FIN 514

Project 3

Project-3: Monte Carlo-Simple Product with 1 underlier

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Executive Summary

We value the Morgan Stanley structured product, due February-2029, with the Dow Jones Industrial Average as underlier, at a target price range of **$9.40 to $9.60**, using a 10*100000-step Monte-Carlo simulation.

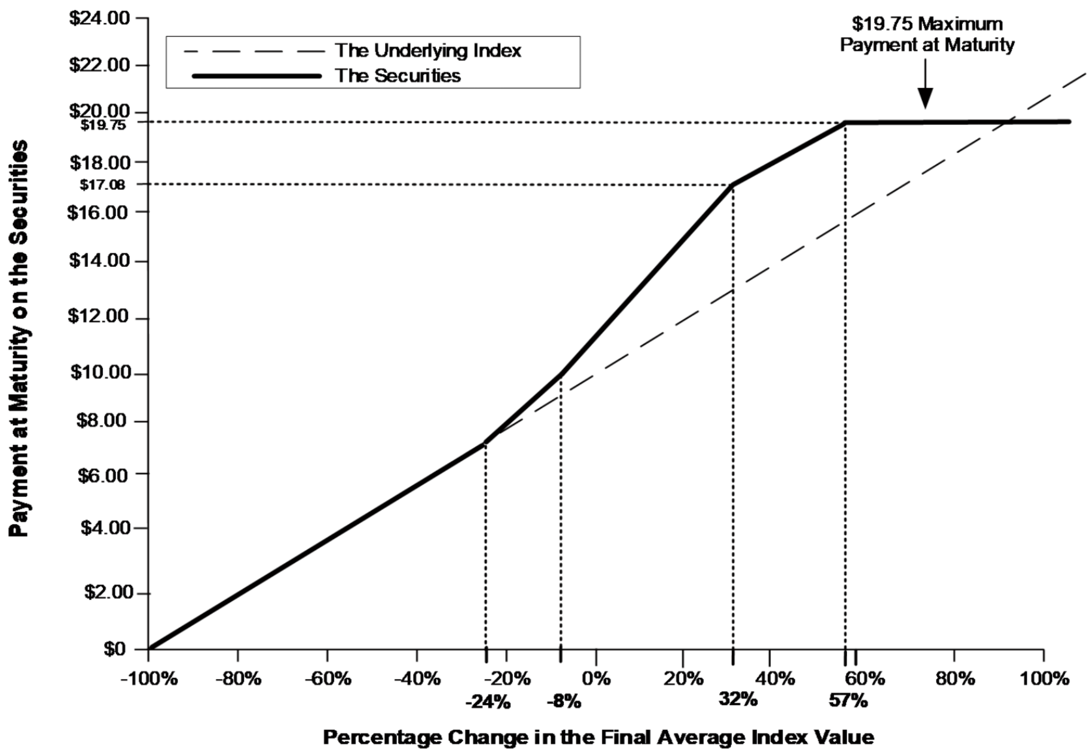
Some additional information about the notes is mentioned before: -

Key Dates	
Pricing Date	02/28/2023
Issue Date	03/03/2023
Initial Averaging Dates	Business days between Feb-24 and Apr-12, 2023 (inclusive)
Final Averaging Dates	Business days between Nov-22, 2028 and Feb-24, 2029.
Maturity Date	02/28/2029

The basic features of the note are: -

1) The note has only one underlying asset. i.e., the Dow Jones Industrial Average Index
2) The final payoff is determined by the average index value over the initial averaging dates, the average index value over the final averaging dates and the index value at maturity.

