

# Electric Vehicle Charge Scheduling on Highway Networks from an Aggregate Cost Perspective

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**Abstract**—In this paper, we attempt to optimally schedule the charging of long-range battery electric vehicles (BEVs) along highway networks, in order to minimize aggregate costs to the system as a whole. Thus, we approach the problem from the perspective of both customers (EV car owners), as well as charging station operators and utilities.

## I. INTRODUCTION

### A. Motivation

The popularity of electric vehicles has been rising rapidly in recent years, driven mainly by the greater driving comfort, improved safety, cheaper maintenance and lower exhaust emissions they offer. As battery and materials costs continue to decline and we transition towards cleaner, more diversified and renewable electricity generation, the use of both plug-in hybrid and battery electric vehicles is likely to expand even more. Increased EV utilization rates also mean that we will need more sophisticated and effective techniques to schedule their charging in real-time - in order to minimize negative effects like congestion, long waiting times for customers or high costs and uneven demand for utilities and station operators.

### B. Literature Review

The work proposed thus far for EV scheduling has been focused on four areas: optimization with respect to load on power distribution networks [5,9,12,13,14], queuing models for input-output dynamics of charging stations [1,2,6,17], game-based approaches for trajectory optimization [2,4,8], and more traditional trajectory optimal control methods [3,7,10,11,15]. Note that power distribution problems either use game-based approaches or traditional optimal control methods but are considered distinct in the objective: the former looks at power quality on networks whereas the latter has objective functions related to time or expected financial costs.

The work done in power distribution considers the effect of EV charging on the grid, mainly with regard to base load. In this sense EVs are treated as dispatchable loads that can fill valley's during otherwise low demand. Other work has considered topological effects of charging station power draws [Do we have one?]. The work described in [5] uses statistical estimation to predict future EV arrivals and the associated charging demands in order to do load-shifting via model predictive control. [9] proposes a mean-field game-based approach for large EV populations. The objective is to fill the overnight valley in base load and does not consider EV routing. The work in [12] coordinates plug-in hybrid EV (PHEV) in order to reduce system power losses, maximize load factor, and minimize load variance. A decentralized approach

is proposed in [13] that focuses on load shifting via dispatch signals provided by a centralized authority. The issue of load shifting is considered in [14] with emphasis on circumventing load forecasting limitations. A locally optimal distributed approach is demonstrated. The work mentioned here focuses on optimization in the time domain—load shifting for valley filling throughout a day.

Many different queuing model approaches have been proposed for optimal EV scheduling. These works generally focus on modeling the arrival and departure/service rates of the EVs. While some approaches utilize probability theory as the foundation for the scheduling others rely on optimization. The work in [1] presents a first-come first-served queuing model with arrival rates corresponding to an exponential distribution and Poisson departure rates in a Markov chain. This does not account for location and path planning. The dynamics of the EV battery are not considered. A similar approach is shown in [2] where the objective is to minimize EV waiting time at charging stations. This makes more considerations around path planning between nodes but limits the road network to a one-dimensional, unidirectional flow. [6] also minimizes EV waiting time using a similar queuing model. Theoretical bounds on waiting time are presented as well as a communication strategy (vehicle-to-charging station). A battery-swapping station approach in [17] utilizes queues and also defines probabilities around customer dissatisfaction.

Trajectory optimization problems have been formulated for electric vehicle charging with an emphasis on finding efficient algorithms and sensible objective functions. In [3] a hierarchical approach is proposed that considers power system network dynamics and battery dynamics. Given a fleet of EVs the highest level problem is station location. The provisioning problem sits below this in which the number of chargers per station is allocated according to a queuing model and given demand. The final layer consists of a global cost minimization. The work in [7] focuses on a graph theoretic framework for EV scheduling. The work exploits previous classical work done in path planning to reduce the complexity of scheduling EV charging in large networks. [10] details heuristic search algorithms for solving the EV-routing problem on a road network with charging stations located at specific nodes. A mixed approach is described in [11] whereby probabilistic tools, power distribution concerns, and an objective function formulated for routing efficiency is formulated. More specifically, the problem maximizes the station profits while maintaining a high equality of service. Finally, the work in [15] utilizes finite horizon and receding horizon approaches to consider the two cases of 1) a priori knowledge of all users and 2) new inputs (vehicles) to the system during the horizon

for vehicle routing on a charging network. The objective function takes into consideration the customer's perspective in an attempt to satisfy demand while keeping the cost low. Here a variable charging rate is implemented, which adds complexity to the required infrastructure and solution space but enables more interesting solutions.

### C. Proposed Contributions (for proposal purposes)

We propose three novel contributions to solving the optimal EV charging problem:

(i) Aggregating costs from the perspective of all three stakeholders or agents participating in the network. Most of the previous work in this area has focused on optimizing the scheduling process separately with respect to each group i.e. either (a) minimizing only the costs, inconvenience and time spent by the customer, or (b) maximizing profits to the station operator or (c) minimizing distribution losses to the grid. In this group, we build a cost function that incorporates all three of the above factors. We hypothesize that by tackling the problem globally, we can arrive at a solution that is also locally optimal for each group individually.

(ii) Formulation of this charging network as a hybrid system for each of the electric vehicles. Each EV transitions between three discrete states of charging, waiting at a station and discharging (while driving). The charging and discharging modes also have their own continuous states associated with the battery dynamics. After solving the original problem using traditional convex optimization techniques, we hope to apply reachability analysis to the hybrid system in order to obtain a fuller set of potentially feasible solutions, through either forward or back-propagation.

