Spatial Distribution of Access to Service: Theory and Evidence from Ridesharing

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Abstract

This paper studies access to services across geographical regions, using both theoretical and empirical analyses. We model and examine the effects of economies of density in ridesharing markets. Our model predicts that (i) economies of density skew access to rideshareing service away from less dense regions, (ii) the skew will be more pronounced for smaller platforms (i.e., "thinner markets"), and (iii) rideshare platforms do not find this skew efficient and thus use prices and wages to mitigate (but not eliminate) it. We show that these insights are robust to whether the source of economies of density is the supply-side or the demand-side. We then calibrate our model using ride-level Uber data from New York City. We devise an identification strategy based on relative flows of rides among regions which allows us to infer unobsrevable potential demand in different boroughs. We use the model to simulate counterfactual scenarios providing insights on platform optimal pricing with and without spatial price discrimination, the role of market thickness, the impact of prices/wages on access to rides, and the effects of minimum-wage regulations on access equity across regions.

Keywords: Ridesharing; Spatial Markets; Transportation; Economies of Density; Market Thickness

1 Introduction

Ridesharing markets are increasingly forming an important and critical part of the transportation networks of major metropolitan areas (Fortune Business Insights (2021)). In these markets, ridesharing firms act as two-sided platforms, intermediating between consumer (passenger) demand, and driver supply. Ridesharing markets are *spatial*. Thus, inequality in access to rides across geographic areas is an important challenge, that has drawn attention from regulators, lawmakers as

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well as consumer and community advocates (Jin et al. (2019), Diao et al. (2021)). Such markets are prone to spatial agglomeration of drivers in high-density regions. While such agglomeration could improve matching locally, it comes at the expense of other, sparser, regions. Crucially, it is challenging to examine whether and to what extent the platform would find such agglomeration efficient: on the one hand, the platform benefits from better local matching in areas with agglomeration; but on the other hand, drivers that agglomerate in denser regions exacerbate the sparsity of the areas they abandon, an externality that the platform internalizes. Finally, it is challenging to empirically measure the extent to which some regions are under-served relative to others, given that unfulfilled demand is unobservable.

This paper studies this issue of inequality in access to ridesharing services across regions within large metropolitan areas. We use a combination of closely connected theoretical and empirical analysis. Our theoretical model examines drivers' and passengers' decision making among a set of regions in a spatial market with a monopolist rideshare platform. Each region has an arrival rate of potential demand for rides that could end in the same region or a different one. Actual demand is a fraction of potential demand, depending on price (and wait time) in region i. A novel feature of our model is that it endows each region with a size rather than considering it a point. We model driver free entry, with each driver choosing whether and in which region to enter the market. Each driver makes this choice to maximize her revenue given other drivers' choices. Revenue in each region is positively related to the wage per ride in that region and negatively related to the "total wait time" each driver has to wait in the region to give a ride to a passenger. Total wait time consists of (i) "idle time," the time it takes for the driver to be assigned to a passenger requesting a ride, and (ii) "pickup time," the time it takes to arrive at the pickup location after being assigned to a passenger. More drivers operating in each region i means a higher expected idle time in i. This leads the drivers to geographically distribute themselves proportionally to the distribution of demand. On the other hand, more drivers in region i means a lower expected pickup time in i, incentivizing drivers to agglomerate. This agglomeration can happen either because drivers directly prefer shorter pickup times (which we define as Supply-Side Economies of Density, or supply-side EOD) or because demand positively responds to short pickup times and supply follows (which we define as Demand-Side Economies of Density, or demand-side EOD). The interplay between the balancing force through idle times and the agglomerating force through pickup times has a key role in our results. The platform decides the price per ride and driver wage per ride in each region i. Any passenger who needs a ride–i.e., each unit of *potential* demand–actually decides to demand a ride if the price is below her willingness to pay (and, under demand-side EOD, if driver arrival time is sufficiently short). Therefore, some proportion of potential demand is realized or fulfilled, based on the availability of driver supply. We define "access" to rides in any region i as the number of realized rides as a fraction of potential demand. We use this model to deliver a number of results on how the spatial distributions of access to rides is shaped in response to the incentives of the platform, drivers, and passengers.

Our theoretical results based on the above model are qualitatively the same in case of either supply-side EOD or demand-side EOD. First, we find that access is skewed in equilibrium in favor of regions with higher potential-demand density. The primary reason for access skew is higher pickup times in sparser regions. Second, we find that platform size plays a significant role in access skew, with smaller platforms obtaining more unequal access. The intuition is that larger platforms have more demand, and correspondingly more driver entry, which leads to lower pickup times across the board. This reduces the importance of pickup times relative to idle times, and leading to more equal access across regions. Our third result finds that a profit-maximizing platform would optimally use prices and wages to mitigate but not fully eliminate the access skew. The platform implements this by offering higher wages to drivers in sparser regions but does only partially passing that extra wage on to passengers. It is helpful to understand why: The platform, like the drivers and passengers, benefits from the decreased pickup times due to EOD, hence the incentive to not fully eliminate the access skew. However, these drivers could have had a larger enhancing impact on pickup times in sparser regions that they avoid. This is effectively an externality on other regions and, by extension, the platform. The platform, hence, stands to gain from mitigating "over-agglomeration" of drivers.

We next take the theoretical model to data from Uber, the largest Rideshare platform in New York City (NYC), from March to June 2019. We leverage ride-level data on rides (including pickup and drop-off location), driver wages and prices across the regions (boroughs) of NYC. The dataset is publicly available from NYC's Taxi and Limousine Commission (TLC), and is rich in frequency of observations, with the complete rides across the regions of the city available collected in a standardised way. We use this data to calibrate our model.

A key part of our calibration is an identification strategy that we develop for inferring the ratio between access to rides in given regions i and i'. This is critical because access (i.e., rides divided by potential demand) is unobservable due to potential demand being unobservable. Our strategy is based on an assumption that overall potential demand for rides from i to i' are equal to those from i' to i during a time period encompassing one or multiple days. This, in turn, is motivated by the assumption that for every trip, there is a "trip back" by the same person shortly after (otherwise, the trip itself is the "trip back" for one that must have happened shortly before). Combined with other supplementary assumptions which we will detail later in the paper, this strategy allows us to prove a powerful result: access ratio between any pair of regions i and i' is equal to the ratio of rides from i to i' to those in the opposite direction. We term this ratio the relative outflow between the pair of regions.

We formally prove that both models with supply-side EOD and demand-side EOD are identified from the data using variation in relative outflows, prices and wages across regions. First, we use our model assumptions (especially free entry/exit of drivers) to recover the number of drivers in each region as a function of the number of rides and the wage rate in the region. Then, we observe that the variation in access ratios between pairs of regions (which we measure using observed relative outflows) can be explained in our model either by variation in regional prices or by variation in density of drivers. Therefore, if there is adequate independent variation between prices and driver density across regions, the contribution of each factor to access is separately identified. The change in relative outflow as prices change across regions informs the price sensitivity, and the variation in the number of drivers and their wages across regions informs us about supply-side EOD or demand-side EOD. We also show that given the EOD parameters, the potential demand is identified by the variation in the number of drivers as well as wage and price levels.

We note that although our identification arguments require only cross-sectional variation across regions, the arguments could also be made based on temporal variation. Our data features both cross-sectional as well as temporal variation in the quantities of interest. Overall, our identification strategy contributes to the literature by noting that we can learn about unfulfilled and fulfilled demand, i.e. potential demand in a region. To do so, we leverage data not only on rides starting in that region, but also on rides *ending* there.[1] This approach allows us to avoid relying on supply-side moments for identification, which would require us to assume that the platform price and wage strategy are optimal.

We embed the above strategy within a formal estimation procedure with two steps. We first represent the number of rides for all origins and destinations as a function of model parameters using the structure of our model. Next, we match those model rides to observed rides (specifically, we match relative flows of rides for all pairs of regions) to recover model parameters. We then estimate the model not only for the extreme cases of supply-side EOD and demand-side EOD, but also for multiple intermediate cases where a fixed "portion" of EOD comes from the supply side and the rest from the demand side. In all of these estimates, we consistently find that the potential-demand density is highest in Manhattan and lowest in the Bronx, with the rank ordering being the same irrespective of the source of EOD. We find that across a wide range of relative strengths of these two sources of EOD, the results are qualitatively the same. The price elasticities obtained by our model are reasonable and in the same range as findings in the literature for the same market.

We use our calibrated model to simulate five counterfactual scenarios. First, we examine how a profit maximizing platform would set prices and wages. We find that the profit maximizing price is about 50% greater than the current price. These higher prices would lead to lower access across all regions. Second, we evaluate the impact of platform size (i.e., market thickness) on the outcomes, by scaling up potential demand across all regions. As predicted by our theory, we observe that as the size increases, driver entry increases and resulting access levels increase in all regions, while the skew in access is simultaneously reduced. To illustrate the magnitude, in one of our specifications, we find that access to rides in Manhattan under optimal pricing increases from 37.0% to 38.7% if Uber's potential demand grows from 80% of the size we estimate to 120% of the size we estimate. This growth in access translates to about 4.6% increase. The same numbers for Queens are 28.8% and 31.6% respectively, which translate to about a 9.7% increase in access. Our third counterfactual restricts the platform to using the same price and wage rates across regions (say, due a citywide regulation). We find that under such a counterfactual, geographical skew in access to rides gets

[1] In the main text of the paper, and then to a greater extent in the appendix, we provide further evidence as to why our identification strategy based on relative flows of rides is a useful one.

exacerbated. More precisely, the ratio between access levels in the borough with the lowest access (Queens) and that with the highest access (Manhattan) drops from about 80% to about 60% if the platform does not use spatially differentiated pricing to mitigate the inequity. This result points to the idea that imposing equality in actions taken by a firm (platform) might exacerbate outcomes (access skew) that policy makers might care about. The fourth counterfactual evaluates what level of region-specific wages/prices will result in equalized access across regions. We find that the platform would need to pay significantly higher wages (charge lower prices) in sparser regions relative to denser ones. Also, as the platform size increases, the wages/prices across the regions required to equalize access would converge. Finally, we examine the impact of minimum wage regulation that is common across regions. We find that such minimum wages might lead to lower access, because even though higher wages would result in more driver supply, the platform would raise prices beyond the level that was profit maximizing without the minimum wage constraint, which in turn would then harm access levels. Additionally, we find that access skew across regions would be lower in the presence of a minimum wage.

In summary, our paper contributes to the study of economies of density in rideshare in at least four important ways. First, our model endows each region with a non-trivial size which allows to capture the notion of pickup time in each area and how it varies with driver density. This, in turn, allows us to model economies of density both on the supply side and on the demand side. Second, our empirical strategy to recover access differences across regions based on relative flows of cross-region rides is powerful in that it helps infer unobservable access levels using only ride-level data. Third, we study (theoretically and empirically) not only driver behavior, but also that of the platform and the potential ways the incentives between the two entities might be (mis)aligned when it comes to economies of density. Finally, we extensively document (both theoretically and empirically) that the major implications of economies of density for rideshare markets–as summarized in our main results–are robust to what portion of economies of density arises from the supply side and what portion from the demand side.

There are a few aspects of our model that generalize beyond ridesharing to spatial markets more broadly. First is the research examining platform incentives. The same mis-alignment between platform and driver incentives that we document in ride share could exist in other spatial markets (between micro-suppliers and a social-planner/market-maker) when it comes to economies of density. Second, in other passenger-transportation markets, leveraging the concept of relative outflows across regions could be applied to infer unfulfilled demand. Third, our results point to the fact that imposing constraints on firm actions (e.g. prices and wages) to make them more equal across regions may backfire by making other outcomes (e.g. access) more unequal across regions. Given that the same notion could apply in other markets, policymakers should adopt a more nuanced role in evaluating whether and how platforms share an incentive to achieve equity goals before imposing regulations.

2 Literature Review

Our paper relates to multiple strands of the literature: (i) the recent and growing literature on the empirical analysis of geographical distribution of supply, and its possible distortion from that of demand, in spatial markets; (ii) the literature on transportation markets (in particular ridesharing); and (iii) the literature that studies the effects of market thickness in two-sided markets.

The empirical literature on the spatial match between supply and demand is new and small. To our knowledge, Buchholz (2018); Brancaccio et al. (2019c) are the only papers directly examining this issue, and papers such as Frechette et al. (2019); Brancaccio et al. (2019a,b) look at related problems. They extend the empirical techniques in the matching literature (see Petrongolo and Pissarides (2001) for a survey) in order to structurally infer the size of unobserved demand (e.g., passengers searching for rides) in different locations of a decentralized-matching market when only the size of supply (e.g., available drivers) and the number of demand-supply matches (e.g., realized rides) are observed. They accomplish this by inverting a matching function that gives the number of rides as a function of searches and vacancies. Our relative-outflows method is complementary. On the one hand, it requires the extra assumption that potential demand for rides from region i to region i' is the same as that for rides in the opposite direction. We justify this in our application by noting that almost all passengers have a home base that they need to return to, if the time period considered spans at least a day. But this assumption would clearly not hold if what is transported is goods rather than passengers. On the other hand, our method (i) requires data only on the number of rides rather than rides *and* vacant supply, search time, etc.; (ii) it applies generally to all passenger-transportation markets regardless of whether the matching system is centralized (e.g., rideshare) or decentralized (e.g., taxicabs);[2] and, finally, (iii) our approach detects skew of supply away from a given region i even if in response to short supply, passengers in i have learned to forego searching (which would make it look like demand is low).

The second strand of the literature to which our paper relates is the set of papers on the functioning of transportation (in particular rideshare) markets. This strand itself can be roughly divided into (at least) two categories. One category is the group of papers focusing on this market as it relates to labor economics.[3] The second category, to which our paper belongs, consists of papers focusing on evaluating the performance of these markets and on market design aspects. Some of those papers, although related to our work in many ways, focus on questions that are inherently not spatial (examples are Cohen et al. (2016); Nikzad (2018); Lian and van Ryzin (2019); Cachon et al.

[2]Another subset of the literature on spatial markets that this paper builds on is the study of location decisions, resulting in agglomeration. Papers such as Ellison and Glaeser (1997); Ahlfeldt et al. (2015); Datta and Sudhir (2011); Holmes (2011); Miyauchi (2018) examine agglomeration of firms or residents. We add to this literature by arguing, empirically and theoretically, that agglomeration is also present in transportation markets. In addition, our comparative static theory results, which characterize how the extent of agglomeration is impacted by different factors, may be applied beyond transportation systems.

[3]For instance, Chen et al. (2017) examine how much workers benefit from the schedule flexibility offered by ridesharing. Cramer and Krueger (2016) study the extent to which ridesharing, compared to the traditional taxicab system, reduces the portion of time drivers are working but not driving a passenger. Chen and Sheldon (2016) examine the reaction of labor supply to the introduction of ridesharing. Buchholz et al. (2018) estimate an optimal stopping point model to study the labor supply in the taxi-cab industry.

(2017); Guda and Subramanian (2019); Asadpour et al. (2019)). Others study questions that are related to the spatial nature of the market (such as Castillo et al. (2017); Frechette et al. (2019)), but they do not examine the spatial distribution of supply and potential mismatches with demand. Many of the papers that do study geographical supply-demand (im)balance in transportation (such as Banerjee et al. (2018); Afèche et al. (2018); Besbes et al. (2018)) focus on the short-run, intra-day, aspects. Some other papers (such as Buchholz (2018); Lagos (2000, 2003); Bimpikis et al. (2019); Shapiro (2018); Lam and Liu (2017); Garg and Nazerzadeh (2019); Ata et al. (2019); He et al. (2020)), however, examine such spatial markets from a long-term perspective. Our paper is complementary to this literature in that it provides a detailed theoretical and empirical investigation of economies of density (arising from both supply and demand sides) and market thickness, while abstracting away from some of the phenomena considered in these papers.

It is worth noting that a large part of this literature has focused on the ways in which ride-share platforms improve upon the traditional taxi system, in particular due to their flexible pricing and superior matching algorithms (Cramer and Krueger (2016); Buchholz (2018); Frechette et al. (2019); Cohen et al. (2016); Shapiro (2018); Castillo et al. (2017); Besbes et al. (2018); Lam and Liu (2017) among others). We add to this literature by arguing that even within the world of rideshare which utilizes central matching, the quality of supply-demand match may be influenced by platform size.

Also close in spirit to our paper in this strand of the literature is Rosaia (2023), which studies the rideshare market in NYC, compiling comprehensive data from both Uber and Lyft, and using exogenous shocks to prices to identify a model of dynamic decision making by drivers under platform competition. Compared to Rosaia (2023), we have a more stylized model that is static and abstracts from competition. We also have a different focus: the consequences of economies of density and market thickness for spatial skew in access to service in favor of denser regions. To this end, we deliver theoretical results on those spatial implications, examine a larger geographical market that encompasses not only dense but also sparser regions and use relative flows of rides to empirically measure the magnitude of access differences, and finally, study counterfactuals that quantify our main theory results and assess relevant policy and business-strategy decisions.[4]

The third set of papers to which we relate is a large, mostly theoretical, literature on the impact of market thickness on the functioning of two-sided platforms in general (such as Akbarpour et al. (2017); Ashlagi et al. (2019)) and transportation markets in particular (such as Frechette et al. (2019); Nikzad (2018)). This literature, to our knowledge, has not examined how the spatial distribution of supply–and its (mis)alignment with that of potential demand–responds to a change in market thickness. Our paper focuses on this, both empirically and theoretically.

[4]To see how the geographical span of the data we use is more amenable to capture economies of density, see Appendix I.

3 Theoretical Model

We develop a model to study the spatial distribution of access to ridesharing services in presence of economies of density. We aim to use to model to understand (i) whether access will be skewed in favor of some regions over others in equilibrium, (ii) whether the skew will be exacerbated or moderated as platform grows in size, (iii) whether the platform would like to use prices and wages as levers to influence the skew of access, and if so, the direction of this influence, and (iv) whether the answers to the previous questions are sensitive to whether economies of density arises from demand or supply side of the market. We abstract away from aspects of the ridesharing market that are not directly relevant to answering the above questions.

We develop a three-stage model where a monopolistic platform optimally sets prices and wages, drivers decides whether to operate and which region to operate, and passengers decide whether to take the ride (see appendix Appendix E for some alternative model setups). We start with the setting where there are no inter-region rides across regions and introduce two variants of the model. First, we model the drivers' equilibrium spatial distribution where the economies of density comes only from the supply-side (referred to as the "supply-side model" or the "model with supply-side EOD"). That is, drivers benefit from a larger number of drivers on the platform, as it reduces the pickup time and thus increases drivers' revenue. Second, we model the equilibrium spatial distribution with only demand-side economies of density, where we assume that ride demand from passengers decreases with the pickup time and increases with the number of drivers (referred to as the "demand-side model" or the "model with demand-side EOD").[5]

We prove the the following set of results hold for both models: (i) Under the platform optimal strategy, the equilibrium spatial distribution of rides is skewed towards regions with higher demand densities. (ii) The skew is more intensified for smaller ride-sharing platforms. (iii) The platform's optimal strategy would involve charging lower margins for rides in regions with lower demand densities. Then we extend the results to the setting with inter-region rides across regions in the supply-side model and the demand-side model separately.

The section is structured as follows. We start with a setting where we assume all rides take place only within regions. Section 3.1 presents the general setup of the model that applies to both the model with supply-side EOD and the model with demand-side EOD. Section 3.2 describes the model with supply-side EOD and characterizes the equilibrium spatial distribution of drivers. Section 3.3 describes the model with demand-side EOD and characterizes the equilibrium spatial distribution of drivers. Section 3.4 extends the two models in Section 3.2 and Section 3.3 to the setting with inter-region rides.

[5]Nevertheless, in both the supply-side model and the demand-side model, the agglomeration of drivers in a region increases the idle time and "turns away" drivers if the region becomes too dense. Therefore, under the assumption of a pool of infinite drivers and free entry of drivers, the equilibrium spatial distribution of drivers is reached when the two forces, the pickup time and the idle time, are balanced.

3.1 Model Setup

We model a market with regions $i \in \{1, ..., I\}$ and a monopolist ridesharing platform serving them. The regions (which, depending on the application, one could think of as neighborhoods, boroughs, etc.) are modeled as circumferences of circles, a la Salop. The time that it takes a driver to travel a full circumference of circle i is denoted t'_i. Let t' denote the vector $(t'_1, ..., t'_I)$. For ease of exposition, we assume t' is measured in hours. Intuitively, t'_i captures the geographical size of the region. For any given numbers of drivers and riders, the drivers and the riders distribute more sparsely in the region as t'_i becomes larger.

In each region, the platform decides price per ride p_i and wage per ride c_i.[6] Let p denote the vector of $(p_1, ..., p_I)$ and c denote the vector of $(c_1, ..., c_I)$. A distribution of drivers is denoted by vector $n = (n_1, ..., n_I)$, where n_i denotes the number of drivers in region i. To ease the analysis, we assume n_i are real numbers as opposed to integers. Let ω_i denote the pickup time of each ride in region i. Passengers arrive at a rate λ_i per unit of time (i.e. λ_i passengers arrive per hour). Demand arrival rate is a function of price per ride p_i and can also be a function of ω_i, depending on our assumptions of the source of economies of density. We discuss this in further details in the subsequent sections. For now, we assume that the demand arrival rate takes the following form:

$$\lambda_i(p_i, \omega_i) = \bar{\lambda}_i f(p_i, \omega_i)$$

$\bar{\lambda}_i$ is the "potential demand" in region i, which captures the total demand of transportation in region i per hour and thus is the maximum number of realized rides achievable in region i for the ridesharing platform. Equivalently, this is the volume of demand when both p_i and ω_i are zero. Function $f(\cdot)$ –which is assumed uniform across regions– captures the fraction of $\bar{\lambda}_i$ that would be willing to pay p_i for a ride given they observe pickup time ω_i, which is the time it takes a driver, after being assigned to a ride request, to drive and arrive at the passenger's pickup location. $f(\cdot)$ takes the values between 0 and 1. Given t_i and $\bar{\lambda}_i$, we define the density of potential demand for region i as $\frac{\bar{\lambda}_i}{t'_i}$. Without loss of generality, we assume that the density of potential demand decreasing in region index: $\forall i < j : \frac{\bar{\lambda}_i}{t'_i} \geq \frac{\bar{\lambda}_j}{t'_j}$. Also, $\bar{\lambda}$ represents the vector $(\bar{\lambda}_1, ..., \bar{\lambda}_I)$.

Given t'_i, λ_i and n_i, there is a wait time for drivers before they can provide a ride to a passenger. This total wait time is denoted by $W_i(n_i)$ and is a function of n_i, the number of drivers present in the region (this notation suppresses the implicit dependence of W_i on prices). Total wait time in each region has two components: idle time and pickup time. Idle time is the time it takes a driver to get assigned to a ride request by a passenger. Idle time in region i is increasing in n_i. That is, the more drivers in region i, the longer it takes for each of them to get assigned to a passenger. Pickup time ω_i in region i is decreasing in n_i. This is because the more drivers in region i, the more densely the region is populated with them. Therefore, each driver becomes less likely to be asked to pick up a passenger who is far away in the region.[7] We assume that each arriving passenger's

[6] We assume that the rides are homogeneous.

[7] We assume that the centralized platform assigns each passenger to the closest driver. See Appendix B for more

location is uniformly distributed on the circumference of the circle. Then the pickup time ω_i as a function of n_i can be written as

$$\omega_i(n_i; t'_i) = \frac{n_i}{4t'_i}$$

The total wait time for each region i is given by:

$$\underbrace{W_i(n_i)}_{\text{Total Wait Time}} = \underbrace{\frac{n_i}{\lambda_i}}_{\text{Idle Time}} + \underbrace{\frac{t'_i}{4n_i}}_{\text{Pickup Time}} = \left(\frac{n_i}{\lambda_i} + \frac{t_i}{n_i}\right) \tag{1}$$

where $t_i = \frac{t'_i}{4}$. In the appendix, we provide a micro-foundation for this functional form. From this point on, we refer to t_i (instead of t'_i) as the size of region i.

We model a game of three stages. In the first stage, the platform optimally decides prices per ride p_i and wages per ride c_i for each region i to maximize the total profit across all regions. In the second stage, drivers from an infinitely large pool simultaneously decide whether to enter the market and, if so, which region to operate in. In the third stage, both price p_i and pickup time ω_i are available to the riders. Then actual demand λ_i is realized.

We solve the equilibrium using backward induction starting from the third stage. In the third stage, given $\bar{\lambda}_i$, p_i and ω_i, the actual demand per hour is $\lambda_i = \bar{\lambda} f(p_i, \omega_i)$. In the second stage, each driver seeks to maximize her expected hourly revenue. The hourly revenue for a driver in each region i equals the wage per ride in that region multiplied by the frequency of rides given by each driver in the region. That is, the revenue will be $\frac{c_i}{W_i(n_i)}$.[8] Let $\bar{c}$ denote the exogenously given reservation value, i.e., the hourly revenue that each driver can make by leaving the market and taking "the outside option".

Now we formally define the partial equilibrium distribution of drivers (given p_i and c_i) as follows:

Definition 1. Under "market primitives" $(\bar{\lambda}, t, \bar{c})$ and given the platform's strategy (p, c), a distribution of drivers $n^* = (n_1^*, ..., n_I^*)$ among the I regions is called an equilibrium if no small mass of drivers[9] currently operating in region i can strictly profit from changing their strategy and operating instead in region j, where $i, j \in I$. That is, $\exists \delta > 0$ $s.t.$ $\forall i, j \in I$, $\forall \delta' \in [0, \delta]$ we have:[10]

$$\frac{c_i}{W_i(n_i^*; p_i)} \geq \frac{c_j}{W(n_j^* + \delta'; p_j)}$$

Also, we call n^* an "all-regions" equilibrium allocation if it is an equilibrium and if $n_i^* > 0$ for all i.

Directly follow the definition above, we have the following statement that gives the necesasry

details.

[8] Note that this formulation abstracts away from the time it takes to drive a passenger to the dropoff location. This assumption simplifies some of our analysis and we do not expect the results to be qualitatively sensitive to it. In the empirical calibration, however, some of the magnitudes we recover might be sensitive to this assumption.

[9] The reason why we focus on "small mass" deviations is that our continuous-mass model of drivers is an approximation for a large population. In this environment, allowing for "large-mass" deviations from any given strategy should be interpreted as drivers *coordinating* a deviation, which we do not allow.

[10] If the drivers are currently operating in region i, then the left hand side of the inequality is replaced by $\bar{c}$. If the drivers are considering switching to outside option, then the right hand side of the inequality is replaced by $\bar{c}$.

condition for the partial equilibrium of driver distribution:

Lemma 1. *Suppose function $W_i(\cdot,\cdot)$ is continuous. For any given (p, c), if $n = (n_1^*, ..., n_I^*)$ with $n_i^* > 0\ \forall i$ is the equilibrium distribution of drivers, then*

$$\frac{c_i}{W_i(p_i, n_i^*)} = \bar{c},\ \forall i \tag{2}$$

Given the drivers' response, in the first stage of the game, the platform optimally chooses (p, c) given market primitives $(\bar{\lambda}, t, \bar{c})$ to maximize the platform profit, which is the sum of the regional profit. The total number of rides per hour in region i, denoted $r_i(n_i)$, is given by the total number of drivers in that region divided by the time each driver has to wait before giving a ride: $r_i(n_i) \equiv \frac{n_i}{W_i(n_i)}$. If the distribution of drivers satisfy eq. (2), then $r_i(n_i^*) = \frac{n_i^*(p_i,c_i)}{c_i/\bar{c}}$ in equilibrium. The platform's profit per hour, which is the object the platform seeks to maximize by choosing regional prices and wages, is given by:

$$\pi(p, c) = \Sigma_{i=1}^{I}(p_i - c_i)r_i(n_i(p_i, c_i))$$

Then the equilibrium of the game is given by $(p^*, c^*, n^*, \lambda^*)$, where p^* and c^* are the prices and wages that maximize platform profit, n^* is the equilibrium driver distribution given (p^*, c^*), and λ^* is the actual demand corresponding to prices p^* and pickup times $\omega_i^*(n^*)$.

Next, we introduce the notion of "access" to rides in region i by $A_i(n_i)$ and define it as the fraction of the potential demand $\bar{\lambda}_i$ that leads to rides. That is:

$$A_i(n_i) \equiv \frac{r_i(n_i)}{\bar{\lambda}_i} \tag{3}$$

3.2 Theory Model with Supply-Side Economies of Density

In this section, we assume that the market involves supply-side economies of density (but does not involve demand-side economies of density). That is, a driver in a given region i can benefit from the presence of other drivers in that region because with higher driver density, she is less likely to be asked to pick up a passenger far away in the region. On the other hand, to abstract away the effect from the demand-side EOD, we assume that passengers' arrival rate does not depend on the number of drivers (and hence on pickup time). Specifically, given $\bar{\lambda}_i$, the demand arrival rate of region i is only a function of p_i, and we assume that $f(p_i) = 1 - \alpha p_i$ so that [11]

$$\lambda_i = \bar{\lambda}_i(1 - \alpha p_i)$$

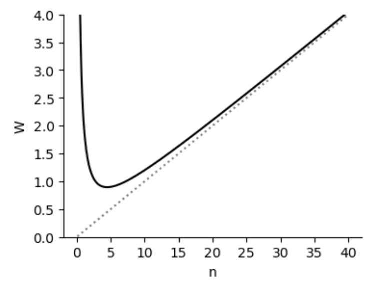
Accordingly, the wait time is given by:

$$W_i(n_i) = \frac{n_i}{\bar{\lambda}(1 - \alpha p_i)} + \frac{t_i}{n_i}, \tag{4}$$

[11] The linear demand model has been used in literature. See e.g., Bimpikis et al. (2019).

where the first component is the idle time and the second component is the pickup time. The wait time curve given by eq. (4) is illustrated in Fig. 1. As can be seen there, $W_i(\cdot)$ is initially decreasing in n_i because the effect of pickup time is dominant. When n_i is large enough, pickup time becomes less important and $W_i(\cdot)$ becomes increasing in n_i due to the effect of idle time.

Figure 1: Wait time as a function of the number of present drivers, given by eq. (4). This is illustrated in a region with $\lambda_i(p_i) = 10$ and $t_i = 2$. The dashed line is $\frac{n}{\lambda}$, depicting how long the wait time would be if pickup time were zero.



In the propositions below, we characterize the equilibrium spatial distribution of supply, and how this distribution changes in response to a changed market thickness.

Lemma 2. *Suppose prices and wages are both positive and flexible and that market primitives are* $(\bar{\lambda}, t, \alpha, \bar{c})$. *Then an equilibrium* (p^*, c^*, n^*) *exists and* n^* *is unique. Also* c_i^* *and* p_i^* *for any* i *with* $n_i^* > 0$ *are unique.*

Proposition 1. *Suppose prices and wages are both flexible and that market primitives are* $(\bar{\lambda}, t, \alpha, \bar{c})$. *Also suppose* (c^*, p^*, n^*) *is an equilibrium. Then for any two regions* i, j *with* $n_i^* \neq 0 \neq n_j^*$, *we have:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow A_i(n_i^*) \geq A_j(n_j^*)$$

where the latter comparison holds with equality only if the first one does. Moreover, regions that are not supplied are those with lowest demand densities:

$$\exists \mu > 0 : \; s.t. \forall i : \; \frac{\bar{\lambda}_i}{t_i} < \mu \Leftrightarrow n_i^* = 0$$

The proof is provided in the appendix, but here we develop an informal intuition for the above proposition in two steps. First, we argue that if all prices and wages were spatially homogeneous, equilibrium driver allocation (fixing those prices and wages) would give higher access to rides in denser regions relative to sparser ones. Next, we argue that the platform does not have the incentive to use prices and wages to fully eliminate such access skew.

For an informal intuition of the first argument in the previous paragraph, consider regions i and j with $\frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j}$ and observe that with spatially homogeneous prices and wages, the total wait time for drivers should be the same between the two regions. This directly implies that the distribution of drivers should be skewed toward i, that is: $\frac{n_i}{n_j} > \frac{\bar{\lambda}_i}{\bar{\lambda}_j}$. To see why, note that if we instead had $\frac{n_i}{n_j} = \frac{\bar{\lambda}_i}{\bar{\lambda}_j}$, then idle times would be equal between the two regions but pickup time in region i would be shorter than that in j, incentivizing drivers to relocate from j to i. But given total driver wait times are equal between the regions, $\frac{n_i}{n_j} > \frac{\bar{\lambda}_i}{\bar{\lambda}_j}$ is equivalent to $\frac{r_i}{r_j} > \frac{\bar{\lambda}_i}{\bar{\lambda}_j}$ which by definition means $A_i > A_j$

The second step in intuiting the proposition is to argue informally that the platform would not benefit from fully eliminating access gap between regions i and j when it can use prices and wages as levers. The simple argument here is that drivers spending long times en route for pickups hurts not only drivers themselves but also the profitability of the platform. As a result, it should not be surprising that the platform's optimal decision, at least in part, agrees with what what drivers would do under spatially uniform prices.

Next, we turn to examining the impact of platform size (i.e. market thickness) on the skew of access to drivers. Before that, we give a formal definition of market thickening.

Definition 2. Consider a market with the primitive of potential demand as $\bar{\lambda}$. We call a market with potential demand $\gamma\bar{\lambda}$ with $\gamma > 1$ and all other primitives remaining the same a "thickening" of the market with potential demand $\bar{\lambda}$.

Intuitively, market thickening increases the potential demand in each region proportionally so that the ratio of potential demand is *preserved* between any two regions. Nevertheless, as our next results shows, making a market thicker will mitigate the skew of supply distribution.

Proposition 2. *Suppose prices and wages are both flexible and that market primitives are* $(\bar{\lambda}, t, \alpha, \bar{c})$*. Also suppose* (c^*, p^*, n^*) *is an equilibrium. Consider a thickening of the market from* $(\bar{\lambda}, t, \alpha, \bar{c})$ *to* $(\gamma\bar{\lambda}, t, \alpha, \bar{c})$ *where* $\gamma > 1$*. Then, the following are true:*

1. *There exists an equilibrium* $(c^{*'}, p^{*'}, n^{*'})$ *under the new primitives* $(\gamma\bar{\lambda}, t, \alpha, \bar{c})$*, where* $n^{*'}$ *is unique. Also* $c_i^{*'}$ *and* $p_i^{*'}$ *for any* i *with* $n_i^{*'} > 0$ *are unique.*

2. *For any* i*:* $n_i^* > 0 \Rightarrow n_i^{*'} > 0$.

3. *For any* i, j*:*
$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow \frac{A_j(n_j^*)}{A_i(n_i^*)} \leq \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} \leq 1$$

4. *There will be equitable access to rides as the market gets sufficiently thick:*
$$\forall i, j : \lim_{\gamma \to \infty} \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} = 1$$

The underlying intuition for the result is that as the market gets thicker (i.e., the platform gets larger,) all regions get denser with drivers. As such, the importance of pickup times relative to idle times decreases in drivers' decision making, leading to a supply distribution that is more balanced with demand.

We now delve deeper into the incentives of the platform and how it aligns with those of the drivers. In interpreting Proposition 1, we argued that the platform, similarly to drivers, does suffer from long pickup times and is, hence, not willing to use prices and wages to fully eliminate access skew among regions of different densities. The next natural question is: does the platform have the incentive to at least mitigate the skew? The following proposition sheds some light on this.

Proposition 3. *Suppose prices and wages are both flexible and that market primitives are* $(\bar{\lambda}, t, \alpha, \bar{c})$*. Also suppose* (c^*, p^*, n^*) *is an equilibrium. For any regions* i, j *with* $n_i^* \neq 0 \neq n_j^*$*, we have:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow \begin{cases} p_i^* \leq p_j^* \\ c_i^* \leq c_j^* \\ p_i^* - c_i^* \geq p_j^* - c_j^* \end{cases}$$

where the latter three comparisons hold with equality only if the first one does.

As this proposition shows, the platform pockets lower margins ($p_i - c_i$) for its services in less dense regions. That is, the platform optimally gives higher wages to drivers in sparser regions but does not pass the entire wage increase on to passengers in the form of higher prices. This is consistent with an incentive to mitigate the access skew across regions. The intuition for why the platform would like to mitigate the access skew has to do with *externalities*. Each driver's decision to operate in a denser region i as opposed to a sparser region j leaves region j further sparse, thereby increasing the pickup time in j and negatively affecting incentives of other drivers to choose j. While the drivers operating in region i makes the region even denser, the positive marginal impact of region i being denser does not compensate the loss from region j being sparser. This externality on other drivers is internalized by the platform through profits, but not directly internalized by the driver. This creates a wedge between platform and driver incentives and leads the platform to use price and wage levers to mitigate such externalities by drivers.

There are a few interesting observations. First, the platform's optimal strategy is more complex than what might seem based on the intuition in the above paragraph. The complexity arises from the fact that the platform simultaneously determines optimal prices and optimal wages. If the platform were only optimizing over wages (under fixed and spatially uniform prices), one could show that it would indeed be optimal to give higher wages in sparser regions, as Proposition 3 predicts. However, if the platform were only optimizing prices under fixed and spatially uniform wages, it would be optimal to charge lower prices in sparser regions, unlike what Proposition 3 states. These results (both of which are formally stated and proved in the appendix) are also in line with mitigating access skew by the platform, given that they both help boost demand/supply in sparser regions. But they go against each other when both prices and wages can be decided by the platform: offering higher wages in sparser regions put some upward pressure on prices. Likewise, lower prices in sparser regions put some downward pressure on wages. One question, which Proposition 3 above helps answer, is "which lever, prices or wages, does the platform use to mitigate the access skew when it can use both prices and wages?" Proposition 3 suggests that in sparser regions, the platform optimally builds economies of density by providing higher wages.

Despite that the platform also charges higher prices, which reduces the economies of density, overall the effect of wage dominates.

In the empirical section, we connect the margin results in Proposition 3 to access. That is, in our counterfactual simulations, we show that access among regions gets more skewed if we only allow the platform spatially uniform prices and wages (hence also spatially uniform margins).

With our analysis of the model with supply-side EOD completed, we now turn to a version of the model where economies of density arises only from the demand side; and we study if the above insights are robust to such a change in the model.

3.3 Theory Model with Demand-Side Economies of Density

The pickup time for a driver is the same thing as total wait time for a passenger riding with that driver, and is decreasing in the number of drivers. As a result, it is natural to model the riders' sensitivity to pickup time as demand-side EOD and examine the consequences of it. This would be a model for demand-side economies of density: higher pickup times in sparser regions lowers demand in those regions, making the market sparser and further increasing local pickup times, and so forth.

In this section, we examine a model of demand-side EOD *only*; that is, we study a model in which pickup times do not directly impact driver behavior or revenue but impact passengers. To incorporate the latter assumption, we need to make demand λ_i not only a function of prices, but also pickup times:

$$\lambda_i(p_i, n_i) = \bar{\lambda}(1 - \alpha p_i)(1 - \beta \frac{t_i}{n_i}),$$

where $\alpha > 0$ measures the price sensitivity and $\beta > 0$ measures the sensitivity to pickup time, thereby capturing the magnitude of demand-side economies of density.

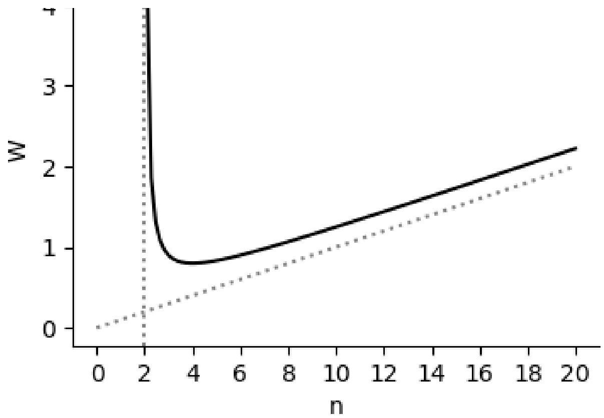
To capture the idea pickup times do not directly impact drivers, we need to assume two things. First, region sizes t_i (and hence pickup times) are so small that driver wait times is approximated by idle time only. Second, in spite of all t_i values being vanishingly small, the sensitivity to pickup times β is sufficiently large so that $\beta \times t_i$ values are non-negligible and, hence, passengers still care about pickup times. In this environment, driver wait times are given by:

$$W_i(n_i) = \frac{n_i}{\lambda_i(p_i, n_i)} = \frac{n_i}{\bar{\lambda}_i(1 - \alpha p_i)(1 - \beta \frac{t_i}{n_i})}, \tag{5}$$

The wait time curve is illustrated in Fig. 2. Similar to the model with supply-side EOD, $W_i(\cdot)$ initially decreases in n_i as the effect of pickup time dominates. As n_i is large enough, $W_i(\cdot)$ becomes increasing in n_i due to the effect of idle time.

Similar to the supply-side model, the driver' hourly payoff is given by $\frac{c_i}{W_i(n_i)}$. The total number

Figure 2: Wait time as a function of the number of present drivers, given by eq. (5). This is illustrated in a region with $\bar{\lambda}_i(1-\alpha p_i) = 10$ and $\beta t_i = 2$.



rides per hour in region i is given by $r_i \equiv \frac{n_i}{W_i(n_i)}$. Access to rides in region i is defined as

$$A_i(n_i) \equiv \frac{r_i(n_i)}{\bar{\lambda}_i} \tag{6}$$

The following proposition characterizes the equilibrium spatial distribution of supply and how this distribution changes in response to a changed market thickness in the setting of demand-side economies of density.

Proposition 4. *Suppose the market primitives are $(\bar{\lambda}, t, \alpha, \beta, \bar{c})$. The prices and wages are both flexible. Under the model with demand-side economies of density, all propositions with the model with supply-side economies of density (Lemma 2, Proposition 1, Proposition 2 and Proposition 3) still hold.*[12]

The key lesson from this proposition is that our main insights are robust to the source of economies of density. In addition to being interesting in its own right, this lesson will play a key role in our empirical analysis: there, we will argue that demand- and supply-side economies of density are not separately identifiable from each other (at least with our data). Hence, robustness of our main insights to the source of economies of density becomes crucial when it comes to practical recommendations to firms or public policy agents.

Next, we take another step to make the model more suitable for empirical analysis: we include inter-region rides.

3.4 Inter-Region Rides

An abstraction in our model so far was that it assumes rides take place only within regions. In this subsection, we first define the equilibrium notion with inter-region rides. Then we show that

[12] One small exception is that in the demand-side model, the first comparison in Proposition 3 (i.e., $p_i^* \geq p_j^*$ always holds with equality.)

Proposition 1 through Proposition 4 hold when we allow some of the rides to be cross-regional.

Let $\bar{\lambda}_{ij}$ denote the potential demand for rides from i to j. By construction, we have $\bar{\lambda}_i = \Sigma_j \bar{\lambda}_{ij}$. Define realized demand $\lambda_{ij}(p_i, n_i)$ in a similar way: $\lambda_{ij}(p_i, n_i) = \bar{\lambda}_{ij} f(p_i, n_i)$. Define r_{ij} as the realized number of hourly rides from i to j. Assume that the proportion of unfulfilled demand does not depend on the destination. That is: $\forall j : \frac{r_{ij}}{\lambda_{ij}} = \frac{r_i}{\lambda_i} = A_i$. This assumption has also been made in Bimpikis et al. (2019).

In this environment, one can think of n_i, the number of drivers in region i, as a stock variable and consider flows in and out of it. Realized r_{ij} values lead to "cross-regional flows" of drivers. By internal, we mean flows of drivers who are already in the market. In addition to that, we now also allow for a (negative or positive) net flow of drivers ρ_i from each region i to the outside option. In this new environment, we need a new notion of equilibrium which is given by Definition 3.

Definition 3. Suppose the "market primitives" $(\bar{\lambda}, t, \bar{c})$ and platform strategy (p, c) are given. Let ρ_i denote the external (positive or negative) net flow of drivers out of each region i. $n^* = (n_1^*, ..., n_I^*)$ denote the distribution of drivers among the I regions. The pair of vectors (n^*, ρ^*) is an equilibrium if it satisfies the following two conditions:

1. **Steady State**: Net flow out of each n_i^* is zero: $\forall i : \ \rho_i^* + \sum_j r_{ij}^* - \sum_j r_{ji}^* = 0$
2. **Optimality**: no small perturbation in rates ρ^* is able to strictly improve driver revenue in any region. Formally, fixing some elapsed time $\bar{T} > 0$, and perturbation $\Delta\rho \neq 0$, the following will hold for $\forall i \in I, \forall T \in (0, \bar{T})$:

$$\frac{c_i}{W_i(n_i^*)} \geq \frac{c_i}{W(n_i^* + T \times \Delta\rho)}$$

Also, we call n^* an "all-regions" equilibrium allocation if (n^*, ρ^*) it is an equilibrium and if $n_i^* > 0$ for all i.

We now show that the results provided in the previous section (i.e., without inter-region rides) all hold with inter-region rides as well. Proposition 5 formalizes this.

Proposition 5. *Suppose market primitives $(\bar{\lambda}, t, \bar{c})$ and platform strategy (c, p) are given. Then for any (n^*, ρ^*) with $n^* >> 0$, the pair (n^*, ρ^*) is a driver equilibrium according to Definition 3 if and only if n^* is a driver equilibrium in the no-inter-region-ride version of the problem.*

The proof of this proposition is straightforward and is provided in the appendix. This result shows that as long as we focus on all-region equilibria, all of our results from Section 3.2 and Section 3.3 should hold. The intuition is simple. If there are too many incoming rides into a region i with overall low payoff, that will lead to many drivers opting to exit the market and pursue an outside option upon dropping a passenger in i. Similarly, attractive payoff in a region i with low inflow of rides will garner local driver entry.

Note that the purpose of Proposition 5 is *not* the provision of conceptually new insights about economies of density (the proposition only claims that the old results hold). The purpose is, rather, to connect our theory to the forthcoming empirical analysis in which we will use relative magnitudes of inter-region rides for the purpose of identifying the strength of economies of density and measuring access skew. The empirical portion of the paper, which we turn to now, will shed more light on how this measurement takes place.

4 Data and Summary Statistics

For the empirical analysis, our main source of data is the ride-level data on Uber in New York City from March to June in 2019. Uber is the largest ridesharing platform whose total number of rides given in NYC per month was about 14.4M in June 2019. The regions are defined as the four boroughs of New York City: Bronx, Brooklyn, Manhattan and Queens[13]. For each ride, we observe the exact date, time, and location both for the pickup and the dropoff. Besides, we observe the wage paid to the driver and the price paid by the passenger.

There are a number of reasons why we chose NYC from March to June 2019 as the setting for our empirical analysis. First, it provides data on both pickup and dropoff locations. Also, starting February 2019, additional information on ride-level prices wages became available. Both of these features play key roles in our analysis. Second, NYC is one of the densest cities in the world. Therefore, if frictions from low density impact the spatial distribution of supply in NYC, it is reasonable to conclude they must be relevant in other markets too.

We aggregate the ride-level data at borough-day level. For each borough i and day d, we compute the average price p_{id} per ride and the average wage c_{id} per ride across all rides picked up from borough i on day d. Fig. 3 shows daily average prices and wages by borough over the course of our data. Besides, we compute the average number of rides picked up from the region per hour, denoted as r_{id}. Table 1 lists the 25th, 50th and 75th percentile of the variables for each borough across days. The table shows considerable variations in all three variables across regions and days. The reservation level of wage, $\bar{c}$, is \$17.22, which is the independent contractor equivalent of an employee's \$15 minimum wage with paid time off, as estimated by Parrott et al. (2019). We set $\bar{c}$ to uniform across boroughs and across days. Additionally, we leverage data on the borough area (in square miles) [14] and then take the square root of it. We denote the square root of the area for each borough as s_i.[15]

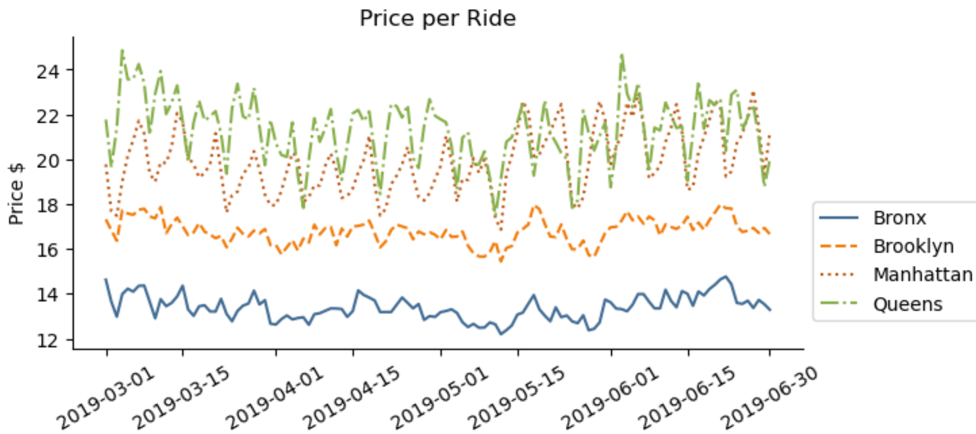
[13] We exclude rides either picked up or dropped off in Staten Island because the rides between Staten Island and other boroughs are scarce on some of the days within our data period. This leads to extreme outliers in the measures relative outflows and biases the estimates. Also, we exclude shared rides, since the data set does not provide separate wage and price data for each co-rider.

[14] https://www.census.gov/quickfacts/fact/table/newyorkcountynewyork,richmondcountynewyork,kingscountynewyork,queenscountynewyork,bronxcountynewyork,newyorkcitynewyork/LND110220

[15] The reason why we think of the square root of the borough area as the size is the following simple hypothetical: if we take a borough and double its size in each dimension (thereby quadrupling its area), the distance between two randomly picked points in the region doubles (and does not quadruple). The notion of measuring the region size in length as opposed to area is also implemented in the micro-foundation for our model which is provided in Appendix B.