The payoff at maturity is given as follows: -



Assumptions

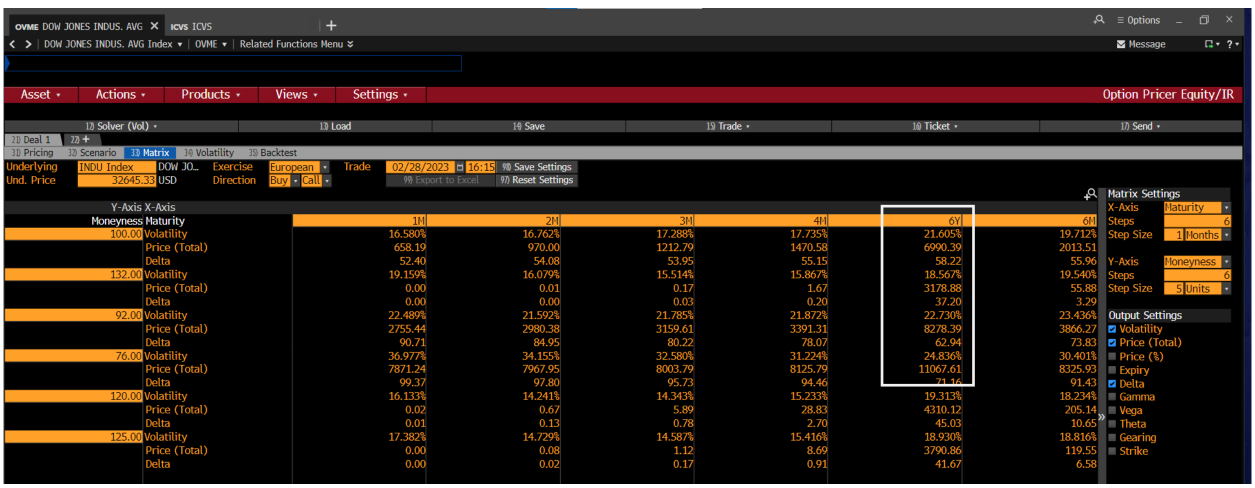
- The dividend amount paid is a constant % of the index value.
- The yield is expected to remain at the Feb-28 level for the next 36 months.

- The period between the initial observation date and the final maturity date is 2192 days. So, we assume that a period of 2192/365 is a better estimate for T instead of 6.
- The risk-free rate for the entire period is considered constant.

Input Data Summary

- **Initial level(S0)—*32656.70***
- **Volatility (Moneyness-100)(sigma)—*21.605%***
- **Volatility (Moneyness-132)(sigma_132)—*18.567%***
- **Volatility (Moneyness-92)(sigma_92)—*22.730%***
- **Volatility (Moneyness-76)(sigma_76)—*24.836%***
- **Risk-free rate(r)—*3.87573%***
- **Dividend Yield(q)—*1.923%***
- **Face value(Face)—$10**
- **T=2192/365**
- **N—100000**
- **Initial_avg_days—36**
- **Final_avg_days—70**

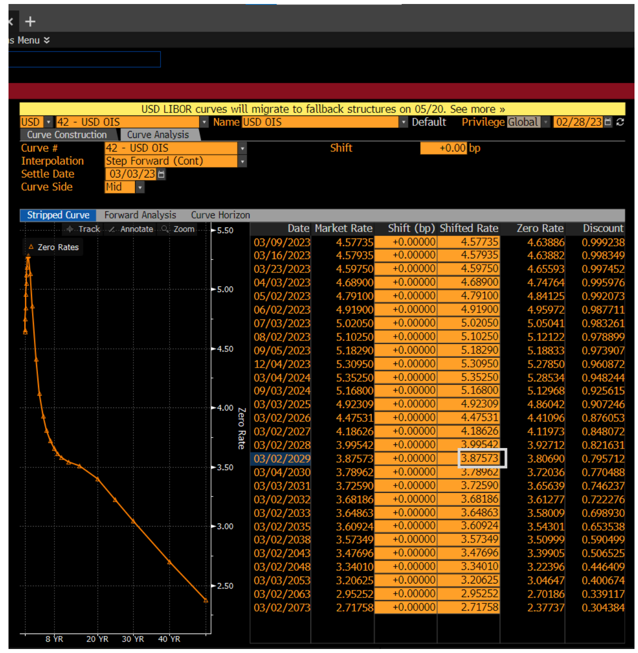
Volatilities at Moneyness of 100, 132, 92, 76



