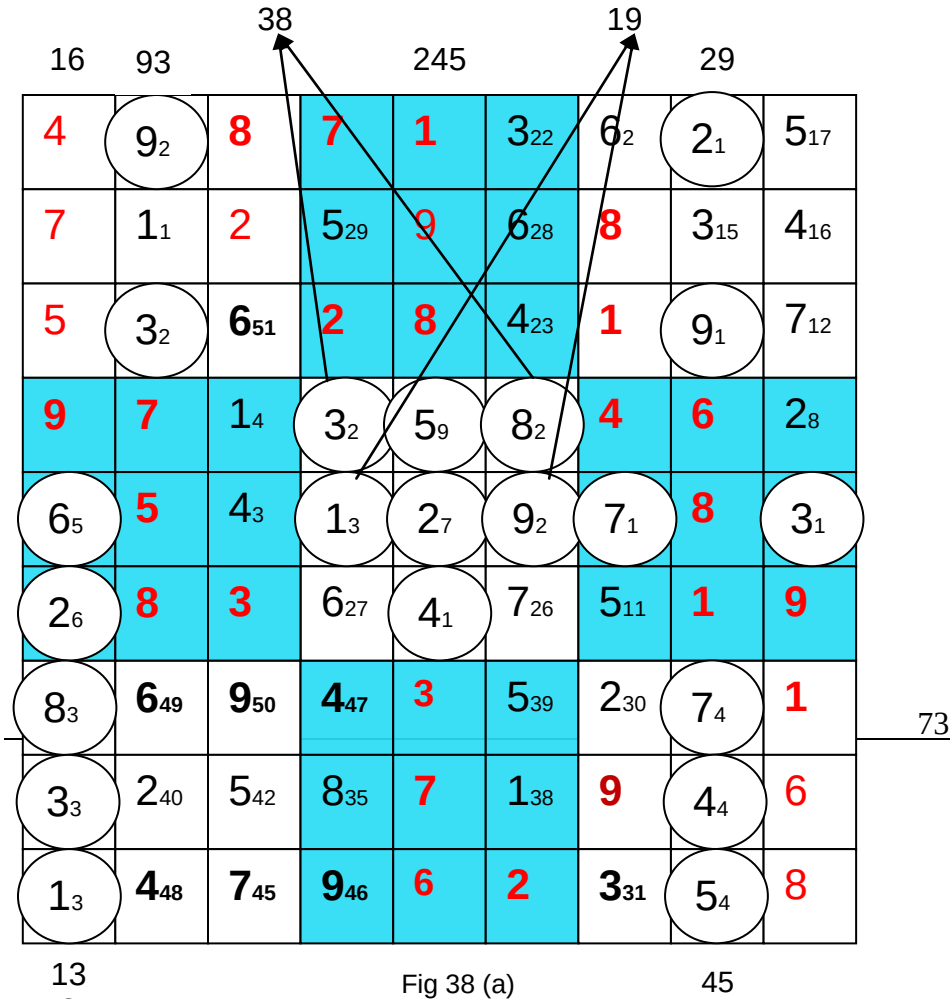


Fig 38 (a)

SUDOKU 17

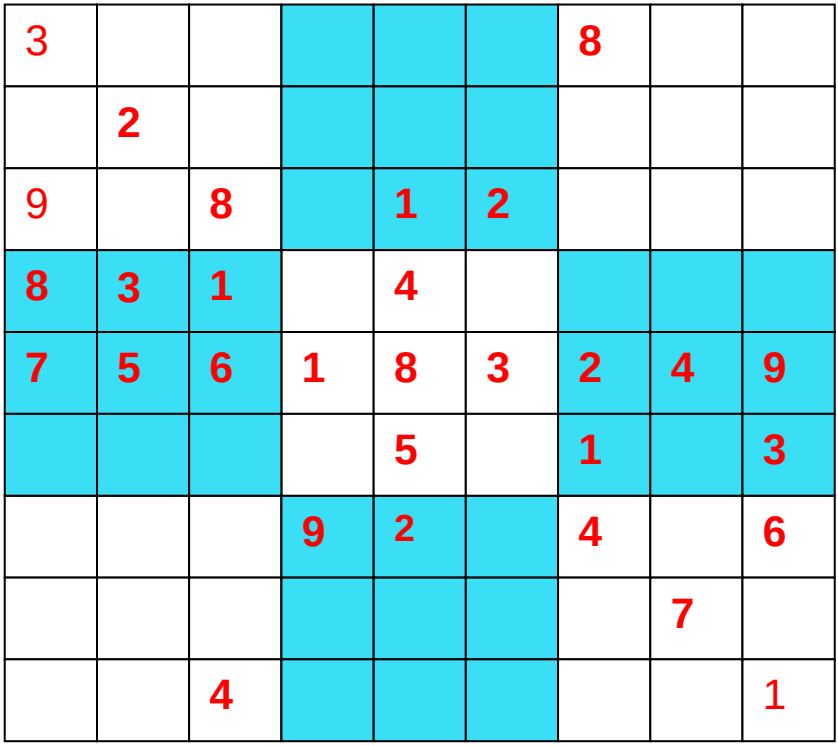


Fig 39

3	1_{30}	5_{37}	6_{36}	7_{35}	4_{23}	8	9_{26}	2_{29}
4_{20}	2	7_{38}	3_{33}	9_{25}	8_{44}	6_{32}	1_{27}	5_{40}
9	6_{31}	8	5_{43}	1	2	7_{39}	3_{28}	4_{22}
8	3	1	2_{2}	4	9_{3}	5_{41}	6_{9}	7_{42}
7	5	6	1	8	3	2	4	9
2_{21}	4_{24}	9_{14}	7_{46}	5	6_{45}	1	8_{1}	3
1_{6}	8_{51}	3_{5}	9	2	7_{48}	4	5_{7}	6
5_{12}	9_{15}	2_{13}	4_{11}	6_{18}	1_{10}	3_{17}	7	8_{4}
6_{19}	7_{50}	4	8_{49}	3_{34}	5_{47}	9_{16}	2_{8}	1

Fig 39 (a)

2_2 because block 2 row line contains 2.

