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TUTORIAL

Divide & Conquer - Quick Sort

Chapter

It is another divide and conquer approach. It is slightly different from merge sort as it will not pick the middle element always. Instead it picks a random element called as pivot element. And then it rearrange the array so that pivot element divide the array in two halves such that all elements lesser than pivot element are on left side of pivot element and all elements greater than the pivot element are on right side of pivot element. It then do the same thing recursively on both these halves. This way it can do the sorting as an in-place sort. Obviously if the division is not proper then complexity may increase. So for quicksort the picking of pivot element is very important and tricky. The best and average case complexity of it is O)n Log n), whereas in worst case it go to O(n^2). But the space complexity is O(Log n) which is better in comparison to merge sort.

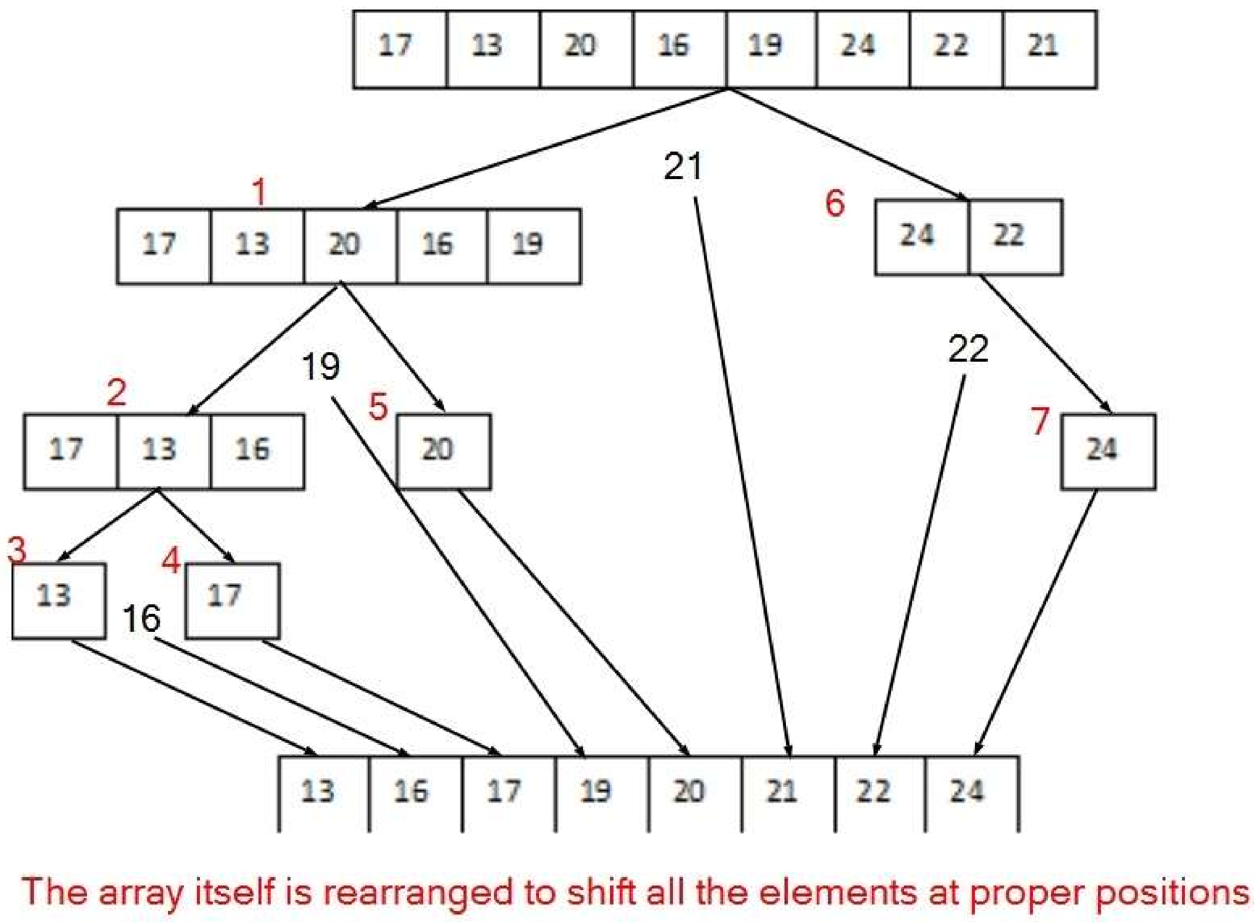
Let's see an example on quick sort.

```
Let's start with array holding [17, 13, 20, 16, 19, 24, 22, 21] 8 values, for the
sake of simplicity,
the pivot is picked as the rightmost element usually (In implementations we can
chose any element randomly but deterministically).

So in 1st iteration the pivot element is 21,
Now the array will be rearranged such that 21 will be at proper position and
all elements lesser will be on left of 21 and all elements greater will be on right
of 21.

There are many ways to do it, so we can choose any method and get it done as below: -
[17, 13, 20, 16, 19, 21, 24, 22]
Now array is splitted into two halves, and quicksort will repeat the same thing on
these two halves recursively,
till the whole array is sorted.
```