Figure 3

(a) Variation of Price per Ride across Days and Boroughs



(b) Variation of Wage per Ride across Days and Boroughs

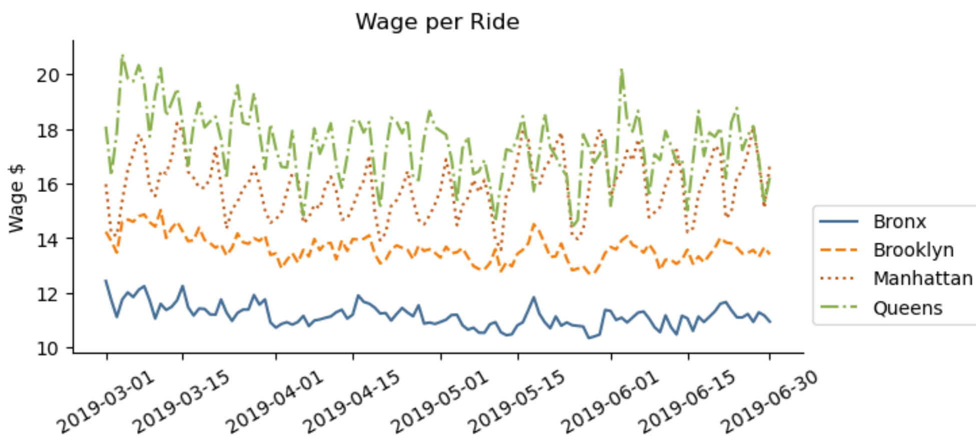
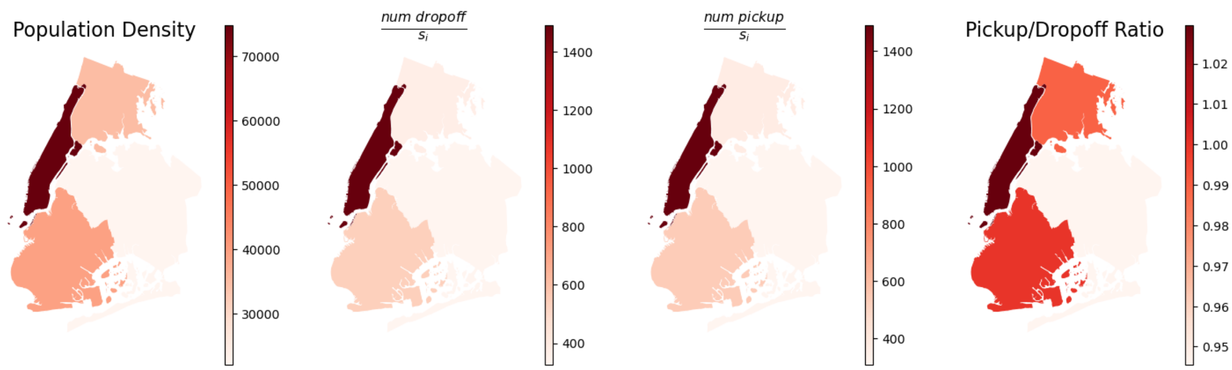


Table 1: 25th, 50th and 75th quartile of Number of Rides, Price, Wage across Days of Each Borough

Borough	**Num Rides** (r)			**Price** (p)			**Wage** (c)		
	q25	median	q75	q25	median	q75	q25	median	q75
Bronx	2181	2334	2654	13.01	13.34	13.71	10.92	11.14	11.39
Brooklyn	3853	4334	5012	16.46	16.82	17.09	13.32	13.59	13.90
Manhattan	6559	7265	7944	18.91	19.78	21.05	15.10	15.92	16.60
Queens	2934	3146	3504	20.31	21.43	22.25	16.63	17.61	18.23

Before getting to the empirical model, we show some data patterns that sheds light on how our theory results are connected to the empirical analysis. Fig. 4 shows suggestive evidence that access is skewed to denser regions. The first plot from left shows the population density (people per square miles) by borough, and the second plot shows the average number of dropoffs per hour normalized by region size s_i as defined before. Both measures can be interpreted as loose proxies of demand for rides. Perhaps unsurprisingly, and as one can see in the figure, the ordering among the four boroughs is the same across the three measures in the left three panels: Manhattan, Brooklyn, The Bronx, Queens. The last panel of Fig. 4 shows the pickup/drop-off ratio for each borough. If every passenger that used Uber to leave a borough also used Uber to return, all these ratios would equal 1. Interestingly, however, these ratios do not equal 1, and the four boroughs are ranked in the exact same order along this dimension as they are along the other three. We interpret this as a sign that access to rides is easier in denser regions, which is in line with what our theory predicted.

Figure 4



Based on the intuition provided by Fig. 4 we now construct a measure that will prove directly useful in our empirical anlayisis. From the inter-region rides r_{ijd}, the average number of rides starting in borough i and ending in borough j per hour on a given day d, we construct the "relative outflow" between i and j from the data as follows:

$$RO_{ijd} = \frac{r_{ijd}}{r_{jid}} \tag{7}$$

Table 2 shows all such pair-wise relative outflows from our data, averaged across days. Interestingly, one can see that for any two boroughs i, j where i is denser than j in the sense of one of the three left panels of Fig. 4, we have $RO_{ij} > 1$. Again, we interpret this as a sign that rides are more accessible in those denser regions, consistent with our theory. Next section further formalizes this idea, clearly sets out the assumptions underneath it, and develops it into an identification/estimation strategy.

Table 2: Average Relative Outflows RO_{ij} across Days

	region j			
	Bronx	Brooklyn	Manhattan	Queens
region i				
Bronx	N/A	0.93	0.93	1.05
Brooklyn	1.07	N/A	0.91	1.14
Manhattan	1.07	1.1	N/A	1.25
Queens	0.96	0.87	0.8	N/A

5 Empirical Model

Our theory model provides an understanding of platform, driver and passenger behavior under profit maximizing assumption for platforms and drivers. We now seek to take the model to data in order to obtain parameter estimates corresponding to model primitives and to run counterfactual analysis to provide policy recommendations.[16] Section 5.1 describes the setting of the empirical model. Section 5.2 presents the empirical model with supply-side EOD. Section 5.3 presents the empirical model with demand-side EOD. Section 5.4 introduces a comprehensive model that incorporates both supply-side and demand-side EOD. Before describing the model, we provide a table of notation in Table 3.

5.1 Connecting the model to data and the role of inter-region rides

This section sets up the empirical calibration problem. It also provides a proposition that will prove useful in finding the solution to the problem.

Start by noting that each driver makes $\frac{1}{W_{id}}$ trips per unit of time. Thus, the number of rides is given by: $r_{id} = \frac{n_{id}}{W_{id}}$. Free entry of drivers ensures that drivers wage is equivalent to the outside option in equilibrium, implying $W_{id} = \frac{c_{id}}{\bar{c}}$. These relationships imply that the number of drivers can be directly estimated on the data from the number of trips, driver wage, and the outside option:

$$n_{id} = \frac{r_{id}}{\frac{c_{id}}{\bar{c}}} \tag{8}$$

Let $\mathcal{D}$ denote the set of observable data and the variables that can be derived from the data as described above: $\mathcal{D} = \{p_{id}, c_{id}, r_{id}, n_{id}\}_{i \in I, d \in D} \cup \{r_{ijd}\}_{i,j \in I, d \in D} \cup \{\bar{c}\}$. The objective of our empirical exercise will be to recover using $\mathcal{D}$ the following set of parameters: $\{\bar{\lambda}_{id}\}_{i \in I, d \in D}, \{t_i\}_{i \in I}, \alpha, \beta$.

In recovering the above parameters of interest, we leverage additional parameterization as well as additional results provided by the structure of our model. Starting by region sizes t_i, we assume $\forall i : t_i = a \times s_i$ where s_i is the square root of borough i's area (hence observable) and a is a parameter to be estimated. This parameter could be thought of as the expected pickup in any region if the region had one driver per unit of size. We assume a is homogeneous across regions.[17] This approach reduces the estimation burden for sizes t_i to only one scalar.

On the front of unobservable potential demand $\bar{\lambda}_{id}$, we leverage an assumption that cross regional demand is balanced: $\forall\ i, j \in I$, $\bar{\lambda}_{ij} = \bar{\lambda}_{ji}$. The motivation for this assumption is that given our time periods are daily, then for every trip there is a "trip back" shortly after. We now show that this assumption, combined with the rest of the structure of our model, creates a tight connection between the unobservable potential demand and observable ride counts.

[16] Note that by taking our theory model to the data, we are effectively imposing the assumptions it entailed. In particular, we impose the assumptions that the outside option payoff $\bar{c}$ is homogeneous across all drivers and the price coefficient α is homogeneous among all passengers.

[17] In reality, a could be heterogeneous due to different regional extents of traffic congestion, etc. We abstract from those in this paper. Note that this abstraction likely understates economies of density, given that denser regions tend to have higher traffic.

Table 3: Table of Notations

Symbol	**Category**		**Meaning**
	Theory Model	Empirical Model	
i	Index	Index	Region(Borough) index
j	Index	Index	Region(Borough) index
d	NA	Index	Day index
t_i	Market Primitive	Model Parameter	Size of region i
a	Market Primitive	Model Parameter	Ratio of t_i to s_i
α	Market Primitive	Model Parameter	Passenger's price sensitivity
β	Market Primitive	Model Parameter	Passenger's sensitivity to pickup time
$\bar{\lambda}_i$, $\bar{\lambda}_{id}$	Market Primitive	Model Parameter	Potential demand of rides per hour of region i (on day d)
s_i	NA	Data	Square root of area of borough i
$\bar{c}$	Market Primitive	Data	Reservation level of wage
p_i, p_{id}	Decision Variable	Data/Decision Variable	Price per ride of region i (on day d)
c_i, c_{id}	Decision Variable	Data/Decision Variable	Wage per ride of region i (on day d)
n_i, n_{id}	Outcome Variable	Data/Outcome Variable	Number of drivers in region i (on day d)
W_i, W_{id}	Outcome Variable	Outcome Variable	Wait time (pickup time plus idle time) per ride in region i (on day d)
r_i, r_{id}	Outcome Variable	Data/Outcome Variable	Number of rides per hour originating from region i (on day d) (destination could be region i or other regions)
r_{ijd}	NA	Data/Outcome Variable	Number of rides picked up from region i and dropped off in region j per hour on day d
RO_{ijd}	NA	Data/Outcome Variable	Relative outflow between region i and region j on day d
$\mathcal{D}$	NA	Data	Set of observable data and the variables can be derived directly from data
A_i, A_{id}	Outcome Variable	Outcome Variable	Access of region i (on day d)

Proposition 6. *Let I denote the set of regions served by the platform. (n, ρ) is a steady-state (not necessarily equilibrium) driver allocation according to Definition 3, under $(c, p, \bar{\lambda}, t)$. Suppose potential demand for rides are "balanced": $\forall\ i, j \in I$, $\bar{\lambda}_{ij} = \bar{\lambda}_{ji}$. Also suppose the proportion of unfulfilled demand does not depend on destination: $\forall\ i, j \in I$, $\forall i, j \in I$, $\frac{r_{ij}}{\bar{\lambda}_{ij}} = \frac{r_i}{\bar{\lambda}_i} = A_i$.*[18] *Then we have:*

$$\forall i, j \in I, \frac{\frac{r_i}{\bar{\lambda}_i}}{\frac{r_j}{\bar{\lambda}_j}} \equiv \frac{A_i}{A_j} = \frac{r_{ij}}{r_{ji}} \equiv RO_{ij}$$

This result shows that the access ratio between two regions is equal to the relative outflow. To

[18] A similar proportionality assumption can be found in Bimpikis et al. (2019). See equation (2) on page 748 and subsequent explanation.

interpret the result, if region i has a lower access than region j, that is, the supply is more skewed to region j rather than region i, then there are more inter-region rides from j to i than those from i to j following our assumptions. This illustrates how observable relative flows RO_{ij} help infer the skew among unobservable access levels A_i and A_j, which in turn helps calibrate the magnitude of economies of density. We carry out this calibration exercise for both the supply-side and the demand-side models.

Note that even though the assumptions made in Proposition 6 are not trivial, the result provides some meaningful advantages. First, the data requirement is mild: only the numbers of rides rather than vacant cars, search behavior, etc. Second, this approach can help identify spatial supply-demand mismatch (i.e., skew in access) in passenger transportation markets regardless of whether the matching technology is centralized or decentralized. The measure of potential demand it helps infer (as we do later in the calibration procedure) includes not just the set of passengers who searched for rides by showing up on their app but failed to obtain a ride, but also those who skipped searching for rides in the first place due to anticipating that one would be difficult to find.[19]

It is also worth noting that to the extent the assumption $\forall\, i, j \in I$, $\bar{\lambda}_{ij} = \bar{\lambda}_{ji}$ on unobservable potential demand may be violated in reality, it is likely violated in a direction that would make our results stronger. To illustrate, a dense and central region like Manhattan is expected to have more alternative transportation options than does Queens (e.g., more available taxicabs, more accessible public transport). This would imply $\bar{\lambda}_{ij} < \bar{\lambda}_{ji}$ where i and j denote Manhattan and Queens respectively. This inequality, combined with the observation that $RO_{ij} > 1$, in turn implies that access ratio $\frac{A_i}{A_j}$ in reality could be even larger (further away from 1) than what we estimate using the assumption $\bar{\lambda}_{ij} = \bar{\lambda}_{ji}$. Additionally, in Appendix F.2, we provide multiple "reduced form" tests for economies of density and the role of market thickness under weaker assumptions.

With this result at hand, we next turn to empirically calibrating our model.

5.2 Empirical Model with Supply-Side Economies of Density

In this section, we adapt the theory model with supply-side economies of density to the empirical setting. Next, we describe the procedure to estimate the model parameters with the data. Finally, we show the identification of the model parameters.

5.2.1 Model

In contrast to the theory section of the paper, here we do not (need to) assume that the platform optimally sets prices and wages in the empirical model. Instead, our focus is on estimating the model parameters from the driver behavior. It is indeed necessary to avoid optimality assumptions

[19] To illustrate, suppose passengers consistently use rideshare to go to region i but never open their apps to find a ride back out of region i because they have learned that supply is scarce in that region. Even hard-to-obtain data on app sessions cannot detect this phenomenon and would, hence, understate low access to rides in region i. But a method based on relative flows of rides could.

on platform prcing decisions if we are to make some recommendations to the platform in our counterfactual analysis.

Recall from the theory section 3.2, for each region i and each day d, given the market primitives $(\bar{\lambda}_{id}, t_i, \bar{c})$ and (p_{id}, c_{id}) chosen by the platform, the equilibrium number of drivers is given by

$$\frac{c_{id}}{W_{id}(n_{id})} = \bar{c}$$

Following the theoretical model, the wait time is given by $W_{id} = \frac{n_{id}}{\bar{\lambda}_{id}(1-\alpha p_{id})} + \frac{t_i}{n_{id}}$.[20]

Also recall that we operationalize the region size parameter t as $t_i = as_i$, where s_i is the square root of the region (borough) size and thus is observable. Thus, given n_{id} generated following eq. (8) and platform choices p_{id} and c_{id}, in the driver free-entry equilibrium, we have the wait time obtained equates the drivers ridesharing wage to the outside option. This leads the following estimating equation:

$$\frac{n_{id}}{\bar{\lambda}_{id}(1-\alpha p_{id})} + \frac{as_i}{n_{id}} = \frac{c_{id}}{\bar{c}} \tag{9}$$

The parameters to be estimated in the supply-side model are, therefore, $(\alpha, a, \{\bar{\lambda}_{id}\}_{i\in I, d\in D})$.

5.2.2 Calibration

Our calibration proceeds in two steps. First, for any candidate pair (α, a), we impose the structure of our model alongside observable data in order to recover the potential demand across each region i and on each day d $(\bar{\lambda}_{id})$. We then use these recovered potential demand values to simulate relative outflows between pairs of regions. These simulated relative outflows are by construction a function of the pair (α, a) that we start with. We match these simulated model predictions to observed relative outflows from data, minimizing the sum of squared distance. This yields estimations $(\hat{\alpha}, \hat{a})$.

Formally: our steps are as follows:

Step 1 We obtain the number of drivers from the data following eq. (8). Using the number of drivers, eq. (9) allows us to express $\bar{\lambda}_{id}$ in terms of data and parameters (a, α) as follows:

$$\bar{\lambda}_{id}(a, \alpha; \mathcal{D}) = \frac{n_{id}}{(1-\alpha p_{id})(\frac{c_{id}}{\bar{c}} - \frac{as_i}{n_{id}})}, \tag{10}$$

Step 2 Given the observables and following step 1, we express the relative outflow given by the empirical model between each pair of regions i, j on each day d as a function of parameters (a, α):

$$RO_{ijd}^{model} = \frac{A_{id}}{A_{jd}} = \frac{r_i/\bar{\lambda}_{id}(a, \alpha; \mathcal{D})}{r_j/\bar{\lambda}_{jd}(a, \alpha; \mathcal{D})},$$

where the first equality is given by Proposition 6. We estimate (a, α) by matching the moments of RO_{ij} for every pair of regions i, j by taking the average across $d \in D$ between the model

[20] We assume that the trip time is 0 as in our theory model.

prediction and the data. Specifically, for each pair of regions i, j, denote $d_{ij}(a, \alpha; \mathcal{D}) = |\frac{1}{|D|}\sum_d RO^{model}_{ijd}(a, \alpha; \mathcal{D}) - \frac{1}{|D|}\sum_d RO^{data}_{ijd}(a, \alpha; \mathcal{D})|$ and let $e(\alpha, a; \mathcal{D})$ be the vector consisting of d_{ij}. We minimize

$$\min_{\alpha, a} d(\alpha, a; \mathcal{D})' d(\alpha, a; \mathcal{D}), \tag{11}$$

The estimates of $\bar{\lambda}_{id}$ are obtained from $\bar{\lambda}_{id}(a, \alpha, \mathcal{D})$ accordingly.

5.2.3 Identification

We informally discuss the identification here, and relegate the formal statement and proof of the identification to Appendix D.2. The model is identified jointly by the variation in the data we use and the assumptions our model imposes on the relationships between different quantities of interest. Below, we lay out the intuition on how this takes place.

First note that one can obtain the number of drivers from the data following eq. (8). Using the number of drivers, in eq. (9) allows us to express $\bar{\lambda}_{id}$ in terms of data and parameters (a, α). Next, observe that each $\bar{\lambda}_{id}$ can be written as an explicit function of data and a, α, β as in eq. (10). Therefore, the identification problem reduces to identify α, and a from the RO moments. We consider the relative outflow across regions for a single day d. For notational brevity, we drop the day subscript d. The model relative outflow can be written as

$$RO^{model}_{ij} = \frac{A_i}{A_j} = \frac{r_i/\bar{\lambda}_i}{r_j/\bar{\lambda}_j} = \frac{\frac{n_i}{c_i}/\bar{\lambda}_i}{\frac{n_j}{c_j}/\bar{\lambda}_j} = \frac{(1-\alpha p_i)(1 - a\frac{s_i \bar{c}}{n_i c_i})}{(1-\alpha p_j)(1 - a\frac{s_j \bar{c}}{n_j c_j})} \tag{12}$$

where the first equality holds following Proposition 6 and the second equality holds given the definition of access. The third equality holds given the assumption of driver equilibrium distribution. The fourth equality is obtained if we substitute $\bar{\lambda}_i$ and $\bar{\lambda}_j$ with a, α and observable data $\mathcal{D}$ following eq. (10). Note that in this equation, everything is data except for a and α.

The formal proof of how matching the moments in eq. (11) identifies both a and α is relegated to Appendix D.2. To see the raw intuition, first let us assume $\alpha = 0$. In this case, roughly, a is identified by the variation in relative outflows: the larger the magnitude of a, the higher the imbalance of flows between denser and less dense regions. [21] Price sensitivity α would be identified in a similar fashion if we assumed $a = 0$; the higher the magnitude of α, the larger the imbalance of flows between more and less expensive regions. Now, note that if there is sufficient independence between regional price variation and regional density variation, both a and α are identified.[22]

One additional observation about our identification is noteworthy. In principle, the price variation across region is prone to endogeneity, which might bias the results. One possible solution to

[21] More specifically, RO_{ij} being close to 1 is consistent with low values of a. As a increases, RO_{ij} would need to diverge from 1. For example, if $\frac{s_i \bar{c}}{n_i c_i} > \frac{s_j \bar{c}}{n_j c_j}$, then RO_{ij} is less than 1 and it decreases as a increases.

[22] Broadly speaking, sufficient independence requires that the comparison of p between two regions cannot inform how $\frac{s\bar{c}}{nc}$ compares between the two regions, and vice versa. Specifically, there exists a pair of regions where both p and $\frac{s\bar{c}}{nc}$ are high for one of the regions; and there also exists another pair of regions where one region has higher p and another region has higher $\frac{s\bar{c}}{nc}$. See the proof in the appendix for more details.

this problem would be to calibrate the parameter α in a way that would lead to the a similar price elasticity of demand to what is found in other research that leverages exogenous variation for the same market (such as Rosaia 2020), instead of directly recovering it from our data. We did not do so, however, given that the α we found indeed implies price elasticity levels that are close to Rosaia 2020 (See Table 5 for the results.).[23]

5.3 Empirical Model with Demand-Side Economies of Density

In this section, we adapt the theory model with demand-side economies of density to the empirical setting. We also describe the estimation procedure and identification. The overall strategy is analogous to the model with supply-side EOD. As such, we skip some of the details.

5.3.1 Model

Similar to the empirical model with supply-side economies of density, we do not assume that the platform optimally sets prices and wages, and we focus on estimating the model parameters from the driver behavior. Following the theoretical model with demand-side economies of density, the wait time is given by $W_{id} = \frac{n_{id}}{\bar{\lambda}_{id}(1-\alpha p_{id})(1-\beta' \frac{t_i}{n_{id}})}$. Recall from our discussion in the theory section that in order to model demand-side-*only* EOD, we need to assume that a is vanishingly small but β' is so large as to make $\beta \equiv \beta' \times a$ of non-trivial magnitude. Thus, in equilibrium, we will have:

$$\frac{c_{id}}{W_{id}(n_{id})} = \bar{c} \Longleftrightarrow \frac{n_{id}}{\bar{\lambda}_{id}(1-\alpha p_{id})(1-\frac{\beta s_i}{n_{id}})} = \frac{c_{id}}{\bar{c}} \tag{13}$$

The parameter β can be interpreted as the reduced-form aggregation of (i) the model parameter of pickup time as a function of region size and the number of drivers (ii) the sensitivity of ride demand to pickup time. Therefore, β captures the demand-side EOD.

In this model, the parameters to be estimated are $(\alpha, \beta, \bar{\lambda})$.

5.3.2 Calibration

We adopt a similar approach to the calibration of the supply-side model. Specifically, we estimate the model parameters following the two steps below:

Step 1 Following eq. (13), We express $\bar{\lambda}_{id}$ in terms of data and parameters (α, β) as follows:

$$\bar{\lambda}_{id}(\alpha, \beta; \mathcal{D}) = \frac{n_{id}}{(1-\alpha p_{id})(\frac{c_{id}}{\bar{c}})(1-\frac{\beta s_i}{n_{id}})} \tag{14}$$

[23] There is another possible concern about our price variation: the price measure we use for our analysis is price per ride, which we directly read off of the data and average out across rides in each region. One could worry that a portion of the price variation might be mechanically arising from the variation in ride lengths (both in distance and in time). Given that we do not model ride lengths, this length-originated price variation may be "artificial" when used in our context. To deal with this issue, we carry out a robustness check in which we construct a price measure that only includes the residual variation after controlling for the length-based variation. We re-estimate the model with this new price measure. The results are robust. See Appendix G for more details on how we implement this.

Step 2 Given the observables and following step 1, we express the relative outflow between each pair of regions i, j on each day d as a function of parameters (α, β):

$$RO_{ijd} = \frac{A_{id}}{A_{jd}} = \frac{r_i/\bar{\lambda}_{id}(\alpha, \beta; \mathcal{D})}{r_j/\bar{\lambda}_{jd}(\alpha, \beta; \mathcal{D})},$$

We estimate (α, β) by matching the moments of RO_{ij} for every pair of regions i, j, where we take the average across $d \in D$ between the model prediction and the data. The estimates of $\bar{\lambda}_{id}$ are obtained from $\bar{\lambda}_{id}(\alpha, \beta, \mathcal{D})$ accordingly.

5.3.3 Identification

The identification follows a similar intuition as the supply-side empirical model. Given eq. (14), the identification problem reduces to identify α and β from the RO moments. Simular to the model with supply-side EOD, we consider the relative outflow across regions for a single day d. Dropping the day subscript d, the model relative outflow can be written as

$$RO_{ij}^{model} = \frac{A_i}{A_j} = \frac{r_i/\bar{\lambda}_i}{r_j/\bar{\lambda}_j} = \frac{\frac{n_i}{c_i}/\bar{\lambda}_i}{\frac{n_j}{c_j}/\bar{\lambda}_j} = \frac{(1-\alpha p_i)(1-\beta\frac{s_i}{n_i})}{(1-\alpha p_j)(1-\beta\frac{s_i}{n_i})} \tag{15}$$

Following a similar intuition as the empirical model with supply-side EOD, it can be seen that as long as there is sufficient independence between regional price variation and regional density variation, both α and β are identified. The formal statement and the proof can be found in the appendix.

Our informal identification argument sheds light on why separately identifying supply-side from demand-side EOD is not possible using relative outflows data: variation in density across regions creates geographical skew in access (and hence in relative outflows) through both of the EOD sources. As a result, one cannot separate out how much each source is contributing to the overall economies of density by just observing the relative flows. This is why in calibration with our data, we need to assume (rather than infer) the relative magnitudes of the two sources. That said, as was discussed in the theory section and as will be further elaborated when we report estimation results and counterfactual simulations, our main insights are robust to the source of EOD.

5.4 The Comprehensive Model

In taking the model to data, we construct a comprehensive model here that includes both supply-side and demand-side economies of density that we have separately examined above. The parameters we seek to include are $(a, \alpha, \beta, \{\lambda_{id}\}_{i\in I, d\in D})$.

We next detail the comprehensive model, where the passenger pickup time both affects the demand arrival rate and drivers' supply decisions. The wait time per ride is given by

$$W_{id}(n_{id}; \bar{\lambda}_{id}, \alpha, \beta, a) = \frac{n_{id}}{\bar{\lambda}_{id}(1-\alpha p_{id})(1-\frac{\beta s_i}{n_{id}})} + \frac{a s_i}{n_{id}} \tag{16}$$

The distribution of drivers on a given day d is obtained as an equilibrium outcome by equating the wait time to the ratio of wage per ride to the outside option:

$$\forall i,\ W_{id}(n_{id}; \bar{\lambda}_{id}, \alpha, \beta, a) = \frac{c_i}{\bar{c}} \tag{17}$$

As detailed earlier, the number of rides is given by $r_{id} = \frac{n_{id}}{W_{id}}$.

The parameters to be estimated are $(\alpha, \beta, a, \bar{\lambda})$. This general model accepts as special cases the two model we examined so far: supply-side-only EOD: $\beta = 0$ and demand-side-only EOD: $a = 0$. As we discussed, separately identifying a and β is not possible using the variation in our data. However, as we showed, each of the two parameters would be identified if we assumed the other was zero. In the same spirit, β and a would be identified if in addition to the structure of our model, we imposed the assumption that the ratio between them was known. This roughly translates to assuming that the relative contributions of the supply-side and the demand-side EOD being known.

Specifically, we evaluate the following ratios: $\frac{a}{\beta} \in \{10, 1, 0.1\}$. A larger ratio implies the domination of the supply-side effects, while a smaller ratio implies the domination of the demand-side effects. Of course a model estimated with each of these assumptions on $\frac{a}{\beta}$ is in nature "arbitrary" and uninformative on its own. But the analysis of this wide range for $\frac{a}{\beta}$ is useful in that it can help to show that our model results are robust across a wide range of possible ratios (as opposed to just the two extreme cases) of the relative importance between supply- and demand-side EOD.[24]

6 Empirical Results

In this section, we present parameter estimates from calibrating different versions of the model (different in terms of the relative importance of supply- v.s. demand-side EOD) to the data. We then use these calibrated parameters to simulate counterfactual scenarios that (i) further establish the robustness of our theoretical findings to cases with both supply-side and demand-side EOD; and (ii) provide additional insights on matters related to public policy and platform strategy.

6.1 Model Estimation

In this section, we present estimates from the model with supply-side EOD, the model with demand-side EOD, as well as the comprehensive model with both sources of EOD. Table 4 presents the estimates of the parameters from the supply-side and the demand-side model. The parameter estimates are qualitatively similar. Table 5 shows the price elasticities implied by the model estimates. As a benchmark, Rosaia (2020) finds a broadly consistent price elasticity ranging from 0.55 to 1.09.[25] Cohen et al. (2016) finds an overall price elasticity of 0.61 for the New York City.

[24] In addition to $\frac{a}{\beta} \in \{10, 1, 0.1\}$, we also ran the analysis with 100 and 0.01. The results were robust.

[25] Rosaia (2020) is an order version of Rosaia (2023). It reports the price elasticities implied by the model estimates.

Table 4: Model Parameters from the model with supply-side EOD and the model with demand-side EOD. We report the point estimates (with t-statistics in parentheses) for the parameters (α, a, β). Since our model estimates $\bar{\lambda}$ for each day and each region, we therefore only present the average of the point estimates of $\bar{\lambda}$ across days for each region.

	Model w/ Supply EOD	Model w/ Demand EOD
α	0.0201 (33.36)	0.0138 (21.17)
a	45.20 (29.82)	N/A
β	N/A	59.43 (26.88)
Average $\bar{\lambda}$ by Regions		
Bronx	4674	3943
Brooklyn	7824	6791
Manhattan	12650	10535
Queens	6565	5629

Table 5: Average demand elasticities across days with respect to prices given by the model with supply-side EOD and the model with demand-side EOD separately. The averages are weighted by the number of trip requests per day.

Region	Model w/ Supply EOD	Model w/ Demand EOD
Bronx	0.37	0.34
Brooklyn	0.51	0.36
Manhattan	0.67	0.40
Queens	0.75	0.55
All Regions	0.60	0.41

Table 6 presents the average (across days) potential demand density and access metrics based on all the models in each region. We find that potential demand density varies significantly across the regions with Manhattan having more than four times the density of Queens. This spatial differential points to the importance of the model developed here modeling and characterizing the differences across regions.

We find that, despite the varying model specifications, allowing for different relative levels of supply-side and demand-side EOD, the access levels following an ordering that is consistent with the demand density. Specifically, Queens is found to have the lowest access and Manhattan the highest access. The results are also consistent with the theory model, which predicts that regions with lower demand density have lower access. Thus we see that even without requiring optimal choices made by the platform, we still find the main theoretical results hold empirically.[26] These results are robust to model specification across the supply-side and the demand-side model.

[26] One thing to note here is that our model estimation does not require that the observed prices and wages are set at the optimal level by the platform, and we do not assume the observed data are at the optimal level. On the other hand, our theory results are derived under the assumption of optimal platform strategy. Despite this, we still observe that the model estimates are largely consistent with the theory results.

Table 6: Model Outcomes (Regions Ordered by Demand Density)

Model Spec. $\frac{a}{\beta}$	**Supply EOD** ∞	**Supply & Demand EOD** 10	1	0.1	**Demand EOD** 0
Demand Density ($\frac{\bar{\lambda}_i}{s_i}$)					
Queens	629.72	626.44	597.92	550.26	539.86
Bronx	719.46	714.56	676.58	619.01	606.90
Brooklyn	939.19	934.72	896.17	830.08	815.22
Manhattan	2655.16	2631.96	2472.72	2255.54	2211.13
Access ($\frac{r_i}{\lambda_i}$)					
Queens	0.49	0.49	0.51	0.56	0.57
Bronx	0.51	0.52	0.55	0.60	0.61
Brooklyn	0.57	0.57	0.60	0.65	0.66
Manhattan	0.58	0.58	0.62	0.68	0.69

Note: The results presented correspond to averages across days across the data time periods.

With model calibration results at hand, we next turn to counterfactual analysis.

6.2 Counterfactual Analysis

We explore the market outcomes under a wide range of scenarios of practical interest to consumers, firms and regulators. The first three counterfactuals build on the insights from the theoretical section of the paper, empirically quantifying their magnitudes. The last two counterfactuals explore some questions related to public policy.

6.2.1 Platform Optimal Strategy with Heterogeneous Prices and Wages

Recall that our estimation did not require any assumption on platform pricing policy. We now use our model to find optimal prices and wages across regions. Fig. 5 presents the results. Different rows respectively show optimal prices, wages, margins and the associated access. Columns correspond to different model specifications: from supply-side-only EOD to demand-side-only EOD.

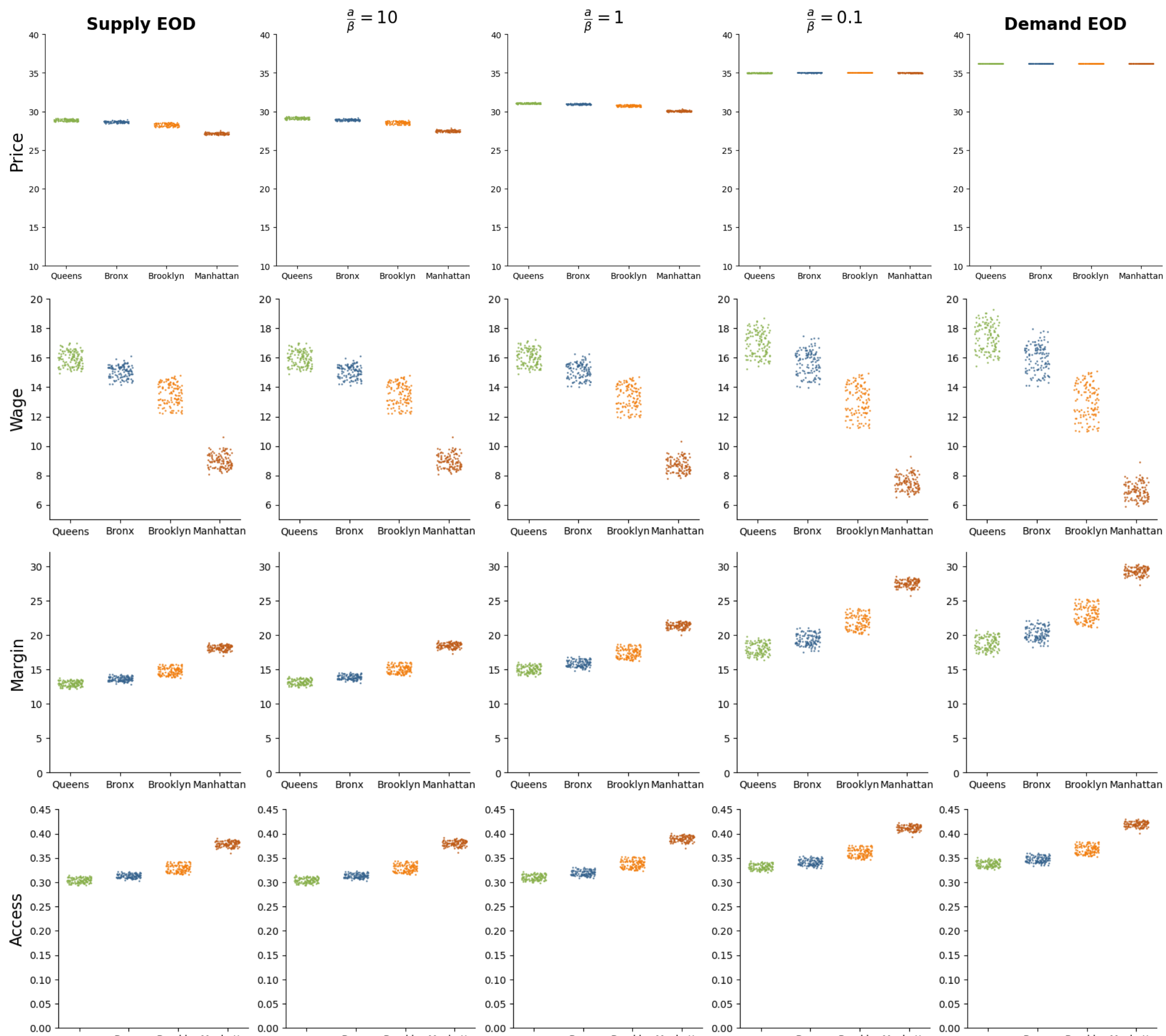
As can be seen in this figure, across all model specifications, two important patterns hold under optimal platform policy on prices and wages. First, lower density regions have lower access. Second, the platform accepts lower margins in those sparser regions (we will discuss soon that the platform does so in order to partially build economies of density locally). These results should not be surprising in the case of pure supply- or demand-side EOD, given that our theory predicted them. However, we did not have theory results on this matter in the case of mixed supply-and-demand EOD; and Fig. 5 shows that our expectations empirically hold.

Fig. 5 also helps get a sense of the magnitudes. First, our simulations recommend an overall price increase by about 50% relative to what Uber charged during the time period of our data. This is in line with what Uber has done more recently. [27] Second, our simulations propose margin

[27] See Appendix H for more details.

difference of \$5-\$12 per ride (depending on specification) across regions. Finally, our simulations suggest that under the platform optimal policy, access levels across regions should cover a range of about 10% width. Note that compared to accesses level calibrated from the model showed in Table 6, the accesses here are at a relatively lower level. This is because our model predicts prices that are higher than the observed levels.

Figure 5: Prices, wages, margins and accesses under optimal platform strategy. In each plot, regions (boroughs) are arranged in ascending order by demand density from left to right. Given the estimated model parameters, we find the optimal prices and wages for each region on each day. Each dot represents the metric for a region of a specific day.



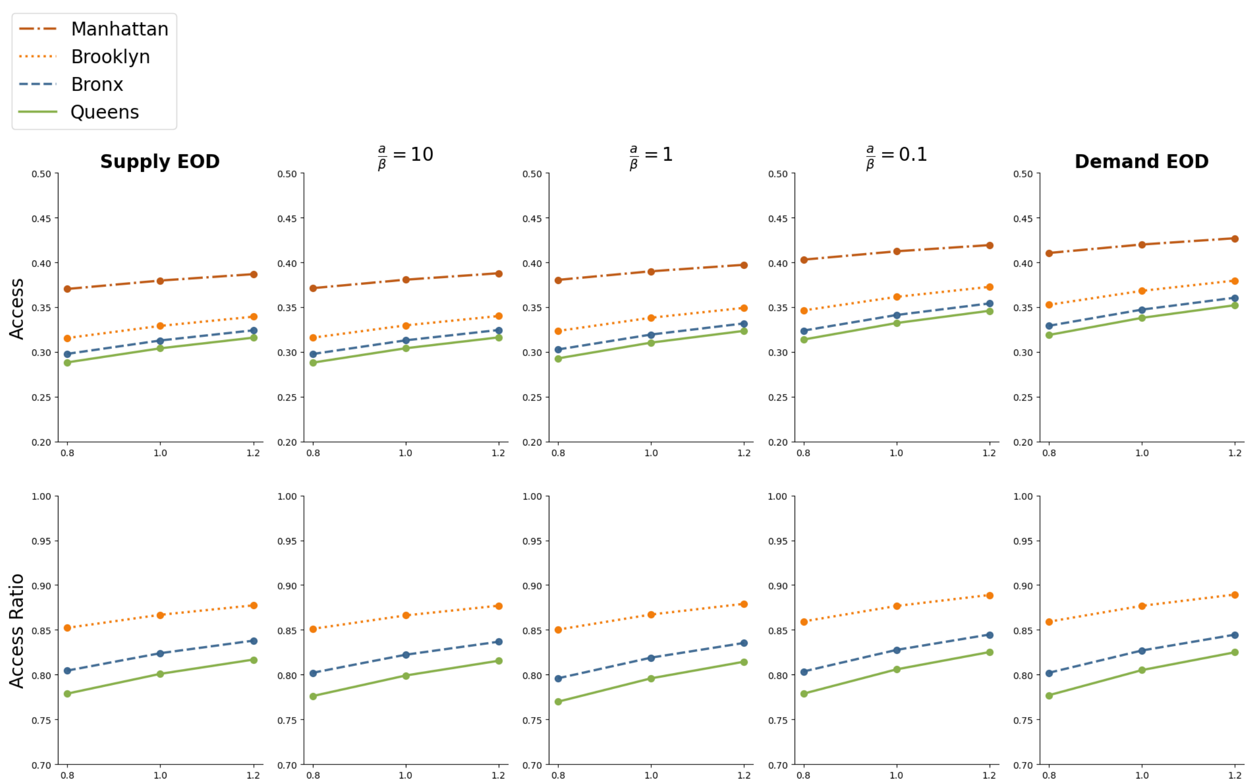
6.2.2 Impact of Market Thickness

To show how degree of access skew changes with different sizes of the platform, we take the average potential demand across days as the base demand levels for each region. Then we scale the demand levels ($\bar{\lambda}_i$) for each region by a uniform factor to simulate a larger platform.

We calculate the optimal prices and wages and derive other market outcomes such as ride volumes and accesses as the platform gets larger. Fig. 6 shows the access levels and access ratios to Manhattan as the platform gets larger with each of the models separately. With each model, the access for each region increases with the platform size. Also, the access skew is mitigated as the platform gets larger, as we observe that the access ratios versus Manhattan, the densest region, approach 1.

We find that the result is robust to the relative strength across the sources of EOD (supply-side/demand-side). We observe that the accesses are at similar levels between the purely supply-side and purely demand-side models, as well as the model with both effects. This shows that our results are consistent both between the theory model and the empirical estimation and between the model with supply-side EOD and the model with demand-side EOD.

Figure 6: Access (in absolute terms and relative to Manhattan) as market thickens.

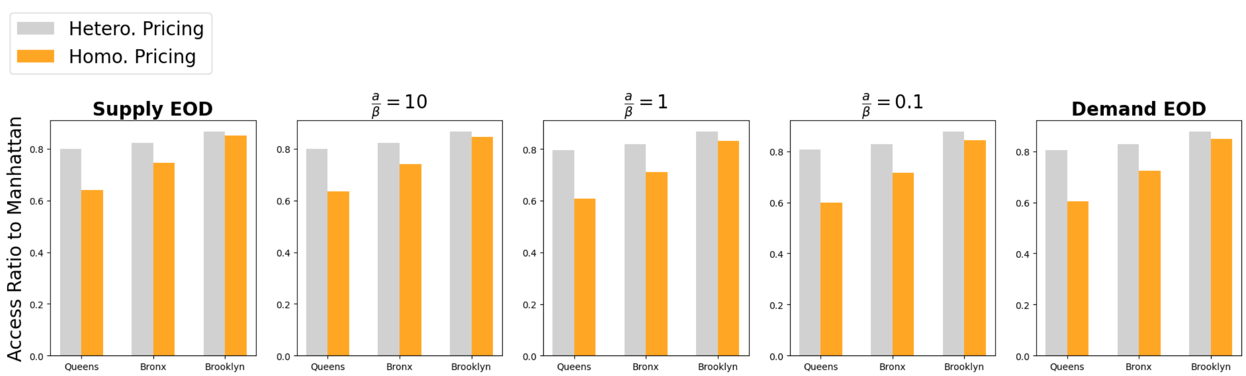


6.2.3 Platform Optimal Strategy with Uniform Prices and Wages

Regulators in areas with multiple regions (e.g. NYC and its boroughs) often impose citywide regulations on prices or wages, which we evaluate here. Proposition 3 and Proposition 4 showed that under pure supply- or demand-side EOD, the platform's optimal strategy involves charging lower margins $p_i^* - c_i^*$ in sparser regions (in Section 6.2.1, we showed that this extends to cases where both EOD sources exist at the same time.) Our interpretation of this margin result was that the platform is trying to build economies of density into sparser regions by increasing driver wages there but not fully passing on that extra wage to passengers. The analysis in this section is complementary in that we examine platform responses and outcomes when the platform is restricted either by regulation or by company policy in setting prices and wages in a spatially flexible manner. We compare the results to the counterfactuals in which Uber can freely optimize across regions (similar to the counterfactual above, we take the average potential demand across days for each region as the base level of potential demand.) Fig. 7 shows the results.

According to Fig. 7, access levels across regions vary more widely if the platform is not allowed to charge borough-specific margins. This is in line with our interpretation that the platform charges smaller margins in sparser regions in order to optimally mitigate the spatial access skew. This insight is robust with respect to the relative importance of the two sources of EOD.

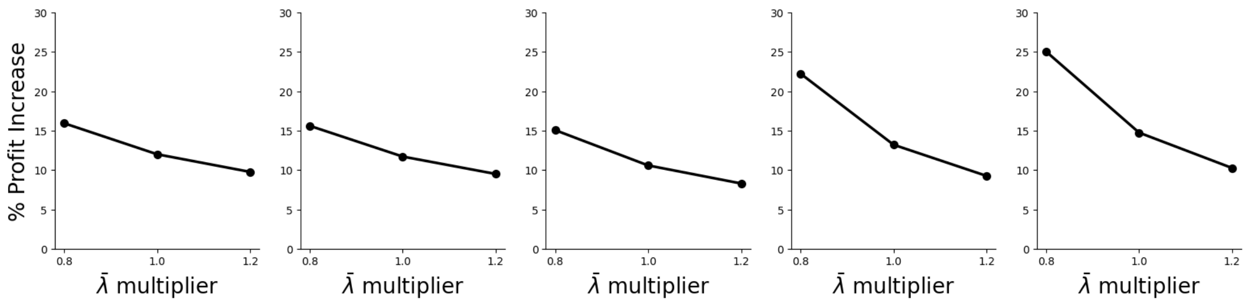
Figure 7: Comparison of access ratios to Manhattan under homogeneous and heterogeneous prices and wages. The regions are ordered by demand density from left to right in each bar plot in ascending order.



Next, we show that the smaller platforms are more adversely affected under the restrictive pricing scheme. We use the same set of demand scaling factors as in Section 6.2.2. Under each of the set of potential demands and with each model, we calculate the optimal uniform price and wage and the market outcomes. Fig. 8 shows the percentage profit gain from the flexible prices and wages compared to the uniform price and wage as a function of the platform size. For our estimated platform size, the percentage profit gain is 10-15% under the heterogeneous prices and wages compared to homogeneous price and wage.[28]

[28] Note that this magnitude might be exaggerated due to the fact that our model of the market does not account for actual trip times. More details on this are provided in the limitations section. Nevertheless, one should note that

Figure 8: Percentage gain in overall profit across all regions for the platform as a function of platform size. The number is calculated as the overall profit under heterogeneous prices and wages less the overall profit under homogeneous price and wage, divided by the latter.



Before turning to the next counterfactual, we make a final point which is worth noting. Although Uber, according to the data, does not use spatially uniform pricing, it uses an almost constant markup of 26%.[29] This implies a pass-through of 1.26 whereas analysis on mitigating the spatial skew suggests that the pass-through should be less than 1.[30]

6.2.4 Equalizing Accesses

Unequal access across regions may be undesirable to a social planner, both due to externality considerations (similar to what was discussed regarding platform incentives) and due to fairness considerations. In this section, we study using counterfactual simulations the extent to which Uber would need to adjust its regional prices/wages in order to achieve uniform access across regions. This average access almost coincides with access level in Brooklyn.

We first study the effect of wages. More specifically, for our estimated potential demands (averaged across days), we keep prices fixed at the optimal level p^* if Uber were to jointly optimize (p^*, c^*). Then we find the regional wage levels c_i that set access in all regions equal to each other (and to the average across regions under (p^*, c^*)). Results are reported in Fig. 9 and Fig. 10. Next, we study the effect of using prices. Here, we fix wages at c^* and modify regional prices to set all access levels to the global average. The results are reported in Figure Fig. 11 and Fig. 12.

There are two main findings. The first one is that as the platform gets larger, it needs a smaller amount of price- or wage- intervention to equalize access. This can be seen from the curves in the bottom panels of figures Fig. 9 and Fig. 11: all curves get closer to zero as we increase the size multiplier. As the figures also show, this insight is robust to the source of EOD.

there is also a reason why our numbers may be an understatement; and that is fixed costs. Our constructed measure of profits does not incorporate such fixed costs. But we know that once we subtract the fixed costs of running a business from "gross" profits under two pricing policies, the ratio between the profitabilities of these policies will be amplified. Given that Uber was not profitable (See Uber annual report 2022: https://investor.uber.com/financials/default.aspx) in spite of charging positive margin, it is likely that it faces significant fixed costs.

[29] Regressing prices on wages without a constant using the rides data returns an intercept of -0.5 and a wage coefficient of 1.26 and an R^2 equal to 0.9787.

[30] Of course the decision on the optimal pass-through by Uber may have non-spatial reasons which we are abstracting away from in this paper.

Our second finding is that the to equalize access under fixed (at optimum) prices, the downward trend of wages as a function of demand density should intensify. This can be seen in Fig. 10. The magnitude of change is about 2-3 dollars. On the other hand, we find that to equalize access under fixed (at optimum) wages, the downward trend of prices as a function of demand density should not only soften, but in fact get reversed into a downward trend. This can be seen in Fig. 12. Here, too, the magnitude of the requisite change is 2-3 dollars. Similar to the previous insight, this insight is robust to the source of EOD.

Figure 9: Wages for equalizing accesses across regions as a function of platform size, with prices fixed for each region. The wage adjustments are computed as the wages that equalize the accesses less c^*.

