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TUTORIAL

Divide & Conquer - Merge Sort

Chapter

Divide and Conquer: - It is an algorithm design paradigm. A divide and conquer algorithm works by repeatedly breaking down a problem into two or more sub-problems of the same or related type, until these become simple enough to be solved directly. The solutions to the sub-problems are then combined to give a solution to the original problem.Some examples of this paradigm are Binary Search, merge Sort, Quick Sort etc.

Merge Sort: - It is a sorting technique based on Divide and Conquer paradigm. It is based on the idea of breaking down a list into several sub-lists until each sublist consists of a single element and merging those sublists in a manner that results into a sorted list. It uses divide-and-conquer as below:

- Divide the array in two parts by finding the middle position between left and right.
- Conquer by recursively sorting the subarrays in each of the two subproblems created by the divide step.
- Combine by merging the two sorted subarrays back into the single sorted subarray array.

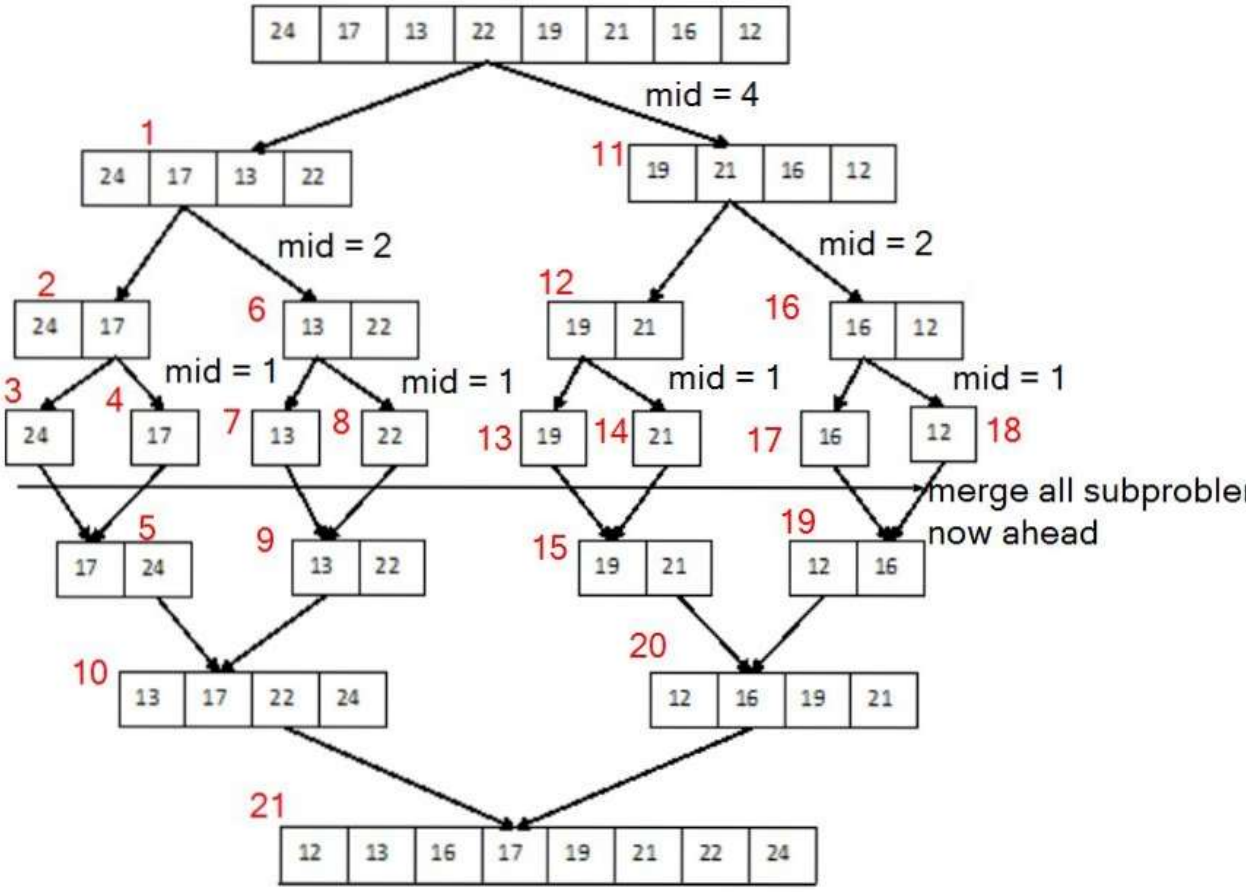
We need a base case. The base case is a subarray containing only one element, that is, when left = right, since a subarray with just one element is already sorted.