Following figure shows the step by step procedure for quick sort on this array: -



The numbers in red color are showing the sequence of steps the quick sort procedure will take to sort this array. The algorithm for quick-sort is as below: -

```
If left < right
   Pick the pivot element from the list
   Split the list into two halves according to the pivot element.
   Sort the left half repeatedly using this approach.
   Sort the right half repeatedly using this approach.
End if
```

Following is the implementation of quicksort in languages: -

Implementation of quicksort

```c
#include<stdio.h>
```

C

```

// To print array of length=size
void printArray(int array[], int size)
{
  int i;
  for (i=0; i < size; i++)
    printf("%d\t", array[i]);
  printf("\n");
}

// To swap two elements
void swap(int* a, int* b)
{
  int t = *a;
  *a = *b;
  *b = t;
}

/* This function takes last element as pivot, and places all smaller
   elements to left of pivot and all greater elements to right of pivot
*/
int partition (int array[], int low, int high)
{
  int pivot = array[high];    // pivot
  int i = (low - 1);  // Index of smaller element
  int j;

  for (j = low; j <= high- 1; j++)
  {
    if (array[j] <= pivot)
    {
      i++;            // increment index of smaller element
      swap(&array[i], &array[j]);    // shift all lesser elements in
left half
    }
  }
  swap(&array[i + 1], &array[high]);    // place pivot element at end
of smaller elements
  return (i + 1);    // the index of pivot element
}

// Recursive function to sort array with quicksort
void quickSort(int array[], int low, int high)
{
  if (low < high)
  {
    int pivot_index;
    pivot_index = partition(array, low, high);
    printf("Pivot element is %d \n",array[pivot_index]);
    printf("Array after pivot partitioning : \n");
    printArray(array,8);

    // Call quicksort() on left half and right half excluding
    // pivot element as it is already at proper position i.e.
    // between lesser and greater elements.
    quickSort(array, low, pivot_index - 1);
    quickSort(array, pivot_index + 1, high);
  }
}
int main()
{
  int array[] = {17, 13, 20, 16, 19, 24, 22, 21};
  int n = sizeof(array)/sizeof(array[0]);
  printf("Given array: \n");
  printArray(array, n);
  quickSort(array, 0, n-1);
  printf("Sorted array: \n");
  printArray(array, n);
  return 0;
}

```

Output: -

```
Given array:
17   13   20   16   19   24   22   21
Pivot element is 21
Array after pivot partitioning :
17   13   20   16   19   21   22   24
Pivot element is 19
Array after pivot partitioning :
17   13   16   19   20   21   22   24
Pivot element is 16
Array after pivot partitioning :
13   16   17   19   20   21   22   24
Pivot element is 24
Array after pivot partitioning :
13   16   17   19   20   21   22   24
Sorted array:
13   16   17   19   20   21   22   24
```