146 not possible in C_{31} and C_{32} so 3_5 at C_{37}. 87 not possible in C_{87} and C_{17} so 1_6 **at** C_{17}.

SUDOKU 18

					1	7	4	
4		3			5	2		
8			4	3				
7				8	4			
2			3		7			4
			2	6				7
				1		4		2
		2	7	4		8		6
	6	4	9					

Fig 40

Solution

4_1 by 2nd vertical column line filling at C_{26}. 1_2 at C_{44} by 2nd column block to block scanning. $\mathbf{7_3}$ at C_{52} by fourth block filling. $\mathbf{2_4}$ at C_{84} by 8th block filling.

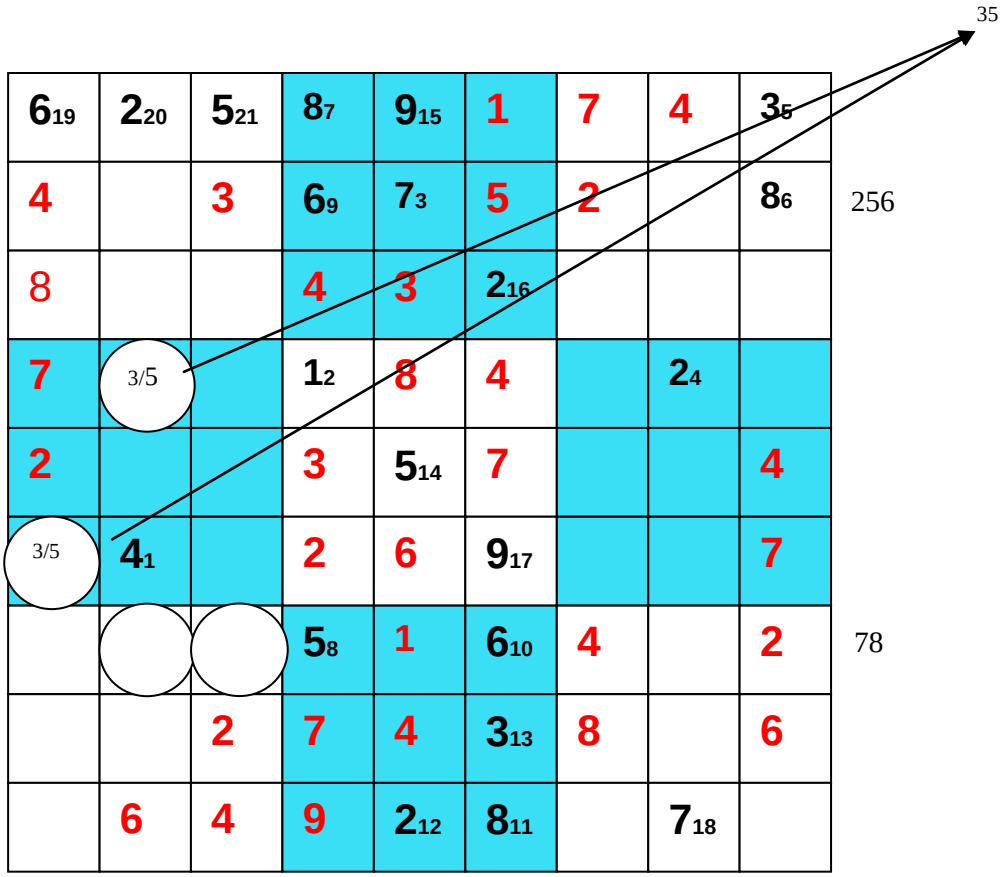
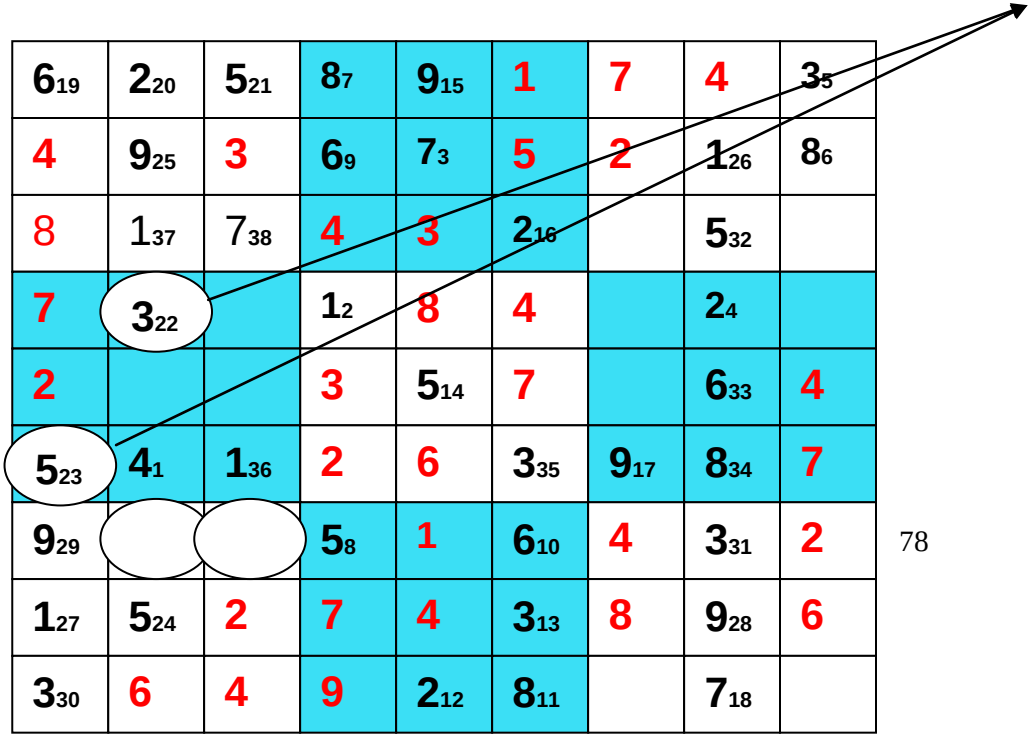


