

Coordination of distributed energy resources for a reliable, resilient, and affordable decarbonized grid

PhD Thesis Defense
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MIT-Pillar
AI Collective



ENVIRONMENTAL
SOLUTIONS
INITIATIVE



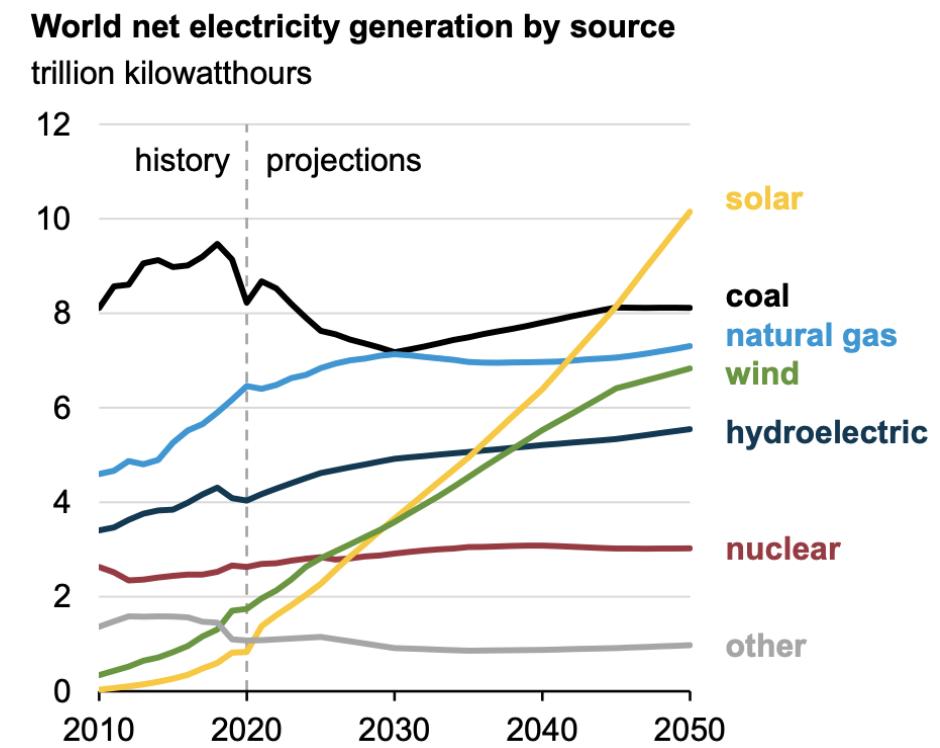
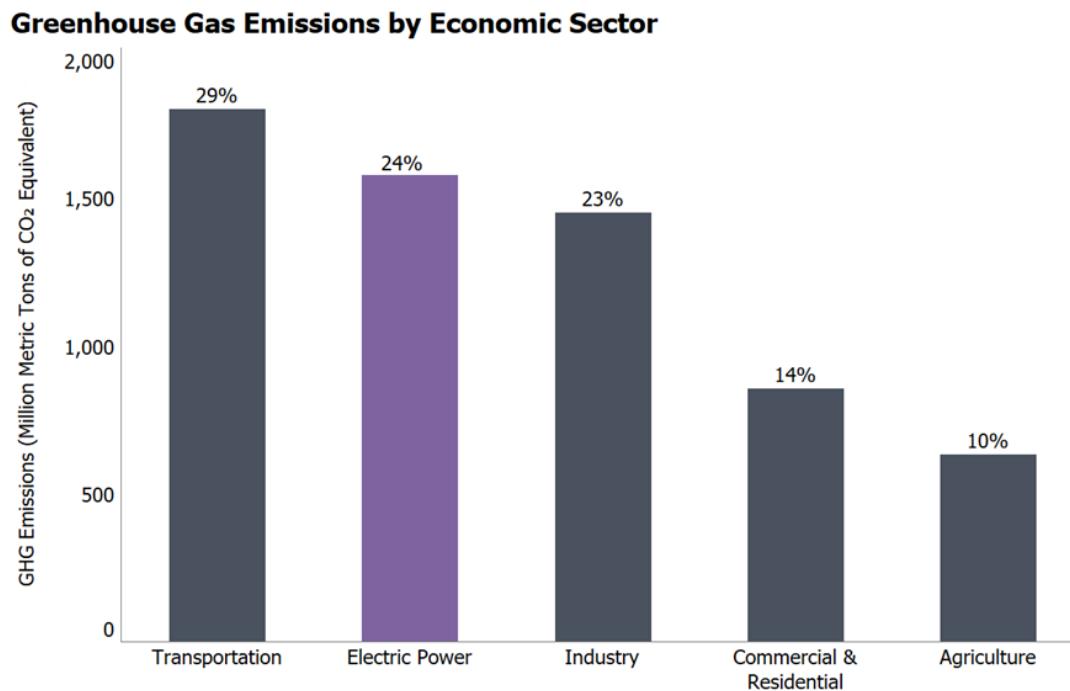
the center for
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science & engineering

Active-Adaptive Control Laboratory
Department of Mechanical Engineering, MIT

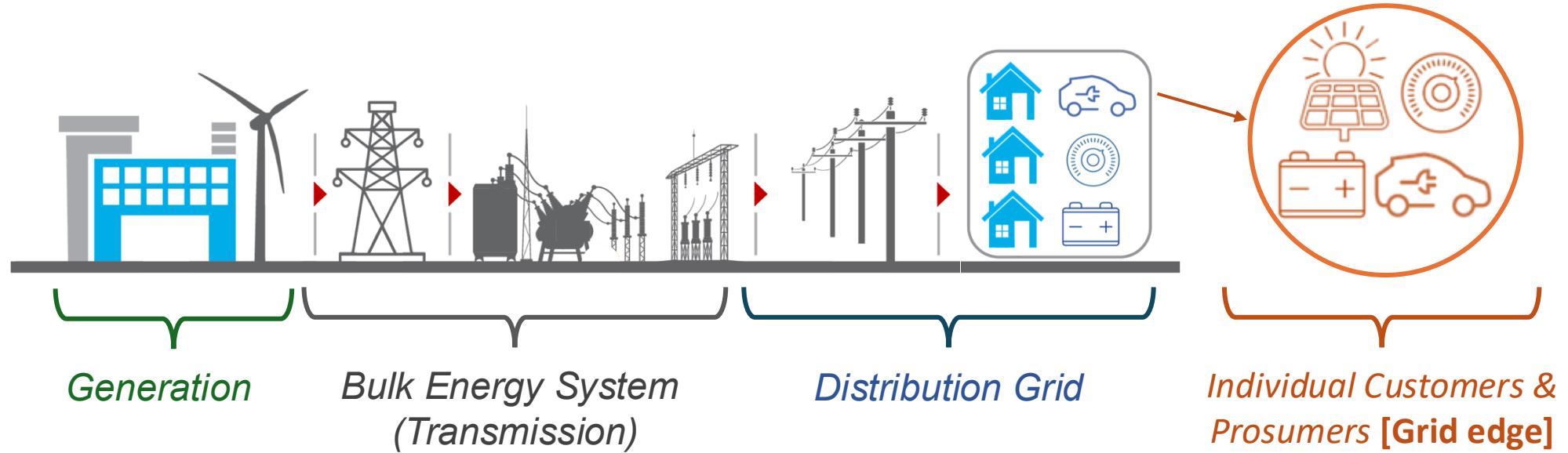


Grid decarbonization for climate action

- Reach net-zero economy-wide by 2050
- 100% clean electricity by 2035

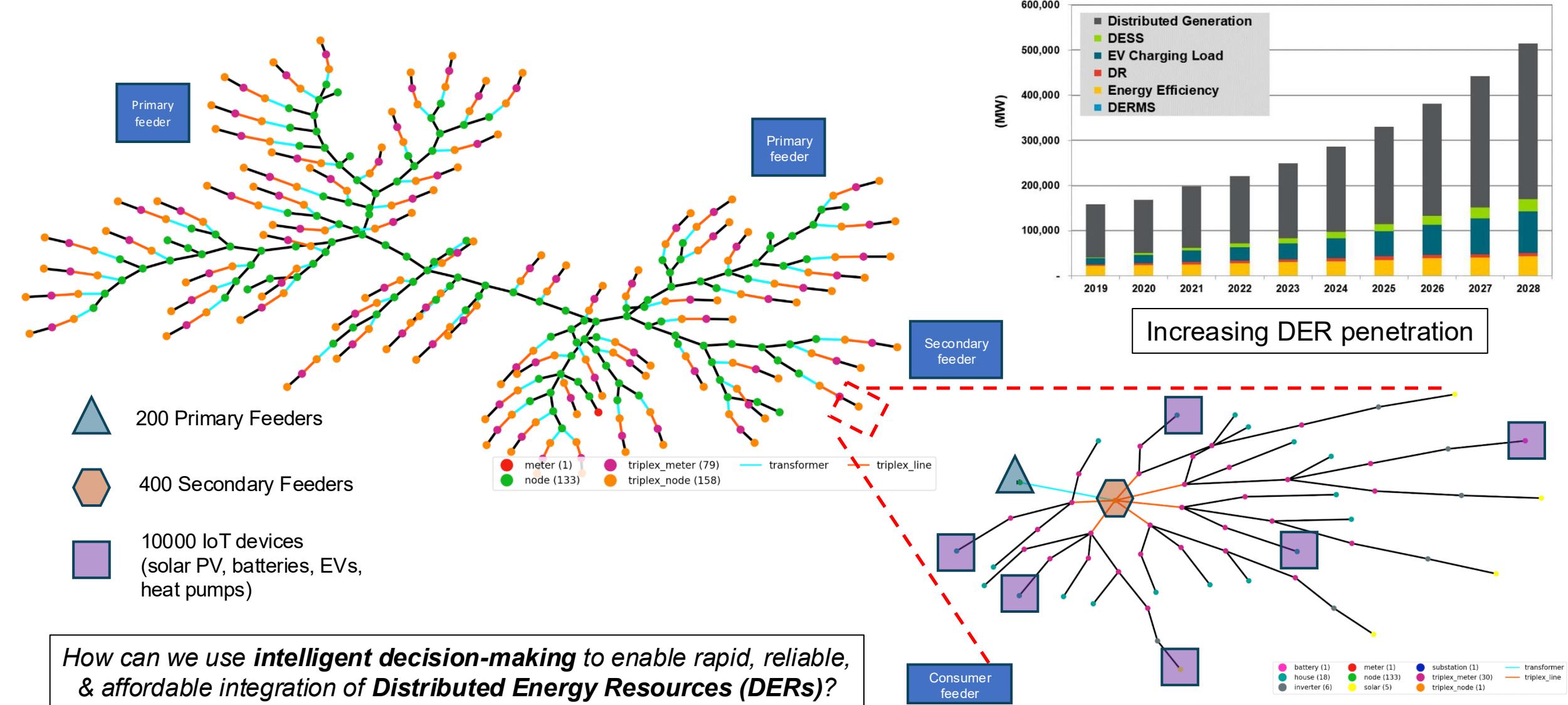


What's the grid edge



- Increasing penetration of **distributed energy resources (DERs)**: Solar PV, heat pumps, batteries, electric vehicles, etc.
- Grid edge is transforming due to many new stakeholders
- Prossumers, DER aggregators, virtual power plants etc.

Grid edge becoming more complex, intelligent, capable



How can we use **intelligent decision-making** to enable rapid, reliable, & affordable integration of **Distributed Energy Resources (DERs)**?

Challenges with the future grid edge

1. DERs are generally autonomous & independently owned:
Utilities or grid operators can't directly control them
2. Decentralized grids will have millions of DERs:
Difficult to coordinate resources & manage grid
3. Renewable intermittency, uncertainty, & variability:
Reliability & stability issues, along with inefficiencies
4. Rapid growth of renewables, storage & electrified demand:
High grid stress hinders decarbonization & raise costs

Solutions from my thesis

1. DERs are generally autonomous & independently owned:
New markets & price signals to incentivize agents
2. Decentralized grids will have millions of DERs:
Hierarchical market designs & scalable optimization tools
3. Renewable intermittency, uncertainty, & variability:
Use transactive framework to provide valuable grid services
4. Rapid growth of renewables, storage & electrified demand:
Coordinate DERs to dynamically increase grid capacity

Overall thesis summary & contributions

[2] Grid services using transactive framework

- Coordinate DERs to provide valuable grid services like voltage regulation
- Derived accurate pricing decomposition
- Generalized to different networks using multiple power flow models

[1] Hierarchical local retail electricity market

- Decentralized and distributed multiobjective optimization algorithms
- Increased efficiency; reduce losses, costs & retail rates

Data-driven decision-making tools & coordination for a decarbonized & distributed grid

[5] Enhance grid hosting capacity

- Apply market-based coordination to increase dynamic hosting capacity & enable flexible interconnection
- Accurately account for uncertainty
- Realistic case studies with varying levels of DER penetration

[3] Game-theoretic analysis & mechanism design

- Extract DER flexibility with Stackelberg incomplete information game
- Detailed flexible DER models with multiperiod optimization & intertemporal constraints
- Derived analytical equilibria with closed form solutions for market operators & agents

[4] Distributed IoT coordination for grid resilience

- Detect & mitigate cyber-physical attacks of different types & scales using local flexibility & grid reconfiguration
- Collaborated with external partners to extensively validate simulation results using industry-grade software & hardware-in-the-loop
- Large-scale simulations with thousands of IoT devices

Research outputs



3 primary
journal papers



4 primary
conference papers



1 primary book
chapter



4 primary
workshop papers



1 secondary
journal paper



3 projects: US DOE,
MIT Energy Initiative

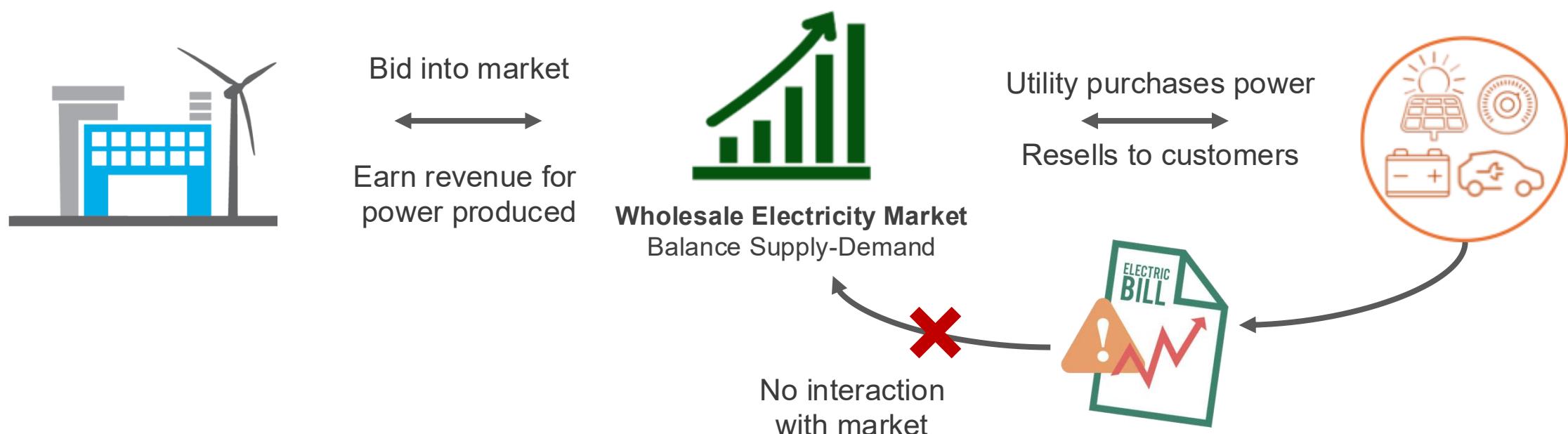
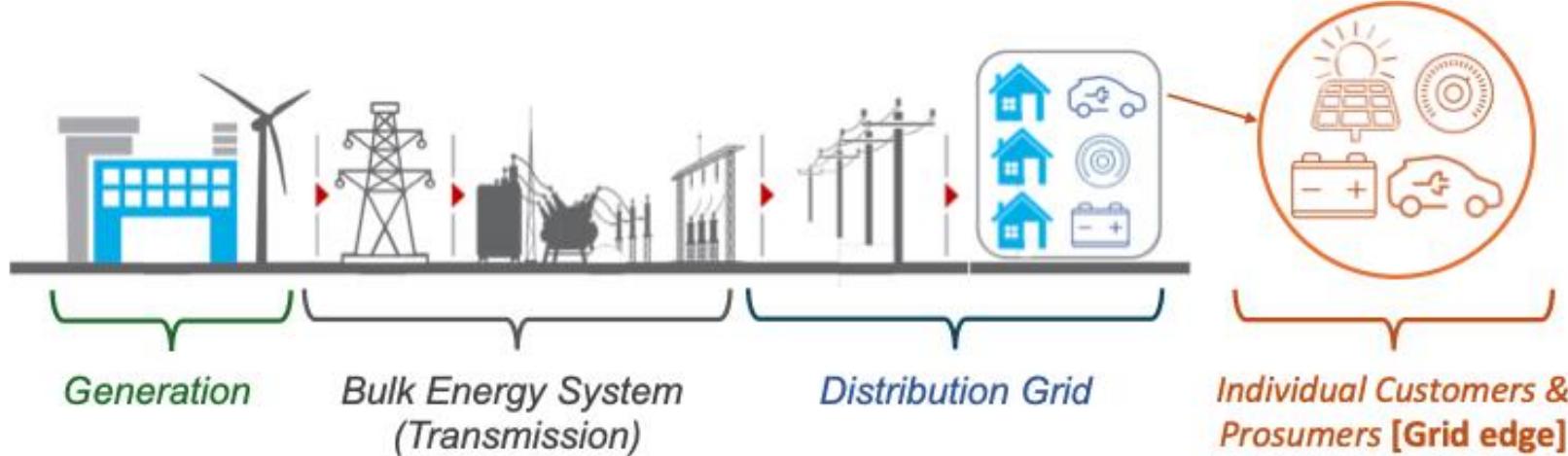


Supervised 4
master's &
undergraduates



5 papers in
preparation

Why do we need local electricity markets?

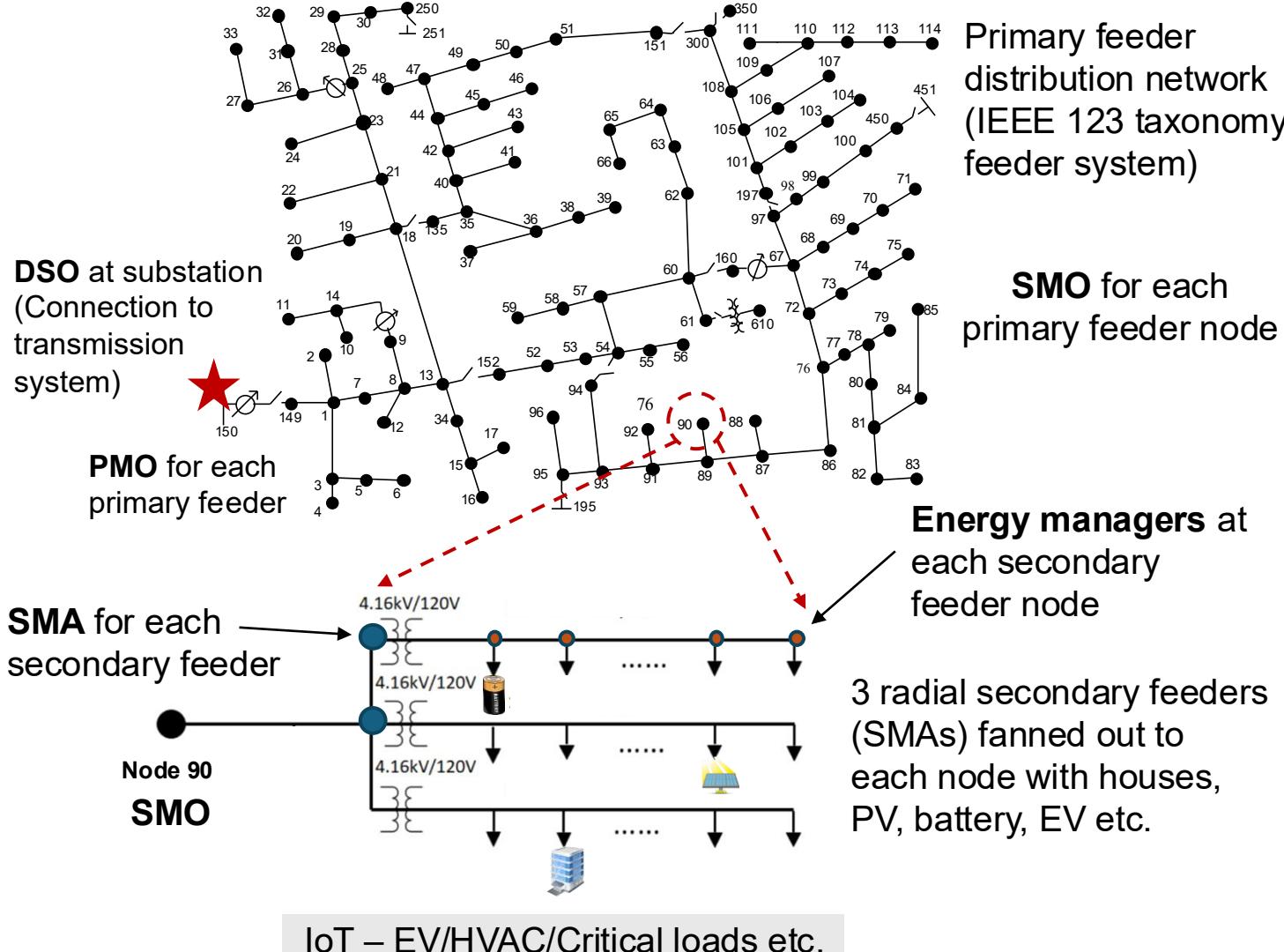


Our proposal: Redesign markets for DERs



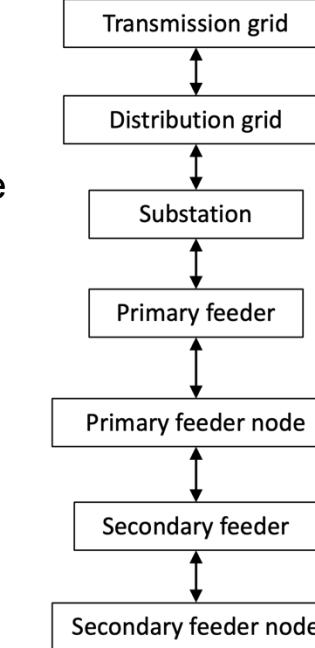
Use markets & prices to influence desired behaviors from various autonomous, independent DERs at grid edge, at fast timescales

Our suite of hierarchical local electricity markets

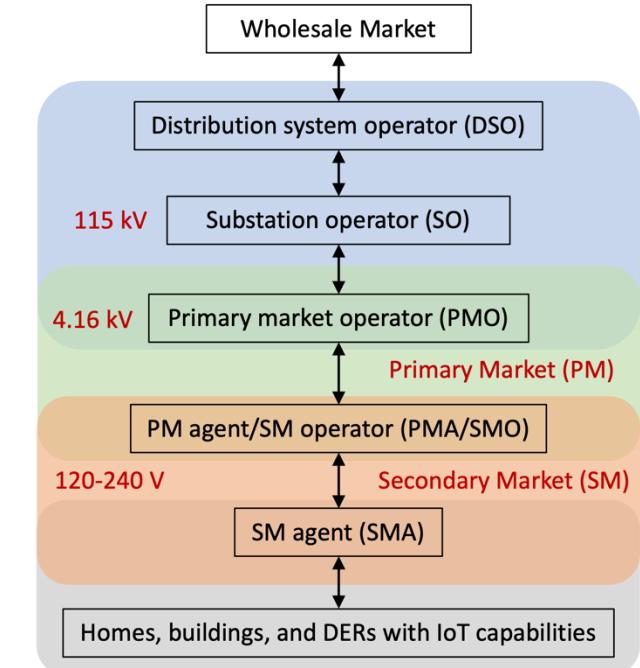


Primary feeder distribution network
(IEEE 123 taxonomy feeder system)

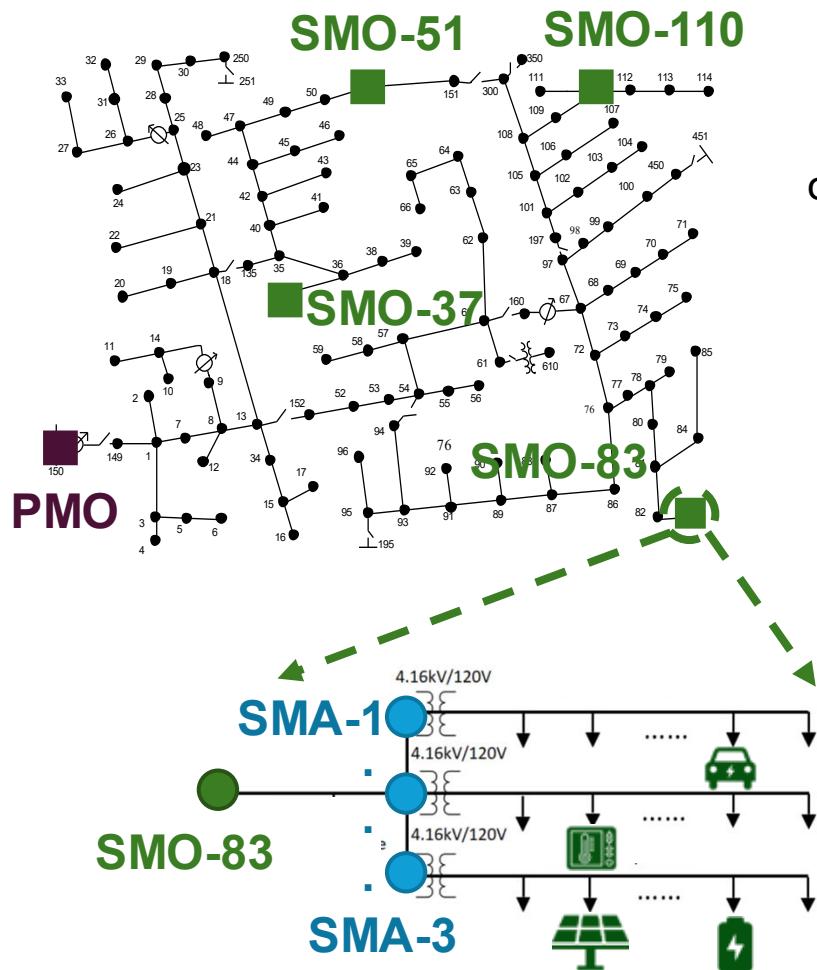
Grid infrastructure



Market infrastructure



Summary of markets



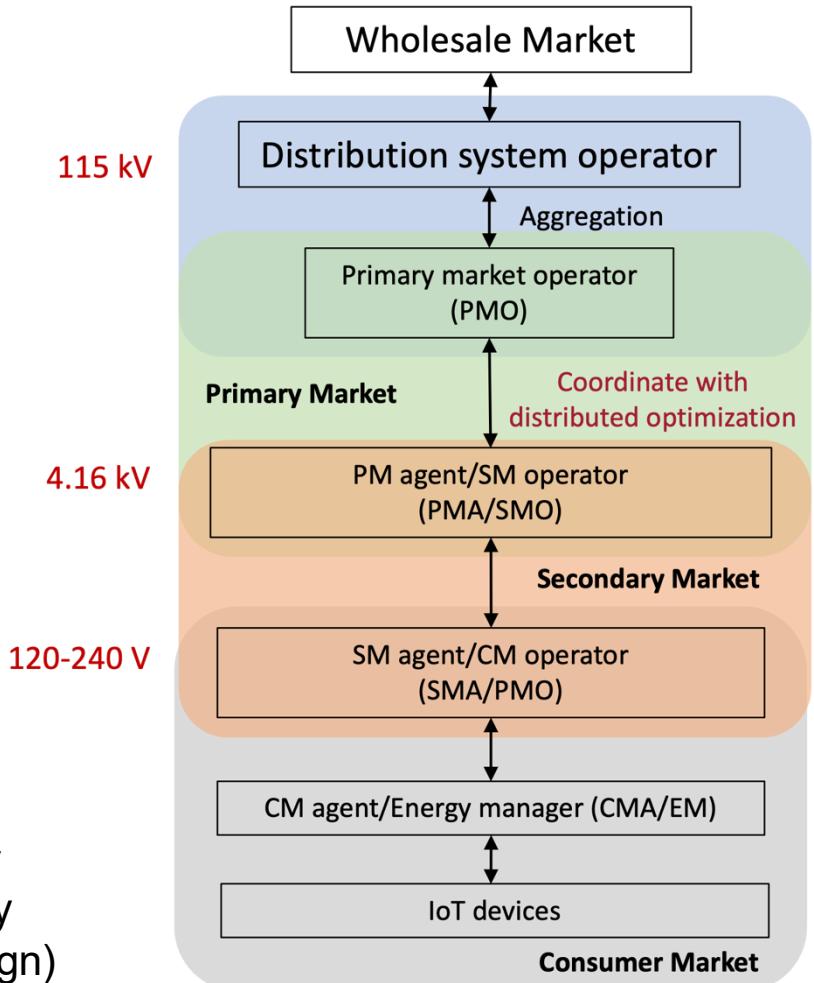
Hierarchical structure:
Accommodate concerns for
market stakeholders and grid
operators at all levels of the grid

Primary market: Power physics
and distribution-level constraints
(Distributed optimization)

Secondary market:
Commitment reliability, utility,
flexibility & budget constraints
(Decentralized optimization)

Consumer market: Prosumer
preferences & end-user privacy
(Game theory & mechanism design)

LEM provides situational awareness at
both primary & secondary feeder levels



Overall thesis summary & contributions

Primary & secondary markets

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Data-driven decision-making tools & coordination for a decarbonized & distributed grid

Consumer market

[3] Game-theoretic analysis & mechanism design

- Extract DER flexibility with Stackelberg incomplete information game
- Detailed flexible DER models with multiperiod optimization & intertemporal constraints
- Derived analytical equilibria with closed form solutions for market operators & agents

[4] Distributed IoT coordination for grid resilience

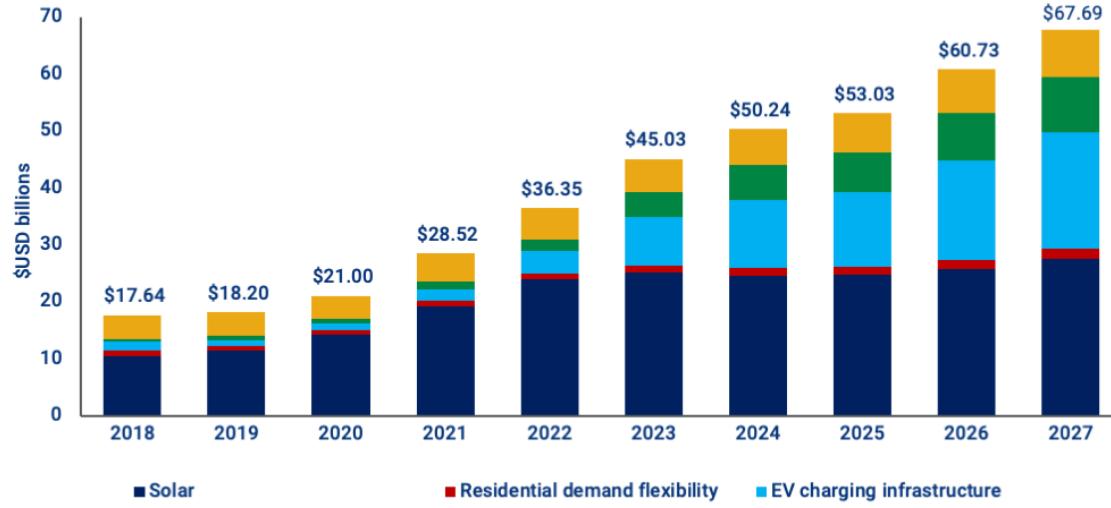
- Detect & mitigate cyber-physical attacks
- Successfully resolve attacks of different types & scales using local flexibility & grid reconfiguration
- Collaborated with external partners to extensively validate simulation results using industry-grade software & hardware-in-the-loop
- Large-scale simulations with thousands of IoT devices

[5] Enhance grid hosting capacity

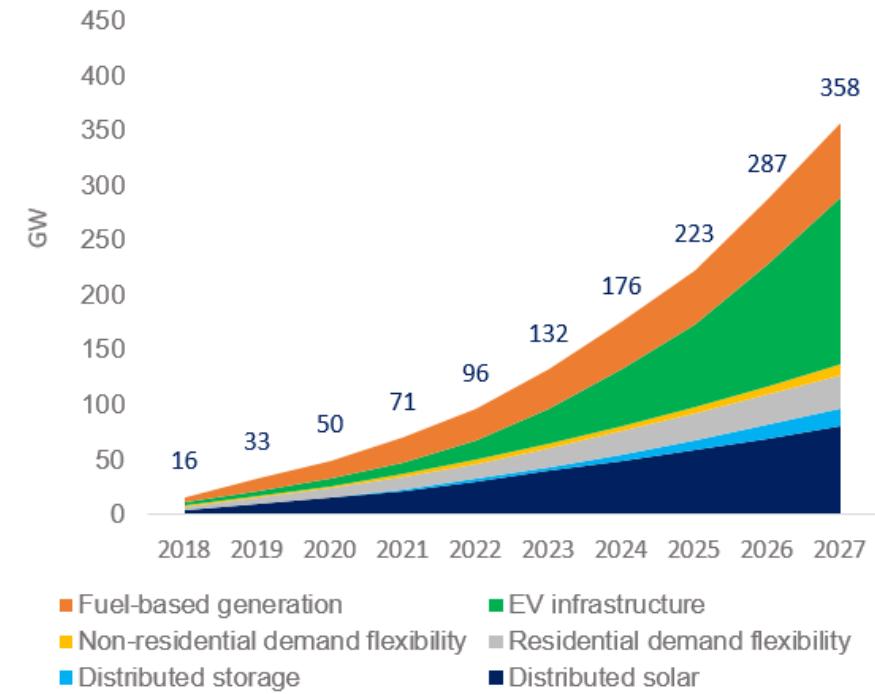
- Apply market-based coordination to increase dynamic hosting capacity & enable flexible interconnection
- Accurately account for uncertainty
- Realistic case studies with varying levels of DER penetration

DERs are growing rapidly

The US DER market will nearly double from 2022 to 2027, reaching US\$68 billion per year



Source: Wood Mackenzie Grid Edge, US Distributed Solar and Energy Storage Service



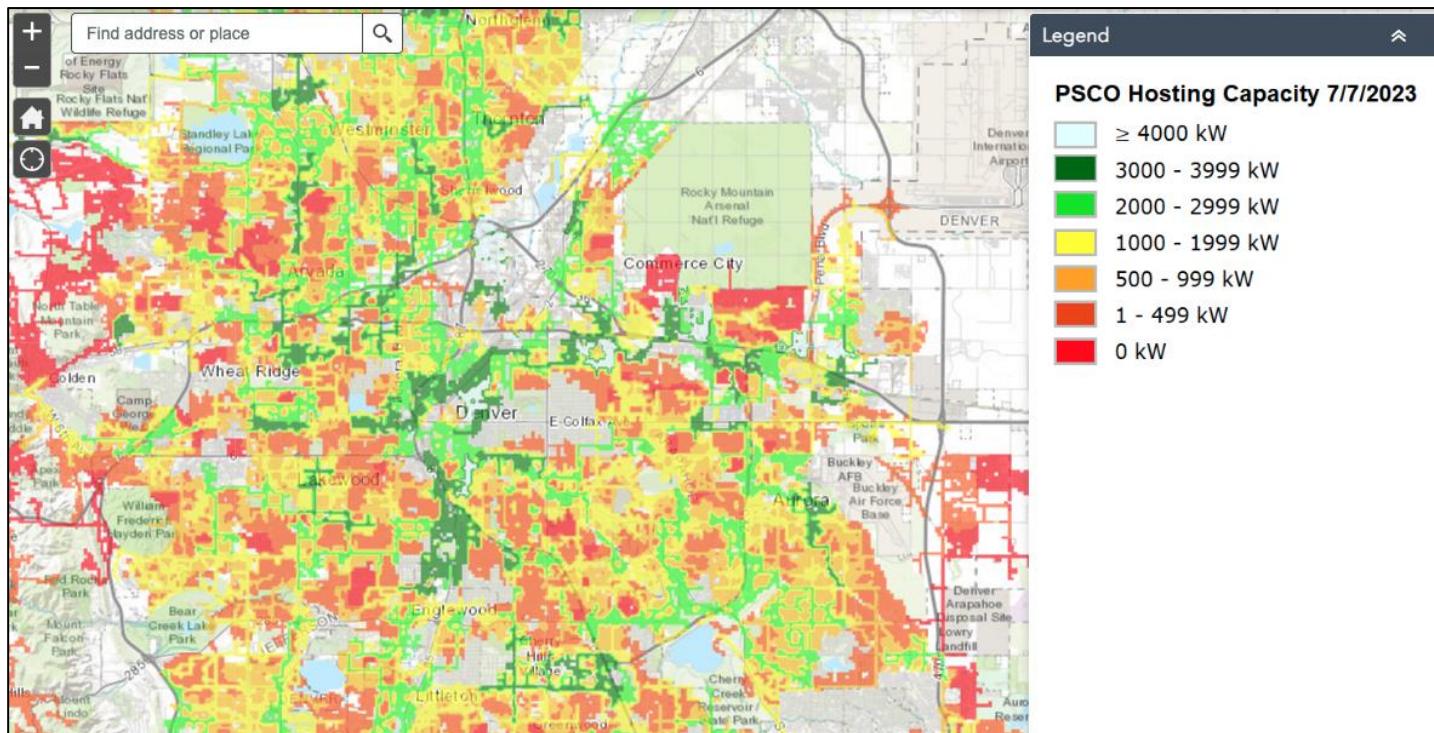
Source: Wood Mackenzie

- New **262 GW** of Distributed Energy Resources between 2023 to 2027
- Almost same amount of growth as utility-scale resources (272 GW) for same period
- Industry is focused largely on ***transmission*** grid capacity & utility-scale interconnection

Need to also ensure ***distribution*** grid has enough capacity to accommodate DER growth

Hosting Capacity (HC)

- Amount of new load or generation that can be interconnected to the distribution system without triggering system upgrades/retrofits
- HC may be limited by voltage, power quality, reliability, thermal or operational constraints
- Many utilities are already severely HC-constrained
→ Limits or delays PV, batteries, EVs, heat pumps

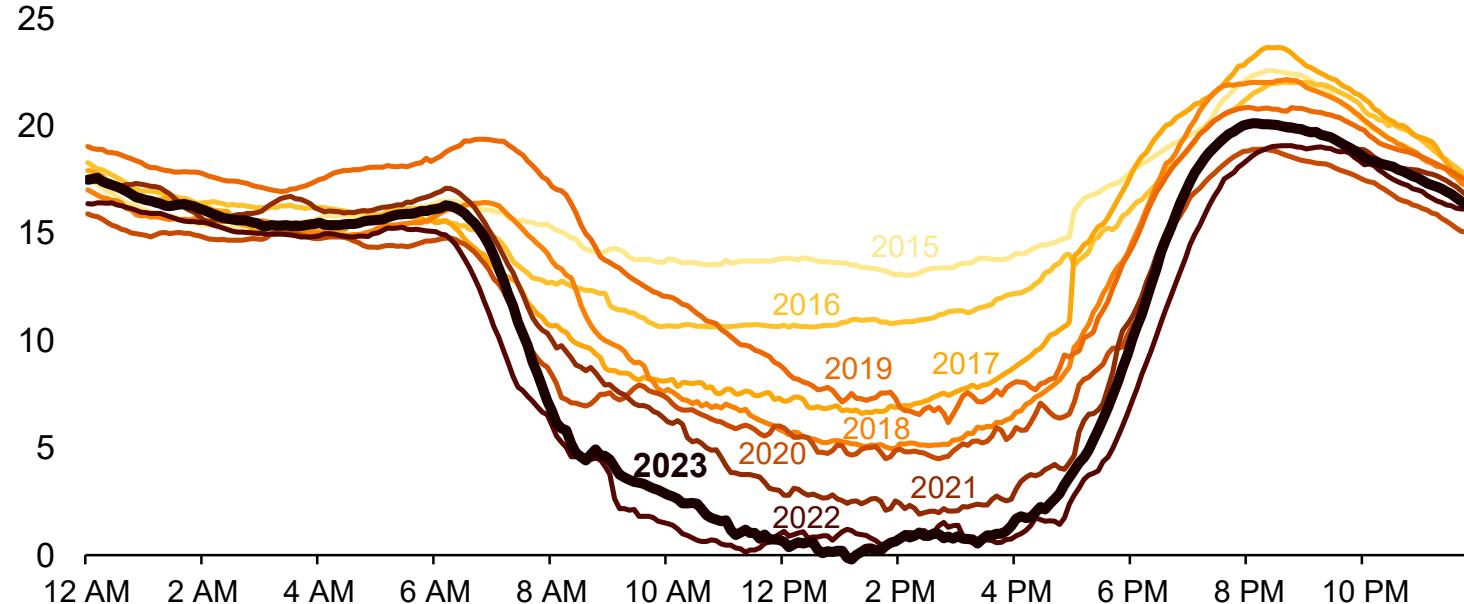


Example HC map for Denver, CO

Grid constraints lead to solar curtailment

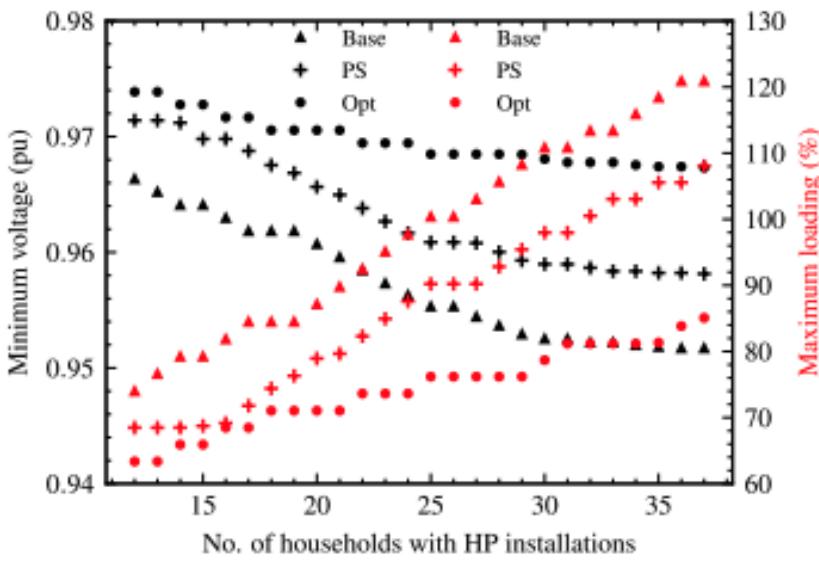
California's duck curve is getting deeper

CAISO lowest net load day each spring (March–May, 2015–2023), gigawatts

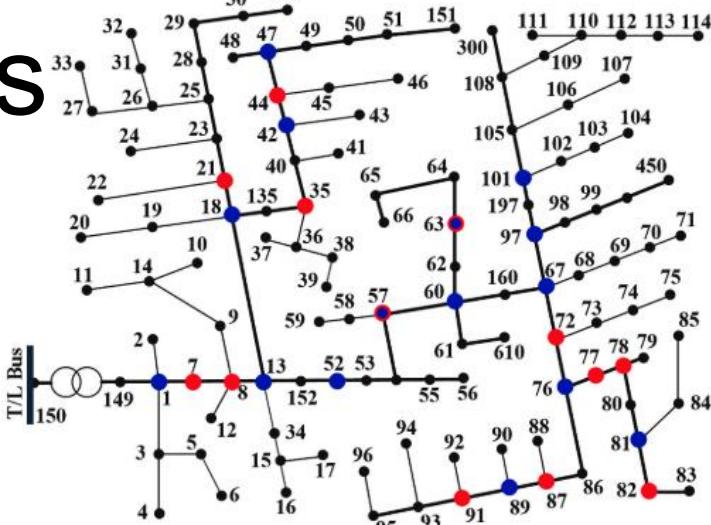


- Excess PV output mid-day is curtailed
- Overbuilding of capacity → Increased capital costs → Higher rates
- Ideally would like to utilize PV output productively (e.g. to charge batteries/EVs)

Example results from prior papers



Towards Congestion Management in Distribution Networks: a Dutch Case Study on Increasing Heat Pump Hosting Capacity



Demand response of HVAC systems for hosting capacity improvement in distribution networks: A comprehensive review and case study

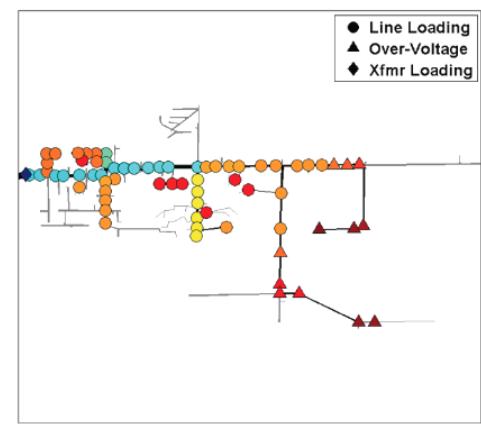


Figure 5. Feeder 1 locational hosting capacity with Volt-Var control on the PV inverter.

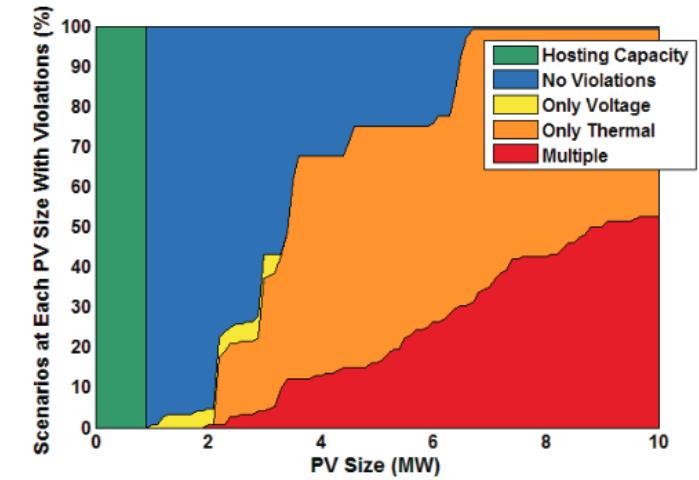


Figure 3. Feeder 1 hosting capacity profile with Volt-Var control of PV inverter.

Improving Distribution Network PV Hosting Capacity via Smart Inverter Reactive Power Support

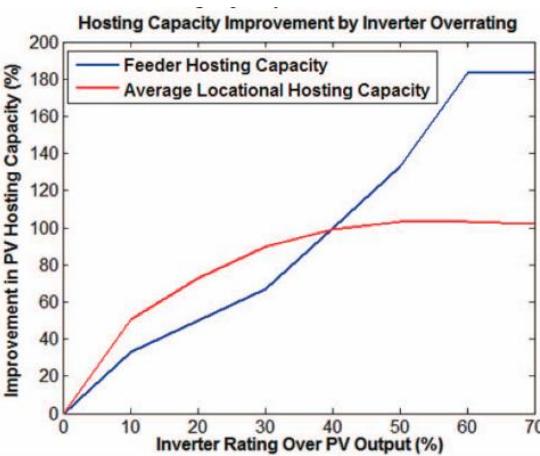


Figure 6. Feeder 1 overall (blue) and locational (red) hosting capacity improvement as a function of PV system inverter oversizing when using Volt-Var control.

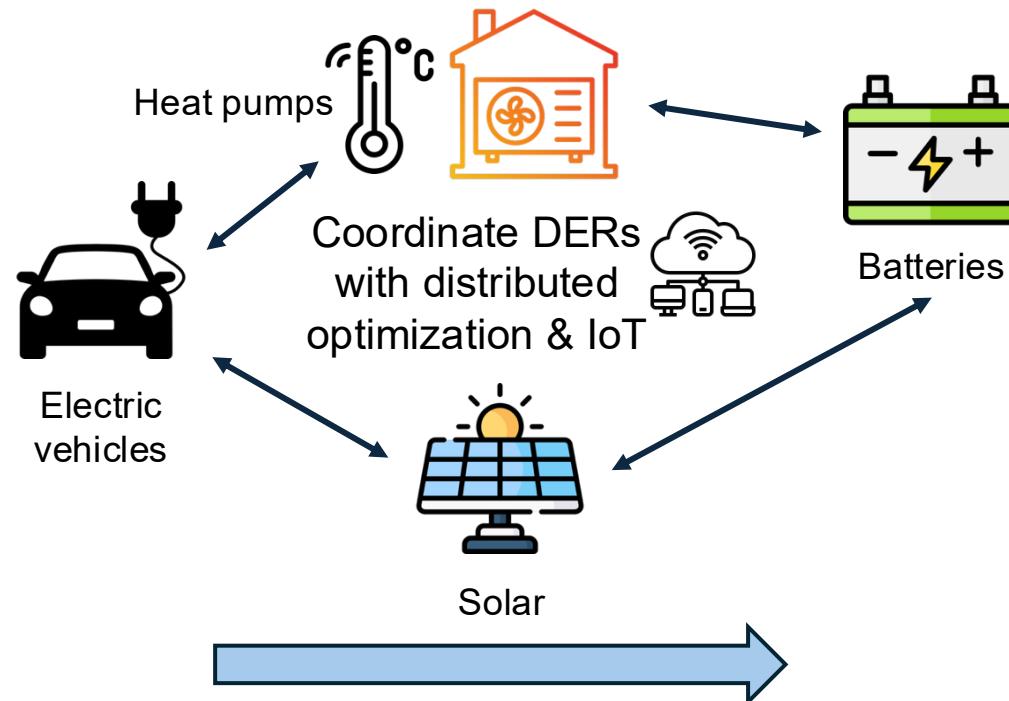
Literature gaps & our contribution

- Majority of prior papers focus on maximizing only PV HC
- Some have assessed HC of either EV or HP (in isolation)
- No prior works have studied HC of multiple DERs simultaneously
- Contribution: Conduct HC analysis while considering all major types of DERs together → Solar PV, EVs, batteries, heat pumps
 - Develop flexible framework to co-optimize various DER types together
 - Accurately model device-level dynamics for true flexibility & controllability
 - Framework also allows incorporation of other loads like data centers

How can we increase hosting capacity

Static hosting capacity

- Assumes that grid edge is not flexible
- Conservative/worst-case approach
- Fixed interconnection agreements
- Most common today



Dynamic hosting capacity

- Assumes that grid edge is flexible
- Optimize DER operation to increase HC
- Flexible interconnection agreements
- Not yet widely adopted

- Leverage *complementary* relationships among different types of DERs (distributed generation, storage, flexible demand)
- Reduce solar curtailment & costs
- Take advantage of massive flexible load growth

Underlying AC power flow constraints

Ohm's Law

$$v_j - v_i = (R_{ij}^2 + X_{ij}^2)l_{ij} - 2(R_{ij}P_{ij} + X_{ij}Q_{ij})$$

Power balance

$$P_{ij} = R_{ij}l_{ij} - P_j + \sum_{k \in \{k_j\}} P_{jk}$$

$$Q_{ij} = X_{ij}l_{ij} - Q_j + \sum_{k \in \{k_j\}} Q_{jk}$$

Apparent power definition
(with conic relaxation)

$$P_{ij}^2 + Q_{ij}^2 \leq v_i l_{ij}, \quad P_{ij}^2 + Q_{ij}^2 \leq \bar{s}_{ij}^2$$

$$0 \leq l_{ij} \leq \bar{s}_{ij}^2 / \bar{v}_i$$

Operating bounds

$$P_j \in [\underline{P}_j, \bar{P}_j], Q_j \in [\underline{Q}_j, \bar{Q}_j], v_j \in [\underline{v}_j, \bar{v}_j]$$

where $l_{ij} = |I_{ij}|^2$ and $v_i = |V_i|^2$

$$\begin{aligned} P_i(t) &= P_i^{PV}(t) + P_i^{BS}(t) + P_i^{EV}(t) + P_i^L(t) \\ Q_i &= Q_i^{PV} + Q_i^{BS} + Q_i^{EV} + Q_i^L \end{aligned}$$

- Quadratic program: Second-order cone program (SOCP) convex relaxation
- For both static & dynamic cases, we run this feasibility optimization problem
- Check whether DER power injections & dispatch results are feasible to satisfy grid physics
- Assume grid is radial & balanced

Dynamic optimization: AC optimal power flow

$$\min f(x)$$

Subject to:

$$v_j - v_i = (R_{ij}^2 + X_{ij}^2)l_{ij} - 2(R_{ij}P_{ij} + X_{ij}Q_{ij})$$

$$P_{ij} = R_{ij}l_{ij} - P_j + \sum_{k \in \{k_j\}} P_{jk}$$

$$Q_{ij} = X_{ij}l_{ij} - Q_j + \sum_{k \in \{k_j\}} Q_{jk}$$

$$P_{ij}^2 + Q_{ij}^2 \leq v_i l_{ij}, \quad P_{ij}^2 + Q_{ij}^2 \leq \overline{s_{ij}}^2$$

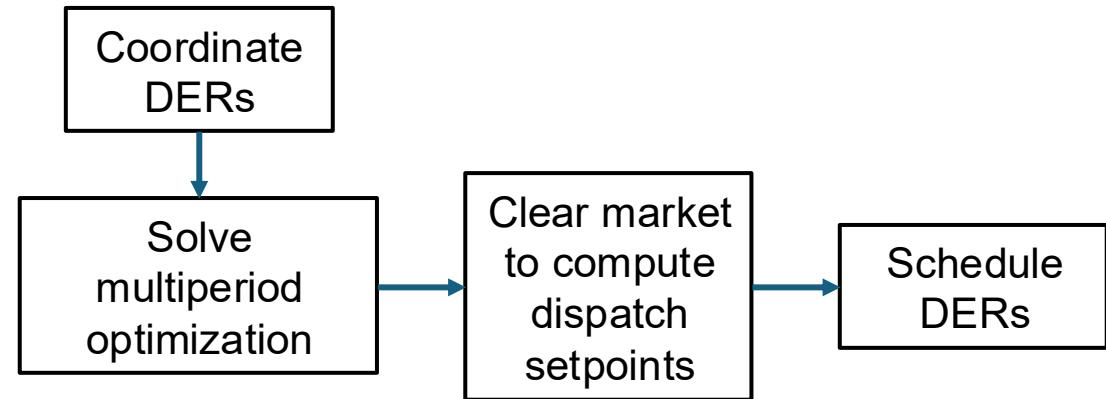
$$0 \leq l_{ij} \leq \overline{s_{ij}}^2 / v_i$$

$$P_j \in [P_j, \bar{P}_j], Q_j \in [Q_j, \bar{Q}_j], v_j \in [\underline{v}_j, \bar{v}_j]$$

where $l_{ij} = |I_{ij}|^2$ and $v_i = |V_i|^2$

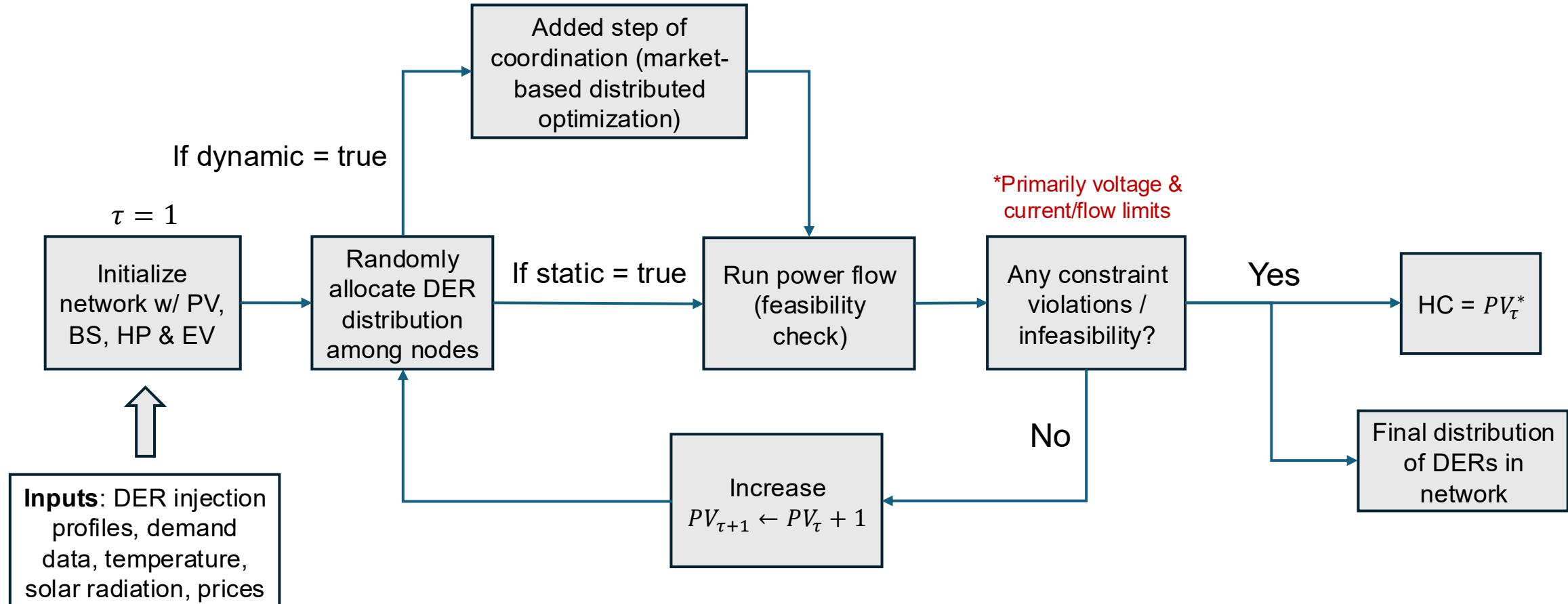
$$\begin{aligned} P_i(t) &= P_i^{PV}(t) + P_i^{BS}(t) + P_i^{EV}(t) + P_i^L(t) \\ Q_i &= Q_i^{PV} + Q_i^{BS} + Q_i^{EV} + Q_i^L \end{aligned}$$

Device-specific constraints (DER models)