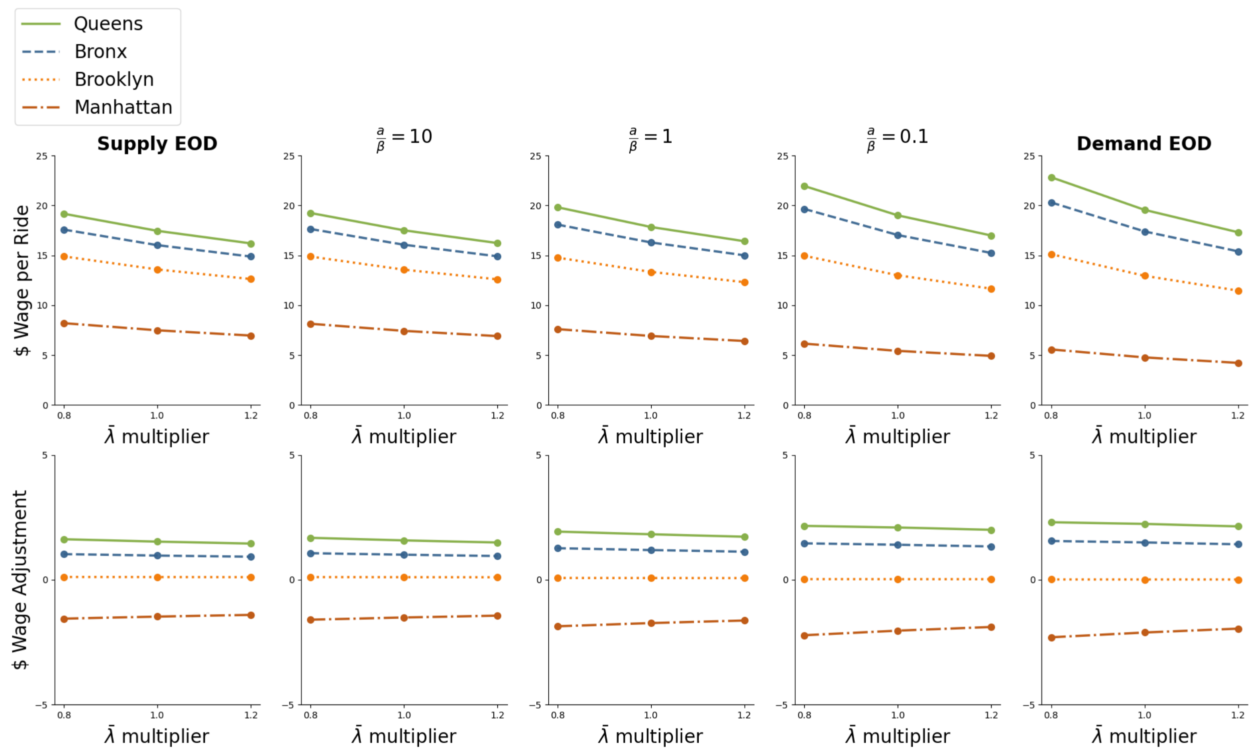


Figure 10: Optimal wages and the wages that equalize accesses, under platform's original size

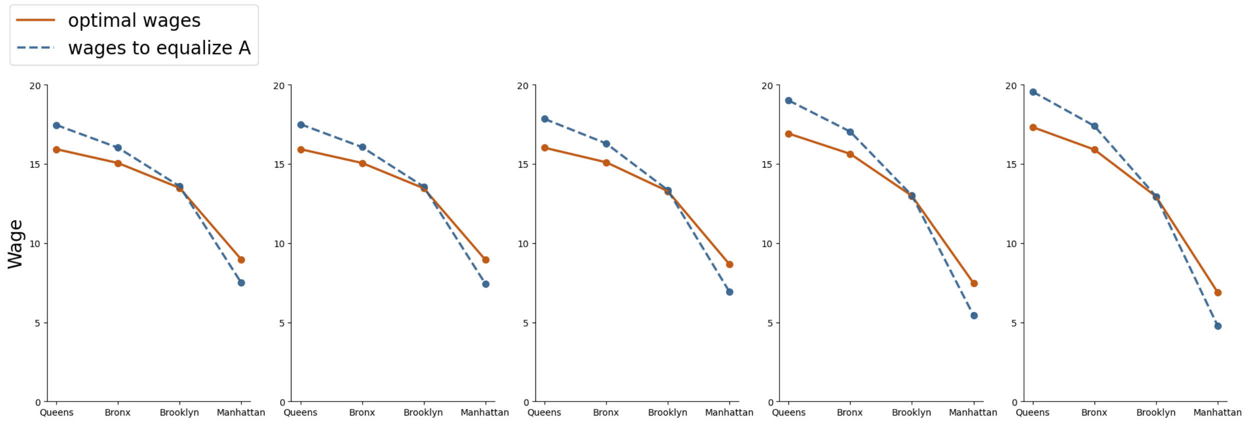


Figure 11: Prices for equalizing accesses across regions as a function of platform size, with wages fixed for each region. The price adjustments are computed as the prices that equalize the accesses less p^*.

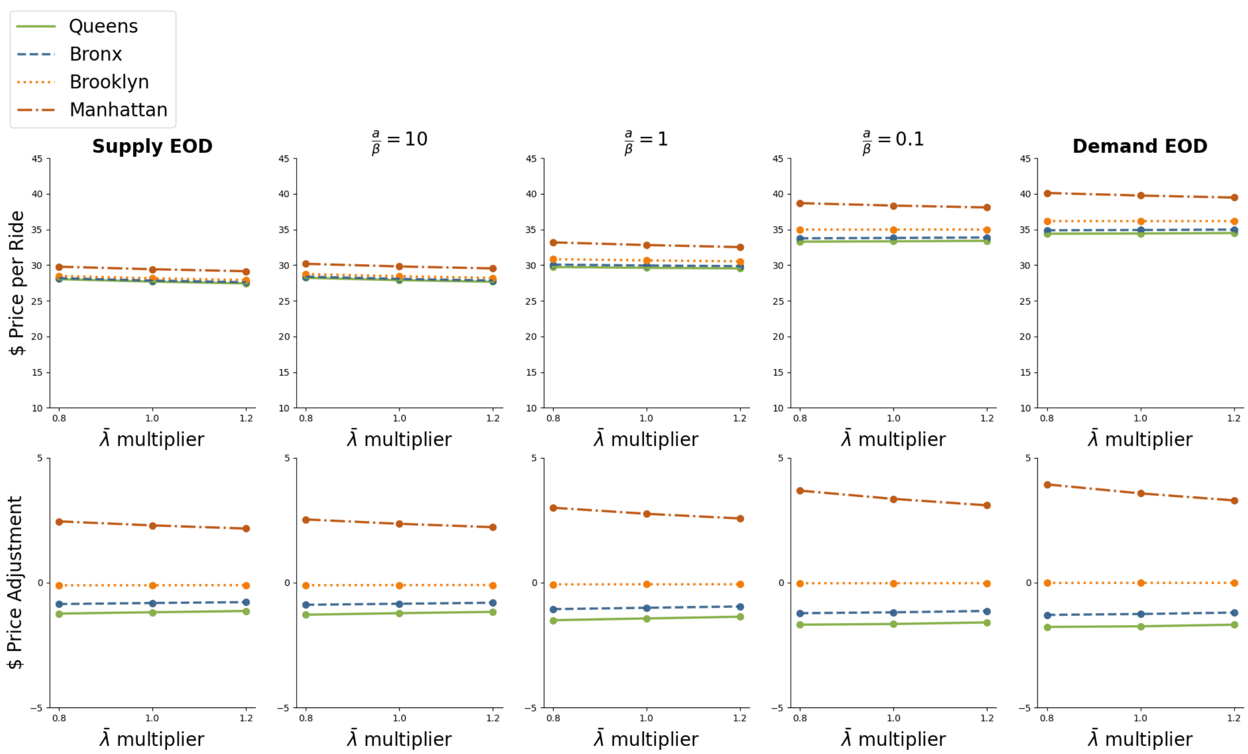
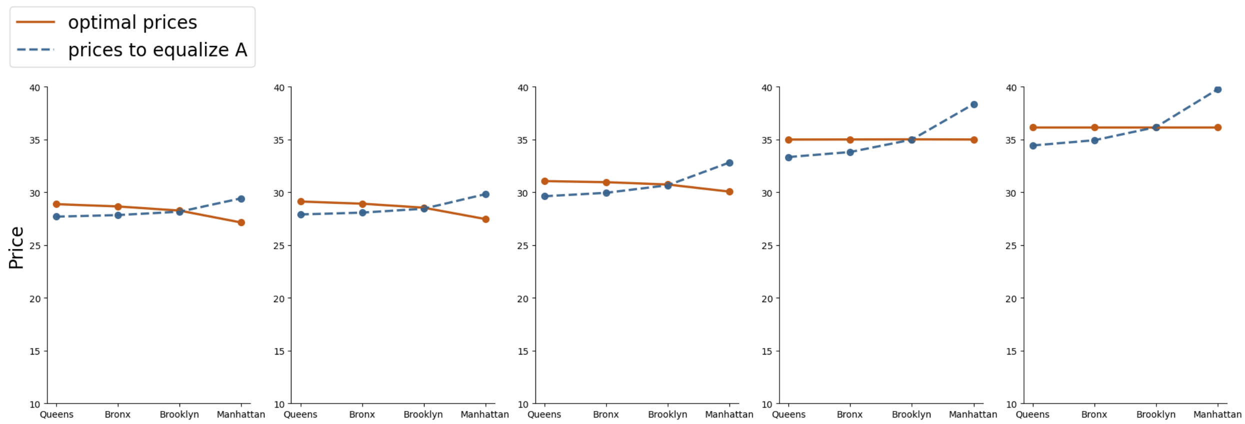


Figure 12: Optimal prices and the prices that equalize accesses, under platform's original size.



6.2.5 Minimum Wage

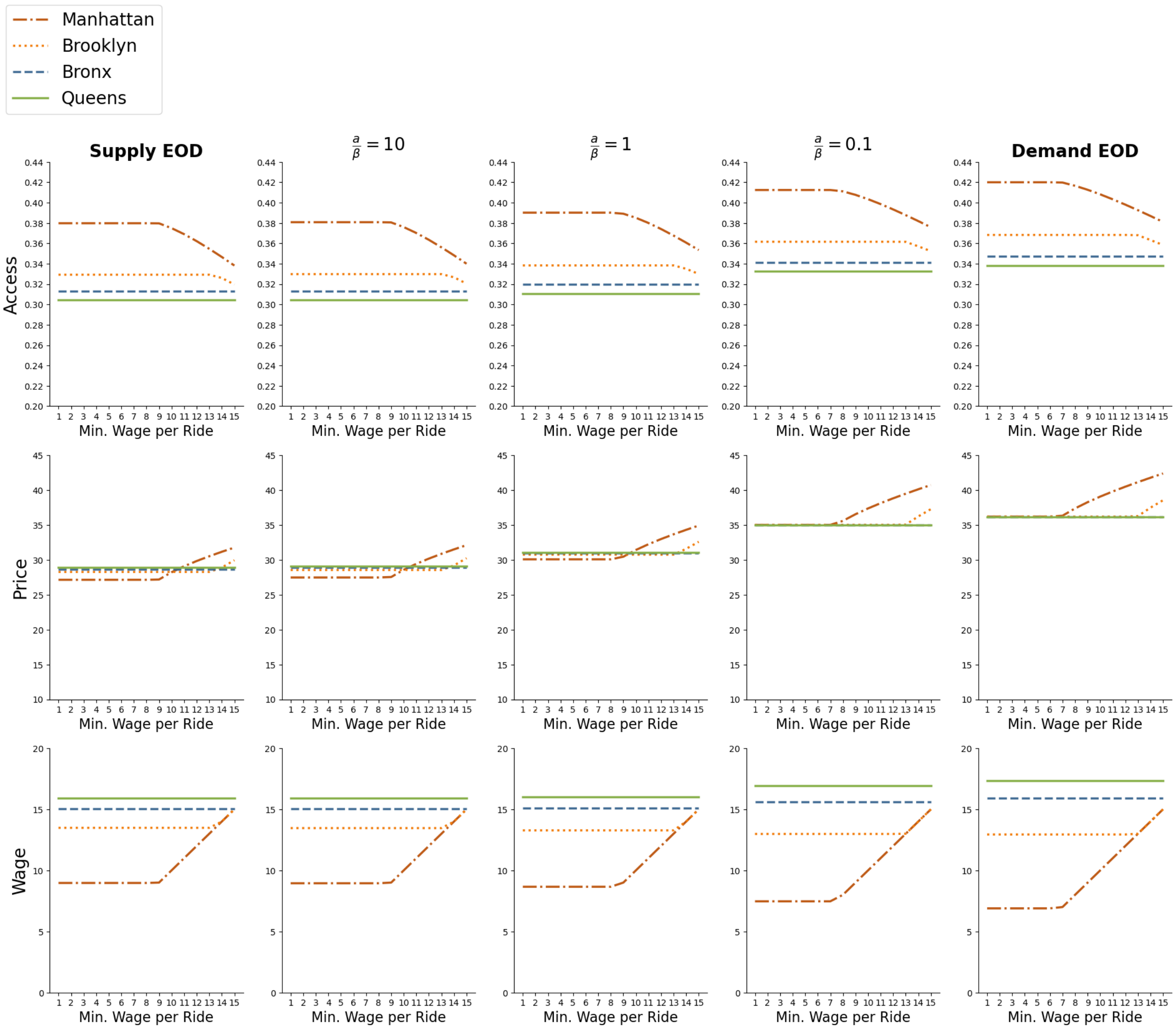
One policy that has been of interest to regulators is the imposition of a uniform minimum wage across regions (citations). We simulate the effects of such policies, varying the level of the minimum wage per ride. Fig. 13 presents our results. We have two main findings. First, if a minimum wage binds in a region, access drops relative to the level under the platform's unrestricted optimal price-wage combination. The reason is nuanced: the direct effect of a minimum wage on access should be an positive one because higher wages attract more drivers. However, the platform also responds to this higher wage by increasing its price, which should harm access. Our simulations reported in Fig. 13 suggest that the negative effect of price increase on access more than offsets the positive effect of the wage increase, and this is robust to the source of EOD. As an example, observe the behavior of dot-dash red lines (representing Manhattan's wage, price, access in different rows) for any column of the figure. We believe a key reason why the effect of price increase dominates that of a wage increase is that our simulations assume prior to the imposition of the minimum wage, the platform was setting prices and wages *optimally*. This implies that further price increase makes demand for rides excessively elastic, thereby sharply dropping ride requests. Also, optimality implies that the platform must have considered the positive effect of wage on access in its initial decision on the wage and, hence, would not gain a first order bump in access upon additional wage increase due to diminishing returns.

Our second finding directly follows from the combination of the first finding and our main result: a minimum wage policy (weakly) reduces both access and access skew across regions. The underlying mechanism is straightforward. The platform gives lower wages for high-density regions, so the minimum wage constraint is unlikely to be binding for low-density regions.

7 Limitations

Before concluding the paper, we discuss some limitations of our study. One limitation is that we decompose drivers' time into pickup time and idle time, abstracting away from the amount of time

Figure 13: Accesses, prices and wages as a function of minimum wage constraints.



that drivers spend carrying passengers to destination. We do not expect this limitation to change any of our results qualitatively. But we do expect that it would attenuate the magnitudes of some of our quantitative findings, given that once accounting for time spent carrying passengers, the pickup-time differences across regions would have lower relative weights.

There are two shortcomings on the data front. First, we lack exogenous price variation. As mentioned before, this issue can lead to biased price coefficients estimated in our demand model(s). Nevertheless, we continued with our estimation given that our elasticity magnitudes are fairly close to Rosaia (2020) who does have access to exogenous variation. Alternatively, we could directly calibrate the price coefficient so that it would imply elasticities close to Rosaia (2020). Second, our relative-outflows identification strategy cannot empirically separate between demand- and supply-side EOD. We showed that this hurdle does not significantly undermine our study given that our results are robust to the source of EOD. Nevertheless, our identification approach would be insufficient (and would need amendments) for studies of consequences of EOD that are sensitive to which side of the market is the main source of EOD.

Finally, we do not examine the role of competition as it relates to spatial location decisions by drivers and spatial price and wage decisions by platforms. While it would be difficult to speculate on the impact of competition without formal modeling, our conjecture is that it would exacerbate the access skew by breaking up the market into multiple pieces, each thinner than what would emerge under monopoly. For more on the role of competition, see Rosaia (2023).

8 Conclusion

This paper examines the effects of economies of density and market thickness on spatial distribution of access to service. We focus on ridesharing and analyze this market both theoretically and empirically. Theoretically, we show that in equilibrium, relative to lower density regions, the regions with higher densities of potential demand get more supply–even after normalizing by their higher demand. We show that this "spatial skew" is more intense for smaller platforms. Finally, we show that a rideshare platform's optimal pricing strategy would involve mitigating, but not fully eliminating, the spatial mismatch between supply and demand. All of these results are robust to whether the source of economies of density is the supply-side or the demand-side. The lack of full alignment between the equilibrium driver distribution (if the platform does not intervene) on the one hand and the platform's optimal distribution on the other comes from externalities that drivers have on each other, which are internalized by the platform.

On the empirical front, we calibrate our model using ride-level data on Uber from New York City. Key in our calibration exercise is the identification of relative access to service across regions by leveraging the "relative outflows" of rides between pairs of regions. The basic idea of our strategy to identify the extent of economies of density could be used beyond our setting to all passenger-transportation markets regardless of whether the matching technology is centralized (e.g., rideshare) or decentralized (e.g., taxicabs). We use our calibrated model to quantitatively speak to a number of questions, including platform optimal prices and wages across regions, the role of platform size (i.e., market thickness), the impact of forcing the platform to optimize prices and wages subject to spatial homogeneity, the magnitude of price/wage adjustment needed to homogenize access across regions, and finally the consequences of minimum wages for the spatial distribution of access to service.

The analysis in this paper can be extended in multiple directions. First, extending the model to markets with decentralized matching such as the taxicab market would provide useful insights on the relative effectiveness of policies in that market relative to ridesharing. Second, research could examine the differing incentives of social planners and platforms in terms of access differences across regions, and the conditions under which they converge and diverge. Third, a useful next step would be to examine the tradeoff between platform size, which we find to have a positive effect here, with the effect of competition, in considering whether platforms should be allowed to coordinate or even merge. Finally, understanding the value of other policy instruments like regulating the total number of drivers in a region or congestion pricing would be of broad interest in spatial markets like ridesharing.

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Appendices

This section provides nine appendices. Appendix A provides anecdotal evidence for the relevance of our model. Appendix B provides a micro foundation based on a circular city model for the total wait time formula that we use throughout the paper. Appendix C gives proofs of the propositions for the theory model. Appendix D presents proofs related to the empirical model. Appendix E discusses other variations of the theory model, characterizes the equilibrium outcome under each different setup, and shows that the main insights carry over. Proofs are also included. Appendix F supplies additional empirical evidence. Appendix G presents the results re-estimated using measures of prices and wages adjusted for trip length. Appendix H shows the price trend after 2019. Appendix I compares the data used in our analysis to that in Rosaia (2023).

A Anecdotal Evidence from Media and Online Forums

This appendix points to a list of anecdotal pieces of evidence (by no means exhaustive) from online rideshare forums on how drivers complain about Lyft's far pickups in suburbs and how they recommend responding to it. The explanations in brackets within the quotations are from us.

- **From the online forum "Uber People,"[31] a thread in the Chicago section:** The title of the thread is "To those who drive Lyft in the suburbs." The thread was started on Dec 19 2016. The first post says "Are the ride requests you get on Lyft always seem to be far away from you location? Seems like they are always 5 miles or more for the pickup location. I got one for 12 miles last night. I drive in the Schaumburg/Palatine area [two northwestern suburbs of Chicago about 30mi away from downtown]."
- **From the same thread:** "iDrive primarily in Palatine. about two out of every five ride requests are for more than 10 minutes away. I ignore those".
- **From the same thread:** "I was a victim of that once. Never again I take a ping more than 10 minutes away in the burbs".
- **From the same thread:** "Yesterday was my 1st day on Lyft. Was visiting in Homer Glen [a village about 30mi southwest of downtown Chicago] & decided to try Lyft for the first time. First ping was 18 minutes away. Dang, I could make it 1/2 way downtown in that time! I ignored the ride request. 2nd ping was also 18 minutes away. Lyft app complained my acceptance rate is too low. I ignored the 2nd ping & went off-line."
- **From the same forum, a thread titled "First 3days of Lyft":** "If your area is spread out...and you have to take those $>$ 10 minute requests, well...I might look for another job."
- **From the same thread:** "Yeah, another (mostly) Lyft-specific problem, especially when working in the suburbs, is you sometimes (fairly frequently, actually) receive trip requests

[31] In spite of what the name suggests, this is a general ridesharing forum, not exclusively about Uber.

that are not close to your current location. I've received requests from passengers 20 miles away."

- **From Chicago Tribune article titled "Lyft takes on Uber in suburbs":** Jean-Paul Biondi, Chicago marketing lead for Lyft is quoted to explain the reason for Lyft's planned expansion into surburbs as follows "The main reason is we saw a lot of dropoffs in those areas, but people couldn't get picked up in those areas." Which is in line with our reasoning that small relative-outflow is a sign of potential demand which does not get served due to under-supply.

- **From the rideshare website "Become a Rideshare Driver":** It says successful Lyft drivers use the following strategy:
  - "The drivers usually run the Lyft app exclusively when they are in the busy downtown or city areas."
  - "Usually in the suburbs, Uber is busier than Lyft, and in such areas, the drivers run both the Uber and Lyft apps."

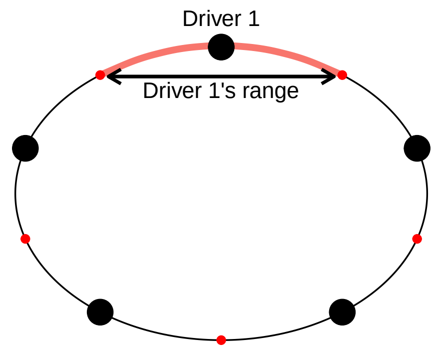
B Micro Foundation for the Total Wait Time Formula

This section provides a simple micro-foundation for the total wait time formula eq. (1) using a circular city model ala Salop. The circular model of regions is illustrated in Figure 14. The n_i drivers (in the figure, $n_i = 5$) are placed on equidistant locations on the circumference. [32] Drivers are matched to arriving passengers via a centralized matching system. Each driver's "range" or "catchment area" will be the arc consisting of all the points on the circle that are closer to that driver than they are to any other driver in region i. Given that the total arrival rate in the region is λ_i, the arrival rate in each driver's catchment area is $\frac{\lambda_i}{n_i}$. Each driver picks up the first passenger that arrives within that driver's catchment area. In practice, ridesharing platforms implement a similar matching rule (Frechette et al. (2019) use a similar approach to model a centralized matching market).

Suppose the time it takes a driver to travel a full circumference to pick up a passenger is t'. The platform allocates an arriving customer to the closest driver. Because drivers are situated at equidistant points on the circumference of the region, their catchment areas include half the distance to their nearest neighbors on both sides. The idle time expected for a customer to arrive in the driver's area is $\frac{n_i}{\lambda_i}$. The distance between drivers is $\frac{l}{n_i}$ where l is the circumference. Since consumer location is uniform, the distance a consumer will be from the driver along the arc is distributed $d \sim U[0, \frac{l}{2n_i}]$, implying that the expected distance is $E[d] = \frac{l}{4n_i}$. Thus, the expected pickup time is $\frac{t'}{4n_i}$.

[32] While we do not model the locational choice of drivers within the region, it is fairly easy to see that equidistant positioning location from neighbors is an equilibrium. While there might be other locational choices that might also be equilibria, we focus on the equidistant positioning equilibrium.

Figure 14: Illustration of Circular Model of each Region i and Driver Allocation



Based on the above, the expected total wait time $W_i(n_i)$ defined from the driver's perspective as:

$$\underbrace{W_i(n_i)}_{\text{Total Wait Time}} = \underbrace{\frac{n_i}{\lambda_i}}_{\text{Idle Time}} + \underbrace{\frac{t'}{4n_i}}_{\text{Pickup Time}} = \left(\frac{n_i}{\lambda_i} + \frac{t}{n_i}\right) \tag{18}$$

where $t = \frac{t'}{4}$. This is exactly the same as eq. (1) except the notation on $\lambda_i(p_i)$ is suppressed to λ_i.

C Proofs of Propositions in the Theory Sections

This appendix provides proofs for all the results we state for the theory model. It consists of three part. Appendix C.1 gives proofs for the model with supply-side EOD and without inter-region rides (i.e. Proposition 1 through Proposition 3). Appendix C.2 presents proofs for the model with demand-side EOD (i.e. Proposition 4). Appendix C.3 provides the proof for the setting with inter-region rides (i.e. Proposition 5).

Before we get to the proofs for the specific models, we first prove Lemma 1 that speaks to the equilibrium driver equilibrium in general.

Proof of Lemma 1. The proof is straightforward. First, observe that, for each region i, $W_i(p_i, n_i)$ must be non-decreasing n_i. Otherwise, because $W_i(p_i, n_i)$ is continuous, there exists a small $\epsilon > 0$ such that $\frac{c_i}{W_i(p_i,n_i^*)} > \frac{c_i}{W_i(p_i,n_i^*+\epsilon)}$, the condition in Definition 1 is violated.

Next, we prove the lemma using contradiction. If $\frac{c_i}{W_i(p_i,n_i^*)} > \bar{c}$, then given $W_i(p_i, n_i)$ is continuous and non-decreasing in n_i, there exists $\delta > 0$, such that $\frac{c_i}{W_i(p_i,n_i^*)} > \frac{c_i}{W_i(p_i,n_i^*+\delta)} > \bar{c}$. That is, a mass of δ drivers will switch from the outside option to driving in region i. If $\frac{c_i}{W_i(p_i,n_i^*)} < \bar{c}$, then the condition in Definition 1 is also violated, so n_i^* cannot be an equilibrium.

C.1 Proofs of the theory model with supply-side EOD

Before starting the proofs, we make an observation. With free entry of drivers and abstraction from inter-region rides, the problem in each region is, in some sense, separated from other regions. As such, from the platform's perspective, maximizing the profit over all regions is the equivalent to maximizing the profit of each region independently. Therefore, when solving for the equilibrium number of drivers for each region, it suffices to look at the equilibrium under the price and wage that maximize the profit of that specific region.

Proof of Lemma 2, Proposition 1, Proposition 2, and Proposition 3.

The overall strategy is to prove the lemma and the propositions in a unified framework. First, we prove a series of lemmas that show that, under the parameters $(\bar{\lambda}, t, \bar{c}, \alpha)$, the lemma and the propositions can be reduced to a numerical problem. That is, we first show using Lemma A1 - Lemma A5 that each of the propositions holds for a general $(\bar{\lambda}, t, \bar{c}, \alpha)$ if and only if it holds for $(\bar{\lambda}, 1, 1, 1)$. Showing that the propositions holds for $(\bar{\lambda}, 1, 1, 1)$ is essentially numerical, which we carry out accordingly in Lemma A6.

Lemma A1. *Assume $\gamma \in \mathbb{R}$ is a strictly positive number. Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha)$, vectors p^*, w^*, and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha') = (\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$, vectors $p^{*'} = p^*$, $c^{*'} = c^*$, and $n^{*'} = \gamma n^*$ constitute an equilibrium (note that here, unlike the previous two propositions, we are not suppressing the * notation for equilibria or the regional indices). Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

Proof of Lemma A1 Let us expand the notation on platform profits and equilibrium numbers of drivers across the market in a way that allows for explicitly tracking the dependence of all of these on market primitives. To be more specific, we denote the equilibrium number of drivers in region i under primitives $(\bar{\lambda}, t, \bar{c}, \alpha)$ and platform strategy (p, c) by:

$$n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha)$$

Similarly, we denote the platform profit by:

$$\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha)$$

Before proving the lemma, we first derive the equilibrium number of drivers given the primitives and the platform strategy. First, note that in equilibrium, the hourly revenue for drivers has to be equal to the reservation value $\bar{c}$. Therefore, we have:

$$\frac{c}{W} = \bar{c}$$

Under the model with supply-side EOD, the wait time W can be expressed in terms of n

$$W(n) = \frac{n}{\lambda} + \frac{t}{n}$$

Replacing W and then solve for the equilibrium n^*, we get two solutions:

$$n^* = \frac{\frac{c}{\bar{c}} \mp \sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\lambda(p)}}}{\frac{2}{\lambda(p)}}$$

Note that only the larger solution (i.e., the one with the + sign) is an equilibrium because at the lower solution, further driver entry will lead to lower overall wait time, increasing driver revenue. As such, we have:

$$n^* = \frac{\frac{c}{\bar{c}} + \sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\lambda(p)}}}{\frac{2}{\lambda(p)}} \tag{19}$$

We now turn to the proof of the lemma, starting with a claim:

Claim 1. *For any given (p, c) where region i attracts no drivers under $(\bar{\lambda}, t, \bar{c}, \alpha)$, we have:*

$$\pi_i(p, c|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha) = \gamma^2\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha).$$

Proof of Claim 1. To see this, note that under $(\bar{\lambda}, t, \bar{c}, \alpha)$, from eq. (19), region i will attract no drivers –hence will generate zero profit– if any only if:

$$(\frac{c}{\bar{c}})^2 - \frac{4t}{\bar{\lambda}(1 - \alpha p)} < 0$$

The corresponding condition under $(\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$ will be:

$$(\frac{c}{\gamma\bar{c}})^2 - \frac{4t}{\gamma^2\bar{\lambda}(1 - \alpha p)} < 0$$

The above two conditions are equivalent. This finishes the proof of the claim. □

Now let us assume p and c are such that region i does get a positive number of drivers. There, the number of drivers present in i under $(\bar{\lambda}, t, \bar{c}, \alpha)$ will be uniquely give by:

$$n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha) = \frac{\frac{c_i}{\bar{c}} + \sqrt{(\frac{c_i}{\bar{c}})^2 - \frac{4t_i}{\bar{\lambda}_i(1-\alpha p_i)}}}{\frac{2}{\bar{\lambda}_i(1-\alpha p_i)}}$$

The same quantity under $(\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$ will be uniquely give by:

$$n_i^*(p, c|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha) = \frac{\frac{c_i}{\gamma\bar{c}} + \sqrt{(\frac{c_i}{\gamma\bar{c}})^2 - \frac{4t_i}{\gamma^2\bar{\lambda}_i(1-\alpha p_i)}}}{\frac{2}{\gamma^2\bar{\lambda}_i(1-\alpha p_i)}}$$

$$= \gamma \frac{\frac{c_i}{\bar{c}} + \sqrt{(\frac{c_i}{\bar{c}})^2 - \frac{4t_i}{\bar{\lambda}_i(1-\alpha p_i)}}}{\frac{2}{\bar{\lambda}_i(1-\alpha p_i)}} = \gamma n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha)$$

Also note that the equilibrium driver wait time W_i in region i under $(\bar{\lambda}, t, \bar{c}, \alpha)$ has to be $\frac{c_i}{\bar{c}}$ in order to equate driver revenues in the region with the reservation value. Similarly, under $(\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$, the driver wait time in region i will be $\frac{c_i}{\gamma\bar{c}}$.

Next, note that for any p, c:

$$\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha) = (p_i - c_i) \times \frac{n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha)}{\frac{c_i}{\bar{c}}}$$

Also:

$$\pi_i(p, c|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha) = (p_i - c_i) \times \frac{n_i^*(p, c|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)}{\frac{c_i}{\gamma\bar{c}}}$$

$$= (p_i - c_i) \times \frac{\gamma n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha)}{\frac{c_i}{\gamma\bar{c}}} = \gamma^2 \pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha)$$

Therefore, the profit functions under $(\bar{\lambda}, t, \bar{c}, \alpha)$ and $(\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$ are fully proportional in each region, which mean they will have the exact same maximizers p^* and c^*. Given these maximizers, we will have the number of drivers in each region i under $(\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha)$ will be given by:

$$n_i^{*'} \equiv n_i^*(p^*, c^*|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha) = \gamma n_i^*(p^*, c^*|\bar{\lambda}, t, \bar{c}, \alpha) \equiv \gamma n_i^*$$

Finally:

$$A_i^{*'} = \frac{n_i^{*'}}{(\frac{c_i^*}{\gamma\bar{c}}) \times \gamma^2\bar{\lambda}_i} = \frac{\gamma n_i^*}{(\frac{c_i^*}{\gamma\bar{c}}) \times \gamma^2\bar{\lambda}_i} = \frac{n_i^*}{(\frac{c_i^*}{\bar{c}}) \times \bar{\lambda}_i} = A_i^*$$

The proof of the lemma is now complete. □

Lemma A2. *Assume $\gamma \in \mathbb{R}$ is a strictly positive number. Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha)$, vectors p^*, w^*, and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha') = (\bar{\lambda}, t, \gamma\bar{c}, \frac{\alpha}{\gamma})$, vectors $p^{*'} = \gamma p^*$, $c^{*'} = \gamma c^*$, and $n^{*'} = n^*$ constitute an equilibrium Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

Proof of Lemma A2. The steps of the proof for this lemma are very similar to those in the proof of Lemma A1. Hence, we skip the details. □

We just point out that there is a simple intuition for this result: a change of primitives from $(\bar{\lambda}, t, \bar{c}, \alpha)$ to $(\bar{\lambda}, t, \gamma\bar{c}, \frac{\alpha}{\gamma})$ should not be expected to change the equilibria because it is effectively a "currency change." The only two places where "money" becomes involved in the model are where demand is determined based on the price of a ride and where supply is determined based on wage. If the reservation hourly revenue is multiplied by γ and, at the same time, the price sensitivity of passengers is divided by the same factor γ, it is as if we are looking at the same market but with a new currency one unit of which is worth $\frac{1}{\gamma}$ times one unit of the old currency. This should not change the equilibrium supply, demand, and access. It should only multiply the equilibrium prices and wages by γ.

Lemma A3. *Assume $\gamma \in \mathbb{R}$ is a strictly positive number. Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha)$, vectors p^*, w^*, and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha') = (\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha)$, vectors $p^{*'} = p^*$, $c^{*'} = c^*$, and $n^{*'} = \gamma n^*$ constitute an equilibrium Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

Proof of Lemma A3. Again, the steps of this proof are very similar to those in the previous two lemmas. As such, we skip the details. □.

Our next lemma combines the previous three.

Lemma A4. *Under primitives* $(\bar{\lambda}, t, \bar{c}, \alpha)$*, vectors* p^**,* c^**, and* n^* *constitute an equilibrium if and only if under primitives* $(\tilde{\lambda}, 1, 1, 1)$ *where* $\tilde{\lambda}_i = \frac{\bar{\lambda}_i}{(\bar{c}\alpha)^2 t_i}$ *for each* i*, the following vectors constitute an equilibrium:* $p^{*'} = \alpha p^*$*,* $c^{*'} = \alpha c^*$*, and* $n^{*'} = \frac{1}{(\bar{c}\alpha)t} n^*$*. Additionally, access to rides in all regions are equal under these two equilibria:* $\forall i : A_i^{*'} = A_i^*$*.*

Proof of Lemma A4. Apply Lemma A1 with $\gamma = \frac{1}{\bar{c}\alpha}$ on primitives $(\bar{\lambda}, t, \bar{c}, \alpha)$. On the resulting primitives, apply Lemma A2 with $\gamma = \alpha$. Finally, on the resulting primitives, apply Lemma A3 with $\gamma = \frac{1}{t}$. This will establish the equivalence claimed in this lemma. □

With this result in hand, we now turn to the main two lemmas that prove Lemma 2 and Proposition 1 through Proposition 3.

Lemma A5. *Lemma 2 and Proposition 1 through Proposition 3 hold under all generic primitives* $(\bar{\lambda}, t, \bar{c}, \alpha)$ *if they hold under primitives in the form of* $(\tilde{\lambda}, 1, 1, 1)$*.*

Proof of Lemma A5. Suppose Lemma 2 and Proposition 1 through Proposition 3 hold for under primitives that take the form of $(\tilde{\lambda}, 1, 1, 1)$. We show it holds for a general $(\bar{\lambda}, t, \bar{c}, \alpha)$. To this end, choose vector $\tilde{\lambda}$ such that $\forall i : \tilde{\lambda}_i = \frac{\bar{\lambda}_i}{(\bar{c}\alpha)^2 t_i}$. Next, we prove each lemma/proposition for general primitives.

Lemma 2:

Given that Lemma 2 holds under $(\tilde{\lambda}, 1, 1, 1)$, there exists a unique equilibrium under $(\tilde{\lambda}, 1, 1, 1)$. Denote it $(\tilde{p}, \tilde{c}, \tilde{n})$. By Lemma A4, there also exists equilibrium (p^*, c^*, n^*) under $(\bar{\lambda}, t, \bar{c}, \alpha)$, which is given by $n^* = \tilde{n}_i(\bar{c}\alpha t_i)$ for each i. And $\tilde{n}$ being unique implies that n^* is unique. Also, for any i for which $n^* > 0$, we know $\tilde{n} > 0$. From the lemma, we know there are unique equilibrium price $\tilde{p}_i$ and wage $\tilde{c}_i$. Then p^* and c^* are also unique.

Proposition 1:

First we show the first half of the proposition that speaks to $n_i^* \neq 0 \neq n_j^*$ holds for the generic primitives. By assumption, this statement holds under $(\tilde{\lambda}, 1, 1, 1)$. By the construction of $\tilde{\lambda}$, it is immediate that:

$$\tilde{\lambda}_i > \tilde{\lambda}_j \Leftrightarrow \frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j}$$

Also by Lemma A4, we know that equilibrium access to rides in each region i is the same under $(\tilde{\lambda}, 1, 1, 1)$ and $(\bar{\lambda}, t, \bar{c}, \alpha)$. It immediately follows that the first half of the statement holds under $(\bar{\lambda}, t, \bar{c}, \alpha,)$.

For the second half of the proposition that speaks to the regions without supply, by assumption, we know there is a μ such that:

$$\forall i : \frac{\tilde{\lambda}_i}{1} < \mu \Leftrightarrow \tilde{n}_i = 0$$

Note that by construction, $\tilde{\lambda}_i < \mu$ is equivalent to $\frac{\bar{\lambda}_i}{t_i} < \mu(\bar{c}\alpha)$. Also note that $n_i^* = 0$ if and only if $\tilde{n}_i = 0$. Therefore, we have:

$$\forall i : \frac{\bar{\lambda}_i}{t_i} < \mu(\bar{c}\alpha) \Leftrightarrow n_i^* = 0$$

This finishes the proof of Proposition 1 under the generic primitives.

Proposition 2:

Key to proving the four statements of Proposition 2 is the observation that the same equivalence established between $(\tilde{\lambda}, 1, 1, 1)$ and $(\bar{\lambda}, t, \bar{c}, \alpha)$ by Lemma A4 may also be established between $(\gamma\tilde{\lambda}, 1, 1, 1)$ and $(\gamma\bar{\lambda}, t, \bar{c}, \alpha)$ for any positive real γ. Even the multipliers $\frac{1}{(\bar{c}\alpha)^2 t}$ remain the same. With this observation, we turn to the proofs.

Statement 1. By Lemma A4, We know $\tilde{p} = \alpha p^*$, $\tilde{c} = \alpha c$, and $\tilde{n} = \frac{1}{(\bar{c}\alpha)t} n^*$. By the first statement being true for $(\bar{\lambda}, 1, 1, 1)$, there exists equilibrium $(\tilde{c}', \tilde{p}', \tilde{n}')$ under the primitives $(\gamma\tilde{\lambda}, 1, 1, 1)$, where $\tilde{n}'$ is unique. Also $(\tilde{p}', \tilde{c}')$ are unique for $\tilde{n}' > 0$. Then by Lemma A4, there exists equilibrium $(p^{*'}, c^{*'}, n^{*'}$ under the primitives $(\gamma\tilde{\lambda}, t, \alpha, \bar{c})$, with $n^{*'} = (\bar{c})\alpha t \tilde{n}'$, $p^{*'} = \frac{1}{\alpha}\tilde{p}$, and $c^{*'} = \frac{1}{\alpha}\tilde{c}$. Therefore , $n^{*'}$ is unique, and $(p^{*'}, c^{*'})$ unique for $n^{*'} > 0$.

Statement 2. By Lemma A4, we know $n_i^* > 0 \Rightarrow \tilde{n}_i > 0$. By the proposition being true for $(\tilde{\lambda}, 1, 1, 1)$, we know that $\tilde{n}_i > 0 \Rightarrow \tilde{n}_i' > 0$ where $\tilde{n}'$ is the equilibrium driver allocation under $(\gamma\tilde{\lambda}, 1, 1, 1)$. Finally, again by Lemma A4, we have $\tilde{n}_i' > 0 \Rightarrow n_i^{*'} > 0$. This finishes the proof of this statement.

Statement 3. Lemma A4 shows that access to rides in all regions is preserved under the transformation from $(\bar{\lambda}, t, \bar{c}, \alpha)$ to $(\tilde{\lambda}, 1, 1, 1)$ and the transformation from $(\gamma\bar{\lambda}, t, \bar{c}, \alpha)$ to $(\gamma\tilde{\lambda}, 1, 1, 1)$. This, together with $\tilde{\lambda}_i \geq \tilde{\lambda}_j \Leftrightarrow \frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j}$ and the assumption that the proposition holds under $(\tilde{\lambda}, 1, 1, 1)$ gives the proof of this statement under $(\gamma\bar{\lambda}, t, \bar{c}, \alpha)$.

Statement 4. The logic for the proof of this statement is identical to that for the proof of the previous one.

Proposition 3:

Note that by construction of $\tilde{\lambda}$ and by Lemma A4, for any i, j the following hold:

1. $\tilde{\lambda}_i \geq \tilde{\lambda}_j \Leftrightarrow \frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j}$ and a similar statement when both inequalities are strict.
2. $\tilde{c}_i \leq \tilde{c}_j \Leftrightarrow c_i^* \leq c_j^*$ and a similar statement when both inequalities are strict.
3. $\tilde{p}_i \leq \tilde{p}_j \Leftrightarrow p_i^* \leq p_j^*$ and a similar statement when both inequalities are strict.
4. $\tilde{p}_i - \tilde{p}_j \leq \tilde{c}_i - \tilde{c}_j \Leftrightarrow p_i^* - p_j^* \leq c_i^* - c_j^*$ and a similar statement when both inequalities are strict.

These four statements, together with the assumption that Proposition 3 of the proposition on economies of density holds under $(\tilde{\lambda}, 1, 1, 1)$ imply that it also holds under $(\bar{\lambda}, t, \bar{c}, \alpha)$.

The proof of Lemma A5 is now complete. □

Our last step in proving Lemma 2 and Proposition 1 through Proposition 3 is to show that it does indeed hold under $(\tilde{\lambda}, 1, 1, 1)$. The convenient feature of this remaining step is that it can be done fully numerically. The following lemma takes on this task.

Lemma A6. *Lemma 2 and Proposition 1 through Proposition 3 holds if* $(\bar{\lambda}, t, \bar{c}, \alpha)$ *takes the specific form of* $(\tilde{\lambda}, 1, 1, 1)$.

Proof of Lemma A6. We start by proving the second part of Proposition 1 that gives conditions for a region not to be supplied, and then move to all others.

Second half of Proposition 1. Here, we are looking for conditions on potential demand $\bar{\lambda}_i$ such that the platform can get a positive number of drivers into region i without sustaining any loss. Note that from our analysis before, we know the region can get a positive number of drivers if:

$$(\frac{c_i}{\bar{c}})^2 \geq \frac{4t_i}{\bar{\lambda}_i(1-\alpha p_i)}$$

From $\bar{c} = t_i = \alpha = 1$ and $\bar{\lambda}_i = \tilde{\lambda}_i$, this turns into:

$$c_i^2 \geq \frac{4}{\tilde{\lambda}_i(1-p_i)}$$

Or, equivalently:

$$\tilde{\lambda}_i \geq \frac{4}{c_i^2(1-p_i)}$$

As such, the minimum requirement volume for $\tilde{\lambda}_i$ depends on the maximum possible value for $c_i^2(1-p_i)$ subject to the constraint that the platform does not run any losses. This happens when $p_i = c_i$. Therefore, the region will get a positive number of drivers if and only if:

$$\tilde{\lambda}_i \geq \frac{4}{\max_{c_i} c_i^2(1-c_i)} = 27$$

This says each region i gets a positive number of drivers if and only if $\tilde{\lambda}_i \geq 27$ which proves the second half of Proposition 1.

Existence of equilibrium in Lemma 2. Now we prove the statement on the existence of the equilibrium in Lemma 2. The proof above shows that if $\tilde{\lambda} < 27$, then the maximum profit is always 0 and it is trivial that (p^*, c^*) exists and $n^* = 0$ exists for $\tilde{\lambda} < 27$. As such, we focus on the case where $\tilde{\lambda} \geq 27$. We show the existence by dividing the parameter space $\mathbf{R}^2$ for (p, c) into four subsets and show that the maximum of the profit function exists in each case. Then combining all cases, there exists (p^*, c^*) and corresponding n^* that maximizes the profit function.

Case (1): If $c_i \leq 0$, then drivers receive zero revenue and no drivers operate in the platform and no rides realized. Thus the equilibrium is $p_i^* \in \mathbf{R}$, $c_i^* \leq 0$, $n_i^* = 0$, which gives $\pi_i^* = 0$.

Case (2): If $p_i \geq 1$, then there is no demand and no rides are realized. Drivers receive zero revenue from the platform and thus the equilibrium is achieved for $p_i^* \geq 1$, $c_i^* \in \mathbf{R}$, $n_i^* = 0$, which gives $\pi_i^* = 0$.

Now we discuss the remaining parameter space for (p, c) where $p_i < 1$ and $c_i > 0$. Recall that the wait time as a function of number of drivers is given by

$$W(n_i) = \frac{n_i}{\tilde{\lambda}_i(1-p_i)} + \frac{1}{n_i}$$

The function $W(n_i)$ (with $n_i > 0$) is decreasing in $(0, \sqrt{\tilde{\lambda}_i(1-p_i)})$, increasing in $(\sqrt{\tilde{\lambda}_i(1-p_i)}, \infty)$ and minimized at $n_i = \sqrt{\tilde{\lambda}_i(1-p_i)}$. The minimum is $W(\sqrt{\tilde{\lambda}_i(1-p_i)}) = \frac{2}{\sqrt{\tilde{\lambda}_i(1-p_i)}}$.

Case (3): If $p_i < 1$ and $0 < c < \frac{2}{\sqrt{\tilde{\lambda}_i(1-p_i)}}$, then $c \le \min_{n_i>0} W(n_i)$. Therefore, $\frac{c}{W(n_i)} < 1 = \bar{c}$, $\forall n_i$. That is, the driver revenue is always lower than the reservation wage. So $n_i^* = 0$ and $\pi_i^* = 0$. The equilibrium is achieved with $(p_i^*, c_i^*) \in \{(p,c) : p < 1, 0 < c < \frac{2}{\sqrt{\tilde{\lambda}_i(1-p_i)}}\}$ and $n_i^* = 0$, which gives $\pi_i^* = 0$.

Case (4): If $p < 1$ and $c \ge \frac{2}{\sqrt{\tilde{\lambda}_i(1-p_i)}}$, then as the roof of Lemma A1 shows, only the larger solution of the equation $W(n_i) = c$ is the equilibrium. The equilibrium number of drivers is given by

$$n_i^*(p_i, c_i) = \frac{c_i + \sqrt{c_i^2 - \frac{4}{\tilde{\lambda}(1-p_i)}}}{\frac{2}{\tilde{\lambda}(1-p_i)}}$$

And the profit function is given by

$$\pi_i(p_i, c_i) = (p_i - c_i)\frac{n_i(p_i, c_i)}{c_i}$$

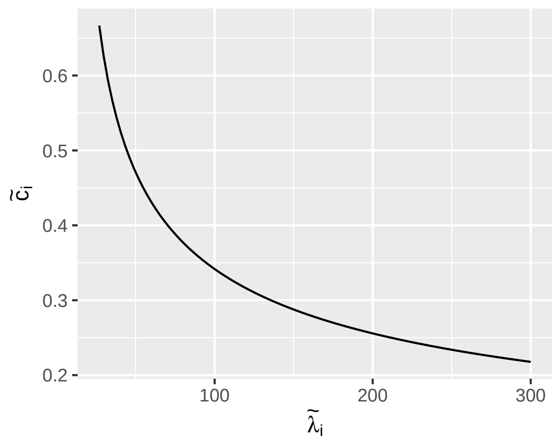
Considering that in all previous cases we have discussed above, it is at least feasible to achieve zero profit. Therefore, to find the maximum, we focus on the parameter space that leads to non-negative profit. As such, we focus on the (p_i, c_i) such that $p \ge c$ and consider the set $\mathcal{X} = \{(p_i, c_i) : 0 \le p_i \le 1, c_i \le p_i, c_i \ge \frac{2}{\sqrt{\tilde{\lambda}_i(1-p_i)}}\}$. The set is non-empty given $\tilde{\lambda} \ge 27$.

Given that the profit function is continuous and the set $\mathcal{X}$ is closed and bounded, there exists (p_i^*, c_i^*) that maximizes the profit function. And it can be seen that the maximum is non-negative (e.g. pick any $(p_i, c_i) \in \mathbf{X}$ with $p_i = c_i$). Therefore, combining all the cases above, the maximum of the profit function always exists, and thus (p^*, c^*, n^*) exist.

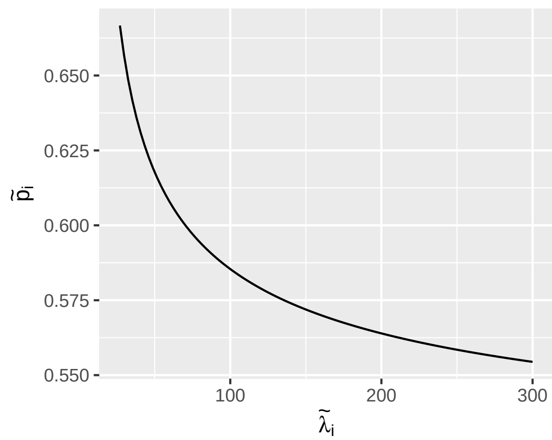
Next, we prove the remaining part of Lemma 2, Proposition 1 and Proposition 3 numerically. To do this, we need to study the equilibrium price $\tilde{p}_i$, equilibrium wage $\tilde{c}_i$, equilibrium number of drivers $\tilde{n}_i$, and equilibrium access $\tilde{A}_i$ in a given region i as a function of $\tilde{\lambda}_i$. The key is that all of the other parameters are equal to 1 and $\tilde{\lambda}_i$ is the only parameter changing. Therefore, for any value that $\tilde{\lambda}_i$ assumes, we have a fully numerical problem that can be solved using a software such as R. Below, we are providing graphs of these equilibrium quantities as functions of $\tilde{\lambda}_i$. Fig. 15 provides these results. First, the figure gives the unique optimal price and wage computed for $\tilde{\lambda}_i \ge 27$, which shows that Lemma 2 holds. Second, access $\tilde{A}_i$ is strictly increasing in density. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a lower access to rides in the equilibrium. This completes the proof of Proposition 1. Additionally, as shown in the figure, optimal price $\tilde{p}_i$ and optimal wage $\tilde{c}_i$ both strictly decrease in density $\tilde{\lambda}_i$. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a higher price and a higher wage in the equilibrium. Also margin $\tilde{p}_i - \tilde{c}_i$ is strictly increasing in density. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a lower margin in the equilibrium. These shows that Proposition 3 holds.