Fig 40 (a)

3_5 at C_{91} by 1st row block to block scanning. 8_6 by 9th vertical column line filling at C_{92} because of presence of 8 at C_{78}, C_{44} and C_{13}. 8_7 at C_{41} by fourth block filling. 5_8 by 4th vertical column line filling at C_{47}. 6_9 also by 4th vertical column line filling at C_{42}. 6_{10} at C_{67} by sixth block filling. 8_{11}, 2_{12} and 3_{13} at C_{69}, C_{59} and C_{68} respectively by sixth block fillings. 5_{14} and 9_{15} by 5th vertical column line filling at C_{55} and C_{51}. 2_{16} and 9_{17} by 6th vertical column line filling at C_{63} and C_{66}. 7_{18} by 9th row line filling at

C_{89}. $\mathbf{6}_{19}$ at C_{11} by first block filling as 6 will occupy either at C_{34} or at C_{35} for block 2. $\mathbf{2}_{20}$ at C_{21} and $\mathbf{5}_{21}$ at C_{31} by first row line filling.



78

Fig 40 (b)

C_{27} and C_{37} are circled for 7 and 8 similarly C_{24} and C_{16} are circled for 3 and 5. Now, $\mathbf{3}_{22}$ at C_{24} is filled by 2nd vertical line /column line filling so this gives $\mathbf{5}_{23}$ at C_{16}. This gives $\mathbf{5}_{24}$ at C_{28} by 2nd column line filling. 9 occupies either C_{87} or C_{88} hence it may occupy either at C_{73} or C_{93} this gives $\mathbf{9}_{25}$ at C_{22}. This gives $\mathbf{1}_{26}$ at C_{82} by 2nd row line filling. $\mathbf{1}_{27}$ by 3rd block filling as 1 will occupy either at C_{79} or C_{99} because of presence of 1 at C_{82}. This gives $\mathbf{9}_{28}$ at C_{88} by 8th row line filling. This gives $\mathbf{9}_{29}$ at C_{17} by 7h

row line filling and **3**$_{30}$ at C_{19} by 3rd block filling. This gives **3**$_{31}$ at C_{87} **by** 7th row line filling. Now, Fig 40(C). This gives **5**$_{32}$ at C_{83} by 8th column line filling. **6**$_{33}$ and **8**$_{34}$ by 8th column line filling at C_{85} and C_{86} respectively. **3**$_{35}$ and 1$_{36}$ by 6th row line filling at C_{76} and C_{36} respectively. 1$_{37}$ at C_{23} and **7**$_{38}$ at C_{33} by 1st block filling.

This gives 7$_{39}$ and 8$_{40}$ at C_{37} and C_{27} respectively against the circled cells C_{37} and C_{27}. **6**$_{41}$, **9**$_{42}$ and **8**$_{43}$ at C_{34}, C_{35} and C_{25} respectively by 2nd block filling.

6$_{44}$ and **9**$_{45}$ by 7th block filling at C_{73} and C_{93} respectively. **1**$_{46}$ by 8th block filling at C_{75}. **1**$_{47}$ and **5**$_{48}$ by 9th block filling at C_{99} and C_{79} respectively. **5**$_{49}$ and **9**$_{50}$ by 8th block filling at C_{94} and C_{74} respectively.

