### Problem Set 4

Total 15 points

Due Friday, XXXXXX 2022 before 11:59pm

**Group:**

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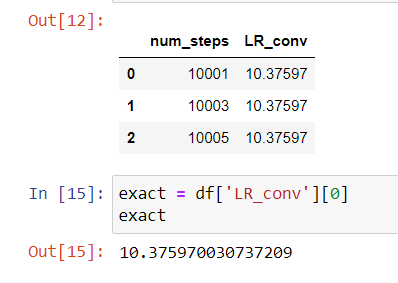
**American Put Option Valuation** **[5 points]**

Consider an American put option where S0 = 100, K = 105, r = 0.04, d = 0, s = 0.3 and T = 0.5.

1. Calculate the value of the American put option to within $0.000001 (using any binomial model you like), to do this you should use more and more timesteps (this should be lots, e.g 9999 or 10001 for LR) until you are happy that you have converged to the correct value.

We will compare the performance of the following binomial lattice methods.

* Cox, Ross and Rubinstein, 1979 (You have code for this from PS3)
* Leisen and Reimer, 1996 (You have code for this from PS3)
* **!NEW! Broadie and Detemple, 1996 (see Lecture 4 notes)**



**For each model perform the following,**

1. Calculate the value of the American put option for time steps ranging from N = 50 to N = 1000. Plot a graph of N (x-axis) against error (y-axis) when compared to your answer from a. Explain the graph you obtain.

From the above picture we can observe that the error rate curves are smooth for BD and LR model when compared to CRR

1. For the ***CRR model***, with N = 100, plot out the position of the early exercise boundary on a graph with time on the x-axis and the boundary level on the y-axis. (The exercise boundary is the value of S, Sf, at which you exercise at Sf and below and hold above Sf). Do you think that the boundary you obtain looks correct compared to the true (non-binomial) boundary? Do you think this will have any effect upon the accuracy of your option value?

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No, the theoritical curve is expected to be smooth; from the above graph we can observe that the boundary curve is cerrated where we notice that the prices of later periods are lower which is not expected and we also notice that there are intermittent jumps at some time points, thus resulting in poor prediction accuracies \*[¶](http://127.0.0.1:8888/notebooks/MSF_Gies_UIUC/Spring%202023/Derivatives/Assignments/PS4/PS4_Q1_CRR.ipynb#*No,-the-theoritical-curve-is-expected-to-be-smooth;-from-the-above-graph-we-can-observe-that-the-boundary-curve-is-cerrated-where-we-notice-that-the-prices-of-later-periods-are-lower-which-is-not-expected-and-we-also-notice-that-there-are-intermittent-jumps-at-some-time-points,-thus-resulting-in-poor-prediction-accuracies-*)

**Continuous barrier options (trigger securities) [5 points]**

Consider a down-and-out call option where **S0 = 100, K = 100, B = 95, r = 0.1, d = 0, s = 0.3 and T = 0.2** – **NOTE THE CHANGE IN PARAMETERS** The down-and-out call is a barrier option where you receive max(ST-K,0) at expiry but if at any time before expiry St < B the option expires worthless.

The analytic formula is given by:



where d1 and d2 are as in the Black-Scholes formulae and



We will now analyse the performance of just the **Leisen and Reimer**, **1996** model:

1. Calculate the value of the Down-and-out call option for time steps ranging from N = 50 to N = 1000 and plot a graph of N (x-axis) against error (y-axis) when compared to the analytic price and explain the graph you obtain.

The error rate is still non linear and not monotonic as expected even with LR model which is efficient in the case of American options.

Therefore, even the best binomial model is not efficient in the case of continuous barrier options

1. Can you explain the error profile that you observe? It may be useful to try to plot the position of the nodes relative to the *barrier* (as in the Lambda graph in Lecture 4) to understand what is happening in this question.

We can notice that the error rates are minimum when lamda values are zero.

1. Can you think of anyway of improving the convergence to the correct value? [I’m looking for ideas here rather than calculations but these may be useful for future valuations]

As the steps are increasing we can notice that the errors are converging, so with a significant number of steps we can improve the convergence,

We can also improve the convergence by selecting the steps in such a way that the lambda is close to zero

Chart

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**Discrete Barrier options (Autocall products) [5 points]**

Consider a discrete down-and-out call option where S0 = 100, K = 100, B =95, r = 0.1, d = 0, s = 0.3 and **T = 0.2** **only now the barrier is only applied on 4 dates, at t = 0.04, 0.08, 0.12 and 0.16 [*Note that it can be difficult to get your code to do this due to problems of real numbers/integers etc.]****.*

This is a discretely monitored barrier option and does not have an analytic formula but following the methods in Kou (2008), the accurate value is 5.6711051343.

We will now analyse the performance of the **Leisen and Reimer**, **1996** model::

* 1. Calculate the value of the discrete Down-and-out call option for time steps ranging from N = 50 to N = 1000 **in steps of 10** (i.e 50, 60, 70, …) to ensure that the barrier times are always matched by a tree step and all the barrier steps are even numbers. Plot a graph of N (x-axis) against error (y-axis) when compared to the analytic price and explain the graph you obtain.
  2. Plot the position of the nodes relative to the barrier (as in the Lambda graph in Lecture 4). When does it appear that the tree gives the most accurate option values?

Chart

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We can notice that the error rates are minimum when lamda values are close to 0.5.