Figure 15: Numerical results that are necessary to prove Lemma A6.

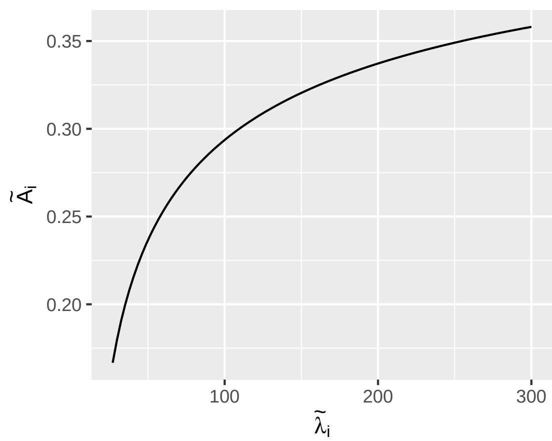
(a) Response of wage to density



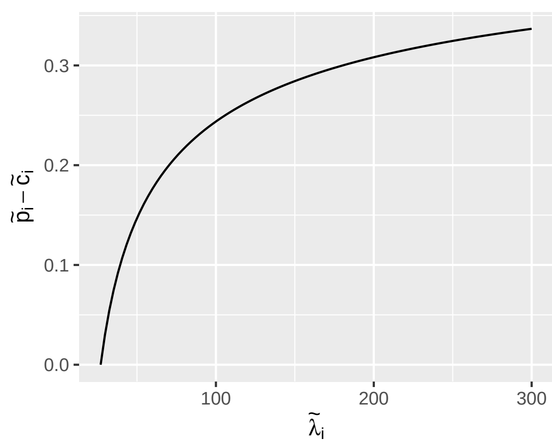
(b) Response of price to density



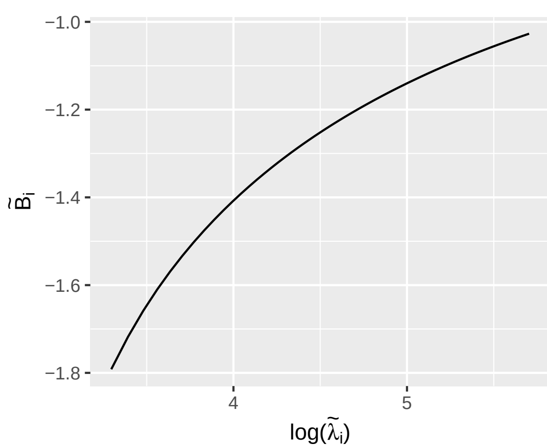
(c) Response of access to density



(d) Response of margin (price less wage) to density



(e) Response of log access to log density



As for Proposition 2, statement 1 follows directly from the second half of Proposition 1. To prove statement 2, first note that the second part of this statement (i.e., $\tilde{\lambda}_i > \tilde{\lambda}_j \Rightarrow \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} < 1$) is implied by the first half of Proposition 1, which we have already proved. We, hence, will only prove:

$$\tilde{\lambda}_i > \tilde{\lambda}_j \Rightarrow \frac{A_j(n_j^*)}{A_i(n_i^*)} < \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})}$$

In order to prove this statement, we first state and prove a lemma. Recall, again, that given the optimization problems in different regions are separate from each other, the equilibrium access $A_i(n_i^*)$ is only a function of $\tilde{\lambda}_i, p_i, c_i$ rather than those at other regions. Therefore, the notation i may be suppressed. In the proofs of some of the last statements we have even suppressed the notation on n^* and denote the equilibrium access in region i simply by A. In our next lemma, we will keep suppressing those notations. We also note that access has been an implicit function of model parameters, in particular $\tilde{\lambda}_i$. In the proof of this lemma, we make the exposition of the dependence of equilibrium access A on parameter $\tilde{\lambda}_i$ explicit by denoting the equilibrium access as $A(\tilde{\lambda}_i)$. In other words, $A(\tilde{\lambda}_i)$ is the equilibrium access in a region with demand $\tilde{\lambda}_i$ when the platform has set the optimal wage c^* and price p^* given density $\tilde{\lambda}_i$ and drivers respond accordingly.

Lemma A7. *Define function $B(\tau)$ as*

$$B(\tau) \equiv \log(A(e^{\tau}))$$

The second statement of Proposition 2 holds if B is a strictly convcave function whenever defined.

Proof of Lemma A7. First notice that B is defined whenever $A > 0$ which is whenever τ is larger than some cutoff.

Next, take two regions i, j with $\tilde{\lambda}_i > \tilde{\lambda}_j$. Set $\tau_i = \log(\tilde{\lambda}_i)$ and $\tau_j = \log(\tilde{\lambda}_j)$. Also set $\beta = \log(\gamma)$ where γ is the scalar larger than one in the statement of the proposition. By strict concavity of B, we have:

$$B(\tau_i + \beta) - B(\tau_i) < B(\tau_j + \beta) - B(\tau_j)$$

This implies:

$$\log[A(e^{\tau_i+\beta})] - \log[A(e^{\tau_i})] < \log[A(e^{\tau_j+\beta})] - \log[A(e^{\tau_j})]$$

$$\Rightarrow \log[\frac{A(e^{\tau_i+\beta})}{A(e^{\tau_i})}] < \log[\frac{A(e^{\tau_j+\beta})}{A(e^{\tau_j})}]$$

$$\Rightarrow \frac{A(\gamma\tilde{\lambda}_i)}{A(\tilde{\lambda}_i)} < \frac{A(\gamma\tilde{\lambda}_j)}{A(\tilde{\lambda}_j)}$$

$$\Rightarrow \frac{A(\tilde{\lambda}_j)}{A(\tilde{\lambda}_i)} < \frac{A(\gamma\tilde{\lambda}_j)}{A(\gamma\tilde{\lambda}_i)}$$

which is exactly the statement we needed to prove. This completes the proof of the lemma. □

Therefore, to prove the second statement of Proposition 2 the only thing we need to prove is strict concavity of the function B. This can be seen from the last panel in Fig. 15 which shows that log of access is concave in log of density.

Finally, statement 3 of Proposition 2 is straightforward and left to the reader (the key would be to note that as the market gets sufficiently large, pickup times in all regions become negligible; hence the pricing problem in all regions boils down to that in a model without economies of density, giving all regions the same price, wage, and access to rides).

This finishes the proof of Lemma A6 and, hence, the proofs for all lemma/propositions of the theory model with supply-side EOD (i.e. Lemma 2 and Proposition 1 through Proposition 3).■

C.2 Proofs of the theory model with demand-side EOD

This appendix proves Proposition 4, which states that an equivalence of Lemma 2 and Proposition 1 through Proposition 3 holds under the theory model with demand-side EOD. The proof strategy is largely similar to that for the model with supply-side EOD, as discussed in Appendix C.1. We prove the propositions in a unified framework, where the propositions can be reduced to a numerical problem. Specifically, Lemma A8 presents similar statements to Lemma A1 through Lemma A3 and shows the sets of primitives that leads to equivalent equilibrium. Lemma A9 and Lemma A10 resemble Lemma A4 and Lemma A5 and states that the proposition can be reduced to a numerical problem, where the proposition holds if it holds for a specific set of primitives. Finally, Lemma A11 shows that the proposition does hold for the set of primitives.

Lemma A8. *Assume $\gamma \in \mathbb{R}$ is a strictly positive number. Then the following statements hold: is equivalent to any of the following statements:*

1. *Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, vectors p^*, c^* and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha', \beta') = (\gamma\bar{\lambda}, \gamma t, \alpha, \beta, \bar{c})$, vectors $p^{*'} = p^*$, $c^{*'} = c^*$ and $n^{*'} = \gamma n^*$ constitute an equilibrium. Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

2. *Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, vectors p^*, c^* and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha', \beta') = (\frac{1}{\gamma}\bar{\lambda}, t, \bar{c}, \frac{1}{\gamma}\alpha, \beta)$, vectors $p^{*'} = \gamma p^*$, $c^{*'} = \gamma c^*$ and $n^{*'} = n^*$ constitute an equilibrium. Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

3. *Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, vectors p^*, c^* and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}, \frac{1}{\gamma}t, \bar{c}, \alpha, \gamma\beta)$, vectors $p^{*'} = p^*$, $c^{*'} = c^*$ and $n^{*'} = n^*$ constitute an equilibrium. Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

4. *Under primitives* $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$*, vectors* p^**,* c^* *and* n^* *constitute an equilibrium if and only if under primitives* $(\gamma\bar{\lambda}, t, \gamma\bar{c}, \alpha, \beta)$*, vectors* $p^{*'} = p^*$*,* $c^{*'} = c^*$ *and* $n^{*'} = n^*$ *constitute an equilibrium. Additionally, access to rides in all regions are equal under these two equilibria:* $\forall i : A_i^{*'} = A_i^*$*.*

Proof. As the proofs of the statements are very similar, we prove the first statement and skip the rest.

Under the model with demand-side EOD, we denote the equilibrium number of drivers in region i under primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ and platform strategy (p, c) by:

$$n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)$$

Similarly, we denote the platform profit of region i by:

$$\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)$$

Note that the equilibrium number of drivers in each region is given by

$$\frac{n}{\bar{\lambda}(1-\alpha p)(1-\beta\frac{t}{n})} = \frac{c}{\bar{c}}$$

Solving the equation, we get two solutions:

$$n^* = \frac{1 \mp \sqrt{1 - \frac{4\beta t}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}}{\frac{2}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}$$

The equilibrium number of drivers n^* takes real value if and only if $4\beta t \le \frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)$. Now we show that the region cannot get a positive number of drivers if $4\beta t_i > \frac{c_i}{\bar{c}}\bar{\lambda}_i(1-\alpha p_i)$.

The hourly wage for the rider is

$$\frac{c_i}{\frac{n_i}{\bar{\lambda}_i(1-\alpha p_i)(1-\frac{\beta t_i}{n_i})}} = \frac{c_i\bar{\lambda}_i(1-\alpha p_i)(1-\frac{\beta t_i}{n_i})}{n_i} < \frac{4\beta t_i(1-\frac{\beta t_i}{n_i})}{n_i}\bar{c} = \frac{4\beta t_i n_i - 4\beta^2 t_i^2}{n_i^2}\bar{c}$$

Given $(n_i - 2\beta t_i)^2 \ge 0$, we have $\frac{4\beta t_i n_i - 4\beta^2 t_i^2}{n_i^2} < 1$. So when $\frac{c_i}{\bar{c}}\bar{\lambda}_i(1-\alpha p_i) < 4\beta t_i$, the hourly wage for working on the ride sharing platform is lower than $\bar{c}$. Therefore, the region cannot get a positive number of drivers.

Note that only the larger solution (i.e., the one with the + sign) is an equilibrium because at the lower solution, further driver entry will lead to lower overall wait time, increasing driver revenue. As such, we have:

$$n^* = \frac{1 + \sqrt{1 - \frac{4\beta t}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}}{\frac{2}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}} \tag{20}$$

We now turn to the proof of the lemma, starting with a claim:

Claim 2. *For any given (p, c) where region i attracts no drivers under $(\bar{\lambda}, t, \bar{c}, \alpha)$, we have:*

$$\pi_i(p, c|\gamma^2\bar{\lambda}, t, \gamma\bar{c}, \alpha) = \gamma\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha).$$

Proof of Claim 2. To see this, note that under $(\gamma\bar{\lambda}, \gamma t, \alpha, \beta, \bar{c})$, from eq. (20), region i will attract no drivers –hence will generate zero profit– if any only if:

$$4\beta t > \frac{c}{\bar{c}}\bar{\lambda}(1 - \alpha p)$$

The corresponding condition under $(\gamma\bar{\lambda}, \gamma t, \alpha, \beta, \bar{c})$ will be:

$$4\beta\gamma t > \frac{c}{\bar{c}}\gamma\bar{\lambda}(1 - \alpha p)$$

The above two conditions are equivalent. This finishes the proof of the claim. □

Now let us assume p and c are such that region i does get a positive number of drivers. There, the number of drivers present in i under $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ will be uniquely give by:

$$n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta) = \frac{1 + \sqrt{1 - \frac{4\beta t}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}}{\frac{2}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}$$

The same quantity under $(\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta)$ will be uniquely give by:

$$n_i^*(p, c|\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta) = \frac{1 + \sqrt{1 - \frac{4\beta t}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}}{\frac{2}{\frac{c}{\bar{c}}\gamma\bar{\lambda}(1-\alpha p)}} = \gamma\frac{1 + \sqrt{1 - \frac{4\beta t}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}}}{\frac{2}{\frac{c}{\bar{c}}\bar{\lambda}(1-\alpha p)}} = \gamma n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)$$

Also note that the equilibrium driver wait time W_i in region i under $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ has to be $\frac{c_i}{\bar{c}}$ in order to equate driver revenues in the region with the reservation value. Similarly, under $(\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta)$, the driver wait time in region i will be $\frac{c_i}{\bar{c}}$.

Next, note that for any p, c:

$$\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha) = (p_i - c_i) \times \frac{n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)}{\frac{c_i}{\bar{c}}}$$

Also:

$$\pi_i(p, c|\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta) = (p_i - c_i) \times \frac{n_i^*(p, c|\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta)}{\frac{c_i}{\gamma\bar{c}}}$$

$$= (p_i - c_i) \times \frac{\gamma n_i^*(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)}{\frac{c_i}{\bar{c}}} = \gamma\pi_i(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta)$$

Therefore, the profit functions under $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ and $(\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta)$ are fully proportional in each region, which mean they will have the exact same maximizers p^* and c^*. Given these maximizers, we will have the number of drivers in each region i under $(\gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta)$ will be given by:

$$n_i^{*'} \equiv n_i^*(p^*, c^* | \gamma\bar{\lambda}, \gamma t, \bar{c}, \alpha, \beta) = \gamma n_i^*(p^*, c^* | \bar{\lambda}, t, \bar{c}, \alpha, \beta) \equiv \gamma n_i^*$$

Finally:

$$A_i^{*'} = \frac{n_i^{*'}}{(\frac{c_i^*}{\bar{c}}) \times \gamma\bar{\lambda}_i} = \frac{\gamma n_i^*}{(\frac{c_i^*}{\bar{c}}) \times \gamma\bar{\lambda}_i} = \frac{n_i^*}{(\frac{c_i^*}{\bar{c}}) \times \bar{\lambda}_i} = A_i^*$$

The proof of the first statement of the lemma is now complete. □

The next lemma combines the statements of the previous one.

Lemma A9. *Under primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, vectors p^*, c^*, and n^* constitute an equilibrium if and only if under primitives $(\bar{\lambda}', t', \bar{c}', \alpha', \beta') = (\frac{1}{\alpha\beta\bar{c}t}\bar{\lambda}, 1, 1, 1, 1)$, vectors $p^{*'} = \alpha p^*$, $c^{*'} = \alpha c^*$, $n^{*'} = \frac{1}{\beta t} n^*$ constitute an equilibrium. Additionally, access to rides in all regions are equal under these two equilibria: $\forall i : A_i^{*'} = A_i^*$.*

Proof. Apply part 3 of Lemma Lemma A8 with $\gamma = \frac{1}{\beta}$, then part 1 of Lemma Lemma A8 with $\gamma = \frac{1}{\beta t}$, part 2 of Lemma Lemma A8 with $\gamma = \alpha$, and then part 4 of Lemma Lemma A8 with $\gamma = \frac{1}{\bar{c}}$.

With the previous lemma in hand, as in the proofs for the model with supply-side EOD, we turn to the two main lemmas that prove Proposition 4.

Lemma A10. *Proposition 4 holds under all generic primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ if it holds under primitives in the form of $(\tilde{\lambda}, 1, 1, 1, 1)$.*

Proof of Lemma A10. The proof is very similar to Lemma A5 with the model with supply-side EOD. Suppose Proposition 4 holds under primitives in the form of $(\tilde{\lambda}, 1, 1, 1, 1)$. That is, Lemma 2 and Proposition 1 through Proposition 3 hold under the model with demand-side EOD. We show that, still with the demand-side EOD model, the lemma and the propositions hold for a general $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$. To this end, choose vector $\tilde{\lambda}$ such that $\tilde{\lambda} = \frac{1}{\alpha\beta\bar{c}t}\bar{\lambda}$. Next, we prove each lemma/proposition for general primitives. We suppress the regional indices below.

Lemma 2:

Given that Lemma 2 holds under $(\tilde{\lambda}, 1, 1, 1, 1)$, there exists equilibrium $(\tilde{p}, \tilde{c}, \tilde{n})$ under $(\tilde{\lambda}, 1, 1, 1, 1)$. By Lemma A9, there also exists equilibrium (p^*, c^*, n^*) under $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, which is given by $p^* = \frac{1}{\alpha}\tilde{p}$, $c^* = \frac{1}{\alpha}\tilde{c}$ and $n^* = (\beta t)\tilde{n}$. Then for $n^* > 0$, we have $\tilde{n}_i > 0$. Then p^* and c^* are unique if $n^* > 0$ because $\tilde{n}_i > 0$ and $(\tilde{p}, \tilde{c})$ are unique.

Proposition 1:

First we show the first half of the proposition that speaks to $n_i^* \neq 0 \neq n_j^*$ holds for the generic primitives. By assumption, this statement holds under $(\tilde{\lambda}, 1, 1, 1, 1)$. By the construction of $\tilde{\lambda}$, it is immediate that:

$$\tilde{\lambda}_i > \tilde{\lambda}_j \Leftrightarrow \frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j}$$

Also by Lemma A9, we know that equilibrium access to rides in each region i is the same under $(\tilde{\lambda}, 1, 1, 1)$ and $(\bar{\lambda}, t, \bar{c}, \alpha)$. It immediately follows that the first half of the statement holds under $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$.

For the second half of the proposition that speaks to the regions without supply, by assumption, we know there is a μ such that:

$$\forall i : \frac{\tilde{\lambda}_i}{1} < \mu \Leftrightarrow \tilde{n}_i = 0$$

By construction, we know $\tilde{\lambda}_i < \mu$ is equivalent to $\frac{\bar{\lambda}_i}{t_i} < \alpha\beta\bar{c}\mu$. Also, Lemma A8, $\tilde{n}_i = 0$ is equivalent to $\frac{1}{\beta t_i} n_i^* = 0 \Leftrightarrow n_i^* = 0$. Therefore, we have:

$$\forall i : \frac{\bar{\lambda}_i}{t_i} < \alpha\beta\bar{c}\mu \Leftrightarrow n_i^* = 0$$

This finishes the proof of Proposition 1 under the generic primitives in the model setting with demand-side EOD.

The logic for the proof that Proposition 2 and Proposition 3 hold under generic case if they hold for $(\tilde{\lambda}, 1, 1, 1, 1)$ is identical to that in the proof of Lemma A5. As such, we skip the rest of the proof.

The proof of Lemma A10 is now complete. □

The last step in proving Proposition 4 is to show that it does indeed hold under $(\tilde{\lambda}, 1, 1, 1, 1)$. As the proof of Lemma A6 for the model with supply-side EOD, this last step is done fully numerically. The following lemma takes on this task.

Lemma A11. *Proposition 4 holds if $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ takes the specific form of $(\tilde{\lambda}, 1, 1, 1, 1)$.*

Proof of Lemma A10. We start by proving the second part of Proposition 1 in the context of model with demand-side EOD, and then move to all others.

Second half of Proposition 1. Here, we are looking for conditions on potential demand $\bar{\lambda}_i$ such that the platform can get a positive number of drivers into region i without sustaining any loss. Note that from eq. (20), we know the region can get a positive number of drivers if:

$$\frac{c_i}{\bar{c}}\bar{\lambda}_i(1 - \alpha p_i) > 4\beta t_i$$

Plugging in $\bar{c} = t_i = \alpha = \beta = 1$ and $\bar{\lambda}_i = \tilde{\lambda}_i$, this turns into:

$$c_i\tilde{\lambda}_i(1 - p_i) \geq 4$$

Or, equivalently,

$$\tilde{\lambda}_i \geq \frac{4}{c_i(1 - p_i)}$$

Then the minimum requirement for $\tilde{\lambda}_i$ depends on the maximum possible value for $c_i(1 - p_i)$ subject to the constraint that $p_i \geq c_i$. This happens when $p_i = c_i$. There fore, the region will get a positive number of drivers if and only if

$$\tilde{\lambda}_i \geq \frac{4}{\max_{c_i} c_i(1-c_i)} = 16$$

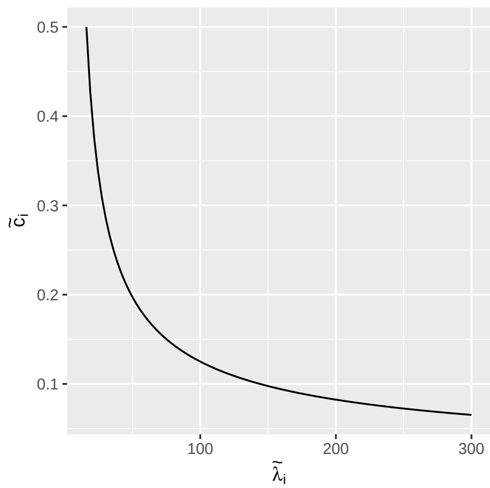
This proves the second part of Proposition 1 for the model with demand-side EOD.

The proof of the existence of the equilibrium in Lemma 2 is also analogous to the proof in the model with supply-side EOD. So we skip the details here. Now, we prove Lemma 2, Proposition 1 and Proposition 3 numerically with the model with demand-side EOD. To do this, we need to study the equilibrium price $\tilde{p}_i$, equilibrium wage $\tilde{c}_i$, equilibrium number of drivers $\tilde{n}_i$, and equilibrium access $\tilde{A}_i$ in a given region i as a function of $\tilde{\lambda}_i$. Similar to the proof of Lemma A6, the key is that all of the other parameters are equal to 1 and $\tilde{\lambda}_i$ is the only parameter changing. Therefore, for any value that $\tilde{\lambda}_i$ assumes, we have a fully numerical problem that can be solved using a software such as R. Fig. 16 provides graphs of these equilibrium quantities as functions of $\tilde{\lambda}_i$.

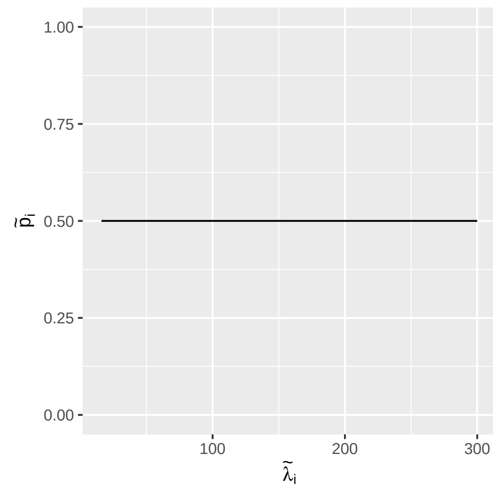
As shown in the figure, unique optimal price and wage are computed for $\tilde{\lambda}_i \geq 16$, which shows that Lemma 2 holds. Second, access $\tilde{A}_i$ is strictly increasing in density. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a lower access to rides in the equilibrium. This, together with the proof of the second half of Proposition 1 given above, completes the proof of Proposition 1. Additionally, optimal wage $\tilde{c}_i$ strictly decreases in density $\tilde{\lambda}_i$, while optimal price $\tilde{p}_i$ remains constant. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a higher wage in the equilibrium. Also margin $\tilde{p}_i - \tilde{c}_i$ is strictly increasing in density. This means if two regions i, j are such that $\tilde{\lambda}_i > \tilde{\lambda}_j$, then region j will have a lower margin in the equilibrium. These shows that Proposition 3 holds with an exception that the optimal price $\tilde{p}_i$ does not change with $\tilde{\lambda}_i$.

Figure 16: Numerical results that are necessary to prove Lemma A11.

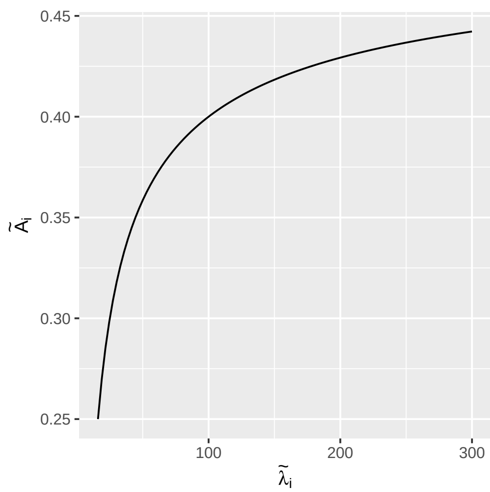
(a) Response of wage to density



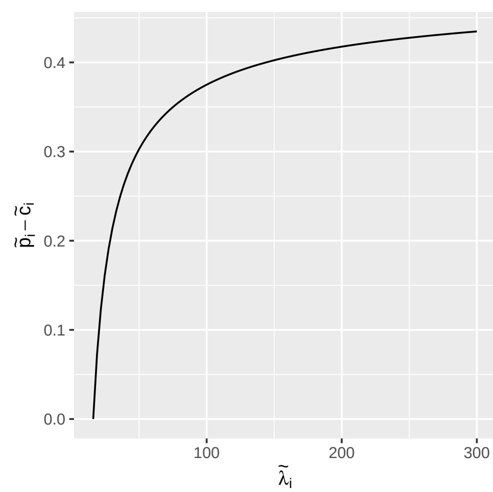
(b) Response of price to density



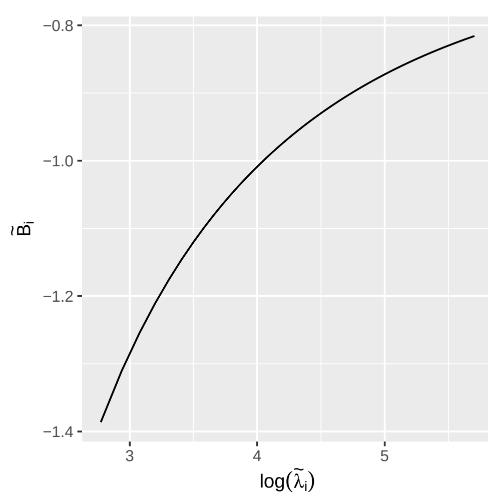
(c) Response of access to density



(d) Response of margin (price less wage) to density



(e) Response of log access to log density



Before showing that Proposition 2 holds with the model with demand-side EOD, we first formalize the result of constant optimal price under $(\tilde{\lambda}, 1, 1, 1, 1)$ in the lemma below and provide the proof of it.

Lemma A12. *Under the theory model with demand-side EOD, for any generic primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$, if the region i has a positive number of drivers under optimal price and wage, then the optimal price for the region is $\frac{1}{2\alpha}$.*

Proof of Lemma A12. From eq. (20), given the primitives $(\bar{\lambda}, t, \bar{c}, \alpha, \beta)$ and (p, c), the equilibrium number of drivers for a region can be written as

$$n(p, c|\bar{\lambda}, t, \bar{c}, \alpha, \beta) = \frac{k + \sqrt{k^2 - 4\beta tk}}{2}, where\ k = \frac{c}{\bar{c}}\bar{\lambda}(1 - \alpha p)$$

To maximize the profit, the problem that the platform solves is the following:

$$(c^*, p^*) = \arg\max_{c,p} \pi(c, p) := (p - c) \times \frac{n^*(p, c)}{c}$$

Take the first-order conditions:

$$\frac{\partial \pi}{\partial p} = \frac{n^*}{c/\bar{c}} + (p - c)\frac{1}{c/\bar{c}}\frac{\partial n^*}{\partial p} = 0 \tag{21}$$

$$\frac{\partial \pi}{\partial c} = -\frac{n^*}{c/\bar{c}} + (p - c)\frac{\frac{\partial n^*}{\partial c}c/\bar{c} - n^*\frac{1}{\bar{c}}}{(c/\bar{c})^2} = 0 \tag{22}$$

From eq. (21), we have

$$n - (p - c)\frac{\partial n^*}{\partial k}\frac{c}{\bar{c}}\bar{\lambda}\alpha = 0 \tag{23}$$

From eq. (22), we have

$$-n^*c + (p - c)(\frac{\partial n^*}{\partial c}c - n^*) = 0$$

$$-n^*p + (p - c)\frac{\partial n^*}{\partial k}\frac{c}{\bar{c}}\bar{\lambda}(1 - \alpha p) = 0 \tag{24}$$

Adding $(1 - \alpha p)$ times eq. (23) and α times eq. (24), we have

$$(1 - \alpha p)n^* - \alpha p n^* = 0$$

$$p = \frac{1}{2\alpha}$$

This finished the proof of the lemma. □

Next we prove that Proposition 2 holds under the model with demand-side EOD. Similar to the model with supply-side EOD, statement 1 follows directly from the second half of Proposition 1. As for statement 2, the second part of this statement (i.e., $\tilde{\lambda}_i > \tilde{\lambda}_j \Rightarrow \frac{A_j(n_j^{*\prime})}{A_i(n_i^{*\prime})} < 1$) is implied by Proposition 1. Then applying Lemma A7, the first part of statement 2 on the role of market thickness holds if $B(\tau) = log(A(e^{\tau}))$ is a strictly concave function. And this is shown in the last plot of Fig. 16.

To see statement 3, from Lemma A12, we have $p^* = \frac{1}{2}$. Then the equilibrium number of drivers of region i is given by

$$\frac{n_i}{\tilde{\lambda}_i(1-\frac{1}{2})(1-\frac{1}{n_i})} = c_i$$

$$n_i^* = \frac{\frac{c_i\tilde{\lambda}_i}{2} + \sqrt{\frac{c_i^2\tilde{\lambda}_i^2}{4} - 2c_i\tilde{\lambda}_i}}{2}$$

As $\tilde{\lambda}_i$ gets larger, n_i^* gets larger. Also note that $\tilde{\lambda}_i$ and n_i^* have the same order. So for any given c_i, the equilibrium number of rides $r_i = \frac{n_i}{\frac{n_i}{\tilde{\lambda}_i} + \frac{t_i}{n_i}}$ approaches infinity as the market gets larger. Therefore, the optimal wage approaches 0. The access ratio of any region approaches to 1.

This finishes the proof of *Proposition* 4. ■

C.3 Proof of Proposition 5

First suppose n^* is *not* a driver equilibrium in the version of the problem with no inter-region rides. This, by definition, means that either a small number δ_0 of new drivers can join one of the regions i from outside and strictly improve their payoffs, of a small δ_0 can relocate from i to i' or to the outside option and have a strictly positive improvement. In that case, one can find the corresponding small perturbation in ρ that would in a given amount of time, change n^* in a way that would increase the payoff in those regions. Similarly, if we assume (n^*, ρ^*) with (for a ρ^* that guarantees stability) is not an equilibrium of the version of the problem with cross-region rides, one can use the small profitable perturbation in ρ^* and use them to construct changes in n^* that yield strictly improved payoffs in some regions in the original version. This finishes the proof of the proposition. ■

D Proofs of Propositions in the Empirical Sections

This appendix supplies proofs relevant in the empirical sections. Appendix D.1 gives the proof of Proposition 6. Appendix D.2 formalizes the statement of the identification results for the model with supply-side EOD, followed by the proof of it. Appendix D.3 formalizes the identification results for the model with demand-side EOD.

D.1 Proof of Proposition 6

This section gives the proof of the Proposition 6 that states the relationship between access ratio and relative outflow. The proof is straightforward and follows directly from the assumptions.

Proof of Proposition 6. Let (n, ρ) be the steady-state driver allocation.

$$\frac{A_i(n)}{A_j(n)} = \frac{r_i/\bar{\lambda}_i}{r_j/\bar{\lambda}_j} = \frac{r_{ij}/\bar{\lambda}_{ij}}{r_{ji}/\bar{\lambda}_{ji}} = \frac{r_{ij}}{r_{ji}}$$

The second equality holds because $\frac{r_{ij}}{\bar{\lambda}_{ij}} = \frac{r_i}{\bar{\lambda}_i}$ and $\frac{r_{ji}}{\bar{\lambda}_{ji}} = \frac{r_j}{\bar{\lambda}_j}$. The third equality holds because $\bar{\lambda}_{ij} = \bar{\lambda}_{ji}$. ■

D.2 Proof of identification of the model with supply-side EOD

This appendix gives a formal statement of the condition of the identification of the parameters in the empirical model with supply-side EOD, followed by the proof. The condition relies on specific data patterns, which are satisfied in the data used for analysis.

First, following the discussion of the identification in the main text, recall that each $\bar{\lambda}_{id}$ can be written as an explicit function of $\mathcal{D}$, a and α as in eq. (10). Therefore, the identification problem reduces to identify α and a from the RO moments. We consider the relative outflow across regions for a single day d. For notational brevity, we drop the day subscript d. The relative outflow derived from the model can be written as

$$RO_{ij}^{model} = \frac{A_i}{A_j} = \frac{r_i/\bar{\lambda}_i}{r_j/\bar{\lambda}_j} = \frac{\frac{n_i}{c_i}/\bar{\lambda}_i}{\frac{n_j}{c_j}/\bar{\lambda}_j} = \frac{(1-\alpha p_i)(1-a\frac{s_i\bar{c}}{n_i c_i})}{(1-\alpha p_j)(1-a\frac{s_j\bar{c}}{n_j c_j})} \tag{25}$$

where the first equality holds following Proposition 6 and the second equality holds given the definition of access. The third equality holds given the assumption of driver equilibrium distribution. The fourth equality is obtained if we substitute $\bar{\lambda}_i$ and $\bar{\lambda}_j$ with a, α and observable data $\mathcal{D}$ following eq. (10).

To uniquely identify the two parameters (α, a), it suffices to match two relative outflows between the data and the model. Therefore, the identification proof is reduced to showing that there exists two pairs of regions, (i, j) and (k, l),[33] such that the following system of equations has a unique solution for (a, α), given everything else is data:

$$RO_{ij}^{model} = \frac{(1-\alpha p_i)(1-a\frac{s_i\bar{c}}{n_i c_i})}{(1-\alpha p_j)(1-a\frac{s_j\bar{c}}{n_j c_j})} = RO_{ij}^{data}$$
$$RO_{kl}^{model} = \frac{(1-\alpha p_k)(1-a\frac{s_k\bar{c}}{n_k c_k})}{(1-\alpha p_l)(1-a\frac{s_l\bar{c}}{n_l c_l})} = RO_{kl}^{data}$$

The proposition below states the condition for the uniqueness of the solution and thus the identification:

Proposition 7. *Under the model with supply-side economies of density, the parameters are uniquely identified if the following condition holds:*

Denote $d_i = \frac{s_i\bar{c}}{n_i c_i}$. There exists two distinct pairs of regions, indexed by (i, j) and (k, l), where k may overlap with i or l may overlap with j, s.t. $(p_i - p_j)(d_i - d_j) > 0$ and $(p_k - p_l)(d_k - d_l) < 0$.

Equivalently, the condition implies that, among the two pairs of regions, one pair has the same ordering for (p_i, p_j) and (d_i, d_j), while the other pair has the opposite ordering. The implication

[33] Note that k or l may overlap with i or j. So we may need only three regions.

is in line with the intuition of independence between regional price variation and regional density variation that we show in the main text. We show the formal proof below.

Proof of Proposition 7. We show the proof of the uniqueness of the solution in the following steps:

Step 0: Note that given the actual demand, $\lambda_i = \bar{\lambda}_i(1 - \alpha p_i)$ is positive for any region i, the parameter α satisfies $\alpha < \min_{i \in I}\{\frac{1}{p_i}\}$. Also, given that, in equilibrium, the idle time can be expressed as $\frac{n}{\lambda_i} = \frac{c_i}{\bar{c}} - \frac{as_i}{n_i}$, the positive idle time implies that $a < \min_{i \in I}\{\frac{c_i n_i}{\bar{c} s_i}\}$. We denote $d_i = \frac{s_i \bar{c}}{n_u c_i}$. Then we have $a < \min_{i \in I}\{\frac{1}{d_i}\}$.

Step 1: Given that the condition, without loss of generality, we assume that $p_i < p_j$, $d_i < d_j$, $p_k < p_l$, and $d_k > d_l$.

Step 2: Consider a function $f(x; k_1, k_2) = \frac{1 - xk_1}{1 - xk_2}$, where $0 < x < \min\{\frac{1}{k_1}, \frac{1}{k_2}\}$. Then $\frac{\partial f(x)}{x} > 0$ if $k_1 < k_2$ and $\frac{\partial f(x)}{x} < 0$ if $k_1 > k_2$.

Denote $D_{ij} = \{p_i, p_j, d_i, d_j\}$ and $D_{kl} = \{p_k, p_l, d_k, d_l\}$. Then we can express RO_{ij}^{model} and RO_{kl}^{model} as functions of (a, α):

$$
\begin{aligned}
RO_{ij}^{model}(a, \alpha; D_{ij}) &= \frac{(1 - \alpha p_i)(1 - ad_i)}{(1 - \alpha p_j)(1 - ad_j)} = f(\alpha; p_i, p_j) f(a; d_i, d_j) \\
RO_{kl}^{model}(a, \alpha; D_{kl}) &= \frac{(1 - \alpha p_k)(1 - ad_k)}{(1 - \alpha p_l)(1 - ad_l)} = f(\alpha; p_k, p_l) f(a; d_k, d_l)
\end{aligned}
$$

Given $\alpha < \min_{i \in I}\{\frac{1}{p_i}\}$, $a < \min_{i \in I}\{\frac{1}{d_i}\}$, $p_i < p_j$, $d_i < d_j$, $p_k < p_l$, and $d_k > d_l$, we have

$$
\begin{aligned}
\frac{\partial RO_{ij}^{model}(a, \alpha; D_{ij})}{\partial \alpha} > 0, \frac{\partial RO_{kl}^{model}(a, \alpha; D_{kl})}{\partial \alpha} > 0 \\
\frac{\partial RO_{ij}^{model}(a, \alpha; D_{ij})}{\partial a} > 0, \frac{\partial RO_{kl}^{model}(a, \alpha; D_{kl})}{\partial a} < 0
\end{aligned}
\tag{26}
$$

Step 3: Suppose there exists a pair of (a, α) that satisfies the system of equations above. That is,

$$
\begin{aligned}
RO_{ij}^{model}(a, \alpha; D_{ij}) = RO_{ij}^{data} \\
RO_{kl}^{model}(a, \alpha; D_{kl}) = RO_{kl}^{data}
\end{aligned}
$$

We show by contradiction that there does not exist a distinct pair of (a', α') that satisfies the system of equations above, where at least one of $a = a'$ and $\alpha = \alpha'$ can not hold.

We consider the four cases below:

(1) Consider $a' > a$ and $\alpha' \geq \alpha$. Following the derivatives in eq. (26), we have $RO_{ij}^{model}(a', \alpha'; D_{ij}) > RO_{ij}^{model}(a, \alpha; D_{ij}) = RO_{ij}^{data}$. So (a', α') cannot be the solution.

(2) Consider $a' > a$ and $\alpha' < \alpha$. Following the derivatives in eq. (26), we have $RO_{kl}^{model}(a', \alpha'; D_{kl}) > RO_{kl}^{model}(a, \alpha; D_{kl}) = RO_{kl}^{data}$. So (a', α') cannot be the solution.

(3) Consider $a' < a$ and $\alpha' \geq \alpha$. Following the derivatives in eq. (26), we have $RO_{kl}^{model}(a', \alpha'; D_{kl}) < RO_{kl}^{model}(a, \alpha; D_{kl}) = RO_{kl}^{data}$. So (a', α') cannot be the solution.

(4) Consider $a' < a$ and $\alpha' < \alpha$. Following the derivatives in eq. (26), we have $RO_{ij}^{model}(a', \alpha'; D_{ij}) > RO_{ij}^{model}(a, \alpha; D_{ij}) = RO_{ij}^{data}$. So (a', α') cannot be the solution.

Therefore, there does not exist (a', α') that solves the system of equations with at least one of $a = a'$ and $\alpha = \alpha'$ can not hold. ■

Given we observe the pattern in the data with (i, j) and (k, l) being Manhattan and Queens, Bronx and Manhattan respectively, the parameters in the model with supply-side EOD are identified in this case.

Two remarks are worth mentioning. First, the proof relies on monotonicity of $f(x)$ given data. Therefore, it should be considered parametric identification. However, the same identification logic would essentially carry through for any non-parametric monotonic function. Second, in practice, we take the average of RO_{ij}^{model} and RO_{ij}^{data} across days. We do not require over-time variation to identify a and α.

D.3 Proof of identification of the model with demand-side EOD

Now we turn to the identification results for the model with demand-side EOD. We formalize the condition of the identification. The statement and the proof of the identification is analogous to those for the model with supply-side EOD. As such, we skip some of the details.

First, note that each $\bar{\lambda}_{id}$ can be written as an explicit function of $\mathcal{D}$, a and α as in eq. (14). Therefore, the identification problem reduces to identify α and β from the RO moments. Similar to the model with supply-side EOD, We consider the relative outflow across regions for a single day d. The relative outflow derived from the model can be written as

$$RO_{ij}^{model} = \frac{A_i}{A_j} = \frac{r_i/\bar{\lambda}_i}{r_j/\bar{\lambda}_j} = \frac{\frac{n_i}{c_i}/\bar{\lambda}_i}{\frac{n_j}{c_j}/\bar{\lambda}_j} = \frac{(1-\alpha p_i)(1-\beta\frac{s_i}{n_i})}{(1-\alpha p_j)(1-\beta\frac{s_i}{n_i})} \tag{27}$$

where the first equality holds following Proposition 6 and the second equality holds given the definition of access. The third equality holds given the assumption of driver equilibrium distribution. The fourth equality is obtained if we substitute $\bar{\lambda}_i$ and $\bar{\lambda}_j$ with a, α and observable data $\mathcal{D}$ following eq. (14).

To uniquely identify the two parameters (α, β), it suffices to match two relative outflows between the data and the model. Therefore, the identification proof is reduced to showing that there exists two pairs of regions, (i, j) and (k, l),[34] such that the following system of equations has a unique

[34]Note that k or l may overlap with i or j. So we may need only three regions.

solution for (α, β), given everything else is data:

$$RO_{ij}^{model} = \frac{(1-\alpha p_i)(1-\beta \frac{s_i}{n_i})}{(1-\alpha p_j)(1-\beta \frac{s_i}{n_i})} = RO_{ij}^{data}$$
$$RO_{kl}^{model} = \frac{(1-\alpha p_k)(1-\beta \frac{s_k}{n_k})}{(1-\alpha p_l)(1-\beta \frac{s_l}{n_l})} = RO_{kl}^{data}$$

The proposition below states the condition for the uniqueness of the solution and thus the identification:

Proposition 8. *Under the model with demand-side economies of density, the parameters are uniquely identified if the following condition holds:*

Denote $d_i = \frac{s_i}{n_i}$. There exists two distinct pairs of regions, indexed by (i, j) and (k, l), where k may overlap with i or l may overlap with j, s.t. $(p_i - p_j)(d_i - d_j) > 0$ and $(p_k - p_l)(d_k - d_l) < 0$.

The proof is analogous to that of Proposition 7. Thus, we skip the proof. Taking to the data, the condition is satisfied if we set (i, j) to be Manhattan and Queens and (k, l) to be Bronx and Queens.

E Other Versions of the Theoretical Model

In this appendix, we explore three other versions of the theoretical model with supply-side EOD. First, we study two models that are similar to the main text, where platform strategy is endogenized. The only difference is that only price *or* wage is endogenized, while the other is kept fixed and uniform across regions. Further, we look at a "degenerated" model where platform strategy on both prices and wages are fixed and the focus is on studying driver behavior. For each of the models, we characterize the equilibrium spatial distribution of supply and provide the proofs. Although the settings of the models in this section deviate from the assumptions we make in the main text, the models still predict similar results.

E.1 Model with endogenous wage and fixed price

In this subsection, we assume that the prices are fixed uniformly across regions and the platform chooses price to maximize the total profit. Besides, we assume that there is only supply-side EOD and that there are no inter-region rides. Also, note that, for this setting, we do not assume $f(p_i) = 1 - \alpha p_i$. Rather, we only need $\lambda_i = \bar{\lambda} f(p_i)$. The following proposition speaks both to the the characterization of the equilibrium spatial distribution of supply, and to how this distribution changes in response to a changed market thickness.

Proposition 9. *Suppose that prices are fixed at $p_i = p$ for all i and that market primitives are $(\bar{\lambda}, t)$. Also suppose the pair of vectors (c^*, n^*) is an equilibrium. Then, the following are true:*

1. *n^* is unique. Also c_i^* for any i with $n_i^* > 0$ is unique. In other words, if (c'^*, n'^*) is another equilibrium, we have $n^* = n'^*$, and for any i with $n_i^* > 0$, we have $c_i^* = c_i'^*$.*

2. *Regions that are not supplied are those with lowest demand densities:*

$$\exists \mu : \ s.t. \forall i : \ \frac{\bar{\lambda}_i}{t_i} < \mu \Leftrightarrow n_i^* = 0$$

3. *For any two regions i, j with $n_i^* \neq 0 \neq n_j^*$, we have:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow A_i(n_i^*) \geq A_j(n_j^*)$$

where the latter comparison holds with equality only if the first one does.

4. *For any regions i, j with $n_i^* \neq 0 \neq n_j^*$, we have:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow c_i^* \leq c_j^*$$

where the latter comparison holds with equality only if the first one does.

In addition, consider a thickening of the market from $(\bar{\lambda}, t)$ to $(\gamma\bar{\lambda}, t)$ where $\gamma > 1$. Suppose that $(c^{*'}, n^{*'})$ is an equilibrium under the new primitives $(\gamma\bar{\lambda}, t)$. Then, the following are true:

1. *For any i: $n_i^* > 0 \Rightarrow n_i^{*'} > 0$.*

2. *For any i, j:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow \frac{A_j(n_j^*)}{A_i(n_i^*)} \leq \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} \leq 1$$

and the latter two inequalities hold strictly only if the first one does.

3. *There will be equitable access to rides as the market gets sufficiently thick:*

$$\forall i, j : \ \lim_{\gamma \to \infty} \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} = 1$$

This proposition has three main messages. First, if wages are flexible, the platform's optimal strategy will involve wage incentives for drivers to operate in areas with lower densities of potential demand. This is, again, in line with the intuition that even though platforms, like drivers, find long pickup times undesirable, they would still like to intervene and make the distribution of drivers across regions less skewed toward busier areas. In other words, the platform will optimally try to "build economies of density" in sparser regions. The second message is that in spite of the platform's intervention, the equilibrium will still exhibit lower access to service in less dense areas compared to denser ones. The third message is that, similar to the case of exogenous and uniform wages, here too an increase in market thickness will lead to a more balanced supply.

In the first glance, the result in Proposition 9 may seem at odds with previously established results in the literature that the optimal response to high demand in a region is a wage increase in that region in order to encourage drivers to relocate to the said region and meet the demand Besbes et al. (2018). Note, however, that the result in Besbes et al. (2018) has to do with a *short term* local demand shock which could only be met if drivers in other regions are incentivized to incur the costs of relocation. Our model, however, is complementary in that it captures steady-state distribution of supply in the market and how it is impacted by economies of density, abstracting away from short run shocks. As such, driver location choice in our model should be thought of as a driver's general strategy for where to drive on a regular basis rather than a (costly) relocation from a region to another. Therefore, our result and the result in Besbes et al. (2018) are not inconsistent with each other. They are, rather, complements to each other, each shedding light on a different aspect of spatial pricing in the market.

E.2 Model with endogenous price and fixed wage

This subsection studies the case where wages are uniformly fixed and the platform is allowed to optimally decide the regional prices. The following proposition presents the characterization of the equilibrium outcomes. For this proposition, we assume $\lambda_i = \bar{\lambda} f(p_i)$ with $f(p_i) \equiv 1 - \alpha p_i$ and $\alpha > 0$.

Proposition 10. *Suppose that wages are fixed at $c_i = c$ for all i and that market primitives are $(\bar{\lambda}, t)$. Also suppose the pair of vectors (p^*, n^*) is an equilibrium. Then, the following are true:*

1. *n^* is unique. Also p_i^* for any i with $n_i^* > 0$ is unique. In other words, if (p'^*, n'^*) is another equilibrium, we have $n^* = n'^*$, and for any i with $n_i^* > 0$, we have $p_i^* = p_i'^*$.*

2. *Regions that are not supplied are those with lowest demand densities:*

$$\exists \mu : \ s.t. \forall i : \ \frac{\bar{\lambda}_i}{t_i} < \mu \Leftrightarrow n_i^* = 0$$

3. *For any regions i, j with $n_i^* \neq 0 \neq n_j^*$, we have:*

$$\frac{\bar{\lambda}_i}{t_i} \geq \frac{\bar{\lambda}_j}{t_j} \Rightarrow p_i^* \geq p_j^*$$

   where the latter comparison holds with equality only if the first one does.

In addition, consider a thickening of the market from $(\bar{\lambda}, t)$ to $(\gamma\bar{\lambda}, t)$ where $\gamma > 1$. Suppose that $(p^{*'}, n^{*'})$ is an equilibrium under the new primitives $(\gamma\bar{\lambda}, t)$. Then, the following are true:

1. *For any i: $n_i^* > 0 \Rightarrow n_i^{*'} > 0$.*

2. *There will be equitable access to rides as the market gets sufficiently thick:*

$$\forall i, j : \ \lim_{\gamma \to \infty} \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} = 1$$

Broadly, Proposition 10 has similar messages to those of Proposition 9. There are, however, two additional points worth noting about Proposition 10. First, unlike Proposition 9, Proposition 10 does *not* claim that as the market gets thicker, access to rides becomes more balanced across regions. One could construct a counter-example for such a claim in the case of fixed wages and endegenous prices. One can show that the comparative static result does hold if the platform is large enough (i.e., if γ is large enough).[35]

The second, and more crucial, point about Proposition 10 is how it should be understood as it relates to surge pricing. Proposition 10 suggests that prices should be higher regions with higher densities of potential demand. This outcome may seem in-line with what one would naturally expect in this spatial market and with results already established in the literature Besbes et al. (2018). Nevertheless, our result holds *only because of the network externalities that arise from pickup times.* That is, regions with higher demand density can attract more drivers which leads to lower pickup times which, in turn, further helps sustain the number of present drivers. Given the diminishing sensitivity of pickup times to the number of drivers in the region, the effect of a price increase in dense regions will be restricted to a decrease in demand. However, in regions with lower demand densities and, hence, longer pickup times, the demand decrease arising from a price increase will have a more substantial adverse consequences through network effects: with every driver leaving the region as a result of the lower demand, the pickup time for the remaining drivers increases, further encouraging drivers to leave. As a consequence, a permanent price increase is a safer action for the platform in denser regions compared to less dense ones. This mechanism is different from Besbes et al. (2018) who focus on short-run demand shocks. In fact, in our model, if we abstract away from pickup time frictions (by setting vector t to zero), the optimal price will be uniform across regions even though some regions have higher demand densities than others.