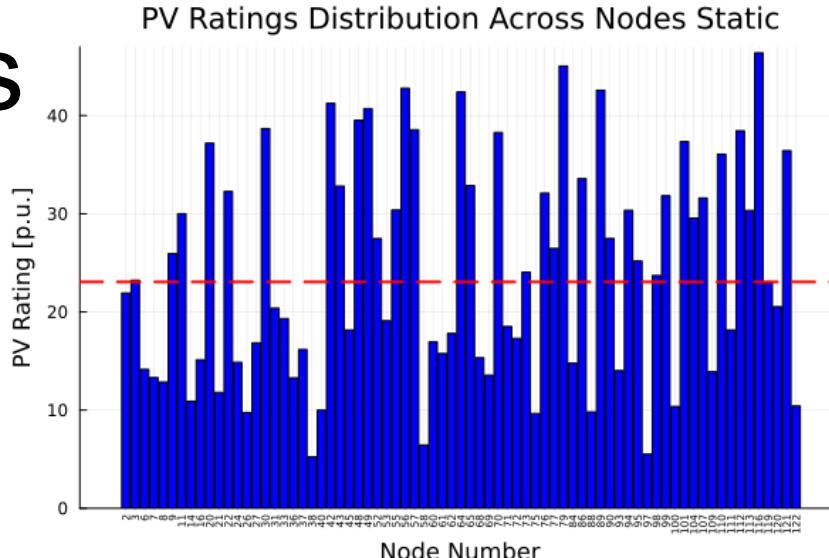
- Detailed DER models: Battery storage (BS), heat pumps (HP) & electric vehicles (EV)
- Intertemporal constraints
- Reactive power support from smart inverters
- Mixed integer quadratic program
- Integer variables to model DERs (e.g. BS charge/discharge, switching between HP cooling/heating)

Deterministic iterative method



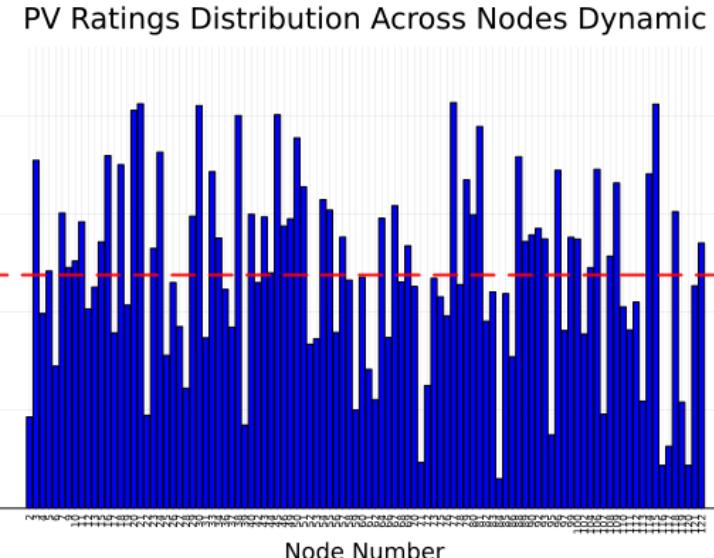
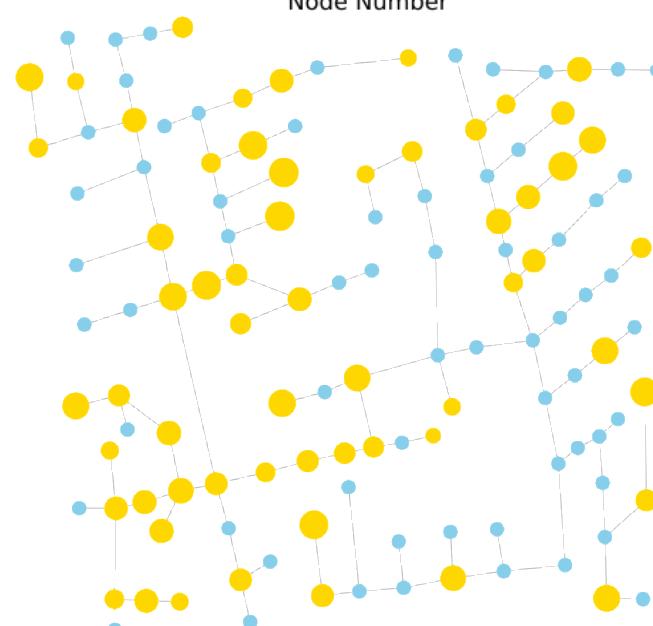
Results

Total PV =
1758 kW
59 nodes
with PV

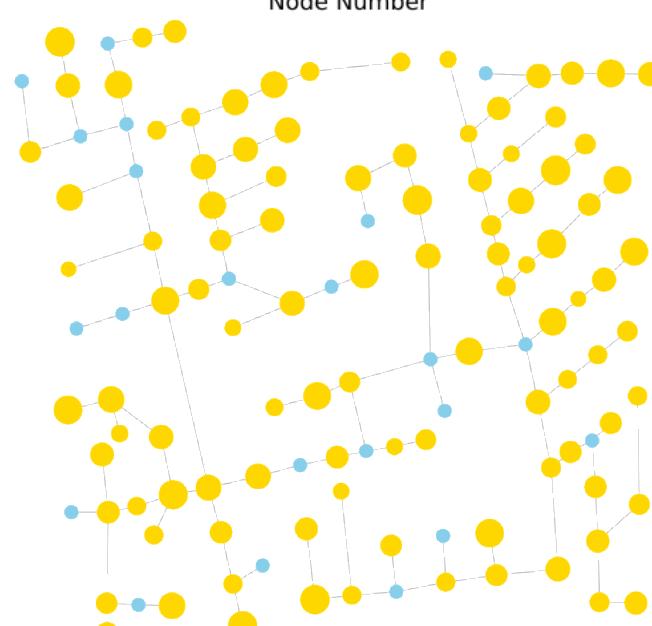


With 5% each
of BS, HP & EV

Nodes with solar PV
• 3 kW → • 42 kW

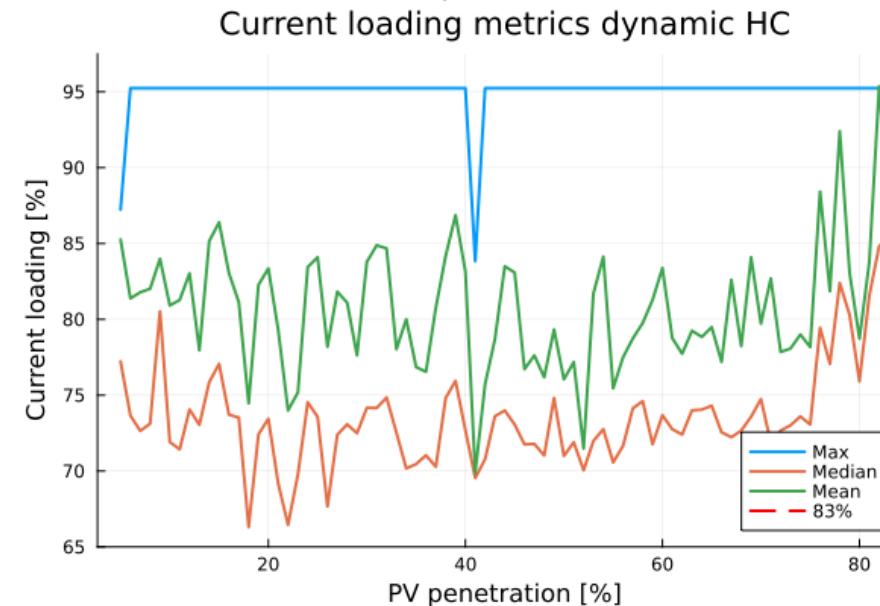
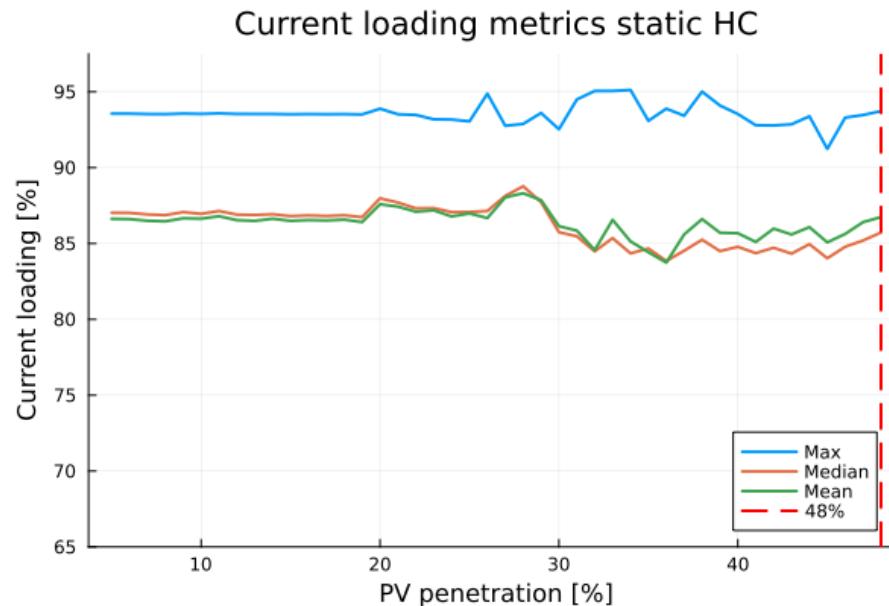
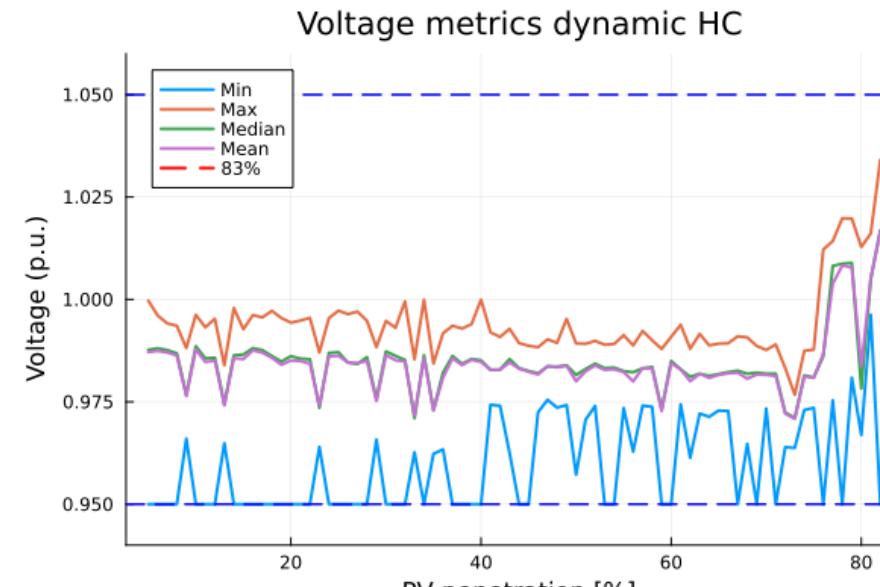
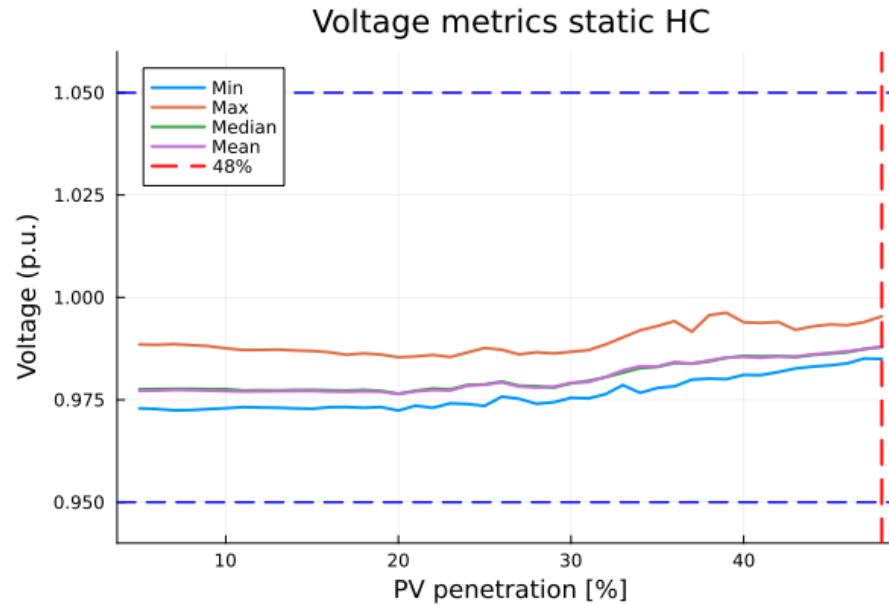


Total PV =
2491 kW
101 nodes
with PV



Coordination allows
~70% relative
increase of solar
penetration
without curtailment

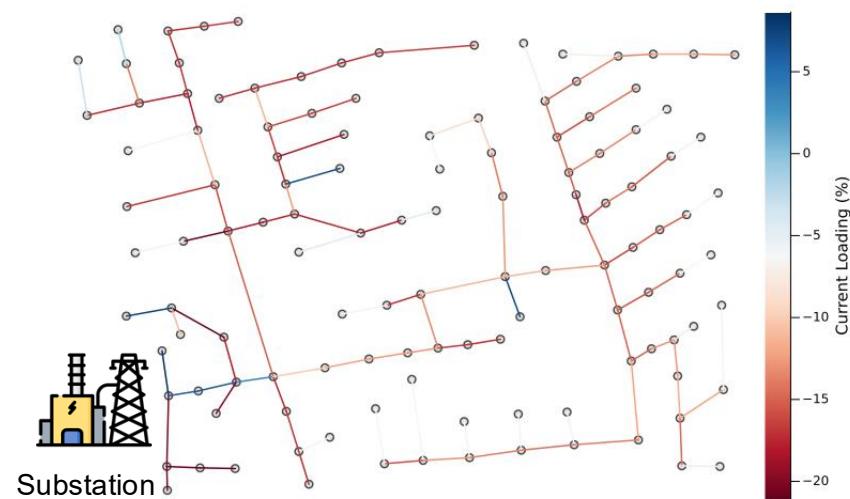
Power flow metrics



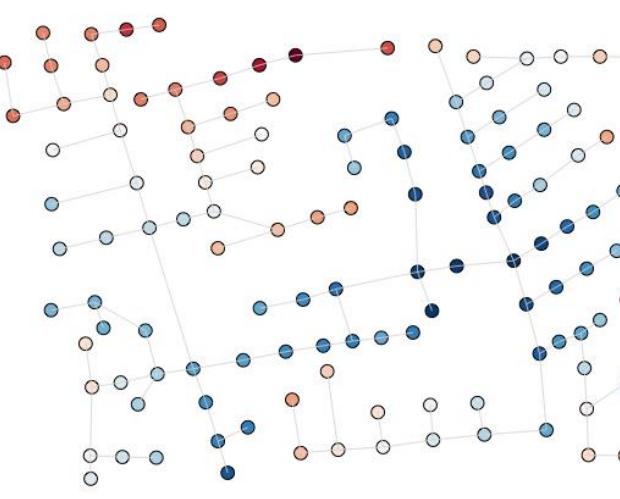
Overvoltage & highly loaded lines are likely limiting factors for hosting capacity

Static at 48 % PV vs Dynamic at 75% PV

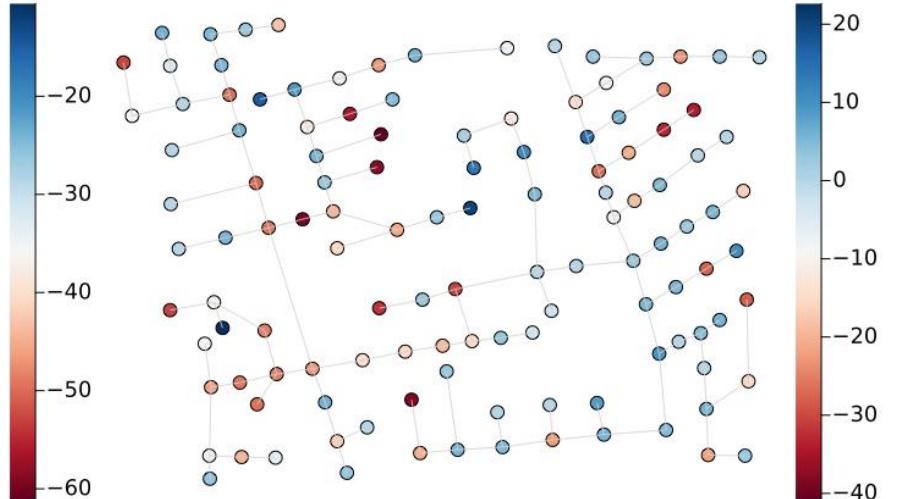
Current loading changes: Dynamic (@ 75%) - Static HC [in %]



Voltage changes: Dynamic (@ 75%) - Static HC [in V]



P injection changes: Dynamic (@ 75%) - Static HC [in kW]

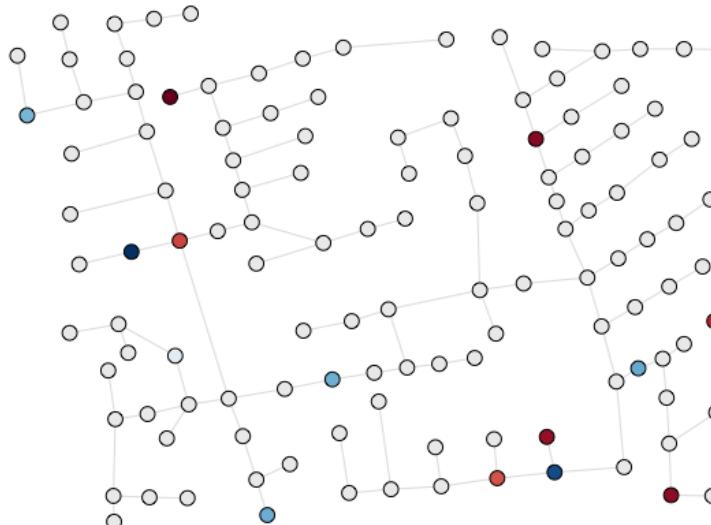


General reduction in overvoltage & current loading with dynamic coordination → Boosts HC

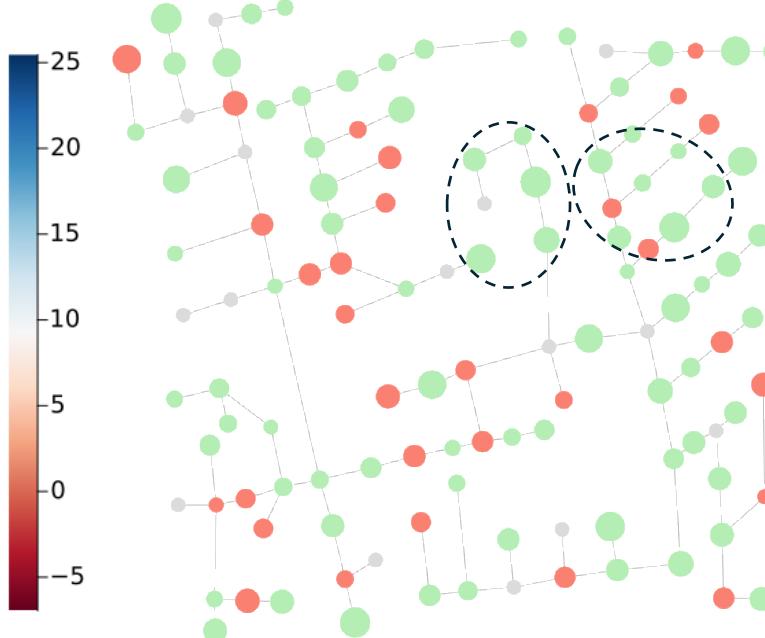
Note: All the following plots are for the dispatch at 12:30PM (peak PV output)

Batteries & heat pumps boost dynamic HC

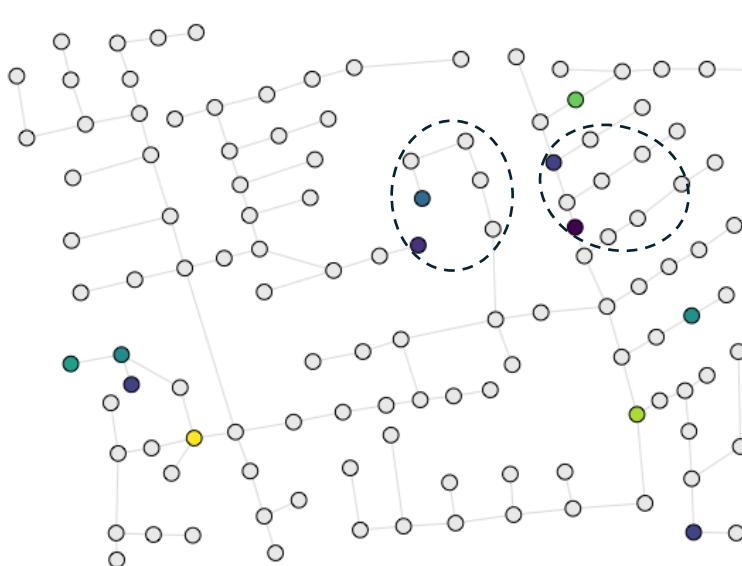
HP demand changes: Dynamic final - Static HC [in kW]



Changes in PV capacities: Static → Dynamic



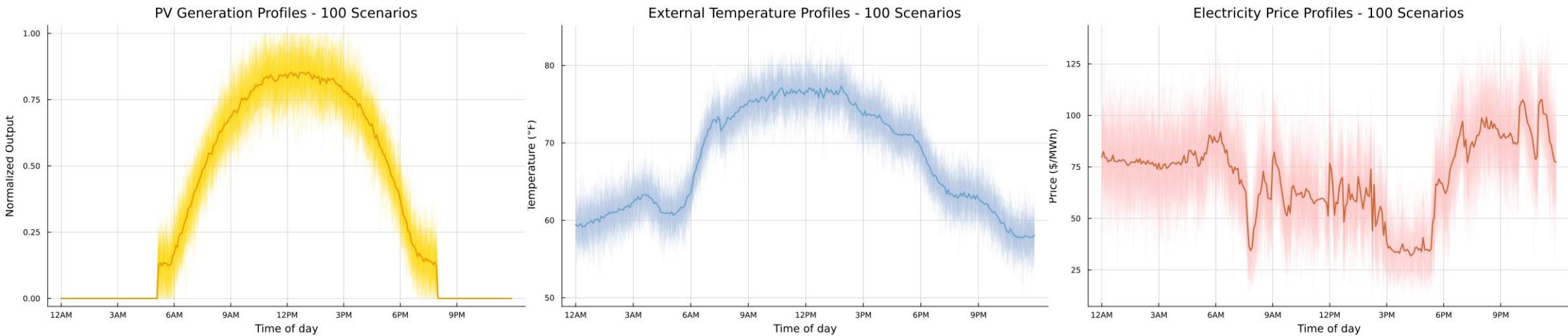
BS injection changes: Dynamic final - Static HC [in kW]



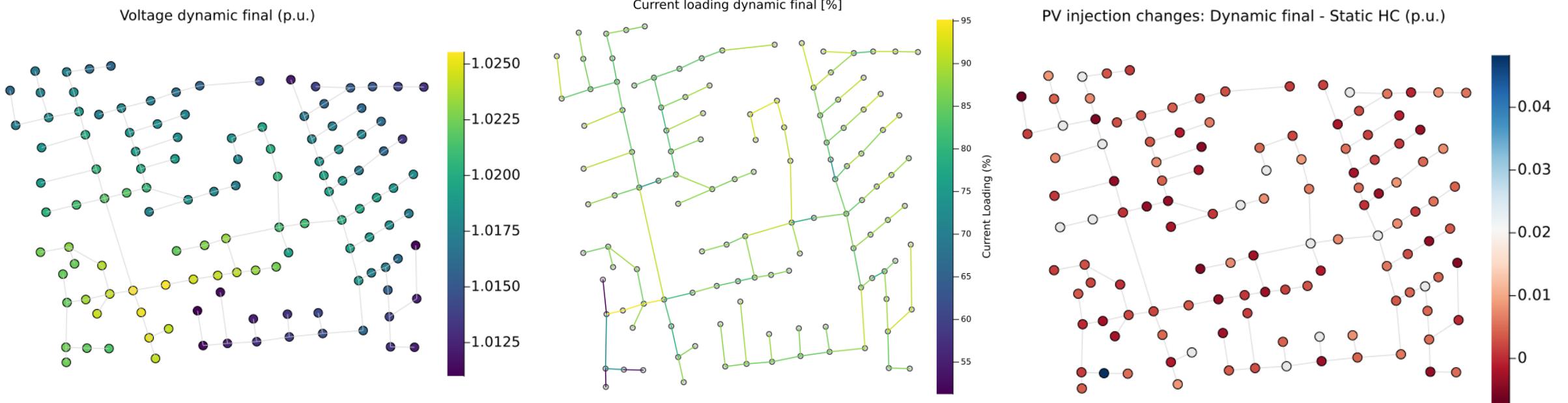
- Total BS penetration = 5% of total max load (2930 kVA) = 147 kW, 441 kWh
- Total HP = 168 kW (5% of homes)
- Total EV = 396 kW, 2700 kWh (5% of homes)
 - Only available for limited time periods
- Coordinated BS charge more & HPs consume more to absorb excess PV
- Co-located BS helps boost PV at same node
- Batteries also support PV at nearby nodes

●	Nodes with increase in PV
●	Nodes with decrease in PV
● → ●	1 kW → 42 kW

Complementarity between DERs



Network at maximum dynamic HC (84%)

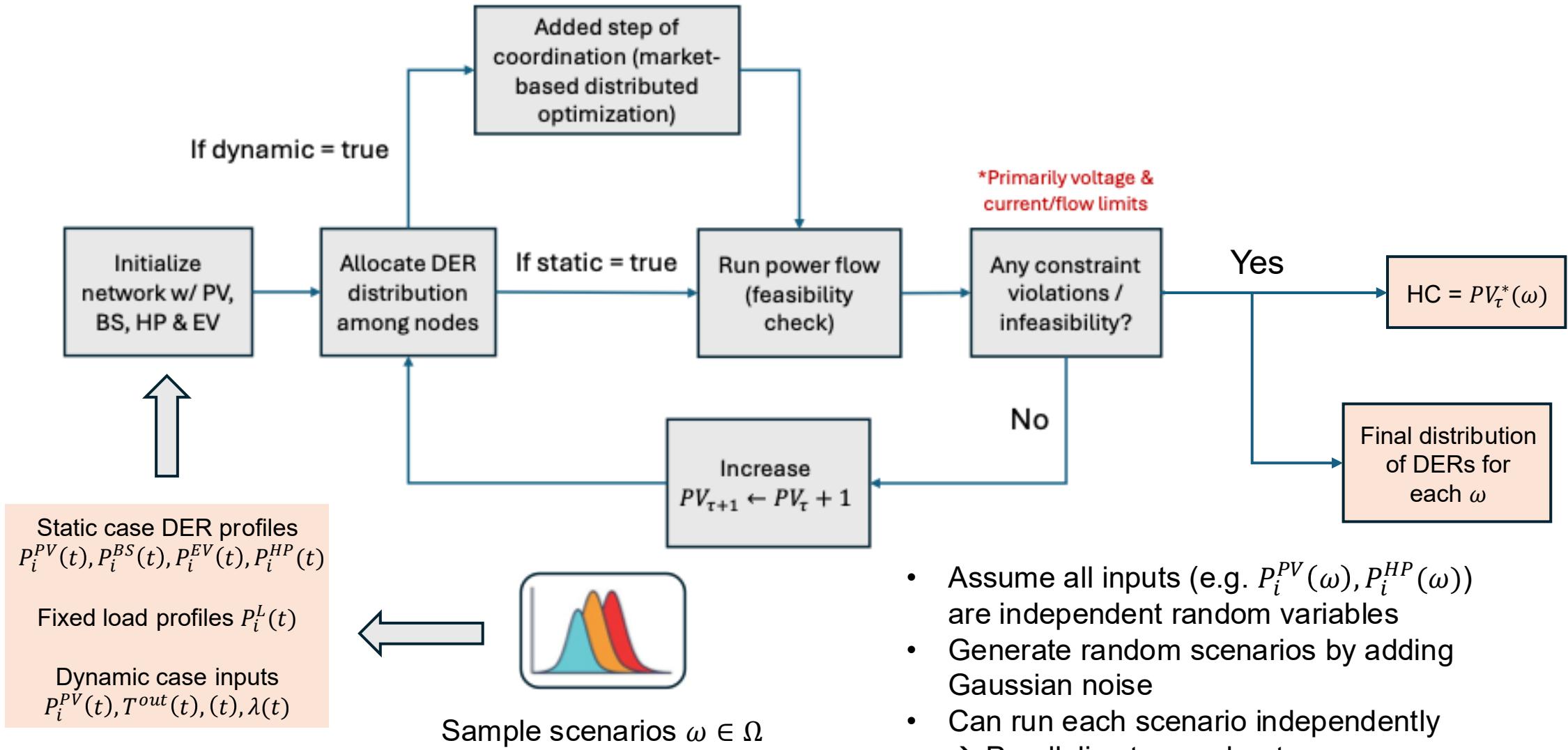


Even as we push PV injections to the maximum feasible level, our method guarantees that voltages and currents are within limits.

Incorporating uncertainty: Non-deterministic HCA

- Many sources of uncertainty affect distribution grid operation
- Static case: Uncertainty in baseline power injection profiles for DERs & fixed loads
- Dynamic case has uncertainty in key inputs:
 - Solar radiation (for PV output)
 - External air temperature (affects HP load & flexibility)
 - Electricity prices (influences power import from transmission grid & BS charging)
 - Fixed load profiles
- Grid planners need to accurately account for uncertainty for more realistic, conservative HC estimates → Lower than deterministic results

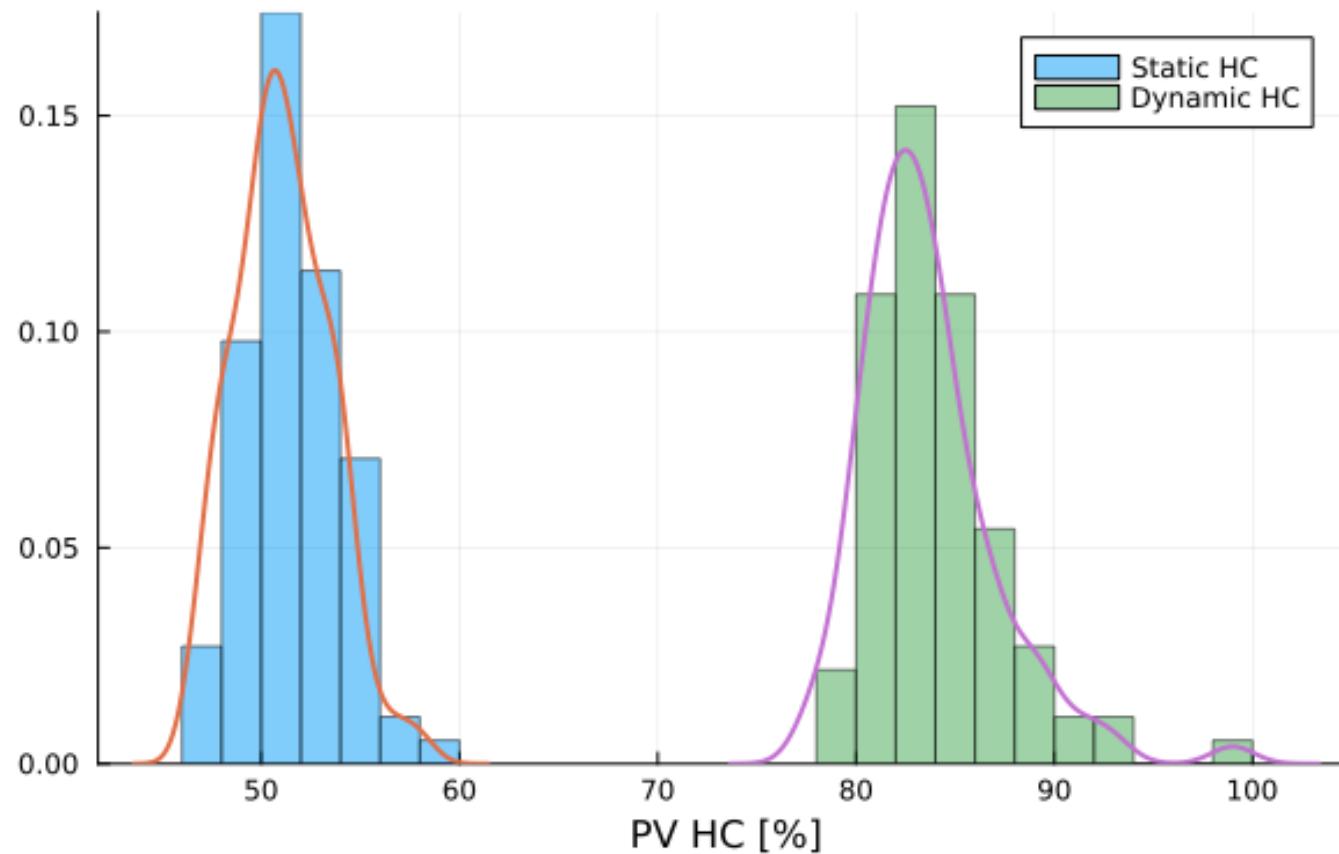
Stochastic iterative method: Monte Carlo sampling



- Assume all inputs (e.g. $P_i^{PV}(\omega), P_i^{HP}(\omega)$) are independent random variables
- Generate random scenarios by adding Gaussian noise
- Can run each scenario independently
→ Parallelize to accelerate

Distributions of HC results across scenarios

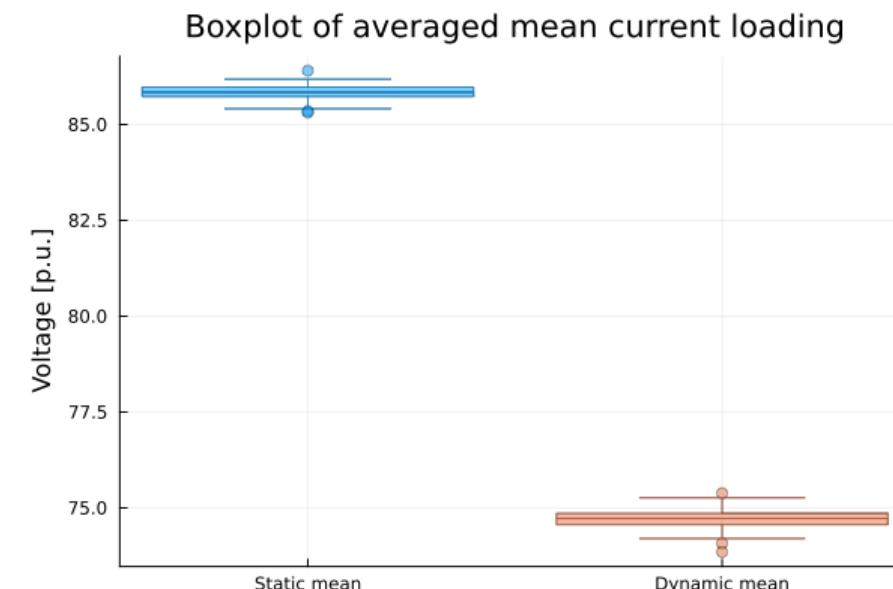
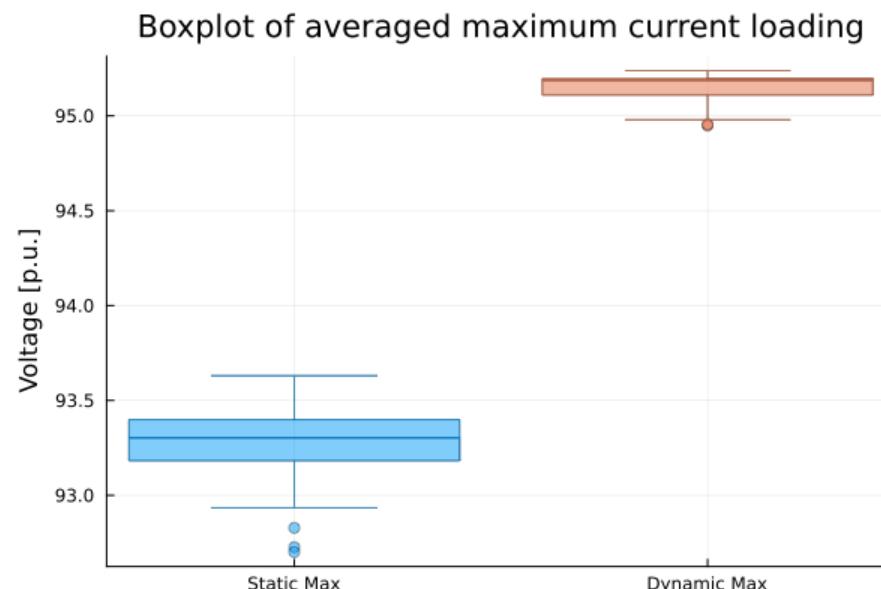
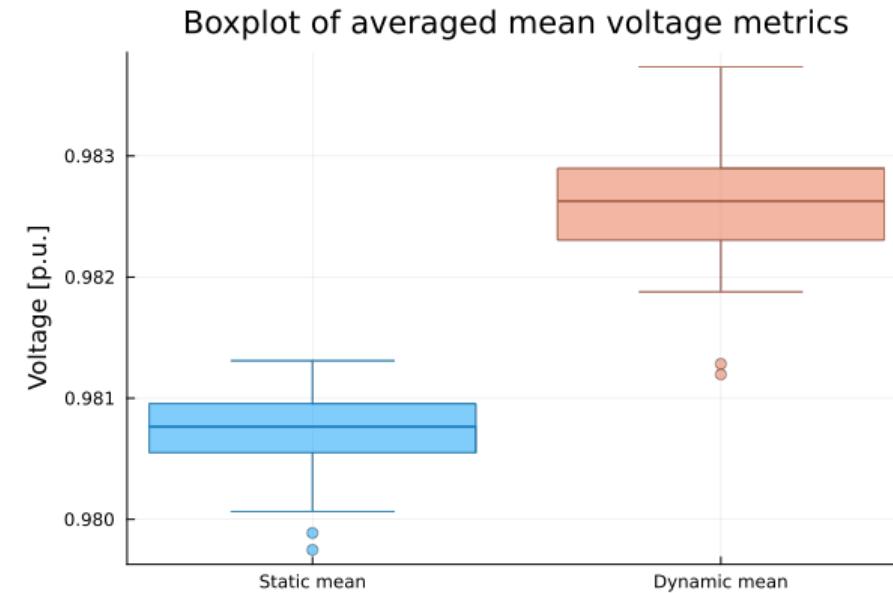
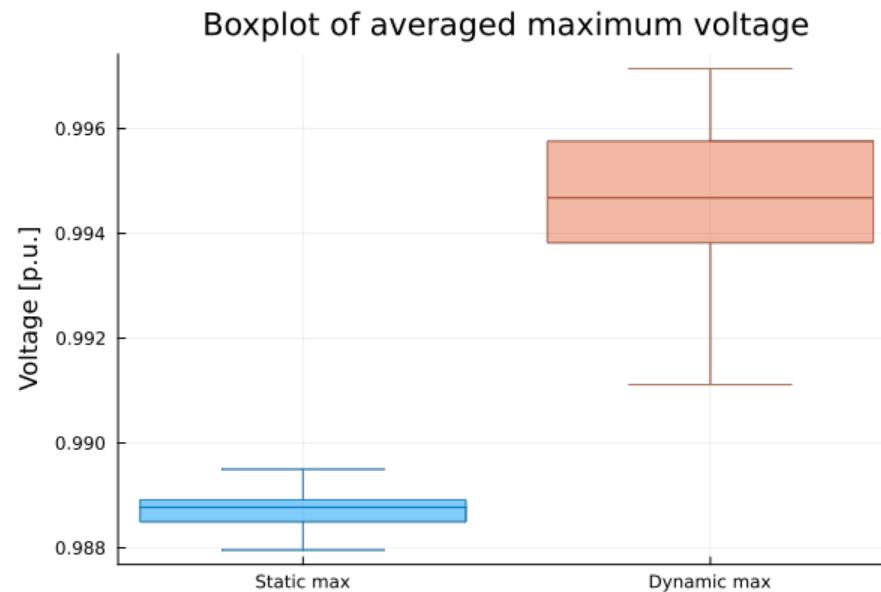
Distributions of PV HC with Kernel Density Estimates



	Static	Dynamic
Mean HC [%]	51.08	83.63
Standard deviation	2.36	3.44

Dynamic coordination boosts HC but has more uncertainty across scenarios than static case

Distributions of power flow metrics



Optimization-based approach

- Iterative approach: Randomly place DERs at various nodes
- New method: Optimal siting and sizing of DERs
 - Determines DER locations & capacities that maximize HC
 - Identify right portfolio of different DER types $\{PV, BS, HP, EV\}$
- Incorporate uncertainty via Stochastic programming (SP)
- Stochastic programs are a better fit than other tools
 - Robust opt (RO): Too conservative, hard to estimate uncertainty sets
 - Chance constraints: Safer than SP, less conservative than RO, but
 - Chance constraints can be hard to compute efficiently
 - Need accurate probability distributions (may not be available)

Two-stage stochastic program (2-SSP)

- Master problem (1st stage): Design, planning & investment

$$\min_{\mathbf{x}} \quad f^I(\mathbf{x}) + \mathbb{E}_{\omega}[V(\mathbf{x}, \omega)]$$

$$\text{s.t.} \quad g_I(\mathbf{x}) = 0,$$
$$h_I(\mathbf{x}) \leq 0$$

1st stage decision variables \mathbf{x} = DER locations & capacities

Grid planner chooses these before seeing input uncertainty

- with the sub-problem (2nd stage): Operation

$$V(\mathbf{x}, \omega) = \min_{\mathbf{y}(\omega)} \quad f^{II}(\mathbf{x}, \mathbf{y}, \omega)$$

$$\text{s.t.} \quad g_{II}(\mathbf{x}, \mathbf{y}, \omega) = 0,$$
$$h_{II}(\mathbf{x}, \mathbf{y}, \omega) \leq 0$$

2nd stage decision variables $\mathbf{y}(\omega)$
= Power flow solutions & DER dispatch

Solved after uncertain scenarios ω are realized

- Here, we use expected value as our *risk measure*:
More risk-averse planners can use Conditional Value at Risk

$$CVaR_{\alpha}(X) = \mathbb{E}[X | X \geq VaR_{\alpha}(X)] = \min_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\alpha} \mathbb{E}[(X - t)^+] \right\}$$

2-SSP for HC analysis: 1st stage

- Maximize penetrations of PV/HP/EV for a given max BS capacity

$$\begin{aligned} \max_{x_i} f^I &= \sum_i x_i^{PV} + x_i^{BS} + x_i^{EV} + x_i^{HP} - f_{cost}(x_i^{PV}, x_i^{EV}, x_i^{HP}, x_i^{BS}) \\ \text{s.t. } \underline{x}_i^{PV} &\leq x_i^{PV} \leq \bar{x}_i^{PV}, \quad \underline{x}_i^{BS} \leq x_i^{BS} \leq \bar{x}_i^{BS}, \quad \underline{x}_i^{EV} \leq x_i^{EV} \leq \bar{x}_i^{EV}, \quad \underline{x}_i^{HP} \leq x_i^{HP} \leq \bar{x}_i^{HP} \\ &\sum_i x_i^{BS} \overline{PL}_{nom,i} \leq \bar{x}^{BS} \overline{PL}_{nom} \end{aligned}$$

x_i^{PV}, x_i^{BS} : PV or BS capacity at node i relative to its nominal nodal load

x_i^{EV}, x_i^{HP} : Proportion of homes at node i that have been electrified

- Coupling with 2nd stage: \boldsymbol{x} sets max DER capacities at each node

$$\begin{aligned} \overline{P}_i^{HP} &= x_i^{HP} n_i^h P_r^{HP}, \quad \overline{P}_i^{EV} = x_i^{EV} n_i^h P_r^{EV}, \quad \overline{E}_i^{EV} = x_i^{EV} n_i^h E_r^{EV} \\ \overline{P}_i^{BS} &= x_i^{BS} \overline{PL}_{nom,i}, \quad \overline{E}_i^{BS} = x_i^{BS} \overline{PL}_{nom,i} E_r^{BS}, \quad \overline{P}_i^{PV} = x_i^{PV} \overline{PL}_{nom,i} \end{aligned}$$

n_i^h : # of homes at node i P_r^{EV}, P_r^{HP} : Rated power of EV/HP units E_r^{BS}, E_r^{EV} : Rated energy storage capacity of BS/EV
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2-SSP combined discrete problem

- Sample average approximation: Discretize problem to consider finite number of scenarios $\{\omega_1, \dots, \omega_K\} \in \Omega$

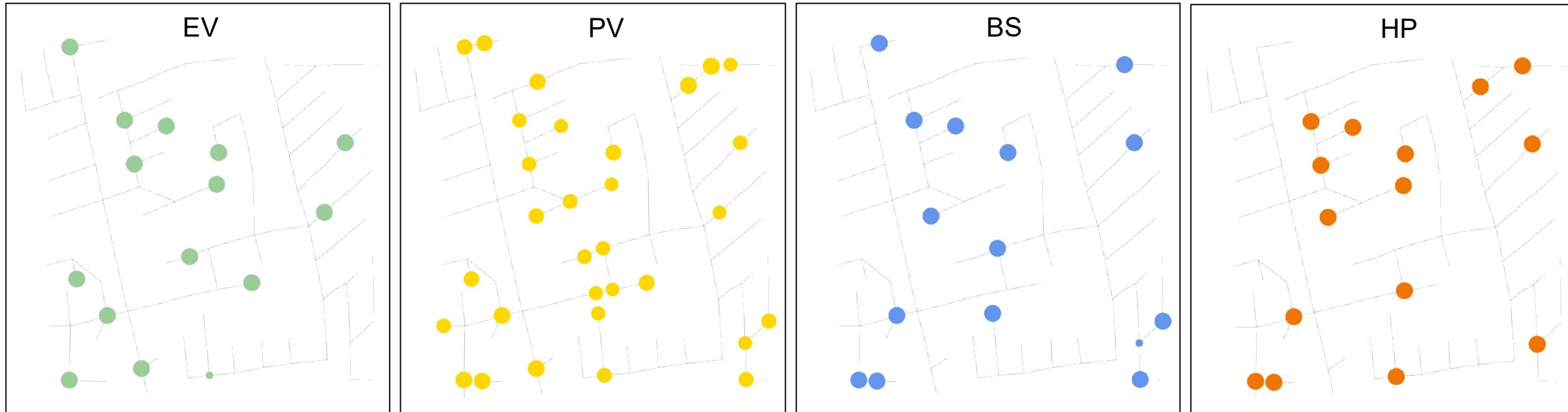
$$\begin{aligned} \min_{\mathbf{x}, \mathbf{y}_1, \dots, \mathbf{y}_K} \quad & f^I(\mathbf{x}) + \sum_{k=1}^K p_k f^{OPF}(\mathbf{x}, \mathbf{y}_k, \lambda) \\ \text{s.t.} \quad & \text{Constraints on } \mathbf{x} \text{ from Stage 1} \\ & h_{pf}(\omega_k, \mathbf{x}, \mathbf{y}_k, t) = 0, \quad g_{pf}(\omega_k, \mathbf{x}, \mathbf{y}_k, t) \leq 0, \quad k = 1, \dots, K \\ & h_{der}(\omega_k, \mathbf{x}, \mathbf{y}_k, t) = 0, \quad g_{der}(\omega_k, \mathbf{x}, \mathbf{y}_k, t) \leq 0, \quad k = 1, \dots, K \end{aligned}$$

- h_{pf}, g_{pf} : Power flow constraints, h_{der}, g_{der} : DER models
- p_k : Scenario probability (assume equally likely $p_k = 1/K$)
- Convert to large-scale *deterministic equivalent*, solve for:
 - Separate sets of 2nd stage decision vars y_k for each scenario
 - 1st stage design solutions x that are feasible for *all* scenarios

Dynamically optimized DER locations & sizes

10% of homes with EVs and HPs, 10% BS penetration, 72% PV penetration, N = 25 scenarios

• 9 kW → ● 75 kW



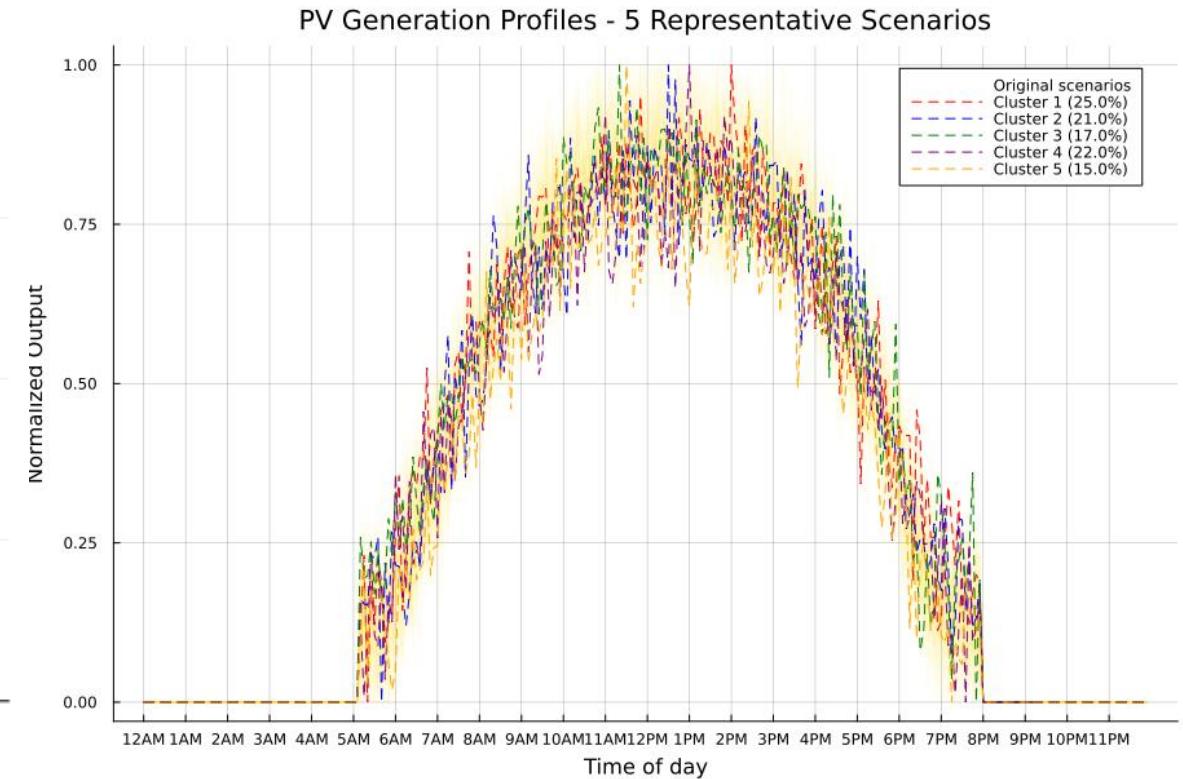
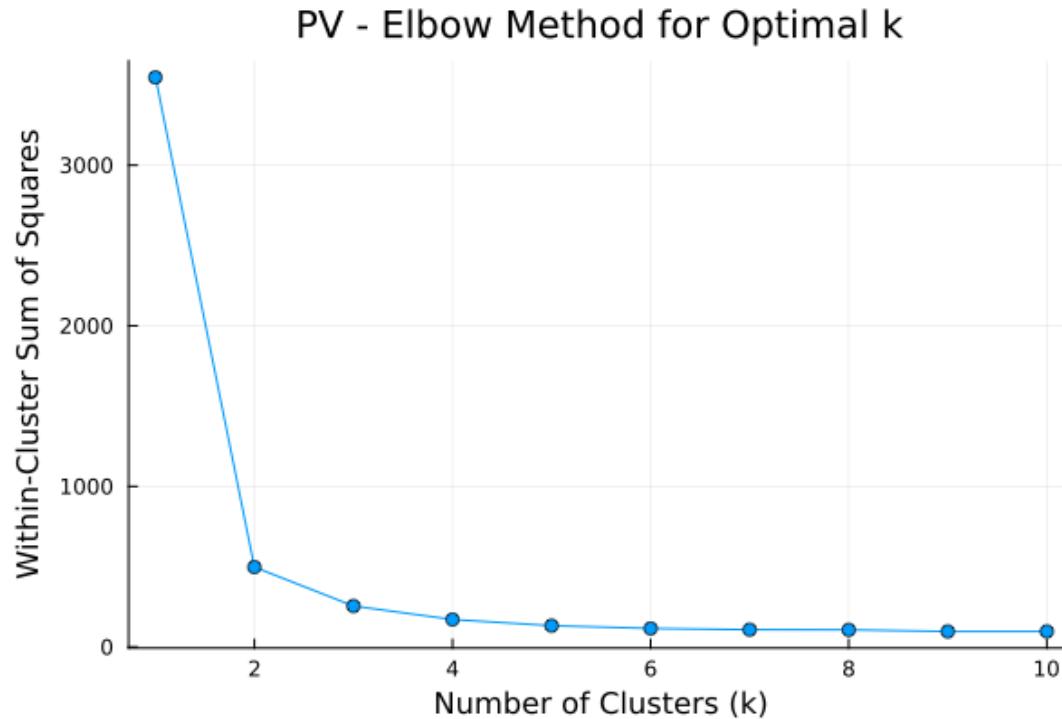
- Generally uniform distribution of capacities over nodes
- Optimization method favors generally co-locating batteries with solar
- Electrified loads (HP, EV) also tend to be co-located with batteries and/or solar
- Co-location enables each node to be more self-sufficient
→ Reduces net injections into the grid or power draw → Less grid stress → Host more PV

Challenges with solving 2-SSP

- Large-scale deterministic equivalent → Slow & expensive
 - Limits no. of scenarios that can be considered
- Decomposition algorithms can reduce computational burden & accelerate solution process
 - Benders decomposition (L-shaped method)
 - Stochastic dual dynamic programming (SDDP)
- But most decomposition tools developed mainly for *linear* programs
 - Challenging to adapt to *mixed integer nonlinear* programs
→ Especially when integer variables are in the 2nd stage rather than 1st
 - Future work: Explore extensions of decomposition methods for MINLPs [1]

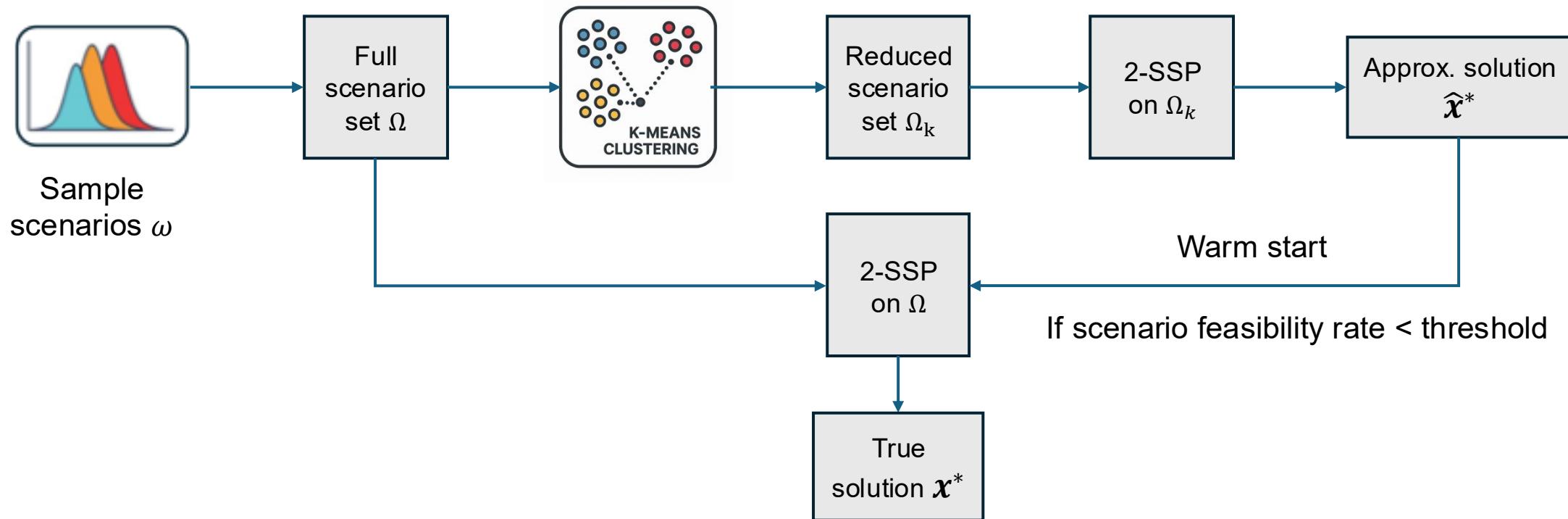
[1] Shixuan Zhang and Andy Sun. Stochastic dual dynamic programming for multistage stochastic mixed-integer nonlinear optimization

Scenario k-means clustering



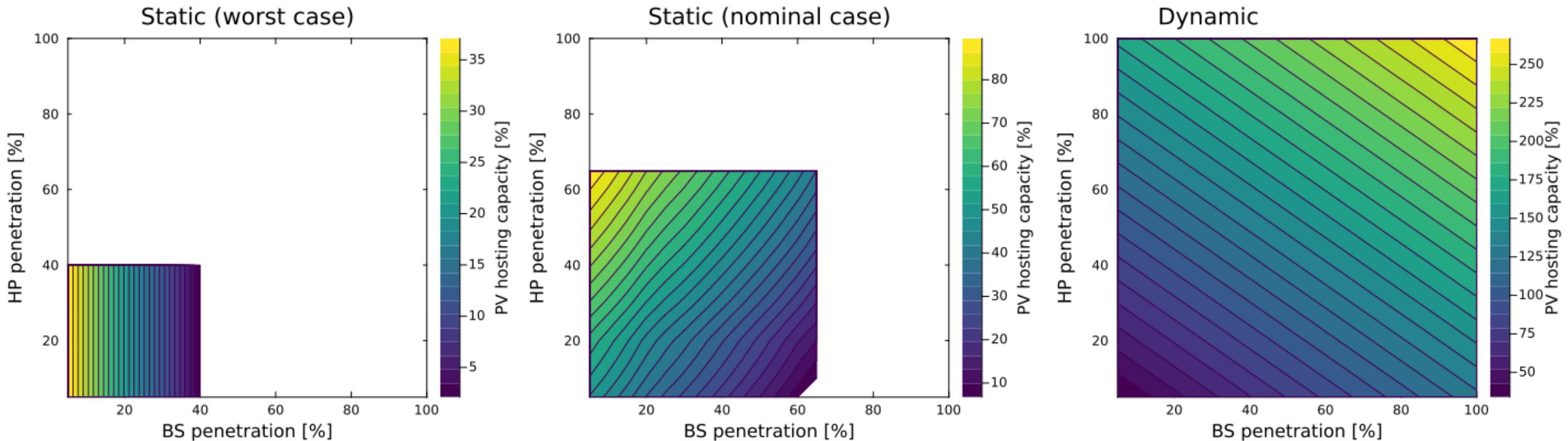
- Apply unsupervised machine learning method
- Select k representative scenarios that capture most of the trends in the timeseries data

Accelerating via scenario reduction



2-SSP enables rapid sensitivity studies

N = 100 scenarios



- Static worst case (high generation, low demand): PV capacity is mainly sensitive to BS rather than HP
→ But uncoordinated BS can further stress grid by discharging when PV output is high
- Static nominal case: BS & HP have competing effects on PV due to grid constraints (substitutes)
- Dynamic: Higher BS & HP penetrations together significantly increase PV (complementary relationship)
- Also plot 2D plots & compute slopes (1st derivative) to estimate marginal value of BS/HP for increasing HC
 - 2nd derivative should likely show diminishing marginal value?