Y-Axis Moneyness	X-Axis Maturity	1M	2M	3M	4M	6Y	6M
100.00	Volatility	16.580%	16.762%	17.288%	17.735%	21.605%	19.712%
	Price (Total)	658.19	970.00	1212.79	1470.58	6990.39	2013.51
	Delta	52.40	54.08	53.95	55.15	58.22	55.96
132.00	Volatility	19.159%	16.079%	15.514%	15.867%	18.567%	19.540%
	Price (Total)	0.00	0.01	0.17	1.67	3178.88	55.88
	Delta	0.00	0.00	0.03	0.20	37.20	3.29
92.00	Volatility	22.489%	21.592%	21.785%	21.872%	22.730%	23.436%
	Price (Total)	2755.44	2980.38	3159.61	3391.31	8278.39	3866.27
	Delta	90.71	84.95	80.22	78.07	62.94	73.83
76.00	Volatility	36.977%	34.155%	32.580%	31.224%	24.836%	30.401%
	Price (Total)	7871.24	7967.95	8003.79	8125.79	11067.61	8325.93
	Delta	99.37	97.80	95.73	94.46	71.16	91.43
120.00	Volatility	16.133%	14.241%	14.343%	15.233%	19.313%	18.234%
	Price (Total)	0.02	0.67	5.89	28.83	4310.12	205.14
	Delta	0.01	0.13	0.78	2.70	45.03	10.65
125.00	Volatility	17.382%	14.729%	14.587%	15.416%	18.930%	18.816%
	Price (Total)	0.00	0.08	1.12	8.69	3790.86	119.55
	Delta	0.00	0.02	0.17	0.91	41.67	6.58

Dividend Yield

OIS Rates



Date	Market Rate	Shift (bp)	Shifted Rate	Zero Rate	Discount
03/09/2023	4.57735	+0.00000	4.57735	4.63886	0.999238
03/16/2023	4.57935	+0.00000	4.57935	4.63882	0.998349
03/23/2023	4.59750	+0.00000	4.59750	4.65593	0.997452
04/03/2023	4.68900	+0.00000	4.68900	4.74764	0.995976
05/02/2023	4.79100	+0.00000	4.79100	4.84125	0.992073
06/02/2023	4.91900	+0.00000	4.91900	4.95972	0.987711
07/03/2023	5.02050	+0.00000	5.02050	5.05041	0.983261
08/02/2023	5.10250	+0.00000	5.10250	5.12122	0.978899
09/05/2023	5.18290	+0.00000	5.18290	5.18833	0.973907
12/04/2023	5.30950	+0.00000	5.30950	5.27850	0.960872
03/04/2024	5.35250	+0.00000	5.35250	5.28534	0.948244
09/03/2024	5.16800	+0.00000	5.16800	5.12968	0.925615
03/03/2025	4.92309	+0.00000	4.92309	4.86042	0.907246
03/02/2026	4.47531	+0.00000	4.47531	4.41096	0.876053
03/02/2027	4.18626	+0.00000	4.18626	4.11973	0.848072
03/02/2028	3.99542	+0.00000	3.99542	3.92712	0.821631
03/02/2029	3.87573	+0.00000	3.87573	3.80690	0.795712
03/04/2030	3.78962	+0.00000	3.78962	3.72036	0.770488
03/03/2031	3.72590	+0.00000	3.72590	3.65639	0.746237
03/02/2032	3.68186	+0.00000	3.68186	3.61277	0.722276
03/02/2033	3.64863	+0.00000	3.64863	3.58009	0.698930
03/02/2035	3.60924	+0.00000	3.60924	3.54301	0.653538
03/02/2038	3.57349	+0.00000	3.57349	3.50999	0.590499
03/02/2043	3.47696	+0.00000	3.47696	3.39905	0.506525
03/02/2048	3.34010	+0.00000	3.34010	3.22396	0.446409
03/03/2053	3.20625	+0.00000	3.20625	3.04647	0.400674
03/02/2063	2.95252	+0.00000	2.95252	2.70186	0.339117
03/02/2073	2.71758	+0.00000	2.71758	2.37737	0.304384

Approach

We used a 100000-iteration Monte-Carlo simulation to determine the value of the product.