Propositions 9 and 10 suggest that there are multiple economic forces governing the platform's optimal behavior, and that these forces may pull the optimal strategy in different directions. To see this, consider the following scenarios. First suppose the platform, in line with Proposition 9, offers higher wages in less dense regions. In this case, when we make the prices also flexible, the platform would have some incentive to also increase the prices in less dense areas because the platform has a higher marginal cost (due to driver wages) in those areas. As a second scenario, suppose the platform, in line with Proposition 10, has decided to offer lower prices in less dense areas. In this case, when wages also become flexible, there is some incentive for the platform to offer lower wages in less dense areas, passing at least part of the regional price cut on to drivers. Third, and last, the

[35] We skip the provision of a counter-example as well as the proof for large γ (both would be available upon request). Instead, we provide some intuition for why the result does not always hold for small γ values. To see the intuition, consider a region i that is just dense enough to attract non-zero supply under the platform's optimal pricing strategy and with a fixed wage c. Now consider the effect of an increase in the region's potential demand arrival rate from $\bar{\lambda}_i$ to $\gamma\bar{\lambda}_i$ with $\gamma > 1$. In response to this demand increase, the platform can decide either (i) to do very little price increase and enjoy the extra volume of demand which also helps with attracting more drivers, or (ii) to increase the price substantially and focus on the margin instead of the volume. If, under the original $\bar{\lambda}_i$, the price elasticity is low at p_i^*, then the platform will choose the latter strategy in response to a scale-up to $\gamma\bar{\lambda}_i$. This could lead to a decrease in access to rides. If the fixed wage c is small enough, indeed the market can get formed at a p_i^* low enough so that it induces a low price elasticity. We have confirmed this intuition using numerical simulations of the market.

platform could adhere to the results from both Proposition 9 and Proposition 10 and offer both lower prices and higher wages in less dense regions. It is not obvious which one of these scenarios (or what combination of them) would prevail when prices and wages are both flexible. Put differently, the platform's incentive to build economies of density pulls the prices down and wages up in sparser areas. But lower prices themselves push wages down; and, likewise, higher wages themselves push prices up. The overall outcome of the interaction among these multiple forces is not clear. As such, the results we present in the main text, where the platform's optimal strategy of both wages and prices are flexible, speaks to this issue.

E.3 Model with fixed price and fixed wage

In this subsection, we assume that price and wage are uniformly fixed across regions. There is a total mass of $N > 0$ drivers who work for the platform, where N is also exogenous. An allocation of drivers is denoted by vector $n = (n_1, ..., n_I)$ such that $\Sigma_{i=1,...,I} n_i = N$. In this case, we modify the definition of equilibrium as follows:

Definition 4. Under "market primitives" $(\bar{\lambda}, N, t)$, an allocation $n^* = (n_1^*, ..., n_I^*)$ of drivers among the I regions is called an equilibrium if (i) $\Sigma_{i=1,...,I} n_i^* = N$, and (ii) no small mass of drivers currently operating in region i can strictly profit from changing their strategy and operating instead in region j, where $i, j \in \{0\} \cup I$ and 0 represents the outside options. That is, $\exists \delta, \forall i, j \in I \cup \{0\}, \forall \delta' \in [0, \delta]$, we have

$$\frac{c_i}{W_i(n_i^*; p_i)} \leq \frac{c_j}{W(n_j^* + \delta'; p_j)}$$

where the fraction for $i(j)$ is replaced by $\bar{c}$ is $i(j)$ is equal to zero.

Also, we call n^* an "all-regions" equilibrium allocation if it is an equilibrium and if $n_i^* > 0$ for all i.

Given that all wages and prices are uniform and fixed, $\exists c, p$ such that $\forall i : c_i = c$ and $p_i = p$. Thus, in this section, we suppress the notation on p_i and denote $\lambda_i(p_i)$ simply as λ_i. Additionally, when c_i is uniform, then the *revenue-maximization objective for drivers boils down to wait-time minimization.* In this section, the "primitives" of the market will be (λ, N, t).

Similar to other versions of the models, we first provide three main results in this section. Our first result describes the equilibria of the game among drivers. Our second result shows that at the equilibrium, the spatial distribution of supply is skewed toward denser regions due to economies of density. The third result examines how the supply distribution responds to a change in "market thickness." Additionally, we give a result that compares the market equilibrium distribution of drivers to the platform-optimal distribution and show that the platform mitigates but not eliminates the skew of supply, which is in line with the counterfactual results in the main text.

Proposition 11. *The following statements are true about the equilibria of the game among drivers.*

1. *An equilibrium n^* always exists. Also, for any subset J of I there is at most one equilibrium n^* under which $n_i^* > 0 \Leftrightarrow i \in J$.*

2. *At any equilibrium n^* such that $n_i^* > 0 \Leftrightarrow i \in J \subset I$, the total wait time (hence the revenue) is equal across regions: $\forall i, j \in J : W_i(n_i^*) = W_j(n_j^*)$.*

3. *Suppose $n^{*'}$ and n^* are two equilibria such that the set of regions served by n^* (i.e., the set $\{i : n_i^* > 0\}$) is a proper subset of the set of regions served by $n^{*'}$. Then driver total wait time is strictly lower (i.e., driver revenue is higher) under $n^{*'}$.*

Note that this proposition not only describes the features of the equilibria, but also provides a partial means for equilibrium selection. Part 3, roughly, says that "larger" equilibria are more profitable for drivers. In particular, if an "all-regions" equilibrium (i.e., n^* with $n_i^* > 0$ for all i) exists, it is the most desirable equilibrium for *all* drivers.

Proposition 12. *Suppose n^* is an equilibrium allocation of drivers under market primitives (λ, N, t). Then the following are true:*

1. *For all regions that get positive supply, supply ratios are skewed towards denser areas:*

$$\forall i < j : \frac{n_i^*}{\lambda_i} \geq \frac{n_j^*}{\lambda_j}$$

with equality only if $\frac{\lambda_i}{t_i} = \frac{\lambda_j}{t_j}$.

2. *The same result holds on access to rides across regions:*

$$\forall i < j : A_i(n_i^*) \geq A_j(n_j^*)$$

with equality only if $\frac{\lambda_i}{t_i} = \frac{\lambda_j}{t_j}$.

To illustrate, this proposition states that if region i has twice as much demand per unit of size as region j, in the equilibrium it may get, say, three times as much supply per unit of size. The basic intuition for why this result is true is the role of pickup times. To see this, consider an allocation n which distributes drivers across regions proportionally to demand arrival rates, meaning $\forall i, j : \frac{n_i}{\lambda_i} = \frac{n_j}{\lambda_j}$. It is easy to show that, under such an allocation, lower density areas have higher total wait times. To see this, note that all regions will have the same idle time $\frac{n_i}{\lambda_i}$. However, for any pair of regions i, j with higher demand density at i (meaning $\frac{\lambda_i}{t_i} > \frac{\lambda_j}{t_j}$), the pickup time at j is strictly larger. This is because $\frac{t_i}{n_i} = \frac{t_i}{\lambda_i} \times \frac{\lambda_i}{n_i}$. But $\frac{t_i}{\lambda_i} < \frac{t_j}{\lambda_j}$ and $\frac{\lambda_i}{n_j} = \frac{\lambda_i}{n_j}$ which, together, yield $\frac{t_i}{n_i} < \frac{t_j}{nj}$. Therefore, if we start from the proportional allocation, there will be an incentive for drivers to relocate from less dense areas to denser ones. This, of course, is only the intuition behind the result. The formal proof has multiple extra steps and is provided in Appendix E.6.

We now turn to studying how the equilibrium spatial distribution of the drivers responds to a change in the platform size (i.e., market thickness). Before that, we extend the definition of "market thickening" as follows:

Definition 5. Consider a market with primitives (λ, N, t). We call a market with primitives $(\gamma\lambda, \gamma N, t)$ with $\gamma > 1$ a "two-sided thickening" of (λ, N, t). Additionally, $(\lambda, \gamma N, t)$ is a "one-sided" thickening of (λ, N, t).

Intuitively, a two-sided thickening increases both the total number of drivers and the demand arrival rate in each area by the same factor $\gamma > 1$. A one-sided thickening only increases the total number of drivers. It is crucial to note that both of these changes *preserve the demand ratios between any two regions.* Nevertheless, as our next result shows, making a market thinner will *skew the supply ratio between any two regions towards the denser one.*

Proposition 13. *Suppose n^* is an equilibrium allocation of drivers under market primitives (λ, N, t). Also assume (λ', N', t') is a one- or two-sided thickening of (λ, N, t). Then there exists an equilibrium allocation $n^{*'}$ under (λ', N', t'), which satisfies the following:*

1. $\{i : n_i^* > 0\} = \{i : n_i^{*'} > 0\}$.
2. *For all i, j with $n_i^* > 0$*
$$\forall i < j : \frac{A_j(n_j^*)}{A_i(n_i^*)} \leq \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} \leq 1$$
where both inequalities are strict when $\frac{\lambda_i}{t_i} > \frac{\lambda_j}{t_j}$.
3. *There will be equitable access to rides as the market gets sufficiently thick:*
$$\forall i, j : \lim_{\gamma \to \infty} \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} = 1$$

The underlying intuition for the result is that as the market gets thicker (i.e., the platform gets larger,) *all regions get denser* with drivers. As such, the importance of pickup times relative to idle times decreases in drivers' decision making, leading to a supply distribution that is more balanced with demand. The proof of this proposition is also given in Appendix E.6. The main technique used to carry out the proof is strong induction in the number of regions. A crucial part of the proof in the induction is to show that, when the market gets thicker, so does any "sub-market" consisting of any arbitrary subset of all the I regions (this will be necessary for the induction step). That is, as the global market gets thicker, the distribution of drivers does shift towards less dense areas *but not so much as to make some of the denser "sub markets" thinner relative to before the global increase in market thickness.* For details, see the following sections.

Finally, we give a result about the difference between the equilibrium distribution of drivers and the platform-optimal distribution of drivers under uniform wages and prices.

Proposition 14. *Suppose that allocation n^* is an equilibrium under market primitives (λ, N, t). Then, for any two regions i, j with $\frac{\lambda_i}{t_i} > \frac{\lambda_j}{t_j}$, if the platform were to optimally reallocate the $n_i^* + n_j^*$ drivers between the two regions, it would choose n_i^{**} and n_j^{**} (subject to $n_i^{**} + n_j^{**} = n_i^* + n_j^*$) such* *that:*

$$\frac{A_j(n_j^*)}{A_i(n_i^*)} < \frac{A_j(n_j^{**})}{A_i(n_i^{**})} < 1.$$

That is, the platform would desire some inequity in access across regions but not as much as the equilibrium allocation among drivers naturally gives rise to. The intuition for this result is that the platform dislikes its drivers having to do long pickups. Therefore, it also prefers some level of geographical "imbalance" between supply and demand. However, the platform internalizes the externalities that drivers leave on each other when deciding where to locate. These externalities come mainly from the fact that when, at the equilibrium, a driver chooses a dense region i over the less dense j, she makes j even sparser which increases the pickup in j. Of course her joining i does slightly decrease the pickup time in i but the effect on i is not as large compared to j, given the diminishing sensitivity of pickup times to the number of drivers present in a region. All of this impacts other drivers and, hence, the platform.

To sum up, in this subsection we show that, in order to avoid longer pickup times, drivers tend to disproportionately locate in regions with higher demand densities. We also show that this may lead to some regions not being served at all. Additionally, we prove that this supply-demand imbalance dwindles as the platform grows. Finally, we show that, if the platform were to optimally allocate the drivers, it would mitigate but not eliminate such supply-demand imbalance. All of these are consistent with the conclusions we draw from the models in the main text.

E.4 Proofs for results with endogenous wage and fixed price

Before starting the proof, note that, similar to the proofs in Appendix C for the models with both endogenous price and endogenous wage, the regions are separable from each other. On the other hand, the proof cannot be reduced to a numerical problem any more. Instead, we employ monotone comparative static techniques from the literature to address the challenges and prove our propositions.

Proof of Proposition 9. We start by proving the statements about economies of density. Then we will move on to statements regarding the role of market thickness.

Proof of Statements 1 and 2 on Economies of Density. To prove the result for the whole market, we can prove it for every region i separately. As such, in what follows, we suppress the index i. As such, in this proof, n^* does not stand for the vector of all driver presence volumes in all regions, but just the number of drivers in region i. We will also suppress some equilibrium notations. For instance, we denote the equilibrium total wait time for drivers by W instead of $W_i(n_i^*)$.

Note that in the equilibrium, the hourly revenue for drivers has to be equal to the reservation value $\bar{c}$. Therefore, we have:

$$\frac{c}{W} = \bar{c} \tag{28}$$

If we replace W in the above with the expression in terms of n^*, and then solve for n^* we get two solutions:

$$n^* = \frac{\frac{c}{\bar{c}} \mp \sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\lambda(p)}}}{\frac{2}{\lambda(p)}}$$

Note that only the larger solution (i.e., the one with the + sign) is an equilibrium because at the lower solution, further driver entry will lead to lower overall wait time, increasing driver revenue. As such, we have:

$$n^* = \frac{\frac{c}{\bar{c}} + \sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\lambda(p)}}}{\frac{2}{\lambda(p)}} \tag{29}$$

Now note that platform profit in the region is given by $\pi = (p - c) \times \frac{n^*}{W}$.

Replacing for W and n^* respectively from eq. (28) and eq. (29), we get:

$$\pi = (p - c) \times \frac{\lambda(p)}{2} \times [1 + \sqrt{1 - \frac{4t}{\lambda(p)}(\frac{\bar{c}}{c})^2}] \tag{30}$$

Next, we make two observations. First, note that if the exogenous price p is such that

$$1 - \frac{4t}{\lambda(p)}(\frac{\bar{c}}{p})^2 < 0 \tag{31}$$

then it means eq. (29) will have a real valued root only if the platform sets $c > p$. That is, the platform can only attract drivers to the region by offering them a wage higher than price p, thereby running a net loss itself. Thus, the platform's optimal action will be to set a wage that will attract no driver if and only if eq. (31) holds. Note that eq. (31) is equivalent to:

$$\frac{\lambda(p)}{t} < 4(\frac{\bar{c}}{p})^2$$

Given that p is fixed in this proposition, then $f(p)$ is also fixed. Therefore, by $\lambda(p) = \bar{\lambda} f(p)$, we have:

$$\frac{\bar{\lambda}}{t} < \frac{4}{f(p)}(\frac{\bar{c}}{p})^2 \tag{32}$$

which exactly says the region will not get any drivers if the density of potential demand falls short of some cutoff as statement 2 of the proposition says.

Our second observation is about the uniqueness of the equilibrium wage when the region does get a positive number of drivers. To ease the notation, denote $\frac{4t\bar{c}^2}{\lambda(p)}$ simply by a. This gives

$$\pi = (p - c) \times \frac{\lambda(p)}{2} \times [1 + \sqrt{1 - \frac{a}{c^2}}]$$

writing the first order condition in terms of c and rearranging yields

$$\sqrt{c^2 - a} = \frac{ap}{c^2} - c$$

This equation can have only one root given that the left hand side is strictly increasing in c while the right hand side is strictly decreasing. Thus, the optimal wage c^* is unique. We know by eq. (29) that this means n^* is unique too. This finishes the proofs of statements 1 and 2 of the proposition. $\square$

Proof of Statement 3 on Economies of Density. Given that the problem in each region is separate from others, this statement is equivalent to the statement that $A(n^*)$ is strictly increasing in $\frac{\bar{\lambda}}{t}$ in each region (Note that we are keeping suppressing the region subscripts). In the rest of the proof, we also suppress the notation on $A(n^*)$ and write it simply as A. Recall that

$$A = \frac{r}{\bar{\lambda}} = \frac{n}{W\bar{\lambda}}$$

Writing out A in terms of the primitives of the model as well as platform strategy parameters p and c, we get:

$$A = \frac{\frac{c}{\bar{c}} + \sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\lambda(p)}}}{\frac{2}{\lambda(p)}} \times \frac{1}{\bar{\lambda} \times \frac{c}{\bar{c}}}$$

Simplifying, we get:

$$A = \frac{1 + \sqrt{1 - \frac{4t}{\bar{\lambda} f(p)} \times (\frac{\bar{c}}{c})^2}}{\frac{2}{f(p)}} \tag{33}$$

This equation shows that A depends on $\frac{\bar{\lambda}}{t}$ but not on either $\bar{\lambda}$ or t separately. Therefore, in order to show A increases as $\frac{\bar{\lambda}}{t}$, we do not need to show that A is both increasing in $\bar{\lambda}$ and decreasing in t. Only one of the two would be sufficient. We choose to focus on the comparative static in t. We will make the same choice in other parts of the proof of this proposition as well as the other remaining propositions.

Our next step is to show that A is strictly decreasing in t. Note that given the uniqueness result in statement 1, there is a 1-to-1 relationship between access A and wage c in the region. Therefore, the platform could be thought of as optimally choosing A as opposed to optimally choosing c. This reduces the problem to a monotone comparative static problem of showing the optimal A is decreasing in the value of t. This allows us to use standard monotone comparative statics theorems a la Milgrom and Shannon. We need to prove that the profit function is strictly submodular in A and t. That is:

$$\frac{\partial^2 \pi}{\partial A \times \partial t} < 0$$

To show this, we first write π out in terms of A instead of c. Recall that platform profit in the region is given by:

$$\pi = (p - c)\frac{n}{W} \tag{34}$$

We need to rewrite so that dependence of π on c (both direct and through n and W) is expressed through dependence on A. First note that $A = \frac{n}{W\bar{\lambda}} = \frac{n}{\frac{c}{\bar{c}}\bar{\lambda}}$. Thus

$$n = A\frac{c}{\bar{c}}\bar{\lambda} \tag{35}$$

Also, writing W out eq. (28) in terms of n and replacing for n from eq. (35), we get:

$$\frac{c}{\bar{c}} = \frac{A\frac{c}{\bar{c}}}{f(p)} + \frac{t}{A\frac{c}{\bar{c}}\bar{\lambda}}$$

Solving for c, we get:

$$c = \bar{c}\sqrt{\frac{\frac{t}{\bar{\lambda}}f(p)}{Af(p) - A^2}} \tag{36}$$

Replacing from eq. (35) and eq. (36) into eq. (34), we get:

$$\pi = (p - \bar{c}\sqrt{\frac{\frac{t}{\bar{\lambda}}f(p)}{Af(p) - A^2}})A\bar{\lambda} \tag{37}$$

Now that we have the right expression, we want to show $\frac{\partial^2 \pi}{\partial A \times \partial t} < 0$. Or equivalently, we want do show that $\frac{\partial \pi}{\partial t}$ is strictly decreasing in A. Taking the partial derivative with respect to t and simplifying yields:

$$\frac{\partial \pi}{\partial t} = -\frac{\bar{c}}{2t}\sqrt{\frac{A\bar{\lambda}tf(p)}{f(p) - A}} \tag{38}$$

Given that $\bar{\lambda}$, $f(p)$, $\bar{c}$, and t are all positive constants, it would suffice to show $-\frac{A}{f(p)-A}$ is strictly decreasing in A. This is immediate by noticing the negative sign, the fact that the numerator is positive and strictly increasing in A, and that the denominator is positive and strictly decreasing in A. This finishes the proof of statement 3 of the proposition. □

Proof of Statement 4 on Economies of Density. Next, we will show that regions with higher density of potential demand $\frac{\bar{\lambda}_i}{t_i}$ will get lower wages c_i when prices are fixed and spatially uniform. Similar to the previous statements, this one can also be proven through a comparative static result focusing on each region. We show that if a region's density of potential demand increases, then the optimal wage chosen by the platform for that region decreases. Again, we will be suppressing notations on region indices as well as equilibrium "*" notations. In the proof for this statement, we take a similar approach to the one we took for the previous statement. We write the profit function fully in terms of wage c by replacing for n from eq. (29). Then we show that

$$\frac{\partial^2 \pi}{\partial c \times \partial t} > 0 \tag{39}$$

which means the optimal wage c^* will be strictly increasing in t and, hence, strictly decreasing in density $\frac{\bar{\lambda}}{t}$.

Replacing from eq. (29) into the eq. (34) and rearranging yields:

$$\pi = (p-c)\frac{1+\sqrt{1-\frac{4t}{\bar{\lambda}f(p)}(\frac{\bar{c}}{c})^2}}{\frac{2}{\bar{\lambda}f(p)}}$$

Differentiating with respect to t and rearranging terms yields:

$$\frac{\partial \pi}{\partial t} = -\frac{(p-c)(\frac{\bar{c}}{c})^2}{\sqrt{(\frac{c}{\bar{c}})^2 - \frac{4t}{\bar{\lambda}f(p)}}}$$

It is easy to see that $\frac{\partial \pi}{\partial t}$ is strictly increasing in c. This is because the two terms in the numerator are strictly decreasing in c, the term in the denominator is strictly increasing in c, and there is a negative sign. This shows that π is a strictly supermodular function in c and t, thereby showing that if t increases, so does the optimal level of c. This finishes the proof of this statement. □

Next we turn to those statements in the proposition that have to do with the role of market thickness.

Proof of Statement 1 on the Role of Market Thickness. This is straightforward given the proof statement 2 on economies of density. By eq. (32), we have

$$n_i^* > 0 \Rightarrow \frac{\bar{\lambda}_i}{t_i} \geq \frac{4}{f(p)}(\frac{\bar{c}}{p})^2$$

$$\Rightarrow \frac{\gamma\bar{\lambda}_i}{t_i} \geq \frac{4}{f(p)}(\frac{\bar{c}}{p})^2 \Rightarrow n_i^{*'} > 0. \quad \square$$

Proof of Statement 2 on the Role of Market Thickness. First note that the second part of the inequality in this statement (i.e., $\frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j} \Rightarrow \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})} < 1$) is implied by statement 3 on economies of density, which we have already proved. We, hence, will only prove:

$$\frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j} \Rightarrow \frac{A_j(n_j^*)}{A_i(n_i^*)} < \frac{A_j(n_j^{*'})}{A_i(n_i^{*'})}$$

In order to prove this statement, we first state and prove a lemma. Recall, again, that given under flexible N the optimization problems in different regions are separate from each other, then the equilibrium access $A_i(n_i^*)$ is only a function of $\bar{\lambda}_i, t_i, p_i, c_i$ rather than those at other regions. Therefore, the notation i may be suppressed. In the proofs of some of the last statements we have even suppressed the notation on n^* and denote the equilibrium access in region i simply by A. In our next lemma, we will keep suppressing those notations. We also note that access has been an implicit function of model parameters, in particular $\frac{\bar{\lambda}f(p)}{t}$ (we showed that equilibrium access is not separately dependent on $\bar{\lambda}$, t, and $f(p)$ otherwise). Denote $\frac{\bar{\lambda}f(p)}{t}$ by d. In the proof of this lemma, we make the exposition of the dependence of equilibrium access A on parameter d explicit by denoting the equilibrium access as $A(d)$. In other words, $A(d)$ is the equilibrium access in a region with density d when the platform has set the optimal wage c^* given density d and price p and drivers respond accordingly.

Lemma A13. *Define function $B(\tau)$ as*

$$B(\tau) \equiv \log(A(e^{\tau}))$$

Then statement 2 of Proposition 9 on the role of market thickness holds if B is a strictly convcave function whenever defined.

Proof of Lemma A13. First notice that B is defined whenever $A > 0$ which is whenever τ is larger than some cutoff.

Next, take two regions i, j with $\frac{\bar{\lambda}_i}{t_i} > \frac{\bar{\lambda}_j}{t_j}$. Set $\tau_i = \log(\frac{\bar{\lambda}_i f(p)}{t_i})$ and $\tau_j = \log(\frac{\bar{\lambda}_j f(p)}{t_j})$. Also set $\beta = \log(\gamma)$ where γ is the scalar larger than one in the statement of the proposition. By strict concavity of B, we have:

$$B(\tau_i + \beta) - B(\tau_i) < B(\tau_j + \beta) - B(\tau_j)$$

This implies:

$$\log[A(e^{\tau_i+\beta})] - \log[A(e^{\tau_i})] < \log[A(e^{\tau_j+\beta})] - \log[A(e^{\tau_j})]$$

$$\Rightarrow \log[\frac{A(e^{\tau_i+\beta})}{A(e^{\tau_i})}] < \log[\frac{A(e^{\tau_j+\beta})}{A(e^{\tau_j})}]$$

$$\Rightarrow \frac{A(\frac{\gamma\bar{\lambda}_i f(p)}{t_i})}{A(\frac{\bar{\lambda}_i f(p)}{t_i})} < \frac{A(\frac{\gamma\bar{\lambda}_j f(p)}{t_j})}{A(\frac{\bar{\lambda}_j f(p)}{t_j})}$$

$$\Rightarrow \frac{A(\frac{\bar{\lambda}_j f(p)}{t_j})}{A(\frac{\bar{\lambda}_i f(p)}{t_i})} < \frac{A(\frac{\gamma\bar{\lambda}_j f(p)}{t_j})}{A(\frac{\gamma\bar{\lambda}_i f(p)}{t_i})}$$

which is exactly the statement we needed to prove. This completes the proof of the lemma. □

Therefore, the only thing we need to prove is strict concavity of the function B. Our next lemma does this.

Lemma A14. *Function B as defined in the previous lemma is strictly concave.*

Proof of Lemma A14. Take τ and $\tau + 2\beta$ for some positive β such that $B(\tau)$ and $B(\tau + 2\beta)$ are both defined (i.e., the corresponding A values are both positive). The proof of the lemma will be complete if we can show:

$$B(\tau + \beta) > \frac{B(\tau) + B(\tau + 2\beta)}{2} \tag{40}$$

Denote $d = e^{\tau}$ and $\gamma = e^{\beta}$. Also denote $\hat{A} = A(d)$. Additionally, from the previous parts of this proposition, we know $A(d\gamma^2) > A(d)$. Therefore, if we write $A(d\gamma^2) = A(d)\zeta^2$ for some positive ζ, then it has to be that $\zeta > 1$. At this point, one can observe that eq. (40) is equivalent to

$$A(d\gamma) > \sqrt{A(d)A(d\gamma^2)} = A(d)\zeta = \hat{A}\zeta \tag{41}$$

To establish the above inequality, we first write out the platform profit function in terms of A (again treating A is the decision variable) and, then, characterize the first order condition which gives $A(d)$ for any d (recall that $A(d)$ is the optimal A under d). This is done in eq. (37). If we differentiate eq. (37) with respect to A, we get:

$$\frac{\partial \pi}{\partial A} = \bar{\lambda}[p - \frac{\bar{c}}{2}\sqrt{\frac{\frac{t}{\bar{\lambda}}}{A}(\frac{f(p)}{f(p) - A})^3}] \tag{42}$$

Equating $\frac{\partial \pi}{\partial A}$ to zero and rearranging yields the following first order condition:

$$\frac{2p}{\bar{c}\sqrt{f(p)^3}} - \sqrt{\frac{\frac{t}{\bar{\lambda}}}{A}(\frac{1}{f(p) - A})^3} = 0 \tag{43}$$

Replacing for $\frac{t}{\bar{\lambda}}$ with $\frac{1}{d}$, we get:

$$\frac{2p}{\bar{c}\sqrt{f(p)^3}} - \sqrt{\frac{1}{dA}(\frac{1}{f(p) - A})^3} = 0 \tag{44}$$

Next, define the function $g(d, A)$ by:

$$g(d, A) \equiv \sqrt{\frac{1}{dA}(\frac{1}{f(p) - A})^3}$$

Therefore, the optimality of $\hat{A}$ under d and the optimality of $\hat{A}\zeta^2$ under $d\gamma^2$ mean:

$$g(d, \hat{A}) = \frac{2p}{\bar{c}\sqrt{f(p)^3}} = g(d\gamma^2, \hat{A}\zeta^2) \tag{45}$$

In order to prove eq. (41), we need to show that under $d\gamma$ and at $A = \hat{A}\zeta$ we have $\frac{\partial \pi}{\partial A} > 0$ (because this would mean the first order condition would hold at some $A > \hat{A}\zeta$, implying exactly eq. (41)). In other words, we need need to show that from eq. (45) one can conclude:

$$g(d\gamma, \hat{A}\zeta) < \frac{2p}{\bar{c}\sqrt{f(p)^3}} \tag{46}$$

To prove this, it would be sufficient to show

$$\log(g(d\gamma, \hat{A}\zeta)) < \frac{\log(g(d, \hat{A})) + \log(g(d\gamma^2, \hat{A}\zeta^2))}{2}$$

Expanding the above expression in terms of g, we get:

$$-\frac{1}{2}\log(dA) - \frac{1}{2}\log(\zeta\gamma) - \frac{3}{2}\log(f(p) - A\zeta) <$$

$$\frac{-\frac{1}{2}\log(dA) - \frac{3}{2}\log(f(p) - A) - \frac{1}{2}\log(dA) - \log(\zeta\gamma) - \frac{3}{2}\log(f(p) - A\zeta^2)}{2}$$

$$\Leftrightarrow -\frac{3}{2}\log(f(p) - A\zeta) <$$

$$\frac{-\frac{3}{2}\log(f(p) - A) - \frac{3}{2}\log(f(p) - A\zeta^2)}{2}$$

$$\Leftrightarrow \log(f(p) - A\zeta) > \frac{\log(f(p) - A) + \log(f(p) - A\zeta^2)}{2}$$

This last inequality holds if we can show that the function $\log(f(p) - Ae^{\nu})$ is strictly concave in ν. To see this, note that:

$$\frac{\partial}{\partial \nu}\log(f(p) - Ae^{\nu}) = \frac{-Ae^{\nu}}{f(p) - Ae^{\nu}} = 1 - \frac{f(p)}{f(p) - Ae^{\nu}}$$

which is strictly decreasing in ν. This finishes the proof of this statement of the proposition. □

Proof of Statement 3 on the Role of Market Thickness. This statement is straightforward is is left to the reader. □

This finishes the proof of Proposition 9. ■

E.5 Proofs for results with endogenous price and fixed wage

The proof is similar to the proof of Proposition 9. We provide the key steps here and skip some of the details.

Proof of Proposition 10.

Proof of Statements 1 and 2 on Economies of Density. The steps of this proof are similar to the corresponding steps in the previous theorem. A given region will have a positive number of drivers in equilibrium if it can attract drivers when the price in the region is equal to the fixed wage c. Based on eq. (30), this means:

$$1 - \frac{4t}{\bar{\lambda}(1 - \alpha c)}(\frac{\bar{c}}{c})^2 \geq 0 \tag{47}$$

or equivalently, when:

$$\frac{4}{(1 - \alpha c)}(\frac{\bar{c}}{c})^2 \leq \frac{\bar{\lambda}}{t}$$

which proves statement 2. Also, eq. (29) holds in this case too, which implies uniqueness of n^* if optimal price p is unique. We now show that optimal price p is indeed unique. More precisely, we want to show that there is a unique p maximizing π in eq. (30) when $\lambda(p) \equiv \bar{\lambda}(1-\alpha p)$. Alternatively, we can show that there is a unique p maximizing $\log(\pi)$. If we write the first order condition for $\log(\pi)$ in terms of p, we get:

$$\frac{\partial \log(\pi)}{\partial p} = \frac{\partial \log(p-c)}{\partial p} + \frac{\partial \log(\bar{\lambda})}{\partial p} +$$

$$\frac{\partial \log(1-\alpha p)}{\partial p} + \frac{\partial \log(1+\sqrt{1-\frac{4t}{\bar{\lambda}W^2}\frac{1}{1-\alpha p}})}{\partial p} = 0$$

Some expanding, simplifying, and rearranging yields:

$$\frac{1}{p-c} + \frac{-\alpha}{1-\alpha p} + \frac{-4t\alpha}{\bar{\lambda}W^2}\frac{1}{1-\alpha p}\frac{1}{\sqrt{1-\alpha p}+\sqrt{1-\alpha p-\frac{4t}{\bar{\lambda}W^2}}}\frac{1}{2\sqrt{1-\alpha p-\frac{4t}{\bar{\lambda}W^2}}} = 0$$

The first order condition can only have one solution given that every one of the three terms in the left hand side expression is strictly decreasing in p. This finishes the proofs of statements 1 and 2 regarding economies of density. □

Proof of Statements 3 on Economies of Density. Similar to the case of the previous proposition, it will suffice to show that the optimal p is strictly increasing in the density of the region. This immediately follows from observing that, in the proof of the previous statement of this theorem, $\frac{\partial \log(\pi)}{\partial p}$ is strictly decreasing in t, meaning $\log(\pi)$ is strictly sub-modular in t and p. This implies that the p that maximizes $\log(\pi)$ (and hence π itself) is strictly decreasing in t. That is, the optimal p is strictly increasing in density of potential demand $\frac{\bar{\lambda}}{t}$. □

Proof of Statement 1 on the Role of Market Thickness. Similar to the equivalent statement in Proposition 9, this statement follows directly from the proofs of statements 1 and 2 on economies of density. □

Proof of Statement 2 on the Role of Market Thickness. Similar to the equivalent statement in Proposition 9, the proof of this statement is straightforward and is left to the reader. □

This completes the proof of the proposition. ■

E.6 Proofs for results with fixed price and fixed wage

This section is organized as follows. In Appendix E.6.1 we build up a set of results that will later be used in proving our propositions. The proof of the preliminary results leverages induction in the number of regions. These preliminary results have two shortcomings. First, they are "all-regions equilibria," meaning equilibria in which each region gets a positive number of drivers. Second, they are developed assuming size t_i is homogeneous across all regions. After building up these preliminary results, our next step in Appendix E.6.2 is to extend the analysis to multiple equilibria and provide proofs of the propositions in the main text regarding driver behavior under fixed and uniform prices and wages under homogeneous t_i. Finally, in Appendix E.6.3, we show that once our propositions are true for homogeneous t_i, they are also true for heterogeneous t_i.

E.6.1 Preliminary results

This section studies the market under fixed prices and wages and fixed N. In these circumstances, for each region i, we have $\lambda_i(p) = \bar{\lambda}_i f(p)$ where p is the uniform price across regions. Therefore, we will suppress the notation on prices and represent the arrival rate of demand in region i by λ_i rather than $\lambda_i(p_i)$. This makes the primitives of the market (λ, N, t) where λ is the vector of all λ_i demand arrival rates, N is the fixed total number of drivers, and t is the uniform region size which is a scalar in this subsection but will be heterogeneous in Appendix E.6.3.

Definition 6. We say allocation n is under-supplied in region j, relative to region i, if we have $A_j(n_j) < A_i(n_i)$. Under fixed prices and wages, this is equivalent to:

$$\frac{n_j}{\lambda_j} < \frac{n_i}{\lambda_i}$$

The "degree of under-supply" in region j relative to region i is defined by $\kappa_{ji} = \frac{A_i(n_i)}{A_j(n_j)} = \frac{\frac{n_i}{\lambda_i}}{\frac{n_j}{\lambda_j}}$.

In general, when developing the preliminary results, we do not use the notation on access and work directly with n_i and λ_i. In Appendix E.6.2 and Appendix E.6.3, we will see how these results imply our results on access in the main text.

Statements of Preliminary Results when there are Only Two Regions In this section, we present our results for the case of $I = 2$. We will present two results. First, if the demand arrival rate in region 1 is strictly larger than that of region 2, then in any all-regions equilibrium, region 2 will be strictly under-supplied. Second, we show that the under-supply problem in region 2 is mitigated as the size of the platform increases, holding fixed the ratio between λ_1 and λ_2.

First we give a result that helps to visually understand an all-regions equilibrium.

Proposition 15. *At any all-regions equilibrium, the wait times in the two regions are equal. Also the wait time for each region is locally increasing in the number of drivers present in that region.*

Proof. If $W_1(n_1) \neq W_2(n_2)$, then, given the wait time functions are continuous, a small mass of drivers can relocate from the region with the higher wait time to the region with the lower wait time and be strictly better off. Thus, at equilirium allocation n^*, we have $W_1(n_1^*) = W_2(n_2^*)$. Next, if at equilibrium, the wait time curve in region i is strictly decreasing, then a small mass of drivers from region j can relocate to i and become strictly better off.■

Next, we introduce a result that speaks to the existence and uniqueness of an all-regions equilibrium.

Proposition 16. *There is exactly one all-regions equilibrium if assumptions (A1) to (A3) hold. Otherwise, there is no all-regions equilibrium.*

(A1) $N \geq \sqrt{\lambda_1 t} + \sqrt{\lambda_2 t}$

(A2,A3) $2\sqrt{\frac{t}{\lambda_j}} \leq \frac{N-\sqrt{\lambda_j t}}{\lambda_i} + \frac{t}{N-\sqrt{\lambda_j t}}$ *for* $j = 1, 2$ *and* $i = 3 - j$

Figure 17 visually illustrates Propositions 15 and 16. In each panel, the wait time curves for the two regions are plotted opposite from each other. In each region, the wait time is initially decreasing in the total number of drivers present in that region due to the decrease it causes in pickup times. But as the region gets more drivers, the effect on pickup time dwindles and overall wait time increases due to increased idle time for drivers.[36] Each point on the horizontal axis of the graph corresponds to a driver allocation between the two regions. One such point is the "demand-proportional" allocation which satisfies $\frac{n_1}{\lambda_1} = \frac{n_2}{\lambda_2}$. This allocation is shown in the figure by a dashed vertical gray line. At each point, the solid blue line shows the total wait time in region 1, and the dashed green line gives the total wait time in region 2.

Translated to these graphical terms, Proposition 15 states that an all-regions equilibrium is a point of intersection between the two wait-time curves, at which both curves are increasing. Among the panels of Fig. 17c, such equilibrium only exists in panel (c).[37] Proposition 16 explains why. In order for an all-regions equilibrium to exist, there should exist allocations for which the total wait time at each region is increasing in the number of drivers present in that region. This is what assumption **(A1)** requires. Graphically, the trough for the wait time curve in region 2 (emphasized by a green circle) should be to the right of the trough of the wait time in region 1 (blue circle). Panel (a) in Fig. 17c lacks this feature and, hence, also lacks an all-regions equilibrium. In addition to **(A1)**, the existence of an all-regions equilibrium would also require that the two wait-time curves do intersect over the range in which they are both increasing. In order for this to happen, we require assumptions **(A2)** and **(A3)**. They require that, under the allocation that minimizes the total wait time in region 1, the total wait time in region 2 be higher than that in region 1. They impose a similar condition on the allocation that minimizes the total wait time in region 2. Graphically, they require that the total-wait-time curve for region 2 (the green dashed line) be above the trough of the wait-time curve in region 1 (the blue circle), and vice versa. Panel (c) satisfies both **(A2)** and **(A3)** and, hence, has an all-regions equilibrium given by the intersection between the two wait-time curves, emphasized by a large black circle. Panel (b), although satisfying **(A1)**, has the wait time curve for region 1 pass below the trough of the wait time curve in region 2. Therefore, there is no all-regions equilibrium in panel (b).

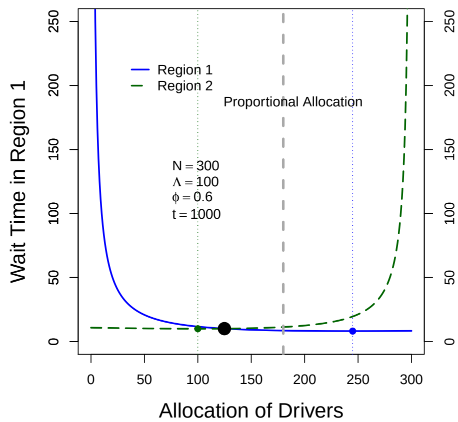
The reason why different panels in Fig. 17c differ in terms of having an all-regions equilibrium is that they pertain to different market primitives (λ, N, t) (in the figure as well as some of the proofs in the appendix, instead of its components λ_1 and λ_2, the vector λ is represented by total demand $\Lambda = \lambda_1 + \lambda_2$ and share of region 1 from demand $\phi = \frac{\lambda_1}{\Lambda}$). The figures are already suggestive of what affects the existence of an all-regions equilibrium (e.g., a large enough N is necessary) or

[36] Total wait time curves being U-shaped has been mentioned in other studies (such as Castillo et al. (2017)). To our knowledge, this curve and the U-shaped assumption on it are used by ride-share platforms in the determination of various strategies including surge pricing.

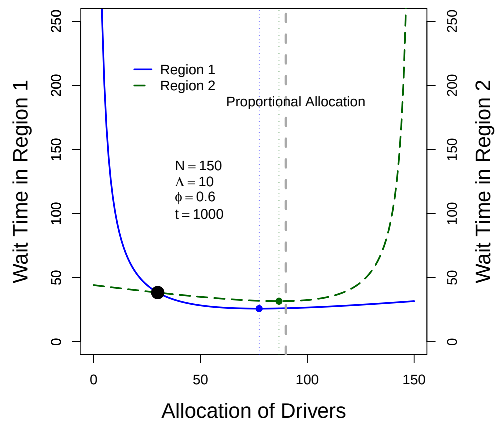
[37] One can verify that in all panels of Fig. 17c, allocations that put all drivers in one of the two regions are in fact equilibria. To illustrate why, note that under allocation $(n_1, n_2) = (N, 0)$, the wait time at region 2 is ∞ due to high pickup time. Thus, no driver has an incentive to move from region 1 to region 2.

where the all-regions equilibrium is located when it exists (to the right of the gray dashed line –i.e., the demand-proportional allocation– instead of on it due to agglomeration of drivers in region 1). Our next results in this section formalize and generalize such observations from the figure and add other results describing the role of market thickness.

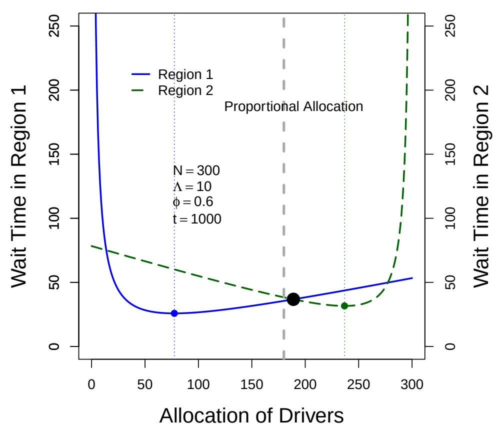
Figure 17: Wait Time and Driver Allocation. An all-regions equilibrium exists only in panel (c)



(a) No All-Regions Equilibrium



(b) No All-Regions Equilibrium



(c) Unique All-Regions Equilibrium

Proposition 17. *Suppose that $\lambda_1 > \lambda_2$ and that an all-regions equilibrium (n_1^*, n_2^*) exists. In that case, the all-regions equilibrium is strictly under-supplied in region 2:*

$$\frac{n_1^*}{\lambda_1} > \frac{n_2^*}{\lambda_2}$$

To illustrate, if region 1 has 80% of the demand, then, in equilibrium, 90% of the drivers might prefer to drive in region 1. This result coincides with our empirical observation that the relative outflow was greater in busier areas than in less busy areas. This can be graphically seen in Fig. 17a panel (c): the equilibrium is to the right of the gray dashed line representing the proportional allocation.

We now turn to our second result which speaks to the impact of market thickness. We prove that "making the market thicker" will decrease the extent of geographical inequity in supply. The next two results show this, respectively, for thickening the market on both sides (increasing the number of drivers and all-regional demand arrival rates) and thickening it on one side (increasing the number of drivers only).

Proposition 18. *Suppose that $\lambda_1 > \lambda_2$, and (n_1^*, n_2^*) is the all-regions EQ under $(\lambda_1, \lambda_2, N, t)$. Consider scaling up the platform size by $\gamma > 1$ to $(\lambda_1', \lambda_2', N', t) = (\gamma\lambda_1, \gamma\lambda_2, \gamma N, t)$. Under these new primitives, an all-regions equilibrium exists and under-supply in region 2 decreases with scaling up, i.e., $\frac{n_1'^*}{N'} < \frac{n_1^*}{N}$ In particular, as $\gamma \to \infty$,the relative under-supply in region 2, κ_{21}, tends to zero.*

Our next proposition proves similar results to those shown in Proposition 18, but this time for thickening the market only on one side.

Proposition 19. *Suppose that $\lambda_1 > \lambda_2$ and (n_1^*, n_2^*) is the all-regions EQ under $(\lambda_1, \lambda_2, N, t)$. If we scale up to $(\lambda_1, \lambda_2, N', t)$ for some $N' > N$, then an all-regions equilibrium still exists. Also, the new equilibrium shows less under-supply of rides in region 2. In particular, as $N' \to \infty$, under-supply in region 2 (and in region 1) tends to zero.*

Statements of Preliminary Results when there are $I \geq 2$ Regions Our main result in this section extends all of the results presented so far from two regions to any number of regions $I \geq 2$. This theorem is powerful in that it provides, among other results, a description of how the market responds to a changed thickness, at the most granular level. That is, it describes what happens to the supply ratio between *any* two regions i, j. As formalized below, the proposition shows that the market responds to a "global thinning" by further agglomerating the supply at the thickest "local markets."

Theorem 1. *In the general version of the game (i.e., $I \geq 2$), the following statements are true:*

1. *For an all-regions equilibrium, the total wait time is equal across all I regions. Also, at the equilibrium allocation, the total-wait-time curve for any region is strictly increasing in the number of drivers present in that region.*

2. *Any all-regions equilibrium $n^* = (n_1^*, ..., n_I^*)$ is unique.*

3. *At any all-regions equilibrium, for any $i < j$, we have $\frac{n_i^*}{\lambda_i} \geq \frac{n_j^*}{\lambda_j}$. The inequality is strict if and only if $\lambda_i > \lambda_j$.*

4. *Suppose an all regions equilibrium $n^* = (n_1^*, ..., n_I^*)$ exists under primitives (λ, N, t) where $\lambda = (\lambda_1, ..., \lambda_I)$. Then, if supply and demand both scale up, that is, under new primitives $(\gamma\lambda, \gamma N, t)$ with $\gamma > 1$, we have:*

- *An all-regions equilibrium* $n^{*'} = (n_1^{*'}, ..., n_I^{*'})$ *exists.*
- *The new equilibrium* $n^{*'}$ *shows less geographical supply inequity than* n^* *in the sense that for any* $i < j$, *we have* $1 \leq \frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}} \leq \frac{\frac{n_i^{*}}{\lambda_i}}{\frac{n_j^{*}}{\lambda_j}}$. *Both inequalities are strict if and only if* $\lambda_i > \lambda_j$.
- *All* $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}}$ *tend to 1 as* $\gamma \to \infty$

5. *The same statement is true if instead of proportionally scaling up both* λ *and* N, *we scale up only* N.

Proofs of Preliminary Results **Proof of Proposition 16.** First, we prove the necessity part, and then sufficiency and uniqueness.

Necessity: We prove necessity of (A1)-(A3) by contradiction. First, if (A1) is not satisfied, then we show that the wait time curves can only intersect when at least one of them is decreasing. To see this, note that taking the first order condition on eq. (1) shows the wait time curve in each region i is minimized at $n_i^{min} = \sqrt{\lambda_i t}$. Thus, condition (A1) simply requires that $N \geq n_1^{min} + n_2^{min}$. Without (A1), there would be no possible allocation of drivers under which the total wait time in each region is increasing in the number of drivers present in that region. Therefore, by Proposition 15, there would be no all-regions equilibrium. Next, suppose condition (A2) were not true. Thus, at the minimum wait time for region 1, i.e. at the allocation ($n_1 = n_1^{min}, n_2 = N - n_1^{min}$), the wait time for region 1 is higher than region 2. Thus, the wait time curves can only intersect in the decreasing region for market 1, which we know cannot be an all-regions equilibrium. The necessity of (A3) is similar to (A2).

Sufficiency: Observe that when (A2) is true, $W_1(n_1^{min}) < W_2(N - n_1^{min})$. Similarly from (A3), we have $W_2(n_2^{min}) < W_1(N - n_2^{min})$. We know that for $n_1 > n_1^{min}$, W_1 is an increasing function, and similar is the case for W_2. Since we have a reversal in relative magnitude for W_1 and W_2, and since the two curves are continuous, we must have an intersection of the curves between n_1^{min} and n_2^{min}, when both wait time curves are increasing. Such an intersection permits no profitable deviation by switching to the other market for any driver, and is thus an all-regions equilibrium.

Uniqueness: Both wait time curves are monotonic for the region $n_1 > n_1^{min}$ and $n_2 = (N - n_1) > n_2^{min}$, implying that there can only be one intersection between the curves when they are both increasing.

Together, these conditions are proven equivalent to existence and uniqueness of an all-regions equilibrium. In such a case, we can characterize the all-regions equilibrium by equating the wait time distributions.[38] ■

To Prove Proposition 17, we first introduce the following Lemma.

Lemma A15. *When (A1)-(A3) are satisfied and when drivers are allocated proportionally to demand, the proportional allocation lies between the minimum wait times for the two regions:* $n_1^{min} < \phi N < N - n_2^{min}$.

where ϕ, as mentioned Appendix E.6.1, is defined as $\phi = \frac{\lambda_1}{\lambda_1+\lambda_2}$. By our assumption $\lambda_1 > \lambda_2$, which came without loss of generality, we have $\phi > \frac{1}{2}$. In graphical terms represented by Fig. 17c, this lemma says the vertical dashed line representing the proportional demand will fall between the troughs of the two wait-time curves.