SUDOKU 19

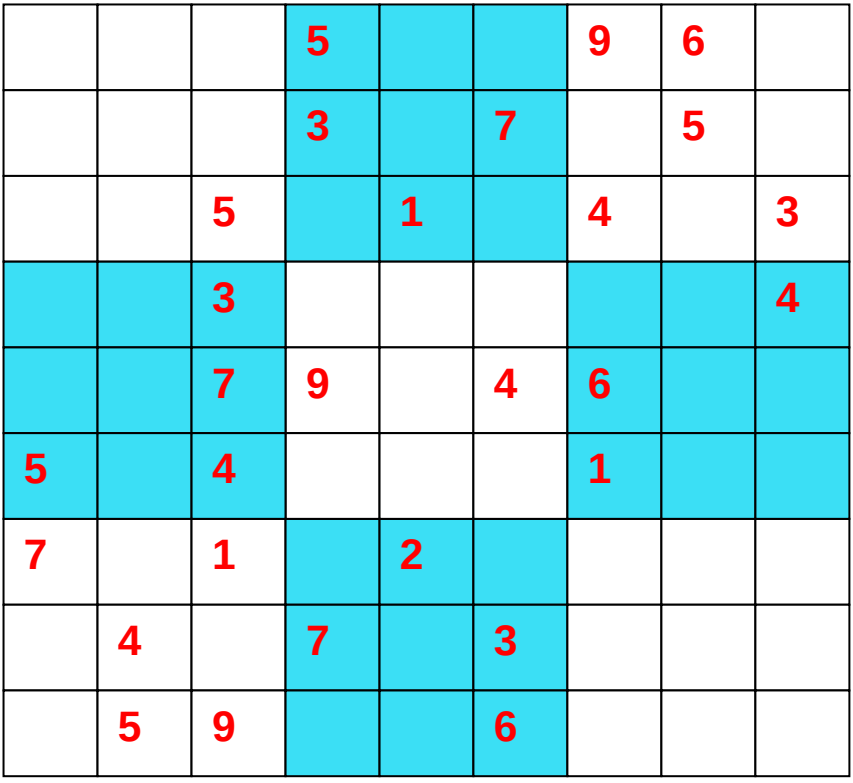


Fig 41

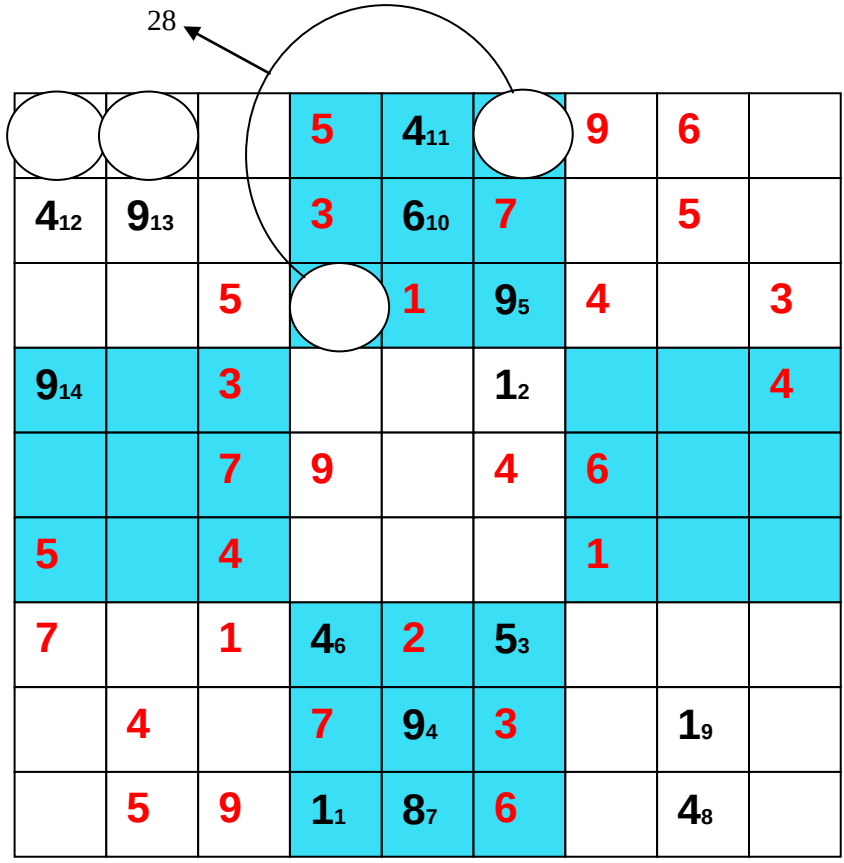


Fig 41 (a)

by 6th block filling at C49. **1**2 at C64 by 2nd column block to block scanning. **5**3 at C67 by 6th column line scanning. **9**4 by 6th block filling at C58. **9**5 at C63 by 2nd column block to block scanning. **4**6 at C37 by 4th column line scanning. **8**7 by 6th block filling at C59. **4**12 at C12 by 1st row block to block scanning

. **4**8 at C89 by 3rd column block to block scanning. **1**9 at C88 by 8th column line scanning. C43 and C61 is circled for digit 2 and 8. So this gives **6**10 at C52. This gives **4**11 at C51 by 4th block filling. **9**13

at C_{22} by 2nd row line scanning. **9_{14}** by 1st column block filling at C_{14} and C_{21} are circled for 13 in block 1.

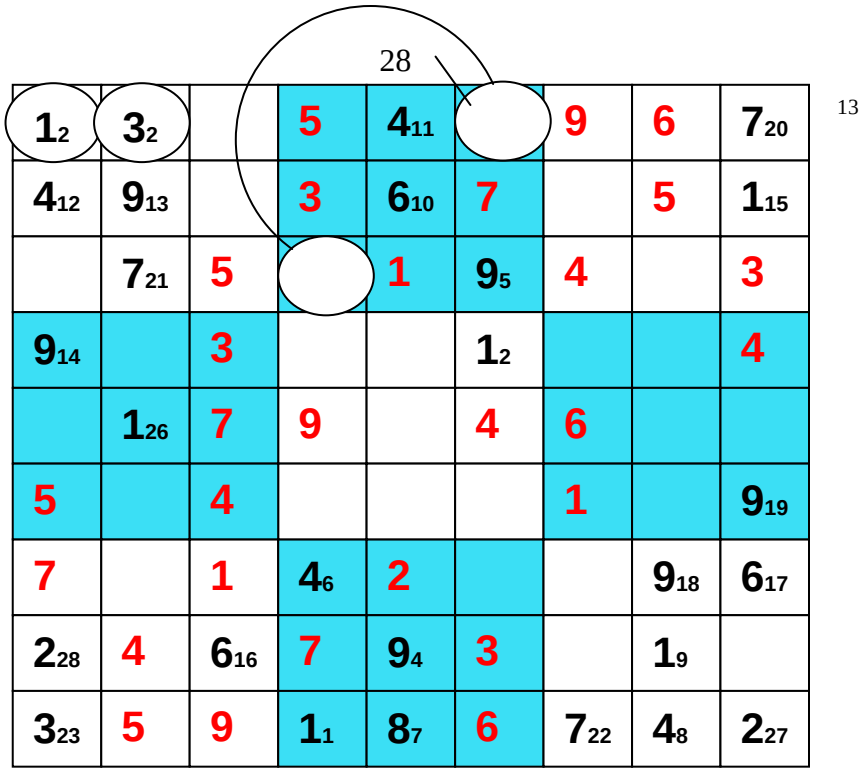


Fig 41 (b)