Takeaways from hosting capacity work

- Market-based distributed coordination of DERs can aid in both distribution grid planning & operation
- Complementary effects between DERs boost grid HC
- Coordination reduces current flows & voltage violations
- Dynamic approach can integrate more DERs while minimizing new physical infrastructure or upgrades
 - Circumvent long permitting & build times
 - Clear DER connection queues → Accelerate grid decarbonization
 - Lower costs for all stakeholders

Overall thesis summary & contributions

[2] Grid services using transactive framework

- Coordinate DERs to provide valuable grid services like voltage regulation
- Derived accurate pricing decomposition
- Generalized to different networks using multiple power flow models

[1] Hierarchical local retail electricity market

- Decentralized and distributed multiobjective optimization algorithms
- Increased efficiency; reduce losses, costs & retail rates

Data-driven decision-making tools & coordination for a decarbonized & distributed grid

[5] Enhance grid hosting capacity

- Apply market-based coordination to increase dynamic hosting capacity & enable flexible interconnection
- Accurately account for uncertainty
- Realistic case studies with varying levels of DER penetration

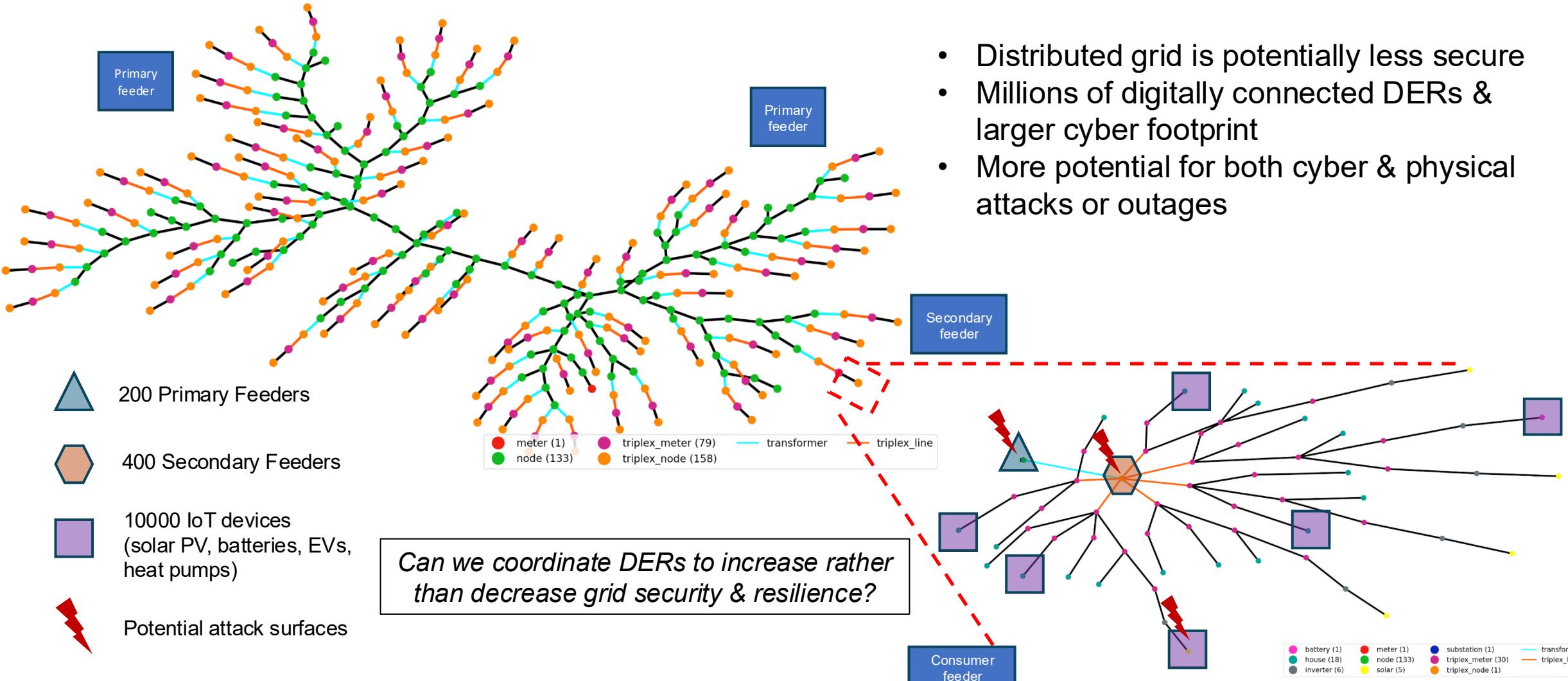
[3] Game-theoretic analysis & mechanism design

- Extract DER flexibility with Stackelberg incomplete information game
- Detailed flexible DER models with multiperiod optimization & intertemporal constraints
- Derived analytical equilibria with closed form solutions for market operators & agents

[4] Distributed IoT coordination for grid resilience

- Detect & mitigate cyber-physical attacks
- Successfully resolve attacks of different types & scales using local flexibility & grid reconfiguration
- Collaborated with external partners to extensively validate simulation results using industry-grade software & hardware-in-the-loop
- Large-scale simulations with thousands of IoT devices

Modern grid edge is more vulnerable



Examples of attacks on the grid

Proposed in literature

1. BlackIoT: Large scale manipulation of 600,000 IoT devices (each controlling 1.5 kW HVAC unit)
→ 900 MW load step [1]
2. MadIoT: Identify most vulnerable nodes & times
→ Achieves same scale attack as 1. but compromising only 150,000 nodes [2]

Real-world attacks

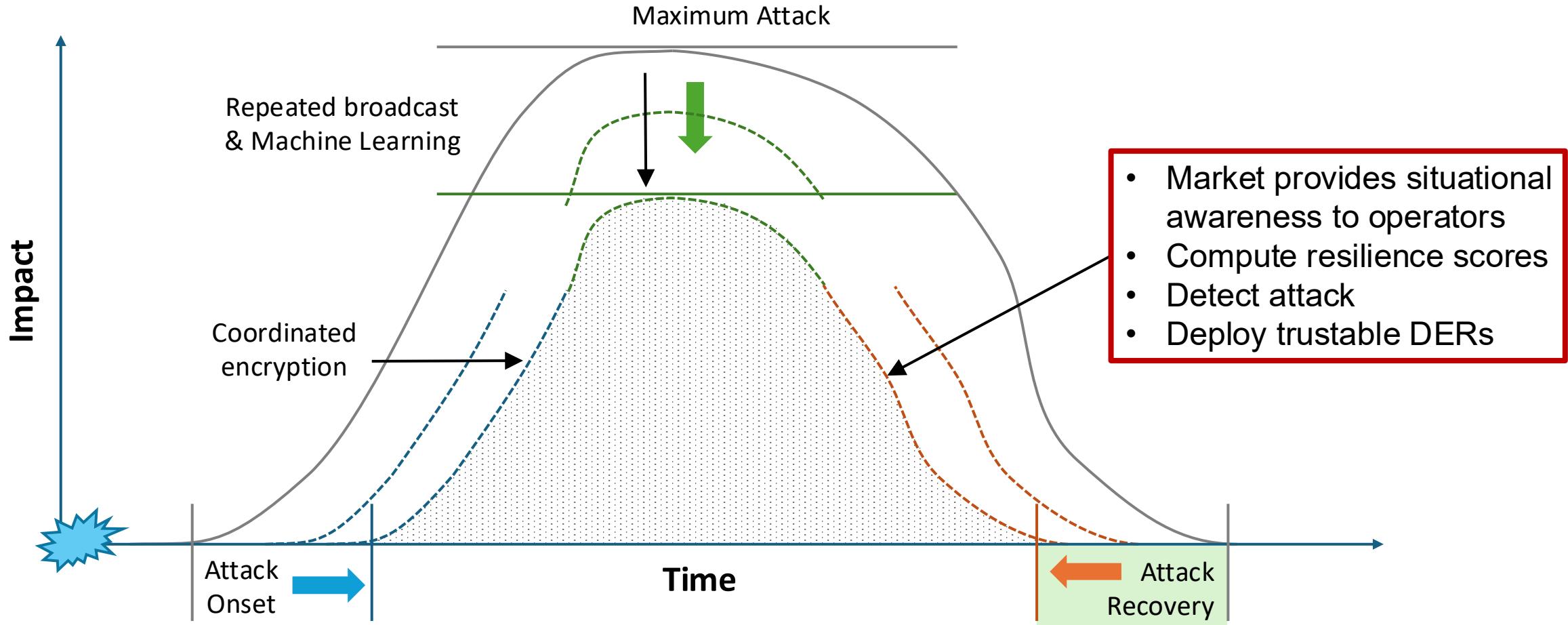
3. Ukraine power grid attacks:
Russian hackers attacked twice (in 2015 & 2016)
30 substations switched off
230,000 customers left without power [3]
4. Recent ransomware attacks on critical infrastructure

[1] Soltan et.al, "BlackIoT: IoT Botnet of High Wattage Devices Can Disrupt the Power Grid" Usenix Security Symposium 2017

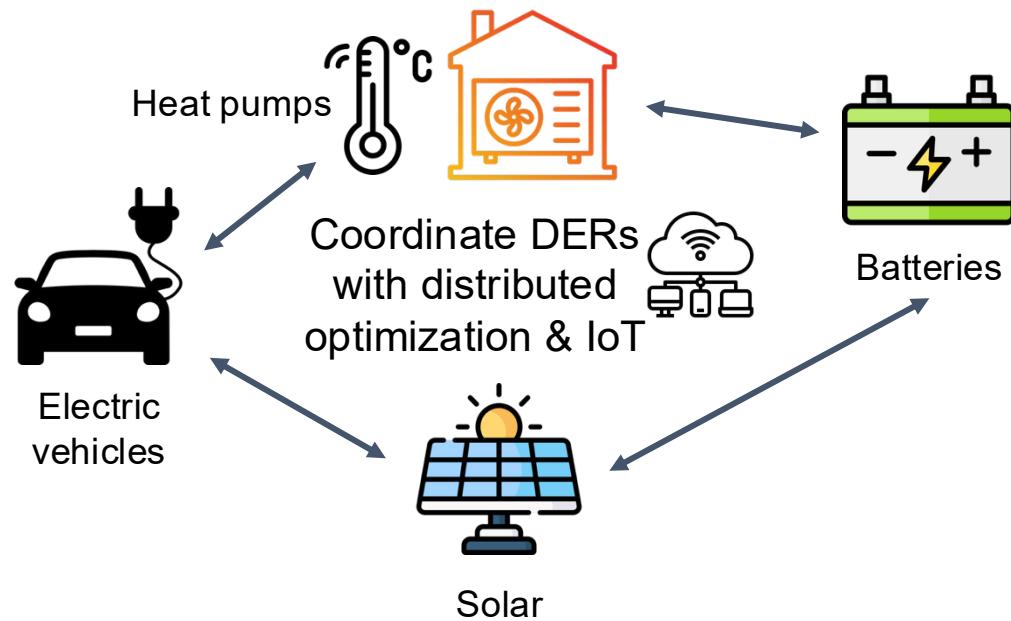
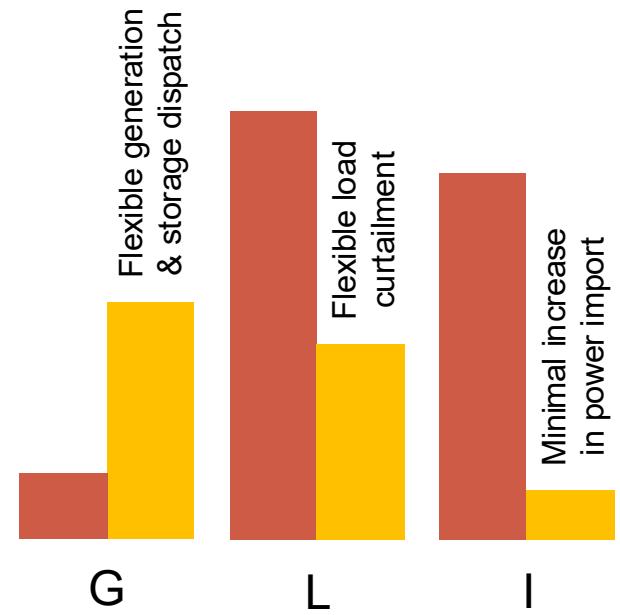
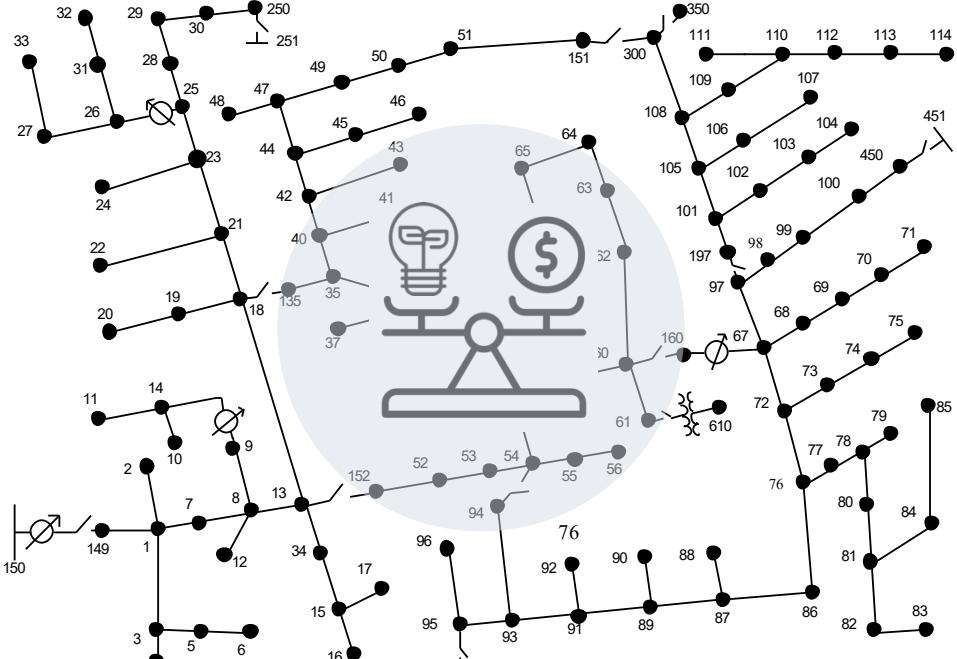
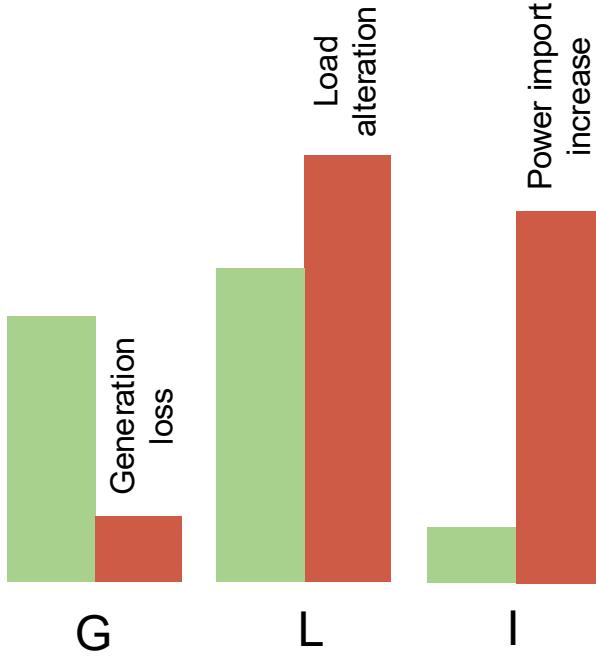
[2] Shekari et.al, "MaDIoT 2.0: Modern High-Wattage IoT Botnet Attacks and Defenses" Usenix Security Symposium 2022

[3] Case, Defense Use. "Analysis of the cyber attack on the Ukrainian power grid." Electricity Information Sharing and Analysis Center (E-ISAC) 388 (2016).

Using markets and IoT-enabled DERs for resilience



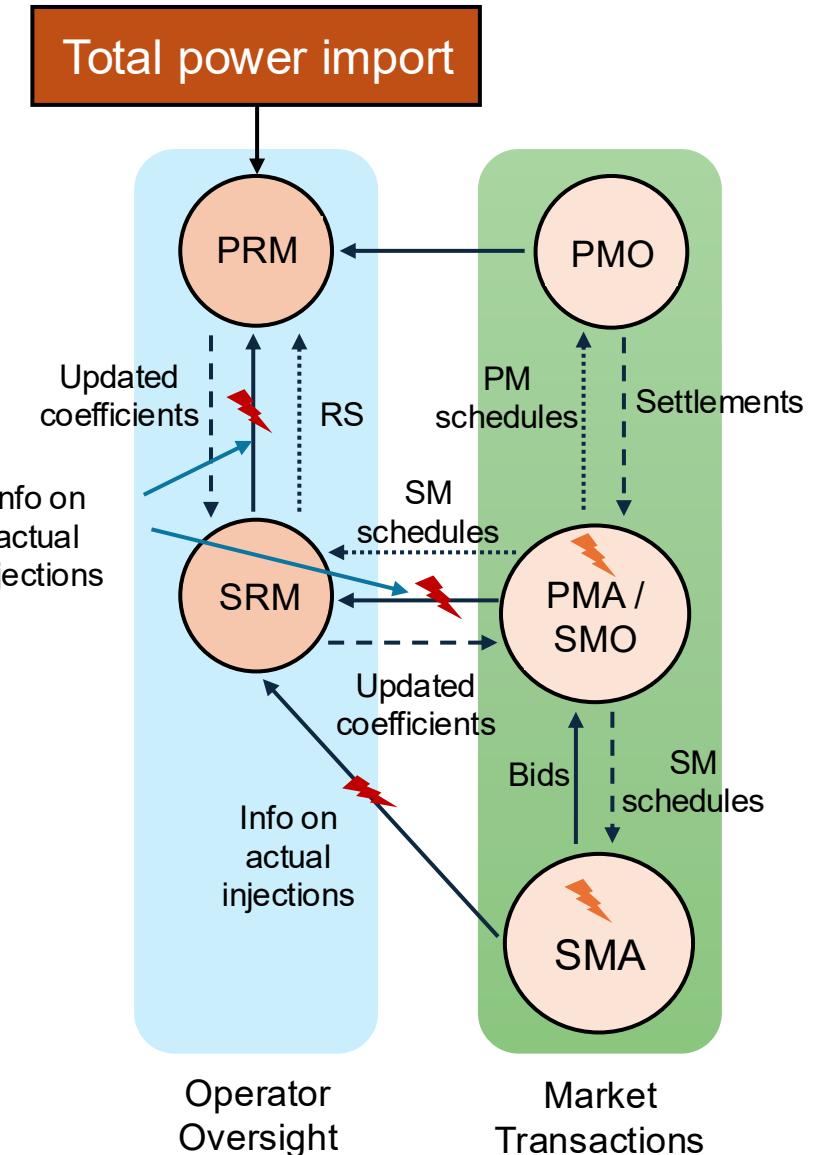
S. M. Dibaji, M. Pirani, D. Flamholz, A. M. Annaswamy, K. H. Johansson, and A. Chakrabortty , “[A Systems and Control Perspective of CPS Security](#),” Annual Reviews in Control, 2019.



G: Generation (inc. storage)
L: Load
I: Power import from main grid

Resilience infrastructure

- Separate grid vs market functionalities
- RM = Resilience manager
 - Monitors grid & provides situational awareness
 - Manages attack mitigation
- MO = Market operator
 - Handles market bidding, clearing, settlement
- Setpoints are corrupted at nodes (⚡)
 - DG: Distributed generation attack
e.g. PV/batteries shut down
 - LA: Load alteration attack
- Simultaneously, key communication links are disrupted (⚡)
- No visibility: PRM doesn't know which nodes have been attacked
- Goal is to provide local resilience: Minimize power import from bulk grid



Attack mitigation via distributed market-based coordination

- PRM does not have direct control or visibility over any SMO's injections
- PRM only monitors net total power injection at substation = \mathbf{P}_{cc}
- Attack changes net injection to $\bar{\mathbf{P}}_{cc}$
- PRM modifies coefficients in objective function $\{\alpha_i, \beta_i, \xi\} \rightarrow \{\bar{\alpha}_i, \bar{\beta}_i, \bar{\xi}\}$
- PMO optimally redispatches PMAs with new, re-weighted objective to mitigate attack

Cost function:
$$\sum_{i=1}^n \left(\frac{1}{2} \alpha_i P_i^{G^2} + \beta_i (P_i^L - P_i^{L0})^2 \right) + \xi \cdot \text{losses}$$

$$\Delta = \mathbf{P}_{cc} - \bar{\mathbf{P}}_{cc}$$

$$Z_i(\delta_i) = 1 + \frac{RS_i \Delta^\top \delta_i}{\mu \sum_i RS_i} \implies \gamma_{i\delta} = \frac{1}{Z_i(\delta_i)}$$

$$\bar{\boldsymbol{\alpha}}_i = \gamma_{i\alpha} \boldsymbol{\alpha}_i, \quad \bar{\boldsymbol{\beta}}_i = \gamma_{i\beta} \boldsymbol{\beta}_i, \quad \bar{\boldsymbol{\xi}} = \left(\frac{\sum_i \gamma_{i\alpha} + \gamma_{i\beta}}{2n} \right)^{-1} \boldsymbol{\xi}$$

Intuition behind coefficient updates

Attack increases net feeder load i.e. $|\bar{\mathbf{P}}_{cc}| > |\mathbf{P}_{cc}| \rightarrow$ Resulting in these coefficient updates:

1. $\gamma_{i\alpha} < 1$: Lower cost coefficients to dispatch more local generation from remaining online SMOs instead of importing power from main grid
2. $\gamma_{i\beta} < 1$: Reduce disutility coefficients to encourage demand response via load shifting/curtailment
3. $\bar{\xi} > \xi$: Penalize electrical line losses more heavily \rightarrow Discourage imports from transmission grid in favor of dispatching more local DERs closer to loads being served.

Cost function:
$$\sum_{i=1}^n \left(\frac{1}{2} \alpha_i P_i^{G^2} + \beta_i (P_i^L - P_i^{L0})^2 \right) + \xi \cdot \text{losses} \quad (1)$$

Assets with higher resilience scores (RS) are used to a greater extent for attack mitigation

$$\Delta = \mathbf{P}_{cc} - \bar{\mathbf{P}}_{cc} \quad (2)$$

$$Z_i(\delta_i) = 1 + \frac{RS_i \Delta^\top \delta_i}{\mu \sum_i RS_i} \implies \gamma_{i\delta} = \frac{1}{Z_i(\delta_i)} \quad (3)$$

$$\bar{\boldsymbol{\alpha}}_i = \gamma_{i\alpha} \boldsymbol{\alpha}_i, \quad \bar{\boldsymbol{\beta}}_i = \gamma_{i\beta} \boldsymbol{\beta}_i, \quad \bar{\boldsymbol{\xi}} = \left(\frac{\sum_i \gamma_{i\alpha} + \gamma_{i\beta}}{2n} \right)^{-1} \boldsymbol{\xi} \quad (4)$$

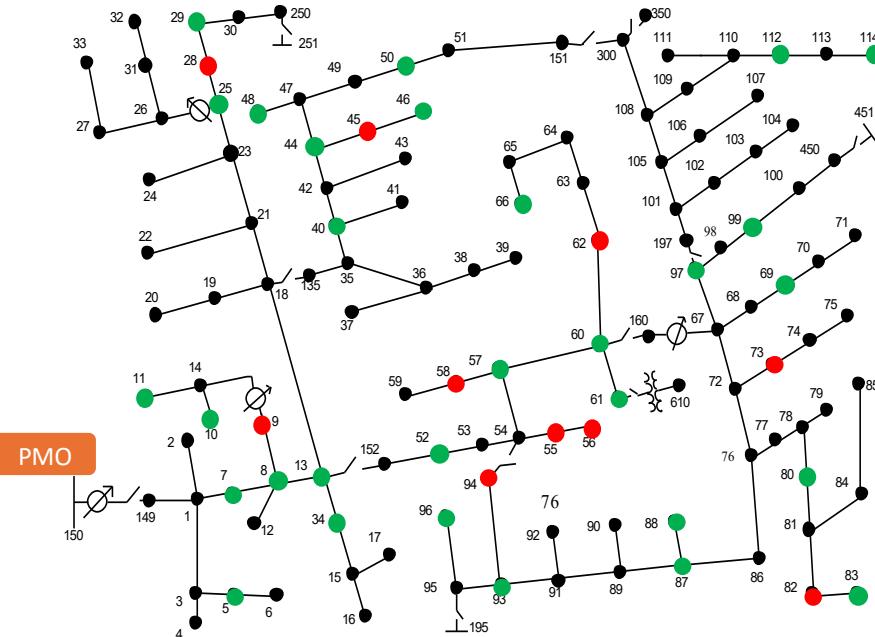
Types of attack scenarios

Table 1. Summary of attack scenarios and use-cases, LA = load alteration attack, DG = distributed generator attack.

Attack Number	Attack type	Attack surface	Grid connection	Power flow model	Grid model	Scale of attack [kW]
1a	LA	PMA	Grid-connected	Current injection	Unbalanced, 3-phase	36
1b	DG	PMA	Grid-connected	Current injection	Unbalanced, 3-phase	45
1c	DG	SMA	Grid-connected	Current injection	Unbalanced, 3-phase	157
2a	DG	PMA	Grid-connected	Branch flow	Balanced, single-phase	261
2b	DG	PMA	Grid-connected	Branch flow	Balanced, single-phase	650
3	DG	PMA	Islanded	Current injection	Unbalanced, 3-phase	2500

- Focus on **disruption** (or denial of service) attacks
- Disconnect generators or corrupt load setpoints
- Key communication links disrupted

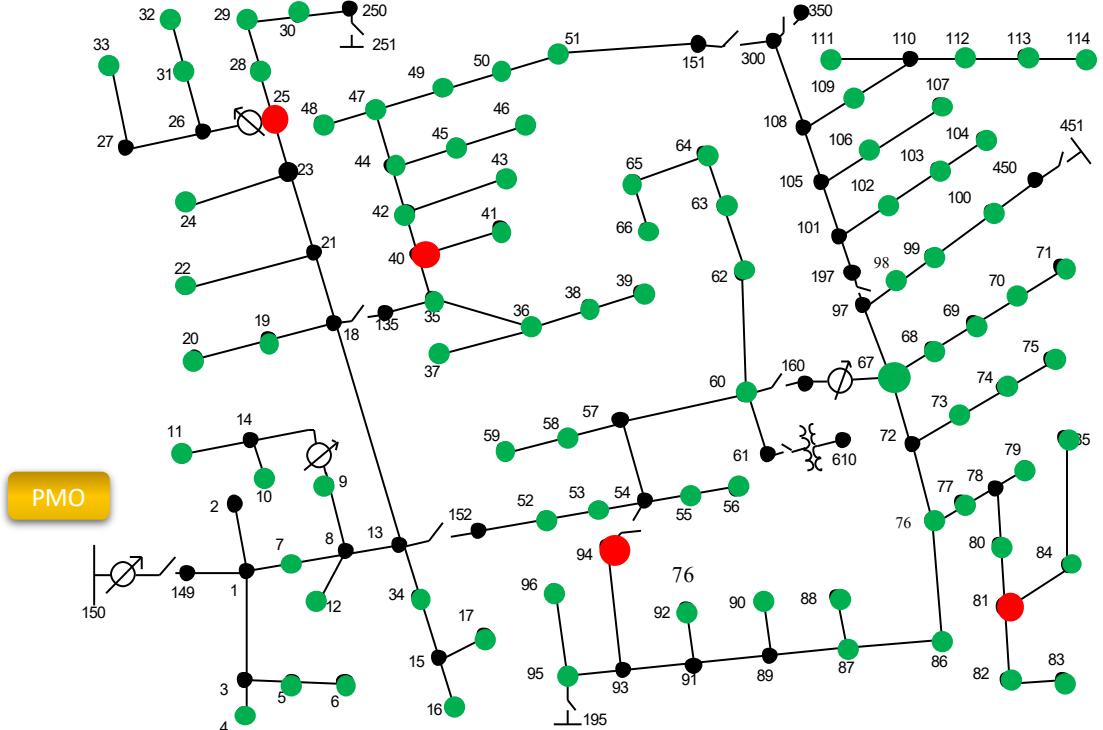
- Situational awareness to operators
- Compute resilience scores
- Detect attack
- Deploy trustable DERs to mitigate



PMA = Primary market agent
SMA = Secondary market agent

Red circle : Attacked Nodes Green circle : Trustable nodes

Large-scale attack (2b) mitigation



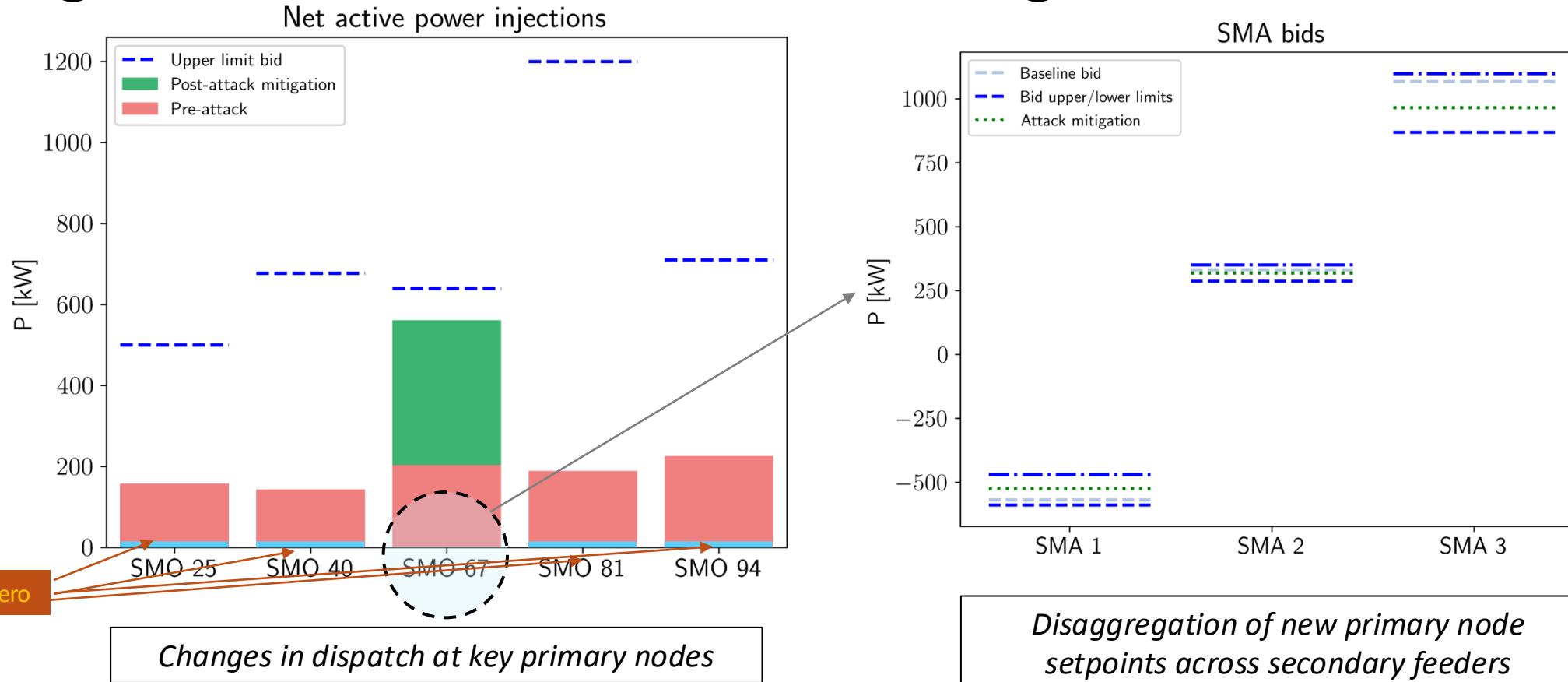
1. Total 641 kW generation loss
2. PRMs alerts other trustable PMAs/SMOs to redispatch their generation assets
3. Trustable PMAs/SMOs will curtail flexible loads to respond & mitigate attack
4. SMOs redispatch SMAs by providing updated setpoints
5. Total import from main grid restored to pre-attack

82 flexible load nodes respond

● : Attacked nodes

● : Trustable nodes

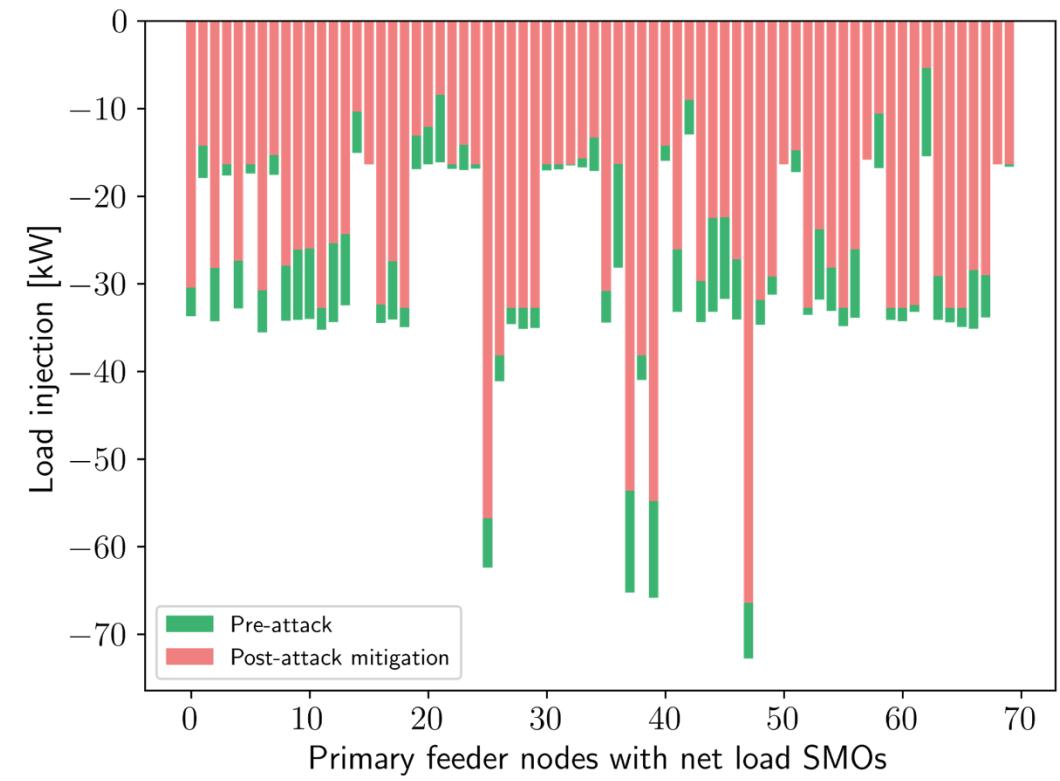
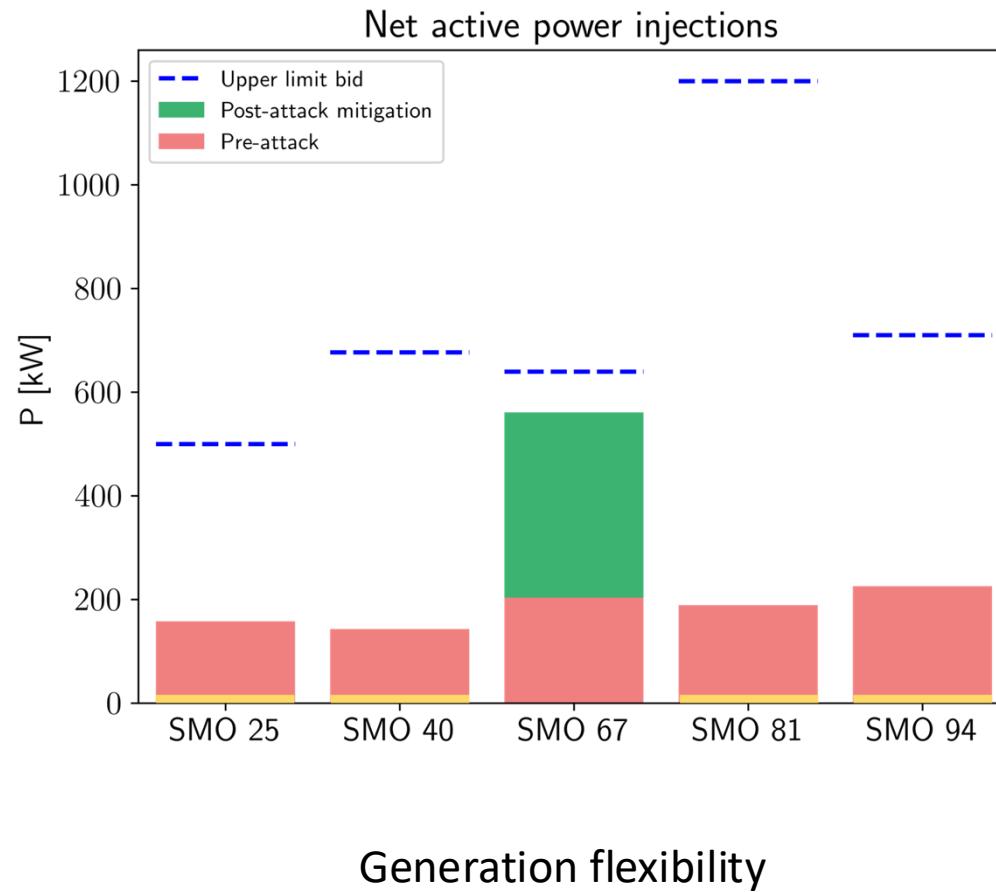
Large scale attack with mitigation



- 4 nodes attacked
 - Physical outage → All drop to zero
 - Cyber attack → Communication with market operator compromised
- Leverage available upward flexibility of remaining generator at SMO 67
- Increase in generator output limited by power flow/network constraints

Leverage load flexibility to fully resolve attack

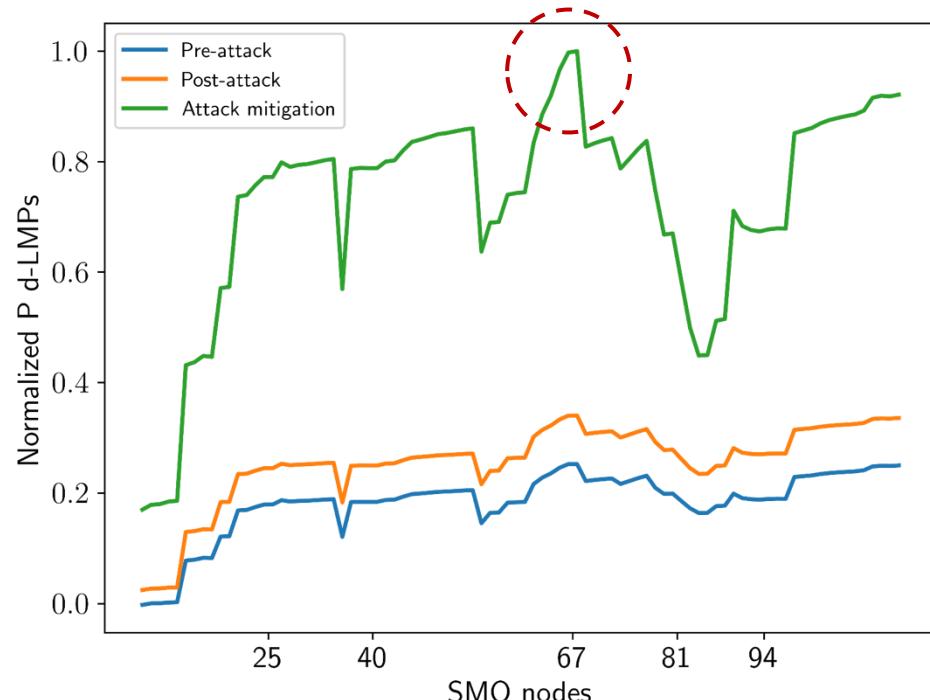
In addition to utilizing extra generation, we also need to shift/curtail some of remaining flexible loads



Demand response flexibility

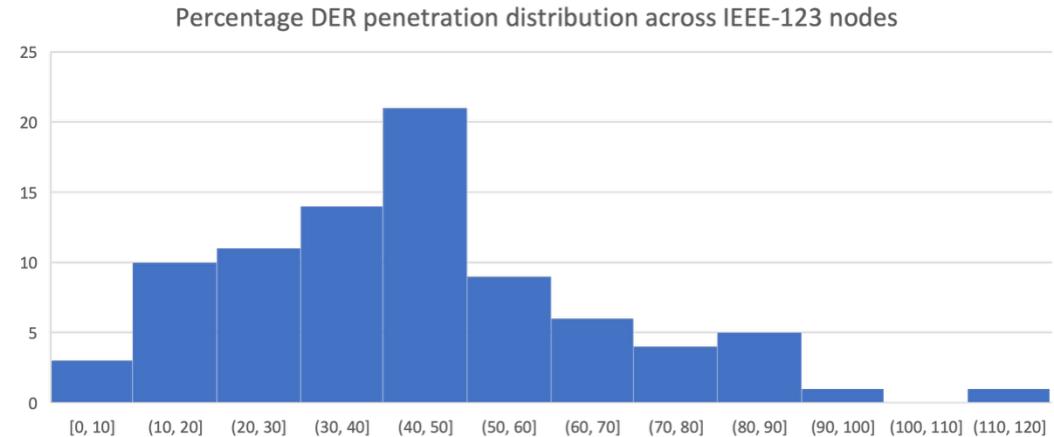
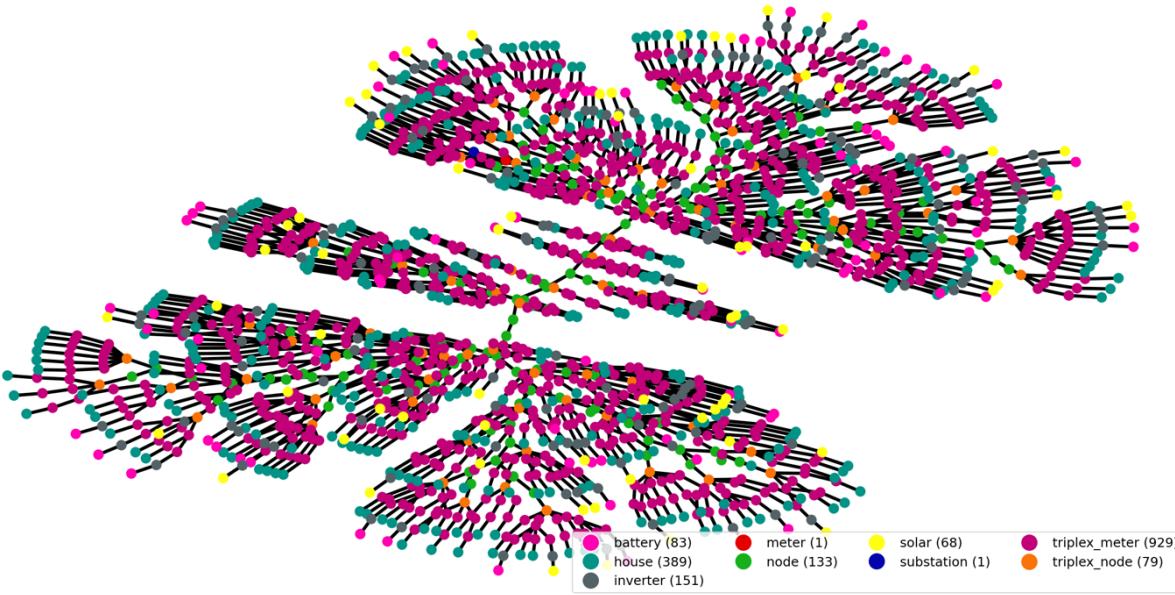
Large scale attack (2b) system impacts

	Pre-attack	Post-attack	Attack mitigation
Power import from main grid [kW]	1,325	1,821 (+37.4%)	1,328
Total cost [\$]	10,752	11,500 (+7%)	14,156 (+31.7%)
Total load [kW]	2,064	2,023 (-0.02%)	1,775 (-14%)



1. Without mitigation, attack would lead to large increase in power import of **600 kW**
2. Mitigation 1st utilizes upward flexibility to increase local generation by **284 kW**
3. Then curtails flexible loads by **307 kW**
4. Minimizes extra power import to only **3 kW**
5. Increases cost for PMO
→ Need to compensate resources for their flexibility

Realistic large-scale simulations & algorithm validation



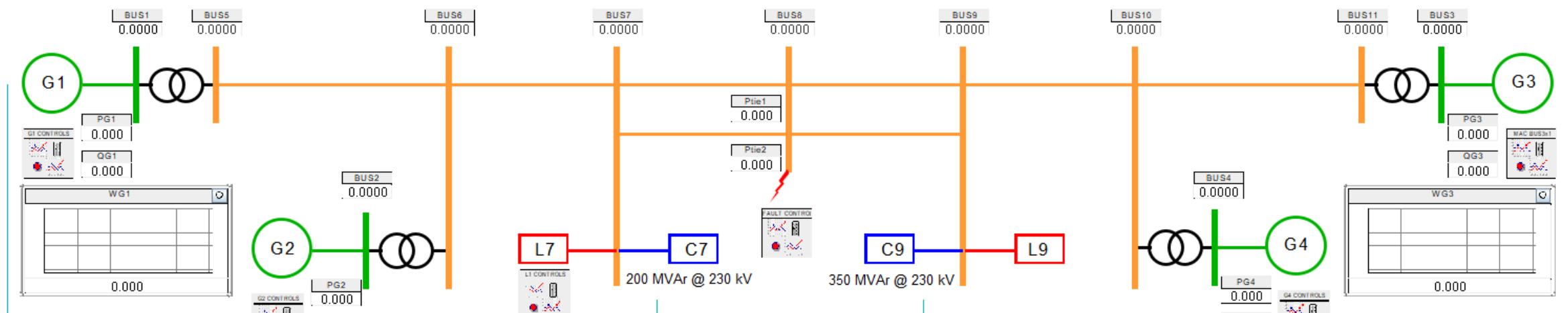
Pacific Northwest
NATIONAL LABORATORY

- Worked closely with external validation partners
- Obtained realistic input data for high DER-penetration systems
- Developed open-source codes for partners in MATLAB, Python and Julia using free packages & solvers
- Tested our algorithms on industry-grade software & hardware at national labs

NREL demonstration of attack 2b*

Two Area Power System

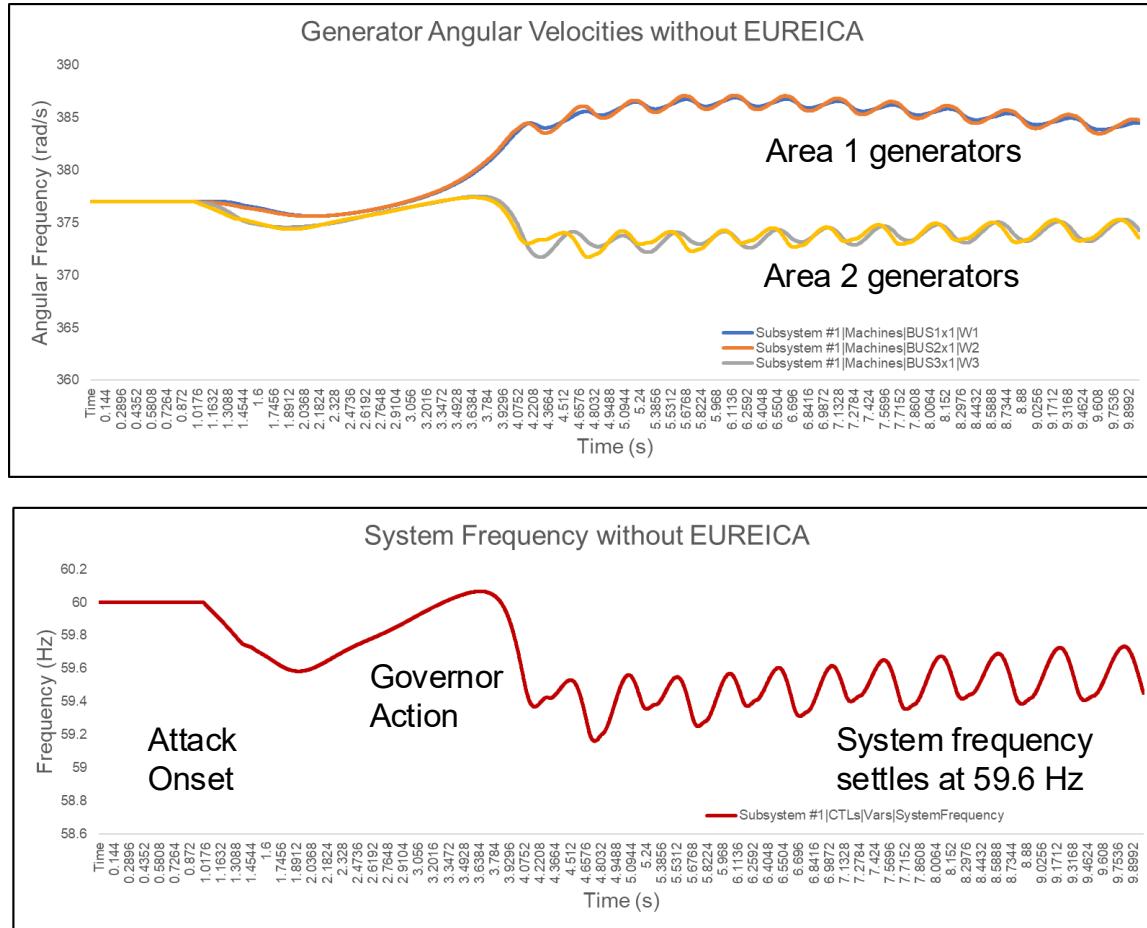
— 20 kV
— 230 kV



- Kundur 2-area system
- **650 kW** generation shortfall replicated in each of the 550 123-node feeders
→ Aggregated **359 MW** loss at transmission level in area 2

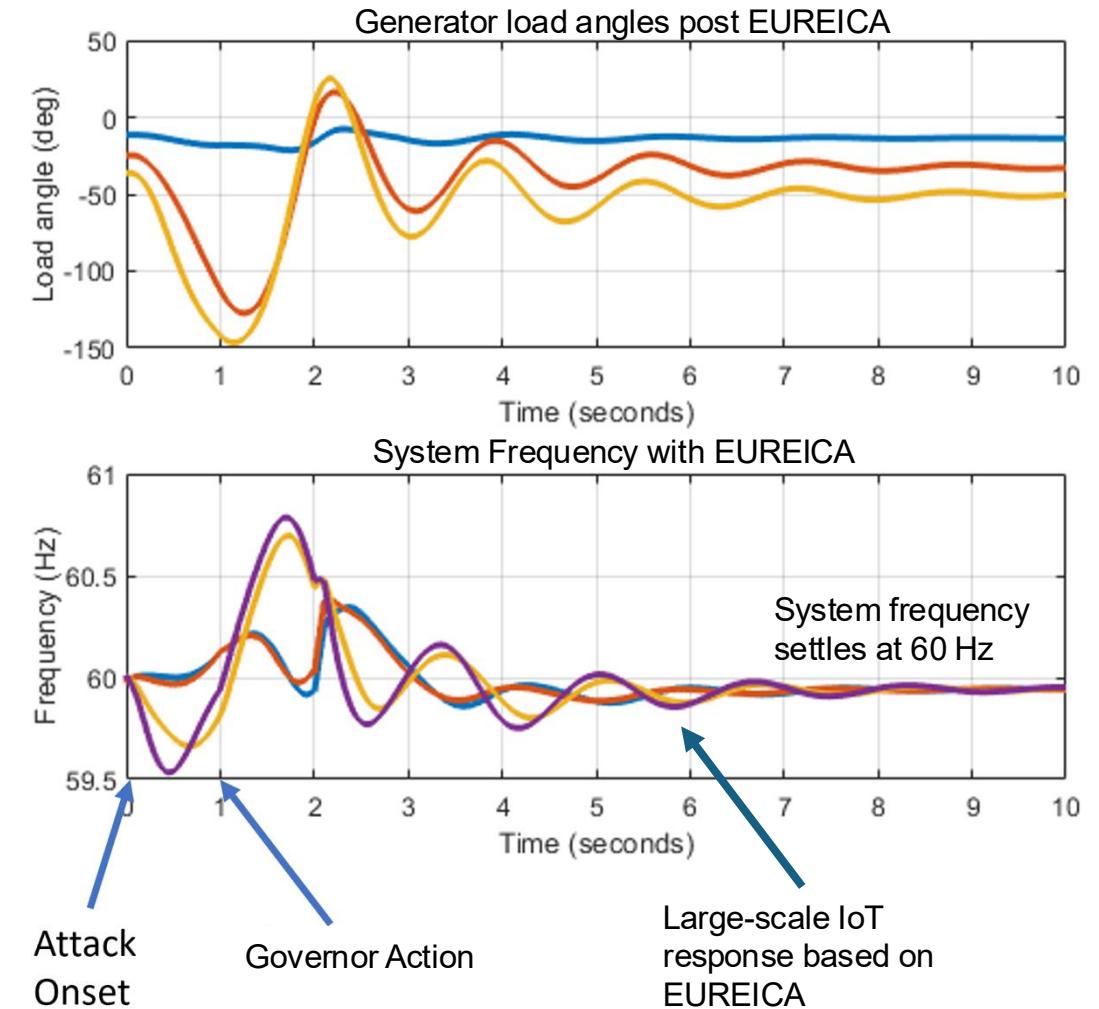
*Performed using our data/algorithms by NREL team

Attack 2b validation at transmission level



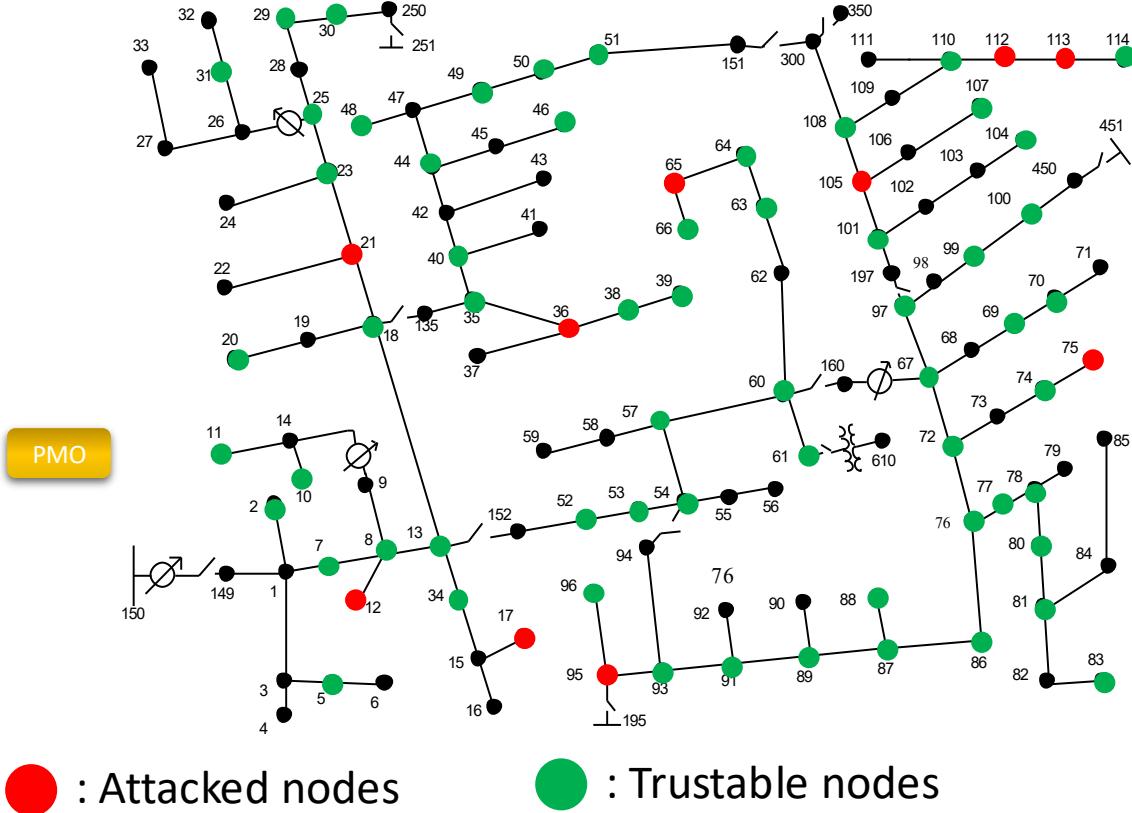
RESPONSE WITHOUT MARKET MITIGATION

*Performed using our data/algorithms by NREL team



RESPONSE WITH MARKET MITIGATION

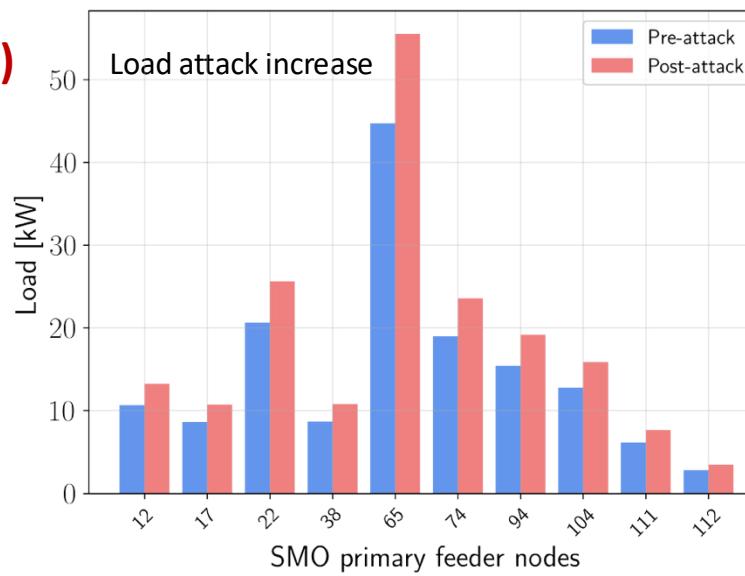
Attack 1a: PMA load alteration



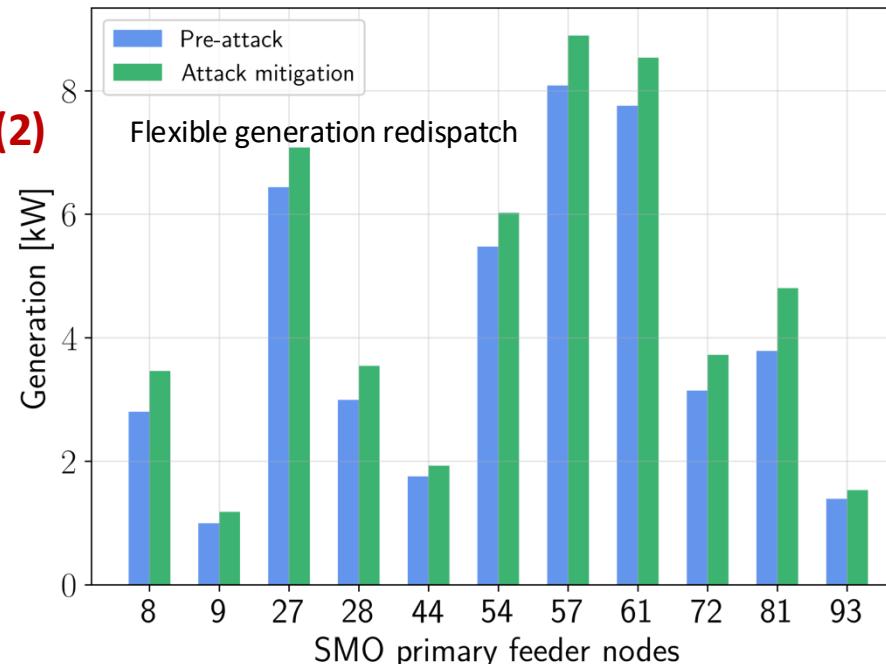
- Increase in load = **36 kW**
- 30 flexible load nodes help with mitigation
- **123 kW decrease in power import**

