EE 325: Probability and Random Processes

Homework for Module 2

Instructions

- Submit the solutions to item nos. 3 above
- There will be viva on these on befor Tuesday, 22 September.

Questions

1. There are several solved examples in the textbook. Make sure you have studied these problems and the solutions.

2. Solve problems 4.2, 4.4, 4.11, 4.16, 4.26, 4.31, 4.33. The solutions to these problems need not be submitted for evaluation. You may be questioned on these in the viva.

3. $F(x)$ is a valid distribution. Which of the following functions of $F(x)$ are also valid distributions. Provide a proof for your claim.

   (a) $aF(x) + (1-a)F(x)$ where $0 \leq a \leq .$

   (b) $(F(x))^r$

   (c) $1 - (1 - F(x))^r$

   (d) $F(x) + (1 - F(x))\log(1 - F(x))$

4. There are two urns—A containing n black balls and B containing n brown balls. At each step, one ball is chosen at random from both urns and swapped, i.e., the one from A is put into B and vice versa. Let $\mathsf{X_m}$ be the number of black balls in urn A after m steps. Observe that this determines the state of the system after m steps, i.e., knowing $\mathsf{X_m}$ describes the composition of both the urns. Obtain the pmf of $\mathsf{X_m}$. This is a model for diffusion.

5. Recall the 'capture-release-recapture' problem: Catch m fish, mark them and release them back into the lake. Allow the fish to mix well and then you catch m fish. Of these p are those that were marked before. Assume that the actual fish population in the lakes is n and has not changed between the catches. Let $P_{m,p}(n)$ be the probability of the event (for a fixed p recatches out of m) coming from n fish in the lake. Generate a plot for $P_{m,p}(n)$ as a function of n for the following values of m and $p : m = 100$ and $p = 10, 20, 50, 75$. For each of these p, use the

plots to estimate (educated guess) the actual value of n. Call these four estimates $\hat{n}_1, \ldots, \hat{n}_4$.

6. Now for each of the four $\hat{n}_i$, simulate the previous problem. One simulation will for case i will do the following. Take $\hat{n}_i$ fish in the pond, mark $m = 100$ of these, mix up the fish and catch $m = 100$ random fish. Repeat the experiment 500 times for each i, calculate the sample average from the 500 experiments and compare with the corresponding p from the previous question. Comment on the comparison between them.

7. A fair die is rolled $6n$ times. Let ρ_n be the probability that there n 6s in the $6n$ rolls. Is ρ_n a monotonic function. Prove your statement.