Proof of Lemma A1. *First, we prove that the proportional allocation line lies between the two minima.* $n_1^{min} = \sqrt{\lambda_1 t}$ *and* $n_2^{min} = \sqrt{\lambda_2 t}$. *Denote the total demand across both locations as* $\Lambda = \lambda_1 + \lambda_2$ *and the fraction of demand in the (higher-demand) location 1 to be* $\phi = \frac{\lambda_1}{\Lambda} > \frac{1}{2}$.

For proportional allocation to be situated between the two minimums on the graph, the following conditions need to hold:

C1: $\phi N > n_1^{min} = \sqrt{\lambda_1 t} = \sqrt{\phi \Lambda t} \implies N > \sqrt{\frac{\Lambda t}{\phi}}$

C2: $(1-\phi)N > n_2^{min} = \sqrt{\lambda_2 t} = \sqrt{(1-\phi)\Lambda t} \implies N > \sqrt{\frac{\Lambda t}{1-\phi}}$

Observe that since $\phi > \frac{1}{2}$, $C2 \implies C1$. *Thus, when the demand is more skewed (higher* ϕ*), we need to have a larger platform size for condition (C2) to be satisfied.*

We now prove that assumption (A1) + (A3) $\implies$ *(C2). Observe that condition that shows up is the following: Assumption (A3) implies*

$$2\sqrt{\frac{t}{(1-\phi)\Lambda}} < \frac{N - \sqrt{(1-\phi)\Lambda t}}{\phi \Lambda} + \frac{t}{N - \sqrt{(1-\phi)\Lambda t}}$$

The second term on the RHS can be bounded as: $\frac{t}{N-\sqrt{(1-\phi)\Lambda t}} < \frac{t}{\sqrt{\phi \Lambda t}}$, *since* $N > \sqrt{\phi \Lambda t} + \sqrt{(1-\phi)\Lambda t}$ *by assumption (A1).*

Thus assumption (A3) implies the following:

$$\frac{N - \sqrt{(1-\phi)\Lambda t}}{\phi \Lambda} > 2\sqrt{\frac{t}{(1-\phi)\Lambda}} - \frac{t}{\sqrt{\phi \Lambda t}} \implies N > \sqrt{\Lambda t}\left(2\phi\sqrt{\frac{1}{1-\phi}} - \sqrt{\phi} + \sqrt{1-\phi}\right)$$

[38] In practice, we obtain the allocation equating wait times, i.e. solving $W_1(n) - W_2(N-n) = 0$, which is equivalent to identifying the roots of the polynomial equation below:

$$-n^3(\lambda_1 + \lambda_2) + n^2(2N\lambda_1 + N\lambda_2) - n(N^2\lambda_1 + 2t\lambda_1\lambda_2) + Nt\lambda_1\lambda_2 = 0$$

By Descartes' rule of signs, this equation (i.e. the numerator) has potentially 3 positive roots. In the case of multiple roots, only the one that lies between the minimum points of the wait time curves where both curves are increasing is the symmetric equilibrium. See Proposition 15.

Next, we prove that the above inequality implies condition (C2), which stated that $N > \sqrt{\frac{\Lambda t}{1-\phi}}$. *Thus, we need to prove the following:*

$$2\phi\sqrt{\frac{1}{1-\phi}} - \sqrt{\phi} + \sqrt{1-\phi} > \sqrt{\frac{1}{1-\phi}} \Leftrightarrow \frac{2\phi - 1}{\sqrt{1-\phi}} - \sqrt{\phi} + \sqrt{1-\phi} > 0$$
$$\Leftrightarrow \sqrt{\phi}(\sqrt{\phi} - \sqrt{1-\phi}) > 0$$

Observe that the last inequality must be true given our assumption that $\phi > \frac{1}{2}$, *so (A1) + (A3)* $\implies$ *(C2). This finishes the proof of the lemma.* ■

Proof of Proposition 17. By Lemma A15, the demand-proportional allocation is in between the minimum wait times for both regions. We show the proportional allocation or any point to the left of it (i.e., an alloction with $n_1 \leq \phi N$) cannot be an all-regions equilibrium. This, combined with the assumption in the proposition that an all-regions equilibrium exists, implies that the all-regions equilibrium should be to the right of the proportional allocation. That is: $\frac{n_1^*}{\lambda_1} > \frac{n_2^*}{\lambda_2}$.

To see why no all-regions equilibrium can be found weakly to the left of the proportional allocation, note that at proportional allocation $n = \phi N$, region 2 wait time is higher than region 1, i.e. $W_2((1-\phi)N) = \frac{N}{\Lambda} + \frac{t}{(1-\phi)N} > W_1(\phi N) = \frac{N}{\Lambda} + \frac{t}{(\phi)N}$ since $\phi > \frac{1}{2}$. As we move left, region 2's wait time increases further, while region 1's wait time decreases until we reach the minimum wait time for region 1, $W(n_1^{min})$. Thus, the divergence between the two regions increases. For the wait time curves to intersect, it must be in region 1's decreasing wait time region. We know from Proposition 15 that such an intersection will **not** be an all-regions equilibrium. Fig. 17c panel (c) should help illustrate this point. This completes the proof of the proposition. ■

Lemma A16. *When an all-regions equilibrium exists for a ridesharing platform with* N *drivers facing demand* $\phi\Lambda$ *and* $(1-\phi)\Lambda$ *in the two regions:*

1. *an all-regions equilibrium also exists when demand is unchanged and there are* $N' = \gamma N$ *drivers where* $\gamma > 1$.

2. *an all-regions equilibrium also exists when both the demand and number of drivers are scaled up by a common factor* $\gamma > 1$ *to* $N' = \gamma N$ *and* $\Lambda' = \gamma\Lambda$.

Proof of Lemma A2. *Consider the equivalent conditions required for the existence of an all-regions equilibrium, characterized by assumptions (A1)-(A3). Below, we show that if the conditions are satisfied for a given* (N, Λ), *then they must be satisfied for (a)* $(N', \Lambda') = (\gamma N, \Lambda)$ *as well as (b)* $(N', \Lambda') = (\gamma N, \gamma\Lambda)$.

First, consider (A1). The proof of (a) is immediate. For (b), we observe that:

$$\gamma N > \sqrt{\gamma}\sqrt{\Lambda t}\left(\sqrt{\phi} + \sqrt{1-\phi}\right)$$

holds since $\gamma > 1$ *and (A1) holds for* (N, Λ).

Next, we prove (A2). The proof of (A3) is similar to that of (A2) and is omitted.

For (A2), first we denote the following function ϕ:

$$\psi(\rho) = \frac{\rho N - \sqrt{\phi \Lambda t}}{(1-\phi)\Lambda} + \frac{t}{\rho N - \phi \Lambda t}$$

We prove that ψ is increasing in ρ, or $\frac{d\phi}{d\rho} > 0$. Observe that:

$$\frac{d\psi}{d\rho} = N\left(\frac{1}{(1-\phi)\Lambda} - \frac{t}{\left(\rho N - \sqrt{\phi \Lambda t}\right)^2}\right)$$

After some algebra and applying (A1), we obtain $\frac{d\psi}{d\rho} > 0$.

Now, for part (a), observe that setting $\rho = \frac{N'}{N} > 1$ implies that, in (A2), the RHS increases and the LHS does not change implying that (A2) still holds for $(N', \Lambda') = (kN, \Lambda)$.

Next, for (b), observe that applying (A2) with $(N', \Lambda') = (\gamma N, \gamma \Lambda)$ gives us:

$$2\sqrt{\frac{t}{\gamma \phi \Lambda}} \;\; < \tfrac{\gamma N - \sqrt{\phi \gamma \Lambda t}}{(1-\phi)\gamma \Lambda} + \tfrac{t}{\gamma N - \phi \gamma \Lambda t} \Leftrightarrow 2\sqrt{\tfrac{t}{\phi \Lambda}} \;\; < \frac{\sqrt{\gamma} N - \sqrt{\phi \Lambda t}}{(1-\phi)\Lambda} + \frac{t}{\sqrt{\gamma} N - \phi \Lambda t}$$

We need to prove the above holds whenever (A1)-(A3) hold.

Since we know that ψ is an increasing function, we know that $\psi(\sqrt{\gamma}) > \psi(1)$ when $\gamma > 1$. But if we write out $\psi(\sqrt{\gamma}) > \psi(1)$, it gives us exactly the expression we needed to be true:

$$2\sqrt{\frac{t}{\phi \Lambda}} < \frac{\sqrt{\gamma} N - \sqrt{\phi \Lambda t}}{(1-\phi)\Lambda} + \frac{t}{\sqrt{\gamma} N - \phi \Lambda t}$$

Thus, when $(N', \Lambda') = (\gamma N, \gamma \Lambda)$, we find that (A2) holds for (N', Λ').

Thus, (A1)-(A3) hold under the conditions detailed in the Lemma. ■

Proof of Proposition 18. The proof of existence of all-regions equilibrium under the new model primitives obtains from Lemma A16 above. To prove that the equilibrium supply ratios tilts towards region 2, we first claim (but skip the straightforward proof) that if all the primitives of the model (λ, N, t) are multiplied by same scaling factor, the existence of an all-regions equilibrium as well as all of the $\frac{n_i^*}{n_j^*}$ ratios (and, by construction, all $\frac{\lambda_i}{\lambda_j}$ ratios) are preserved. Therefore, in this proof, instead of a multiplication of N and λ by a factor of $\gamma > 1$, we focus on fixing N and λ and, instead, replacing t by $t\frac{1}{\gamma}$.

For the all-regions equilibrium (n_1^*, n_2^*), define $\alpha = \frac{n_1^*}{N}$. We know from Proposition 17 that $\alpha > \phi$. The quilibrium condition, written in terms of α will be:

$$W^d(\alpha) \equiv W_1(\alpha N) - W_2((1-\alpha)N) = -\frac{(1-\alpha)N}{(1-\phi)\Lambda} + \frac{(\alpha)N}{\phi \Lambda} + \frac{t}{(1-\alpha)N} + \frac{t}{(\alpha)N} = 0 \qquad (48)$$

where W^d represents the difference between the total wait times between the two regions, which should be zero at the equilibrium. We now use the implicit function theorem to show that α increases as we increase t, which would prove the proposition.

$$\frac{d\alpha}{dt} = -\frac{\dfrac{\partial W^d}{\partial t}}{\dfrac{\partial W^d}{\partial \alpha}} = \frac{(1-\alpha)\alpha(2\alpha-1)(1-\phi)\phi\Lambda}{(\alpha-1)^2\alpha^2N^2 + (2(\alpha-1)\alpha+1)(\phi-1)\phi\Lambda t} \tag{49}$$

The numerator is positive since $\alpha > \frac{1}{2}$. Thus, the sign of $\dfrac{d\alpha}{dt}$ is determined by the denominator. Below, we prove that the denominator is positive as well. The argument takes the following steps:

1. Define the denominator as $g(\alpha) = (\alpha-1)^2\alpha^2N^2 + (2(\alpha-1)\alpha+1)(\phi-1)\phi\Lambda t$.
2. Observe that $g'(\alpha) = -2(2\alpha-1)\left((1-\alpha)\alpha N^2 + (1-\phi)\phi\Lambda t\right) < 0$, implying that $g(\alpha)$ is a decreasing function.
3. Since $\alpha \in \left[\phi, 1 - \frac{n_2^{min}}{N}\right]$, the inequality $g(\alpha) \geq g\left(1 - \frac{n_2^{min}}{N}\right)$ has to hold.
4. We prove that $\min g(\alpha) = g\left(1 - \frac{n_2^{min}}{N}\right) > 0$.

$$g\left(1 - \frac{n_2^{min}}{N}\right) = \tfrac{\phi^2\Lambda t}{N^2}\left(N^2 - 2N\sqrt{\phi\Lambda t} + (2\phi-1)\Lambda t\right) \tag{50}$$

$$= \tfrac{\phi^2\Lambda t}{N^2}\left((N - \sqrt{\phi\Lambda t})^2 - t\Lambda(1-\phi)\right) \tag{51}$$

where the term in parentheses is positive directly from assumption **(A1)**.

Thus, we know that $\dfrac{d\alpha}{dt} > 0$ implying that as t increases, the proportion of supply going to the higher-demand region is greater. ■

Proof of Proposition 19. Note that a scale-up in N can be thought of as a scale-up in $(\lambda_1, \lambda_2, N)$, followed by a scale back down in (λ_1, λ_2). From Proposition 18, we know that the first scale-up (i) preserves the existence of an all-regions equilibrium and also (ii) makes it strictly less under-supplied in region 2. Therefore, the proof of Proposition (19) will be complete if we show that the second scale back down also preserves the existence of an all-regions equilibrium and makes it less under-supplied inn region 2.

To see this, suppose (n_1^*, n_2^*) is the all-regions equilibrium under $(\lambda_1, \lambda_2, N, t)$. Let $\lambda_i' = \frac{\lambda_i}{\gamma}$ for $i \in \{1, 2\}$ and some $\gamma > 1$. We will now show that under $(\lambda_1', \lambda_2', N, t)$, there is an all-regions equilibrium with strictly less under-supply in region 2 than what is implied by (n_1^*, n_2^*).

Lemma A17. *The following statements are true about the "old" equilibrium allocation (n_1^*, n_2^*) under the "new" parameters $(\lambda_1', \lambda_2', N, t)$:*

1. *The total wait function $W_2(n)$ is strictly increasing at $n = n_2^*$.*
2. *At the allocation (n_1^*, n_2^*), the wait time in region 1 is strictly higher than that in region 2. That is, $W_1(n_1^*) > W_2(n_2^*)$.*

3. *The total wait function $W_1(n)$ is strictly increasing at $n = N \times \frac{\lambda'_1}{\lambda'_1+\lambda'_2}$.*

4. *At the allocation proportional to demand, the wait time in region 2 is strictly larger than that in region 1. That is, if we set $n_i = N \times \frac{\lambda'_i}{\lambda'_1+\lambda'_2}$, then $W_2(n_2) > W_1(n_1)$.*

Proof of Lemma A17. We start by statement 1. To see this, first note that from the assumption that (n_1^*, n_2^*) was the all-regions equilibrium under $(\lambda_1, \lambda_2, N, t)$, we know n_2^* has to be strictly larger than where the old W_2 function reached its trough. That is, $n_2^* > \sqrt{\lambda_2 t}$. Now, given $\lambda'_2 < \lambda_2$, we it is also the case that $n_2^* > \sqrt{\lambda'_2 t}$. Therefore, the new W_2 function is also strictly increasing at $n = n_2^*$.

We now turn to statement 2. Given that (n_1^*, n_2^*) was the all-regions equilibrium under the old parameters, the total wait times in the two regions were equal to each other. That is:

$$\frac{n_1^*}{\lambda_1} + \frac{t}{n_1^*} = \frac{n_2^*}{\lambda_2} + \frac{t}{n_2^*} \tag{52}$$

Given Proposition (17), we know that $n_1^* > n_2^*$, therefore: $\frac{t}{n_1^*} < \frac{t}{n_2^*}$. This latter inequality, combined with equality (52), implies $\frac{n_1^*}{\lambda_1} > \frac{n_2^*}{\lambda_2}$. The sign of this inequality is preserved if we multiply both sides of it by the positive number $\gamma - 1$. That is: $(\gamma - 1) \times \frac{n_1^*}{\lambda_1} > (\gamma - 1) \times \frac{n_2^*}{\lambda_2}$. The size of the inequality is also preserved when we add equal numbers to both sides. Those equal numbers are the two sides of equation (52). This will give us:

$$(\gamma - 1) \times \frac{n_1^*}{\lambda_1} + \frac{n_1^*}{\lambda_1} + \frac{t}{n_1^*} > (\gamma - 1) \times \frac{n_2^*}{\lambda_2} + \frac{n_2^*}{\lambda_2} + \frac{t}{n_2^*} \tag{53}$$

Therefore:

$$\gamma \times \frac{n_1^*}{\lambda_1} + \frac{t}{n_1^*} > \gamma \times \frac{n_2^*}{\lambda_2} + \frac{t}{n_2^*} \tag{54}$$

which gives us:

$$\frac{n_1^*}{\frac{\lambda_1}{\gamma}} + \frac{t}{n_1^*} > \frac{n_2^*}{\frac{\lambda_2}{\gamma}} + \frac{t}{n_2^*} \tag{55}$$

which, by definition, means:

$$\frac{n_1^*}{\lambda'_1} + \frac{t}{n_1^*} > \frac{n_2^*}{\lambda'_2} + \frac{t}{n_2^*} \tag{56}$$

This proves statement 2.

Next, we turn to statement 3. The argument is similar to that for statement 1. $N \times \frac{\lambda_1}{\lambda_1+\lambda_2}$ was larger than the trough of W_1 under the old parameters. Given that the trough gets smaller under the new parameters, it will keep being smaller than $N \times \frac{\lambda_1}{\lambda_1+\lambda_2}$.

Finally, statement 4 is obvious from the proof of Proposition (17). □

Now notice that Lemma (A17) completes the proof of the proposition. Given that under the allocation (n_1^*, n_2^*), we have $W_1 > W_2$, and under the allocation fully proportional to demand, we have $W_1 < W_2$, and given that both W_1 and W_2 are continuous functions, there should be an allocation $(n_1^{*'}, n_1^{*2})$ in between the two such that $W_1(n_1^{*'}) = W_2(n_2^{*'})$. This was achieved by statements 2 and 4. Now by statement 1, W_2 is strictly increasing at $n = n_2^{*'}$ because $n_2^{*'} > n_2^* > \sqrt{\lambda_2' t}$. Also, by statement 3, W_1 is increasing at $n = n_1^{*'}$ because $n_1^{*'} > N\frac{\lambda_1}{\lambda_1+\lambda_2} > \sqrt{\lambda_1' t}$. This implies that $(n_1^{*'}, n_2^{*'})$ is the all-regions equilibrium under parameters $(\lambda_1', \lambda_2', N, t)$. Now, given $n_1^{*'} < n_1^*$ and $n_2^{*'} > n_2^*$, it follows that:

$$\kappa^{*'} = \frac{\frac{n_1^{*'}}{\lambda_1'}}{\frac{n_2^{*'}}{\lambda_2'}} < \frac{\frac{n_1^*}{\lambda_1'}}{\frac{n_2^*}{\lambda_2'}} = \frac{\frac{n_1^*}{\lambda_1}}{\frac{n_2^*}{\lambda_2}} = \kappa^*$$

which finishes the proof of the proposition.■

Proof of Theorem (1). Before stating the induction hypothesis, we add one statement to the five statements of Theorem (1). The inclusion of this statement and leveraging it in the induction process will be helpful for the proof. We call it statement 6.

Statement 6. Suppose an all region equilibrium $n^* = (n_1^*, ..., n_I^*)$ *exists under primitives* (λ, N, t) *where* $\lambda = (\lambda_1, ..., \lambda_I)$*. Then, demand arrival rates are scaled down, that is, under new primitives* $(\frac{\lambda}{\gamma}, N, t)$ *with* $\gamma > 1$*, we have:*

- *An all-regions equilibrium* $n^{*'} = (n_1^{*'}, ..., n_I^{*'})$ *exists.*
- *The new equilibrium* $n^{*'}$ *shows less geographical supply inequity than* n^* *in the sense that for any* $i < j$*, we have* $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}} \leq \frac{\frac{n_i^*}{\lambda_i}}{\frac{n_j^*}{\lambda_j}}$*. The inequality is strict if and only if* $\lambda_i > \lambda_j$*.*

In words, this statement simply says the geographical supply inequity decreases if, all else fixed, all demand arrival rates proportionally decrease. The intuition is that this makes idle times relatively more important than pickup times.

We can now state the strong induction hypothesis.

Induction Hypothesis. Take some natural number $I_0 > 2$. If all statements of Theorem (1), including statement 6 added above, are correct for $I \in \{2, ..., I_0 - 1\}$, then they are also all correct for $I = I_0$.

Now, in order to prove the theorem, we need to take two steps. First, we should prove the basis of the induction process. That is, we must show the theorem holds under $I = 2$. Second, we need to prove the induction hypothesis. As for the first step, note that propositions (15) through (19) do this job. The only statement that is not explicitly proven by those theorem is statement 6. However, the proof of statement 6 was the main building block of the proof of Proposition (19).[39]

We now turn to the second and main step of this proof, which is to show that the induction hypothesis is correct (Note that some of the statements are not really proven based on the induction. Nevertheless, we present all of the proofs in this inductive framework since we believe having one induction as well as one non-induction section for the proof will just make it harder to read).

[39] Also, propositions (15) through (19) assume that $\lambda_1 > \lambda_2$ and, hence, leave out the case where $\lambda_1 = \lambda_2$. But the proofs for the case where $I = 2$ and $\lambda_1 = \lambda_2$ are straightforward and we leave them to the reader.

Proof of Statement 1. If the total wait time in region i is strictly higher than that in region j, then given the continuity of these wait-time functions, a small enough mass of drivers can leave region i for j and strictly benefit from that, violating the equilibrium assumption. To see why they are increasing, suppose on the contrary, that at the equilibrium allocation, for region i, the total wait time is strictly decreasing in the number of drivers in that region. Since drivers are equal across all regions in equilibrium, drivers from any other region j will have the incentive to relocate to region i, given that (i) currently region i has the same total wait as they do; and (ii) once they move to region i, the total wait time of that region will decrease. This is a violation of the equilibrium assumption. Therefore it has to be that at the equilibrium, the wait times are all increasing in the number of drivers at all regions.□

Proof of Statement 2. Suppose, on the contrary, that there are two different all-regions equilibria n^* and $\bar{n}$. Given the two vectors are different, there has to be a region i such that $n_i^* \neq \bar{n}_i$. Without loss of generality, assume $n_i^* < \bar{n}_i$. Given that, from statement 1, we know the total wait time is increasing at n_i^*, and given the fact that the wait time function, once it becomes increasing, it remains strictly increasing, we can say $W_i(n_i^*) < W_i(\bar{n}_i)$.

Now, again from statement 1, we know two things. First, $\forall j : \quad W_j(n_j^*) = W_i(n_i^*) \quad \& \quad W_j(\bar{n}_j) = W_i(\bar{n}_i)$, which implies: $\forall j : \quad W_j(n_j^*) < W_j(\bar{n}_j)$. Second, we know that the total wait time function at each region j must be strictly increasing after it hits its trough (which happens weakly before n_j^*). This implies that in order for $\forall j : \quad W_j(n_j^*) < W_j(\bar{n}_j)$ to hold, it must be that $\forall j : \quad n_j^* < \bar{n}_j$. Therefore:

$$\Sigma_{j=1,...,I_0}\bar{n}_j > \Sigma_{j=1,...,I_0}n_j^*$$

But this cannot be given that both of the sums should be equal to N.□

Proof of Statement 3. Note that the definition of equilibrium is that no driver should have the incentive to relocate from one region to another. This definition, by construction, implies that if $n^* = (n_1^*, ..., n_{I_0}^*)$ is an equilibrium under (λ, N, t), then once we fix $\tilde{N} = n_i^* + n_j^*$ for some i, j with $i < j$, then the allocation (n_i^*, n_j^*) is itself an equilibrium of the two-region game with primitives $(\lambda_i, \lambda_j, \tilde{N}, t)$. Thus, by Proposition (17) (or alternatively, by the base of the induction), we know that if $\lambda_i > \lambda_j$, then $\frac{n_i^*}{\lambda_i} > \frac{n_j^*}{\lambda_j}$. Also in case $\lambda_i = \lambda_j$, it is fairly straightforward to verify that $\frac{n_i^*}{\lambda_i} = \frac{n_j^*}{\lambda_j}$. To see this, note that in that case, $\frac{n_i^*}{\lambda_i} = \frac{n_j^*}{\lambda_j}$ if and only if $n_i^* = n_j^*$. It is easy to see that $n_i^* = n_j^*$ is an equilibrium given that it gives the two regions the same total wait time and that at it, the total wait times must be increasing according to previous statements.□

Proof of Statement 4. Before we start the proof of this statement, we note that, similar to the case of Proposition (18), we can work with primitives $(\lambda, N, \frac{t}{\gamma})$ instead of $(\gamma\lambda, \gamma N, t)$. As a reminder, this is because there is a one-to-one and onto mapping between the equilibria under the two primitives, which preserves all of the $\frac{n_i^*}{\lambda_i}$ values.

We start by proving the first statement. That is, if an all-regions equilibrium exists under (λ, N, t), then one does under $(\lambda, N, \frac{t}{\gamma})$ as well. To see this, let us assume that under the "old" primitives (λ, N, t), the all-regions equilibrium allocation n^* is such that $\forall i \in \{1, ..., I_0\} : \quad W_i(n_i^*) = w$. We know this common w must exist from statement 1, and we know it is unique from statement 2.

We show existence of an equilibrium allocation under the new primitive by first describing two "partial equilibrium" allocations. We construct the first partial equilibrium allocation $\bar{n} = (\bar{n}_1, ..., \bar{n}_{I_0})$ by fixing $\bar{n}_1 = n_1^*$ and assuming the rest of values $(\bar{n}_2, ..., \bar{n}_{I_0}$ to be the equilibrium allocation of drivers among regions 2 to I_0 under primitives $((\lambda_2, ..., \lambda_{I_0}), N - n_1^*, \frac{t}{\gamma})$. In words, this allocation fixes the number of drivers in region 1 (i.e., the region with the highest demand arrival rate λ_1) at its value under the old primitives but allows the drivers of all other regions to reshuffle themselves among those regions. The second partial equilibrium allocation $\tilde{n}$ fixes $\tilde{n}_1 = N \times \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}$, and assumes the rest of values $(\tilde{n}_2, ..., \tilde{n}_{I_0}$ to be the equilibrium allocation of drivers among regions 2 to I_0 under primitives $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}), \frac{t}{\gamma})$. In words, this allocation fixes the total number of drivers in region 1 at the value it would take if drivers were to be allocated fully proportional to demand arrival rates. It then allows the rest of the drivers to reshuffle themselves among other areas under the new primitives. We will use these two partial equilibrium allocations to prove existence of an all-region equilibrium. But first we need to prove the existence of these partial equilibrium allocations themselves. Lemma A18 below does this job.

Lemma A18. *Partial equilibrium allocations $\tilde{n}$ and $\bar{n}$ described above exist, are unique, and allocate a strictly positive number of drivers to each region.*

Proof of Lemma (A18). We first start from $\bar{n}$. Note that the assumption of n^* being the equilibrium allocation under (λ, N, t), by construction implies that $(n_2^*, ..., n_{I_0}^*)$ is the unique all-region equilibrium allocation under $((\lambda_2, ..., \lambda_{I_0}), N - n_1^*, t)$. Now, given that by our induction assumption all results (including statement 4) hold for $I_0 - 1$ regions, if the primitives remain the same except that t is divided by some $\gamma > 1$, a unique all-region equilibrium will still exist. This is what we were denoting $\bar{n}_2$ through $\bar{n}_{I_0}$.

Next, we turn to $\tilde{n}$ and construct it from $\bar{n}$. We just showed that $(\bar{n}_2, ..., \bar{n}_{I_0})$ is the unique all-regions equilibrium under $((\lambda_2, ..., \lambda_{I_0}), N - n_1^*, t)$. Also note that by statement 3, we know $n_1^* > \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i})$, which implies $N - n_1^* < N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}))$. Therefore, primitives $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}), \frac{t}{\gamma})$ can be constructed from primitives $((\lambda_2, ..., \lambda_{I_0}), N - n_1^*, t)$ by increasing the total number of drivers. Given that $\bar{n}$ was the unique all-regions equilibrium allocation under $((\lambda_2, ..., \lambda_{I_0}), N - n_1^*, t)$, and given the induction assumption on statement 5 for $I = I_0 - 1$ regions, we can say that primitives $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}), \frac{t}{\gamma})$ also have a unique all-regions equilibrium allocation. This is exactly what was denoted $\tilde{n}_2, ..., \tilde{n}_{I_0}$. This completes the proof of the lemma.□

We now use these two partial equilibrium allocations to show that a unique all-regions equilibrium allocation exists under primitives $(\lambda, N, \frac{t}{\gamma})$. Our next step is to prove the following useful lemma.

Lemma A19. *At the partial equilibrium allocation $\bar{n}$, the total wait time in region 1 is larger than that in any other region. Conversely, at the partial equilibrium allocation $\tilde{n}$, the total wait time in region 1 is smaller than that in any other region.*

Proof of Lemma (A19). To see why the result holds for $\bar{n}$, note that under the old equilibrium n^* and old primitives (λ, N, t), all of the wait times were equal. This means for any $i > 1$ we had

$$\frac{n_1^*}{\lambda_1} + \frac{t}{n_1^*} = \frac{n_i^*}{\lambda_i} + \frac{t}{n_i^*}$$

But given that for all $i > 1$ we have $n_1^* \geq n_i^*$ we get the following inequality under the new primitives $(\lambda, N, \frac{t}{\gamma})$:

$$\frac{n_1^*}{\lambda_1} + \frac{\frac{t}{\gamma}}{n_1^*} \geq \frac{n_i^*}{\lambda_i} + \frac{\frac{t}{\gamma}}{n_i^*}$$

Next, note that the main and only difference between allocations n^* and $\bar{n}$ is that under $\bar{n}$, drivers reshuffle among regions 2 to I_0 in order to reduce their total wait times. Therefore, there has to be at least one region j such that:

$$\frac{\bar{n}_j}{\lambda_j} + \frac{\frac{t}{\gamma}}{\bar{n}_j} \leq \frac{n_j^*}{\lambda_j} + \frac{\frac{t}{\gamma}}{n_j^*}$$

Combining the above two, we get:

$$\frac{\bar{n}_j}{\lambda_j} + \frac{\frac{t}{\gamma}}{\bar{n}_j} \leq \frac{n_1^*}{\lambda_j} + \frac{\frac{t}{\gamma}}{n_1^*}$$

But the total wait time under $(\lambda, N, \frac{t}{\gamma})$ is equal across regions 2 through I_0 under allocation $\bar{n}$. Therefore, the above inequality holds not only for a specific j , but under any $j > 1$. This proves the lemma for $\bar{n}$ given that $\bar{n}_1 = n_1^*$.

Next, we prove the lemma for $\tilde{n}$. We first show that the wait time in region 1 is smaller than that in region 2 if both get drivers proportional to their demand arrival rates. We then show that the wait time in region 2 under $\tilde{n}_2$ is larger than the wait time in region 2 if region 2 were to get drivers proportional to its demand arrival rate. These two statements, combined, will prove our intended result. To see the first claim, note that the wait time in region 1, if it gets $N \times \frac{\lambda_1}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}$ drivers, will be:

$$w_1 = \frac{N \times \frac{\lambda_1}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}}{\lambda_1} + \frac{\frac{t}{\gamma}}{N \times \frac{\lambda_1}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}}$$

which gives:

$$w_1 = \frac{N}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i} + \frac{\frac{t}{\gamma} \times \Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}{N \lambda_1} \tag{57}$$

Similarly, if region 2 were to get $N \times \frac{\lambda_2}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}$ drivers, its total wait time will be:

$$w_2 = \frac{N}{\Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i} + \frac{\frac{t}{\gamma} \times \Sigma_{i \in \{1,\ldots,I_0\}} \lambda_i}{N \lambda_2} \tag{58}$$

It is easy to see that the first terms of w_1 and w_2 are the same, and the second term is larger in w_2 given that $\lambda_1 \geq \lambda_2$. Now note that under allocation $\tilde{n}$, the wait time in region 1 is indeed w_1. So, it remains to show that $W_2(\tilde{n}_2) \geq w_2$. To show this, we make two observations (and prove them both shortly). First, $\tilde{n}_2 \geq N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}$. This simply says under $\tilde{n}$, region 2 is getting more drivers than it would if drivers were to be allocated to regions proportionally to their demand rates. Second, the total wait time function in region 2 is increasing between $N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}$ and $\tilde{n}_2$. Together, these two observations imply $W_2(\tilde{n}_2) \geq w_2$, as desired. Therefore, we have shown that $W_2(\tilde{n}_2) \geq w1$. But given that $(\tilde{n}_2, ..., \tilde{n}_{I_0})$ was an all-regions equilibrium under $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}, \frac{t}{\gamma})$, we know that for any $i, j > 1 : \quad W_i(\tilde{n}_i) = W_j(\tilde{n}_j)$. This, combined with $W_2(\tilde{n}_2) \geq w_1$, completes the proof of the lemma, of course with the exception of the two observations made in this paragraph. We now turn to proving those observations and finish the proof of the lemma.

The first observation was that $\tilde{n}_2 \geq N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}$. To see why this is true, note that $\tilde{n}$ is the all-regions equilibrium under $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}), \frac{t}{\gamma})$. Therefore, by our induction assumption on statement 3, region 2 will get disproportionately more drivers relative to all other regions, because it has the highest λ_i amongst regions $2, ..., I_0$. That is $\forall i > 2 : \quad \frac{\tilde{n}_2}{\lambda_2} \geq \frac{\tilde{n}_i}{\lambda_i}$. It is then easy to show that:

$$\frac{\tilde{n}_2}{\lambda_2} \geq \frac{\Sigma_{i=2,...,I_0}\tilde{n}_i}{\Sigma_{i=2,...,I_0}\lambda_i} \tag{59}$$

But we know, from the primitives, that $\Sigma_{i=2,...,I_0}\tilde{n}_i = N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}) = N \times \frac{\Sigma_{i=2,...,I_0}\lambda_i}{\Sigma_{i=1,...,I_0}\lambda_i}$. Now, plugging this into (59) and rearranging, we get $\tilde{n}_2 \geq N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}$, which is exactly our first observation.

We now turn to the proof of the second observation. That is, we want to show that the total wait time function in region 2 is increasing between $N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}$ and $\tilde{n}_2$. To see this, note that the wait time curve in region 2 takes the form that was depicted in figure (17). In particular, it is a curve with only one trough; and once past the trough, the curve will remains strictly increasing indefinitely. Thus, to prove that the wait-time is increasing over the interval $[N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i}, \tilde{n}_2]$, it is sufficient to show that the smallest point in this interval is past the trough. One can show the trough happens at $n_2 = \sqrt{\frac{t}{\gamma}\lambda_2}$. Therefore, what we need to show is:

$$N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i} \geq \sqrt{\frac{t}{\gamma}\lambda_2} \tag{60}$$

In order to prove this, we first assume, to the contrary, that $N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i} < \sqrt{\frac{t}{\gamma}\lambda_2}$; then we arrive at a contradiction with the result that $\tilde{n}$ is the all-regions equilibrium under $((\lambda_2, ..., \lambda_{I_0}), N \times (1 - \frac{\lambda_1}{\Sigma_{i=1,...,I_0}\lambda_i}, \frac{t}{\gamma})$. Note that we are assuming, without loss of generality, $\lambda_2 \geq \lambda_i$ for any $i > 2$. Therefore, given that all λ_i are positive, for any $i > 2$, we have $\frac{\lambda_i}{\lambda_2} \leq \sqrt{\frac{\lambda_i}{\lambda_2}}$. Thus, if we multiply the left hand side of the inequality $N \times \frac{\lambda_2}{\Sigma_{i\in\{1,...,I_0\}}\lambda_i} < \sqrt{\frac{t}{\gamma}\lambda_2}$ by $\frac{\lambda_i}{\lambda_2}$ and the right hand side by $\sqrt{\frac{\lambda_i}{\lambda_2}}$, then the direction of the inequality should not not change. Therefore, not only for region 2, but also for any region $i \geq 2$, we will have:

$$N \times \frac{\lambda_i}{\Sigma_{j\in\{1,...,I_0\}}\lambda_j} < \sqrt{\frac{t}{\gamma}\lambda_i}$$

Now, if we sum over all $i = 2, ..., I_0$ on both sides of the inequality above, we get:

$$N \times \Sigma_{i=2,...,I_0} \frac{\lambda_i}{\Sigma_{j\in\{1,...,I_0\}}\lambda_j} < \Sigma_{i=2,...,I_0}\sqrt{\frac{t}{\gamma}\lambda_i}$$

Rearranging, we get:

$$N \times \left(1 - \frac{\lambda_1}{\Sigma_{j\in\{1,...,I_0\}}\lambda_j}\right) < \Sigma_{i=2,...,I_0}\sqrt{\frac{t}{\gamma}\lambda_i}$$

But $N \times (1 - \frac{\lambda_1}{\Sigma_{j\in\{1,...,I_0\}}\lambda_j})$ is the total number of drivers in regions 2 through I_0. That is: $N \times (1 - \frac{\lambda_1}{\Sigma_{j\in\{1,...,I_0\}}\lambda_j}) = \Sigma_{j=2,...,I_0}\tilde{n}_j$. Therefore, we get:

$$\Sigma_{j=2,...,I_0}\tilde{n}_j < \Sigma_{i=2,...,I_0}\sqrt{\frac{t}{\gamma}\lambda_i}$$

which implies there should be at least one $j \geq 2$ such that $\tilde{n}_j < \sqrt{\frac{t}{\gamma}\lambda_j}$. But this means that for that region j, the wait time function is decreasing at $n = \tilde{n}_j$ contradicting the result that $\tilde{n}_j$ is part of an all-regions equilibrium. This completes the proof of the second observation, and hence that of lemma (A19).□

Next, we use lemma (A19) to construct an all-regions equilibrium under primitives $(\lambda, N, \frac{t}{\gamma})$. This will be a constructive proof to the existence portion of statement 4. To this end, we start from the first partial equilibrium $\bar{n}$, gradually shifting drivers from region 1 to other regions until we are left with $\tilde{n}$ drivers in region 1. That is, for any $\hat{n}_1 \in [\tilde{n}_1, \bar{n}_1]$ we consider the partial equilibrium $\hat{n} = (\hat{n}_1, ..., \hat{n}_{I_0})$ such that the $(\hat{n}_2, ..., \hat{n}_{I_0})$ is the all-regions equilibrium allocation under primitives $(\lambda_2, ..., \lambda_{I_0}), N - \hat{n}_1, \frac{t}{\gamma})$. The argument for why such partial equilibrium exists for any $\hat{n}_1 < \bar{n}_1$ is similar the argument given in proof of lemma (A18) for $\tilde{n}_1$.

Now, note that by lemma (A19), the total wait time in region 1 is larger than that in other regions when $\hat{n}_1 = \bar{n}_1$ and it is smaller in region 1 than it is in other regions when $\hat{n}_1 = \tilde{n}_1$. Therefore, there should be some $\hat{n}_1 \in [\tilde{n}_1, \bar{n}_1]$ for which the total wait time in region 1 is equal to the total wait time in all of the other regions, which themselves are equal to each other by $\hat{n}$ being a partial equilibrium the way defined above.[40] We claim such allocation $\hat{n}$ is the all-regions equilibrium of the whole market (that is, under primitives $(\lambda, N, \frac{t}{\gamma})$). The proof for this claim is as follows:

We know that under allocation $\hat{n}$ all regions have the same total wait time. We also know, by $(\hat{n}_2, ..., \hat{n}_{I_0})$ being the all-regions equilibrium under primitives $(\lambda_2, ..., \lambda_{I_0}), N - \hat{n}_1, \frac{t}{\gamma})$, that the total wait time in each region $i > 1$ is increasing at $n = \hat{n}_i$. Thus, the only thing that remains to be shown is that for $i = 1$ too the total wait time curve is increasing at $n = \hat{n}_1$. To this end, as argued before in a similar case, we need to show that $\hat{n}_1 \geq \sqrt{\frac{t}{\gamma}\lambda_1}$. Note that given $\hat{n}_1 \geq \tilde{n}_1$, it would

[40] Note that in order to make this argument we also need to know that as we move $\hat{n}_1$ within $[\tilde{n}_1, \bar{n}_1]$, the total wait time in region 1 as well as the common total wait time in the other regions both move continuously. This is true by construction for region 1, since the total wait time function is continuous. For other regions, this needs to be shown that as we add drivers to the collection of these regions, the equilibrium total wait time moves continuously. We skip the proof of this claim here, but can provide it upon request.

suffice to show $\tilde{n}_1 \geq \sqrt{\frac{t}{\gamma}\lambda_1}$. We show this latter inequality by borrowing from what we already did in the proof of the last observation we made as part of proof of lemma (A19). There, we proved inequality (60) holds. Now, given that we have been assuming (without loss of generality) that $\lambda_1 \geq \lambda_2$, and given that all λ_i are positive numbers, we get: $\frac{\lambda_1}{\lambda_2} \geq \sqrt{\frac{\lambda_1}{\lambda_2}}$. Therefore, if we multiply the left-hand side of equation (60) by $\frac{\lambda_1}{\lambda_2}$ and the right hand side by $\sqrt{\frac{\lambda_1}{\lambda_2}}$, the sign of the inequality should not change. This operation gets us:

$$N \times \frac{\lambda_1}{\Sigma_{i \in \{1,...,I_0\}} \lambda_i} \geq \sqrt{\frac{t}{\gamma}\lambda_1}$$

which is exactly what we were after. This shows that $\hat{n}$ is the all-regions equilibrium, completing the proof of the first part of statement 4 in the theorem.□

Now that we have shown the all-region equilibrium $\hat{n}$ under primitives $(\lambda, N, \frac{t}{\gamma})$ exists, we show that it indeed shows less geographical supply inequity than the old equilibrium n^*. As the first step towards this goal, note that for any $j > i > 1$, we can show the result holds based on our induction assumption. More precisely, we know that $(n^*_2, ..., n^*_{I_0})$ is the all-regions equilibrium under primitives $((\lambda_2, ..., \lambda_{I_0}), N - n^*_1, t)$. We also know that $(\hat{n}_2, ..., \hat{n}_{I_0})$ is the all regions equilibrium under primitives $((\lambda_2, ..., \lambda_{I_0}), N - \hat{n}_1, \frac{t}{\gamma})$. The move from primitives $((\lambda_2, ..., \lambda_{I_0}), N - n^*_1, t)$ to primitives $((\lambda_2, ..., \lambda_{I_0}), N - \hat{n}_1, \frac{t}{\gamma})$ involves two steps. The first step is to divide t by some $\gamma > 1$. The second step is to add $n^*_1 - \hat{n}_1$ drivers. Based on our induction assumption, both statements 4 and 5 of Theorem (1) hold for $I_0 - 1$ regions. Therefore, for any $j > i > 1$ we have:

$$\frac{\frac{\hat{n}_i}{\lambda_i}}{\frac{\hat{n}_j}{\lambda_j}} \leq \frac{\frac{n^*_i}{\lambda_i}}{\frac{n^*_j}{\lambda_j}}$$

with the inequality strict if $\lambda_i > \lambda_j$. Now the only thing that remains to show is that we can say the same not only for $j > i > 1$, but also for $j > i = 1$. In order to show this, we consider three cases.

Case 1: for every $j > 1$, we have $\hat{n}_j > n^*_j$. In this case, the result is becomes trivial given that we know $\hat{n}_1 \leq n^*_1$.

Case 2: for at least two distinct $j, j' > 1$, we have $\hat{n}_j \leq n^*_j$ and $\hat{n}_{j'} \leq n^*_{j'}$. We start with j and note that the allocation of drivers in all regions other than j –i.e., allocation $(\hat{n}_1, ..., \hat{n}_{j-1}, \hat{n}_{j+1}, ..., \hat{n}_{I_0})$ is the all-region equilibrium under primitives $((\lambda_1, ..., \lambda_{j-1}, \lambda_{j+1}, ..., \lambda_{I_0}), N - \hat{n}_j, \frac{t}{\gamma})$. Also note that allocation $(n^*_1, ..., n^*_{j-1}, n^*_{j+1}, ..., n^*_{I_0})$ is the all-region equilibrium under primitives $((\lambda_1, ..., \lambda_{j-1}, \lambda_{j+1}, ..., \lambda_{I_0}), N - n^*_j, \frac{t}{\gamma})$. Note that the former primitives can be obtained from the latter by two moves. First, going from t to $\frac{t}{\gamma}$ for some $\gamma > 1$; and second, changing the total number of drivers from $N - n^*_j$ to the (by assumption) larger number of $N - \hat{n}_j$. Based on our induction assumptions, we know that both of these moves reduce the geographical supply inequity. Therefore, now we can claim the following for any $i \neq j$:

$$\frac{\frac{\hat{n}_1}{\lambda_1}}{\frac{\hat{n}_i}{\lambda_i}} \leq \frac{\frac{n_1^*}{\lambda_1}}{\frac{n_i^*}{\lambda_i}}$$

with the inequality strict if $\lambda_1 \neq \lambda_i$. This covers all of the comparisons that we needed with the exception of the comparison between region 1 and region j itself. But we can prove the inequality for that case as well, by going through the exact same process as above, except excluding region j' this time instead of region j. This finishes the proof of statement 4 of the theorem under case 2.

Case 3: for exactly one region $j > 1$, we have $\hat{n}_j \leq n_j^*$. In this case, we can go through the same process as that described in case 2, to show for any $i \neq j$:

$$\frac{\frac{\hat{n}_1}{\lambda_1}}{\frac{\hat{n}_i}{\lambda_i}} \leq \frac{\frac{n_1^*}{\lambda_1}}{\frac{n_i^*}{\lambda_i}}$$

with the inequality strict if $\lambda_1 \neq \lambda_i$. This time, however, we are not able to use a similar argument for the to show the result holds between regions 1 and j. The following lemmas, however, demonstrate a different way to prove the result for this specific comparison.

Lemma A20. *Under the conditions of case 3, we have $\hat{n}_1 \geq \frac{n_1^*}{\sqrt{\gamma}}$ and $\hat{n}_j \geq \frac{n_j^*}{\sqrt{\gamma}}$.*

Proof of Lemma (A20). We only show $\hat{n}_1 \geq \frac{n_1^*}{\sqrt{\gamma}}$. The argument for $\hat{n}_j \geq \frac{n_j^*}{\sqrt{\gamma}}$ is the same.

We start by observing that the total wait time w_1 in region 1 under $n_1 = \frac{n_1^*}{\sqrt{\gamma}}$ is given by:

$$\begin{aligned}
w_1 &= \frac{n_1}{\lambda_1} + \frac{\frac{t}{\gamma}}{n_1} \\
&= \frac{\frac{n_1^*}{\sqrt{\gamma}}}{\lambda_1} + \frac{\frac{t}{\gamma}}{\frac{n_1^*}{\sqrt{\gamma}}} \\
&= (\frac{n_1^*}{\lambda_1} + \frac{t}{n_1^*})\frac{1}{\sqrt{\gamma}} \\
&= \frac{w^*}{\sqrt{\gamma}}
\end{aligned} \tag{61}$$

where w^* is the common total wait time among all regions under primitives (λ, N, t) and the all-regions equilibrium n^* given those primitives.

Next, we show that under the *new primitives* $(\lambda, N, \frac{t}{\gamma})$, but at the *old equilibrium allocation* n^*, the total wait-time in any region i is weakly larger than $\frac{w^*}{\sqrt{\gamma}}$. To see this, we write out one such total wait time:

$$\frac{n_i^*}{\lambda_i} + \frac{\frac{t}{\gamma}}{n_i^*}$$

Note that because n^* is the all-region equilibrium under the old primitives, it must be that for all $i:\quad n_i^* \geq \sqrt{t\lambda_i}$. This gives $\frac{n_i^*}{\lambda_i} \geq \frac{t}{n_i^*}$, or, alternatively: $\frac{t}{n_i^*} \leq \frac{1}{2}(\frac{n_i^*}{\lambda_i} + \frac{t}{n_i^*}) = \frac{w^*}{2}$.

Therefore, we can write:

$$
\begin{aligned}
\frac{n_i^*}{\lambda_i} + \frac{\frac{t}{\gamma}}{n_i^*} &= (\frac{n_i^*}{\lambda_i} + \frac{t}{n_i^*}) - \frac{t}{n_i^*}(1 - \frac{1}{\gamma}) \\
&= w^* - \frac{t}{n_i^*}(1 - \frac{1}{\gamma}) \\
&\geq w^*(1 - \frac{1}{2}(1 - \frac{1}{\gamma})) \\
&= w^* \times (\frac{1 + \frac{1}{\gamma}}{2}) \\
&> w^* \times (\sqrt{1 \times \frac{1}{\gamma}}) \\
&= \frac{w^*}{\sqrt{\gamma}} \qquad (62)
\end{aligned}
$$

Equations (61) and (62), together, tell us that for any i, we have $\frac{n_i^*}{\lambda_i} + \frac{\frac{t}{\gamma}}{n_i^*} > w_1$. Now notice that the total wait time under the new primitives at the old equilibrium alloction in any region is increasing. This is simply because $\forall i: \quad n_i^* \geq \sqrt{t\lambda_i} > \sqrt{\frac{t}{\gamma}\lambda_i}$. This, combined with the fact that there is at least one region i with $\hat{n}_i > n_i^*$,[41] tells us:

$$
\hat{w} \equiv \frac{\hat{n}_i}{\lambda_i} + \frac{\frac{t}{\gamma}}{\hat{n}_i} > \frac{n_i^*}{\lambda_i} + \frac{\frac{t}{\gamma}}{n_i^*} > w_1
$$

where $\hat{w}$ is defined as the common total wait time among all regions under the new primitives and new equilibrium allocation.

What $\hat{w} > w_1$ tells us is that if we reduce the number of drivers in region 1 to $n_1 = \frac{n_1^*}{\sqrt{\gamma}}$, the total wait time in region 1 falls below the equilibrium total wait time. But this means it has to be that $\hat{n}_1 > n_1 = \frac{n_1^*}{\sqrt{\gamma}}$. To see why, consider two scenarios. First, if $n_1 < \sqrt{\frac{t}{\gamma}\lambda_1}$, then by $\hat{n}_1 \geq \sqrt{\frac{t}{\gamma}\lambda_1}$, we get $\hat{n}_1 > n_1$. Next, if $n_1 \geq \sqrt{\frac{t}{\gamma}\lambda_1}$, then the wait time curve is strictly increasing when moving up from n_1, which means at some point past n_1, it hits the higher wait time $\hat{w} > w_1$. That point would be $\hat{n}_1$. Thus, the lemma has been proven for region 1. The proof for region j is exactly the same. $\square$

We now present the another useful lemma which helps us better understand what happens to the two regions 1 and j.