Attack 1a results

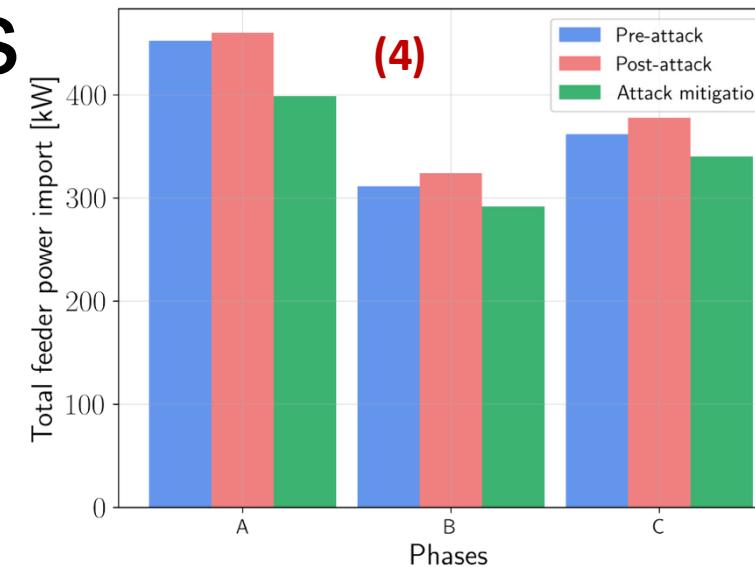
(1)



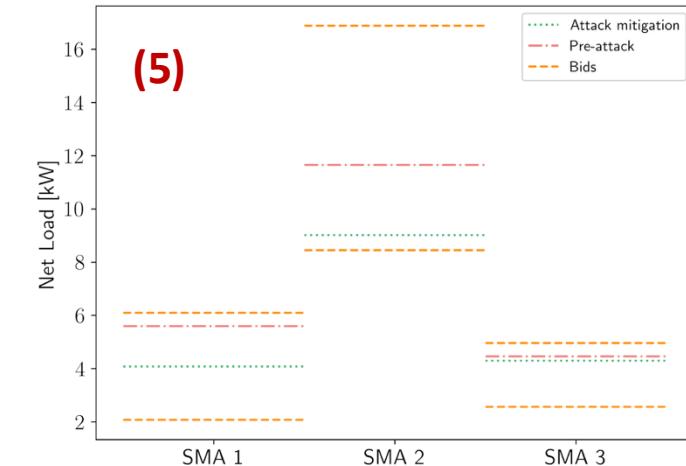
(2)



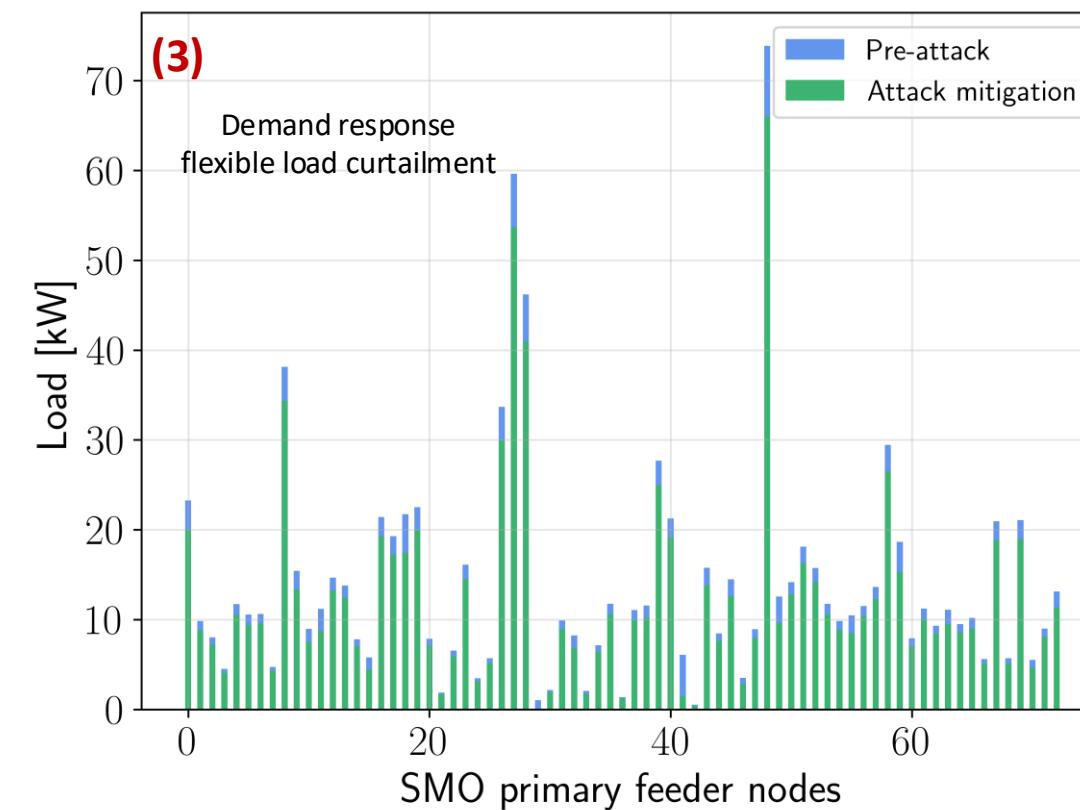
(4)



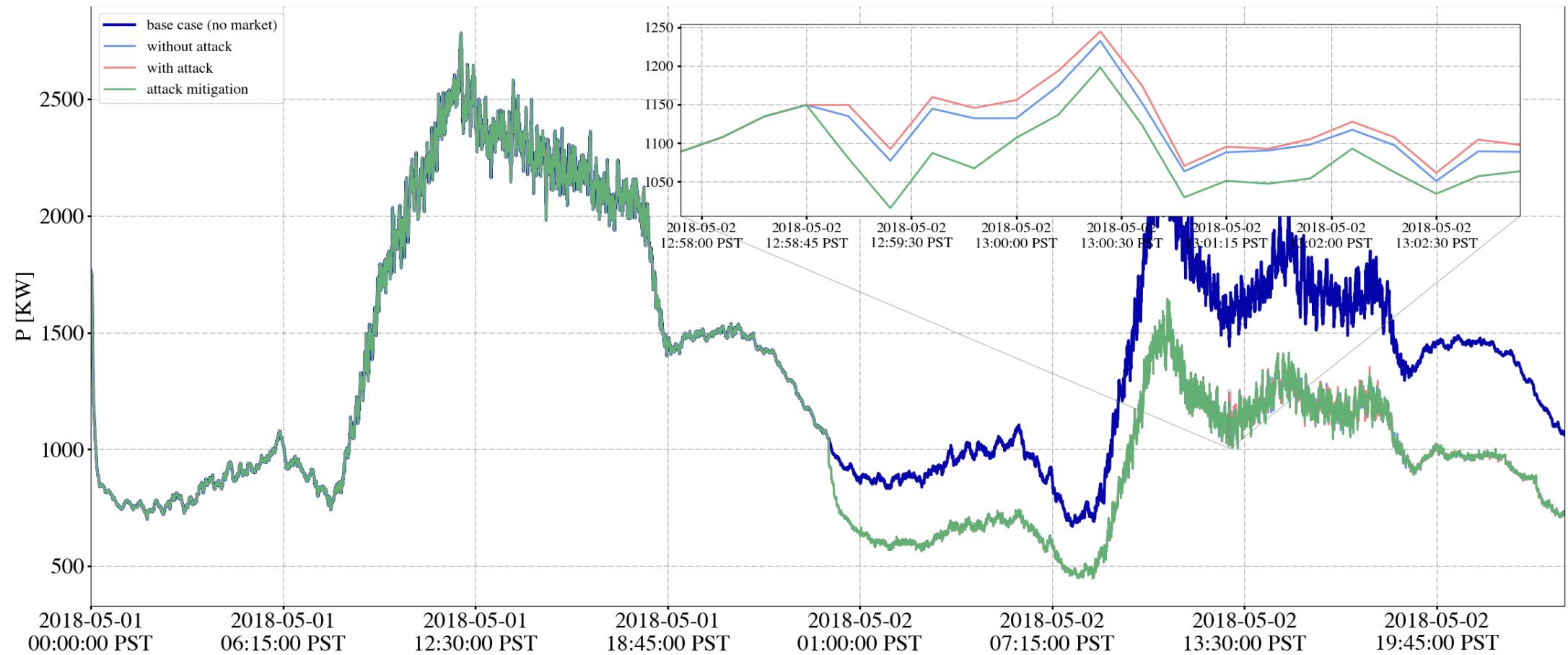
SMA bids and solutions for SMO 35



(3)

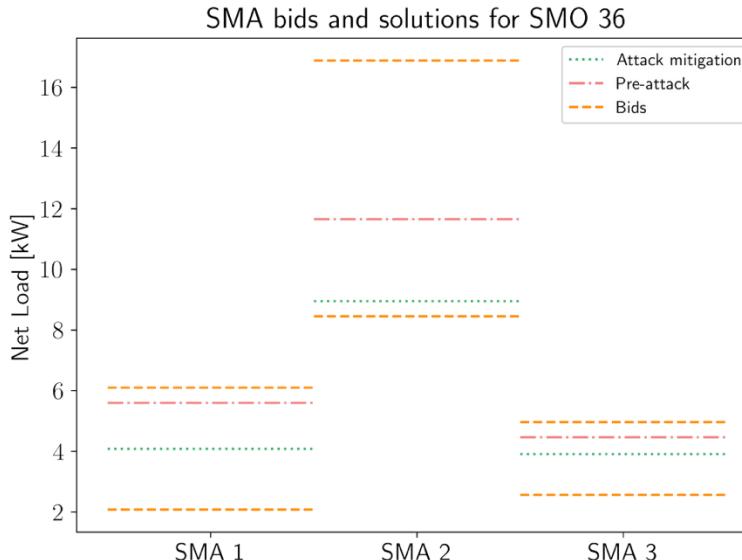


Attack 1a validation with PNNL co-simulation*

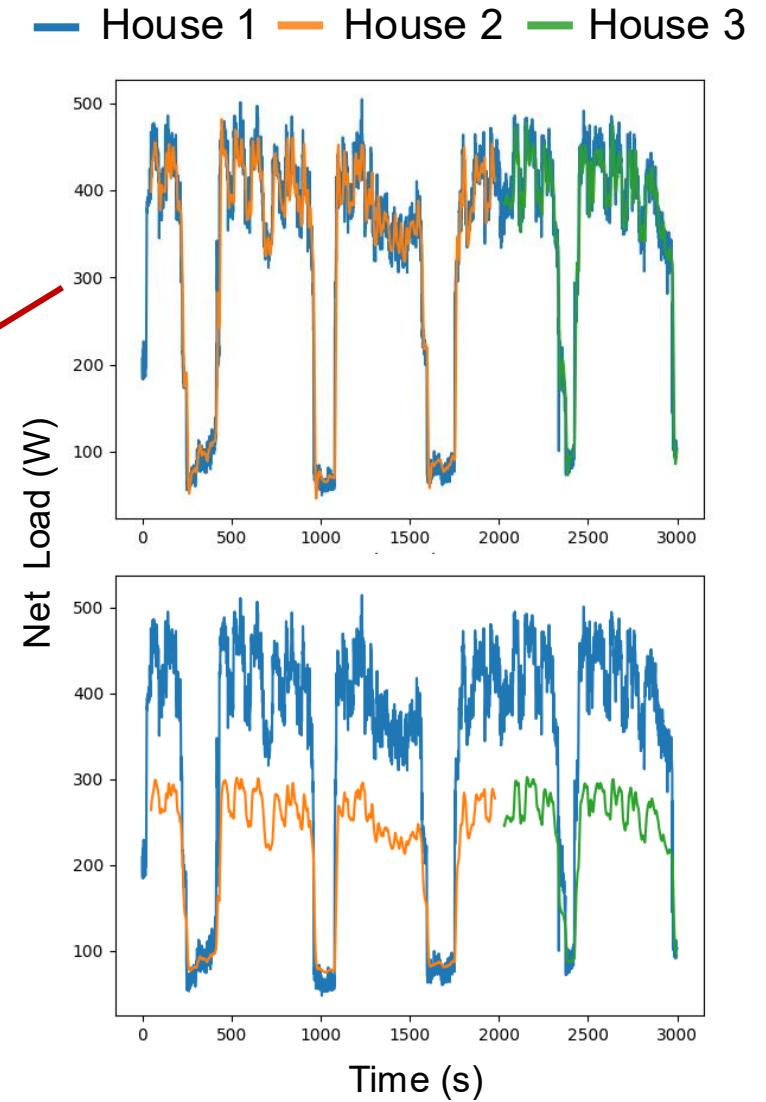
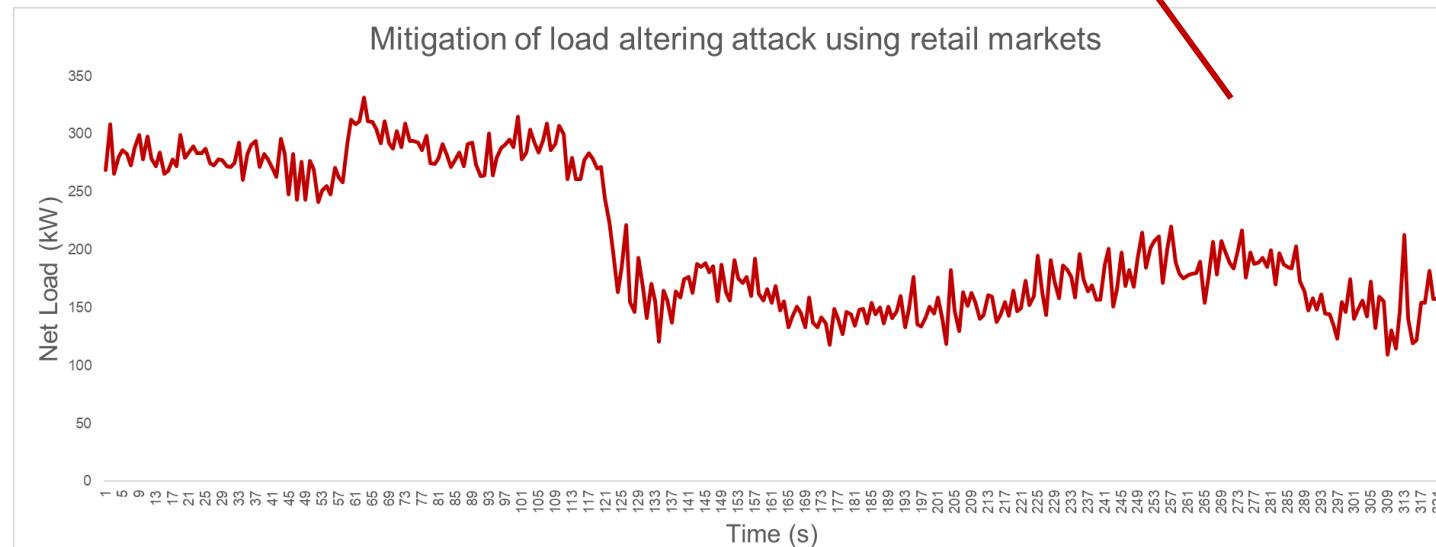


*Performed using our data/algorithms by PNNL team

Attack 1a NREL validation: Typhoon HIL

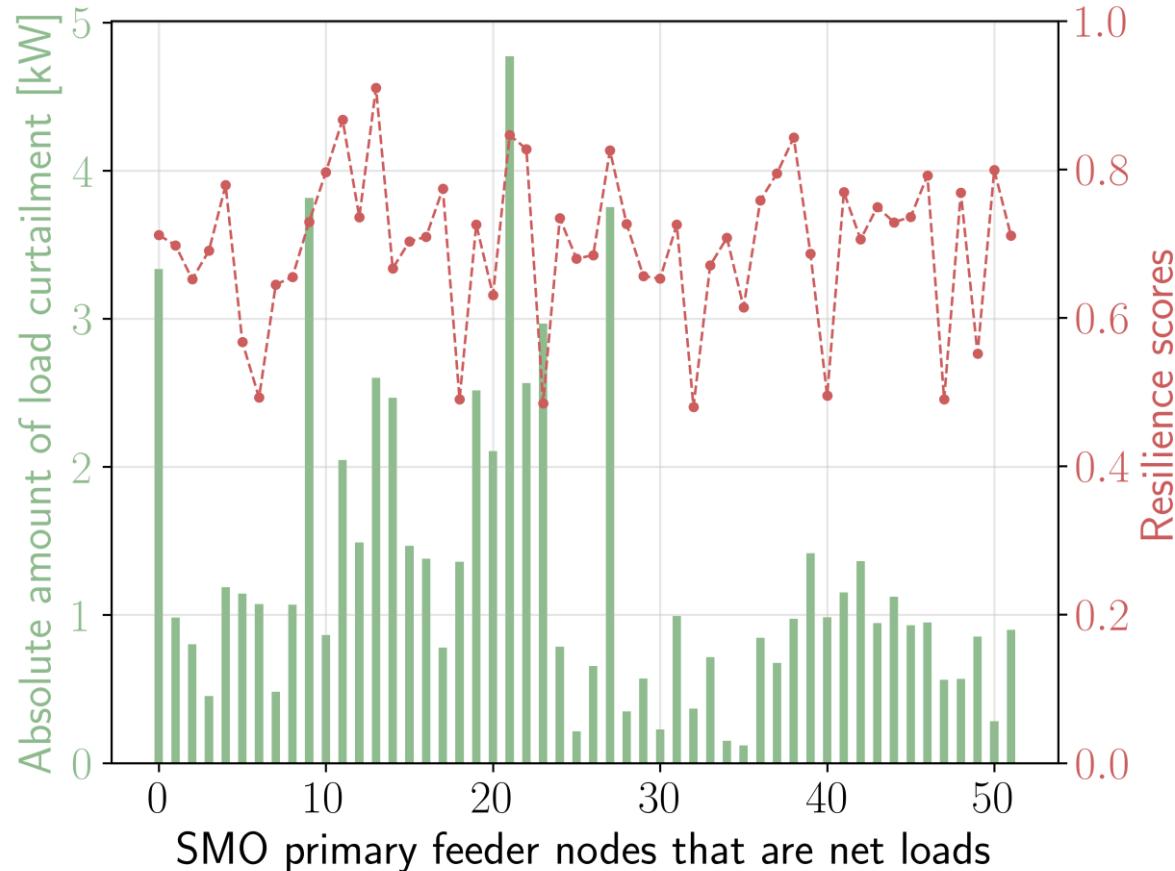


- Hardware-in-the-loop validation using Raspberry Pi controllers on HVAC units
- Market clearing every minute
 - Instantaneous thermostat response
 - Immediate drop in net load
- Load curtailment at IoT device level ranges from 0.2 to 0.5 kW reduction per house
- 130 kW decrease in import after mitigation



*Performed using our data/algorithms by NREL team

Effects of resilience scores on flexible load curtailment in primary market



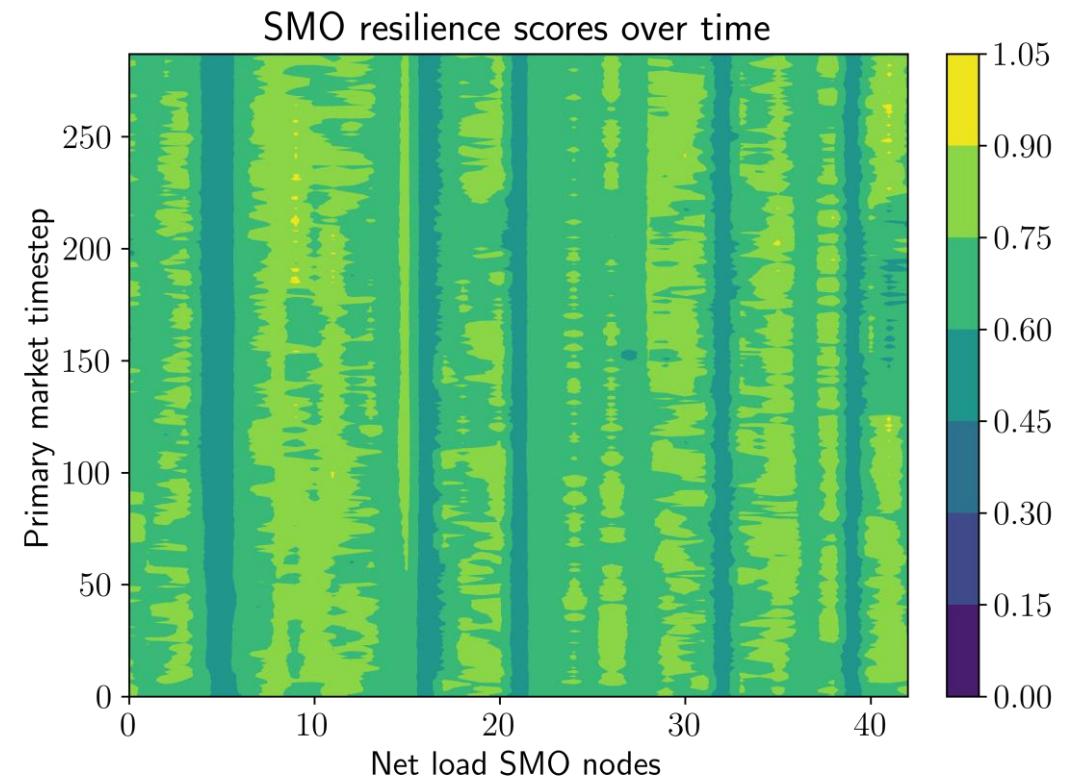
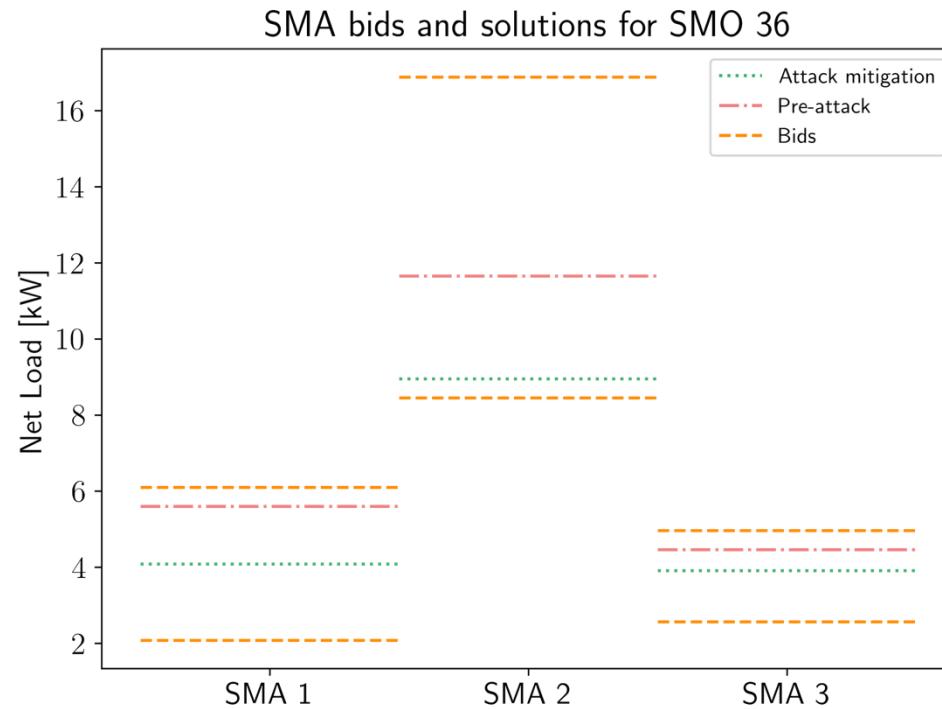
- PMO utilizes more flexibility from SMO nodes with higher scores
- More resilient assets contribute more to load curtailment & attack mitigation

- **IoT trustability score (TS):** Captures possibility of agents being compromised. Based on IoT network traffic patterns & cyber anomalies or vulnerabilities
- **Market commitment score (CS):** Measures how reliably agents will follow through & meet their contractual commitments
- **Resilience score (RS)** combines CS & TS to provide overall situational awareness
 $0 \leq RS \leq 1$ (Higher RS = more resilient)

Resilience score effects on secondary market

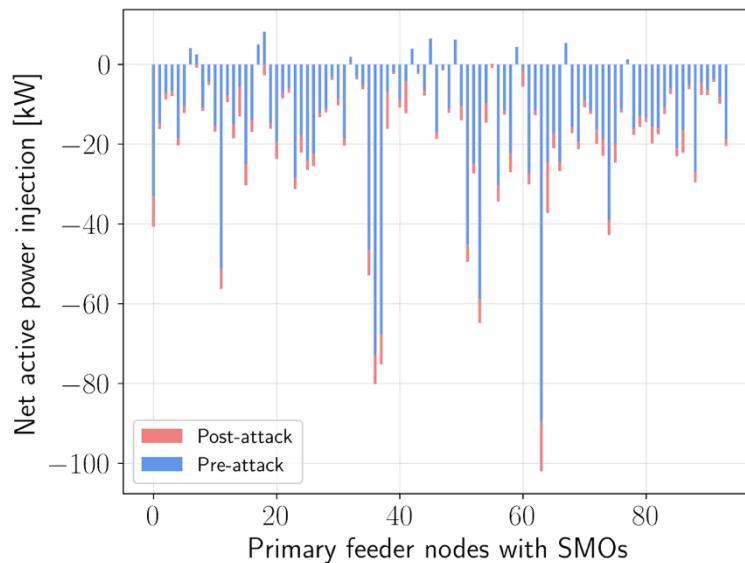
- Distribute flexibility (curtailment) among SMAs based on their RS
- Allocate more flexibility to SMAs with higher RS

SMA	RS
SMA 1	0.947
SMA 2	0.985
SMA 3	0.493

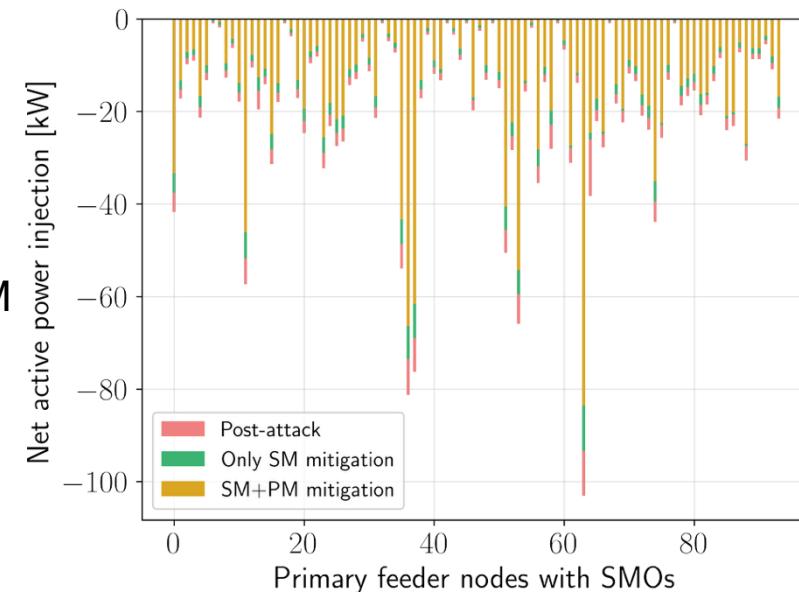


Attack 1c: Attack individual SMA nodes directly

1. More distributed (but smaller in scale) attack



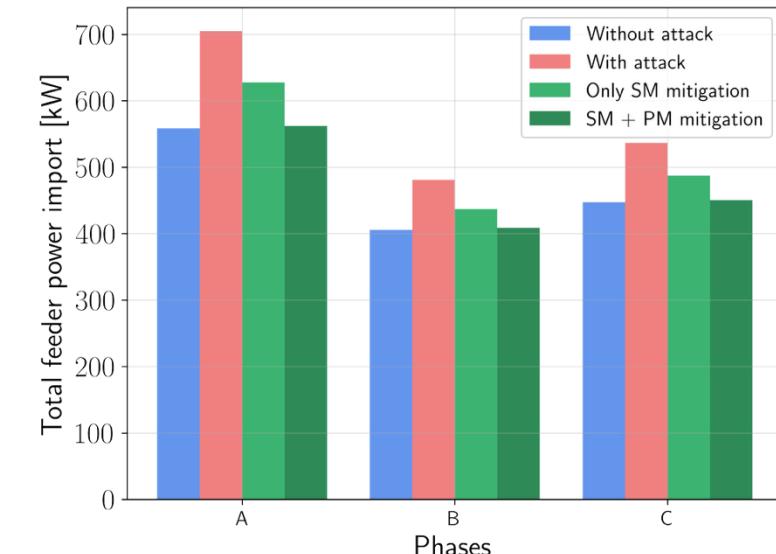
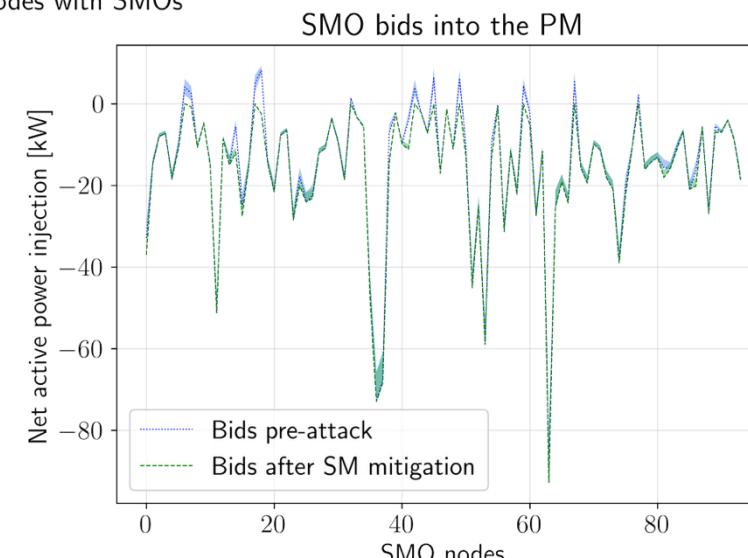
3. Followed by PM
redispatch to fully
resolve attack



4. SM + PM redispatch
restores feeder power
import to pre-attack level



2. SM redispatch 1st
partially mitigates
attack impact



Conclusions

- Developed new local electricity market for distribution grid
- Hierarchical structure accounts for all constraints & user preferences
- Distributed & decentralized optimization improve scalability & privacy
- Leveraged transactive method & pricing to incentivize agents
- Market successfully coordinates distributed energy resources to provide valuable grid services, improve reliability, & lower costs
- Applied game theory to account for strategic behavior & developed mechanism to extract flexibility from DERs
- Successfully mitigated cyberphysical attacks to strengthen grid resilience
- DER coordination significantly boosts grid hosting capacity

Publications

- Vineet Nair et al. “Resilience of the Electric Grid through Trustable IoT-Coordinated Assets.” Proceedings of the National Academy of Sciences (PNAS 2025).
- Vineet Nair, Priyank Srivastava, and Anuradha M. Annaswamy. “Enhancing power grid resilience to cyber-physical attacks using distributed retail electricity markets.” 15th ACM/IEEE International Conference on Cyber-Physical Systems (ICCPs 2024).
- Vineet Nair, and Anuradha M. Annaswamy. “A game-theoretic, market-based approach to extract flexibility from distributed energy resources.” 5th IFAC Workshop on Cyber-Physical Human Systems (CPHS 2024).
- Vineet Nair et al. “Federated Learning Forecasting for Strengthening Grid Reliability and Enabling Markets for Resilience.” International Conference On Electricity Distribution (CIRED) 2024 USA Workshop.
- Lucas Pereira, Vineet Nair, et al. “Accurate Federated Learning With Uncertainty Quantification For Distributed Energy Resource Forecasting Applied To Smart Grids Planning And Operation: The ALAMO Vision.” International Conference On Electricity Distribution (CIRED) 2024 Vienna Workshop.
- Vineet Nair, “Enhanced Physics-informed Neural Networks for high-order power grid dynamics.” NeurIPS 2024 Workshop on Tackling Climate Change with Machine Learning.
- Vineet Nair and Anuradha Annaswamy, “Local retail markets for distribution grid services.” 7th IEEE Conference on Control Technology and Applications (CCTA), 2023.
- Priyank Srivastava, Rabab Haider, Vineet Nair, Venkatesh Venkataraman, Anuradha Annaswamy, and Anurag Srivastava, “Voltage regulation in distribution grids: A survey”. Annual Reviews in Control, 2023.
- Anuradha M. Annaswamy and Vineet Nair. “Human Behavioral Models Using Utility Theory & Prospect Theory.” In Cyber-Physical-Human Systems: Fundamentals & Applications, UK: Wiley, in Press (2023).
- Vineet Nair, Venkatesh Venkataraman, Rabab Haider, and Anuradha Annaswamy, “A Hierarchical Local Market for a DER-Rich Grid Edge.” IEEE Transactions on Smart Grid, 2022.
- Thomas Lee, Vineet Nair, and Andy Sun, “Impacts of Dynamic Line Ratings on the ERCOT Transmission System.” 54th IEEE North American Power Symposium (NAPS 2022)
- Vineet Nair and Lucas Pereira, “Improving accuracy & convergence of federated learning edge computing methods for generalized DER forecasting applications in power grids.” NeurIPS 2022 Tackling Climate Change with Machine Learning workshop.

Working papers

- Vineet Nair, Morteza Vahid Ghavidel, and Anuradha Annaswamy, “Dynamic resource coordination can significantly increase power grid hosting capacity to accommodate more renewables, storage, and electrified load growth.” In preparation for Joule (2025).
- Vineet Nair, Jesús Rodríguez-Molina, Juan Garbajosa, and Anuradha Annaswamy, “Blockchain-enabled energy price formation in local electricity market via energy traceability with Smart Contracts.” In preparation for IEEE Internet of Things journal (2025).
- Luca Hartmann, Vineet Nair, Florian Dorfler, and Anuradha Annaswamy, “Circuit-aware distributed model predictive voltage control for distribution grids.” In preparation for Control Engineering Practice (2025).
- Danielle Knutson, Vineet Nair, and Anuradha Annaswamy, “Understanding technical & socioeconomic drivers behind the spatial distribution & heterogeneity of distributed energy resources in California.” In preparation (2025).
- Layla Araiinejad*, and Vineet Nair*, “The potential for nuclear fusion to sustainably & reliably power AI data centers.” In preparation (2025).
- Vineet Nair, “Optimal transmission switching and grid reconfiguration for transmission systems via convex relaxations.” Under revision for Electricity (2025).



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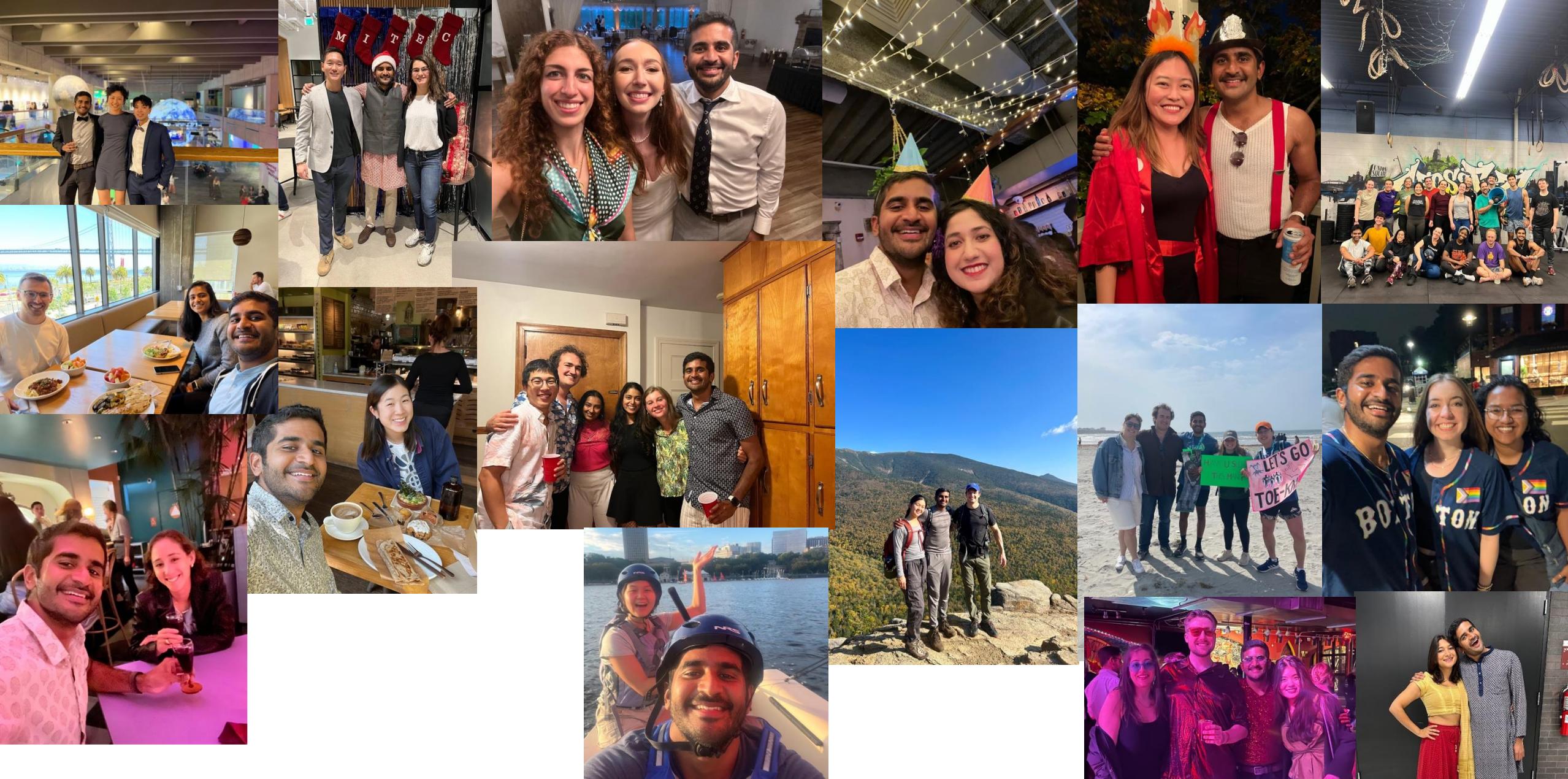
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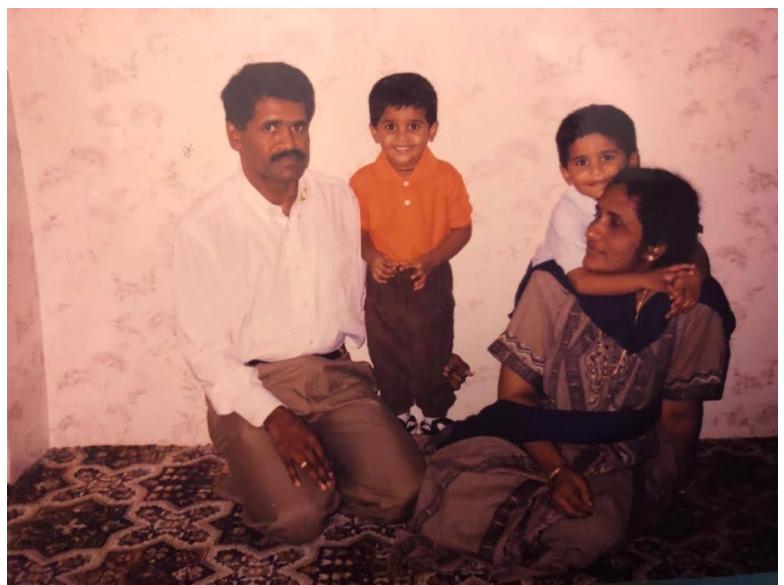
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Appendix

A distributed paradigm

Urban distribution network: ~2 million nodes [1]



At grid's edge

Intelligence + Control



119m smart meters in 2022 [2]

Communication



Internet of things

Renewables



Projected 6.7 GW solar in
New England by 2028 [3]

Faster Timescales



Solar + Wind
Demand Response
milliseconds - minutes

- Resource Coordination: Distributed Optimization + Control
- Multiple stakeholders are present

[1] M. Zhao et al. Trans. on Computer-Aided Design of Integrated Circuits and Systems, Feb. 2002

[2] EIA

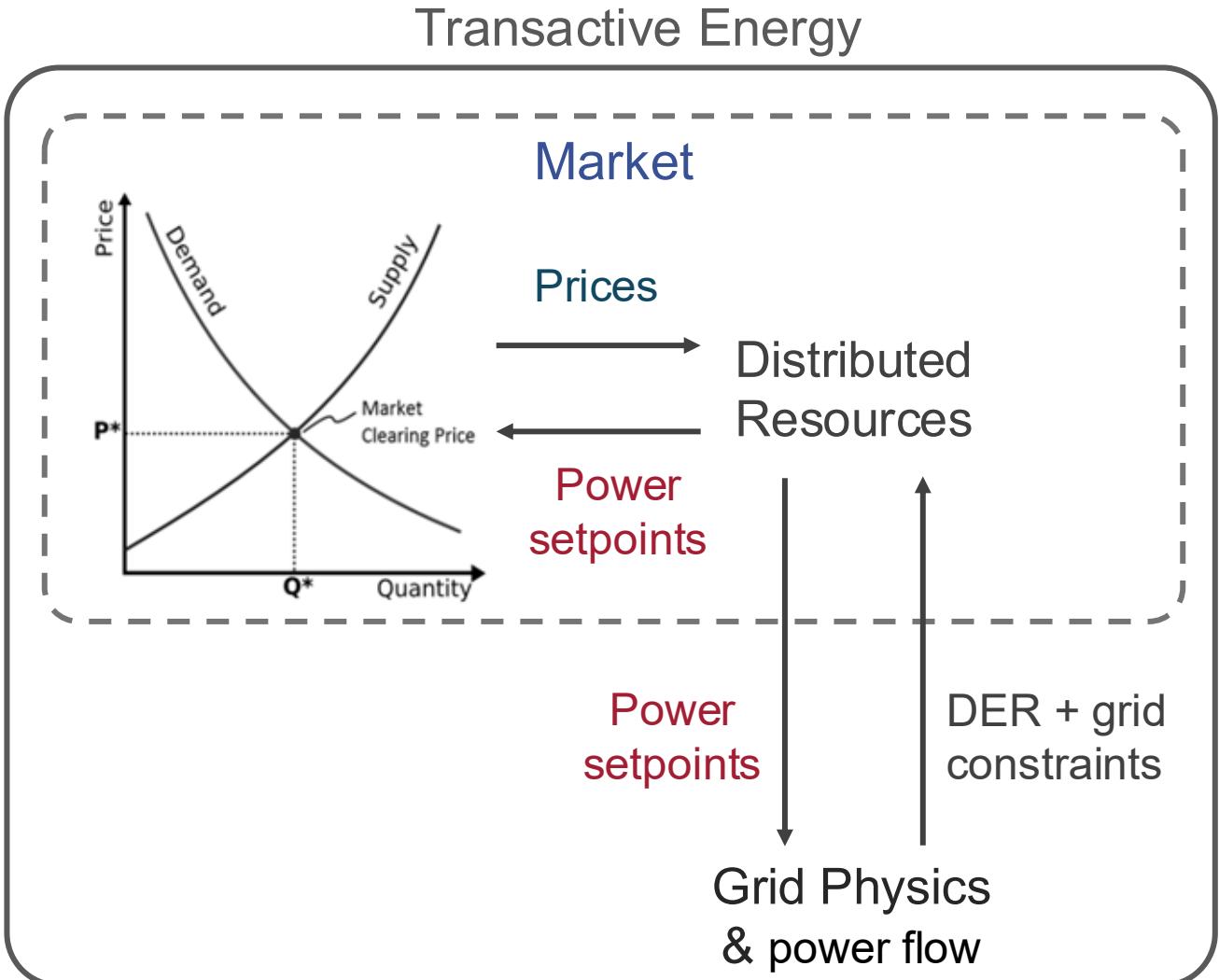
[3] ISO-NE <https://www.iso-ne.com/about/what-we-do/in-depth/solar-power-in-new-england-locations-and-impact>.

Transactive Energy

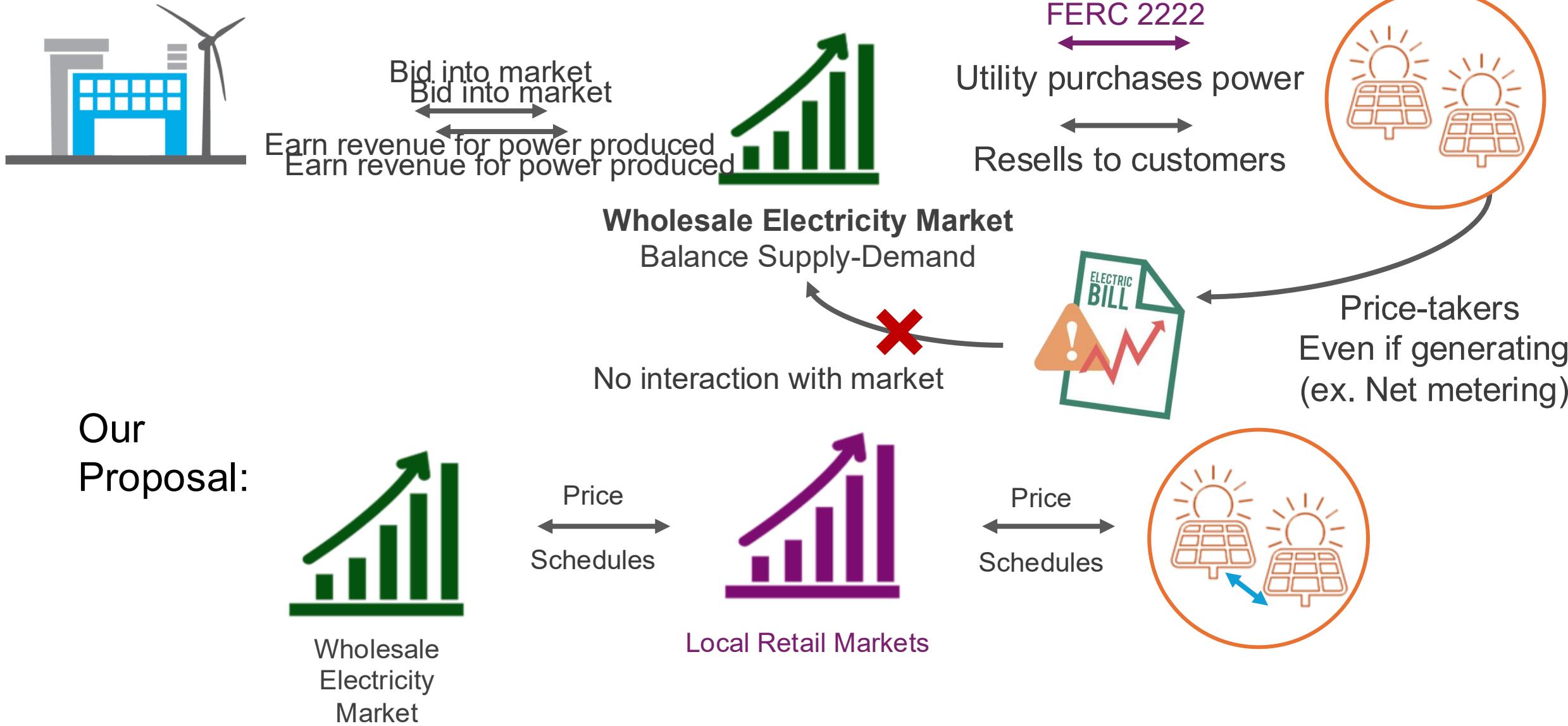
Use markets & prices to influence desired behaviours from various **autonomous, independent agents at the grid edge, at fast timescales**

Efficient integration of Distributed Energy Resources possible with a transactive design:

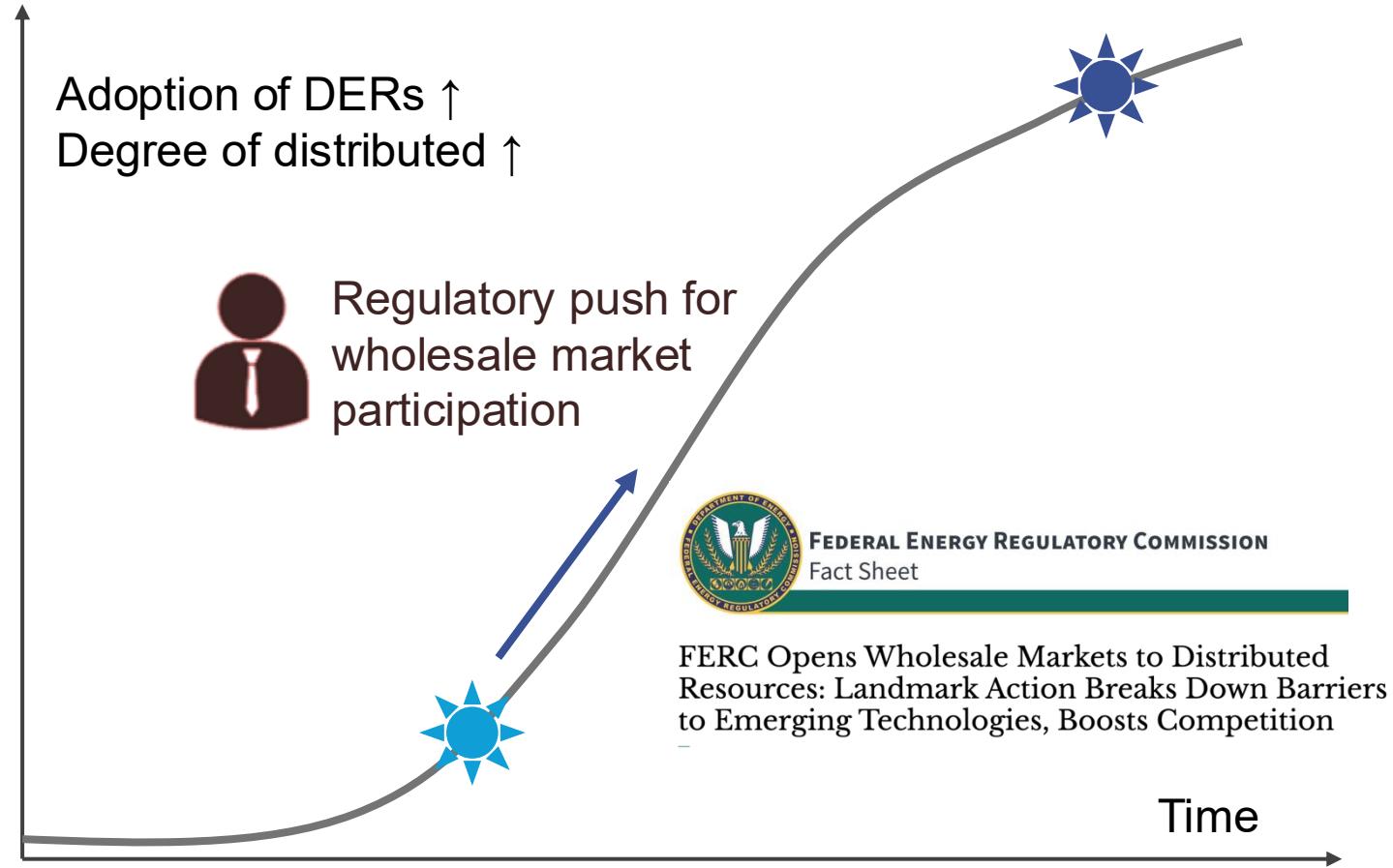
- **Flexible loads** (thermostats, water heaters)
- Distributed generation (rooftop **solar, wind**)
- Storage (**EVs, batteries**)



Primer on electricity markets



Why local retail electricity markets?

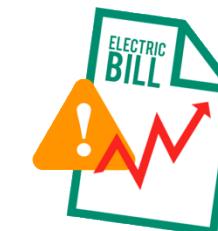


Fully integrate DERs into the network
using **distributed & decentralized local retail markets**

Gaps in Existing Programs



Wholesale



Retail

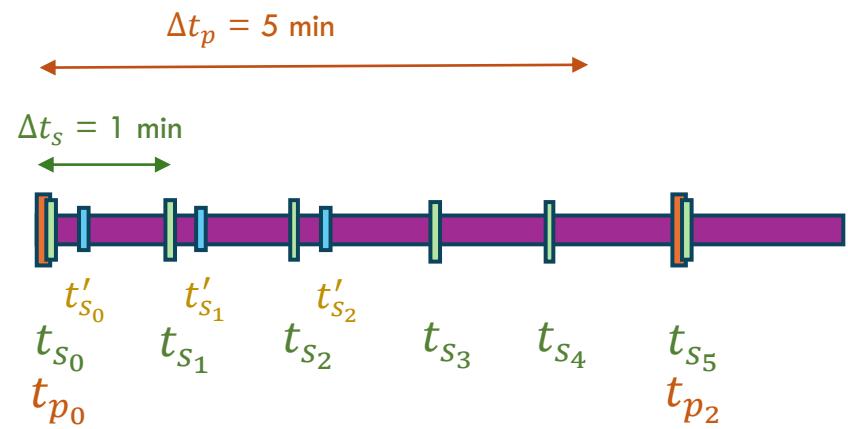


Vary spatially + temporally

- Retail and wholesale differ
- Prohibitive costs for small resources
- Limiting participation requirements [1]
- Retail markets do not exist
- Fixed retail prices are inefficient
- Over- or under-compensation [2,3]

[1] J. Gundlach and R. Webb. "Distributed energy resource participation in wholesale markets: Lessons from the California ISO", 2018
[2] Newell, S and Ahmad Fi "Dynamic Pricing: Potential Wholesale Market Benefits in New York" *The Brattle Group* (2009).
[3] L.V. Wood. "Why net energy metering results in a subsidy: The elephant in the room." 2016

Secondary market (SM): Flexibility in bids



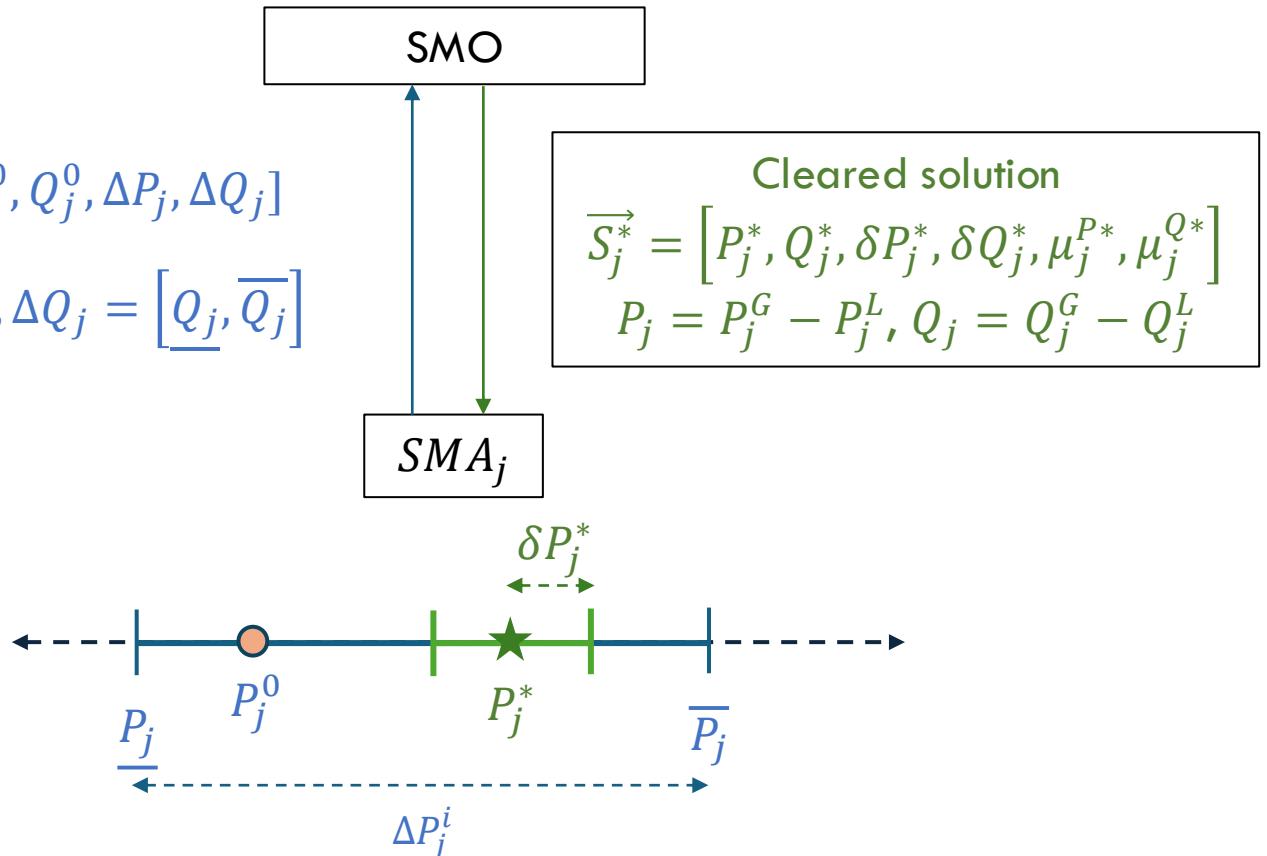
$$\text{Bid } \vec{B}_j = [P_j^0, Q_j^0, \Delta P_j, \Delta Q_j]$$

$$\Delta P_j = [P_j, \bar{P}_j], \Delta Q_j = [Q_j, \bar{Q}_j]$$

Cleared solution

$$\vec{s}_j^* = [P_j^*, Q_j^*, \delta P_j^*, \delta Q_j^*, \mu_j^{P*}, \mu_j^{Q*}]$$

$$P_j = P_j^G - P_j^L, Q_j = Q_j^G - Q_j^L$$



1. t_{s_0} : **Bidding** for $[t_{s_0}, t_{s_1}]$ period
2. t'_{s_0} : **Scheduling** (market clearing) for $[t_{s_0}, t_{s_1}]$
3. t'_{s_1} : **Settlements** (financial transactions) for $[t_{s_0}, t_{s_1}]$

All bids are for 1 period into the future & based on load or generation forecasts

Secondary market: Optimization problem

- IoT trustability score (TS):**
Captures possibility of agents being compromised. Based on IoT network traffic patterns & cyber anomalies or vulnerabilities
 - Commitment score (CS):**
Measures how reliably agents will follow through & meet their contractual commitments
 - Resilience score (RS)** combines both CS & TS to provide overall situational awareness
- $0 \leq RS \leq 1$ (RS closer to 1 = more resilient)

Subject to:

- Device operating and flexibility limits (P and Q)
- Budget balance, price cap for retail prices
- Lossless power balance
- Hierarchical approach* to solve multi-objective problem
- Can extend this to multiple phases

$$\min_{\vec{S}_j^i} \sum_{j \in \mathcal{N}_{J,i}} \{f_{j,1}^i, f_{j,2}^i, f_{j,3}^i, f_{j,4}^i\}$$

$$f_{1,j}^i \succ f_{2,j}^i \succ f_{3,j}^i \succ f_{4,j}^i$$

$$f_{j,1} = -RS_j^i((P_j^i - P_j^{i0})^2 + (Q_j^i - Q_j^{i0})^2)$$

$$f_{j,2}^i = \mu_j^{iP} P_j^i + \mu_j^{iQ} Q_j^i$$

$$f_{j,3}^i = -(\delta P_j^i + \delta Q_j^i)$$

$$f_{j,4}^i = \beta_j^{iP} (P_j^i - P_j^{i0})^2 + \beta_j^{iQ} (Q_j^i - Q_j^{i0})^2$$

Flexibility bids

SMO



Power setpoints,
retail prices

SMA

Max aggregate reliability

Min net cost to SMO

Max aggregate flexibility

Min disutility for SMA flexibility

$$\min_{\vec{S}_j^i} F_k = \sum_{j \in \mathcal{N}_{J,i}} f_{j,k}^i(\vec{S}_j^i) \quad \forall k = 1, 2, 3, 4$$

$$\text{s.t. } f_{j,\ell}^i(\vec{S}_j^i) \leq (1 + \epsilon) \sum_{j \in \mathcal{N}_{J,i}} f_{j,\ell}^i(\vec{S}_j^{i*}) = (1 + \epsilon)F_\ell^*,$$

$$\forall \ell = 1, 2, \dots, k-1, \quad k > 1$$

and all other constraints

Connecting secondary market to primary

- Before each primary clearing period, SMO i aggregates schedules across all of its SMAs j from latest secondary clearing
- SMO uses this combined solution to bid into primary market
- Use this to solve ACOPF at primary level

$$P_i^0(t_p) = \sum_{j \in \mathcal{N}_{J,i}} P_j^{i*}(t_p - \Delta t_s), Q_i^0(t_p) = \sum_{j \in \mathcal{N}_{J,i}} Q_j^{i*}(t_p - \Delta t_s)$$

$$\Delta P_i = \left[\underline{P}_i = \sum_{j \in \mathcal{N}_{J,i}} P_j^{i*} - \delta P_j^{i*}, \bar{P}_i = \sum_{j \in \mathcal{N}_{J,i}} P_j^{i*} + \delta P_j^{i*} \right]$$

$$\Delta Q_i = \left[\underline{Q}_i = \sum_{j \in \mathcal{N}_{J,i}} Q_j^{i*} - \delta Q_j^{i*}, \bar{Q}_i = \sum_{j \in \mathcal{N}_{J,i}} Q_j^{i*} + \delta Q_j^{i*} \right]$$

Primary market: AC optimal power flow

- Branch flow model
- Balanced, single-phase, radial network
- 2nd order cone convex relaxation

$$\min f(x)$$

Subject to:

$$v_j - v_i = (R_{ij}^2 + X_{ij}^2)l_{ij}$$

$$-2(R_{ij}P_{ij} + X_{ij}Q_{ij})$$

$$P_{ij} = R_{ij}l_{ij} - P_j + \sum_{k \in \{k_j\}} P_{jk}$$

$$Q_{ij} = X_{ij}l_{ij} - Q_j + \sum_{k \in \{k_j\}} Q_{jk}$$

$$P_{ij}^2 + Q_{ij}^2 \leq v_i l_{ij}$$

$$P_j \in [P_j, \bar{P}_j], Q_j \in [Q_j, \bar{Q}_j]$$

$$v_j \in [v_j, \bar{v}_j]$$

where $l_{ij} = |I_{ij}|^2$ and $v_i = |V_i|^2$.