(iii) For computational purposes, we will also need to reformulate this problem as a mixed integer program using innovative methods, in order to develop a real-time control law that directs two decision variables for each car (i)  $\in \{0, 1, 2\}$  whether to charge, wait at or leave a given node  $i$  and (ii)  $\in \{0, 1, 2, \dots, n\}$  what edge  $j$  to travel along from the current node to the next, with  $n$  being the total number of links that end at node  $i$ . We hope to draw inspiration from the use of mixed integer linear programming (MILP) for trajectory optimization of UAVs and aircrafts in [18] and [19], and apply it to solving our new mixed integer quadratic program.

## II. TIMELINE (FOR PROPOSAL PURPOSES)

Having finalized our system model and our formulation of the mixed integer quadratic program, we will now proceed to solving the preliminary problem using the Gurobi solver in MATLAB by early-to-mid April. In doing so, we shall also refine and formalize our problem description more, and design a centralized control scheme that determines the optimum trajectory for EVs, and also dictates the actions of all agents (EVs, charging stations) at least within a particular neighbourhood of the network. Solving the optimal control problem for each sub-region will also result in a larger network that's optimal as a

whole. These optimal trajectories will also need to be updated at each time step to correct model mismatch, thereby requiring some form of MPC. We will also be testing our model using real traffic data on a few of the highway networks presented in [3]. We then plan to spend the rest of April trying out more experimental, hybrid-systems based approaches such as reachability, at least on a high level.

## III. MODEL

The following model definitions and formulations are preliminary efforts to begin to formalize the MIQP problem. The exact notation has to be finalized and thus subscripts and functions are at times ambiguous; these will be formalized according to common conventions once the problem becomes more clear and the tractability of it helps to define the feasibility.

### A. Definitions

$$c = \{c | c \in \{1, \dots, c, \dots, p\}\},$$

where  $p$  is the number of cars

$$k = \{k | k \in \{1, \dots, k, \dots, Hp\}\}$$

where  $Hp$  is the prediction horizon

$$i = \{i | i \in \{1, \dots, i, \dots, N\}\}$$

$v_{0,i} :=$  is the starting node for car  $i$

$v_{0,f} :=$  is the ending node for car  $i$

### B. Network Model

The network model will be an undirected graph representing a highway network. Nodes may or may not have an EV charging station on it and the edges/links connecting these represent highway roads. The weights of the edges will be related to the energy and time to traverse the edge. The adjacency matrix of the graph is only important insofar as it dictates how one can travel from one node to another (i.e. the graph is not fully connected).

$$C_{\text{electricity}} = \sum_{\nu} \sum_t c(t, \text{location}) x(SOC_f - SOC_i) \bar{E}_{\text{capacity}} \quad (1)$$

$$C_{\text{station}} := (\sum P_{\text{charge}})^T U (\sum P_{\text{charge}}) \text{ (TBD)} \quad (2)$$

### C. EV Model

The EV model will define three discrete states from a hybrid theoretic perspective. Namely, there is a charging mode, driving mode, and a waiting mode (at the charging station). This allows for description of the battery state of charge as a piece-wise function. The costs to the customers will be the time charging, the time to drive along a specified path, the degradation of the battery, and the time to wait in a queue at a full charging station.

$$\begin{aligned}
C_{cstmr} &:= C^{t_{charging}}(\Delta SOC) + C^{t_{driving}}(e_j) \\
&\quad + C^{t_{degrade}}(\Delta SOC) + C^{t_{waiting}}(l_{queue}) \\
C^{t_{driving}}(e_{ij}) &:= \begin{cases} \frac{(E_c - E_i)}{P_{max}} + \frac{1}{n} \log\left(\frac{m_i - n_i E_c}{m_i - n_i E_f}\right), & E_i < E_c \\ \frac{1}{n} \log\left(\frac{m_i - n_i E_i}{m_i - n_i E_f}\right), & \text{else} \end{cases} \\
C_{degrade} &:= \left( a(\mathbb{E}(P_{ow}))^2 + b * \mathbb{E}(P_{ow}) + c \right) \mathbb{E}(t_{charging}) \\
C_{t,waiting} &:= fn(\text{arrival rate, blocking probability}) \text{ [TBD]}
\end{aligned}$$

#### IV. OPTIMIZATION PROBLEM

$$\min_{x, r \in \{0,1\}^{Hp}} C_{cstmr} + C_{electricity}(t, j) + C_{station}(P_i) \quad (3)$$

$$\text{s.t. } \underline{SOC} \leq SOC_{k,c} \leq \overline{SOC}, \forall c \quad (4)$$

$$0 \leq P_{k,i}^{charge} \leq \overline{P_i^{charge}} \quad (5)$$

$$SOC_j + SOC_i - \frac{e_{ij} \Delta t}{e_{ij}} \geq m, \forall (i, j) \in \mathcal{E} \quad (6)$$

$$SOC_{0,c} = 1, \forall c \quad (7)$$

$$\sum_k t_{k,c} \leq \alpha \underline{T_c}, \alpha > 1, \forall c \quad (8)$$

$$v(t=0) = v_{0,c}, \forall c \quad (9)$$

$$v(t=t_f) = v_{f,c}, \forall c \quad (10)$$

$$(11)$$

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