The output above shows the pivot element and splitting of array in each iteration. The algorithm divide the original array into two halves. The recurrence equation for quicksort is as below: -

```
T(N) = T(i) + T(N - i -1) + cN
```

The time to sort the file is equal to

```
the time to sort the left partition with i elements, plus
the time to sort the right partition with N-i-1 elements, plus
the time to build the partitions
```

So in worst case the pivot is smallest or largest element of the array so the recurrence equation becomes

```
T(N) = T(N-1) + N which belongs to O(n^2) class of complexity, whereas in best and
average case the equation becomes: -
T(N) = 2.T(N/2) + N which is same as Merge sort and belongs to O(n Log n) class.
```

The selection of pivot element and adjusting the array according to pivot element is a key part in quicksort. The selection of pivot element should be random in implementations to avoid the worst case partition. Also we can implement the partition() function in multiple ways. The main objective of partition() function is to place the pivot element at a position such that all lesser elements will be on left side of pivot and all greater elements must be on right side of pivot element. So that the pivot should be at its proper position in final ordering. We can achieve it in multiple ways as described below: -

a. We can start from the leftmost element and keep track of index of smaller (or equal to) elements as pivot say small_idx. While traversing, if we find a smaller than pivot element, we increment small_idx and swap the element with arr[small_idx]. Otherwise we ignore current element. At last we can swap the pivot element with element at array[pivot_idx]. It can be explained as:

```
array[] = {30, 80, 20, 40}
Indexes: 0  1  2  3
low = 0, high = 3, pivot = arr[high] = 40
Initialize index of smaller element, small_idx = -1
Traverse elements from j = low to high-1   // j will loop from 0 to 2
j = 0 : Since array[j] <= pivot, increment small_idx and swap(arr[small_idx], arr[j])
small_idx = 0
array[] = {30, 80, 20, 40} // No change as 10 is already the first element
j = 1 : Since array[j] > pivot, do nothing.      // No change in i and arr[]
j = 2 : Since array[j] <= pivot, increment small_idx and swap(arr[small_idx], arr[j])
small_idx = 1
array[] = {30, 20, 80, 40}    // We swap 80 and 20
j = 3 : We come out of loop because j is greater than high-1. Finally we place pivot
at correct position by swapping array[small_idx+1] and pivot
array[] = {30, 20, 40, 80}    // 80 and 40 Swapped
Now 40 is at its correct place. All elements smaller than 40 are before it and all
elements greater than 40 are after it.
```

b. Another way is keep the left pointer to first index of array and right pointer to last index of the array. Now keep incrementing left till we get elements lesser than pivot. And keep decrementing right till we get elements greater than pivot. Now either the two indexes will cross each other. If they do then stop, otherwise swap the elements at these two index and continue. Following is an sample implementation of it:

```
pivot = array[end]
left = start
right = end - 1
completed = False
while not completed:
      while left <= right and array[left] <= pivot:
          left = left + 1
      while array[right] >= pivot and right >=left:
          right = right -1
      if right < left:
          Completed = True
      Else:
              swap (array[left], array[right])
end
swap(array[left], pivot)
```

Similarly, there can be more ways to implement the partition() function. Each implementation is correct if it achieves the goal of partition() function. You can try any other approach also.

Properties of Quick sort

Worst Case Time Complexity: O(n^2). If pivot element is smallest or largest element of the array.

Best and Average Case Time Complexity: O(n Log n). When the pivot element is median of the array, which split the array from middle.

Auxiliary Space: O(Log n), as array will be partitioned in the same but space is required for recursive calls only.

Sorting In Place: Yes

Stable: Generally not but stable version exist, that depends on implementation.

One of the fastest algorithms on average. Commercial applications use Quicksort whereas generally quicksort is not used in applications which require guaranteed response time. Because of its popularity, most programming languages providing a default sort function implements quicksort in best possible way like qsort() function C language.