- Current injection model
- Unbalanced, 3-phase, radial/meshed network
- McCormick envelopes convex relaxation

$$\min_x f(x)$$

$$I^R = \text{Re}(YV)$$

$$I^I = \text{Im}(YV)$$

$$P_i^\phi = V_i^{\phi,R} I_i^{\phi,R} + V_i^{\phi,I} I_i^{\phi,I} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

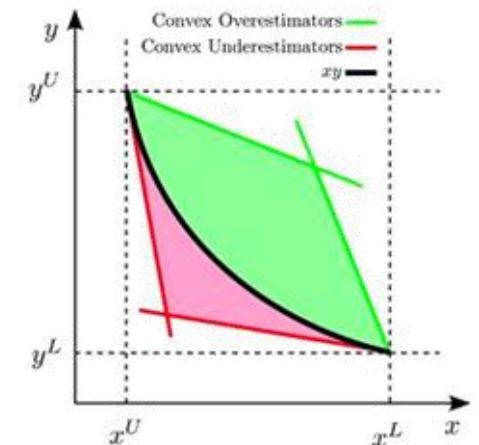
$$Q_i^\phi = -V_i^{\phi,R} I_i^{\phi,I} + V_i^{\phi,I} I_i^{\phi,R} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

$$\underline{V_i^\phi} \leq (V_i^{\phi,R})^2 + (V_i^{\phi,I})^2 \leq \overline{V_i^\phi}^2 \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

$$\underline{P_i^\phi} \leq P_i^\phi \leq \overline{P_i^\phi} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

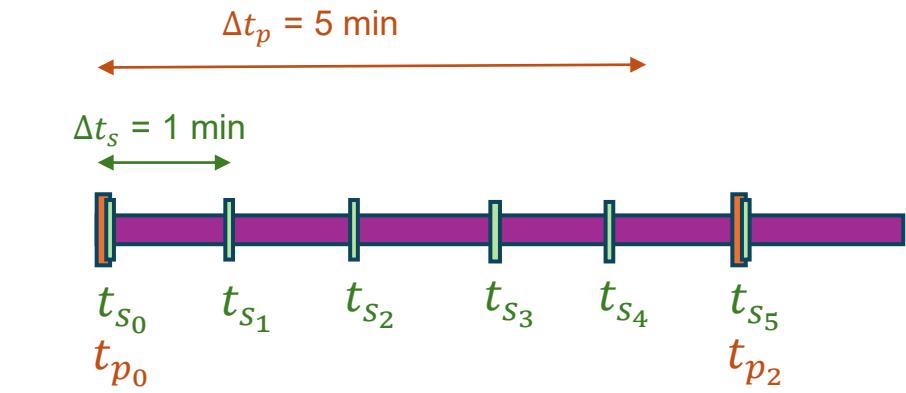
$$\underline{Q_i^\phi} \leq Q_i^\phi \leq \overline{Q_i^\phi} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

$$(I_{ij}^R)^2 + (I_{ij}^I)^2 \leq \overline{I_{ij}}^2 \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P}$$

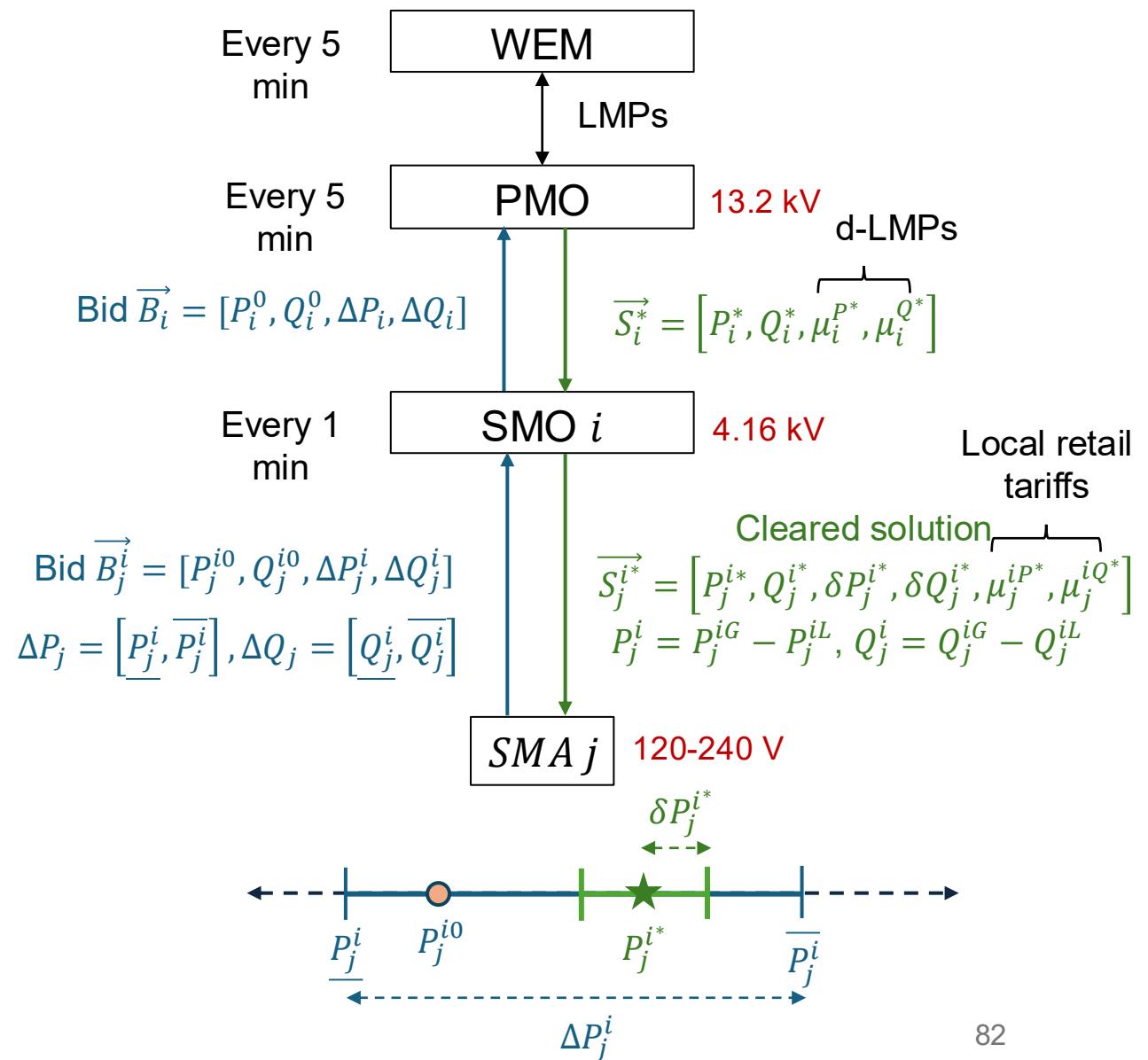


Iterative preprocessing
to set tight V & I bounds

Primary + secondary markets summary



t_{p_0} : Bidding by SMO for $[t_{p_0}, t_{p_1}]$ period
 → PMO clears
 t_{s_0} : Bidding by DCAs for $[t_{s_0}, t_{s_1}]$ period
 → Scheduling by SMO
 t_{s_1} : Billing (contract enforcement) by SMO for $[t_{s_0}, t_{s_1}]$
 $[t_{s_1}, t_{s_4}]$: SMA bidding & SMO clearing continue
 t_{s_5} : SMO aggregates DCA schedules
 → Uses this to bid to PMO for $[t_{p_1}, t_{p_2}]$



Apply 3-phase market for voltage regulation

1. Regulate voltage about set points:

$$|V| = 1 \text{ p.u.} \rightarrow V_i^{\phi, R^*} = 1, V_i^{\phi, I^*} = 0$$

$$f^{Volt, \phi}(x) = \sum_{i \in \mathcal{I}} \sum_{\phi \in \mathcal{P}} \left[\left(V_i^{\phi, R} - V_i^{\phi, R^*} \right)^2 + \left(V_i^{\phi, I} - V_i^{\phi, I^*} \right)^2 \right]$$

2. Minimize line losses

$$f^{Loss, \phi}(x) = \sum_{ij \in \mathcal{E}} \sum_{\phi \in \mathcal{P}} R_{ij} |I_{ij}^{\phi}|^2 = R_{ij} \left(I_{ij}^{\phi, R^2} + I_{ij}^{\phi, I^2} \right)$$

3. Minimize disutility

$$f^{Disutil, \phi}(x) = \beta_i^P (P_i^{L, \phi} - P_i^{L0, \phi})^2 + \beta_i^Q (Q_i^{L, \phi} - Q_i^{L0, \phi})^2$$

4. Minimize generation costs

$$f^{Volt, \phi}(x) = \sum_{i \in \mathcal{I}} \sum_{\phi \in \mathcal{P}} \begin{cases} \alpha_i^{P, \phi} P_i^{G, \phi} + \alpha_i^{Q, \phi} Q_i^{G, \phi}, \\ \lambda_i^P P_i^{G, \phi} + \lambda_i^Q Q_i^{G, \phi}, \end{cases} \quad \text{if } i \text{ is PCC}$$

Overall social welfare objective function

$$\begin{aligned} \min_y f^{S-W}(x) = & w_1 f^{Disutil, \phi}(x) + w_2 f^{Gen-Cost, \phi}(x) \\ & + w_3 f^{Volt, \phi}(x) + w_4 f^{Loss, \phi}(x) \end{aligned}$$

- Mult-objective optimization
 $w_1 + w_2 + w_3 + w_4 = 1, 0 \leq w_i \leq 1$
- All quantities (P, Q, V, I, R) in p.u.
 → Similar magnitude terms
- Adjust relative objective weights to reflect DSO's priorities
- Can extend to other grid service applications
 e.g. Conservation voltage reduction

Accurately pricing different grid services

Dual variables of equality constraints

→ Decompose distribution locational marginal prices (dLMP)

$$\min_x f(x)$$

KCL & KVL combined
(Ohm's law)

$$\left\{ \begin{array}{l} I^R = \text{Re}(YV) \\ I^I = \text{Im}(YV) \end{array} \right\}$$

Value of voltage support $\text{Re}(\lambda_V)$

$I = YV$
complex

Power balance

$$\left\{ \begin{array}{l} P_i^\phi = V_i^{\phi,R} I_i^{\phi,R} + V_i^{\phi,I} I_i^{\phi,I} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \\ Q_i^\phi = -V_i^{\phi,R} I_i^{\phi,I} + V_i^{\phi,I} I_i^{\phi,R} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \\ \underline{V_i^\phi} \leq (V_i^{\phi,R})^2 + (V_i^{\phi,I})^2 \leq \overline{V_i^\phi}^2 \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \\ P_i^\phi \leq \bar{P}_i^\phi \leq \overline{P_i^\phi} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \\ Q_i^\phi \leq \bar{Q}_i^\phi \leq \overline{Q_i^\phi} \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \\ (I_{ij}^R)^2 + (I_{ij}^I)^2 \leq \overline{I_{ij}}^2 \quad \forall i \in \mathcal{N}, \phi \in \mathcal{P} \end{array} \right\}$$

P & Q “energy”
prices λ_P, λ_Q

Price decomposition → Value of grid services

$$\mathcal{L} = f^{obj}(x) + \lambda_P^\top P_{balance} + \lambda_Q^\top Q_{balance} + \lambda_I^\top (I - YV) + \lambda_{ineq}^\top (RHS_{ineq} - LHS_{ineq})$$

$$\begin{aligned}\lambda_I^\top (I - YV) &\equiv \lambda_V^\top (ZI - V) = \lambda_V^\top (Y^{-1}I - Y^{-1}YV) = \lambda_V^\top Y^{-1}(I - YV) \\ \implies \lambda_I^\top &= \lambda_V^\top Y^{-1} \implies \lambda_V = Y^\top \lambda_I\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial V} = \frac{\partial f^{obj}(x)}{\partial V} - \lambda_I^\top Y = \frac{\partial f^{obj}(x)}{\partial V} - \lambda_V$$

At optimality $\frac{\partial \mathcal{L}}{\partial V^*} = \frac{\partial f^{obj}}{\partial V^*} - \lambda_V = 0 \implies \frac{\partial f^{obj}}{\partial V^*} = \lambda_V^*$

Cost of satisfying constraints
(in terms of degradation in
objective functions)

“Bundled” tariff

$$\begin{aligned}\lambda_{eq} &= (\lambda_P^* P^* + \lambda_Q^* Q^* + \bar{\lambda}_V^* \Delta V^*) / P^* \\ \Delta V^* &= |V^{R^*} - 1| + |V^{I^*}|\end{aligned}$$

Distributed optimization: Proximal atomic coordination (PAC)

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & \sum_{i=1}^n f_i(x_i) \\ \text{s.t.} \quad & Gx = b, \\ & Hx \leq d \end{aligned}$$

Atomization

$$\begin{aligned} \min_{a_j} \quad & \sum_{j=1}^k f_j(a_j) \\ \text{s.t.} \quad & G_j a_j = b_j, \\ & H_j a_j \leq d_j, \\ & B_j a = 0 \quad \forall j \in k \end{aligned}$$

Augmented Lagrangian

$$\begin{aligned} \mathcal{L}(a, \eta, \nu) &= \sum_{j \in K} [f_j(a_j) + \eta_j^T (G_j a_j - b_j) + \nu^T B^j a_j] \\ &\triangleq \sum_{j \in K} \mathcal{L}_j(a_j, \eta_j, \nu) \end{aligned}$$

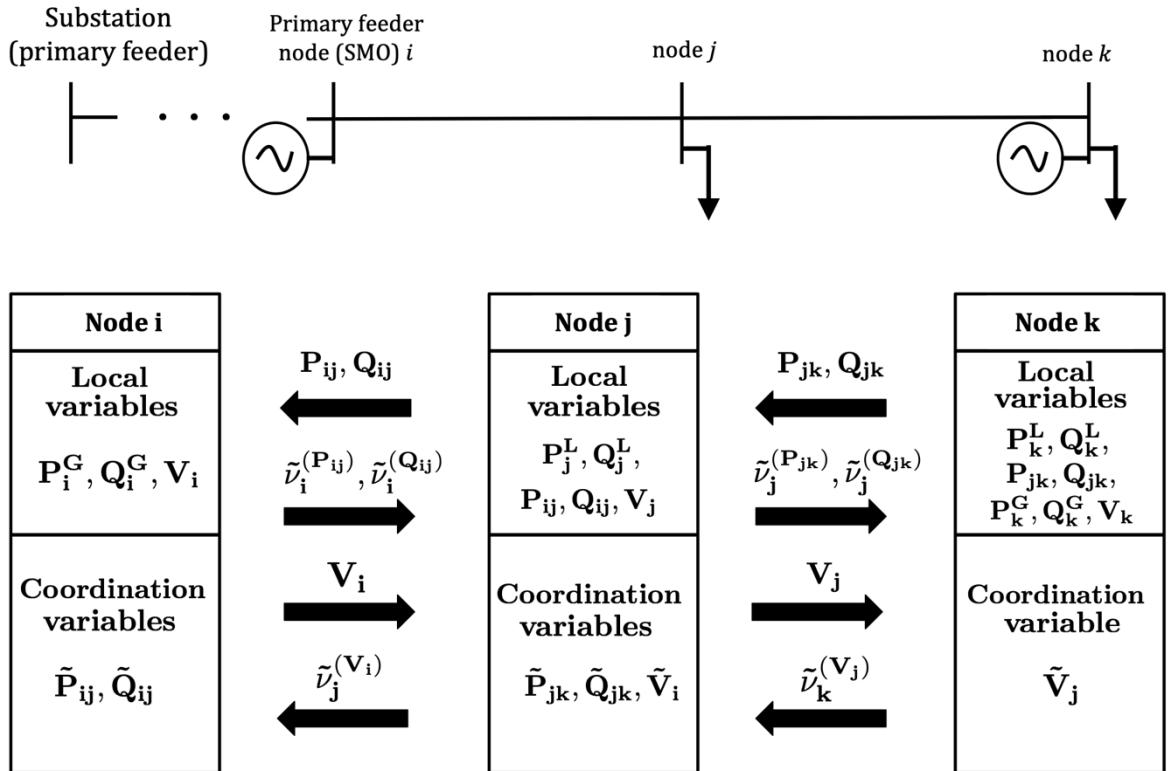
$$a_j[\tau+1] = \underset{a_j}{\operatorname{argmin}} \left\{ \mathcal{L}_j(a_j, \hat{\mu}_j[\tau], \hat{\nu}[\tau]) + \frac{1}{2\rho} \|a_j - a_j[\tau]\|_2^2 \right\}$$

$$\mu_j[\tau+1] = \mu_j[\tau] + \rho \gamma_j \tilde{G}_j a_j[\tau+1] \text{ and } \hat{\mu}_j[\tau+1] = \mu_j[\tau+1] + \rho \hat{\gamma}_j[\tau+1] \tilde{G}_j a_j[\tau+1]$$

Communicate $\{a_j[\tau+1]\}$ with neighbours, for all j

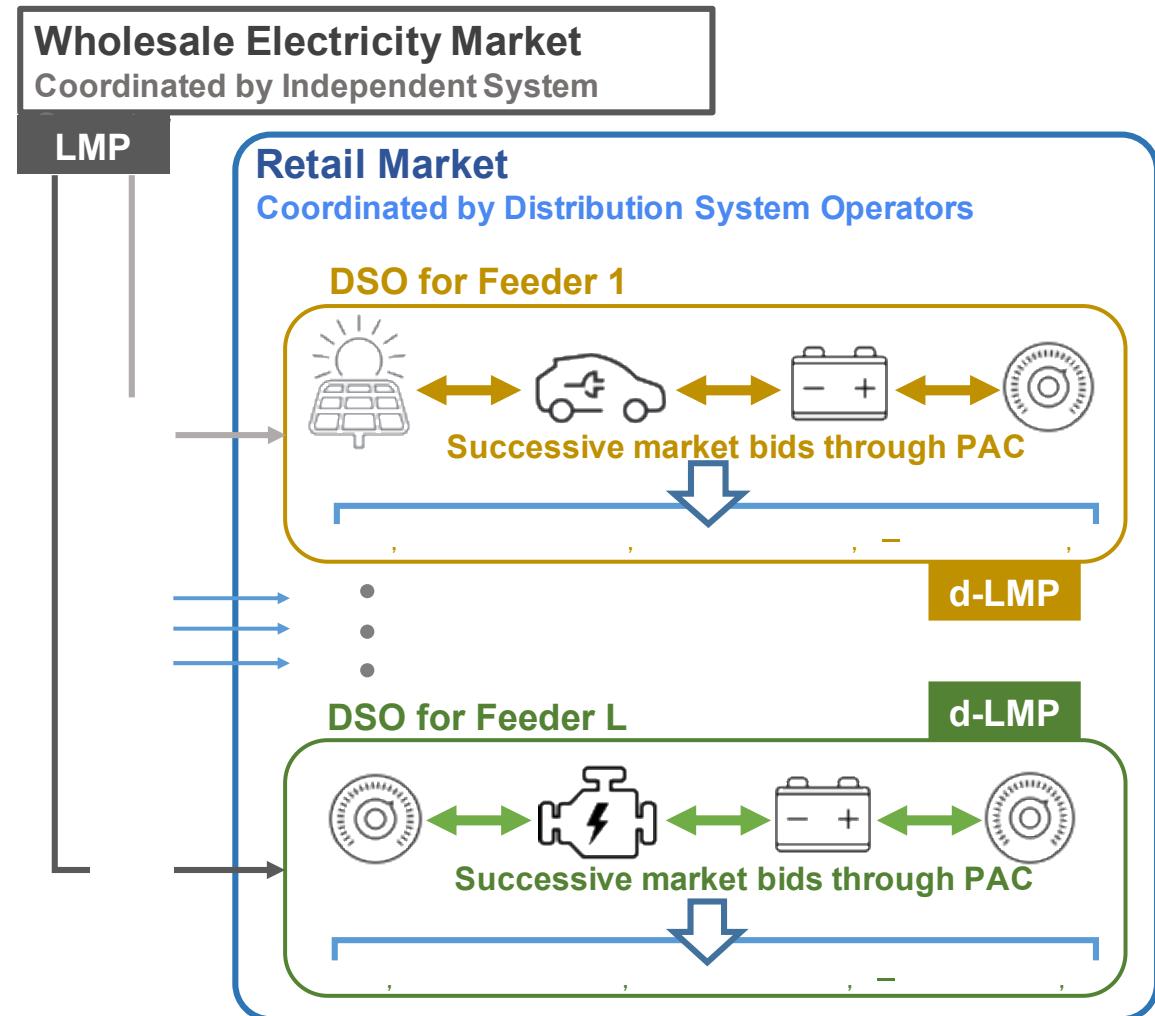
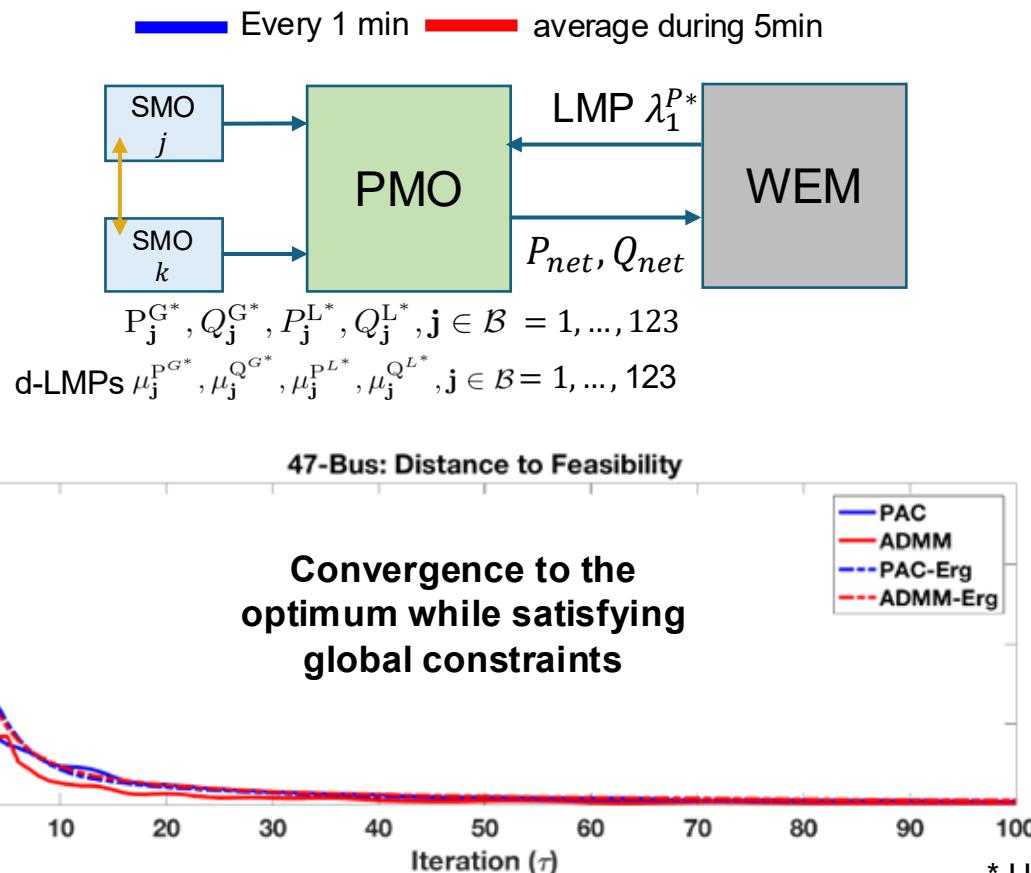
$$\nu_j[\tau+1] = \nu_j[\tau] + \rho \gamma_j [B]_j a[\tau+1] \text{ and } \hat{\nu}_j[\tau+1] = \nu_j[\tau+1] + \rho \hat{\gamma}_j[\tau+1] [B]_j a[\tau+1]$$

Communicate $\{\hat{\nu}_j[\tau+1]\}$ with neighbours, for all j



Primary market clearing using SMO bids & PAC

- Fully distributed
- Computationally tractable
- Reduced communication requirements
- Preserve data privacy (for dual variables)



Enhanced version: NST-PAC

$$\begin{aligned} a_j[\tau + 1] = \underset{a_j}{\operatorname{argmin}} & \left\{ \mathcal{L}_j(a_j, \hat{\eta}_j[\tau], \hat{\nu}[\tau]) \right. \\ & + \frac{\rho_j \gamma_j}{2} \|G_j a_j - b_j\|_2^2 + \frac{\rho_j \gamma_j}{2} \|B_j a_j\|_2^2 \\ & \left. + \frac{1}{2\rho_j} \|a_j - a_j[\tau]\|_2^2 \right\} \end{aligned}$$

$$\hat{a}_j[\tau + 1] = a_j[\tau + 1] + \alpha_j[\tau + 1] (a_j[\tau + 1] - a_j[\tau])$$

$$\eta_j[\tau + 1] = \hat{\eta}_j[\tau] + \rho_j \gamma_j (G_j \hat{a}_j[\tau + 1] - b_j)$$

$$\hat{\eta}_j[\tau + 1] = \eta_j[\tau + 1] + \phi_j[\tau + 1] (\eta_j[\tau + 1] - \eta_j[\tau])$$

Communicate \hat{a}_j for all $j \in [K]$ with neighbors

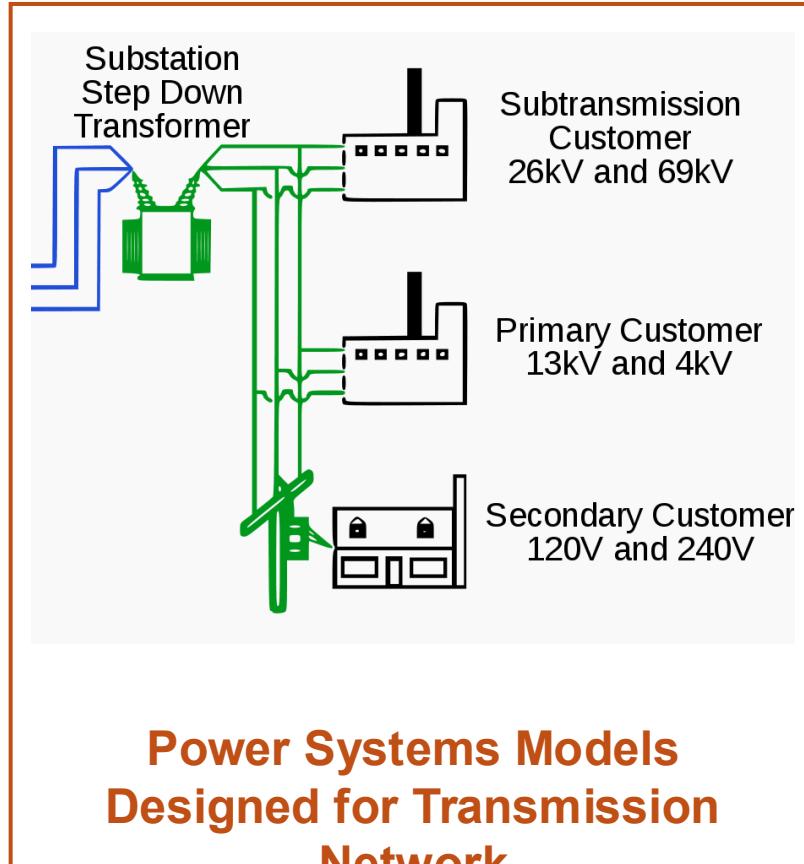
$$\nu_j[\tau + 1] = \hat{\nu}_j[\tau] + \rho_j \gamma_j B_j \hat{a}_j[\tau + 1]$$

$$\hat{\nu}_j[\tau + 1] = \nu_j[\tau + 1] + \theta_j[\tau + 1] (\nu_j[\tau + 1] - \nu_j[\tau])$$

Communicate $\hat{\nu}_j$ for all $j \in [K]$ with neighbors

- Use nonlinear regularization terms instead of linearized (PAC)
- Both primal & dual variable updates use Nesterov acceleration
- Privacy for both primals/duals
 - Use time-varying & atom-specific step-sizes

Distribution system models



Distribution Grid:

Mixed topologies: Meshed and radial

Unbalanced networks

- Lines are not transposed – lines are unbalanced
- Have many single, two, & three-phase lines
- Unbalanced loads

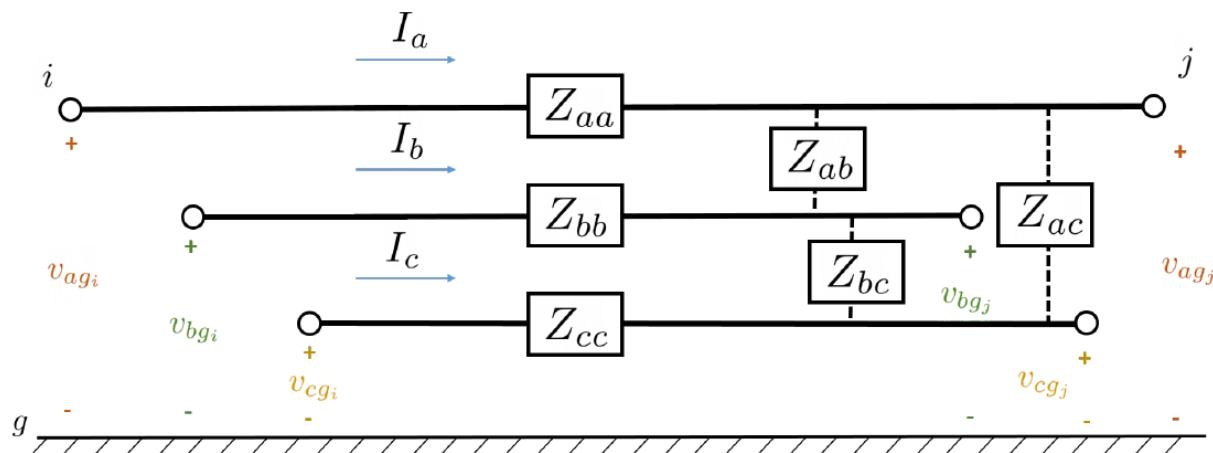
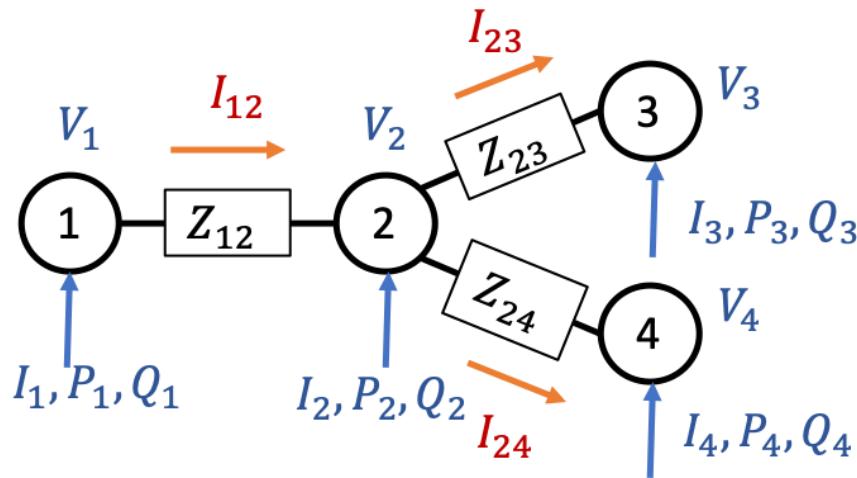
Emerging features:

- High DER penetration along grid's edge
- Energy injection into the grid (DG and storage)

Want power systems model with:

- **Applicability:** To unbalanced and meshed networks
- **Simplicity:** Linear constraints better than quadratic constraints
- **Computational tractability:** Applicable to large networks with limited pre/post-processing time

Review: Notation



Notation

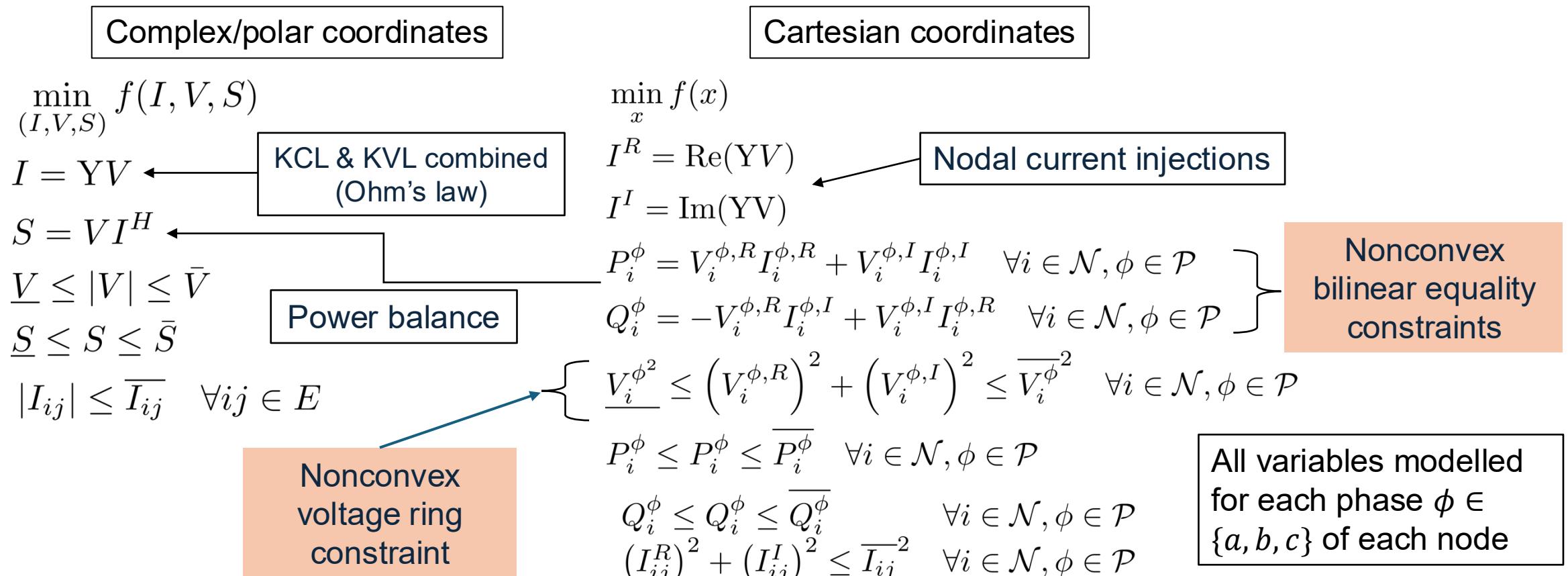
- Nodal variables: Current injections, voltages, power injection (P & Q)
- Line variables: Current flow
- Each line has series impedance, $Z_{ij} = R_{ij} + jX_{ij}$

3-phase impedance matrix

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Admittance-based Current Injection model

For unbalanced, multiphase, radial or meshed distribution networks



McCormick envelopes convex relaxation

$$P_j = \operatorname{Re}(V_j I_j^*) = \operatorname{Re}(V_j) \operatorname{Re}(I_j) + \operatorname{Im}(V_j) \operatorname{Im}(I_j)$$

$$Q_j = \operatorname{Im}(V_j I_j^*) = -\operatorname{Re}(V_j) \operatorname{Im}(I_j) + \operatorname{Im}(V_j) \operatorname{Re}(I_j)$$

Consider bilinear form: $w = xy$

Defined over set: $S \subset \mathbb{R}^3 = \{x, y : x \in [\underline{x}, \bar{x}], y \in [\underline{y}, \bar{y}]\}$

Then we introduce a new variable, w , and we define MCE envelope as:

$$(5a) w \geq \underline{x}\underline{y} + \bar{x}\bar{y} - \underline{x}\bar{y}$$

$$(5b) w \geq \bar{x}\underline{y} + \underline{x}\bar{y} - \bar{x}\bar{y}$$

$$(5c) w \geq \underline{x}\underline{y} + \bar{x}\bar{y} - \underline{x}\bar{y}$$

$$(5d) w \geq \bar{x}\underline{y} + \underline{x}\bar{y} - \bar{x}\bar{y}$$

Thus, we can relax the power balance constraints as:

$$P = a + b$$

$$Q = -c + d$$

$$\underline{V^R} \leq V_r \leq \bar{V^R}$$

and

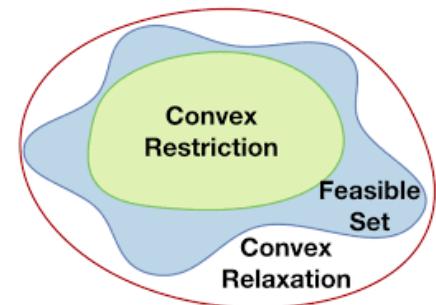
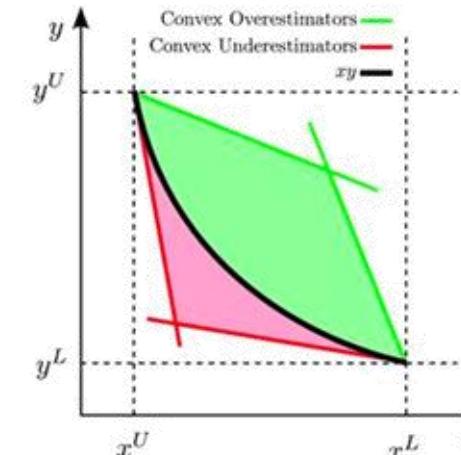
$$\underline{V^I} \leq V^I \leq \bar{V^I}$$

$$\underline{I^R} \leq I_r \leq \bar{I^R}$$

and

$$\underline{I^I} \leq I^I \leq \bar{I^I}$$

Create a convex constraint,
using McCormick Envelopes



Iterative
preprocessing to set
tight V & I bounds



More accurate
convex relaxation

And corresponding MCE constraints on $\{a, b, c, d\}$

McCormick, G.P. "Computability of global solutions to factorable nonconvex programs: Part i - convex underestimating problems." Mathematical Programming. 1976

*Ferro, G. et al. (2022). A Current Injection Based Method for Unbalanced and Meshed Distribution Networks. Under preparation.

Iterative preprocessing for tight V/I bounds

- Need tight bounds on nodal current injections and voltage bounds
- Tighter convex relaxation → More accurate results
- Use iterative, sequential bound tightening approach
- Solve series of simpler optimization problems to find lower/upper bounds

e.g., to find $\overline{I^R}$

$$\max_{\{P, Q, v, \delta\}} h_1(P, Q, v, \delta)$$

$\underline{\delta} \leq \delta \leq \bar{\delta}$ Voltage phase angle

$\underline{v} \leq v \leq \bar{v}$ Voltage magnitude

$\underline{P} \leq P \leq \bar{P}$

$\underline{Q} \leq Q \leq \bar{Q}$

$$h_1 = I^R = \operatorname{Re} \left[\left(\frac{P + jQ}{ve^{j\delta}} \right)^H \right] = \frac{P \cos(\delta)}{v} + \frac{Q \sin(\delta)}{v}$$

e.g., to find $\overline{V^I}$

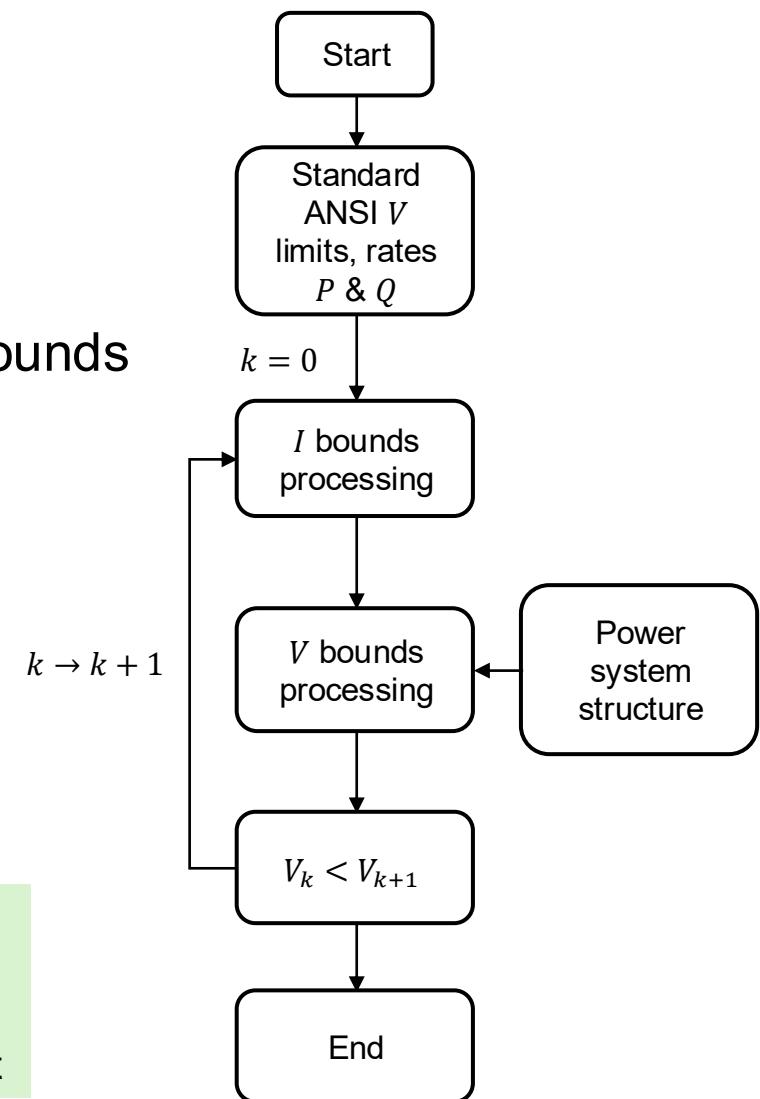
$$\max_{\{V_i^I, I_i^R, I_i^I\}} V_i^I$$

$$V_i^I = \mathbf{R}_{il} I_{l'}^I - \mathbf{X}_{il} I_{l'}^R$$

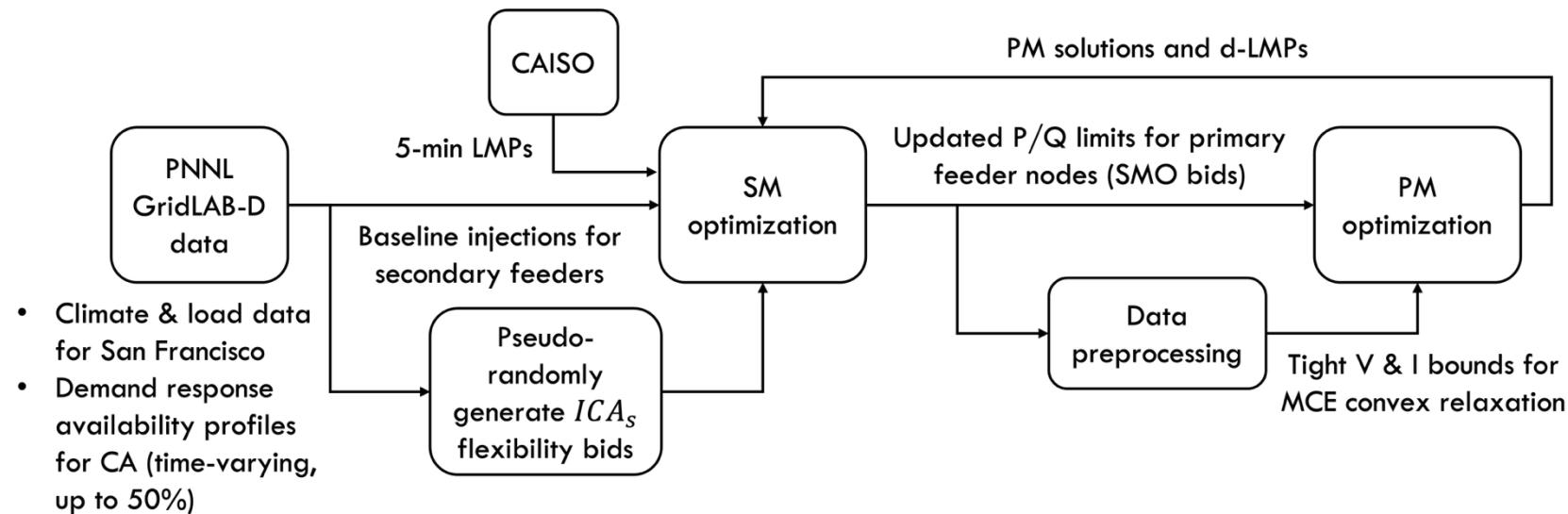
$$\underline{I_l^R} \leq I_l^R \leq \overline{I_l^R}$$

$$\underline{I_l^I} \leq I_l^I \leq \overline{I_l^I}$$

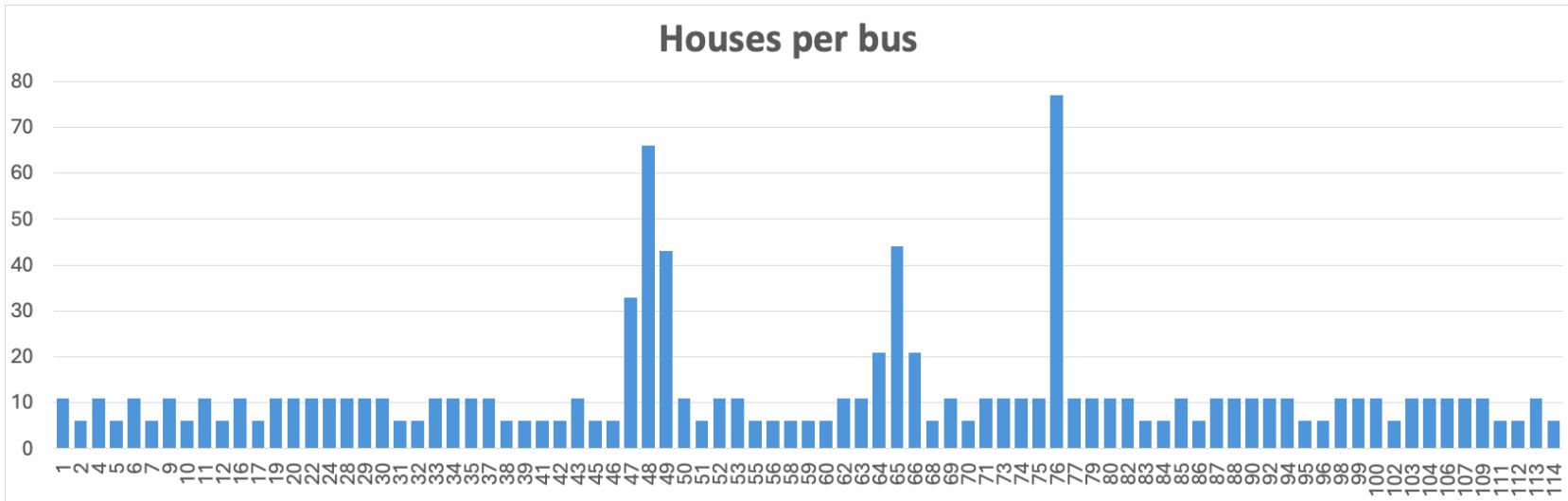
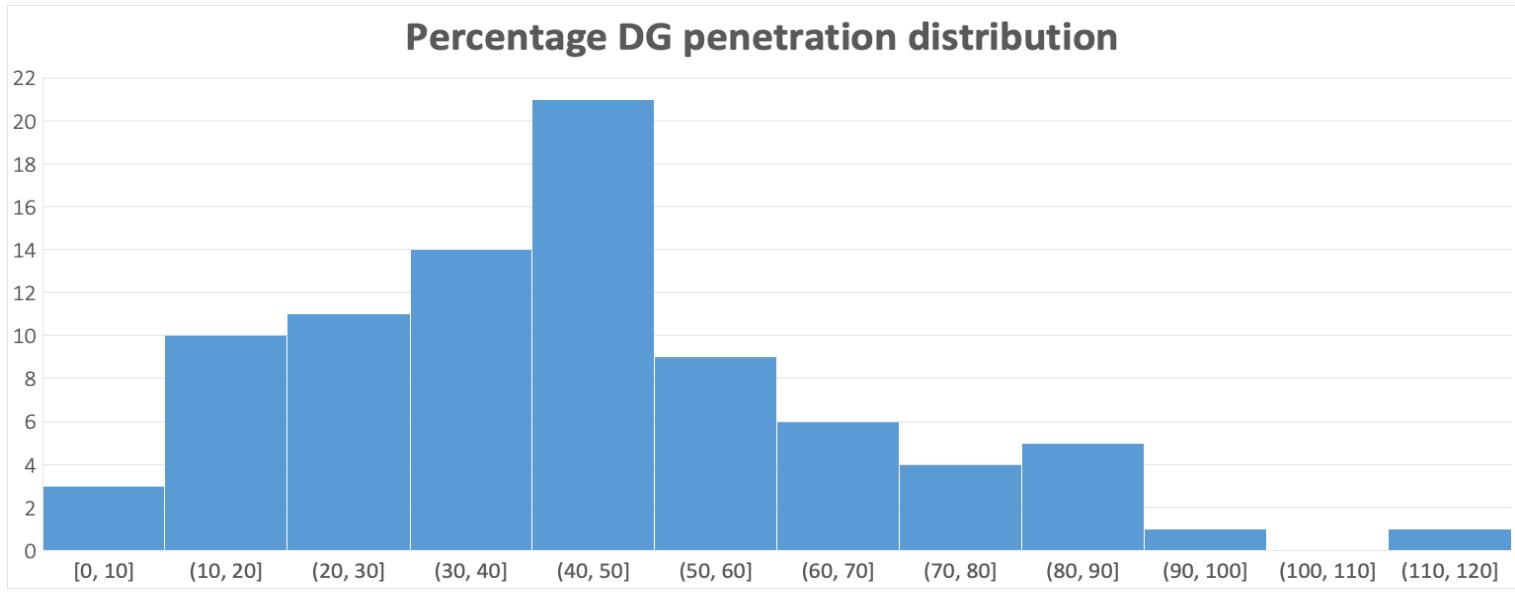
- Can derive closed form analytical solutions
- Solutions also satisfy nonconvex voltage ring constraint



Co-simulation of primary & secondary markets

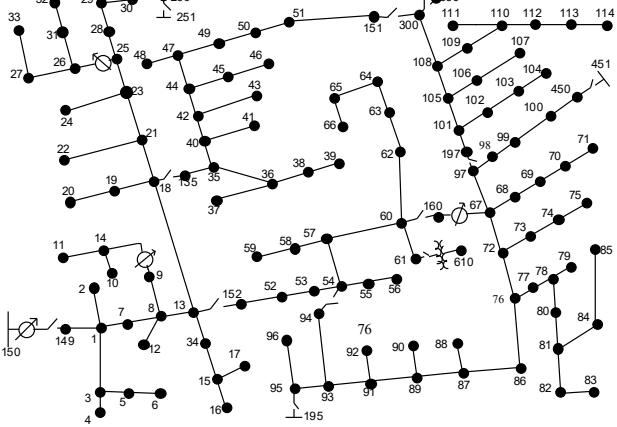


		Number	Capacity	Primary feeder nodes
Standard IEEE 123-node system	Spot loads	85	3,985.7 kVA	85
Our modified DER-rich 123-node system	Houses - Demand response (HVACs in all, WHs in 348)	1008	variable (20-30 % critical)	85
	Distributed generators (DGs)	380	1,745.8 kVA (~44%)	82
	PVs	207	880.84 kVA	68
	Batteries	173	865 kVA	63

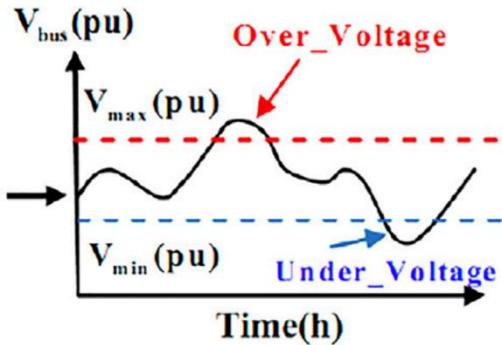
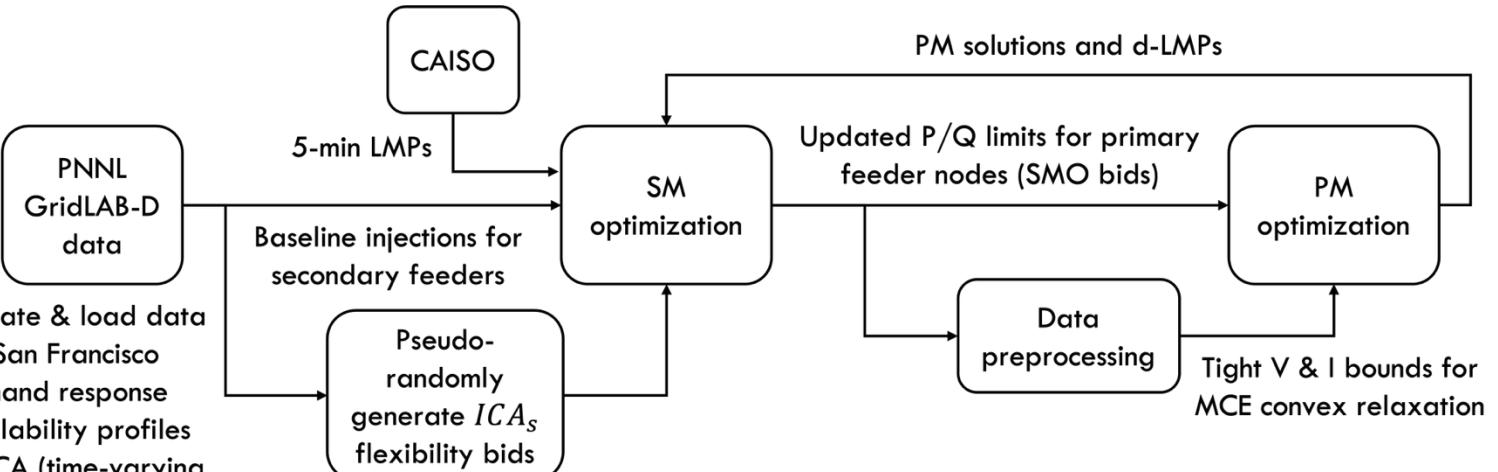