1. We are aware of the errors in valuation that crop up in s Monte-Carlo model, as stock prices are generated using random distributions.
2. Due to that, we have run a 100000 step Monte Carlo simulation, and reiterated it 10 time to get the most accurate value.
3. We have extracted the OIS rates, dividend yield & ADM stock volatilities from Bloomberg terminal.
4. We finally run a sensitivity analysis and run the model using all the different volatilities we extracted from Bloomberg.

Algorithm

For each of the N iterations of the simulation, we follow these steps.

1. We initialize in_av and f_av to store the initial and final average index levels.
2. We run an iteration to for the range of the initial averaging dates to compute the stock price using
$$S_t = S_0 * e^{\left[\left(r-q-\frac{1}{2}\sigma^2\right)*t+\sigma*\sqrt{t}*\emptyset\right]}$$
$$where\ \emptyset = random\ variable$$
3. We add the sum of index levels and then divide them by initial_avg_days to get the average initial index level.
$$in_{av} = in_{av} + S_t$$
We close the inner loop and obtain the initial average index level.
$$in_{av} = \frac{in_{av}}{initial_{avg_{days}}}$$

4. We run steps 2-3 for the final averaging value as well (f_{av}).
5. We use the formula in step-2 to obtain the final DJIA level at maturity.
6. We set the Upper strike index value as:
$$usi = 1.32 * in_{av}$$
7. We set the Lower strike index value as:
$$lsi = 0.92 * in_{av}$$
8. We set the Downside threshold value to:
$$dtv = 0.76 * in_{av}$$
9. We set the index upper side return and index lower side return as follows:
$$r_{usi} = \frac{f_{av} - usi}{in_{av}}$$
$$r_{lsi} = \frac{f_{av} - lsi}{in_{av}}$$
10. Obtain the final payoff value of the product.
$$if\ f_{av} > usi:$$
$$V_i = min(17.08 + 10 * r_{usi} * 1.068 , 19.75)$$
$$else\ if\ f_{av} <= usi\ and\ f_{av} > lsi:$$
$$V_i = 10 + 10 * r_{lsi} * 1.77$$
$$else\ if\ f_{av} <= lsi\ and\ f_{av} >= dtv:$$
$$V_i = 10 + 10 * 0.08 * 1.50 * (\frac{S_{final}}{S_0} - 1)$$
$$else:$$
$$V_i = 10 + 10 * \left(\frac{S_{final}}{S_0} - 1\right)$$
11. We discount the final payoff to determine the value of the option.
$$Option_{value} = e^{-r*T} * V_i$$
12. We repeat the entire process for N number of iterations, and 10 trials.

Sensitivity Analysis

We compute the value of the note by considering the volatilities at a moneyness of 100, 132, 92 and 76 (Upper strike index, Lower strike index, Downside threshold value). This gives us our target price range.

The matrix below shows us the entire range of values.

Moneyness	Implied Volatility (%)	Estimated value of the note ($)
76	24.836%	$9.07
92	22.730%	$9.27
100	21.605%	$9.38
132	18.567%	$9.66

Morgan Stanley deems the fair value of the product to be **$9.778**.

Potential sources of error: -

- Risk-free rates at different periods may be different.
- The implied volatility values may be inaccurate for the DJIA.
- Dividend is never actually proportional to the index level.
- The Monte Carlo simulation process is based on the expectation of payoffs. The error tends to go down with increasing number of price paths.