Lemma A21. *Consider a market with two regions 1 and 2 only. Allocation n^* is an equilibrium in this market under primitives $(\lambda_1, \lambda_2, N, t)$ if and only if allocation $\frac{n^*}{\sqrt{\gamma}}$ is an equilibrium under primitives $(\lambda_1, \lambda_2, \frac{N}{\sqrt{\gamma}}, \frac{t}{\gamma})$.*

Proof of Lemma (A21). Follows directly from definitions.$\square$

[41] This is true because there are at least three regions; and besides regions 1 and j, case 3 assumes $\hat{n}_i > n_i^*$ for all i.

Now, lemmas (A20) and (A21) show us a clear way to complete the last piece of the inductive proof of statement 4 in the theorem. Based on lemma (A20), we know $\hat{n}_1 + \hat{n}_j > \frac{n_1^*+n_j^*}{\sqrt{\gamma}}$. Now define $N^* = n_1^* + n_j^*$ and $\hat{N} = \hat{n}_1 + \hat{n}_j$. We know that (n_1^*, n_j^*) was the all-regions equilibrium under primitives $(\lambda_1, \lambda_j, N^*, t)$. Thus, by lemma (A21) we can claim that $(\frac{n_1^*}{\sqrt{\gamma}}, \frac{n_j^*}{\sqrt{\gamma}})$ is the all-regions equilibrium under primitives $(\lambda_1, \lambda_j, \frac{N^*}{\sqrt{\gamma}}, \frac{t}{\gamma})$.

On the other hand, we know that $(\hat{n}_1, \hat{n}_j)$ is the all-regions equilibrium under primitives $(\lambda_1, \lambda_j, \hat{N}, \frac{t}{\gamma})$. Given that we showed $\hat{N} > \frac{N^*}{\sqrt{\gamma}}$, and given that by our strong induction assumption statement 5 is correct for all two-region cases, we can write:

$$\frac{\frac{\hat{n}_1}{\lambda_1}}{\frac{\hat{n}_j}{\lambda_j}} \geq \frac{\frac{\frac{n_1^*}{\sqrt{\gamma}}}{\lambda_1}}{\frac{\frac{n_j^*}{\sqrt{\gamma}}}{\lambda_j}} = \frac{\frac{n_1^*}{\lambda_1}}{\frac{n_j^*}{\lambda_j}}$$

with the inequality strict whenever $\lambda_1 > \lambda_j$. This completes the proof of case 3, and hence finishes the inductive proof of statement 4 of Theorem (1) with the exception of the last claim about $\gamma \to \infty$, which we turn to next.

To see why for any $i < j$ we have $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}} \to 1$ as $\gamma \to \infty$, assume first on the contrary, that this claim is not true. That is $\exists i < j$ such that $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}}$ does not approach 1 as $\gamma \to \infty$. We use this assumption to get a contradiction. Note that given the other claims in statement 4 of the theorem, we know that $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}}$ monotonically decreases as γ increases. Therefore, the only possibility for it to not approach 1, is for it to approach a number strictly above one. Denote that number by $\kappa > 1$.

Also note that as $\gamma \to \infty$, the equilibrium number of drivers in none of the regions tends to zero. This is because (i) as immediately implied by the other claims in statement 4, the number of drivers in the lowest demand region $n_I^{*'}$ is increasing in γ; and (ii) the number of drivers in any other region is always weakly larger than $n_I^{*'}$. This, along with the fact $\gamma \to \infty$ is equivalent to $t \to 0$, means that the total wait time in each region k will tend to the idle time in that region. Therefore, $\frac{\frac{n_i^{*'}}{\lambda_i}}{\frac{n_j^{*'}}{\lambda_j}} \to \kappa > 1$ implies regions i and j have different limiting total wait times at the equilibrium, which contradicts statement 1. This finishes the proof of the statement. □

Proof of Statement 6. The steps of this proof closely (almost exactly) follow the steps of the proof of statement 4. We skip it but can provide the detailed proof upon request. □

Proof of Statement 5. Similar to the corresponding two-region case (i.e., proof of Proposition (19). This statement can be proven in a straightforward manner once we have proven statements 4 and 6. To be more precise, if we know that geographical supply inequity decreases in the sense defined in the statement of the theorem both (i) when we proportionally scale-up N and the vector λ and (ii) when we scale down the vector λ, it follows that the geographical supply inequity also decreases when we only scale up N, which is a certain combination of (i) and (ii).$\square$

The above proofs show that (i) the theorem holds for $I = 2$ and that (ii) the theorem holds for any $I_0 > 2$ if it holds for all $I \in \{2, ..., I_0 - 1\}$. This means our proof is complete.$\blacksquare$

E.6.2 Proofs under the assumption that region size is homogeneous

Proof of Proposition 11. Proof of Statement 1. To see that an equilibrium always exists, note that all N drivers being in one region i is always an equilibrium. This is because under this allocation, pickup times in all other regions are extremely large, incentivizing drivers to remain in i.

To see uniqueness of equilibrium allocation n^* conditional on the set of served areas $J = \{i : n_i^* > 0\}$, note that allocation n_J^* (i.e., restriction of n^* to regions in J) is an all-regions equilibrium under primitives (λ_J, N, t). We know from Theorem 1 that this equilibrium is unique. $\square$

Proof of Statement 2. This also follows immediately from Theorem 1 and the observation that n_J^* is an all-regions equilibrium under (λ_J, N, t). $\square$

Proof of Statement 3. Suppose $J = \{i : n_i^* > 0\}$ and $J' = \{i : n_i^{*'} > 0\}$. Also suppose $J \subsetneq J'$. Given that the total number of drivers under n^* and $n^{*'}$ are the same, and given $J \subsetneq J'$, there has to be one region $i \in J'$ such that $n_i^* > n_i^{*'}$.

Observe that n_J^* is the all-regions equilibrium under (λ_J, N, t) and $n_{J'}^{'*}$ is the all-regions equilibrium under $(\lambda_{J'}, N, t)$. Therefore, by Theorem 1, we know that in region i, total wait time is strictly increasing in n_i for any $n_i > n_i^{*'}$. This, combined with $n_i^* > n_i^{*'}$, implies that the total wait time in region i is strictly higher under n^* relative to $n^{*'}$. Given the equivalence of toatl wait time across regions with positive supply, this means the total wait time in any served region under n^* is strictly higher than that in any served region under $n^{*'}$.$\square$

This finishes the proof of the proposition (albiet still under the assumption that t is uniform across regions). $\blacksquare$

Proof of Proposition 12. Denote the set up regions that get positive supply under $n^*$by J. Statement 1 directly follows from Theorem 1 once we note that n_J^* is the all-regions equilibrium under primitives (λ_J, N, t). Statement 2 follows directly from statement 1 given that under uniform prices and wages, equilibrium wait times are uniform across regions as shown by Theorem 1. More precisely, we know $A_i(n_i^*) = \frac{n_i^*}{W_i(n_i^*)\lambda_i}$ and the same holds for region j. Now, given $W_i(n_i^*) = W_j(n_j^*)$, statement 2 directly follows from statement 1. $\blacksquare$

Proof of Proposition 13. Again, the proofs of all sections directly follow from Theorem 1 and the observation that once we restrict attention to the set $J = \{i : n_i^* > 0\}$ of regions, then (i) n_J^* is an all regions equilibrium, and (λ'_J, N', t') is still a one (or two) sided thickening of (λ_J, N, t). $\blacksquare$

Proof of Proposition 14. Before proving the proof of the proposition, we prove the following lemma.

Lemma A22. $n_1^{**} > n_2^{**}$.

Proof of Lemma A22. Suppose, on the contrary, that $n_i^{**} < n_j^{**}$. In this case, we can show the platform will be better off swapping the allocation of drivers between the two regions. To see this, note that $n_i^{**} < n_j^{**}$ implies:

$$(\frac{1}{\lambda_i} + \frac{t}{n_j^{**2}}) \times (\frac{1}{\lambda_j} + \frac{t}{n_j^{**2}}) < (\frac{1}{\lambda_i} + \frac{t}{n_i^{**2}}) \times (\frac{1}{\lambda_j} + \frac{t}{n_i^{**2}})$$

By $\lambda_i > \lambda_j$ we get:

$$\frac{\frac{1}{\lambda_j} - \frac{1}{\lambda_i}}{(\frac{1}{\lambda_i} + \frac{t}{n_j^{**2}}) \times (\frac{1}{\lambda_j} + \frac{t}{n_j^{**2}})} > \frac{\frac{1}{\lambda_j} - \frac{1}{\lambda_i}}{(\frac{1}{\lambda_i} + \frac{t}{n_i^{**2}}) \times (\frac{1}{\lambda_j} + \frac{t}{n_i^{**2}})}$$

Therefore:

$$\frac{1}{\frac{1}{\lambda_i} + \frac{t}{n_j^{**2}}} - \frac{1}{\frac{1}{\lambda_j} + \frac{t}{n_j^{**2}}} > \frac{1}{\frac{1}{\lambda_i} + \frac{t}{n_i^{**2}}} - \frac{1}{\frac{1}{\lambda_j} + \frac{t}{n_i^{**2}}}$$

$$\Rightarrow \frac{1}{\frac{1}{\lambda_i} + \frac{t}{n_j^{**2}}} + \frac{1}{\frac{1}{\lambda_j} + \frac{t}{n_i^{**2}}} > \frac{1}{\frac{1}{\lambda_i} + \frac{t}{n_i^{**2}}} + \frac{1}{\frac{1}{\lambda_j} + \frac{t}{n_j^{**2}}}$$

$$\Rightarrow \frac{n_j^{**}}{\frac{n_j^{**}}{\lambda_i} + \frac{t}{n_j^{**}}} + \frac{n_i^{**}}{\frac{n_i^{**}}{\lambda_j} + \frac{t}{n_i^{**}}} > \frac{n_i^{**}}{\frac{n_i^{**}}{\lambda_i} + \frac{t}{n_i^{**}}} + \frac{n_j^{**}}{\frac{n_j^{**}}{\lambda_j} + \frac{t}{n_j^{**}}}$$

But this last statement precisely says that the platform can strictly increase the total number of rides given by swapping the allocation of drivers between regions i and j from (n_i^{**}, n_j^{**}) to (n_j^{**}, n_i^{**}). Given that prices and wages are spatially uniform, this means the platform will also strictly increase its profit through this action, which is a contradiction. Therefore, it has to be that $n_1^{**} \geq n_2^{**}$.

Now we also show that $n_1^{**} = n_2^{**}$ is not the case either. To see this, note that the total platform profit from these two regions is given by:

$$(p - c) \times (\frac{n_i^{**}}{\frac{n_i^{**}}{\lambda_i} + \frac{t}{n_i^{**}}} + \frac{n_j^{**}}{\frac{n_j^{**}}{\lambda_j} + \frac{t}{n_j^{**}}})$$

Subject to a constant $n_i^{**} + n_j^{**}$, maximizing this profit through the first order condition will give:

$$\sqrt{n_i^{**}} W_i = \sqrt{n_j^{**}} W_j \tag{63}$$

where W_i is a suppressed notation for $W_i(n_i^{**})$, and same for W_j. If $n_1^{**} = n_2^{**}$, then we will have $W_i = W_j$ which means the platform optimal allocation coincides with the driver equilibrium. But in that case, we know for sure that $n_1^{**} \neq n_2^{**}$ due to the fact that supply is skewed toward region i This finishes the proof of the lemma. □

Now with Lemma A22 in hand, we prove the main statements in the proposition. We first show

$$\frac{A_j(n_j^*)}{A_i(n_i^*)} < \frac{A_j(n_j^{**})}{A_i(n_i^{**})}$$

.

To see this, note that by eq. (63) and Lemma A22, we have $W_i(n_i^{**}) < W_j(n_j^{**})$. This means if we start from the driver equilibrium allocation which sets $W_i(n_i^*) = W_j(n_j^*)$, we will have to reallocate some drivers from region i to region j in order to get to (n_1^{**}, n_j^{**}). Therefore, $n_j^{**} > n_j^*$ and $n_i^{**} < n_i^*$. Based on this, it is easy to see that $A_j(n_j^{**}) > A_j(n_j^*)$ and $A_j(n_i^{**}) < A_j(n_i^*)$. It follows that $\frac{A_j(n_j^*)}{A_i(n_i^*)} < \frac{A_j(n_j^{**})}{A_i(n_i^{**})}$.

Now we show

$$\frac{A_j(n_j^{**})}{A_i(n_i^{**})} < 1.$$

To see this, note that, by eq. (63), we have:

$$\sqrt{n_i^{**}} \times (\frac{n_i^{**}}{\lambda_i} + \frac{t}{n_i^{**}}) = \sqrt{n_j^{**}} \times (\frac{n_j^{**}}{\lambda_j} + \frac{t}{n_j^{**}})$$

$$\Rightarrow \frac{\sqrt{{n_i^{**}}^3}}{\lambda_i} + \frac{t}{\sqrt{n_i^{**}}} = \frac{\sqrt{{n_j^{**}}^3}}{\lambda_j} + \frac{t}{\sqrt{n_j^{**}}}$$

Which, by Lemma A22, implies

$$\frac{\sqrt{{n_i^{**}}^3}}{\lambda_i} > \frac{\sqrt{{n_j^{**}}^3}}{\lambda_j} \tag{64}$$

Now note that:

$$A_i(n_i^{**}) = \frac{n_i^{**}}{W_i \lambda_i} = \frac{1}{1 + \frac{t\lambda_i}{{n_i^{**}}^2}} = \frac{1}{1 + \frac{t\lambda_i}{\sqrt{{n_i^{**}}^3}\sqrt{n_i^{**}}}}$$

By eq. (64) and Lemma A22, we get:

$$A_i(n_i^{**}) < \frac{1}{1 + \frac{t\lambda_j}{\sqrt{{n_j^{**}}^3}\sqrt{n_j^{**}}}} = A_j(n_j^{**})$$

which completes the proof of the proposition. ■

E.6.3 Proofs under the assumption that region size is heterogeneous

We have shown so far that all of the results we proposed under fixed and uniform prices and wages hold when there is no size heterogeneity across regions. We now show that all of these four propositions still hold even if we relax that assumption and allow each region to have its own size

t_i.[42] The key to this extension is the following lemma.

Lemma A23. *Suppose that there is a "quantum region size" Δ. Consider market primitives (λ, N, t) such that each region's size t_i can be written as a multiplier of this quantum size. That is: $t_i = x_i\Delta$ where $x_i \in \mathbb{N}$. Also, consider market primitives $(\hat{\lambda}, N, \hat{t})$ such that*

$$\hat{t} = (\underbrace{\Delta, ..., \Delta}_{x_1 \text{ times}}, \underbrace{\Delta, ..., \Delta}_{x_2 \text{ times}}, ..., \underbrace{\Delta, ..., \Delta}_{x_I \text{ times}})$$

and

$$\hat{\lambda} = (\underbrace{\frac{\lambda_1}{x_1}, ..., \frac{\lambda_1}{x_1}}_{x_1 \text{ times}}, \underbrace{\frac{\lambda_2}{x_2}, ..., \frac{\lambda_2}{x_2}}_{x_2 \text{ times}}, ..., \underbrace{\frac{\lambda_I}{x_I}, ..., \frac{\lambda_I}{x_I}}_{x_I \text{ times}})$$

Then:

1. *allocation n is an equilibrium under (λ, N, t) if and only if allocation*

$$\hat{n} = (\underbrace{\frac{n_1}{x_1}, ..., \frac{n_1}{x_1}}_{x_1 \text{ times}}, \underbrace{\frac{n_2}{x_2}, ..., \frac{n_2}{x_2}}_{x_2 \text{ times}}, ..., \underbrace{\frac{n_I}{x_I}, ..., \frac{n_I}{x_I}}_{x_I \text{ times}})$$

   is an equilibrium under primitives $(\hat{\lambda}, N, \hat{t})$.

2. *allocations $\hat{n}$ that take the above form (i.e., with equal number of drivers among all regions j with the same $\hat{\lambda}_j$) are the only possible equilibria under $(\hat{\lambda}, N, \hat{t})$ and the only possible allocations that can maximize the platform profit under $(\hat{\lambda}, N, \hat{t})$.*

3. *platform profit from allocation n under market primitives (λ, N, t) is equal to its profit from allocation $\hat{n}$ under market primitives $(\hat{\lambda}, N, \hat{t})$.*

4. *Access to ride in any region i under allocation n and primitives (λ, N, t) is equal to access to rides in any region j with $\hat{\lambda}_j = \frac{\lambda_i}{x_i}$ under allocation $\hat{n}$ and primitives $(\hat{\lambda}, N, \hat{t})$.*

Proof of Lemma A23. The proof is straightforward. It is immediate that $\hat{n}$ sums up to N. It is also immediate that in each region j with demand arrival rate $\hat{\lambda}_j = \frac{\lambda_i}{x_i}$, the total wait time is given by:

$$\hat{W_j}(\hat{n}_j) = \frac{\hat{n}_j}{\hat{\lambda}_j} + \frac{\hat{t}_j}{\hat{n}_j} = \frac{\frac{n_i}{x_i}}{\frac{\lambda_i}{x_i}} + \frac{\Delta}{\frac{n_i}{x_i}} = \frac{n_i}{\lambda_i} + \frac{\Delta x_i}{n_i} = \frac{n_i}{\lambda_i} + \frac{t_i}{n_i} = W_i(n_i)$$

[42] We would like to emphasize that we believe this two step approach (first working with homogeneous t and then considering its heterogeneity) was not necessary. We believe it is likely that the proofs we provided so far would work with heterogeneous t with minimal changes in the main steps. Nevertheless, we decided to take the two step approach for two reasons. First, the proofs are already involved and directly dealing with heterogeneity in t would have led to much less "clean" proofs. The second reason has to do with the progress on the proofs. Our original proofs were based on homogeneous t but later we noticed that the homogeneity assumption was not necessary in any way.

It is also immediate to see that $\hat{W}_j(\cdot)$ is increasing at $\hat{n}_j$ if and only if $W_i(\cdot)$ is increasing at n_i^*. Finally, by Theorem 1, it can be seen that no equilibrium under $(\hat{\lambda}, N, \hat{t})$ can give two different volumes of drivers to two regions of the same size that have the same arrival rate of demand.

Next, note that by Proposition 14, if two regions have the same size and the same demand density, then the platform would also optimally want the same number of drivers in those regions.

Additionally, given what we established regarding the wait-time equivalence between the two regions, it is straightforward to verify that platform profits are equal between n and $\hat{n}$.

Finally, regarding access, note that

$$\hat{A}_j(\hat{n}_j) = \frac{\hat{n}_j}{\hat{\lambda}_j \hat{W}(\hat{n}_j)} \frac{\frac{n_i}{x_i}}{\frac{\lambda_i}{x_i} W(n_i)} = A_i(n_i)$$

This establishes the lemma. □

This lemma is powerful in that it shows any market primitives with heterogeneous t can, with a small enough Δ, be approximated with some market primitives with homogeneous t without loss of the equilibria. In other words, the lemma shows there is a 1-to-1 mapping between the equilibria under $(\hat{\lambda}, N, \hat{t})$ and (λ, N, t). This allows us to complete the proofs of Proposition 11 through Proposition 14 –which were stated for heterogeneous t but so far proven only for homogeneous t. Here, we only provide the basic steps of the proof and avoid going through all the steps, which are notationally cumbersome but conceptually straightforward (full steps would available upon request). The basic idea for proving Proposition 11 through Proposition 14 for general (λ, N, t) with heterogeneous t is as follows: If all t_i are dividable by a common Δ, then use that Δ and take the following steps:

1. Using Lemma A23, "transform" the allocations and primitives in the proposition from the (λ, N, t) space to the $(\hat{\lambda}, N, \hat{t})$ space.
2. Given $\hat{t}$ is homogeneous, apply the proposition.
3. Again, using the provisions of Lemma A23, show that the proposition holding under $(\hat{\lambda}, N, \hat{t})$ means it also holds under (λ, N, t).

If there is no common Δ by which all t_i are dividable, then each x_i will be the quotient in the division of t_i by Δ. This means the proposition holding for $(\hat{\lambda}, N, \hat{t})$ will not imply it also holds for (λ, N, t). It will, rather, hold for an approximation of (λ, N, t). But the approximation will get more and more accurate as we make Δ smaller. Thus, the proof will be obtained as $\Delta \to 0$.

F Additional Empirical Evidence

In this section, we provide additional empirical evidence for our theory. Before getting to the empirical analysis, we provide testable implications from our theoretical model in Appendix F.1. Next, Appendix F.2 analyzes the model prediction on relative outflows using data on Uber, Lyft, and

Via from New York City. Further, using individual driver behavior data from Austin, Appendix F.3 provides suggestive evidence for supply-side EOD – the role of pickup times on driver location decisions. Appendix F.4 shows evidence of EOD in a decentralized market – the taxicab market.

F.1 Testable Implications

Before getting to the empirical evidence, we provide testable implications of our model. The first proposition is a special case with two-regions of Proposition 6 in the main text. In the context of a two-region version of our model, we show that even though potential demand values $\bar{\lambda}$ are unobservable, one may still infer something about how access to rides compares across different regions by looking at the flows of realized rides. To this end, define the "relative outflow" of rides in each region i as $RO_i = \frac{r_i^{\rightarrow}}{r_i^{\leftarrow}}$ where $r_i^{\rightarrow}$ is the realized number of rides exiting i, meaning $r_i^{\rightarrow} = \Sigma_{j \neq i} r_{ij}$ and $r_i^{\leftarrow}$ is the realized number of rides entering i.[43]

Proposition 20. *Suppose $N = 2$ and (n, ρ) is a steady-state (not necessarily equilibrium) driver allocation under $(c, p, \bar{\lambda}, t)$. Also suppose potential demands for rides are "balanced:" $\bar{\lambda}_{12} = \bar{\lambda}_{21}$. Then we have:*

$$\frac{A_1(n)}{A_2(n)} = RO_1$$

In other words, the ratio between access to rides in the two regions can be measured without observing $\bar{\lambda}$ values, and just by looking at the "relative outflow" of rides. Under its balancedness assumption, and combined with our previous propositions, this result predicts that regions with higher access to rides will have more outflows of realized rides than inflows.

Proposition 21. *Suppose $N = 2$, and (n, ρ) is a steady state allocation under $(c, p, \bar{\lambda}, t)$. Also suppose (n', ρ') is a stable allocation under possibly different potential demands and platform strategies but in the same market $(c', p', \bar{\lambda}', t)$. Assume neither of the two potential demands $\bar{\lambda}$ or $\bar{\lambda}'$ has to be balanced but they are "similarly unbalanced": $\frac{\bar{\lambda}_{12}}{\bar{\lambda}_{21}} = \frac{\bar{\lambda}'_{12}}{\bar{\lambda}'_{21}}$. Then we have:*

$$\frac{A_1/A_2}{A'_1/A'_2} = \frac{RO_1}{RO'_1}$$

In this result, the two different primitives $(c, p, \bar{\lambda}, t)$ and $(c', p', \bar{\lambda}', t)$ may represent two different platforms in the same city or the same platform in the same city in two different times. This result says that even without a balancedness assumption, as long as potential demands are "similarly unbalanced," one can still use ride flows for comparing unobservable access levels. Combined with our previous propositions, this result predicts that as a platform gets larger, its relative outflow in busier areas decreases.

[43] Note that the definition of the relative outflow here is slightly different than the definition in eq. (7). Here, the definition is based on a single region i, while the eq. (7) defines relative outflow only if two regions are specified. Nevertheless, the interpretations are similar – both captures the ratio of outgoing rides to incoming rides. The single-region-specific version of definition is necessary to carry out the empirical tests using regressions that we will show later in this section.

F.2 Empirical Analysis of Rideshare in NYC

The purpose of this section is to provide additional empirical analysis that does not require calibrating a model while still provides useful insights on the relevance and validity of the theoretical model. More specifically, we would like to test (i) whether access to service decreases as density of potential demand decreases and (ii) whether the gap between access to service in higher and lower density areas is wider for smaller platforms. These are the two implications of the theoretical model which were robust to the sources of EOD.

Direct empirical tests of spatial inequity in access to rides are impossible without calibrating a model as we do in Section 5. This is because in any region i, access to rides A_i is defined as $\frac{r_i}{\bar{\lambda}_i}$, the fraction of potential demand $\bar{\lambda}_i$ that end up with rides. The number of rides r_i is observable in some datasets but potential demand $\bar{\lambda}_i$ is unobservable even by rideshare platforms themselves. The closest thing to $\bar{\lambda}_i$ on which data is available (usually only to rideshare platforms) is the number of people who opened the rideshare app regardless of whether they ended up with a ride. Note, however, that this does not provide a reliable measure of $\bar{\lambda}_i$ given that, in the long run, those who need rides in region i may respond to persistently high prices and/or wait times in the region by *not going on the app in the first place.* In other words, in the long run, only the portion of $\bar{\lambda}_i$ that expects to find a ride may go on the app. Therefore, r_i divided by the total frequency of app sessions in region i may be fairly homogeneous even if there is substantial heterogeneity in $\frac{r_i}{\bar{\lambda}_i}$ across regions i. This unobservability of $\bar{\lambda}_i$ makes a direct test of heterogeneity in $\frac{r_i}{\bar{\lambda}_i}$ challenging to carry out.

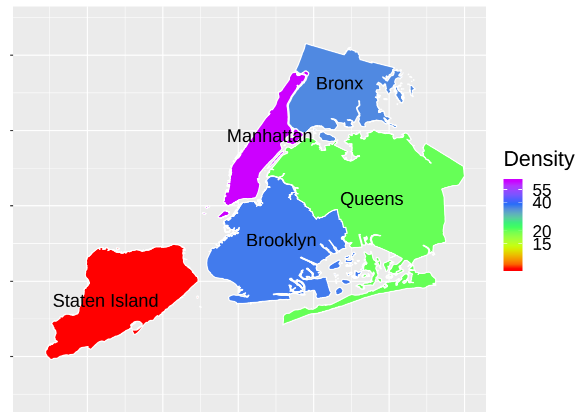
Our solution to this challenge is to devise an indirect tests based on Proposition 20 and Proposition 21. To this end, we leverage a feature of passenger transportation markets which (i) we believe is a powerful tool in empirically identifying spatial supply-demand mismatches, and which (ii) has been overlooked in the literature. In passenger transportation markets, it is reasonable to assume that *for every trip, there is a "trip back" by the same person.* Roughly, this suggests that if passengers consistently use platform k less often to exit region i than they do to enter it, then it means *the same population* that chooses platform k over other options to enter i is systematically more likely to end up having to choose other options over platform k to exit.[44] Under assumptions that we will be more specific about later in this section, and after controlling for some possible confounds, we conclude from such a data pattern that access to (inter-region) rides with platform k is lower in region i than outside of it.[45]

Based on the strategy above, we devise regression analyses to test both whether there is ge-

[44] Crucially, this gap between the inflow and the outflow is not impacted by whether passengers try and fail to find outgoing rides or they have learned to not even go on the app to search.

[45] Note that our theoretical model does not capture flows of rides across regions. Consequently, it does not capture relative outflows directly. Nevertheless, we do not see this as a weakness of the model. Our theory is focused on how to *explain* the impacts of economies of density and market thickness on spatial distribution of supply, whereas our empirical analysis will be focused on how to *identify* those impacts. Relative outflows have a key role in the identification; therefore they are at the heart of our empirical analysis. They do not, however, have a crucial role in the underlying mechanism. Therefore, our theoretical model abstracts away from them. Put in simpler terms, relative outflow patterns are informative consequences of economies of density rather than key antecedents of it.

Figure 18: Population densities of the five boroughs of NYC as of April 2019 in thousands/sq mile. The color scale is logged.



ographical heterogeneity in access to rideshare across regions and whether this heterogeneity is exacerbated when the platform is smaller (i.e., the market is thinner). Before getting to the regressions, however, we discuss our data and provide several visualizations that convey the main intuition behind our subsequent empirical analysis.

F.2.1 Data and Summary Statistics

For the additional tests, we supplement the data used in the main text with other multiple sources of data. For the analysis here in Appendix F.2, we use ride-level data on all rides with three platforms Uber, Lyft, and Via within the New York City proper area from July 2017 to December 2019. Uber, is the largest platform whose total number of rides given in NYC per month grew from 8.7M in the beginning of our dataset to 15.6M. Lyft is the second largest platform with the highest growth rate. Its size, over the same time period, grew from 2.2M rides/month to 5.2M rides/month in NYC. Via is the smallest platform with a size that oscillated around 0.9M rides/month over the course of our data.

For each ride from these three platforms, we observe the exact date, time, and location both for the pickup and the dropoff. We augment. The "location" in our data is one of 263 "taxi zones" that partition NYC. Each such zone belongs to one of the five "boroughs" of NYC: Manhattan, Brooklyn, The Bronx, Queens, and Staten Island. These boroughs and their population densities are shown in Fig. 18 . In addition, we leverage a zoning districts data (provided by NYC Planning Labs) which contains information on the "zoning type" for each taxi zone. The possible types are: residential, commercial, park, and manufacturing. We use these data to provide empirical tests of our theory.

Before getting to the additional empirical analysis, we visualize some data patterns that speak to the tests we will be carrying out in the subsequent sections.

Fig. 19 represent the "relative outflows" across different taxi zones in NYC for Uber, Lyft, and Via. Panel (a) corresponds to July 2017 and Panel (b) depicts one year later, July 2018. The relative outflow for any platform k in region i during some period d is defined by the number of rides with k that exit i during period d divided by the number of rides with k entering i during the same period. Two patterns are noteworthy in these figures. First, the relative outflow tends to be higher in busier areas of the city, and it decreases as we move towards the outer, less busy regions. Second, the gap between the relative outflows of busier and less busy areas is wider for smaller platforms than it is for larger ones (Fig. 19 visualizes this by showing that the heat maps of relative outflows for smaller platforms are more "colorful"). For instance, the relative outflow for Lyft in Staten Island is was close to 60% during July 2017, suggesting that out of every 100 passengers who chose Lyft over other options to enter that regions, close to 40 had to use other options to leave. The same gap between Staten Island and Manhattan, however, is not observed for Uber during the same period. Neither is it observed for Lyft in July 2018 when Lyft was a larger platform. A much wider such gap is observed for Via which was much smaller in size than both Uber and Lyft. See Appendix F.2.4 for Fig. 22 which is similar to Fig. 19 but shows the relative outflows at the borough level instead of the taxi zone level. There we also discuss some of the less intuitive patterns (such as higher relative outflows for some zones in Staten Island and Queens).

In subsequent sections we will formally argue that that the observed relative-outflow gap among regions has to do with economies of density. We do so by controlling for a variety of possible confounds. Before getting to that, more formal, analysis, however, we show additional data patterns in this section that are informative about the nature of the observed gap between the relative outflows in different areas.

Fig. 20 depicts the hourly patterns of relative outflows during July 2017 for Lyft from three different zones. Panel (b) is Alphabet City, a residential area in lower Manhattan. Similar to other residential areas (such as Riverdale in the Bronx, depicted in panel (c)), the relative outflow in Alphabet City peaks in the morning and then gradually decreases. This is the opposite of the pattern observed for a commercial area like Greenwhich Village North, also in Manhattan. In spite of having a similar hourly pattern to Riverdale, however, Alphabet City has a high relative outflow, much more similar to Greenwhich Village North than to Riverdale. Fig. 20, hence, is suggestive that the overall high relative outflow in Manhattan is not merely an outcome of the concentration of commercial areas in Manhattan. If anything, we will show later, that the relative outflow in Manhattan is high *in spite of* (rather than because of) its commercial areas.

Fig. 21 offers a different illustration by, this time, fixing the location and varying the platform. This figure focuses on Staten Island and exhibits relative outflows for Uber and Lyft during July of 2017 (the first month of our data) and those for Uber, Lyft, and Via one year later. As can be seen from this figure, smaller platforms have relative outflows (in Staten Island) that are *consistently* lower than those of larger platforms. This happens even though the non-rideshare outside options are the same for the passenger of all three platforms. This figure suggests that the persistent differences across relative outflows of rideshare platforms are because of systematic differences in "access" rather things such as the schedules of passengers, or outside options.

Next, we turn to carrying out the empirical tests.

Figure 19: Relative outflows for Lyft, Uber, and Via. Panel (a) is July 2017 and Panel (b) is July 2018. The figure depicts two important patterns: (i) relative outflow is highest in Manhattan and decreases as we move towards the outer areas; (ii) the gap between relative outflows of Manhattan and outer boroughs is wider for smaller platforms. Both patterns are in line with our theory model. See Appendix F.2.4 for the same figure at the borough level. **Note:** Via was not operating in Bronx and Staten Island during July 2017.

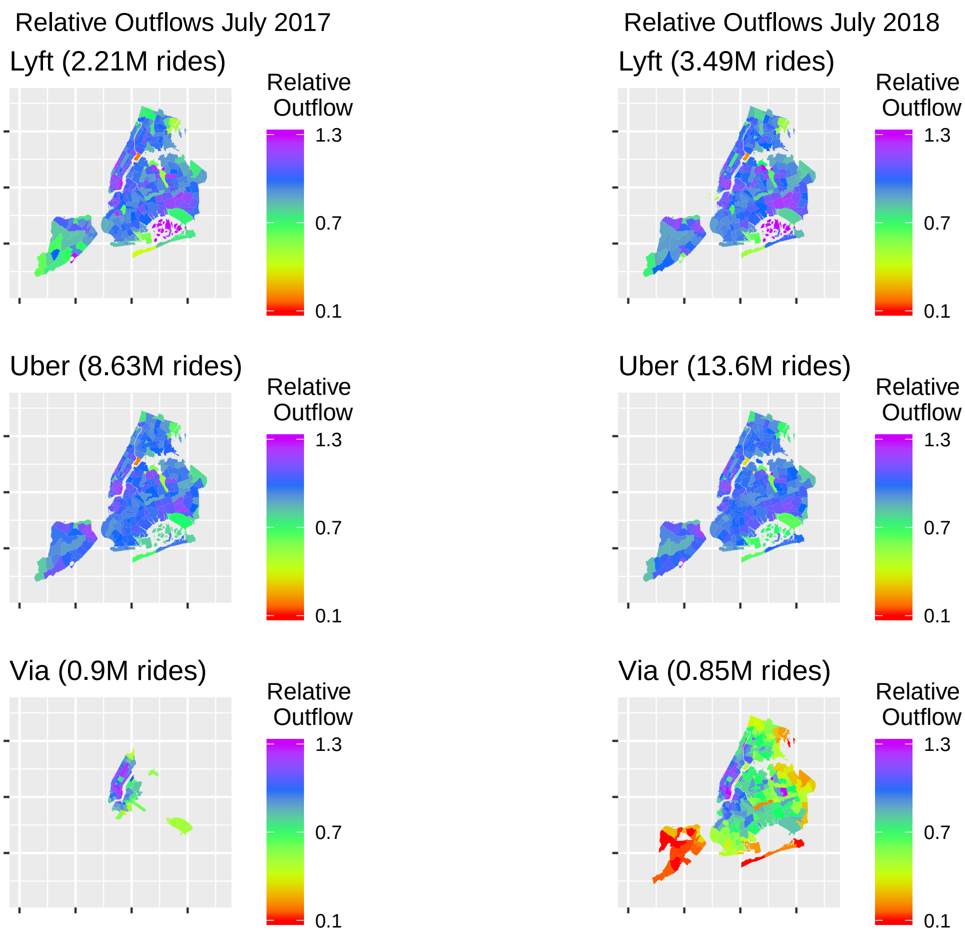


Figure 20: Hourly flows in absolute and relative terms during July 2017 for Lyft from three zones zones. The hourly relative-outflows pattern in Alphabet City Manhattan (residential) is similar to that of Riverdale in Bronx (residential) and different from Greenwhich Village in Manhattan (commercial). Nevertheless, the overall relative outflow in Alphabet City is high, unlike Riverdale but similar to (even slightly higher than) Greenwhich Village.

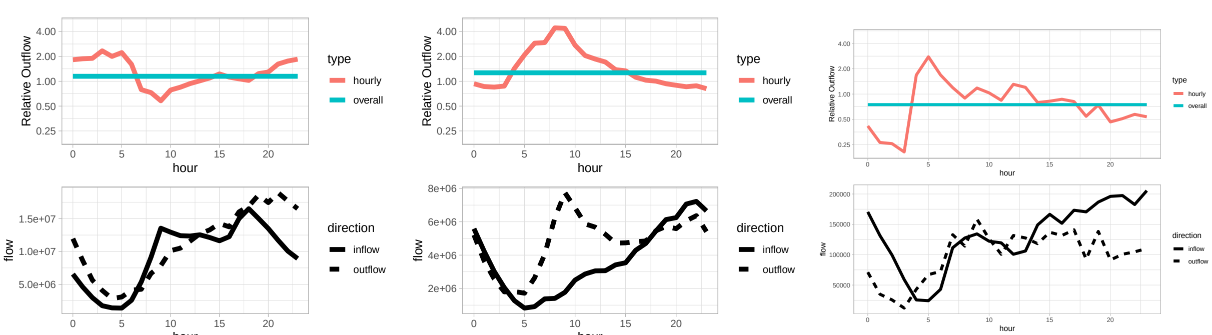
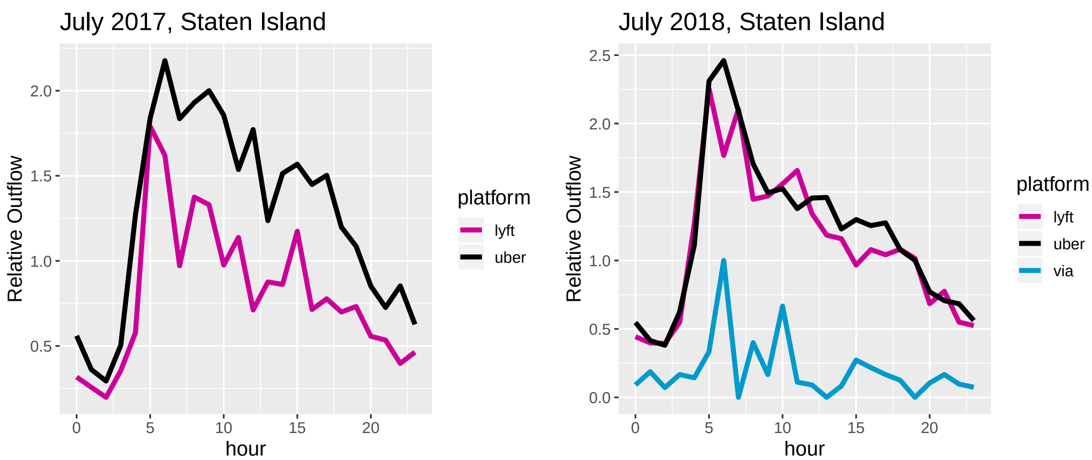


Figure 21: Cross-platform hourly comparisons of relative outflows in Staten Island. During July 2017 –panel (a)– Lyft's relative outflow is consistently smaller than Uber's. During July 2018 (when Lyft had grown substantially in size) the two platforms' relative outflows are very close to each other throughout the day –panel (b). Via (which started operating in Staten Island in 2018,) on the other hand, has a consistently smaller relative outflow compared to Uber and Lyft. This figure suggests the cross platform differences in relative outflows are unlikely to be an artifact of a spike that takes place during a certain, small, time window.



F.2.2 Testing for Economies of Density

If all of our quantities of interest were observed, we would ideally directly test our theoretical result: access to rides $A_i = \frac{r_i}{\lambda_i}$ is higher in regions that have higher densities of potential demand $D_i = \frac{\bar{\lambda}_i}{t_i}$. The challenge, as mentioned before, is that potential demand $\bar{\lambda}_i$ is essentially unobservable.

Our solution, inspired by Proposition 20, Proposition 21 and the discussion on the data patterns, is to turn to relative outflows and devise a test that approximates, as closely as possible, a test of a positive association between A_i and D_i. Recall that we defined the relative outflow for a single region i as:

$$RO_i \equiv \frac{r_i^{\rightarrow}}{r_i^{\leftarrow}} \tag{65}$$

where $r_i^{\rightarrow}$ is the rate of outgoing rides from region i and $r_i^{\leftarrow}$ the rate of incoming ones. Similarly, we define the density of dropoffs in region i by $D_i^{\leftarrow} \equiv \frac{r_i^{\leftarrow}}{t_i}$ where t_i is the size of region i in sqaure miles.

Proposition 20 suggests that we should expect higher RO_i values for denser regions. The regression specification that we use for testing our hypothesis is as follows:

$$\log(RO_{ikd}) = \alpha \log(D_{ikd}^{\leftarrow}) + \beta X_{ikd} + \epsilon_{ikd} \tag{66}$$

Here, RO and $D^{\leftarrow}$ have been indexed not only by region i but also by platform k and time duration d. In this setting, i denotes a "taxi zone" as described previously. Also d in this regression is a year-month combination. The coefficient of interest is α, which according to the predictions of our model, is expected to be positive and significant. Finally, X_{ikd} is a set of some controls that we use in order to deal with possible confounds and aid the interpretation of the test results as a sign of economies of density.

Before presenting the results of this regression, we make two points regarding their interpretation. First, Proposition 20 may leave the impression that in order in order to interpret RO_i as a measure of access to rides in i relative to outside of i, it is necessary that potential demands for rides be balanced. That is:

$$\bar{\lambda}^{\rightarrow}_{ikd} \equiv \bar{\lambda}^{\leftarrow}_{ikd} \tag{67}$$

where $\bar{\lambda}^{\rightarrow}_{ikd}$ is the potential demand for rides with platform k that exit i during period d (a month in this case) and $\bar{\lambda}^{\leftarrow}_{ikd}$ is the potential demand for rides entering it.

Of course eq. (67) would be too strong of a formalization for our assumption that "for every ride there is a ride back by the same person shortly before or after." But Proposition 22 shows that, with the right controls, eq. (67) is *not necessary* for our intended interpretation of the results from regression 66.

Proposition 22. *Suppose that vector Q is a partition of all ikd observations in the data based on some characteristics.*[46] *Also suppose that $\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}$ depends only on characteristics Q. That is, for some function g, we have:*

$$\forall ikd : \frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}} = g(Q_{ikd})$$

Under these conditions, the following regression will lead to the exact same estimated α as regression 66 if controls in X_{ikd} include fixed effects at the level of Q or finer.

$$\log(\frac{A^{\rightarrow}_{ikd}}{A^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \beta X_{ikd} + \epsilon_{ikd} \tag{68}$$

The proof can be found in Appendix F.2.5. Proposition 22 shows that X_{ikd} helps not only with controlling for omitted factors that may be impacting access to rides, but also with correcting for possible error in measurement of $\frac{A^{\rightarrow}_{ikd}}{A^{\leftarrow}_{ikd}}$ using RO_{ikd}. In these ways, the controls help rule out a number of possible alternative hypotheses to the effect of density on access. For instance, it is in principle possible (though not likely) that Manhattan provides worse public transit options than the outer boroughs, prompting passengers to turn to rideshare. This can lead to $\bar{\lambda}^{\rightarrow}_{ikd} > \bar{\lambda}^{\leftarrow}_{ikd}$ when i is in Manhattan, implying that a higher relative outflow in Manhattan is just an artefact of higher demand for (rather than better access to) rideshare there. This issue, as Proposition 22 shows, can be taken care of with borough fixed effects if we assume the gap between $\bar{\lambda}^{\rightarrow}_{ikd}$ and $\bar{\lambda}^{\leftarrow}_{ikd}$ can be explained by boroughs.

The second point we need to make about the interpretation before proceeding to results is that our ideal regressor, as mentioned before, would be $D_{ikd} = \frac{\bar{\lambda}_{ikd}}{t_i}$ but it is essentially unobservable (even by platforms.) In choosing a proxy for it, we decided to use the density of incoming rides $D^{\leftarrow}_{ikd}$. We made this choice for two reasons. First, as mentioned before, those who enter region i

[46] To illustrate, if Q partitions the data based on borough and platform, it means $\forall\, ikd \,\&\, i'k'd' : Q_{ikd} = Q_{i'k'd'}$ if and only if $k = k'$ and regions i and i' are within the same borough.

must have a need to exit shortly before or after. As a result, incoming rides seem like a reasonable proxy for potential demand. Second, working with incoming rides means we are testing the following hypothesis: regions with more incoming rides per square mile have more outgoing rides per incoming rides. That is, $r^{\leftarrow}_{ikd}$ is in the numerator on the right-hand-side and in the denominator on the left-hand-side. This mimics what we would have if we were able to directly observe A_{ikd} and D_{ikd} which would involve $\bar{\lambda}_{ikd}$ in the numerator on the right-hand-side and in the denominator on the left-hand-side. In both of these cases, a positive association between the two objects would be counter-intuitive but possible to explain using economies of density. That said, we did try using other proxies than $D^{\leftarrow}_{ikd}$ for D_{ikd} and the results were robust.[47] We now proceed to presenting the results.

Results: In different specifications, we allow X_{ikd} to capture a variety of fixed effects such as borough fixed effects, platform fixed effects, zone type fixed effects, year-month fixed effects, and interactions among the above. Results of this regression analysis have been reported in Table 7.

Table 7: Effect of dropoff density in a region on its relative outflow (i.e., pickups per dropoff) is always positive and significant. This is robust to a rich set of fixed effects specifications.

	Dependent variable: log relative outflow				
	(1)	(2)	(3)	(4)	(5)
log dropoff density	0.072***	0.081***	0.069***	0.075***	0.074***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fixed Effects†	Constant	B	P	Z	B×P×Z
Observations	21,357	21,357	21,357	21,357	21,357
R^2	0.299	0.378	0.385	0.369	0.495
Adjusted R^2	0.299	0.377	0.385	0.369	0.494

*p<0.1; **p<0.05; ***p<0.01

†: P:Platform, B:Borough, Z:Zone-type

Table 7 shows that the coefficient of interest, α, is always positive and significant under multiple fixed effects specifications. The simplest column to interpret in this table is column (1) which pertains to the specification with no fixed effects. The positive and significant estimate for α in this column simply means $D^{\leftarrow}_{ikd}$ and RO_{ikd} are positively correlated across observations.

As mentioned before, the controls X_{ikd} have two roles. First, they help deal with the issue that RO_{ikd} may not directly measure $\frac{A^{\rightarrow}_{ikd}}{A^{\leftarrow}_{ikd}}$ due to unbalancedness in potential demand. In addition, these controls also help deal with alternative hypotheses to the role of economies of density, even if $\bar{\lambda}^{\rightarrow}_{ikd} = \bar{\lambda}^{\leftarrow}_{ikd}$. The variations that our robustness columns (i.e., (2) through (5)) in Table 7 leverage should help alleviate concerns about such alternative hypotheses.

For instance, a model with borough fixed effects would estimate α only based on within-borough variation in relative outflows as a function of dropoff density, which would not pick up differences

[47] We tried using $D^{\rightarrow}_{ikd}$ and the results were robust. We also tried using $\Sigma_k D^{\leftarrow}_{ikd}$ which means using the total number of incoming rides across all platforms as a proxy for potential demand for each of them. The results were robust again.

between Manhattan and the outer boroughs. This rules out the alternative hypothesis that the relative outflows results arise from heterogeneity across boroughs in their provision of alternative transportation options. Likewise, a zone-type fixed effects model would estimate α from the variation in dropoff density among regions that are of the same type (e.g., residential). To illustrate, recall Fig. 20. A regression with zone-type fixed effects compares overall relative outflows between the two residential zones Alphabet City (panel b) and Riverdale (panel c) to each other in order to infer α, the effect of dropoff density; but it does *not* compare either of the two to Greenwhich Village (panel a) which is mostly commercial. The robustness of our results to this specification rules out the possibility that the observed relative outflows are only an artefact of differential access to outside options across different zone types.[48]

More sophisticated controls help alleviate concerns about more sophisticated alternative hypotheses. For instance, it is conceivable that the overall balance of flows of rides on a *daily or monthly level* gets impacted not by differential densities but by the *hourly level* complexities in the movement of passengers within the city. As an example, in the morning there is a net flow of passengers from the outer boroughs toward Manhattan when individuals commute to work. In the afternoon, the flow is reversed when they return home. Thus, if supply of rideshare is more limited in the morning compared to the evening (e.g., perhaps because some rideshare drivers are at their "first jobs" in the morning), then this can explain why access to rideshare –and, hence, the relative outflow of rides– is higher in Manhattan independently of density. Which controls help with rule out this alternative depends on our assumptions. If we assume this hour-level complexity impacts the relative flow of Manhattan compared to other boroughs homogeneously across platforms, then it is ruled out simply by the borough fixed effects in column (2) of Table 7. If we believe, however, that the effect of this mechanism differs across platforms, we will need $B \times P$ or, like column (5) of the table, $B \times P \times Z$.

Other robustness checks. The fixed effects specifications shown in Table 7 are only a subset of those we examined. In particular, we examined interacting all of the fixed effects columns in that table with year-month (YM) fixed effects. The most general case (i.e., B×P×Z×YM) would have about 1600 separate fixed effects. Under all of our additional regressions, the result was robust. the results also seem robust to functional form assumptions (we tried using the relative outflow itself instead of the log and all results were robust). Finally, we ran the model with only subsets of the full data by taking one platform off each time. Again, α always came back positive and significant.

Before closing this section, we test another prediction by the model. Our model predicts that the positive effect of density D_i on access A_i diminishes as D_i grows large enough. Again directly testing this is not feasible given A_i and D_i are both unobservable due to unobservability of $\bar{\lambda}_i$. Therefore, again, we test this in the context of the relationship between RO_i and $D_i^{\leftarrow}$. To this end,

[48] We would like to add that in fact Manhattan has a higher relative outflow *in spite of* having a concentration of commercial areas rather than because of it. To see this, we note that in column (4) of Table 7, the fixed effects coefficient on commercial areas (not reported in the table) comes back smaller than those of the other zone types (i.e., residential, manufacturing, and park). This means all else (including density of dropoffs) equal, a commercial area is expected to have a lower relative outflow for rideshare compared to other zone types. We believe this should not be surprising given the relative abundance of non-rideshare options in commercial areas.

we change the regression in eq. (66) by adding $\log(D_i^{\leftarrow})^2$ as another regressor:

$$\log(RO_{ikd}) = \alpha_1 \log(D_{ikd}^{\leftarrow}) + \alpha_2 \log(D_{ikd}^{\leftarrow})^2 + \beta X_{ikd} + \epsilon_{ikd} \tag{69}$$

If our model's prediction of diminishing sensitivity to density is empirically relevant, we should expect a positive α_1 and a negative α_2 in the estimation results.[49] As Table 8 shows, the empirical results are robustly in line with the model prediction.