Co-simulation of primary + secondary markets

Data from modified DER-rich IEEE-123 GridLAB-D model



- Climate & load data for San Francisco
- Demand response availability profiles for CA (time-varying, up to 50%)

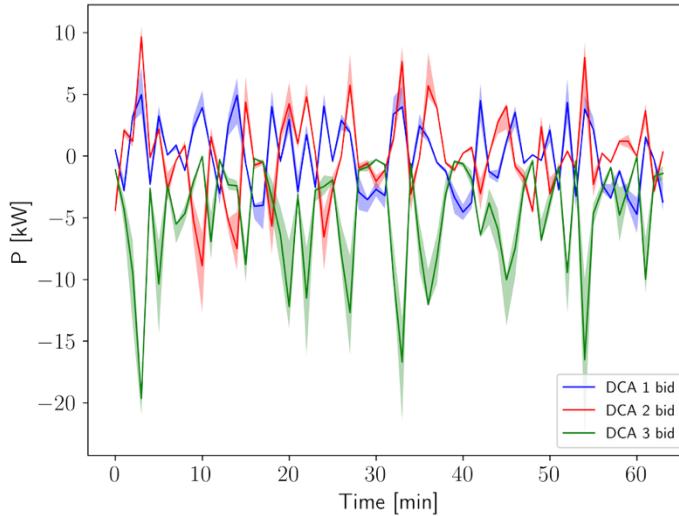


- Accelerated by parallelizing 85 independent SM clearings across all SMO nodes on HPC cluster (MIT Supercloud)
- Mitigate voltage issues common in low-medium voltage distribution grids, for e.g.
 - High PV penetration → Over-voltage
 - Demand spikes from HVAC → Under-voltage

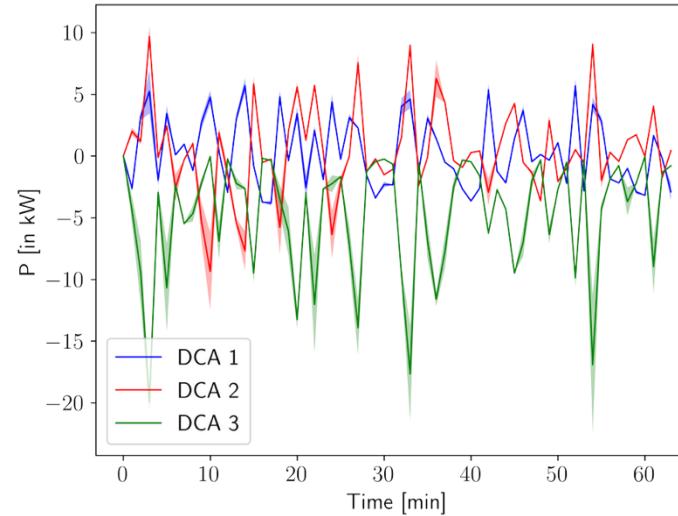
Type	Number	Capacity
DERs	380	1,745.8 kVA (~44%)
PVs	207	880.84 kVA
Batteries	173	865 kVA
Spot loads	85	3,985.7 kVA
Houses	1008	4-10 kW (variable)
Flexible loads	1-2 per house	10-50% flexibility (variable)

Example secondary market results

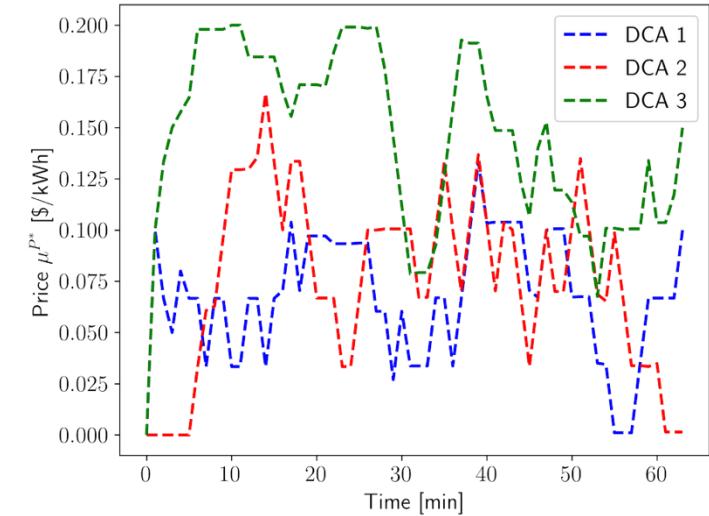
SMA bids into SM at node 7



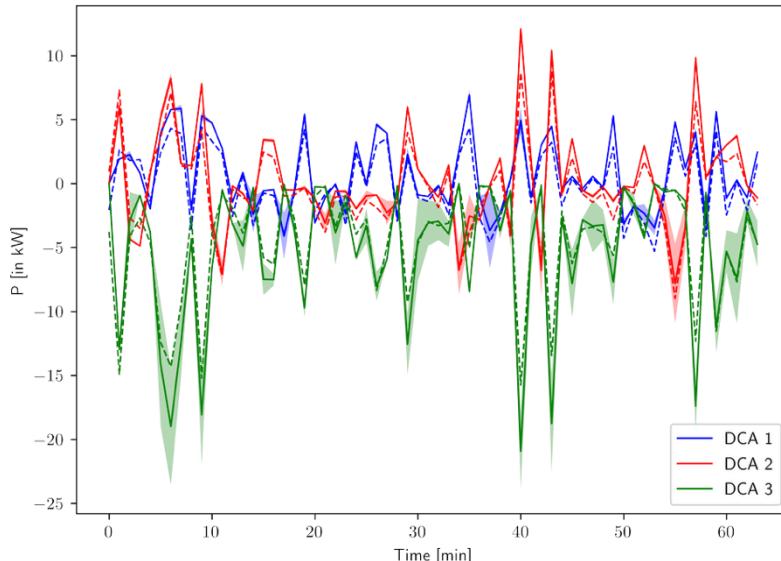
SMA schedules from SMO 7



Local retail tariffs



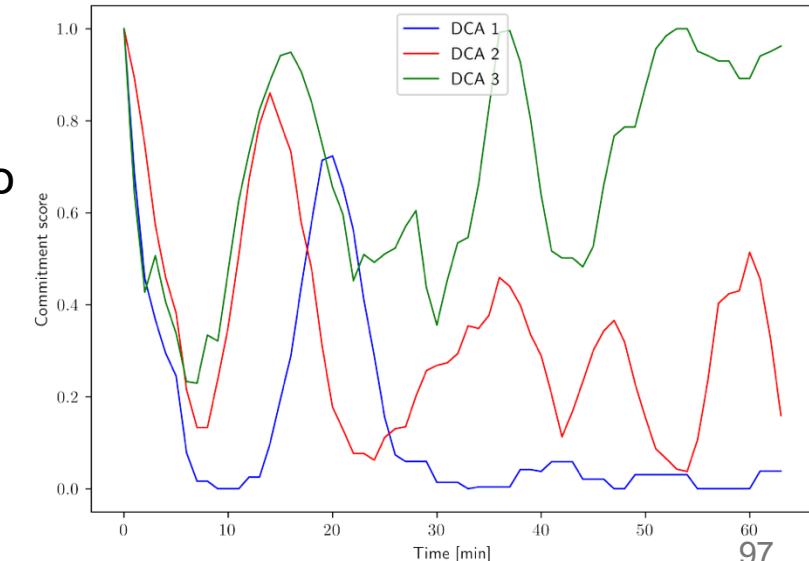
Net active power injection schedules & responses for SMO 17



Use SMAs' actual responses to update commitment scores

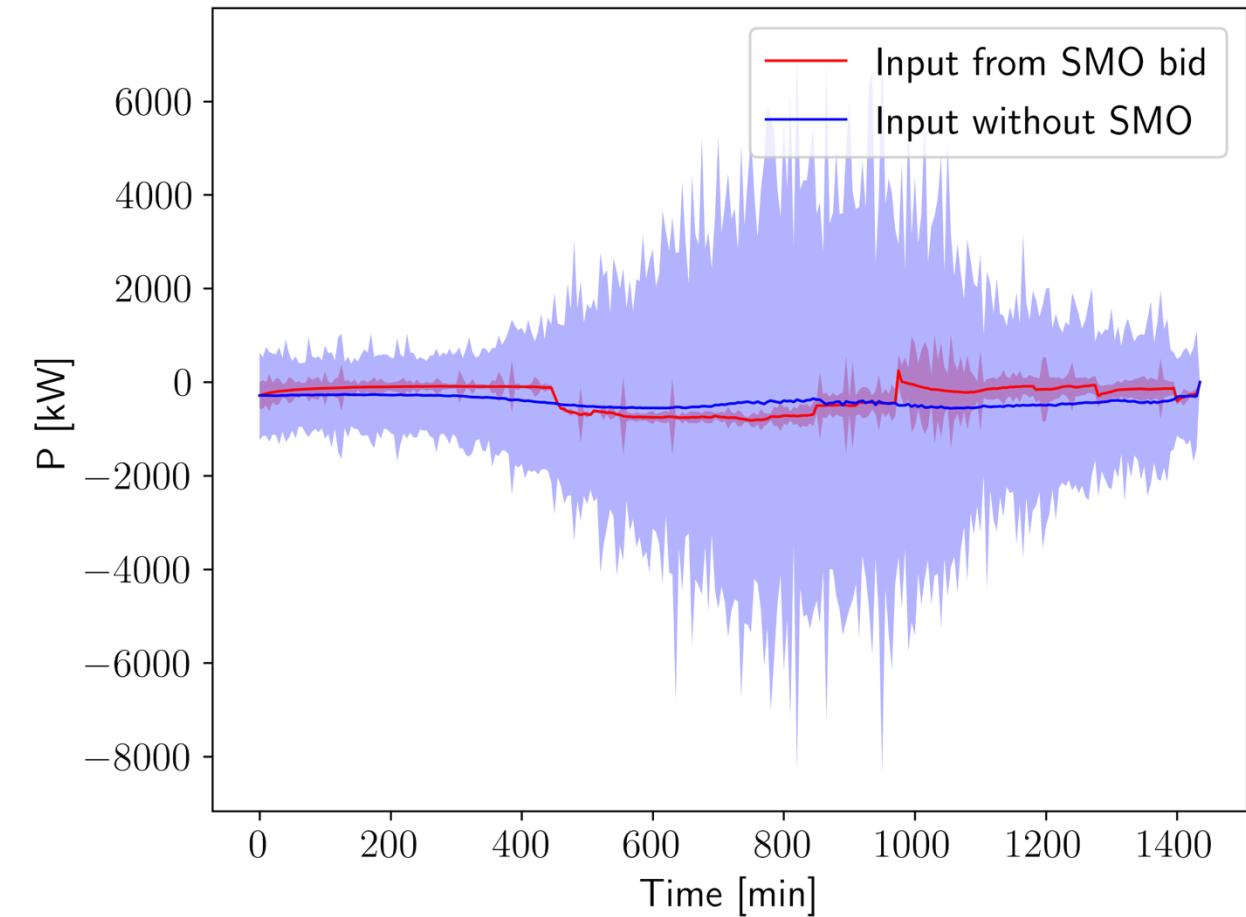


Commitment score of each DCA over time

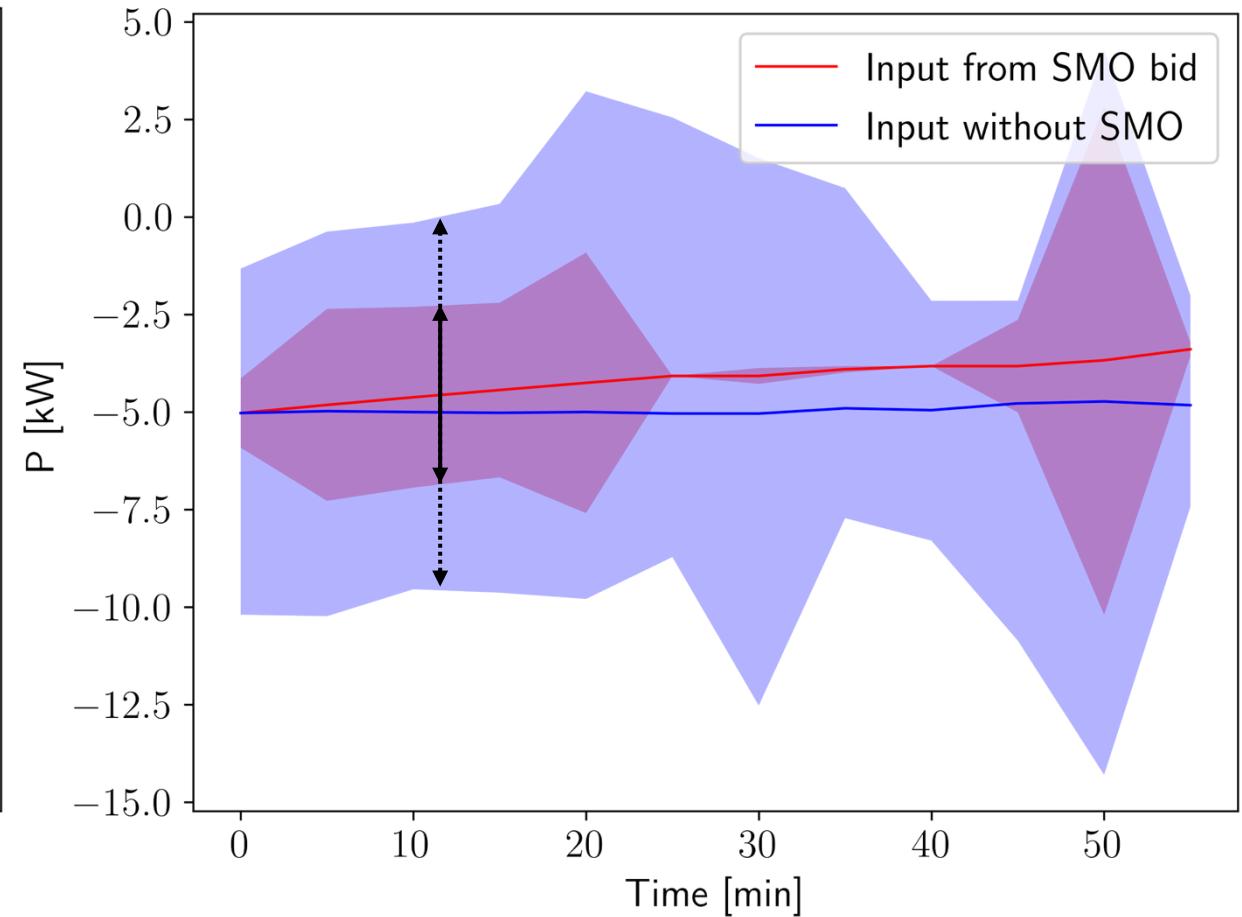


Role of hierarchy in LEM

Aggregated inputs to primary market over all primary feeder nodes

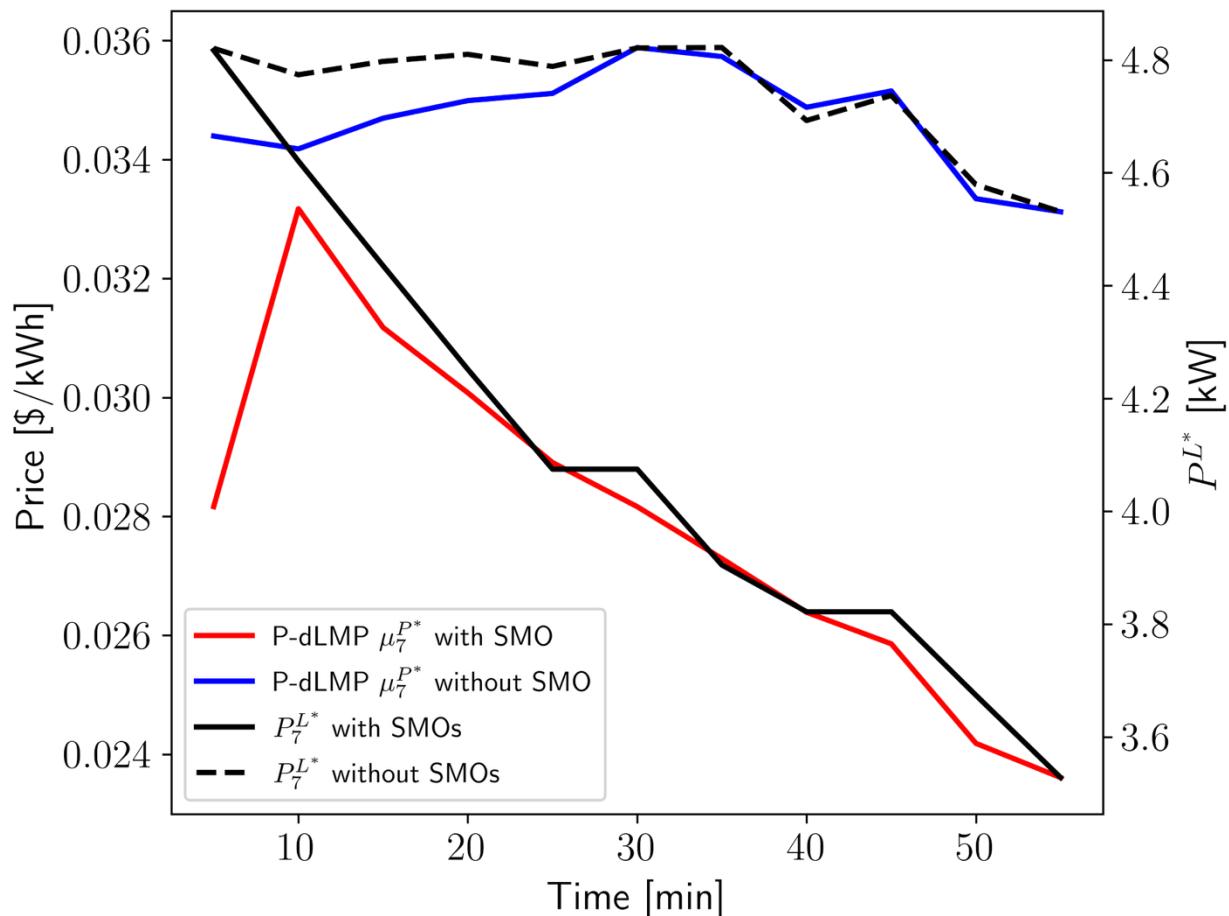


Inputs to primary market for SMO at node 7



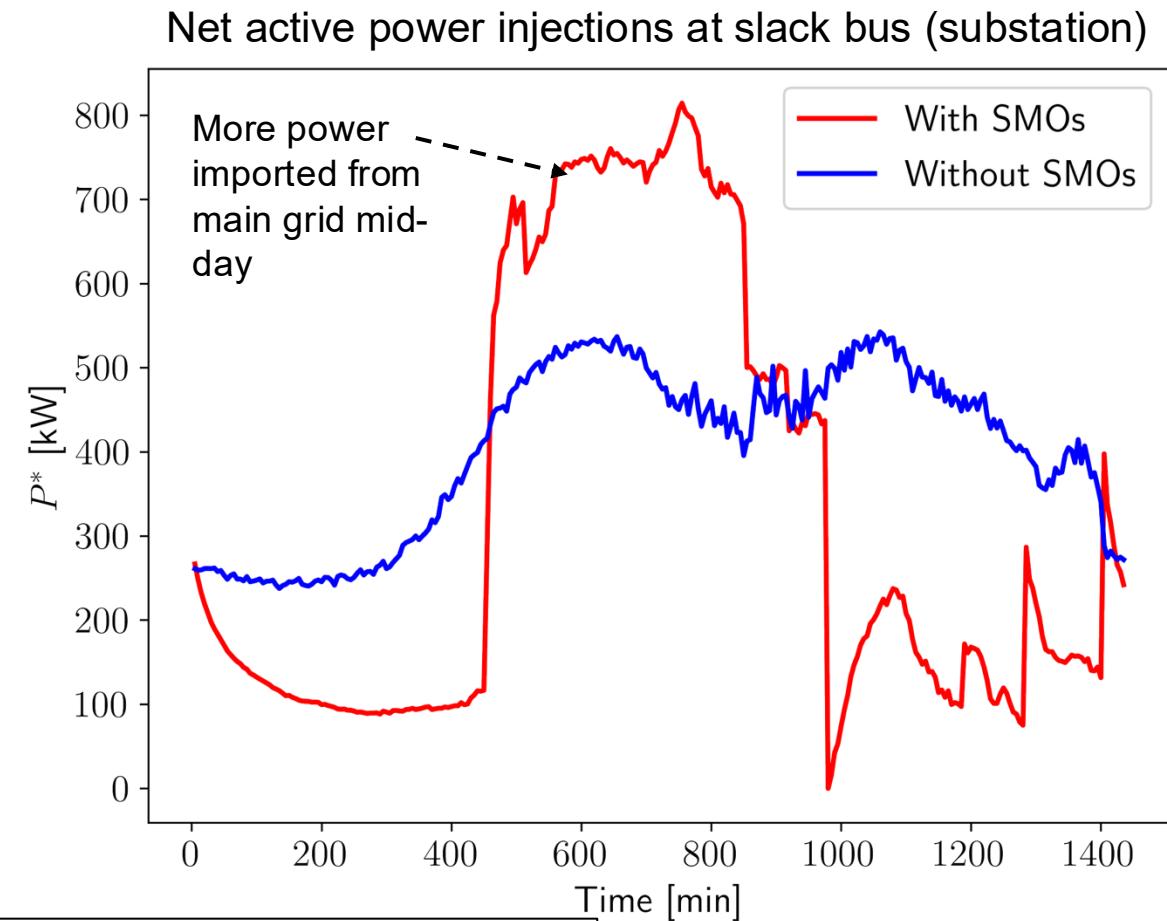
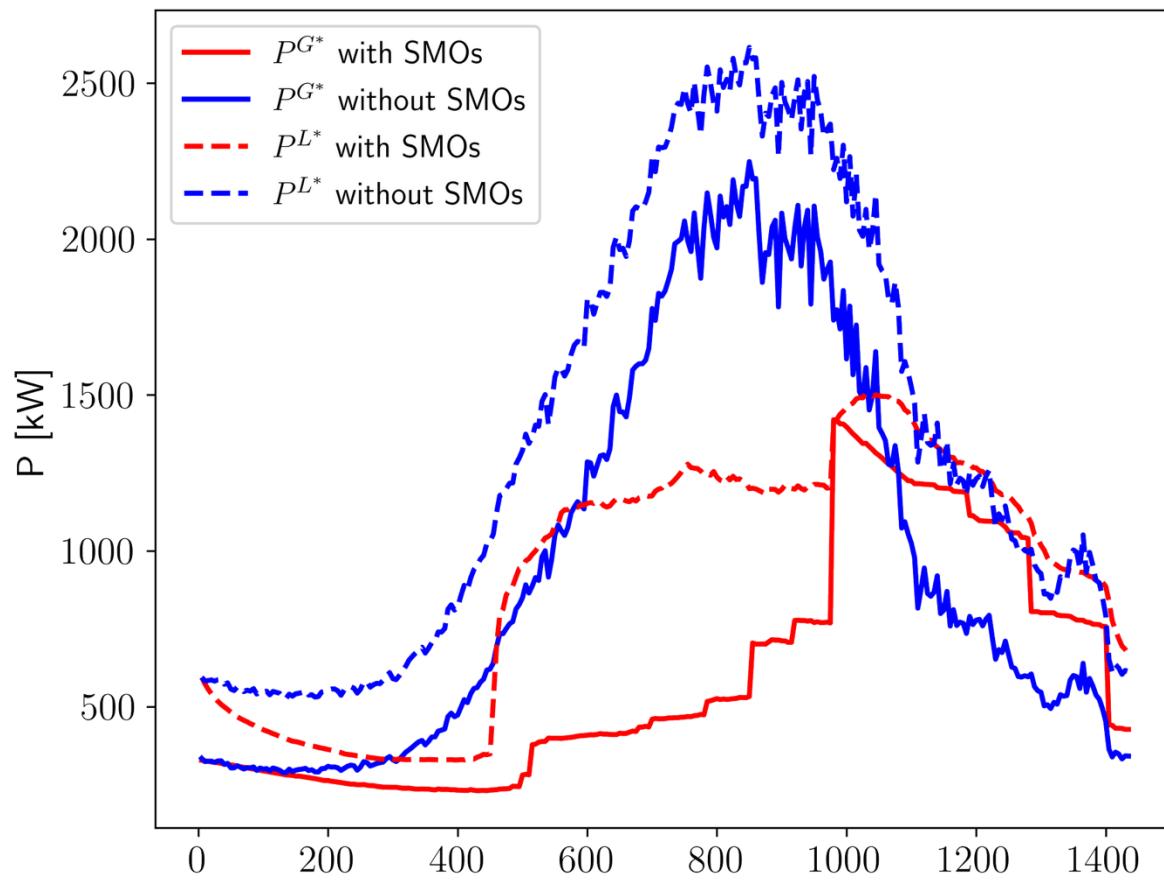
Primary market clearing

Primary market solutions for node 7



SM able to leverage available load flexibility more resulting in lower dLMPs

Effects of secondary market on primary market

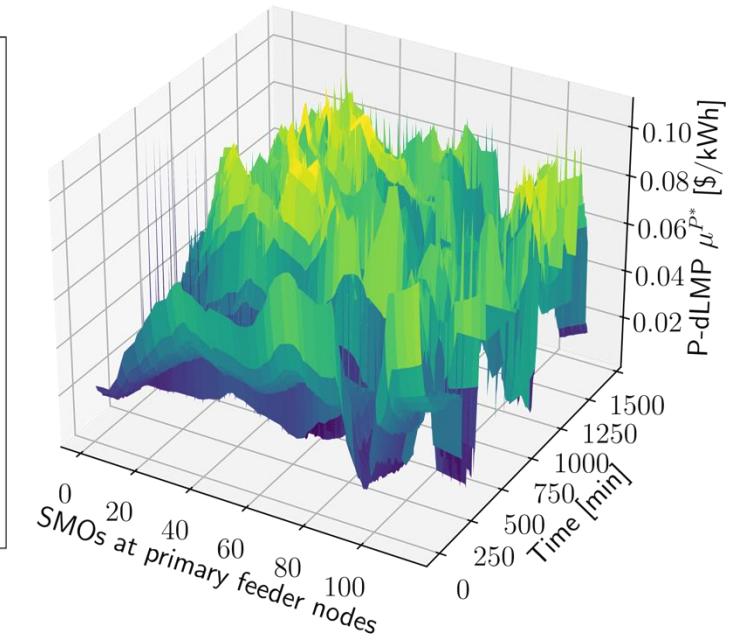
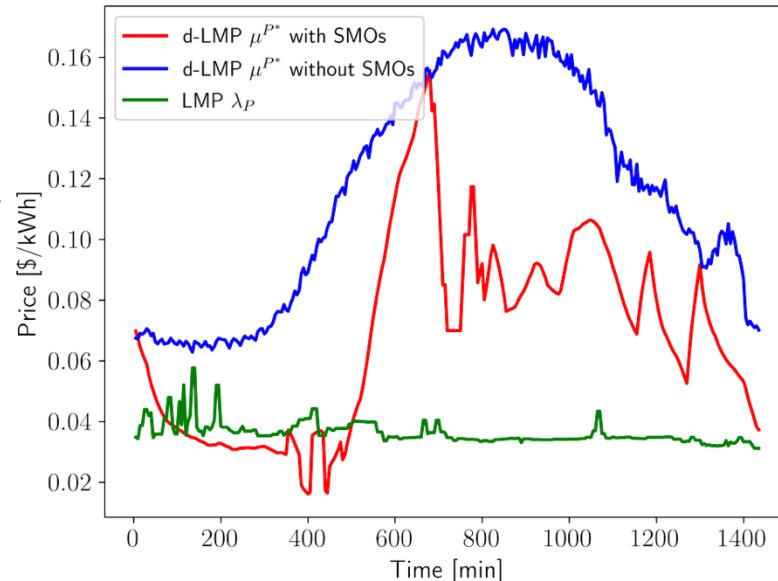


SMO coordinates & aggregates local DERs more effectively to:

- Achieve an optimal combination of local generation & power purchased from bulk transmission network
- Lower congestion costs & line losses
- Improve efficiency of distribution network

Retail prices across PM and SM

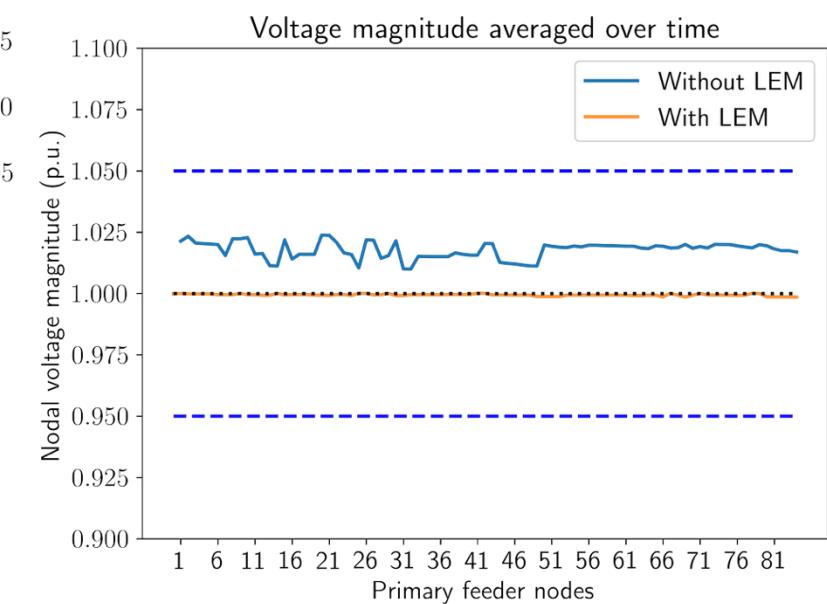
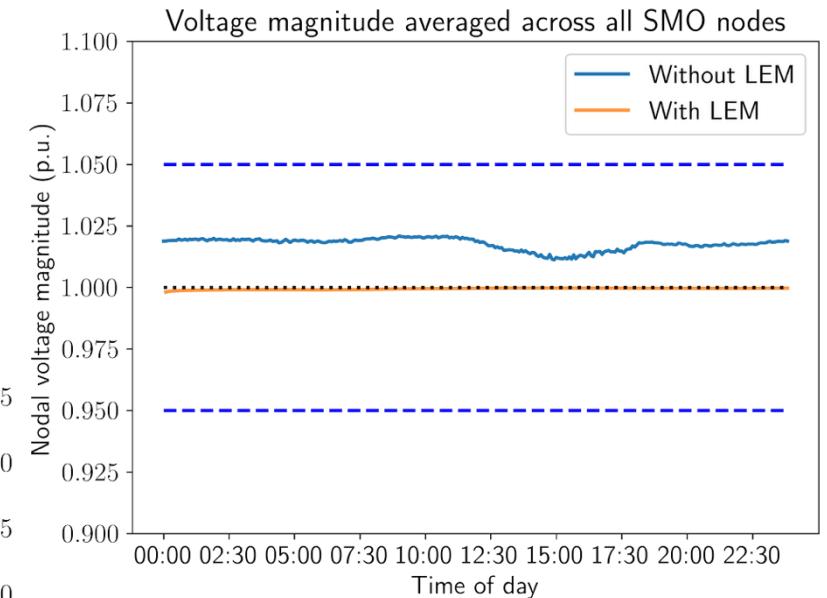
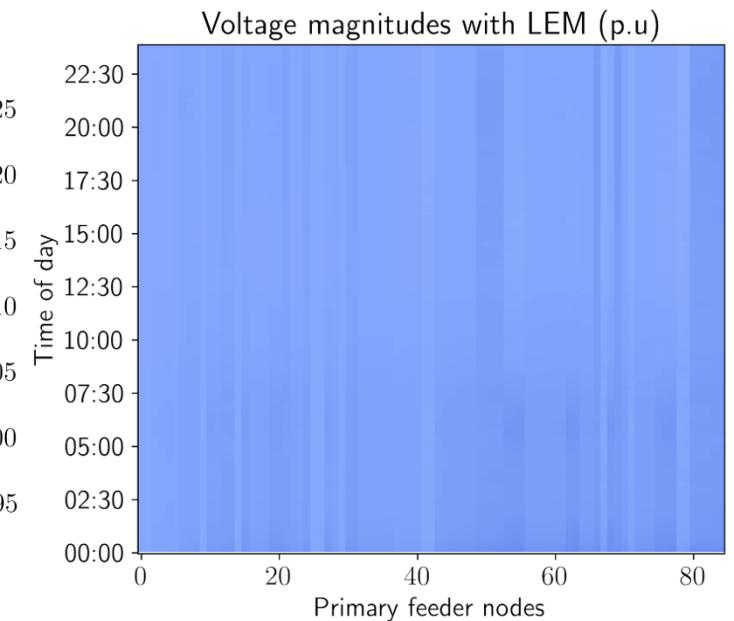
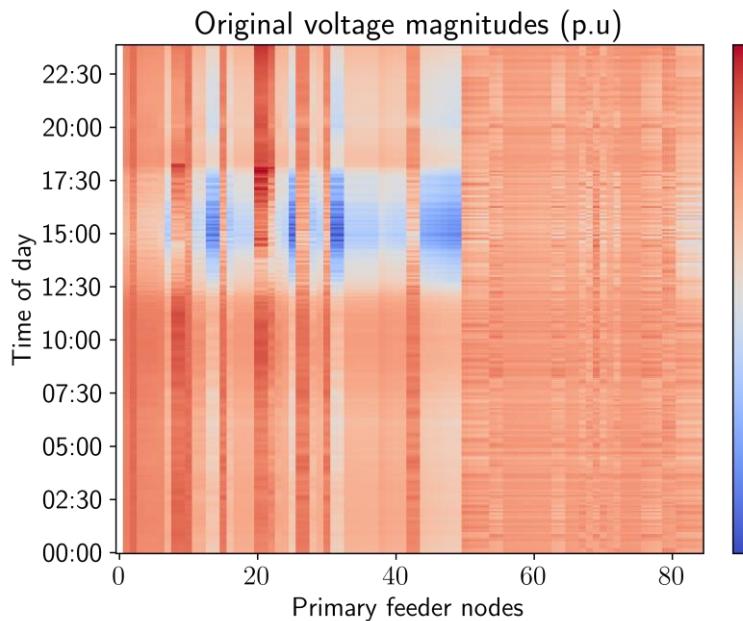
- PM and SM provide granular spatially and temporally varying prices
- Average dLMP across network better reflects real-time operational flexibility with SMO
→ Lower overall costs
- Lower costs from hierarchical LEM
 - ~45% ↓ dLMP relative to PM only
 - ~30% ↓ local tariff relative to PM only
 - ~50% ↓ local tariff relative to no LEM



[\$/kWh]	Hierarchical LEM	Primary LEM only	No LEM
Avg. dLMP	0.064	0.116	N/A
Avg. local tariff	0.082	0.116	0.129 [1]

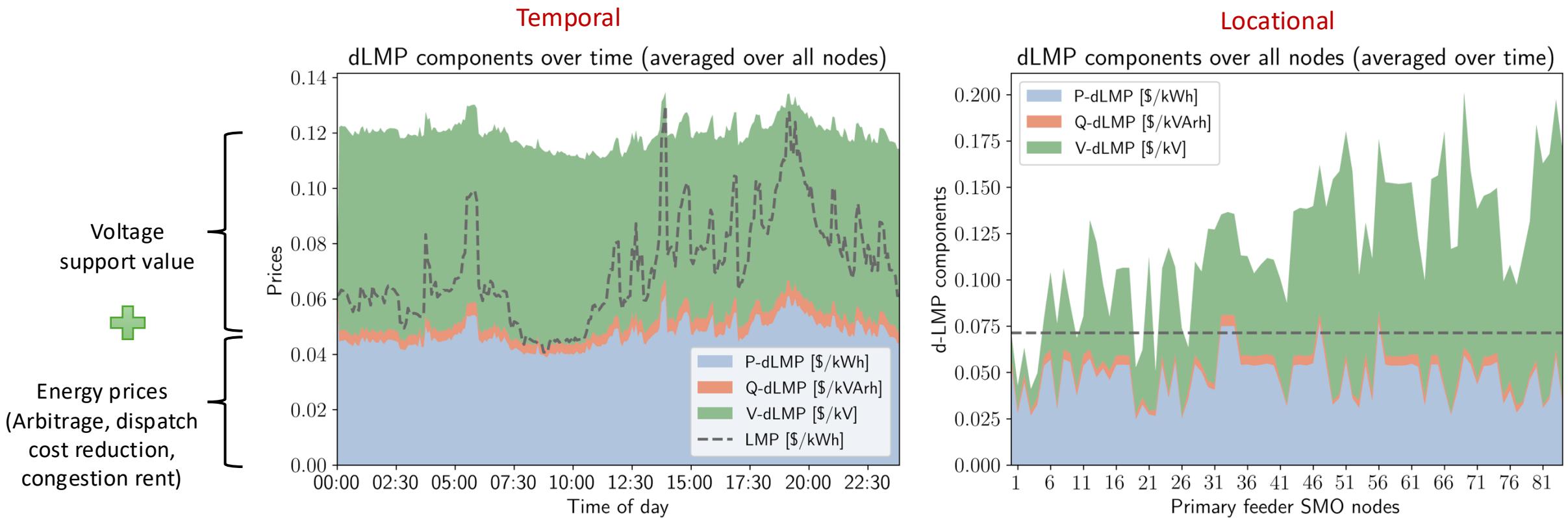
[1] Eversource MA

Markets for voltage regulation

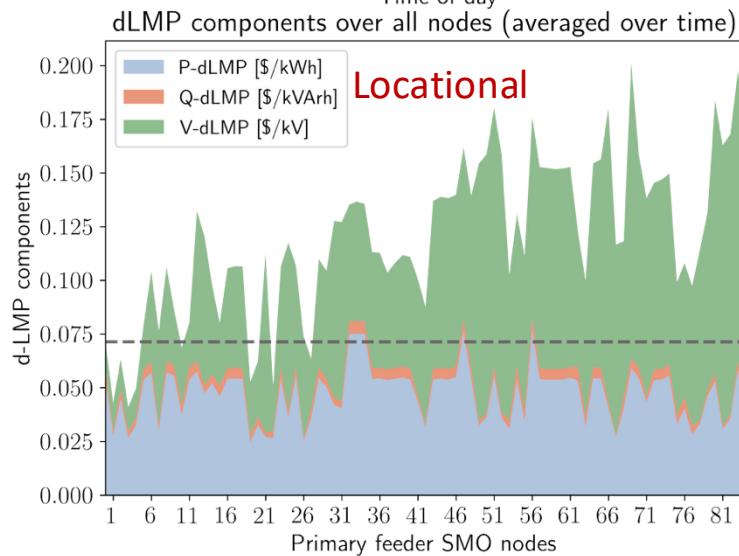
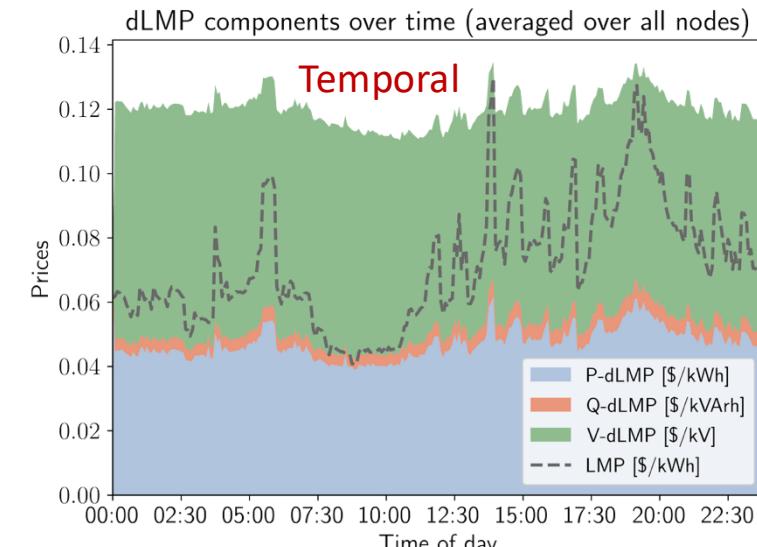


LEM (SM + PM) improves overall voltage profile → More uniform + closer to 1 p.u.

Accurate grid service pricing



Towards accurate grid service pricing & attribution



Voltage support value

+ Energy prices (Arbitrage, dispatch cost reduction, congestion rent)

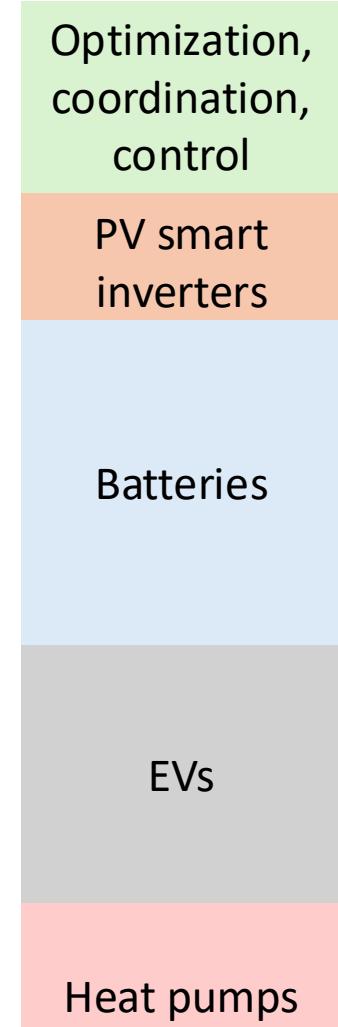
+ Frequency regulation value

+ Hosting capacity enhancement (integrate more PV, EV, heat pumps)

Accurately estimate contribution of each resource

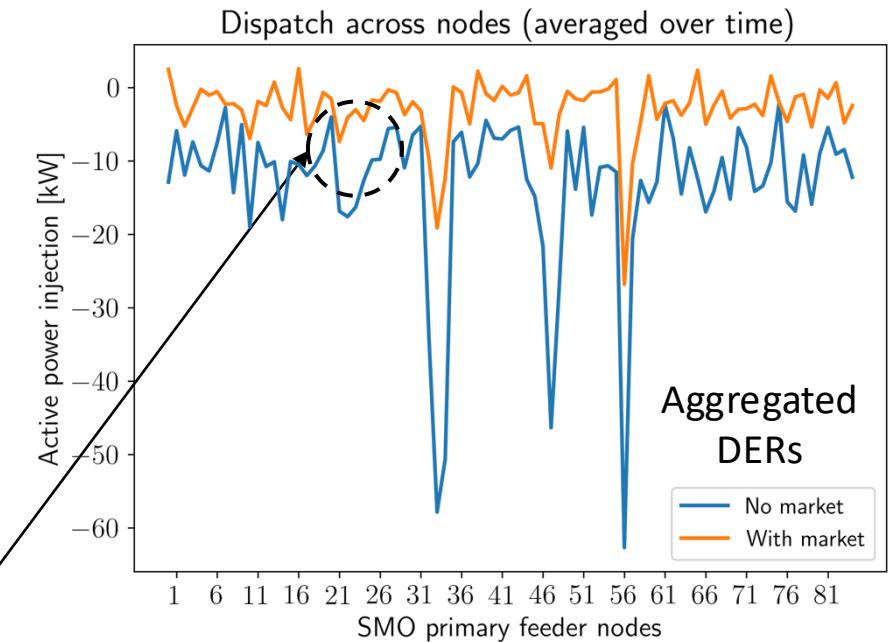
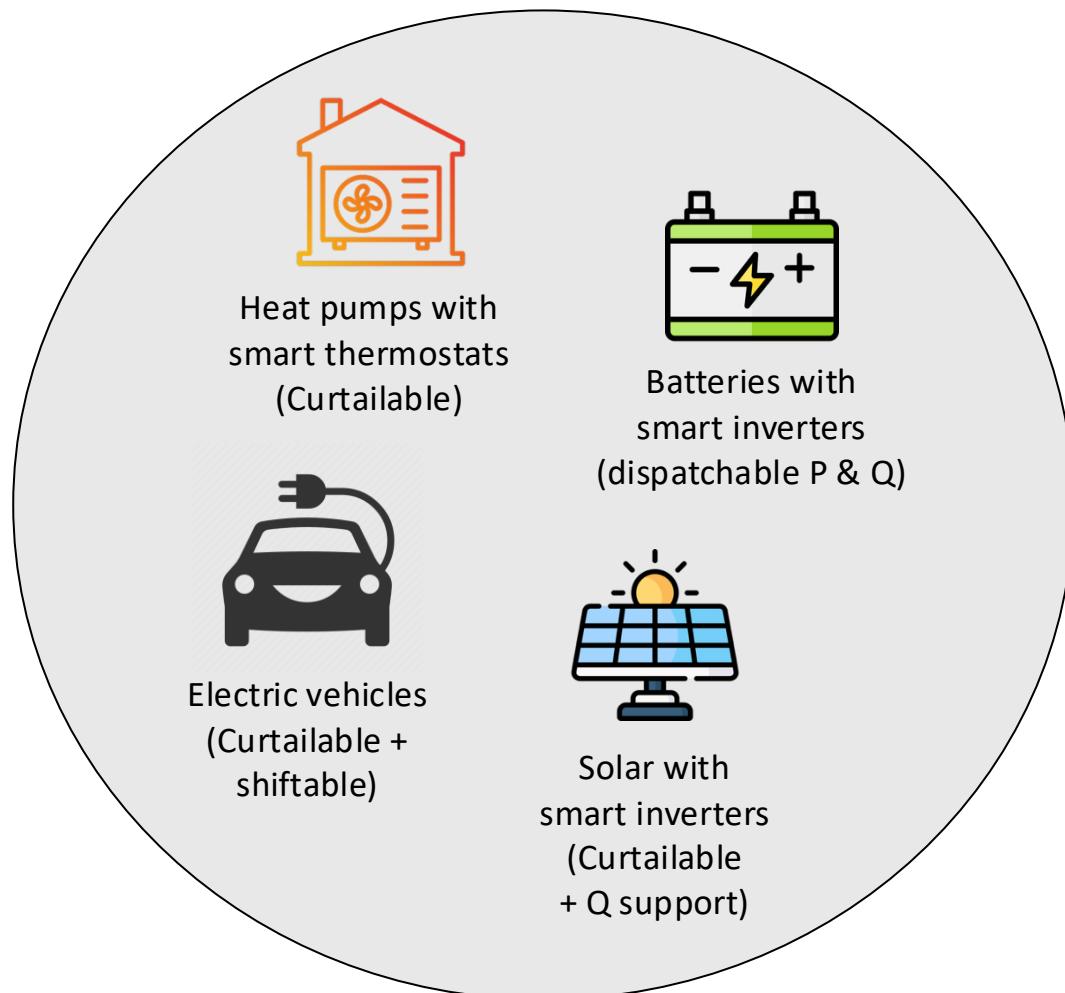
Overall marginal value of DERs for grid planning & operation

Sensitivity studies



Flexible DERs enable voltage regulation

- Market aggregates many DERs @ home/building level
- Reduces net load through flexibility



Max capacity ranges

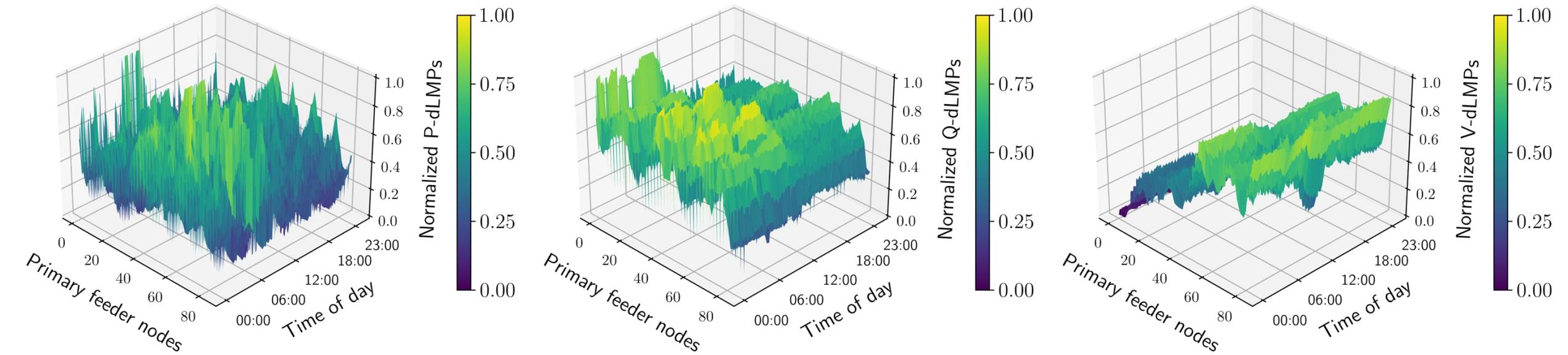
Batteries: 4-8 kW, 10-20 kWh

Solar PV: 4-10 kW

Heat pumps: 5-8 kW

EVs (level 1/2 chargers): 2-10 kW, 20-100 kWh

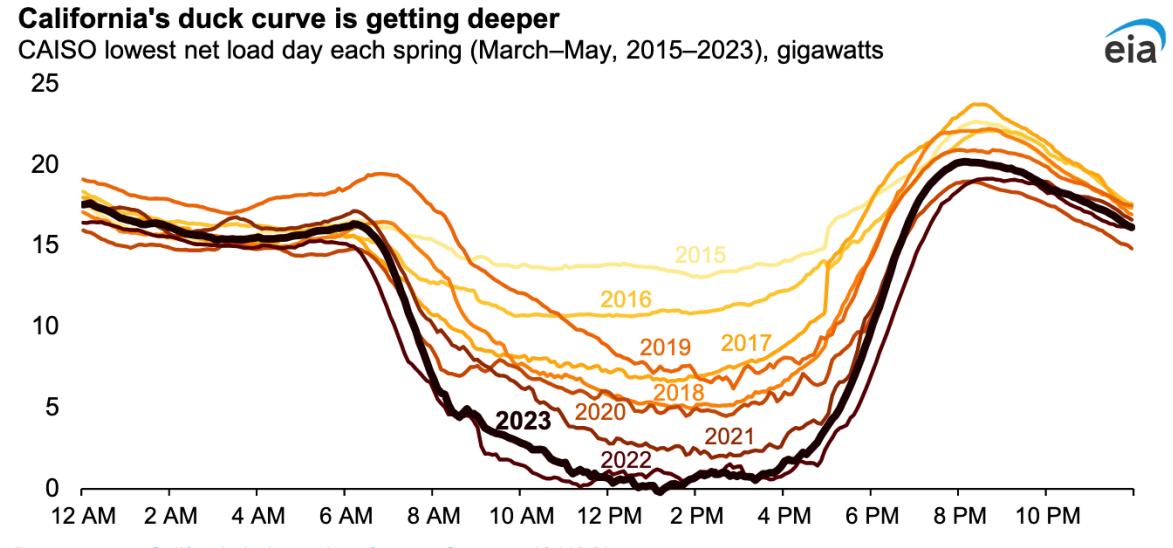
Spatial-temporal dLMP distributions



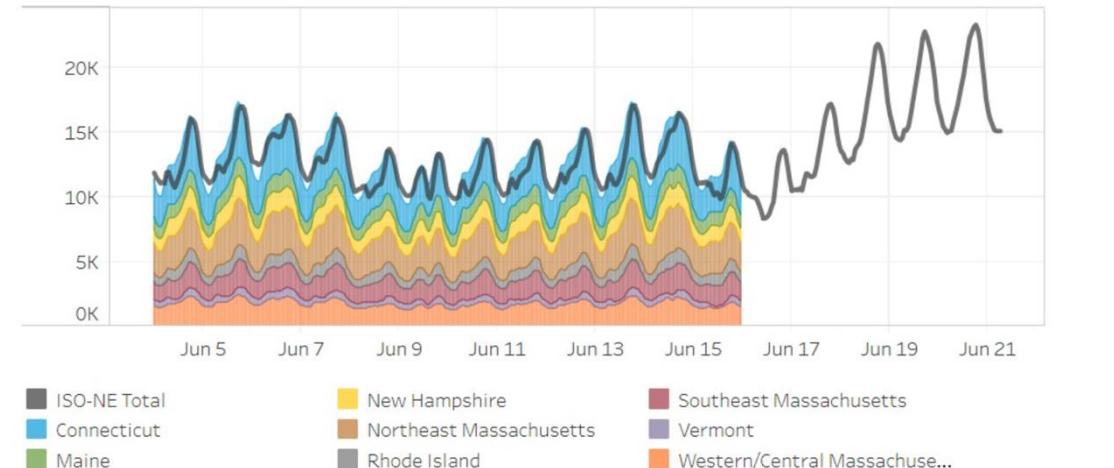
A game-theoretic market-based approach to extract flexibility from distributed energy resources

Challenges for the future decarbonized grid

- Intermittency & variability of renewables → Reliability & stability issues
- Rapid load growth (e.g. heat pumps, EVs, data centers) → Stress on grid
- Extreme weather events
- Voltage & frequency issues due to lower inertia

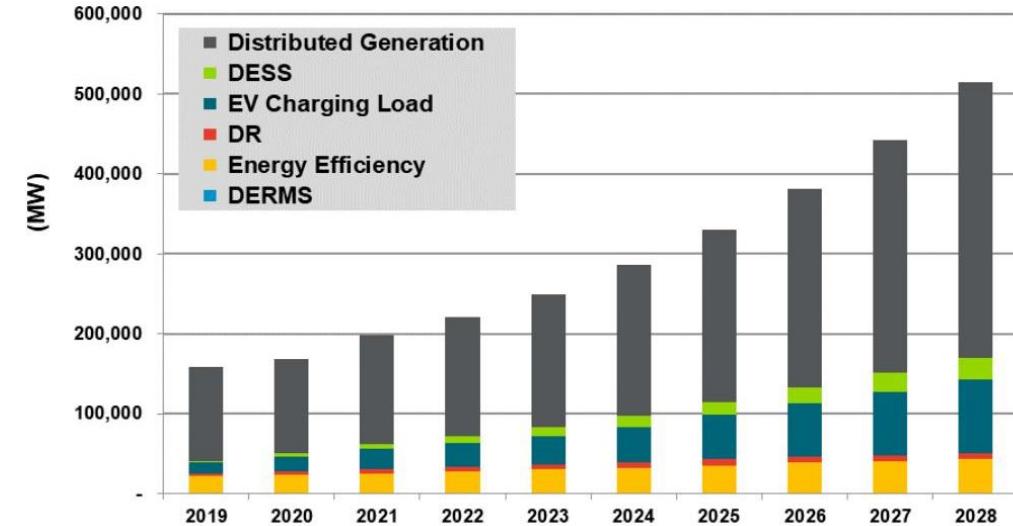


Data source: California Independent System Operator (CAISO)
ISO-NE hourly electricity demand
megawatthours (MWh)



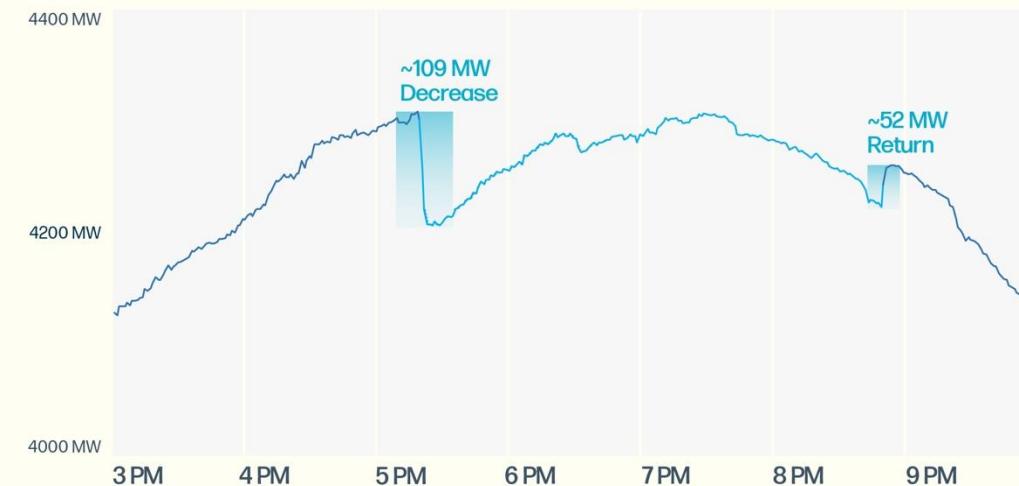
Potential solution: Flexibility

- Distributed energy resources (solar PV, batteries, flexible loads) are flexible
- DERs can provide flexibility as a grid service
- Flexibility can potentially:
 - Improve voltage & frequency profiles
 - Mitigate impacts of extreme weather
 - Improve dispatch efficiency to lower operating costs + prices



Customer Actions - July 8, 2024

PGE customers are making a big difference by shifting or reducing their energy use