Now fig 42b

1_{15} at C_{92} by 1st row block to block scanning. **6_{16}** at C_{38} by 3rd column line scanning. **6_{17}** at C_{97} by 3rd row block to block scanning. **9_{18}** at C_{87} by 3rd row block to block scanning. **9_{19}** at C_{96} by 3rd column block to block scanning. **7_{20}** at C_{91} by 1st row line scanning. **7_{21}** at C_{23} by 1st row block to block scanning. **7_{22}** at C_{79} by 3rd column block to block scanning. **3_{23}** at C_{19} by 1st column line scanning. **3_{24}** and **1_{25}** at C_{21} and C_{11} respectively by 1st column

block to block scanning and 1st block filling. **1**$_{26}$ at C_{25} by 1st column block to block scanning. **2**$_{27}$ at C_{99} by 9th row line scanning and **2**$_{28}$ at C_{18} by 3rd row block to block scanning.

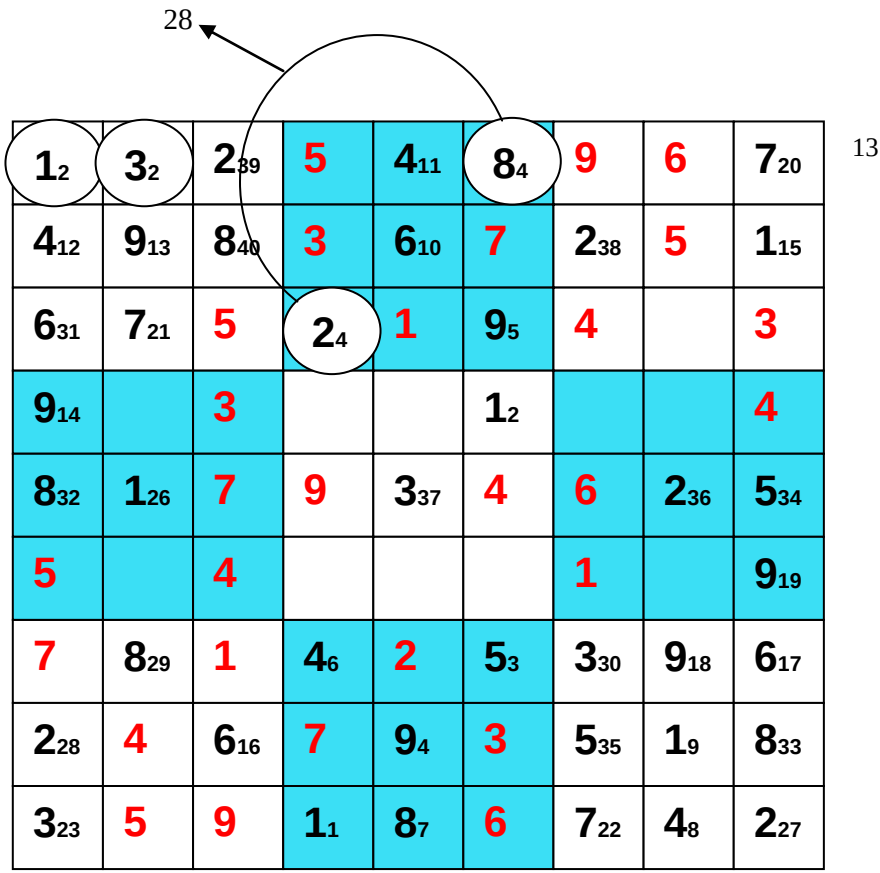


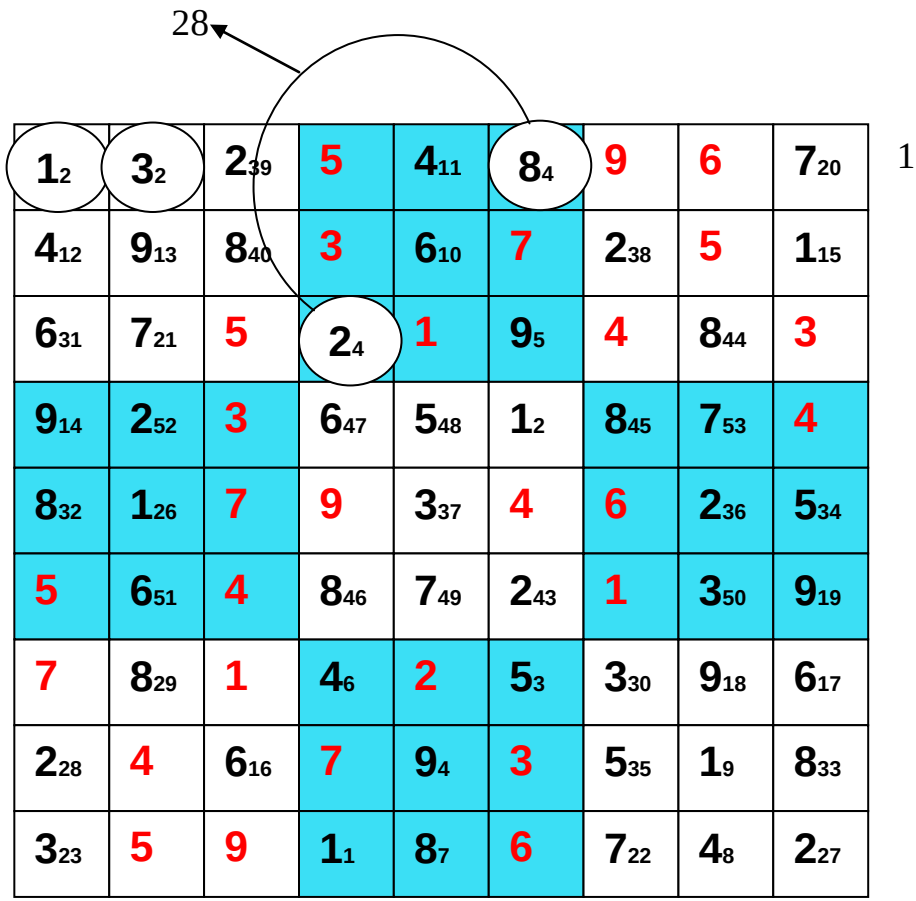
Fig 41 (c)

