

4/10/2023

(1)

Remember about learning dif eqs

u : Diff Functions from $[t_0, T]$ to \mathbb{R}^n :
 $t \rightarrow u(t)$

p : Parameters $p \in \mathbb{R}^2$

F : Zeros of $F \equiv$ solve the dif eq with params p

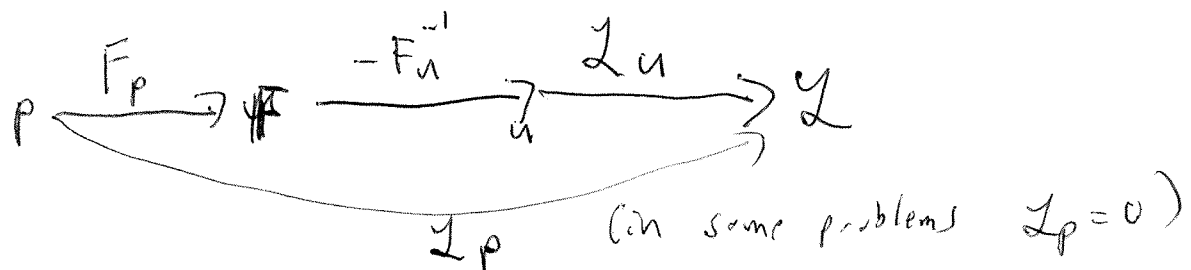
$$F(u, p) = u'(t) - f(u, p, t)$$

To specify the ode we need f and
 boundary condition $u(t_0) = 0$

\mathcal{L} : Loss

$$\mathcal{L}(u, p) = \text{scalar}$$

Remember p determines $\mu(t)$ determines $\mathcal{L}(u(t), p)$
 and we wish to optimize over p .



(2)

Linear operators

F_p : takes a small change to $p \in \mathbb{R}^2$
and sees what happens to F
(a continuous function from $[t_0, T]$ to \mathbb{R}^n)

F_u : takes a small change to u / at time t it is
and sees what happens to F / an $n \times n$ matrix

Following Chris we take

$$L(u, p) = \int_{t_0}^T g(u, p) dt = \int_{t_0}^T g(u(t), p) dt$$

$$L_p = \int_{t_0}^T g_p(u(t), p) dt \quad g: \mathbb{R}^n \times \mathbb{R}^2 \rightarrow \mathbb{R}$$

L_u : measures how loss function reacts to
small changes to u at time $t = g_u(u(t), p)$

$$\text{Inner product } \langle u, v \rangle = \int_{t_0}^T u(t)^T v(t) dt$$

We want to derive

$$\frac{dL}{dp} = \underbrace{L_p}_{\int_{t_0}^T g_p(u, p) dt} - L_u(F_u^{-1} F_p) = L_p - \underbrace{(L_u F_u^{-1})}_{\lambda} F_p$$

$$\nabla_p L = L_p^T - F_p^T \underbrace{F_u^{-T} L_u}_{\lambda(t)}$$

(3)

Easy one

$$F_p^T \lambda = \cancel{F_p} \quad \langle F_p, \lambda \rangle = \int \lambda^T(t) f_p \, dt$$

What are F_M , F_u^T , F_u^{-T} ? $F_M^{-T} L_M$

$$F_u[du] = (du)' - f_u du$$

$$F_u: \quad v(t) \rightarrow \underbrace{v'(t)}_{\substack{\text{Function} \\ \text{of } \mathbb{R}^n}} - \underbrace{f_u(t)}_{\substack{\text{Function} \\ \text{of } \mathbb{R}^n}} v(t)$$

$$- \underbrace{f_u(t)}_{\substack{n \times n \\ \text{matrix} \\ \text{at time} \\ t}} \underbrace{v(t)}_{\substack{n \\ \text{vector} \\ \text{at} \\ \text{the} \\ t}}$$

$$F_u = \frac{d}{dt} - \underbrace{f_u}_{\substack{\text{non at} \\ \text{the } t}} \\ F_u^T = -\frac{d}{dt} - f_u^T$$

$$F_u^T \text{ is therefore } -\frac{d}{dt} - f_u^T$$

$$\text{Solve } F_u^T \lambda = L_M^T$$

$$\text{Means } -\frac{d\lambda}{dt} - f_u^T \lambda(t) = g_M^T(t)$$

$$\text{Analysis of } \left(\frac{d}{dt} \right)^T \text{ requires } \lambda(T) = 0$$