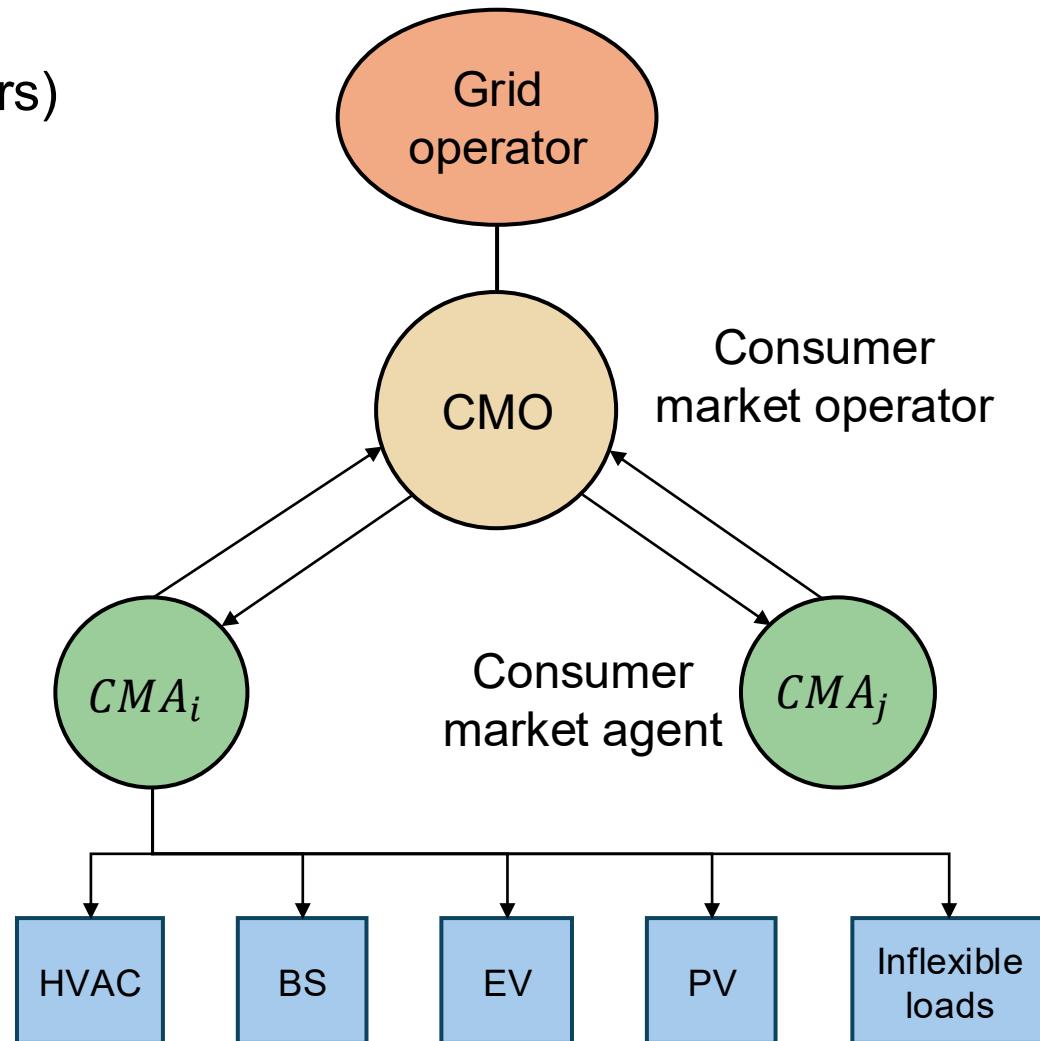
Literature review + contributions

- Rich literature on applications of **game theory** (Saad et al. 2012, Fadlullah et al. 2011), & **mechanism design** (Eid et al. 2016) to electricity markets
- Common modeling tools: **Stackelberg** (Maharjan et al. 2013) & **coordinated (coalitional)** games (Turdybek et al. 2024, Saeian et al. 2022)
- Some have also proposed **distributed algorithms** to solve such games (Li et al. 2011, Anoh et al. 2020) OR utilized **Vickrey-Clarke-Groves** mechanisms (Nekouei et al. 2015)
- We build upon this work:
 1. Analyze how game theory and mechanism design can inform development of markets closest to end-users (electricity consumers & prosumers)
 2. More accurately model physical dynamics & constraints of DERs
 3. Propose new approach to aggregate & maximize flexibility
 4. Using different types of tariffs to charge or compensate agents

Consumer market (CM) formulation

- Formulate as bilevel, multi-stage Stackelberg game between CMO (leader) & CMAs (followers)

1. CMA coordinates its DERs via multiperiod optimization (MPO) to determine its maximum possible flexibility ranges given physical constraints of its DERs
2. CMO solves welfare maximization problem to clear CM & set prices, given all CMA bids
3. Given CM prices, CMAs solve utility maximization problem to determine optimal flexible injection setpoints



Stage I: Multiperiod optimization to coordinate DERs

- Each CMA maximizes **net power injection** into grid & **flexibility** extracted from its DERs while minimizing **costs & disutility**

$$\min_{P_i^d(t), \delta_i^d(t)} \sum_{t \in \mathcal{H}} \sum_{d \in \mathcal{D}} -\delta_i^d(t) + f_i^d(P_i^d) + f_i^{util}(P_i^d) - P_i^{total}(t)$$

$$\text{s.t. } \underline{P_i^d(t)} \leq P_i^d(t) - \delta_i^d(t), \quad P_i^d(t) + \delta_i^d(t) \leq \overline{P_i^d(t)}$$

Net injection =
Generation - Load

All device-specific state constraints for each DER $d \in \mathcal{D}_i$

$$P_i^{total}(t) = \sum_{i \in \mathcal{D}_i} P_i^d(t) - P_i^{fixed}(t), \quad \epsilon_1 |P_i^d| \leq \delta_i^d \leq \epsilon_2 |P_i^d|$$

$$\mathcal{D}_i \subseteq \{BS, EV, HVAC, PV\}, \quad \epsilon_1 < \epsilon_2$$

- Reformulation of absolute injections for $d \in \{BS, EV\}$

$$P_i^d = P_i^{d,+} - P_i^{d,-}, \quad |P_i^d| = P_i^{d,+} + P_i^{d,-}, \quad z_i^d \in \{0, 1\}$$

$$0 \leq P_i^{d,+} \leq z_i^d \overline{P_i^d}, \quad 0 \leq P_i^{d,-} \leq (1 - z_i^d) \underline{P_i^d}$$

Note: $|P_i^{HP}| = -P_i^{HP}$ and $|P_i^{PV}| = P_i^{PV}$

Device-specific DER constraints & costs: BS & EV

- State of charge dynamics with inter-temporal constraints & cycling costs

$$SOC_i^{BS}(t+1) = (1 - \delta_{BS}^i)SOC_i^{BS}(t) - \frac{P_i^{BS}(t)\Delta t \eta_i^{BS}}{\bar{E}_i^{BS}}$$

$$\underline{P}_i^{BS} \leq P_i^{BS}(t) \leq \bar{P}_i^{BS}, \underline{SOC}_i^{BS} \leq SOC_i^{BS}(t) \leq \bar{SOC}_i^{BS}$$

$$SOC_i^{BS}(0) = SOC_i^{BS}(T)$$

$$f_i^{BS}(P_i^{BS}) = \alpha_{cyc} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (P_i^{BS}(t+1) - P_i^{BS}(t))^2$$

- EV model similar to BS

- Tracking objective to achieve desired SOC by specific time (e.g. 90% by 9am)

$$f_i^{EV} = \alpha_{cyc} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (P_i^{EV}(t+1) - P_i^{EV}(t))^2$$

- Additional restriction when EV is unavailable during time window

$$+ \xi_{ev}(SOC_i^{EV}(t^*) - SOC_i^{EV*})^2$$

$$(e.g. 9am-5pm while at work) P_i^{EV}(t) = 0 \forall t \in [t_1, t_2]$$

DER constraints & costs: Heat pump & solar

- **HP:** Thermal dynamics, temperature comfort limits, setpoint tracking

$$\text{Cooling mode } (T_i^{out} > T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1 - \theta_i) (T_i^{out}(t) + \rho_i P_i^{HP}(t))$$

$$\theta_i = e^{\frac{-\Delta t}{R_i^{th} C_i^{th}}} \approx 1 - \frac{\Delta t}{R_i^{th}, C_i^{th}}, \rho_i = R_i^{th} \eta_i$$

$$\text{Heating mode } (T_i^{out} < T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1 - \theta_i) (T_i^{out}(t) - \rho_i P_i^{HP}(t))$$

$$-P_{rated,i}^{HP} = \underline{P}_i^{HP} \leq P_i^{HP}(t) \leq 0, \underline{T}_i^{in} \leq T_i^{in}(t) \leq \overline{T}_i^{in}$$

$$f_i^{HP} = \xi_{ac} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (T_i^{in}(t) - T_i^{in*})^2$$

- **PV:** Non-dispatchable, can be curtailed
- CMA also utilizes as much PV output as possible (when available) to charge BS & EV, by minimizing this objective:

$$f_i^{util}(t) = (P_i^{PV} + P_i^{BS} + P_i^{EV})^2 \forall \{t : P_i^{PV}(t) \neq 0\}$$

$$0 \leq P_i^{PV}(t) \leq \alpha^{PV}(t) \overline{P}_i^{PV}$$

$$f_i^{PV} = \xi_{pv} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (\alpha^{PV}(t) \overline{P}_i^{PV} - P_i^{PV}(t))^2$$

Stage II: CMA welfare maximization problem

- CMA aggregates schedules across all DERs

$$P_i^0 = \sum_{d \in \mathcal{D}_i} P_i^{d*}, \underline{P}_i = P_i^0 - \sum_{d \in \mathcal{D}_i} \delta_i^{d*}, \overline{P}_i = P_i^0 + \sum_{d \in \mathcal{D}_i} \delta_i^{d*}$$

- Maximize social welfare s.t. flexibility constraints, given prices for electricity $\mu(t)$ & flexibility $\tilde{\mu}(t)$ set by CMO

$$\max_{P_i} U_i^{cma}(P_i, \tilde{\mu}, \mu) = \tilde{\mu}(P_i - P_i^0) + \mu P_i - \gamma_i (P_i - P_i^0)^2 \text{ s.t. } \underline{P}_i \leq P_i \leq \overline{P}_i, P_i \geq P_i^0$$

- Analytically solve game via KKT conditions

$$P_i^*(\tilde{\mu}, \mu) = \begin{cases} \overline{P}_i & \text{if } \frac{\gamma_i(\overline{P}_i - P_i^0)}{\tilde{\mu} + \mu} < \frac{1}{2} \\ P_i^0 + \frac{\tilde{\mu} + \mu}{2\gamma_i} & \text{otherwise} \end{cases}$$

- CMA submits optimal bid $\{P_i^0, P_i^*, [\underline{P}_i, \overline{P}_i]\}$ to CMO

Stage III: CMO optimization to set optimal prices

- CMO aims to schedule its CMAs to track desired flexible setpoint $\tilde{P}(t)$ requested by LEM s.t. to budget balance

$$\min_{\tilde{\mu}(t), \mu(t)} -U^{cmo}(P_i(t), \mu(t), \tilde{\mu}(t)) = \left(\sum_{i \in \mathcal{C}} P_i(t) - \tilde{P}(t) \right)^2$$

$$\text{s.t. } P_i^0(t) \leq P_i(t) \leq \bar{P}_i(t) \forall i$$

$$\tilde{\mu}(t) \sum_{i \in \mathcal{C}} (P_i(t) - P_i^0(t)) + \mu(t) \sum_i P_i(t) = \pi(t) \sum_i P_i(t)$$

- Analytically solve for optimal prices for power & flexibility

$$\tilde{\mu}(P_t - P_t^0) + \mu P_t = \pi P_t, \quad P_t^* = \sum_{i \in \mathcal{C}} P_i^* = P_t^0 + \gamma_t \frac{\mu + \tilde{\mu}}{2} = \tilde{P} \implies \mu^* = \frac{\pi \tilde{P}}{P_t^0} - \frac{2(\tilde{P} - P_t^0)^2}{\gamma_t P_t^0}$$

$$\tilde{\mu}^* = \frac{P_t^*(\pi - \mu^*)}{P_t^* - P_t^0} = \frac{\tilde{P}(\pi - \mu^*)}{\tilde{P} - P_t^0} = \frac{\tilde{P} (2(\tilde{P} - P_t^0) - \pi \gamma_t)}{\gamma_t P_t^0}, \quad \gamma_t = \sum_{i \in \mathcal{C}} \frac{1}{\gamma_i}$$

Positivity of prices

- Following condition required for $\mu^*(t), \tilde{\mu}^*(t) > 0$

$$\begin{cases} \tilde{P} > P_t^0 + \frac{\pi\gamma_t}{2} & \text{if } \tilde{P}, P_t^0 < 0 \\ \max\left(a_1, P_t^0 + \frac{\pi\gamma_t}{2}\right) \leq \tilde{P} \leq a_2 & \text{if } \tilde{P}, P_t^0 > 0 \end{cases} \quad a_1 = P_t^0 + \frac{\pi\gamma_t}{4} - \frac{1}{2}\sqrt{\frac{\pi\gamma_t}{2}\left(4P_t^0 + \frac{\pi\gamma_t}{2}\right)}$$
$$a_2 = P_t^0 + \frac{\pi\gamma_t}{4} + \frac{1}{2}\sqrt{\frac{\pi\gamma_t}{2}\left(4P_t^0 + \frac{\pi\gamma_t}{2}\right)}$$

- Generally holds true given that:

- CMO is generally a net load $\rightarrow \tilde{P}, P_t^0 < 0$
- CMO provides upward flexibility to reduce net load $\rightarrow \tilde{P} > P_t^0$
- $\frac{\pi\gamma_t}{2}$ is small for most realistic prices π & disutility coefficients γ_i

Equilibrium

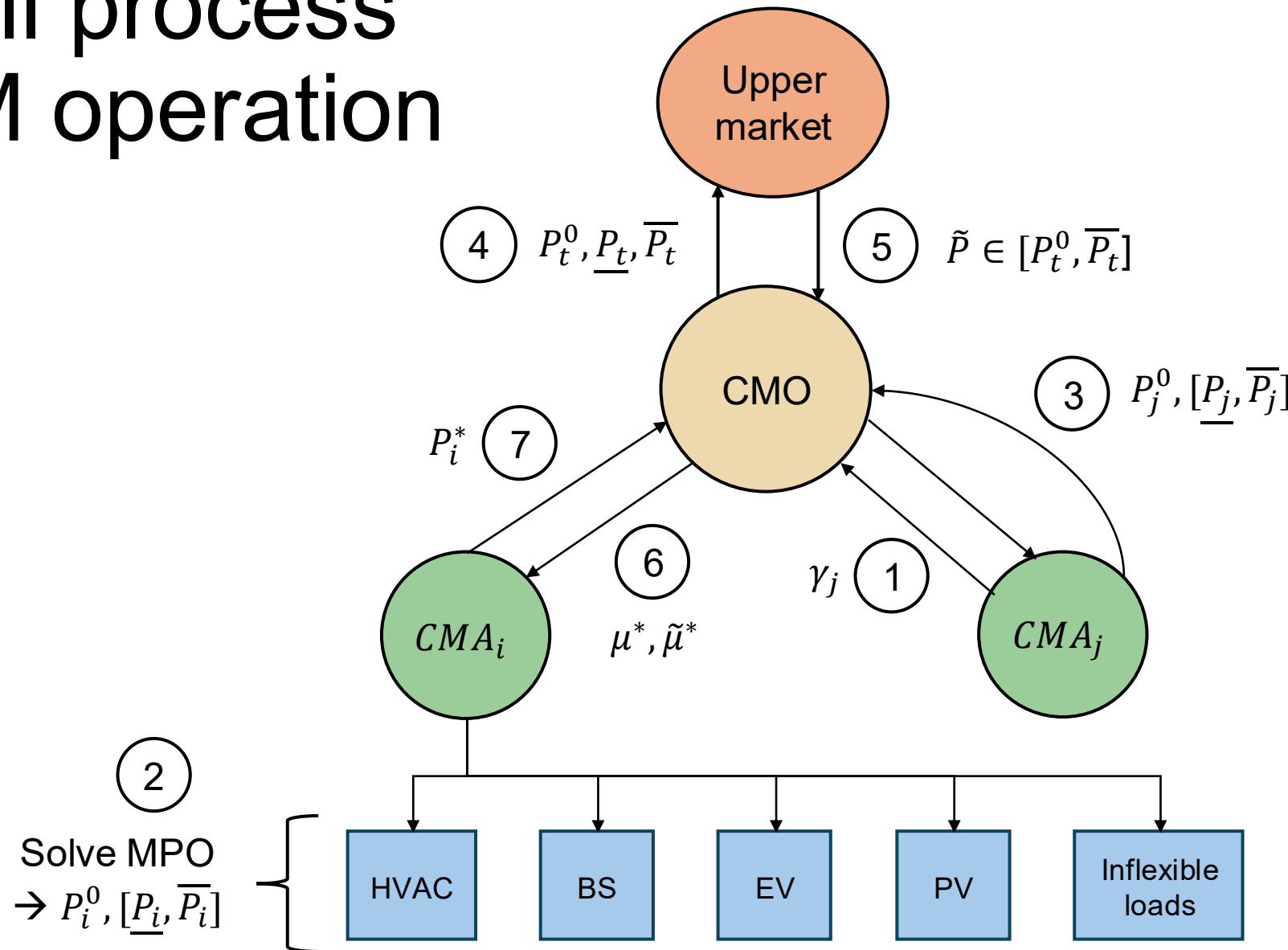
- Note: Nash's theorem states existence of mixed strategy NE for any finite game and Glicksberg's theorem extends this to infinite setting with continuous strategies
- Our assumptions & simplifications allow us to show existence & uniqueness of pure strategy equilibrium with closed form solutions
- Optimal prices $\mu^*, \tilde{\mu}^*$ set by CMO will induce optimal bids P_i^* from all CMAs that lead to an equilibrium in pure strategies
- Set of bids & prices $\{P_i^* \forall i, \mu^*, \tilde{\mu}^*\}$ correspond to unique **Nash equilibrium** amongst CMAs and a unique **Stackelberg equilibrium** between all CMAs & the CMO

$$U_i^{cma}(P_i, \tilde{\mu}(P_i, P_{-i}), \mu(P_i, P_{-i})) \equiv U_i^{cma}(P_i, P_{-i}, \tilde{\mu}, \mu)$$

$$U_i^{cma}(P_i^*, P_{-i}^*, \tilde{\mu}^*, \mu^*) \geq U_i^{cma}(P_i, P_{-i}^*, \tilde{\mu}^*, \mu^*) \quad \forall P_i \in [\underline{P}_i, \bar{P}_i]$$

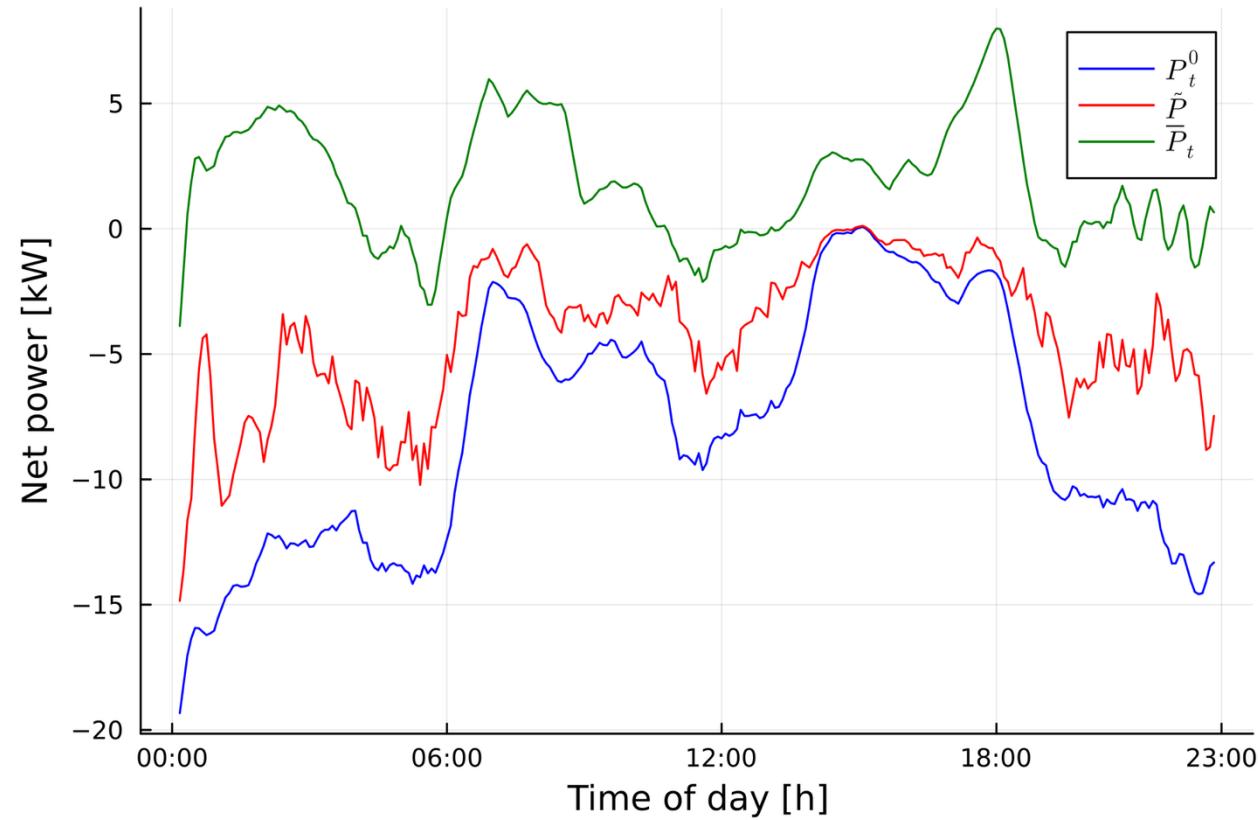
$$U^{cmo}(P_i^*, P_{-i}^*, \mu^*, \tilde{\mu}^*) = 0 \geq U^{cmo}(P_i^*, P_{-i}^*, \mu, \tilde{\mu}) \quad \forall \mu, \tilde{\mu}$$

Overall process for CM operation

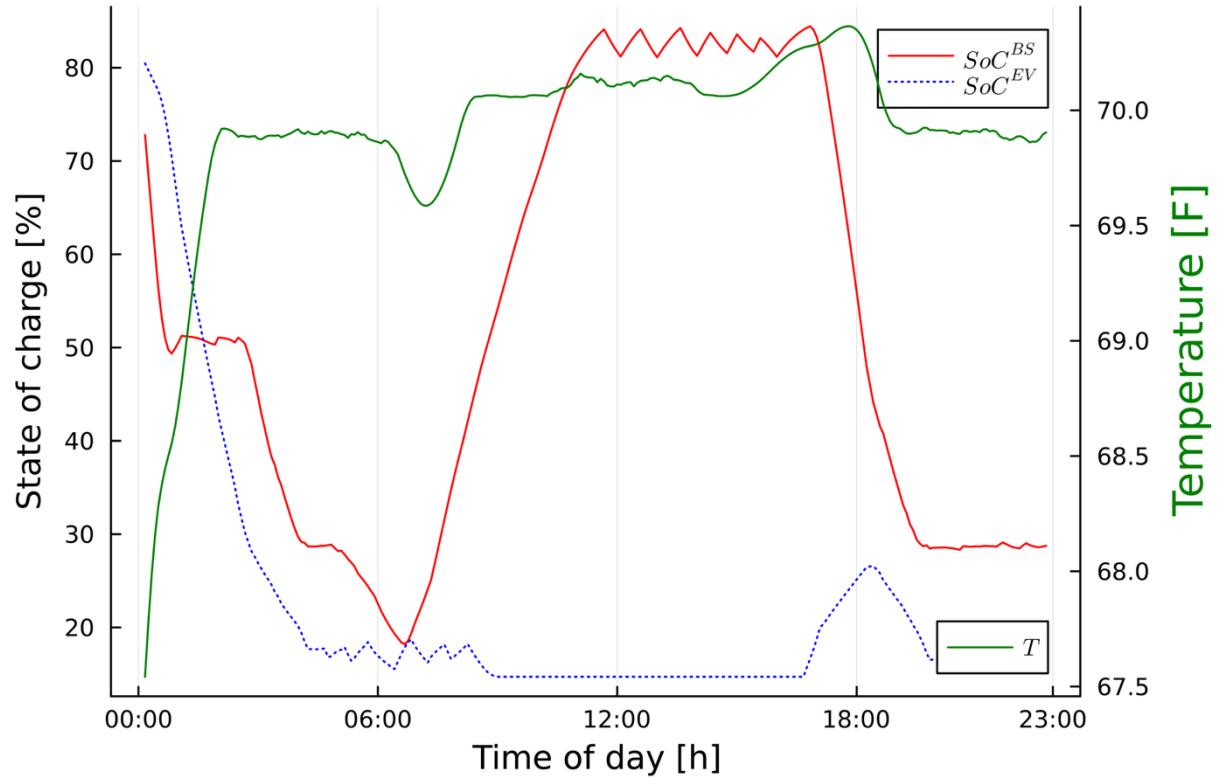
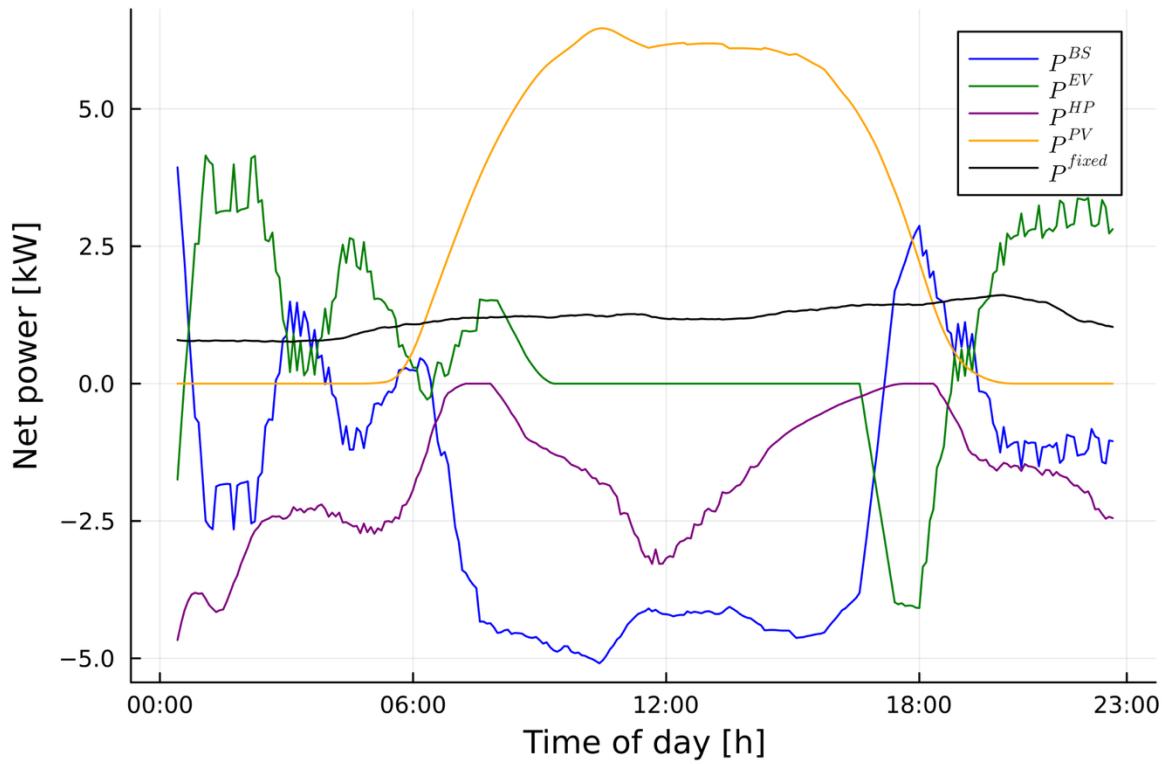


Simulation results on small example CM: Injections

- Simulated 1 CMO & 3 CMAs, using temperature/solar data from California & main grid prices from CAISO
- CMO remains net load throughout the day, with net load lower mid-day during peak solar PV output
- Grid operator requests varying amounts of load flexibility or demand response throughout the day
- CMO is able to successfully aggregate flexibilities of its CMAs to satisfy the regulation signal from the grid operator

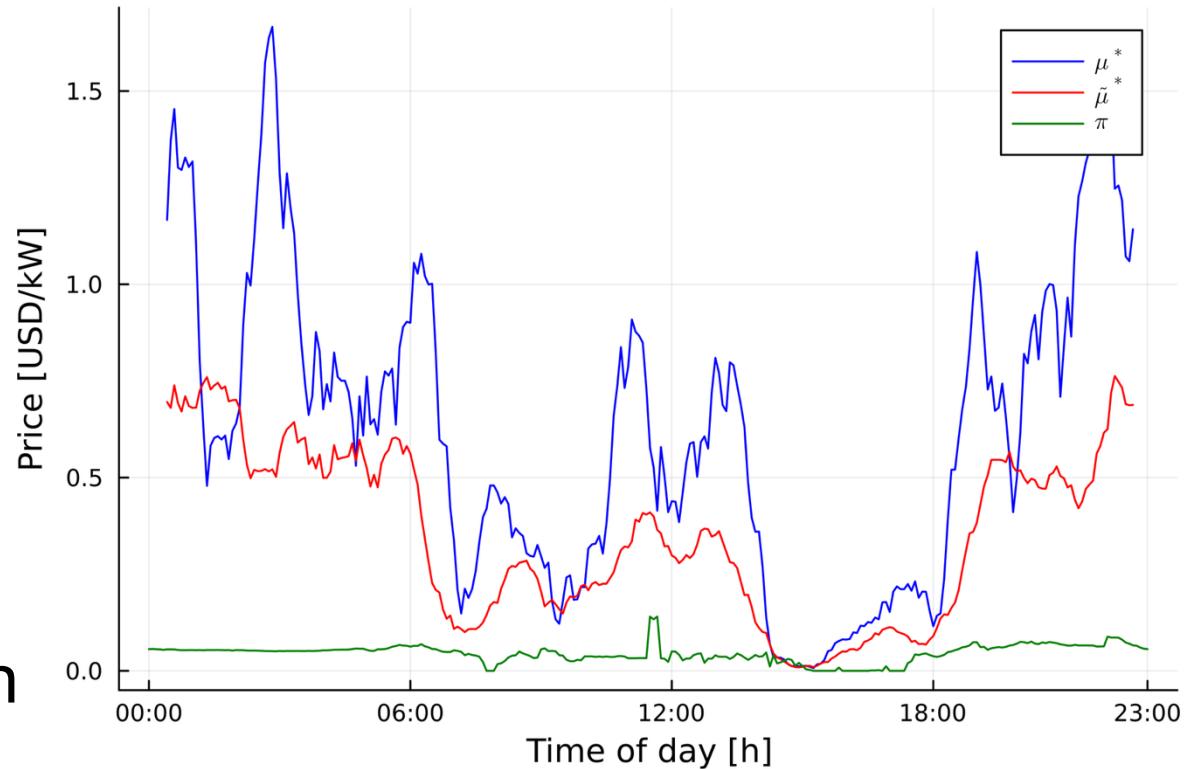


DER coordination for CMA 1



Electricity & flexibility prices

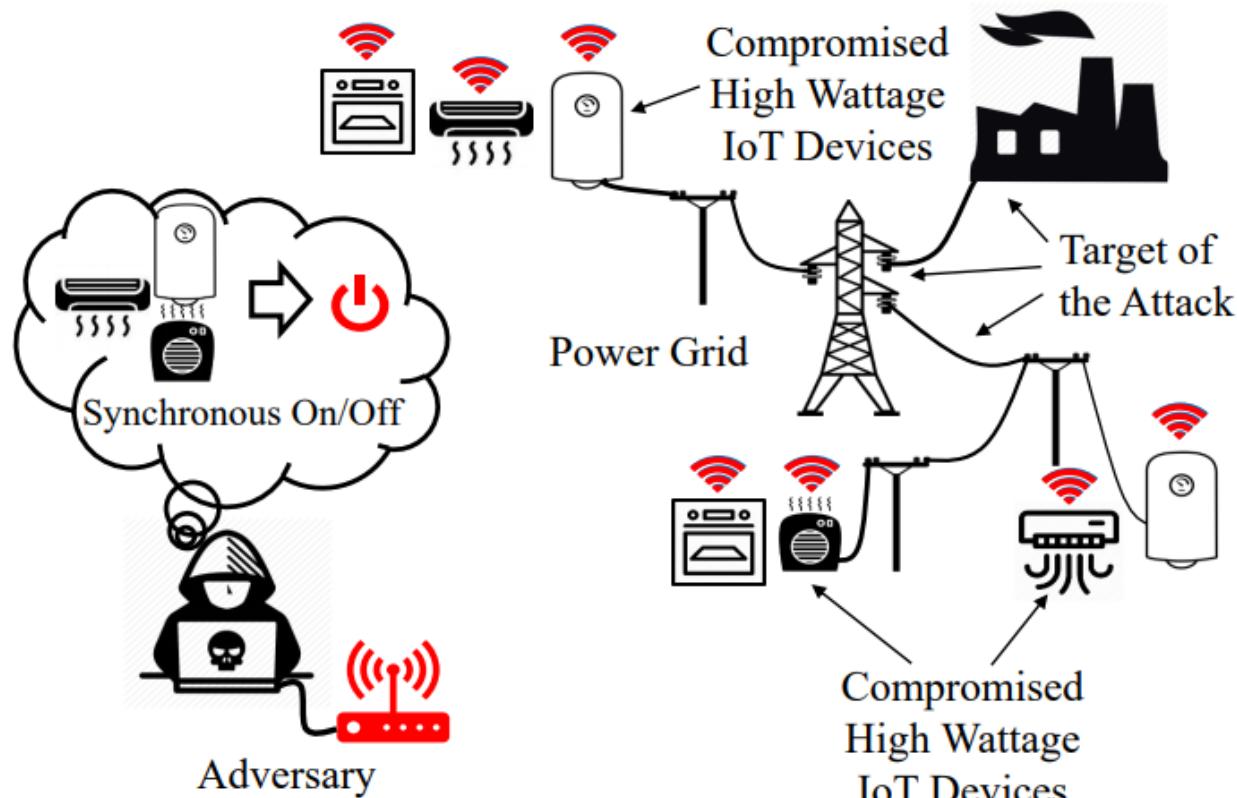
- CM prices are about an order of magnitude higher than the grid price π
- For CMO to provide required flexibility, it has to increase its prices & also compensate CMAs and DERs sufficiently, which raises costs
- Possible approach to mitigate price impacts: Varying prices μ_i^* , $\tilde{\mu}_i$ for each CMA i instead of common CM rate
 - Pros: More efficient, equitable, fair tariffs
 - Cons: Makes equilibrium analysis more challenging



Conclusions

- Game-theoretic market mechanism for DER flexibility
- Hierarchical approach: Satisfy all DER physical constraints
- Market operator sets strategy-proof optimal prices for both electricity & flexibility
- Pricing induces truthful flexibility bids from all agents
- Meet total required flexibility (net load curtailment)
- Established unique equilibria among market participants with closed form analytical solutions
- Future work: Relax model assumptions, explore other types of equilibria

Example I: BlackIoT - Load alteration using IoT-networks

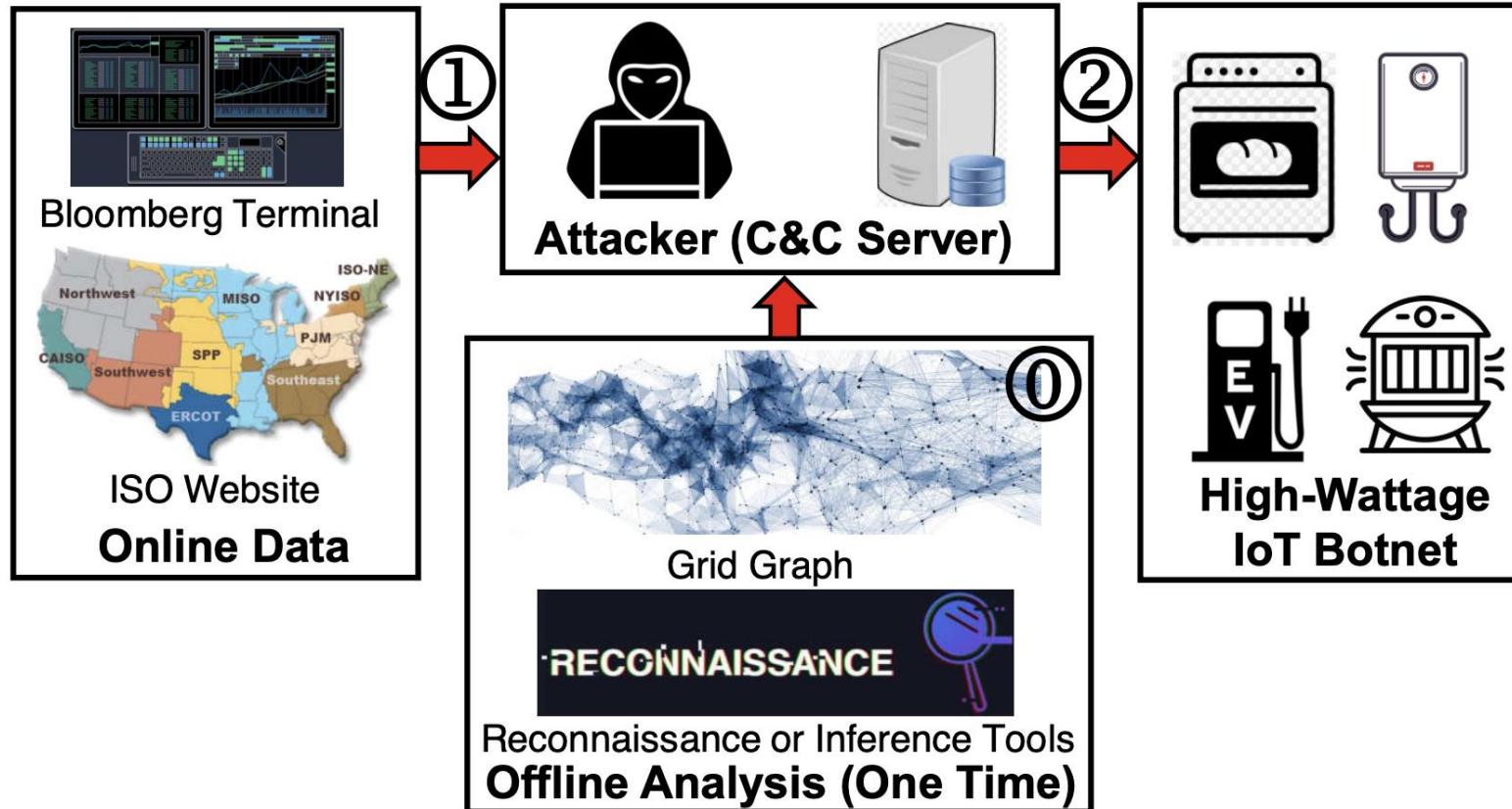


- Large scale manipulation of IoT devices – *botnets*, like Mirai botnets
- A 900MW step change in load with a tightly coordinated 600,000 IoT devices each controlling a 1500W HVAC unit

[1] Soltan et.al, "BlackIoT: IoT Botnet of High Wattage Devices Can Disrupt the Power Grid" Usenix Security Symposium 2017

[2] Huang et.al, "Not Everything is Dark and Gloomy: Power Grid Protections Against IoT Demand Attacks" Usenix Security Symposium 2018

Example II: MaDloT - Strategic Manipulation of Demand

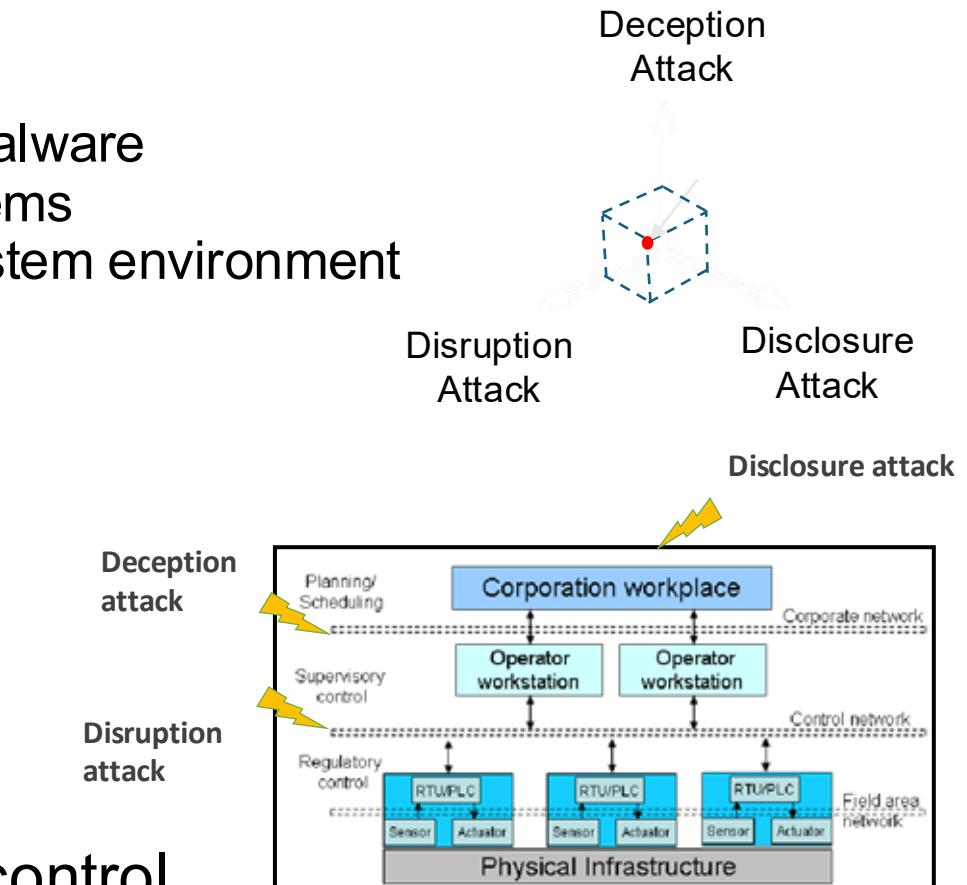


- Identify the most vulnerable nodes and time
- Only need to compromise 150,000 nodes now – much less than the previous attack

[1] Shekari et.al, “MaDloT 2.0: Modern High-Wattage IoT Botnet Attacks and Defenses” Usenix Security Symposium 2022

Example III: Ukraine Attack in 2015-16

- **Confidentiality Attack (Disclosure):**
 - Attack introduced via phishing emails containing malware
 - Enabled attacker communication with hacked systems
 - Enabled attacker to steal critical data and study system environment
- **Integrity Attack (Deception):**
 - Accessed control level over compromised VPN
 - Spoofed control commands
- **Availability Attack (Disruption):**
 - Overwrote substation firmware, permanently ensuring remote inoperability of breakers
- **30 substations switched off**
- **230,000 customers left without power**
- The 2016 attack also corrupted transmission control

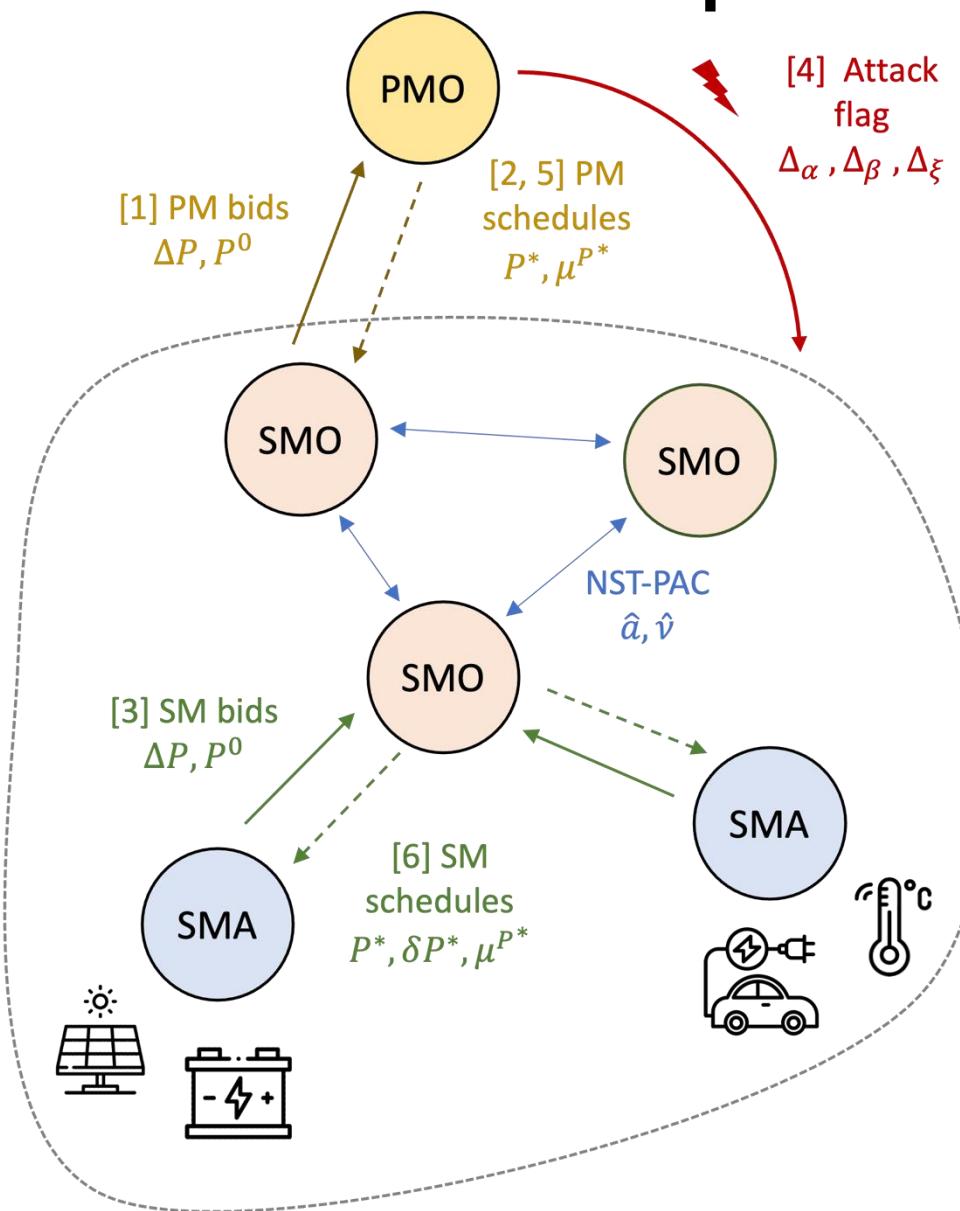


LEM to improve cyber-physical resilience

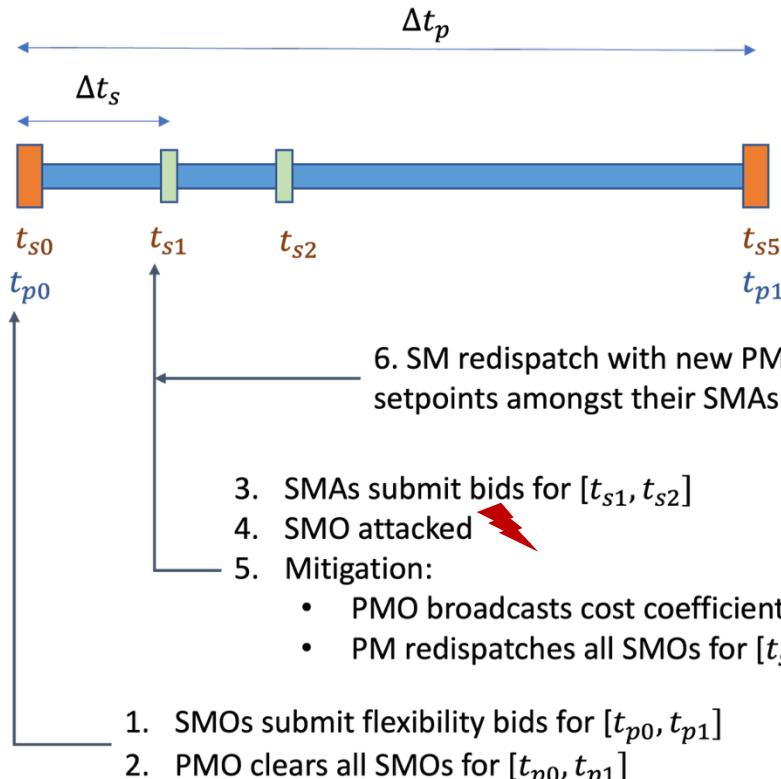
- Decarbonized power grid will necessarily include an increased cyber footprint, allowing a multitude of attack surfaces, cyber and physical
- Ukraine power grid attacks and other recent attacks on infrastructure (e.g. Colonial pipeline) underscore that such threats are real
- Combined presence of both cyber & physical attacks requires new tools for analysis of the emerging grid rich in DERs

Can we use market-based coordination of IoT devices & DERs (w/o direct control) to increase rather than decrease **resilience**?

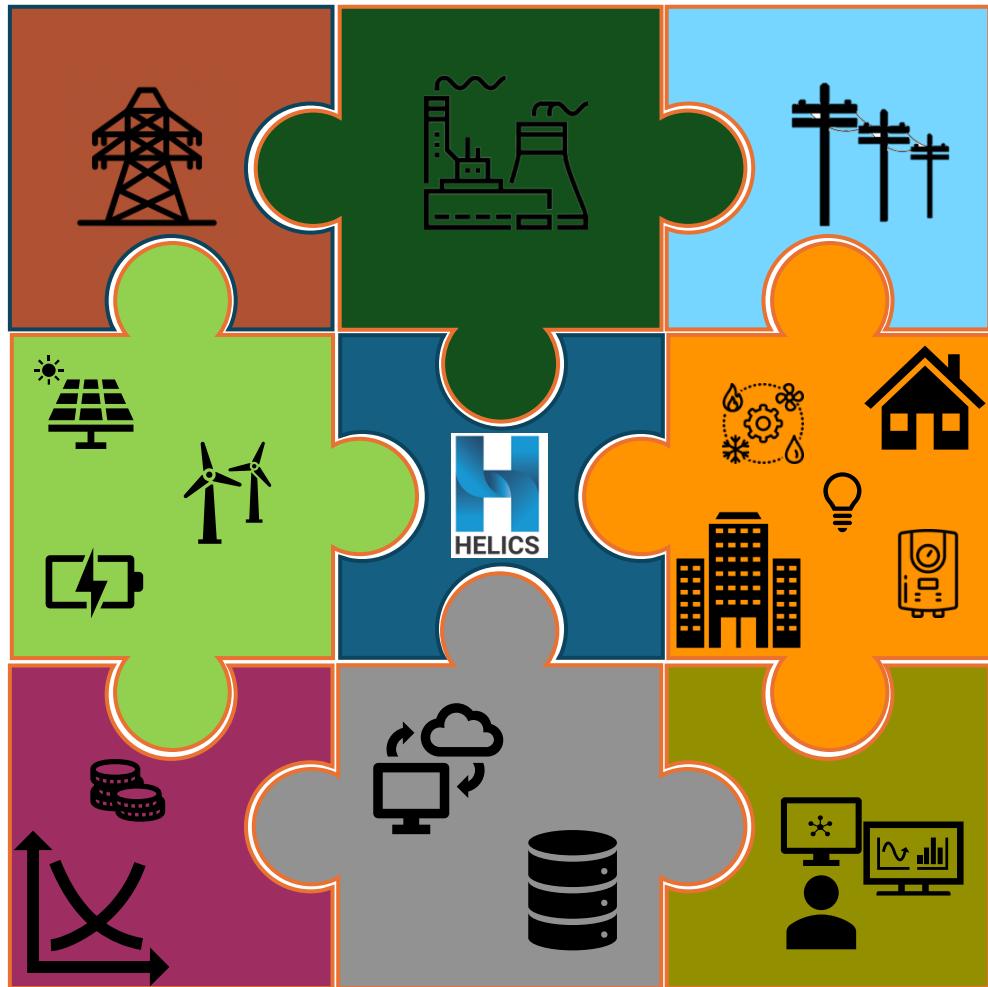
SM & PM response



Mitigation involves both SM & PM redispatch



PNNL co-simulation: Analysis technique for large complex systems



- HELICS - Hierarchical Engine for Large-scale Infrastructure Co-Simulation
- Multi-lab DOE-sponsored
- Highly-scalable
- Libraries and language bindings to integrate simulators (federates) written in:
 - C
 - C++
 - Python
 - MATLAB
 - Java

<https://www.helics.org>

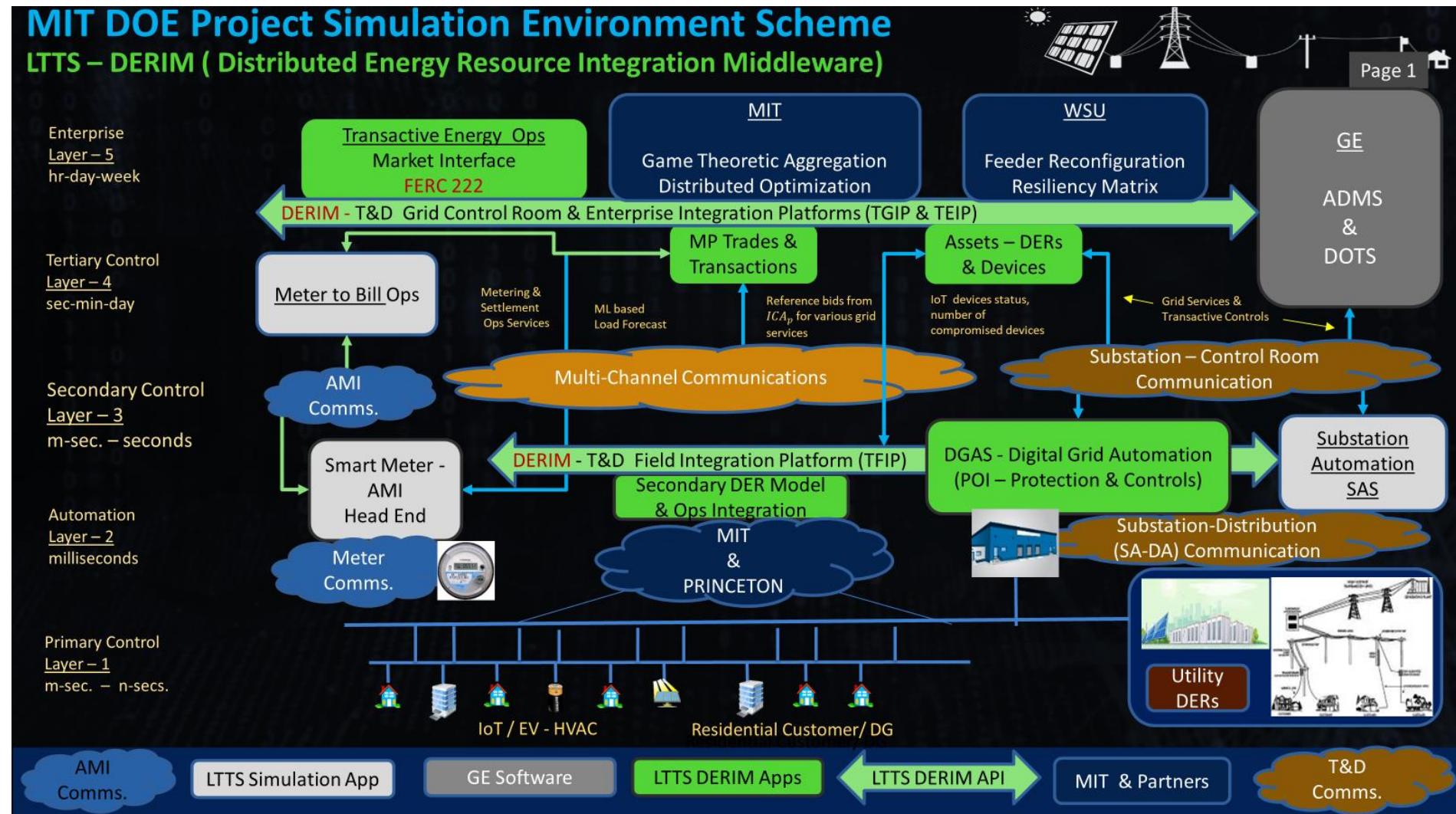
<https://github.com/GMLC-TDC/>

<https://helics.readthedocs.io>

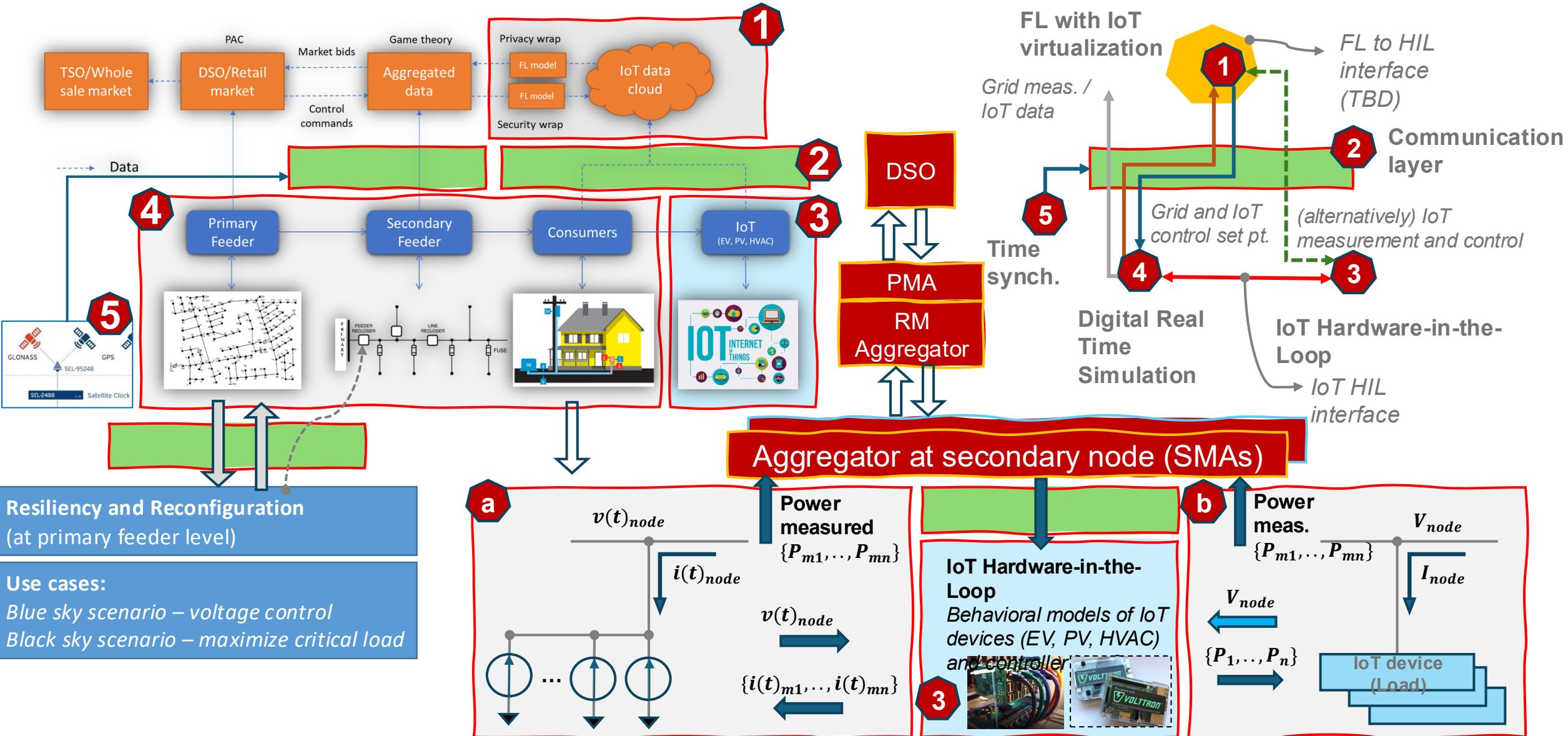
<https://helics.readthedocs.io/en/latest/user-guide/index.html>