Now fig 42c

8$_{29}$ **at** C_{27} by 3rd block filling. **3**$_{30}$ **at** C_{77} by 9th block filling. **6**$_{31}$ at C_{13} by 1st row block to block scanning. **8**$_{32}$ **by** 1st column or vertical line filling at C_{15}. This gives **8**$_{33}$ at C_{98} by 9th vertical or column line filling. And hence 5_{34} is filled at C_{95} again by 9th

vertical line filling. Now we get **5**$_{35}$ at C_{78} by 9th block filling. Now we get **2**$_{36}$ **and 3**$_{37}$ **by** 5th row line filling at C_{85} and C_{55} respectively. This helps in getting **2**$_{38}$ at C_{72} by

3rd column block filling. This helps in getting **2**$_{39}$ and **8**$_{40}$ at C_{31} and at C_{32} by 1st block filling. This gives **8**$_{41}$ and **2**$_{42}$ by 4th block filling at C_{61} and at C_{43}.



1$_{2}$	**3**$_{2}$	**2**$_{39}$	**5**	**4**$_{11}$	**8**$_{4}$	**9**	**6**	**7**$_{20}$
4$_{12}$	**9**$_{13}$	**8**$_{40}$	**3**	**6**$_{10}$	**7**	**2**$_{38}$	**5**	**1**$_{15}$
6$_{31}$	**7**$_{21}$	**5**	**2**$_{4}$	**1**	**9**$_{5}$	**4**	**8**$_{44}$	**3**
9$_{14}$	**2**$_{52}$	**3**	**6**$_{47}$	**5**$_{48}$	**1**$_{2}$	**8**$_{45}$	**7**$_{53}$	**4**
8$_{32}$	**1**$_{26}$	**7**	**9**	**3**$_{37}$	**4**	**6**	**2**$_{36}$	**5**$_{34}$
5	**6**$_{51}$	**4**	**8**$_{46}$	**7**$_{49}$	**2**$_{43}$	**1**	**3**$_{50}$	**9**$_{19}$
7	**8**$_{29}$	**1**	**4**$_{6}$	**2**	**5**$_{3}$	**3**$_{30}$	**9**$_{18}$	**6**$_{17}$
2$_{28}$	**4**	**6**$_{16}$	**7**	**9**$_{4}$	**3**	**5**$_{35}$	**1**$_{9}$	**8**$_{33}$
3$_{23}$	**5**	**9**	**1**$_{1}$	**8**$_{7}$	**6**	**7**$_{22}$	**4**$_{8}$	**2**$_{27}$

Fig 41 (d)

Now fig 42d

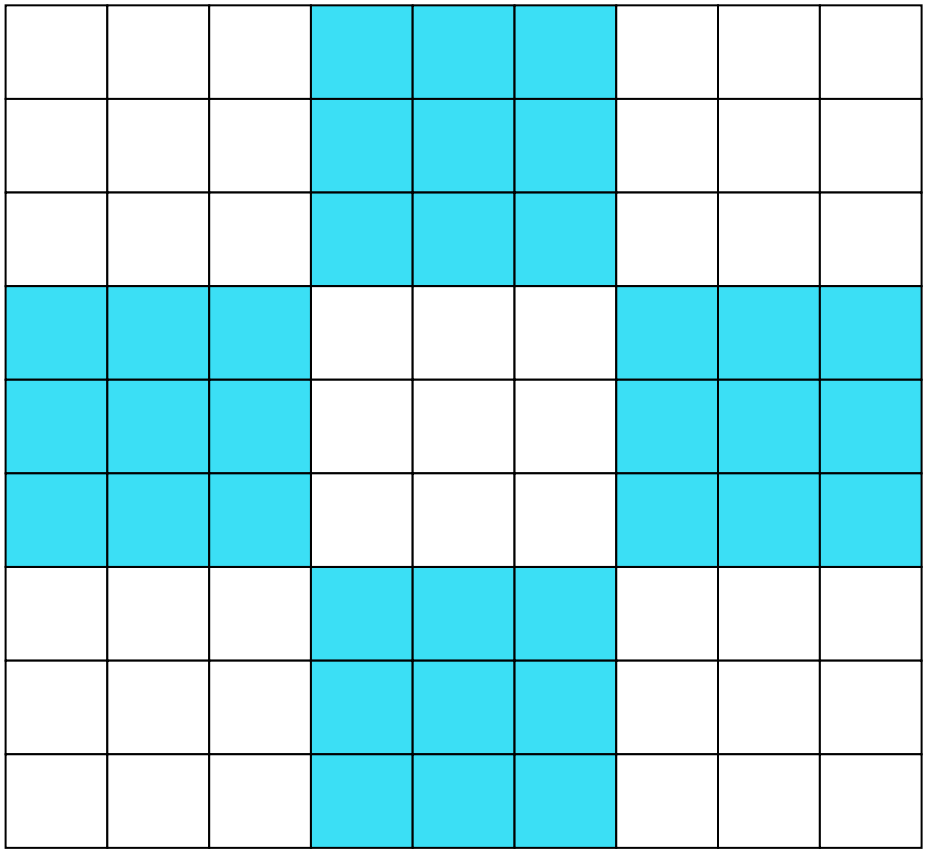
2$_{43}$ by filling of 6th column line at C$_{66}$. **8$_{44}$** by 7th block filling at C$_{83}$. **8$_{45}$** by 8th block filling at C$_{74}$ and **8$_{46}$ by** filling of 5th block at C$_{46}$. **6$_{47}$ by filling of 5th block at** C$_{34}$.