Let's see an example.

```
Let us start with array holding [24, 17, 13, 22, 19, 21, 16, 12],
so that the first subarray is actually the full array, array[0..7] (left=0 and right=7).

This subarray has more than two elements, and so it's not a base case.
In the divide step, we compute middle=3 and divide the array in two parts as array1 [0 to 3]
and array2 [4 to 7].

Now array1 [24, 17, 13, 22] is divided using same approach.
It will be divided in two array from middle as array11 [0 to 1] and array12 [2 to 3].

Now array11[24, 17] will be divided in two arrays array111[0 to 0] and array112[1 to 1].
```

```
These two arrays are containing 1 element each 24 & 17 respectively, so they are already
sorted.
They will be merged so that array11[0 to 1] will contain items sorted as array11[17, 24] after
merging step.

The algorithm go back to do same steps on array12[13, 22] array then on array2[19, 21, 16, 12]
array.
And at last, whole array will be sorted in this manner.
```

Following figure shows the step by step procedure for merge sort on this array: -



The numbers in red color are showing the sequence of steps the merge sort procedure will take to sort this array. The algorithm for merge sort is as below: -

```
If left < right
   Find the middle point to divide the array into two halves: m = (l+r)/2
   Call mergeSort for first half
   Call mergeSort for second half
   Merge the two halves sorted in step c and d:
End if
```

Following is the implementation of Merge sort: -

Implementation of Merge sort

C

```c
#include<stdio.h>

void print_array(int array[],int left,int right)
{
  int i;
  for (i=left; i <= right; i++)
    printf("%d\t", array[i]);
  printf("\n");
}

void merge(int array[], int left, int mid, int right)
{
  int i, j, k;
  int n1 = mid - left + 1;
  int n2 =  right - mid;

  int Left[n1], Right[n2];

  /* Copy data to temp arrays L[] and R[] */
  for (i = 0; i < n1; i++)
    Left[i] = array[left + i];
  for (j = 0; j < n2; j++)
    Right[j] = array[mid + 1+ j];

  i = 0; // Initial index of first subarray
  j = 0; // Initial index of second subarray
  k = left; // Initial index of merged subarray
  while (i < n1 && j < n2)
  {
    if (Left[i] <= Right[j])
      array[k++] = Left[i++];
    else
      array[k++] = Right[j++];
  }

  /* Copy the remaining elements of L[], if there are any */
  while (i < n1)
    array[k++] = Left[i++];

  /* Copy the remaining elements of R[], if there are any */
  while (j < n2)
    array[k++] = Right[j++];
}

void mergeSort(int array[], int left, int right)
{
  int i;
  if (left < right)
  {
    int mid = left + (right - left) / 2;
    printf("middle element is %d at index %d: \n",array[mid],mid);
    // Sort left and right halves
```

```
52
53      printf("Left half : \n");
54      print_array(array,left,mid);
55      mergeSort(array, left, mid);
56      printf("Right half : \n");
57      print_array(array,mid+1,right);
58      mergeSort(array, mid+1, right);
59
60      merge(array, left, mid, right);
61      printf("After merging sorted sub-array : \n");
62      print_array(array,left,right);
63    }
64  }
65
66
67  int main()
68  {
69    int array[] = {24, 17, 13, 22, 19, 21, 16, 12};
70    int array_size = sizeof(array)/sizeof(array[0]);
71    int i;
72
73    printf("Given array is :\n");
74    print_array(array,0,array_size-1);
75    mergeSort(array, 0, array_size - 1);
76
77    printf("\nSorted array is :\n");
78    print_array(array,0,array_size-1);
79    return 0;
80  }
81
```

Output: -

```
Given array is :
24   17   13   22   19   21   16   12
middle element is 22 at index 3:
Left half :
24   17   13   22
middle element is 17 at index 1:
Left half :
24   17
middle element is 24 at index 0:
Left half :
24
Right half :
17
After merging sorted sub-array :
17   24
Right half :
13   22
middle element is 13 at index 2:
Left half :
13
Right half :
22
After merging sorted sub-array :
13   22
```

```
After merging sorted sub-array :
13   17   22   24
Right half :
19   21   16   12
middle element is 21 at index 5:
Left half :
19   21
middle element is 19 at index 4:
Left half :
19
Right half :
21
After merging sorted sub-array :
19   21
Right half :
16   12
middle element is 16 at index 6:
Left half :
16
Right half :
12
After merging sorted sub-array :
12   16
After merging sorted sub-array :
12   16   19   21
After merging sorted sub-array :
12   13   16   17   19   21   22   24

Sorted array is :
12   13   16   17   19   21   22   24
```

The output above shows the divide and merge step in each iteration. The algorithm always divide the original array into two halves. The recurrence equation for merge sort is as below: -

```
T(n) = 2.T(n/2) + n
```

Every time 2 problems of size n/2 are done so 2.T(n/2) and at each step n elements to be merged so n is added at end.

We can solve this equation using any recurrence solving method and complexity of this algorithm is **O(n.Log n)**.

Properties of Merge sort

Worst and Average Case Time Complexity: O(n Log n).

Best Case Time Complexity: O(n Log n). No specific best case for Merge sort.

Auxiliary Space: O(n), as we have to divide the array into two halves and repeat the procedure.

Sorting In Place: Yes

Stable: Yes

It is popularly used when sorting data is huge in size as it can be performed as an external sort easily. Also when data is not in form of array for example, to sort two linked list of size m,n respectively merge sort is popularly used.