LTDES & GE validation: Industry-grade software

- **DERIM:** DER Integration Middleware
- **DOTS:** Utility Distribution Operations Training Simulator
- **ADMS:** Advanced distribution management system

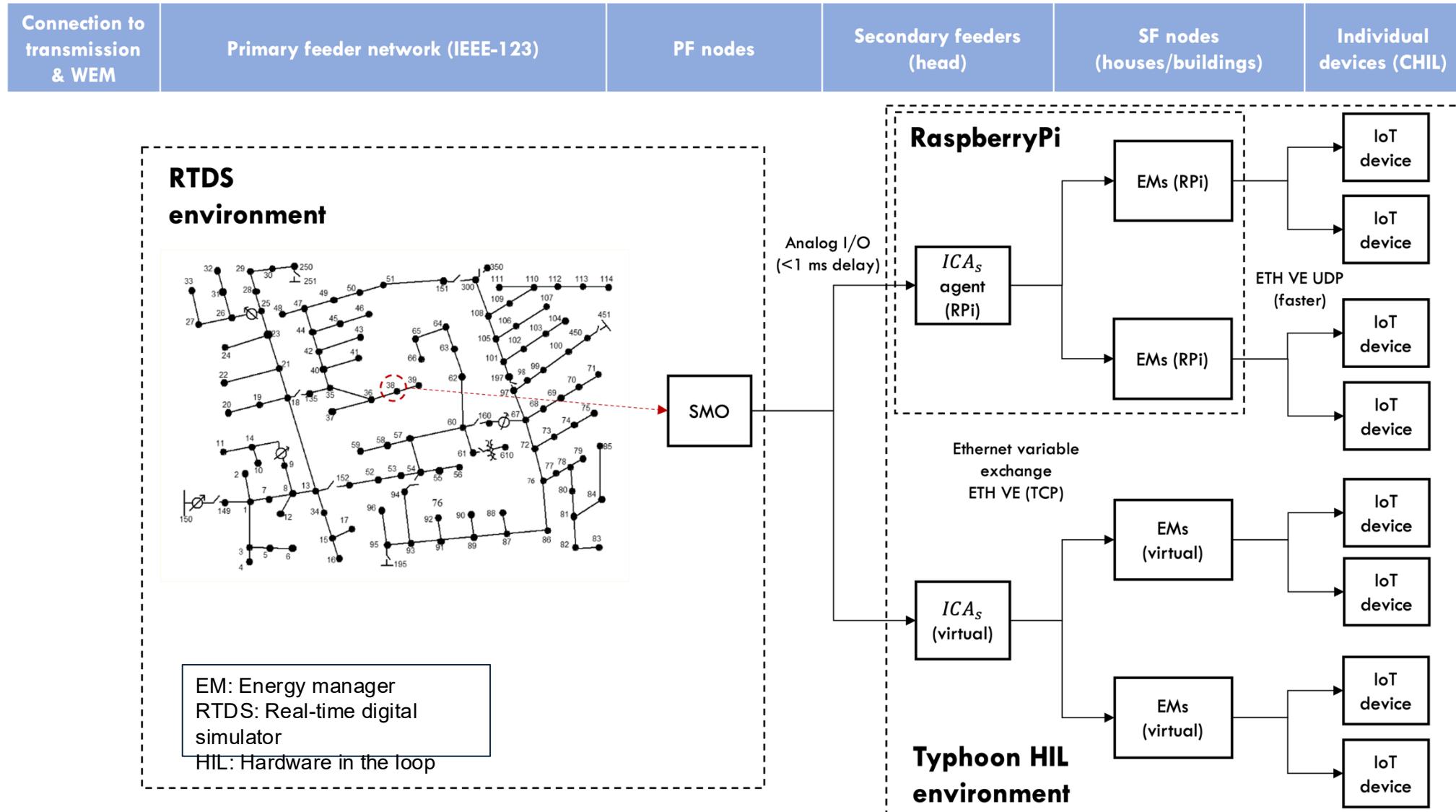


NREL: Real-time HIL Validation



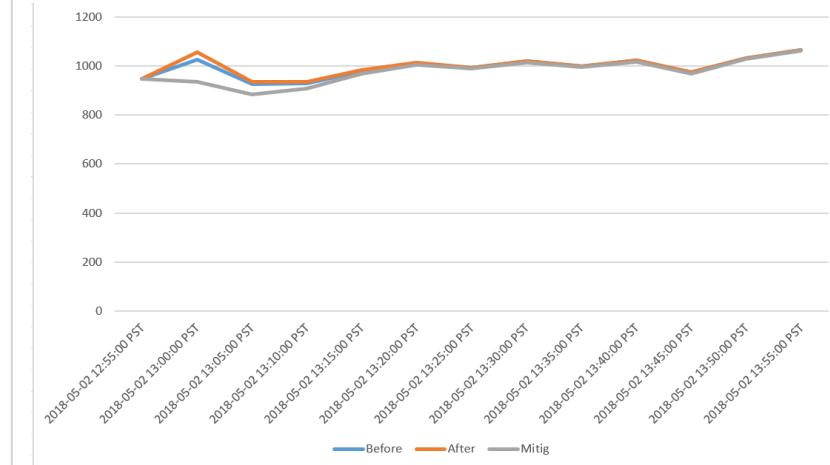
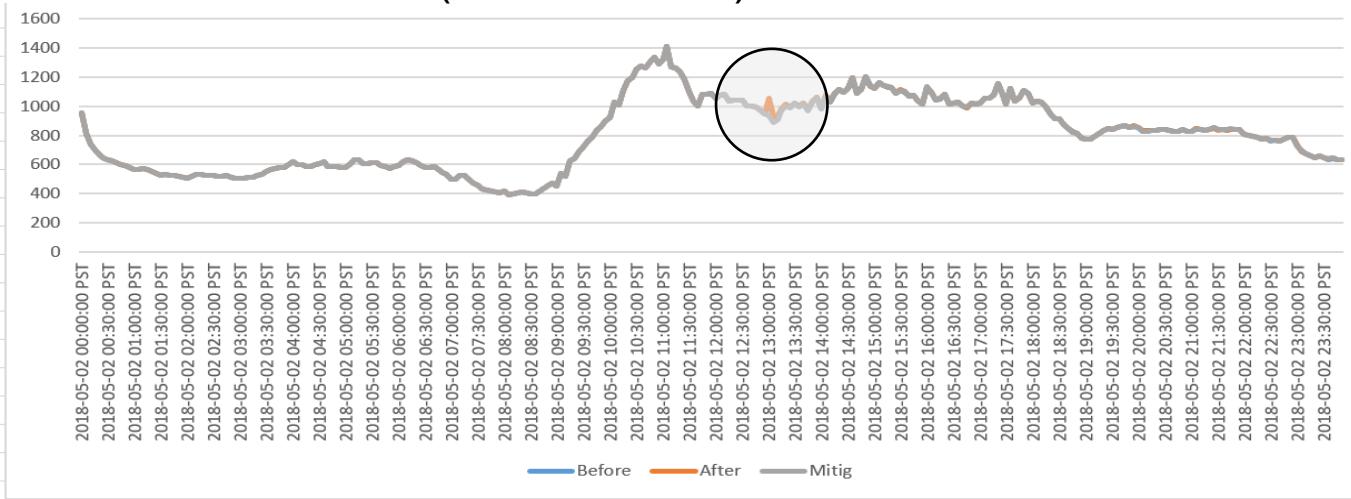
NREL HIL validation setup

Physical/Grid locations

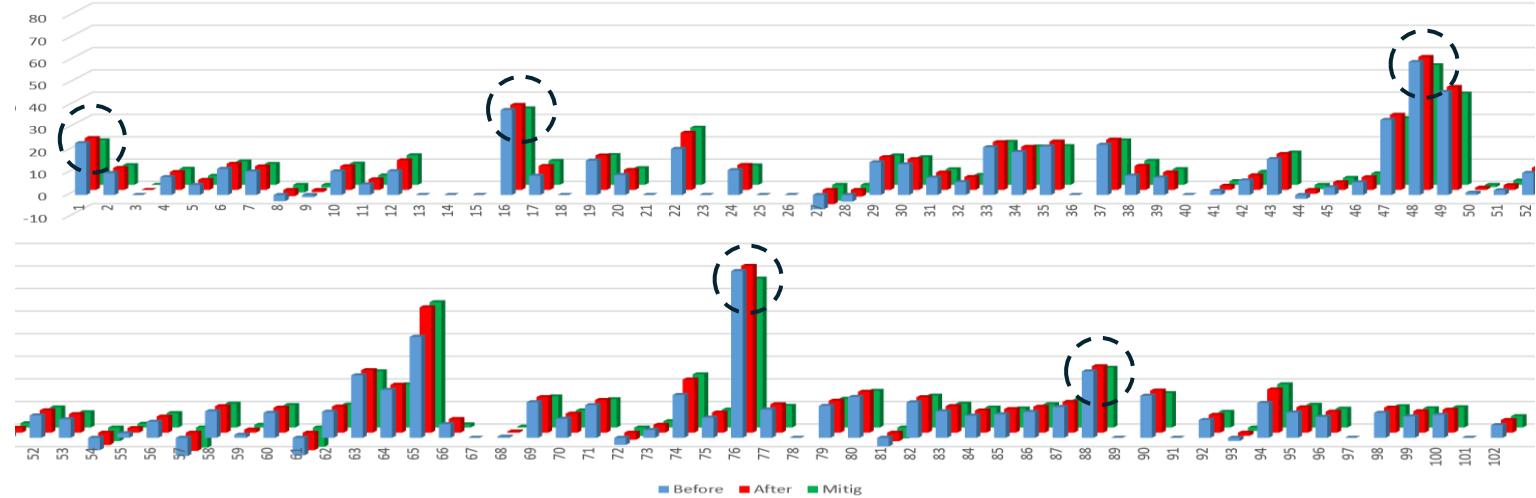


Attack 1a validation with GE & LTDES

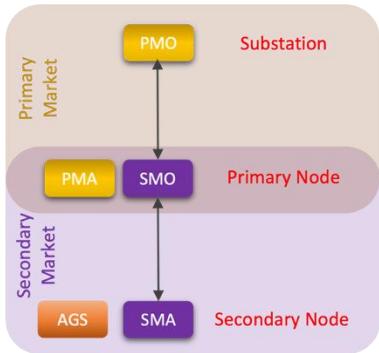
Power Flow (Active Power) result at Substation



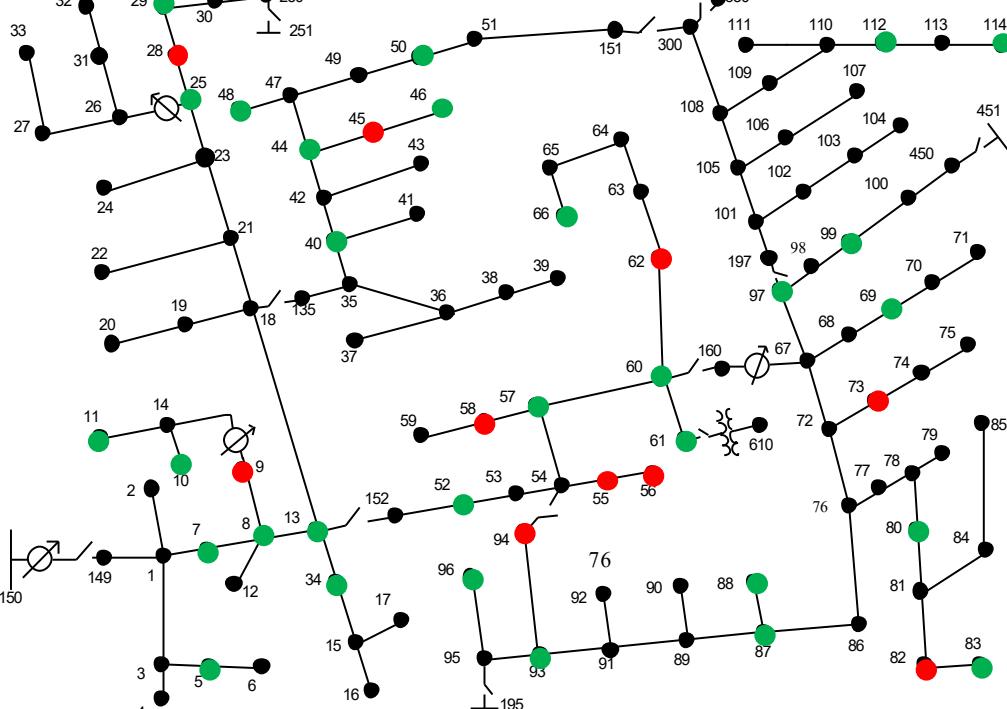
- Without market mitigation: Feeder demand jumped by 37 kW
- With market mitigation: Feeder demand cut by 94 kW
- Nodes with large loads contribute most to mitigation



Attack 1b: PMA generation attack



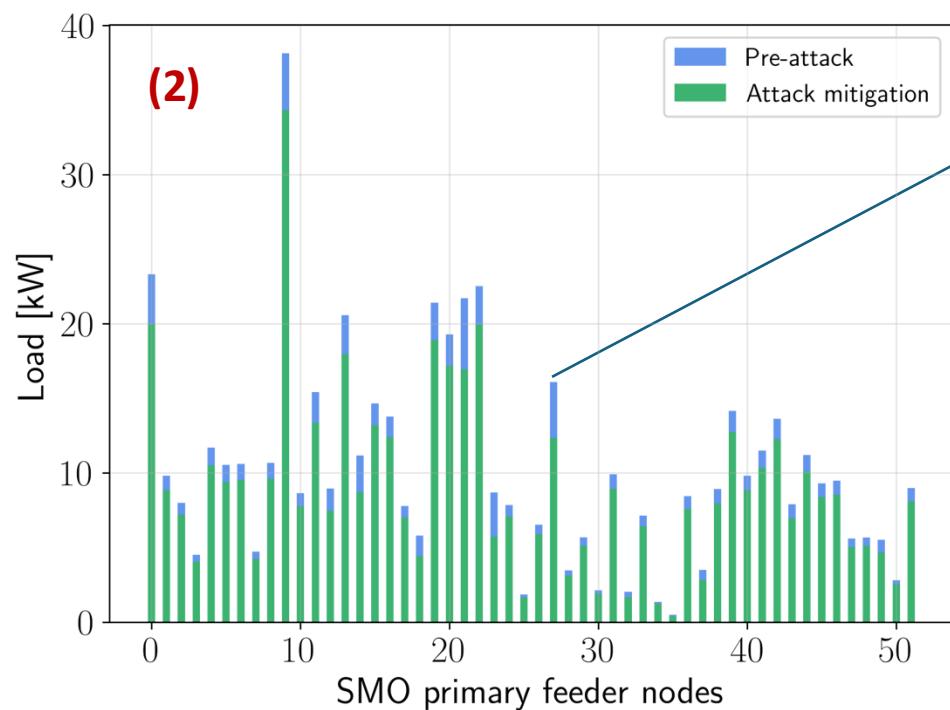
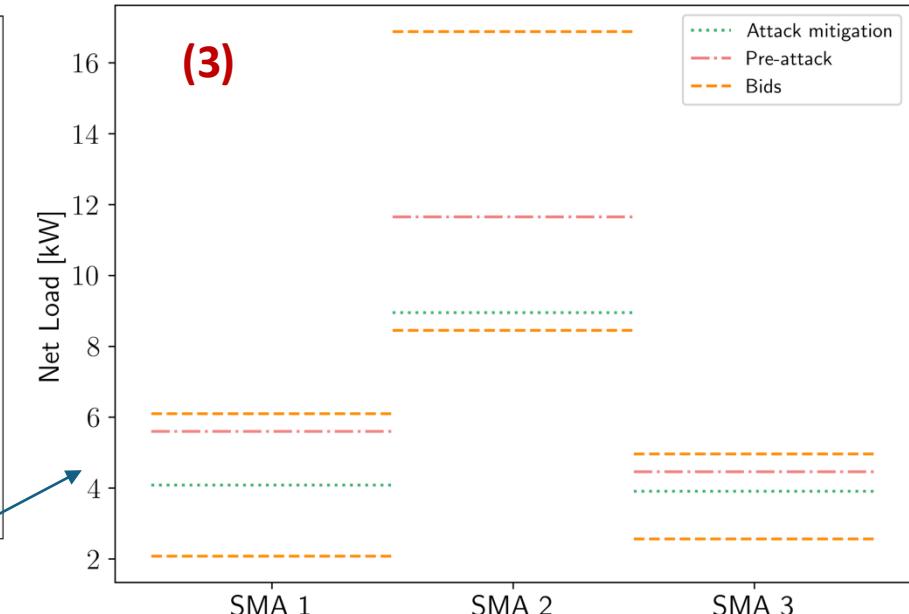
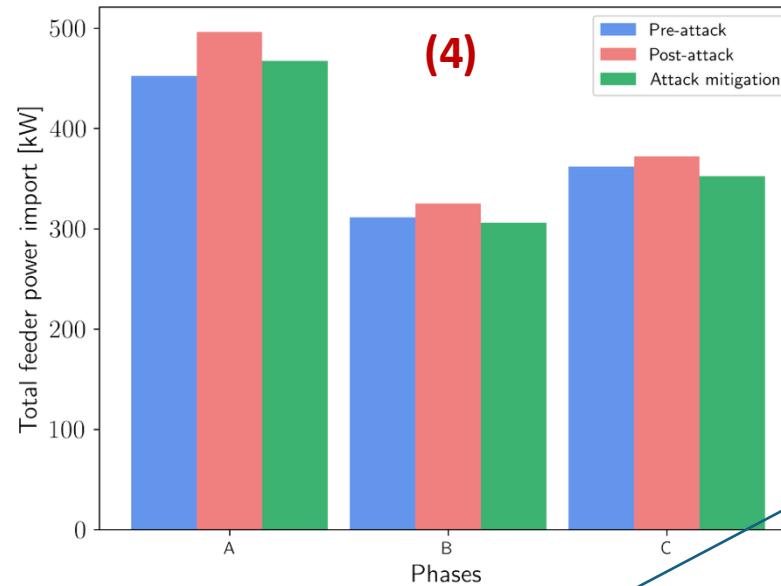
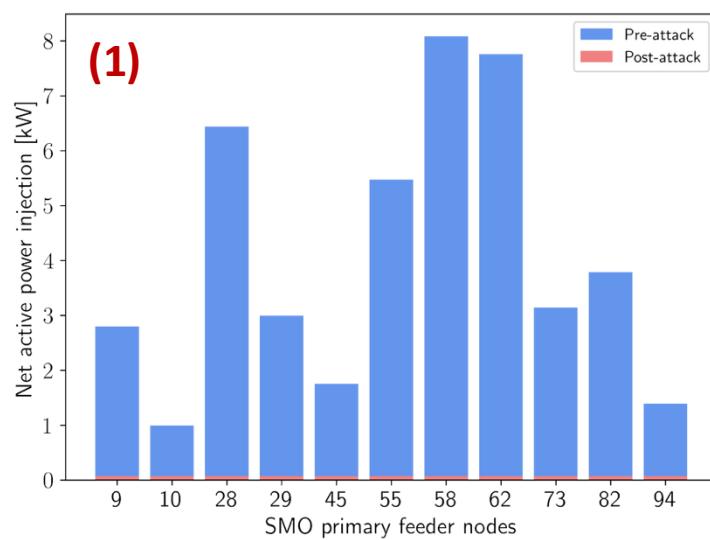
Attack would have resulted in an increase in power import by 68 kW



Red circle : Attacked Nodes Green circle : Trustable EUREICA-Nodes

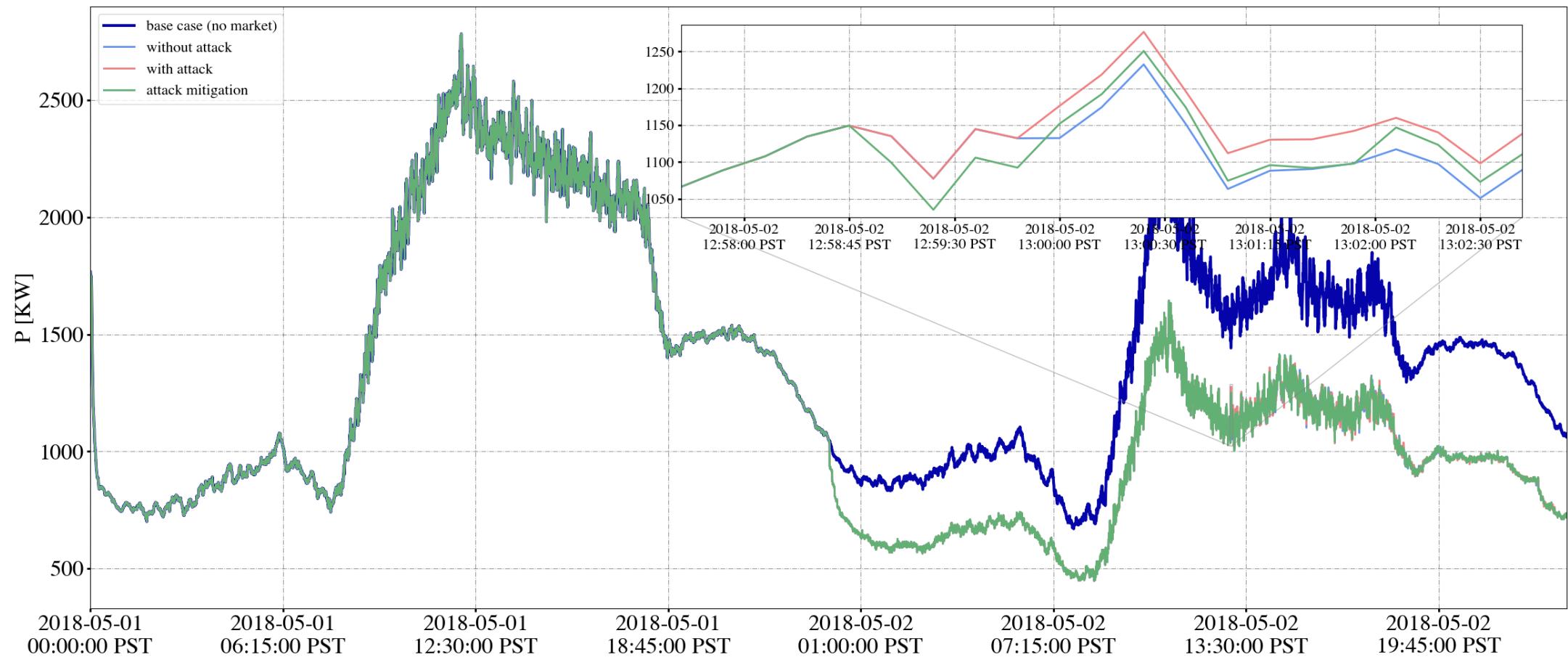
1. 45 kW net-generation compromised (~50 homes)
2. PRM alerts other trustable PMAs/SMOs to redispatch their generation assets
3. Trustable PMAs/SMOs will curtail flexible loads to respond & mitigate attack
4. SMOs redispatch SMAs who provide correct setpoints
5. Total import from the main grid stays at the same level
 - 9, 28, 45, 55, 56, 58, 62, 73, 82, 94
 - 5 7 8 10 11 13 25 29 34 40 44 46 48 50 52
 - 57 60 61 66 69 80 83 87 88 93 96 97 99 112 114

SMA bids and solutions for SMO 36



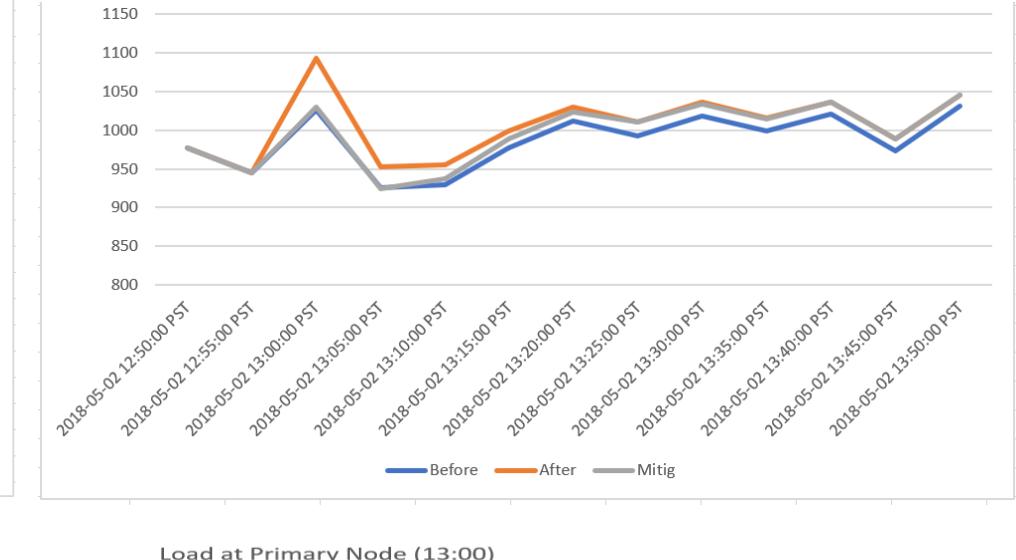
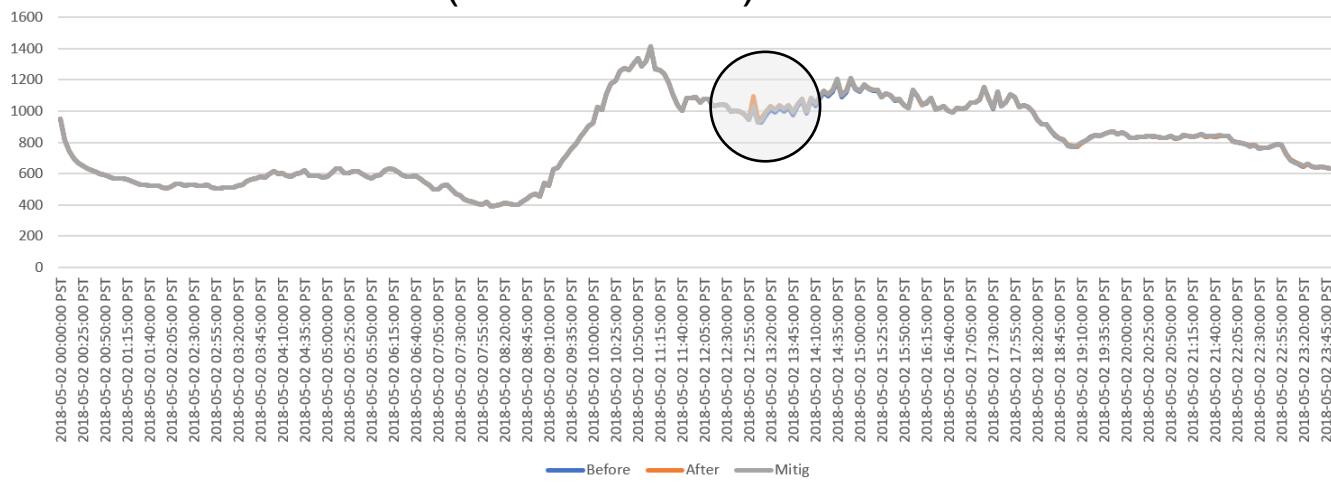
Metric	Value [kW]
Total load without attack	1167.52
Total load with attack	1190.44
Total load after attack mitigation	1123.31
Minimum SMO load curtailment	0.12
Maximum SMO load curtailment	4.77
Total import w/o attack	1125.91
Total import w/ attack	1193.87
Total import w/ attack mitigation	1126.35

Attack 1b validation with PNNL

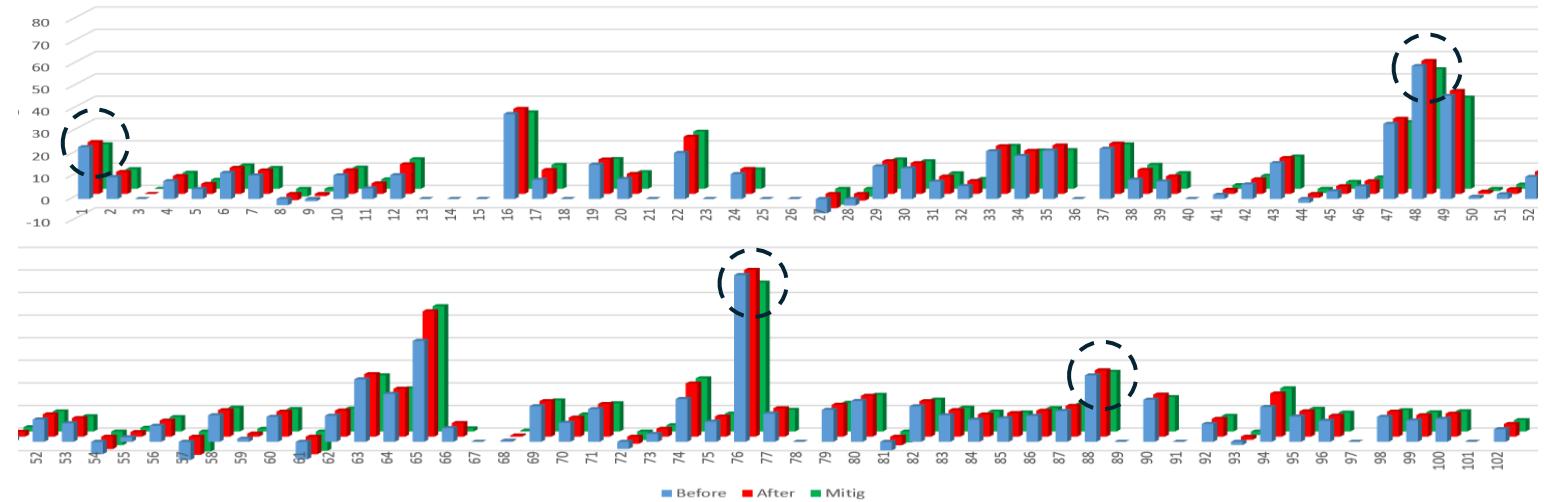


Attack 1b validation with GE & LTDES

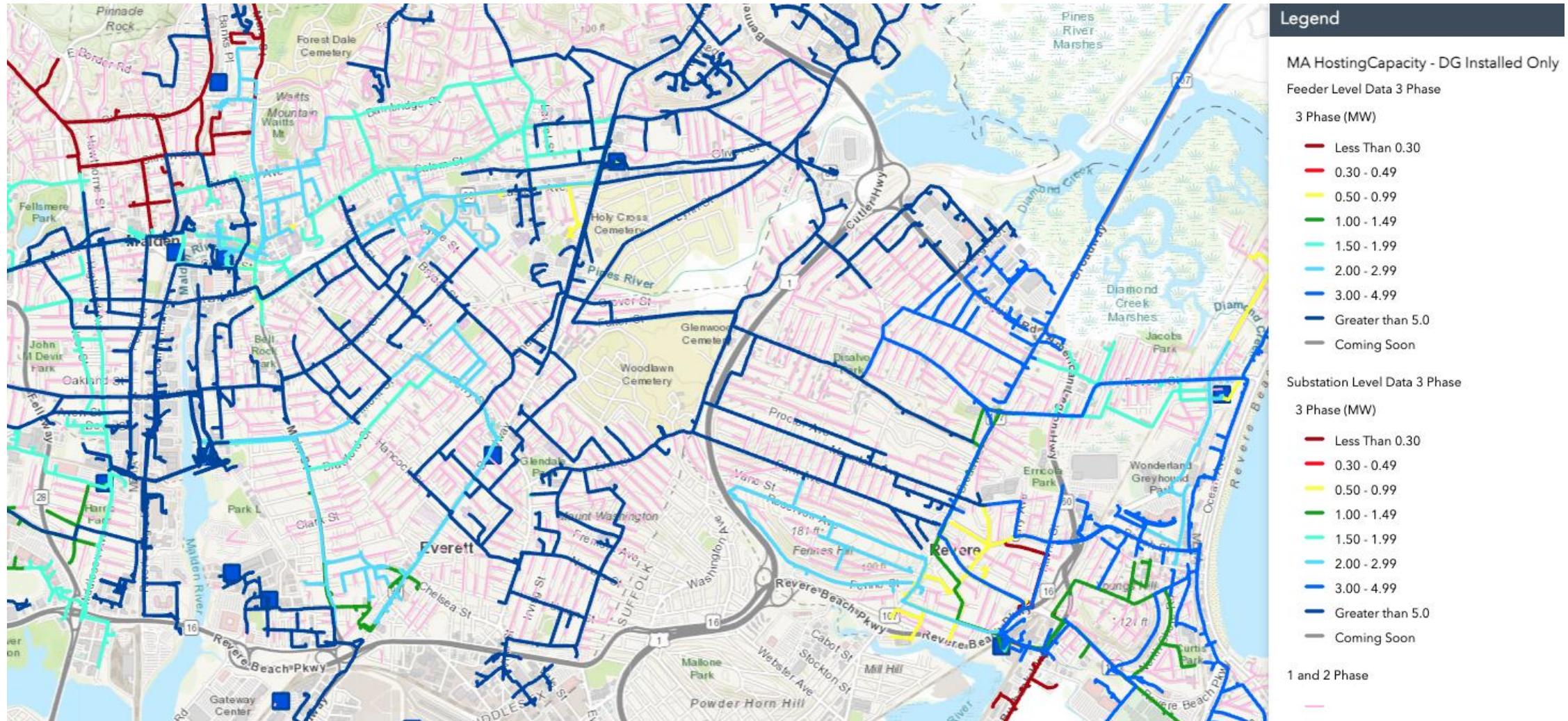
Power Flow (Active Power) result at Substation



- Without market mitigation: Feeder demand jumped by 68 kW
- With market mitigation: No impact on feeder demand
- Nodes with large loads contribute most to mitigation

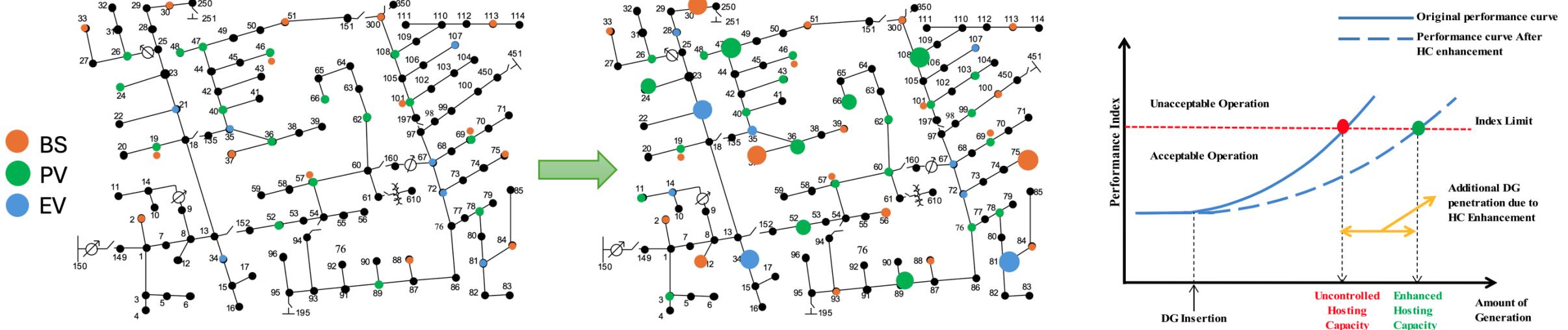


HC map example for Boston: National Grid



HC shown in terms of line capacities

Coordination to increase hosting capacity



Ismael et al. State-of-the-art of hosting capacity in modern power systems with distributed generation. Renewable Energy 2019.

- Market-based distributed real-time DER coordination & flexible interconnection
 - Use flexibility to increase dynamic hosting capacity, reduce solar curtailment & costs
 - Storage (both co-located & others in network) to complement PV
 - Demand response (EV shifting, heat pumps) to modify net feeder load
 - Accelerate DER integration → Exploit synergies between PV deployment & flexible load growth

Device-specific DER constraints: BS & EV

- BS model for state of charge dynamics

$$SOC_i^{BS}(t+1) = (1 - \delta_{BS}^i)SOC_i^{BS}(t) - \frac{P_i^{BS}(t)\Delta t\eta_i^{BS}}{\bar{E}_i^{BS}}$$

$$\underline{P}_i^{BS} \leq P_i^{BS}(t) \leq \bar{P}_i^{BS}, \underline{SOC}_i^{BS} \leq SOC_i^{BS}(t) \leq \bar{SOC}_i^{BS}$$

$$SOC_i^{BS}(0) = SOC_i^{BS}(T)$$

$P_i^{BS,EV} > 0 \Rightarrow$ Discharging
 $P_i^{BS,EV} < 0 \Rightarrow$ Charging

- EV battery model is similar to above

- Additional restriction when EV is unavailable during specific time window (s) e.g. between 9am-5pm while at work, $P_i^{EV}(t) = 0 \forall t \in [t_1, t_2]$
- V1G mode (managed charging): While connected to the grid, EV is only allowed to charge, $P_i^{EV} \leq 0 \forall t$
- V2G mode (vehicle-to-grid): While EV is grid-connected, it is allowed to either charge or discharge (it can inject power back into grid), i.e. P_i^{EV} can be +ve or -ve

DER constraints: HP, solar, smart inverters

HP cooling mode: $(T_i^{out} > T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1 - \theta_i) (T_i^{out}(t) + \rho_i P_i^{HP}(t))$

$$\theta_i = e^{\frac{-\Delta t}{R_i^{th} C_i^{th}}} \approx 1 - \frac{\Delta t}{R_i^{th}, C_i^{th}}, \rho_i = R_i^{th} \eta_i$$

HP heating mode: $(T_i^{out} < T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1 - \theta_i) (T_i^{out}(t) - \rho_i P_i^{HP}(t))$

$$-P_{rated,i}^{HP} = \underline{P}_i^{HP} \leq P_i^{HP}(t) \leq 0, \underline{T}_i^{in} \leq T_i^{in}(t) \leq \overline{T}_i^{in}$$

$$\text{PV curtailment limit: } 0 \leq P_i^{PV}(t) \leq \alpha^{PV}(t) \overline{P}_i^{PV}$$

- Reactive power control for inverter-based resources (IBRs) = PV, BS, EV chargers
- Power factor control with smart inverters

$$Q_i^{IBR} = P_i^{IBR} \tan(\cos^{-1}(pf))$$

$$\underline{pf} \leq pf \leq \overline{pf} \text{ (e.g. } 0.8 \leq pf \leq 1)$$

$$-P_i^{IBR} \tan(\cos^{-1}(\underline{pf})) \leq Q_i^{IBR} \leq P_i^{IBR} \tan(\cos^{-1}(\underline{pf}))$$

Modeling BS & EV

Introduce additional variables to prevent simultaneous charging & discharging at any given time step

$$\begin{aligned} |P_i^{BS}| &= P_i^{BS,+} + P_i^{BS,-}, \quad P_i^{BS} = P_i^{BS,+} - P_i^{BS,-} \\ 0 \leq P_i^{BS,+} &\leq z_i P_i^{BS}, \quad 0 \leq P_i^{BS,-} \leq (1 - z_i) P_i^{BS} \\ z_i &\in \{0, 1\} \\ -|P_i^{BS}| \tan(\cos^{-1} \underline{pf}) &\leq Q_i^{BS} \leq |P_i^{BS}| \tan(\cos^{-1} \underline{pf}) \end{aligned}$$

Objective function for dynamic case

$$\min f^{OPF} =$$

minimize PV curtailment

$$\sum_{i \in bus} \sum_{t \in t_{H_p}} \beta_{pv} (P_i^{PV}(t) - \bar{P}_i^{PV} \alpha^{PV}(t))^2$$

minimize thermal line losses

$$\sum_{(i,j) \in \mathcal{E}} \sum_{t \in t_{H_p}} l_{ij} R_{ij}$$

maximize PV usage for BS/EV charging

$$+ \sum_{i \in bus} \beta_{coupl,bs} \alpha^{PV}(t) P_i^{BS}(t) + \beta_{coupl,ev} \alpha^{PV}(t) P_i^{EV}(t)$$

Prefer charging of EV & BS when PV output is available

minimize absolute thermal discomfort + HP temp tracking

$$+ \sum_{i \in bus} \sum_{t \in t_{H_p}} \beta_T (T_i^{in}(t) - T_i^*)^2$$

minimize HP and BS cycling

$$+ \sum_{i \in bus} \sum_{t \in t_{H_s}} \beta_{hp} (P_i^{HP}(t+1) - P_i^{HP}(t))^2 + \beta_{bs} (P_i^{BS}(t+1) - P_i^{BS}(t))^2$$

Avoid excessive cycling between ON/OFF or charge vs discharge modes → Extend battery & HP lifetime

minimize EV cycling and track desired SOC

$$+ \sum_{i \in bus} \sum_{t \in t_{H_s}} \beta_{ev1} (P_i^{EV}(t+1) - P_i^{EV}(t))^2 + \beta_{ev1} (SOC_i^{EV}(t^*) - SOC_i^{EV*})^2$$

Tracking objective to achieve desired SOC by specific time (e.g. 90% by 9am for work)

minimize cost of power import at PCC (at LMP rate)

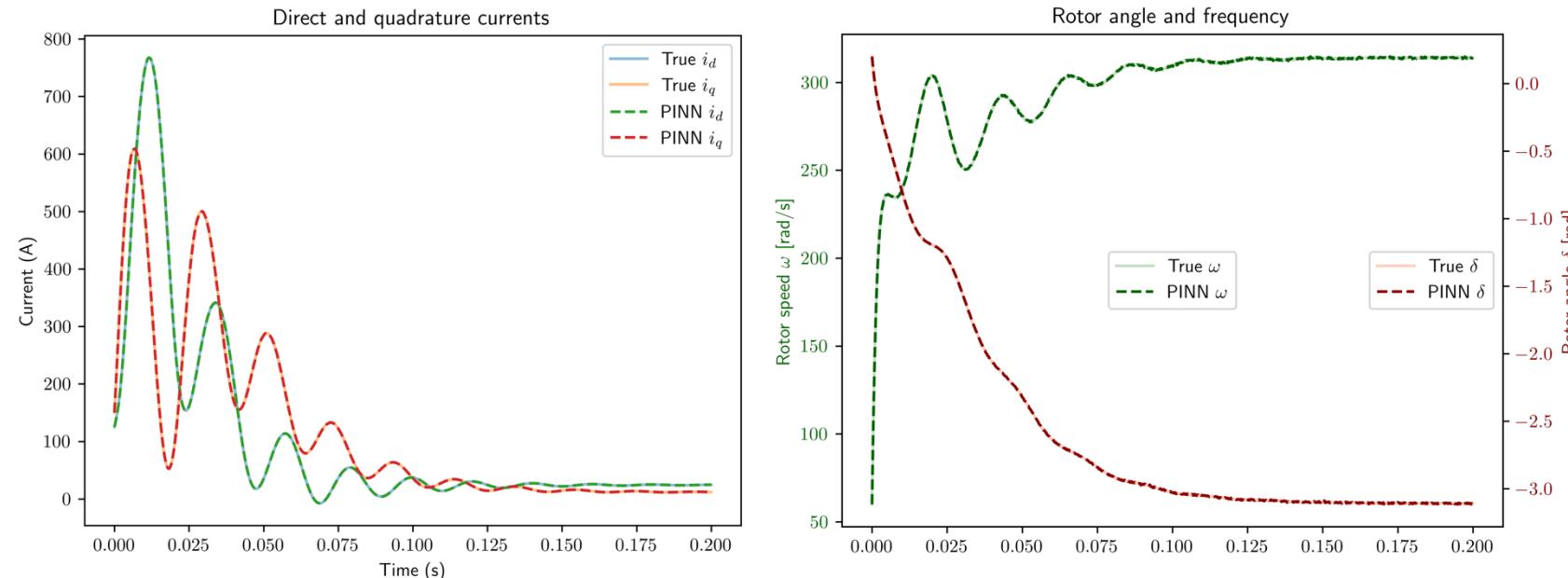
$$+ \sum_{t \in t_{H_p}} \lambda(t) P_{1,t}$$

Some policy recommendations

- FERC order 2222 enabled DERs to participate in wholesale electricity markets through aggregation
 - Significant delays in implementation & low participation rates
→ Need to continue pushing utilities and grid operators
- Utility regulatory reform & market redesign to align incentives
 - Performance-based or service-based rates
 - Current approach of setting rates based on recovering capital costs
→ Not sustainable for future DER-rich grid
 - Results in infrastructure overbuilding, inefficiencies, and higher costs for ratepayers
- Improved rate design will also encourage utilities to focus more on digital solutions & grid modernization solutions to increase hosting capacity
- Regulatory push for distribution-level retail markets to allow DERs to participate & provide services by using existing flexibility

Physics-informed neural networks (PINNs)

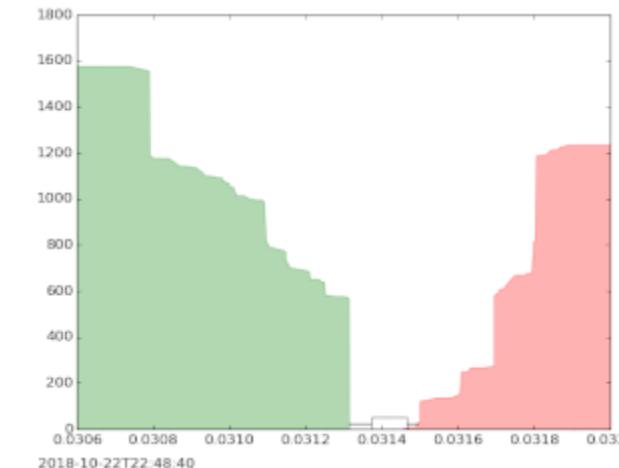
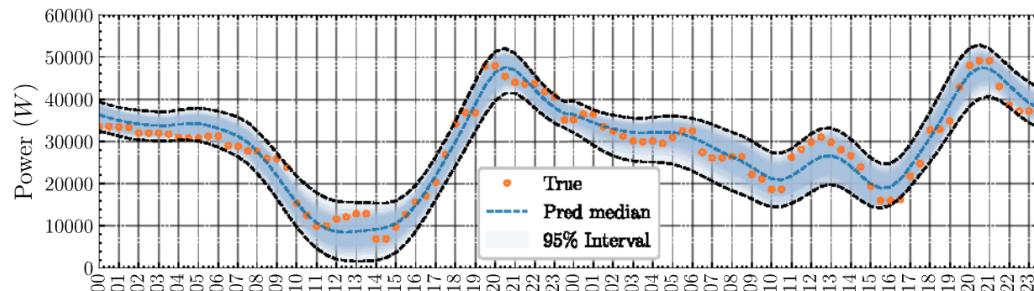
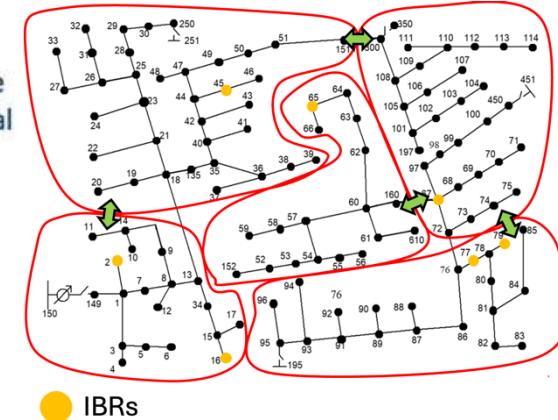
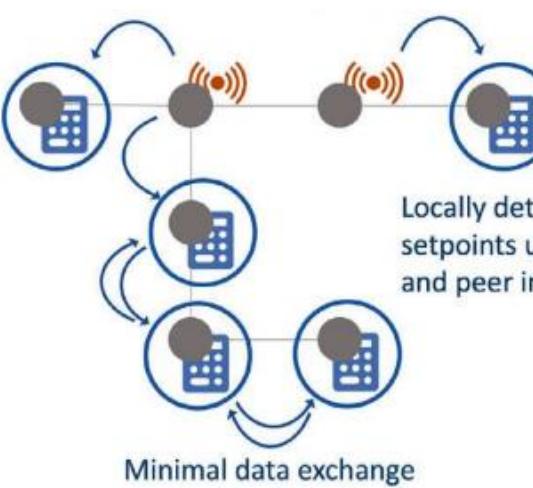
- Accelerate critical dynamic simulations & study grid stability
- 1st study to develop PINNs for **high-order** and **high-dimensional** ordinary differential equations in power systems
e.g. synchronous generators, inverters



Google[x]
pillar

Other projects

1. Circuit-aware clustered & distributed model predictive voltage control for inverter-based resources - *MIT Applied Energy Symposium 2024*
2. Improved probabilistic timeseries forecasting for DERs - *CIRED US 2024, CIRED EU 2024, NeurIPS Climate Change AI workshop 2022*
3. Blockchain to implement secure distributed optimization & transparent smart contracts for local electric markets - *IEEE Internet of Things journal (in prep.)*
4. Market-based coordination to increase grid hosting capacity



Distributed model predictive voltage control

- Primary control with passive grid circuit dynamics

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{i}(t) \\ \mathbf{V}(t) \end{bmatrix} = \begin{bmatrix} -Z & B^\top \\ -\bar{G}B & -\bar{G}Y_{\text{load}} \end{bmatrix} \begin{bmatrix} \mathbf{i}(t) \\ \mathbf{V}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -CK_q \Delta q(t) \end{bmatrix}$$

- Derive coupled cluster dynamics
- Define DMPC optimization problem
- Bring distributed optimization into standard separable form for solver

$$\min_{U_c(k) \in \mathcal{U}_c} \sum_{c \in \mathcal{C}} f_c(U_c(k))$$

$$\text{s.t. } \sum_{c \in \mathcal{C}} \mathcal{A}_c U_c(k) = \mathbf{d}$$

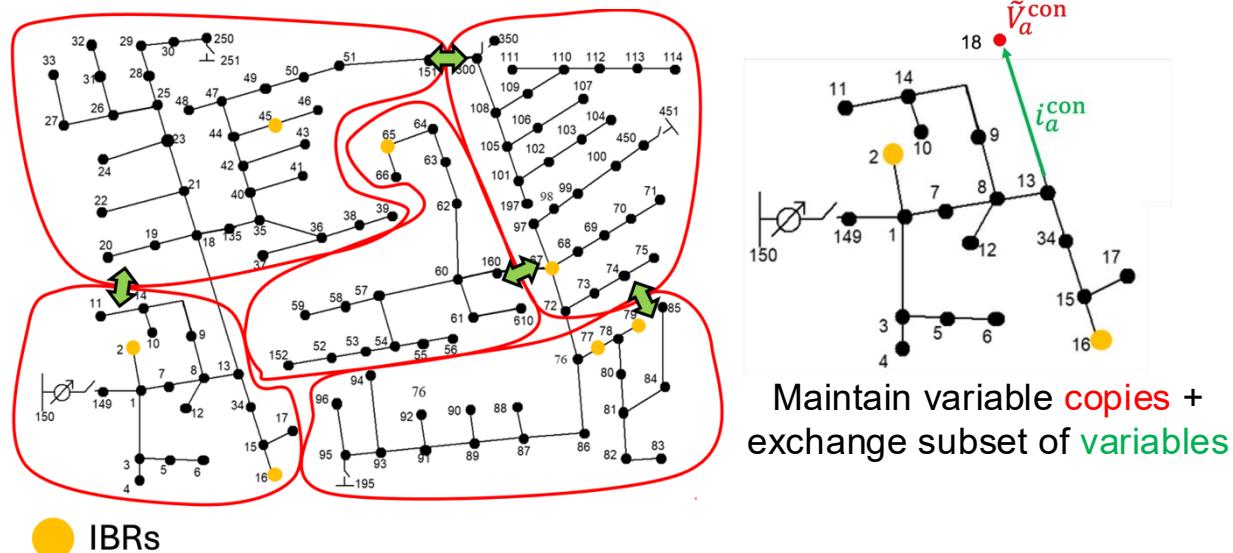
$$\mathbf{x}_c(\ell+1) = \mathcal{A}_c \mathbf{x}_c(\ell) + M_c \mathbf{u}_c(\ell) + \mathbf{c}_c \forall \ell, \forall c \in \mathcal{C},$$

$$\tilde{\mathbf{v}}_{c_i}^{\text{con}}(\ell) = \tilde{\mathbf{B}}_{c_j, c_i}^{\text{con}} \tilde{\mathbf{x}}_{c_j}(\ell), \forall (c_i, c_j) \in \mathcal{E}_c, \forall \ell,$$

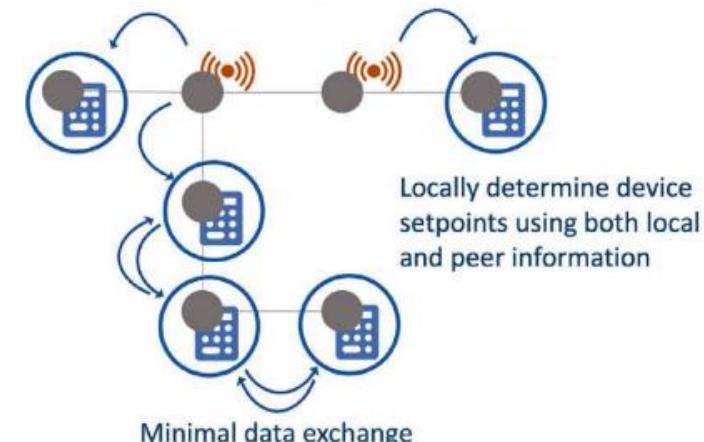
$$\tilde{\mathbf{v}}_{c_j}^{\text{con}}(\ell) = \tilde{\mathbf{B}}_{c_i, c_j}^{\text{con}} \tilde{\mathbf{x}}_{c_i}(\ell), \forall (c_i, c_j) \in \mathcal{E}_c, \forall \ell,$$

- Solve using algorithm based on alternating direction method of multipliers [1]

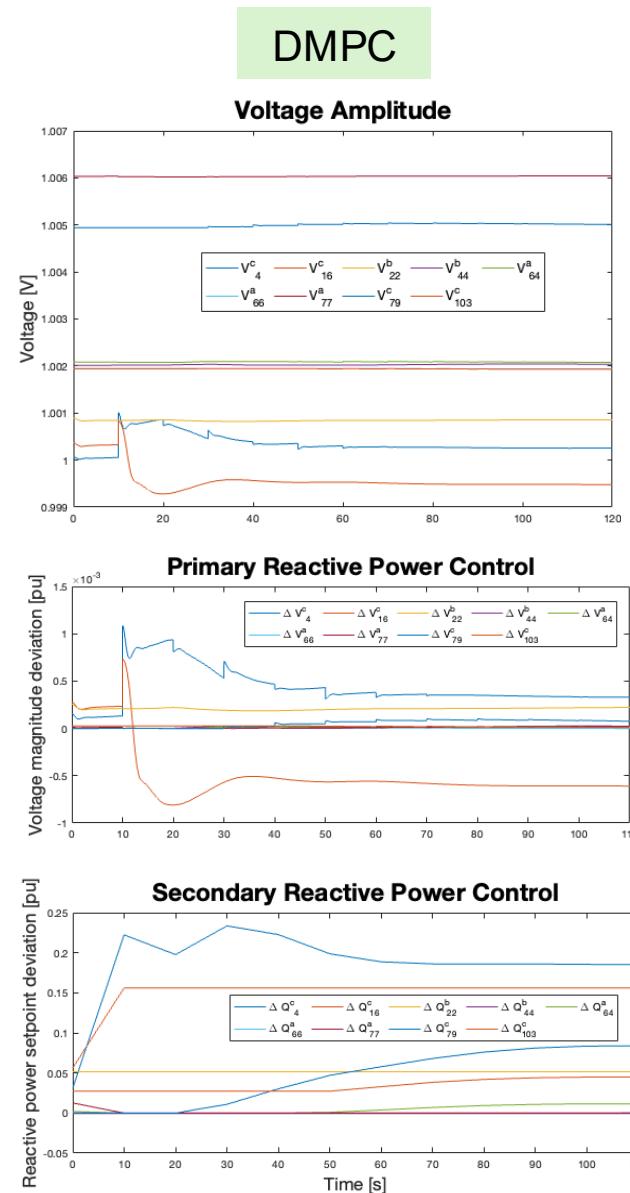
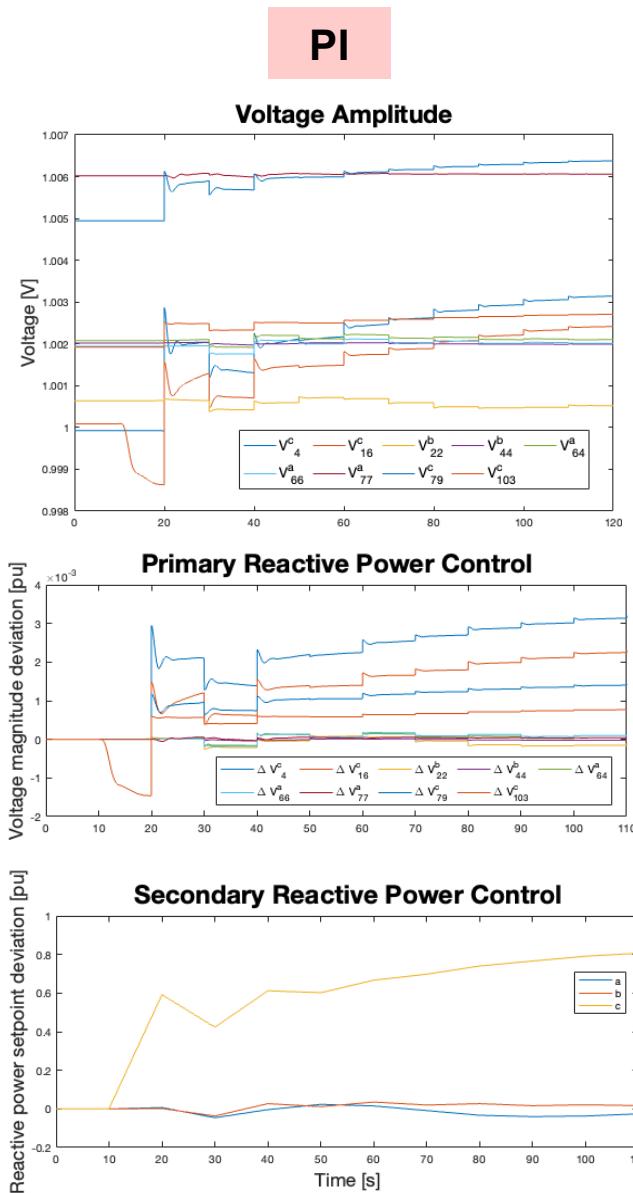
[1] A. Falsone, et al., "Tracking-ADMM for distributed constraint-coupled optimization," *Automatica*, Volume 117, 2020.
Hartmann et al., MITAB 2024; Srivastava et al., 2023



Cluster dynamics
 Consensus constraints
 for voltage copies



DMPC outperforms averaging PI control



- Simulate large load step disturbance
- Stable response without parameter tuning
- Reduces oscillations
- Relies more on on IBRs closer to load step
- More efficient dispatch → Lower losses