5$_{48}$ at C$_{54}$ by 4th row line filling as it is not possible in C$_{24}$ and C$_{84}$. **7$_{49}$ by** 5th block filling at C$_{56}$. **3$_{50}$**, **6$_{51}$**, **2$_{52}$** and **7$_{53}$** are **filled** at C$_{76}$, at C$_{26}$, at C$_{24}$, and at C$_{84}$ respectively by second row block to block scanning, 3rd block filling, 3rd block filling and 8th block filling.

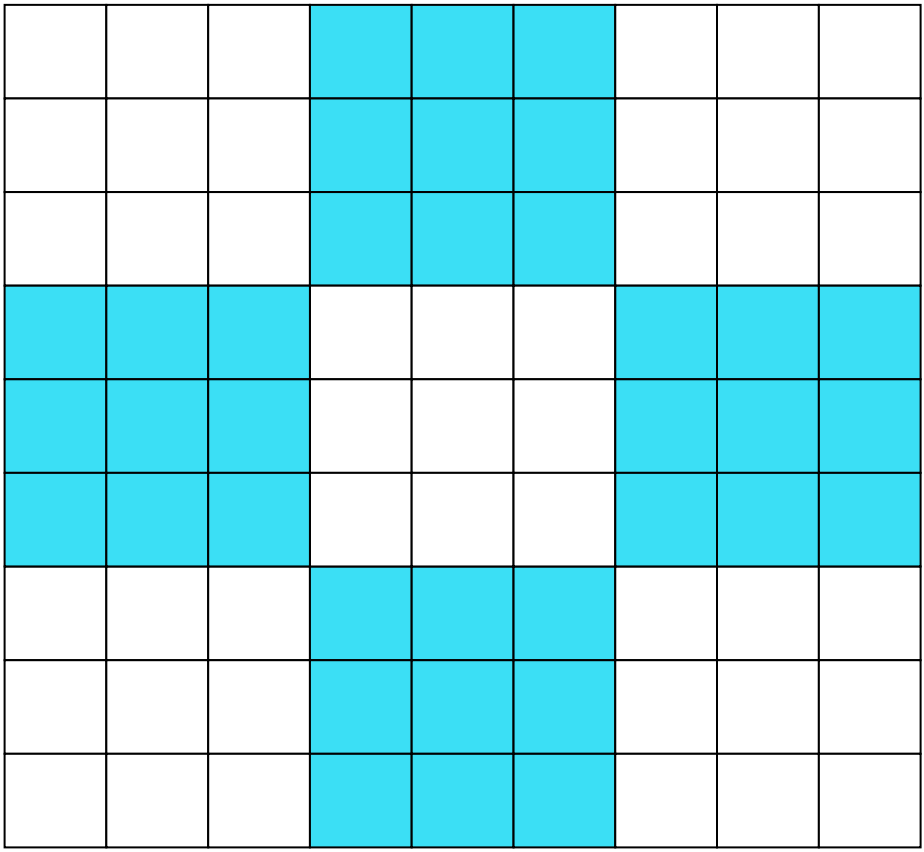
Chapter 5

Practice Squares

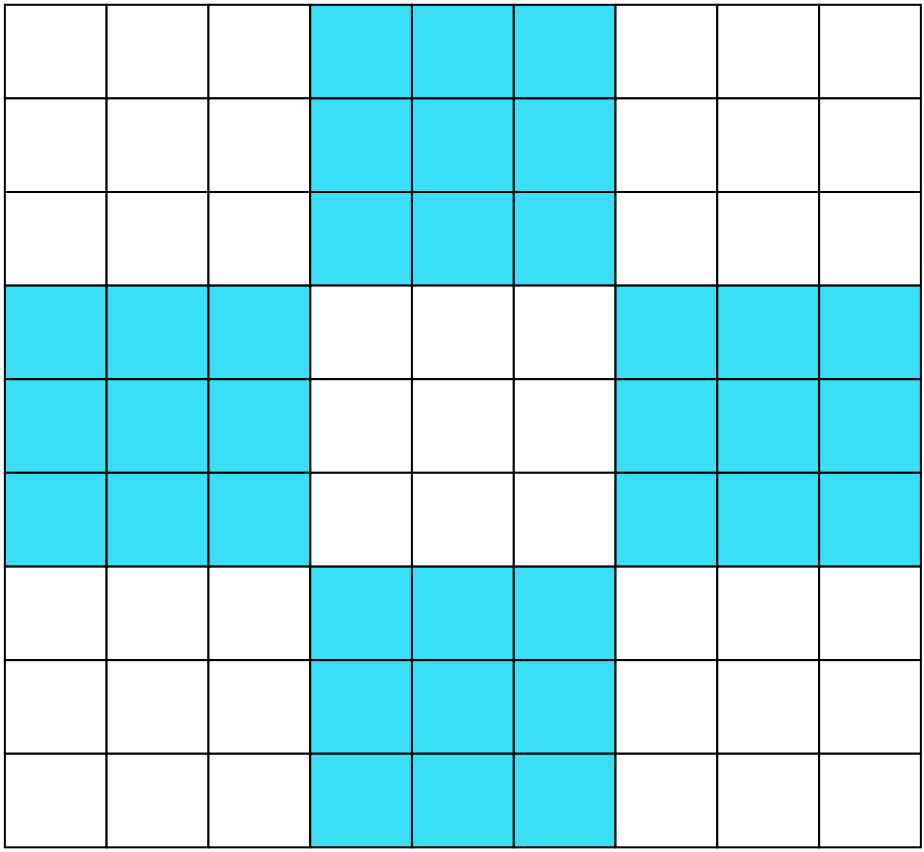
PRACTICE SQUARES



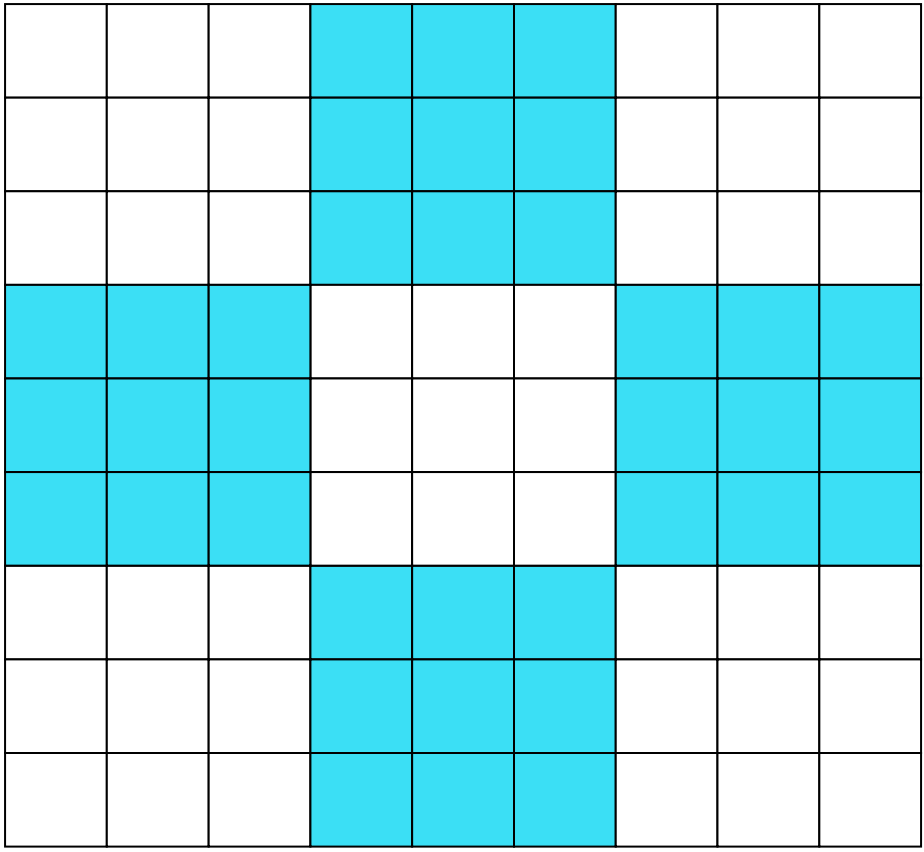
Practice 01



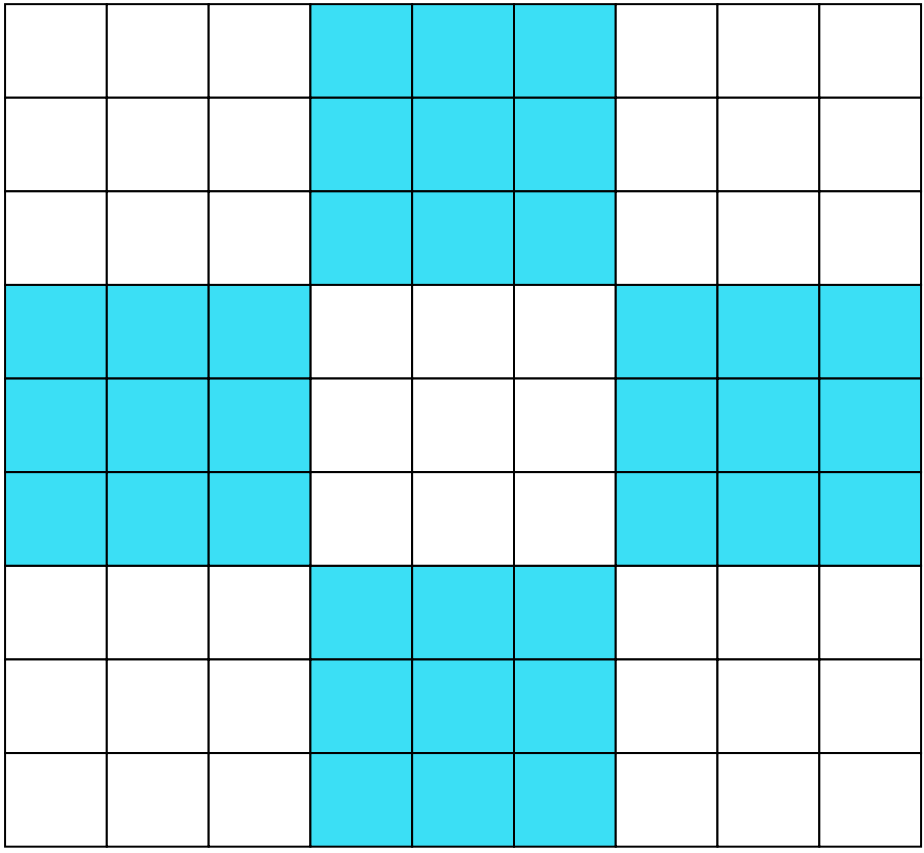
Practice 02



Practice 03



Practice 04



Practice 05