Table 8: A square density term added to regressions in Table 7. The effect of the square density term is robustly negative and significant, in line with model predictions.

	Dependent variable: log relative outflow				
	(1)	(2)	(3)	(4)	(5)
log dropoff density	0.327***	0.311***	0.297***	0.331***	0.104***
	(0.007)	(0.007)	(0.007)	(0.007)	(0.009)
(log dropoff density)^2	−0.008***	−0.007***	−0.007***	−0.008***	−0.001***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0003)
Fixed Effects†	Constant	B	P	Z	B×P×Z
Observations	21,357	21,357	21,357	21,357	21,357
R^2	0.342	0.408	0.415	0.406	0.495
Adjusted R^2	0.342	0.407	0.415	0.406	0.494

Note: *p<0.1; **p<0.05; ***p<0.01

†: P:Platform, B:Borough, Z:Zone-type

Next, we turn to testing for the role of market thickness in determining the spatial distribution of supply.

F.2.3 Testing for the Role of Market Thickness (i.e., Platform Size) in Economies of Density

A second crucial prediction of our model was that the gap between access to rides in busy areas and access to rides in less busy areas is wider for smaller platforms. To this end, we run a regression on rideshare platforms' rides in NYC at the borough level. Specifically, we analyze the following specification:

$$RO_{ikd} = \alpha_0 + \alpha_1 \log(\rho_i) + \alpha_2 \log(S_{kd}) + \alpha_3 \log(S_{kd}) \log(\rho_i) + \nu_{ikd} \tag{70}$$

where RO_{ikd} is the relative outflow for platform k at borough i on date d (note that this regression, compared to previous ones, is coarser on i but finer on d). Also ρ_i is the population density of borough i.[50] Finally, S_{kd} is the size of platform k on date d, which is measured by the total number

[49] A more precise test of diminishing sensitivity of access to density would use $D_{ikd}^{\leftarrow}$ and $D_{ikd}^{\leftarrow 2}$ in the regression rather than $\log(D_{ikd}^{\leftarrow})$ and $\log(D_{ikd}^{\leftarrow})^2$. Nevertheless, we used the latter to keep eq. (69) compatible with eq. (66). We have examined the regression with $D_{ikd}^{\leftarrow}$ and $D_{ikd}^{\leftarrow 2}$ and the results were robust.

[50] In this section, we use population density instead of dropoffs density. Population densities allow us to define busier and less busy regions in a way that is constant across platforms and time. This allows us to focus on cross-platform

of rides given by that platform in NYC during the month in which date d occurs. Tables (9) and (10) report the results from this regression. The first table reports results when we either do not include any fixed effects in the regression or we do have fixed effects but they are not interacted (that is, platform fixed effects, year-month fixed effects, or borough fixed effects). Table 10 incorporates a richer set of fixed effects. It starts, in its first three columns, with fixed effects on (i) platform interacted by year-month, (ii) borough interacted by platform, and (iii) borough interacted by year-month. It then incorporates these three pairs into one single regression. Finally, the last column has interaction fixed effects among boroughs, platforms, and years (not year-month in this column).

The coefficient of interest is α_3, the interaction coefficient between platform size and borough population density. Based on the predictions of our theoretical model, we would expect a negative estimated value for α_3, indicating that as a platform gets smaller, access to its supply in lower density areas falls further behind that in denser areas. As can be seen from tables (9) and (10), this is exactly the result that we get from the empirical analysis and it is robust the controls.

Table 9: Relative Outflow regression with single fixed effects. The coefficient of interest is the interaction coefficient which is robustly negative and significant.

	Dependent variable: Relative Outflow			
	(1)	(2)	(3)	(4)
log(population density)	2.154^{***}	2.222^{***}	2.141^{***}	-
	(0.041)	(0.040)	(0.041)	
log(size)	0.492^{***}	0.483^{***}	0.490^{***}	0.448^{***}
	(0.009)	(0.014)	(0.009)	(0.008)
log(population density)× log(size)	-0.126^{***}	-0.130^{***}	-0.125^{***}	-0.113^{***}
	(0.003)	(0.003)	(0.003)	(0.002)
Fixed Effects†	Constant	P	YM	B
Observations	7,709	7,709	7,709	7,709
R^2	0.595	0.624	0.598	0.725
Note:		*p<0.1; **p<0.05; ***p<0.01		

†: P:Platform, B:Borough, YM:Year-Month

We end this subsection by emphasizing (without introducing a formal proposition) that, similar to regression eq. (66), regression eq. (70) does not require the assumption in eq. (67) for RO_{ikd} to be interpreted as a representation of access to rides in i relative to outside of i. A similar proposition to Proposition 22 can be proven showing that, with the right controls, α_3 remains intact if one replaces RO_{ikd} in the left hand side of eq. (70) with $\frac{A^{\rightarrow}_{ikd}}{A^{\leftarrow}_{ikd}}$. Moreover, note that our coefficient of interest in this section is different from that in the previous section. Here we are interested in α_3, the *interaction* coefficient between platform size and borough population density. Therefore, even without controls, we do not expect this coefficient to be impacted by the possibility that $\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}$ can be region-specific as long as it is not region-platform specific. In other words, even if things such

(and within-platform over time) comparisons in size as they impact the spatial distribution of relative outflows.

Table 10: Relative Outflow regression with interaction Fixed Effects. The coefficient of interest is the interaction coefficient which is robustly negative and significant.

	Dependent variable: Relative Outflow			
	(1)	(2)	(3)	(4)
log(population density)	2.182^{***}	-	-	-
	(0.040)			
log(size)	-	0.596^{***}	0.444^{***}	0.402^{***}
		(0.036)	(0.008)	(0.061)
log(population density)× log(size)	-0.128^{***}	-0.167^{***}	-0.112^{***}	-0.110^{***}
	(0.003)	(0.011)	(0.002)	(0.018)
Fixed Effects†	P×YM	B×P	B×YM	B×P×Y
Observations	7,709	7,709	7,709	7,709
R^2	0.629	0.829	0.738	0.835
Note:		*p<0.1; **p<0.05; ***p<0.01		

†: P:Platform, B:Borough, YM:Year-Month, Y:Year

as availability of public transportation options can create an imbalance on the flow of rides to and from a region i, it should not impact α_3 (even if we do not use additional controls) as long as public transportation options are assumed to create the same amount of flow imbalance in i for all platforms.

To see how these controls help rule out alternative hypotheses, we review two such alternatives. First, in principle it might be that Lyft has lower relative outflows in the outer regions because Lyft drivers are more likely to live in busier areas and, hence, prefer to driver there. Column 2 of Table 10 rules this out by exploiting only within-platform variation in each borough. Such variation can be seen in Fig. 19 as well: Lyft's relative outflows become more balanced geographically as it grows in size. A second, more important, possible issue is the fact that Via is a much different platform in structure than Uber and Lyft; therefore, its more concentrated relative outflows may be a consequence of its different structure rather than its size. Again, our controls that exploit only within-platform (such as columns 1 and 2) variation in each borough show that Via's possible differences from Uber and Lyft is not the main driver of our results.

Finally, in addition to the arguments above, Fig. 21 should be useful in illustrating that our results are not an artifact of a differences in the hourly transportation patterns of the users of these platforms. As can be seen in the figure, comparisons among different platforms' relative outflows in Staten Island is remarkably consistent over time of day, both during July 2017 and a year later. This rules out the possibility that, for instance, Lyft had a lower relative outflow in Staten Island in July 2017 because Lyft users tended to need to exit that region at certain times of the day when rideshare was less available relative to other options.

We finish this section with two further notes.

Note on possible role of competitive forces. We finish this section by briefly arguing against the hypothesis that the observed cross-platform geographical differences in access to service is an artifact of platforms' strategic spatial positioning to avoid competition from one another (i.e., tacit collusion). There are multiple arguments against the role of competitive forces in shaping the observed spatial differences. First, if competitive forces pushed the platforms to each focus on some areas and be less accessible in other areas, then we would see a different "area of focus" for each platform. However, in the data, *all platforms* are more accessible in busier regions compared to the outer areas; it is just the extent of the difference between more and less busy regions that differs by platform. Second, if the platforms were dividing the market among themselves, it would be more natural to expect larger and older platforms such as Uber to claim more attractive markets such as Manhattan. However, Via's supply is showing the highest degree of concentration in the Manhattan area.

Note on comparison to taxicabs. Appendix F.4 provides a relative-outflows analysis of the NYC Yellow Taxicab rides and shows that economies of density is a substantially more pronounced problem in that market compared to rideshare. There are many factors, including the decentralized nature of decision making, lack of real-time information about demand and supply conditions on both sides etc.

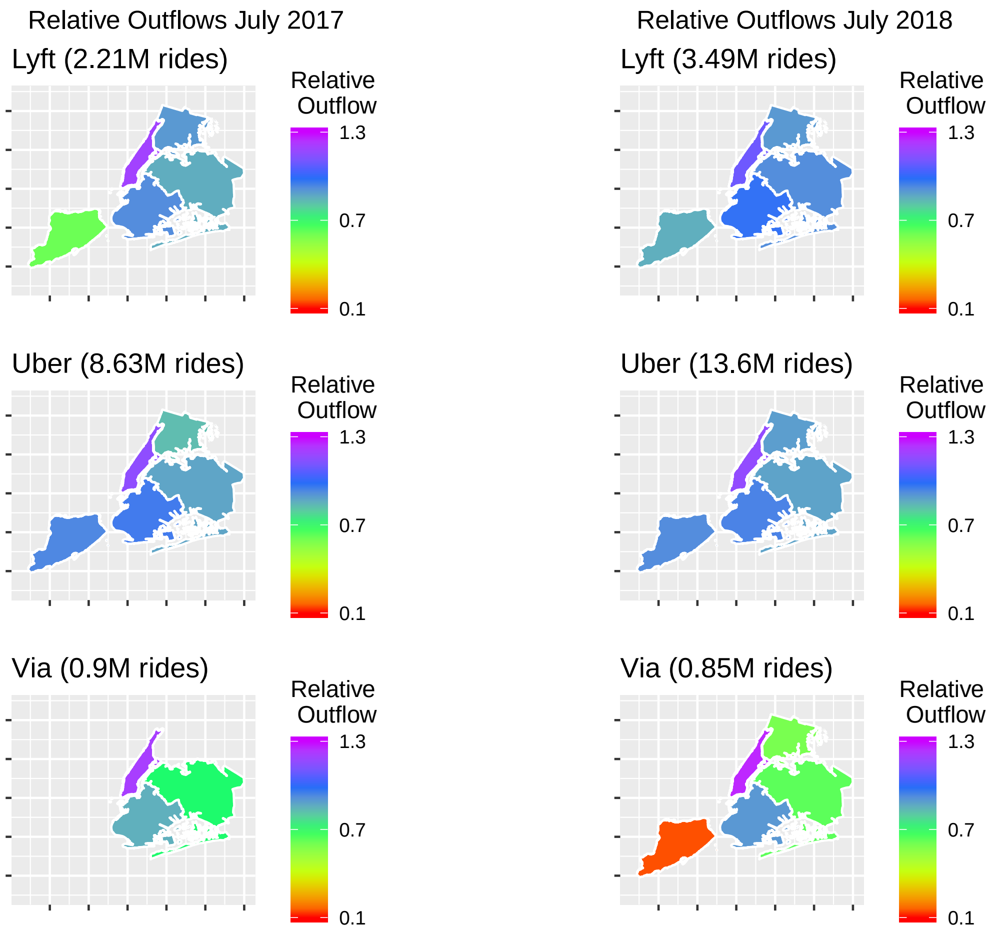
F.2.4 Relative Outflows at the Borough Level

In this section, we present relative outflows at the borough level for Uber, Lyft, and Via. The observed patterns are similar to those at the zones level: (i) busier boroughs have higher relative outflows; and (ii) the relative-outflow gap is wider for smaller platforms. This figure should also help with the understanding of some of the less intuitive patterns in the zones-level relative outflows in Fig. 19. In Fig. 19, one can see that some relative outflows in parts of Staten Island and Queens are higher than some in Manhattan. Here, we would like to point out why these patterns emerge and explain why the potential bias they cause is in fact against our results (meaning the magnitudes of our results would be even larger if we "corrected" for these patterns).

We believe the large relative outflows in Staten Island and Queens arise from two sources. First, in Queens, the areas with larger relative outflows tend to be close to airports. We believe this is likely a consequence of the fact that rideshare companies have historically been more restricted when it comes to picking up passengers at airports. Therefore, drivers who drop off passengers at airports may need to look for new passengers nearby which creates economies of density in those regions. The second, and more important, reason for larger relative outflows in some "unexpected" regions has more to do with the nature of intra-city transportation. As an extreme case, suppose all rides took place within boroughs, meaning each ride's origin was located in the same borough as its destination. In that case, one would expect the highest relative outflow in each borough to be above 1 and the lowest relative outflow in each borough to be below 1 (this is because rides are balanced overall). Therefore, each borough's highest relative outflow would be higher than any other borough's lowest relative outflow, *no matter how the densities of the two boroughs compare.* Of course the assumption that all rides are within-borough is an extreme assumption but if a reasonably large fraction of rides happen within boroughs, we should expect similar relative outflows patterns. Consistent with our hypothesis about this explanation, these patterns are not observed when we look at the borough-level relative outflows graphs in Fig. 22.

Note that these patterns (especially the second one) can be reasonably controlled for using borough fixed effects. We would also like to note that any potential bias these patterns may cause in our results would be against our hypothesis that α in eq. (66) and eq. (68) is positive. This is in line with the observation that once we control for boroughs, the estimated α increases.

Figure 22: Relative outflows for Lyft, Uber, and Via at the borough level. Similar patterns to those at the zone level can be observed.



F.2.5 Proof of Proposition 22

The basic idea behind this proof is that the difference between RO_{ikd} and $\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\lambda^{\rightarrow}_{ikd}}$ "gets absorbed" by the Q-level fixed effects, keeping intact all of the other coefficients as well as the error terms. Here, we provide a proof for the case where fixed effects embedded in controls X_{ikd} are exactly at the Q level. We will skip the proof for the case where the fixed effects are at a level finer than Q but $\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\lambda^{\rightarrow}_{ikd}}$ can be determined fully based on Q. That proof is conceptually the same (because finer fixed effects will absorb the differences more easily) but the notations will be much less clean.

Now, assume the matrix of control variables X is given by:

$$X = [F^Q \quad \bar{X}]$$

where F^Q is the set of columns giving the Q-level fixed effects and $\bar{X}$ consists of the rest of the controls. That is, each column in F^Q corresponds to one possible value q that Q_{ikd} can assume; and each row ikd of F^Q in the column corresponding to q is 1 if $Q_{ikd} = q$ and zero otherwise.

Consequently, also assume $\beta = [\beta_F \quad \beta_{\bar{X}}]$. We now introduce the following lemma.

Lemma A24. *Coefficients α and $\beta = [\beta_F \quad \beta_{\bar{X}}]$ and error terms ϵ_{ikd} satisfy eq. (66) if and only if coefficients α, $\tilde{\beta}$, and error terms ϵ_{ikd} satisfy eq. (68), where $\tilde{\beta}$ is defined as $[\beta_G \quad \beta_{\bar{X}}]$, and β_G is a vector constructed from β_F by subtracting $\log(g(q))$ from any element of β_F from the column in F^Q that corresponds to value q.*

Proof of Lemma A24.

$$\log(RO_{ikd}) = \alpha \log(D^{\leftarrow}_{ikd}) + \beta X_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(RO_{ikd}) = \alpha \log(D^{\leftarrow}_{ikd}) + \beta_F \times F^Q_{ikd} + \beta_{\bar{X}} \times \bar{X}_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(RO_{ikd}) - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + (\beta_F \times F^Q_{ikd} - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}})) + \beta_{\bar{X}} \times \bar{X}_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(RO_{ikd}) - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + (\beta_F \times F^Q_{ikd} - \log(g(Q_{ikd}))) + \beta_{\bar{X}} \times \bar{X}_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(RO_{ikd}) - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \beta_G \times F^Q_{ikd} + \beta_{\bar{X}} \times \bar{X}_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(RO_{ikd}) - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \tilde{\beta} X_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(\frac{r^{\rightarrow}_{ikd}}{r^{\leftarrow}_{ikd}}) - \log(\frac{\bar{\lambda}^{\rightarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \tilde{\beta} X_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(\frac{r^{\rightarrow}_{ikd}}{\bar{\lambda}^{\rightarrow}_{ikd}}) - \log(\frac{r^{\leftarrow}_{ikd}}{\bar{\lambda}^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \tilde{\beta} X_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(A^{\rightarrow}_{ikd}) - \log(A^{\leftarrow}_{ikd}) = \alpha \log(D^{\leftarrow}_{ikd}) + \tilde{\beta} X_{ikd} + \epsilon_{ikd} \Leftrightarrow$$

$$\log(\frac{A^{\rightarrow}_{ikd}}{A^{\leftarrow}_{ikd}}) = \alpha \log(D^{\leftarrow}_{ikd}) + \tilde{\beta} X_{ikd} + \epsilon_{ikd}$$

which finishes the proof of the lemma. □

Note that according to Lemma A24, each error term ϵ_{ikd} remains intact under this equivalence. It follows immediately that the unique estimated value for α, which is part of estimated parameters minimizing the GMM error in eq. (66), will also be the unique estimated value for α in eq. (68) as well. ■

F.3 Empirical Analysis of RideAustin

This subsection supplements the empirical analysis using data on individual driver behavior from RideAustin. Appendix F.3.1 provides suggestive evidence for the mechanism of supply-side economies of density: drivers' location choice in response to pickup times. Appendix F.3.2 provides further evidence by exploring the driver behavior after app-turnoff and during the idle time when the app is on. Appendix F.3.3 carries out relative outflows analyses using the Austin data.

F.3.1 Evidence from Individual Driver Behavior in Austin

We use data from "Ride Austin," a non-profit rideshare platform from June 2016 till April 2017. During this period, Ride Austin was the sole rideshare platform operating in the city due to a one-year exit by Uber and Lyft. Using this data, we show that drivers pay substantial attention to pickup times when deciding whether to stop operating in an area.

Data Description and Summary Statistics. Our data encompasses all of the rides given by Ride Austin from early June 2016 till mid April 2017. For each ride, we observe the exact data and time the ride request was sent through the app and the driver was assigned, the time the driver arrived at the passenger's location, and the time s/he dropped the passenger off at the destination. We also observe the corresponding locations. Moreover, we observe the surge multiplier applied to that ride by the platform. Finally, in addition to a driver ID that is constant for each driver for the duration of our data, we have a driver ID in our dataset which resets every time the driver turns her/his app off and and back on.

Note that using the above dataset, important variables for our analysis can be constructed: One can construct the idle time for the driver by calculating the difference between when s/he is dispatched and the last dropoff s/he made. Pickup time is constructed using the difference between when the driver was dispatched and when s/he picked the passenger up. Finally, one can observe when a driver turned her/his app off after a ride, by looking at whether her/his ID was reset. Table (11) provides a summary.

One caveat is that we can only observe that the driver turned her/his app off after a ride, but we do not observe the exact time when the app was turned off. That is, it is unclear whether the driver turned off the app only after they had waited for a long time or they immediately shut the app off. As a consequence, the idle time generated could be endogenous. Given this limitation of the data, the analyses provided here are only suggestive of the supply-side EOD.

Analysis of Individual Driver Behavior. We next turn to analyzing whether pickup times impact drivers' decision making on whether to turn their app off in a region after they drop a

Table 11: Summary statistics of variables of interest from the Ride Austin Data. Ride level statistics are calculated using the full, ride level data (820K observations). For the district level statistics, the data is first aggregated at the district level (11 observation) and then means and variances are calculated. "App Turnoff" is an indicator variable of whether the driver turns off the app upon finishing a ride.

	Ride Level		District Level	
variable	mean	std	mean	std
Pickup Time (minutes)	6.33	4.34	7.15	0.96
Idle Time (minutes)	9.55	18.04	12.74	5.88
Surge Factor	1.10	0.39	1.07	0.04
App Turnoff	0.40	0.49	0.42	0.03

passenger. We do this while controlling for idle times and prices as well as various fixed effects. This is implemented using the following logit regression:

$$O_\eta = \alpha_0 + \alpha_1 p_\eta + \alpha_2 W_\eta^{idle} + \alpha_3 W_\eta^{pickup} + \varepsilon_\eta \tag{71}$$

In this equation, η indexes an individual ride. Variable O_η is 1 if the driver turned her/his app off after finishing the ride. Variables p_η, W_η^{idle}, and W_η^{pickup} are, respectively, the surge multiplier, idle time, and pickup time that the driver *expects* to have for her/his next ride if s/he keeps her app on and keep searching for rides upon finishing η. We use expected values instead of actual values for two reasons. First, in the case of rides η after which the driver does turn off her/his app (i.e., $O_\eta = 1$), one cannot observe what the surge factor and pickup and idle times would have been had s/he decided to keep searching. Second, even in the case of rides η after which the driver keeps searching in the area (i.e., $O_\eta = 0$), it is reasonable to assume s/he only does not exactly know these quantities for her next rides with full precision. The expectations for each of these quantities are calculated by taking the average among all Ride Austin rides started at the same "time-of-week" and in the same "tile location" as ride η ended. Our definition of time of weak is a slight modification of that in Chen et al. (2017); and our tiles are a set of 1-square-mile squares partitioning the city of Austin and its greater area.

Table 13 presents the results of analyzing regression 71 using different fixed effects specifications. We use a combination of area fixed effects and time-of-week fixed effects. Area fixed effects divide the city into 11 parts: 10 official districts and what we consider the outskirt which is any location not belonging to one of the 10 districts. Time-of-weak indicators were described in the previous paragraphs. More details about these two fixed effects can be found in Figure 23.

Interpretation and Robustness. Regression results shown in table 13 all have the expected signs. All else equal, drivers are more likely to turn off their apps when surge factor is lower or wait time (idle time or pickup time) is higher. Most specifications suggest drivers dislike pickup times more than idle times. This is natural: pickup times are essentially idle times plus fuel costs. Finally, it is not surprising to see that once controlling for time and location fixed effects, the effect of price is attenuated.

Figure 23: Figures/Time-of-week and district fixed effects

bucket name	description
Weekday Regular	Mon-Fri: 10am-4pm
Weekday Morning Rush	Mon-Fri: 8am-10am
Weekday Afternoon Rush	Mon-Fri: 4pm-9pm
Friday Night	Fri: >4pm, Sat: <2am
Saturday Morning	Sat: 2am-1pm
Saturday Rush	Sat: >1pm, Sun <2am
Quiet	All other times

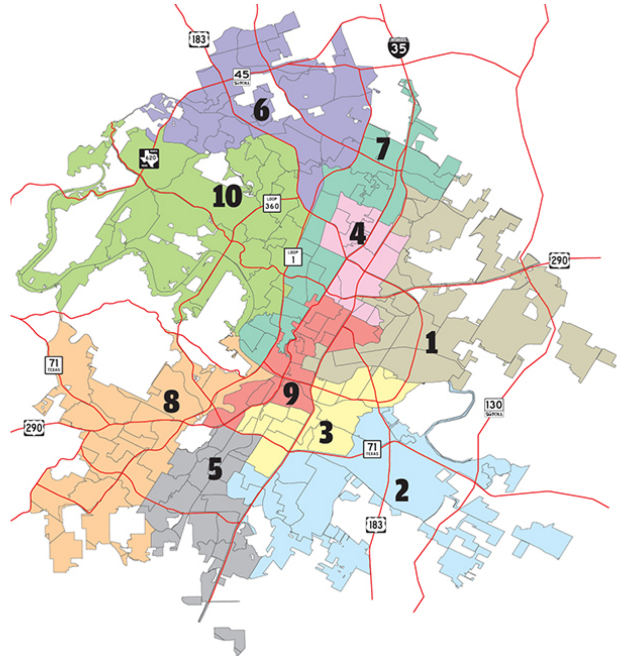


Table 13: Results from regressing app turnoff decisions on pickup times, controlling for prices, idle times, and different fixed effects.

	Dependent variable: App Turned Off				
	(1)	(2)	(3)	(4)	(5)
Pickup Time	0.004***	0.004***	0.012***	0.007***	0.007***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Idle Time	0.003***	0.006***	0.001***	0.001***	0.001***
	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)
Surge Factor	−0.289***	−0.228***	−0.123***	−0.050***	−0.056***
	(0.010)	(0.010)	(0.013)	(0.014)	(0.016)
Constant	0.657***	-	-	-	-
	(0.012)				
Fixed Effects†	N	L	T	T+L	T×L
Observations	820,343	820,343	820,343	820,343	820,343
Log Likelihood	−575,179	−574,801	−572,880	−572,693	−572,618
Akaike Inf. Crit.	1,150,367	1,149,630	1,145,784	1,145,429	1,145,421

Note: *p<0.1; **p<0.05; ***p<0.01

†: N: None, L: Location, T: Time

It is also worth noting that the individual-driver nature of the dataset is reassuring when it comes to possible endogeneity problems. This is because we do not need to worry about reverse causality: no individual driver's behavior should change the expected prices and wait times in regions. The only possible concern would be omitted variables which we control for using fixed effects based on fine partitioning of time and space. Obvious concerns like rush hour and commute traffic would be accounted for by these fixed effects.

According to the results from the column (4) of table 13, drivers would like a 7-minute reduction in pickup times almost as much as they would like a surge factor of 2. According to this column, and comparing with district-level summary statistics from table 11, drivers care about a one-standard deviation change in pickup times almost four times as much as they do about a one-standard-deviation change in the surge factor. The magnitude of the effect according to other columns of the table is smaller. Nevertheless, in all of them, pickup times have a non-trivial impact on drivers' decision making on where (not) to operate.

Though the use of multiple fixed effects specification along with the individual nature of the data is reassuring on the front of robustness, we do perform another check. We restrict the dataset to observations from only the month of March in 2017 (the latest month from which we have full data) and run the same regression. We do this in order to make sure our results are not confounded by underlying macroeconomic or demographic changes during the (almost) one-year duration of our data. Results from this additional analysis are broadly similar to those from table 13. To the extent that there is any difference, it slightly strengthens the role of pickup times. Table 14 shows the results.

This concludes our empirical analysis of Ride Austin. Appendices F.3.3 and F.3.2 provide further analysis in support of economies of density for Ride Austin. Appendix F.3.3 carries out relative outflows analysis for Austin and Appendix F.3.2 shows evidence that drivers move toward busier regions both when they have their app off and when they have it on.

F.3.2 Individual Driver Behavior after App-Turnoff and during Idle Time

This section supplements the empirical analysis of individual driver app-turnoff behavior in Austin in the previous subsection. Specifically, we provide the following analyses. First, we study what happens after app turnoffs and show two main things: (i) drivers are more likely to exit the market for the day upon an app turnoff if the turnoff is taking place in the outer regions of the city, and (ii) drivers are more likely to turn their apps off and relocate to busier regions of the city than the other way around.

Second, we track the movements of drivers between rides *while their apps are on* and show that, when idle, drivers move toward the city center and search for rides.

App turnoff, market exit, and location Here, we run the following logit regression:

$$E_\eta = a_0 + a_1 \times C_\eta + \epsilon_e ta \tag{72}$$

where each η is a ride at the end of which the driver turned off their app. Also, E_η is whether the driver exited the supply for the day upon turrning off the app, and C_η is a binary measure of whether the droppoff after which the app was turned off was centrally located in the city. It takes

Table 14: Repeating the logit regression analysis of app turnoff decisions, restricting the data to March 2017 only

	Dependent variable: App Turned Off				
	(1)	(2)	(3)	(4)	(5)
Pickup Time	0.009^{***}	0.004^{**}	0.015^{***}	0.007^{***}	0.006^{***}
	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)
Idle Time	0.003^{***}	0.004^{***}	0.001^{***}	0.001^{***}	0.002^{***}
	(0.0002)	(0.0003)	(0.0003)	(0.0004)	(0.0004)
Surge Factor	-0.173^{***}	-0.118^{***}	-0.101^{***}	-0.041	-0.032
	(0.022)	(0.023)	(0.030)	(0.032)	(0.037)
Constant	0.520^{***}	-	-	-	-
	(0.027)				
Fixed Effects†	N	L	T	T+L	T×L
Observations	167,838	167,838	167,838	167,838	167,838
Log Likelihood	−118,836	−118,762	−118,628	−118,579	−118,521
Akaike Inf. Crit.	237,681	237,553	237,279	237,201	237,224

Note: *p<0.1; **p<0.05; ***p<0.01

†: N: None, L: Location, T: Time

the value of 1 if ride η was closer than the median ride to the geo locations of downtown Austin (-97.7444 in longitude and 30.2729 in lattitude) and 0 otherwise. We expect the a_1 coefficient to be negative and significant indicating that an app turnoff in the central areas of the city is more likely to be a simple break, whereas one in the outer regions is more likely to be an exit. Here are the regression results, confirming our hypothesis.

Table 15: The result shows that drivers are more likely to exit the market when they end up dropping off a passenger in outer regions.

	Dependent variable: driver exit
a_1	-0.044^{***}
	(0.001)
a_0	0.332^{***}
	(0.001)
Log Likelihood	−444,688.200
Akaike Inf. Crit.	889,380.400

Note: *p<0.1; **p<0.05; ***p<0.01

Drivers' move across the city upon app turnoff Here, we examine drivers who turn off their apps and do not exit the market for the day. In particular, we study whether such drivers are more likely to move to busier regions of the city and restart operating or the other way around. To this end, we perform an analysis that is akin to our relative outflows study of rides, but this time we carry it out for inactive drivers. More specifically, we take different threshold distance values τ and for each τ we calculate the τ-level driver-relative-outflow as follows:

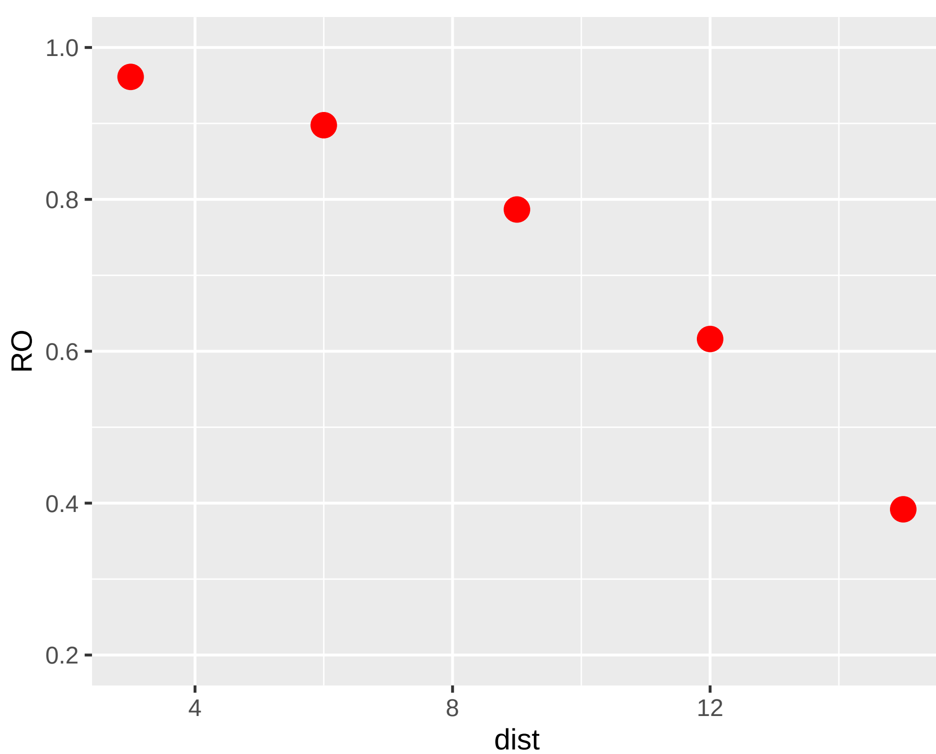
1. We divide the total number of times a driver turns their app off within the τ-mile distance of Austin downtown and then restarts operating *in the same day* outside of the τ-mile radius of downtown. We call this the τ-level outflow.

2. We calculate the τ-level inflow by doing the opposite of the above.

3. We then calculate the relative outflow by dividing the outflow by the inflow.

We expect this driver level relative outflow to be below 1 for different values of τ. That is, we expect the number of drivers who turn their apps off in busy regions and restart operating in less busy ones to be smaller than those who do the opposite. We expect the wedge between these two flows to be bigger (hence, the relative outflow to be farther away from 1) for larger values of τ. Figure 24 below confirms this expectation:

Figure 24: Driver-level relative outflows analysis. The horizontal axis depicts τ, in miles, as defined above and the vertical axis shows the τ-level driver relative outflow. As expected, the results are always below 1.



Driver idle movement while app is on In this section, we show evidence that drivers tend to move toward the city center while they are idle and searching for rides and their app is on. To this end, for each ride η after whose completion the driver did *not* turn their app off, we define the following: $\Delta_\eta^{\leftarrow}$ is defined to be the how far the destination of η was from Austin downtown, subtracted from how far the pickup location for the ride after η was from downtown. A positive value for $\Delta_\eta^{\leftarrow}$ indicates that the driver moved closer to the city center while idle and searching for rides. We perform a simple t-test to examine whether the mean of $\Delta_\eta^{\leftarrow}$ is positive and statistically significant. The mean is given at 0.265 miles with the 95% CI at $(0.26, 0.27)$. This number has a considerable magnitude, given that the average ride in the city moves away from downtown by a distance slightly below 0.2 miles. (Note that the direction of the average ride should not be surprising given economies of density.)

F.3.3 Relative Outflows Analysis in Austin

In this section, we carry out relative outflows analyses using our Austin data. More specifically, we do this at two levels. First, we calculate densities of rides and relative outflows at a geographically granular but time-wise less granular level. That is, our geographical units will be 1-mile by 1-mile tiles from a city grid published by the city of Austin,[51] while our time windows will be one month long each. We take regressions to examine the relationship between the density of incoming rides and the relative outflow at each tile during each month, both with and without city district fixed effects (see Figure 23 for Austin's city districts). Our second analysis makes the time window narrower (a day instead of a month) but uses the 10 districts of the city as the geographical units instead of 1 square mile tiles. We run this second analysis both with and without monthly fixed effects.

More precisely, the regression we consider is as follows:

$$log(RO_{it}) = \alpha_0 + \alpha_1 \times log(D_{it}^{\leftarrow}) + \epsilon_{it}$$

where i is the region (tile or district) and t is the time (month or day respectively). The above equation depicts the regression with no fixed effects. But as mentioned above, two of our four specifications have fixed effects. Table 16 presents the results.

As can be seen from these results, the coefficient of interest is always positive and significant as expected. Note, again, that this positive association between $\log(RO)$ and $\log(D^{\leftarrow})$ exists in spite of the fact that incoming rides appear with negative sign in the former and with positive sign in the latter.

Finally, before closing this section, we show this positive association visually, by presenting heat maps of relative outflows and dropoff densities at the 1-mile by 1-mile tile level aggregated over the duration of our data. Figure 25 shows these quantities. As can be seen there, central areas of the city have both higher dropoff rates and higher relative outflows. These figures also indicate that the access difference to supply across regions are meaningful: relative outflows in the central areas are about 50% larger than their surrounding areas and 2-3 times larger than those in the outer regions. This suggests the likelihood of fulfillment of potential demand is significantly smaller in

[51] See https://data.austintexas.gov/Locations-and-Maps/City-Grid/ty45-9ayu

Table 16: Relative outflow regression analysis results in Austin. Results, both at the tile-month level and at the district-day level, show that the coefficient of interest α_1 is positive and significant.

	Dependent variable: $\log(RO)$			
	tile-month level		district-day level	
	(1)	(2)	(3)	(4)
inflow density	0.034***	0.027***	0.052***	0.028***
	(0.006)	(0.007)		
Constant	−0.199***	-	−0.136***	-
	(0.025)		(0.010)	
Fixed Effects†	N	D	N	YM
Observations	3,388	3,388	3,084	3,084
R^2	0.011	0.014	0.054	0.097
Adjusted R^2	0.011	0.011	0.054	0.094

Note: *p<0.1; **p<0.05; ***p<0.01
†: D: District, YM: Year-month, N: none

less dense regions.

F.4 Discussion: Evidence of Economies of Density in the Taxicab Market?

We conjecture that economies of density also exist in the taxicab market and that they lead to agglomeration of supply in busier regions. Our theoretical model, however, does not directly speak to the taxicab market. Our model assumes a central dispatch structure for the matching of drivers to riders; and one of the key forces behind our results is the pickup time. In the taxi market, on the other hand, there are search frictions; but once a cab and a passenger find each other, they are not far apart (i.e., no pickup time).

Unlike our theory model, our empirical approach (i.e., relative outflows analysis) can be applied regardless of the type of the matching mechanism in the market. Thus, we perform it on data on Yellow Taxi rides in NYC during January 2009 (which is the first month for which data on taxicab rides are available from the TLC website) to gain insight into whether economies of density are at work there. Fig. 26 suggests a strong positive association between relative outflows RO_i and dropoffs densities $D_i^{\leftarrow}$ across regions i. Strikingly, numerous zones in the outer boroughs have relative outflows that are below 10%. This means in those regions (which are regions with smaller dropoff rates per square mile), more than 90% of those who enter using a taxi ride end up exiting using a different transportation option.

Figure 25: Panel (a) dropoff densities in different 1-mile by 1-mile squares across the city. Panel (b) does the same for relative outflows. Both quantities are larger in the middle and drop as we move towards outer areas.

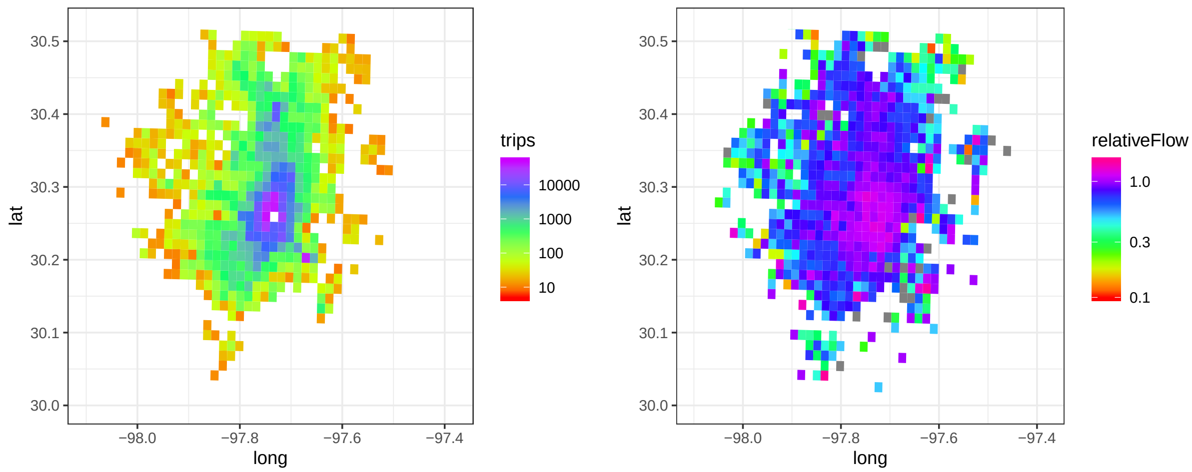
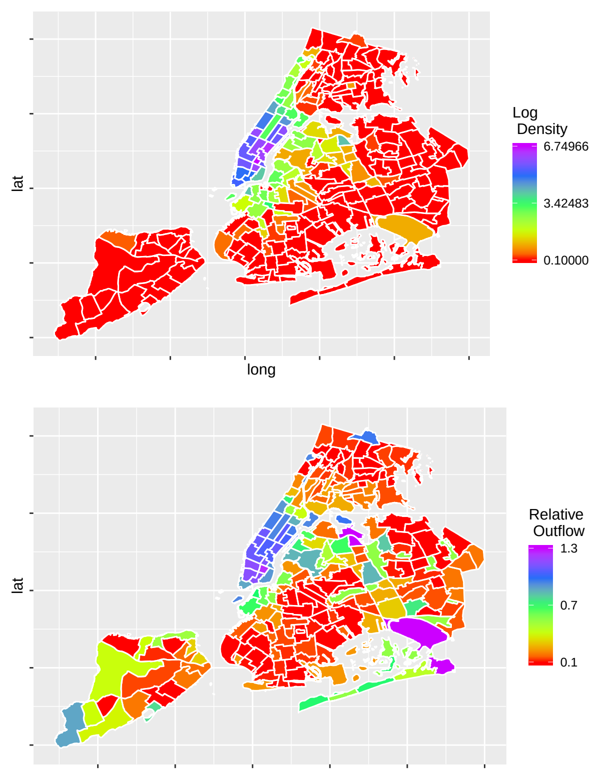


Figure 26: Dropoff densities in panel (a) and relative outflows in panel (b) by taxi zone across NYC for Yellow Cab rides during January 2009 (before Green Taxi was launched to serve the outer boroughs). The two quantities seem spatially highly correlated. We interpret this to indicate concentration of supply especially in mid and lower Manhattan due to economies of density.



Next, we carry out the same regression analysis as in eq. (66) for Yellow Taxi rides. The results, under all regression specifications with regards to borough and zone type come back positive and significant. Also the magnitude is much higher compared to the results in Table 7, suggesting a stronger economies of density effect in the taxicab market compared to the rideshare market.

Table 17: Results from regressing relative outflows on dropoff densities for yellow taxi rides in January 2009 (before the green taxi program started). The regression specification is the same as eq. (66). Similar to the results from the same regression on rideshare data, here too the coefficient α on dropoff density is positive and significant and this is robust to different fixed effects specifications. The magnitude of the estimated α is much higher here compared to rideshare, suggesting more pronounced economies of density in the taxicab market.

	Dependent variable: log relative outflow			
	(1)	(2)	(3)	(4)
log dropoff density	0.184***	0.172***	0.183***	0.165***
	(0.024)	(0.039)	(0.024)	(0.041)
Fixed Effects†	Constant	B	Z	B×Z
Observations	194	194	194	194
R^2	0.233	0.833	0.792	0.839
Adjusted R^2	0.229	0.828	0.786	0.819

Note: *p<0.1; **p<0.05; ***p<0.01
†: B:Borough, Z:Zone-type

Our results in this section suggest that economies of density, and the agglomeration that it leads to, have an important role in shaping the spatial distribution of supply in the taxicab market (Note that January 2009 was before "Green Taxis" were established. Thus, the lower access to yellow taxis in the outer boroughs cannot be because of the presence of green taxis). This points to a future research direction on the modeling and empirical analysis of transportation markets with decentralized matching systems such as taxi cabs: the incorporating of economies of density and its implications. Much of the literature on transportation markets with decentralized matching (e.g., Lagos (2000, 2003); Buchholz (2018); Brancaccio et al. (2019a,c)) is built on the assumption that the "matching function" between passengers and drivers is homogeneous of degree one. That is, the number of rides originated in region i doubles if both the number of passengers searching for drivers and the number of drivers searching for passengers in i double. This abstracts away from economies of density. This would likely lead to an attribution of the relative outflows patterns in Figure 26 to substantially lower need for cab rides entering Manhattan than rides exiting it.[52]

Capturing economies of density in decentralized transportation markets (i.e., allowing the number of matches in a region to *more than double* if demand and supply both double) will also have policy implications. This is because the policy maker may have an incentive to encourage supply in sparer areas in an effort to build economies of density there, whereas individual taxi drivers will not have the same incentive (they do not internalize the density-related externality they leave on others). This incentive mis-alignment in the taxi market will require remedies, which will likely take a similar form to the theoretical results in our rideshare model, and are worth further studying

[52]If the matching-function-inversion approach from this literature is applied to recover search volumes for rides across regions, we would indeed conjecture that the result would point to fewer searches for rides from the outer regions. We would like to emphasize, however, that fewer searches do not necessarily indicate lower potential demand. In fact, based on our relative outflows analysis, those fewer searches would at least in part indicate that potential riders in outer regions forgo searches for taxicabs in anticipation of their likely failure.

both theoretically and empirically.

G Estimation using Adjusted Prices and Wages

This appendix addresses the potential concern that the price variation of the price measure adopted in the main analysis could be mechanically arising from the variation in ride length and thus the identification may not be valid. To this end, we generate alternative measures of prices and wages adjusted for ride length (including both distance and time). Next, we calibrate the model using the adjusted prices and wages. The results are robust to those reported in in Section 6 of the main text. Below, we detail the construction of the adjusted measures and present the results. For brevity, we only show selected estimation and counterfactual results in this appendix.

To control for trip time and trip distance, we first compute the average trip time and distance for each borough i and each day d, denoted as $TripTime_{id}$ and $TripDistance_{id}$. Then we run the following regressions of price/wage on the trip time and the distance at borough-day level:

$$p_{id} = \beta_1 TripTime_{id} + \beta_2 TripDistance_{id} + \epsilon_{id}^{p}$$

$$c_{id} = \beta_1' TripTime_{id} + \beta_2' TripDistance_{id} + \epsilon_{id}^{c}$$

Next, we take the average of $TripTime_{id}$ and $TripDistance_{id}$ across all boroughs and all days, denoted as $\overline{TripTime}$ and $\overline{TripDistance}$. Then the adjusted prices and wages can be generated as

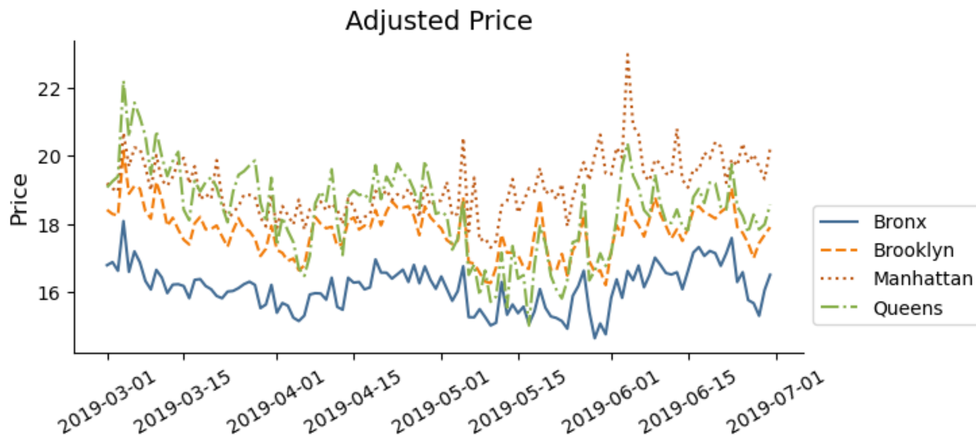
$$\tilde{p}_{id} = \hat{\beta}_1 \overline{TripTime} + \hat{\beta}_2 \overline{TripDistance} + \hat{\epsilon}_{id}^{p}$$

$$\tilde{c}_{id} = \hat{\beta}_1' \overline{TripTime} + \hat{\beta}_2' \overline{TripDistance} + \hat{\epsilon}_{id}^{c}$$

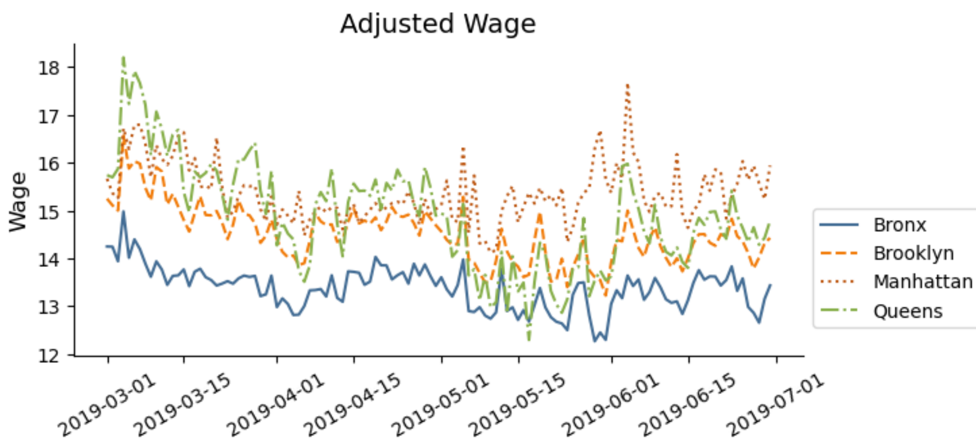
Fig. 27 below shows how the adjusted prices and wages vary across time and boroughs.

Figure 27: Adjusted Prices and Wages

(a) Variation of Price per Ride across Days and Boroughs



(b) Variation of Wage per Ride across Days and Boroughs



We take $\tilde{p}_{id}$ and $\tilde{c}_{id}$ instead of p_{id} and c_{id} directly averaged from data to the calibration, while keeping all other data input to be the same. As such, variations in trip length do not carry over to variations in the adjusted price and wage measures. Table 18 presents the model estimates from the model with supply-side EOD and the model with demand-side EOD using these adjusted measures of price and wage instead of the original ones. Table 19 reports the demand elasticities implied by the estimates. The numbers reported in the two tables are similar to those in Table 4 and Table 5 of the main text respectively.

Table 18: Model Parameters from the model with supply-side EOD and the model with demand-side EOD. We report the point estimates (with t-statistics in parentheses) for the parameters (α, a, β). Since our model estimates $\bar{\lambda}$ for each day and each region, we therefore only present the average of the point estimates of $\bar{\lambda}$ across days for each region.

	Model w/ Supply EOD	Model w/ Demand EOD
α	0.0214 (18.79)	0.0164 (12.51)
a	53.35 (25.71)	N/A
β	N/A	61.80 (24.67)
Average $\bar{\lambda}$ by Regions		
Bronx	4848	4182
Brooklyn	8420	7333
Manhattan	12940	11156
Queens	6943	6008

Table 19: Average demand elasticities across days with respect to prices given by the model with supply EOD and the model with demand EOD separately. The averages are weighted with respect to the number of trip requests per day.

Region	Trip Time (Min)	Model w/ Supply EOD	Model w/ Demand EOD
Bronx	14.51	0.53	0.50
Brooklyn	16.61	0.61	0.49
Manhattan	18.59	0.69	0.48
Queens	19.00	0.65	0.62
All Regions	17.59	0.64	0.51

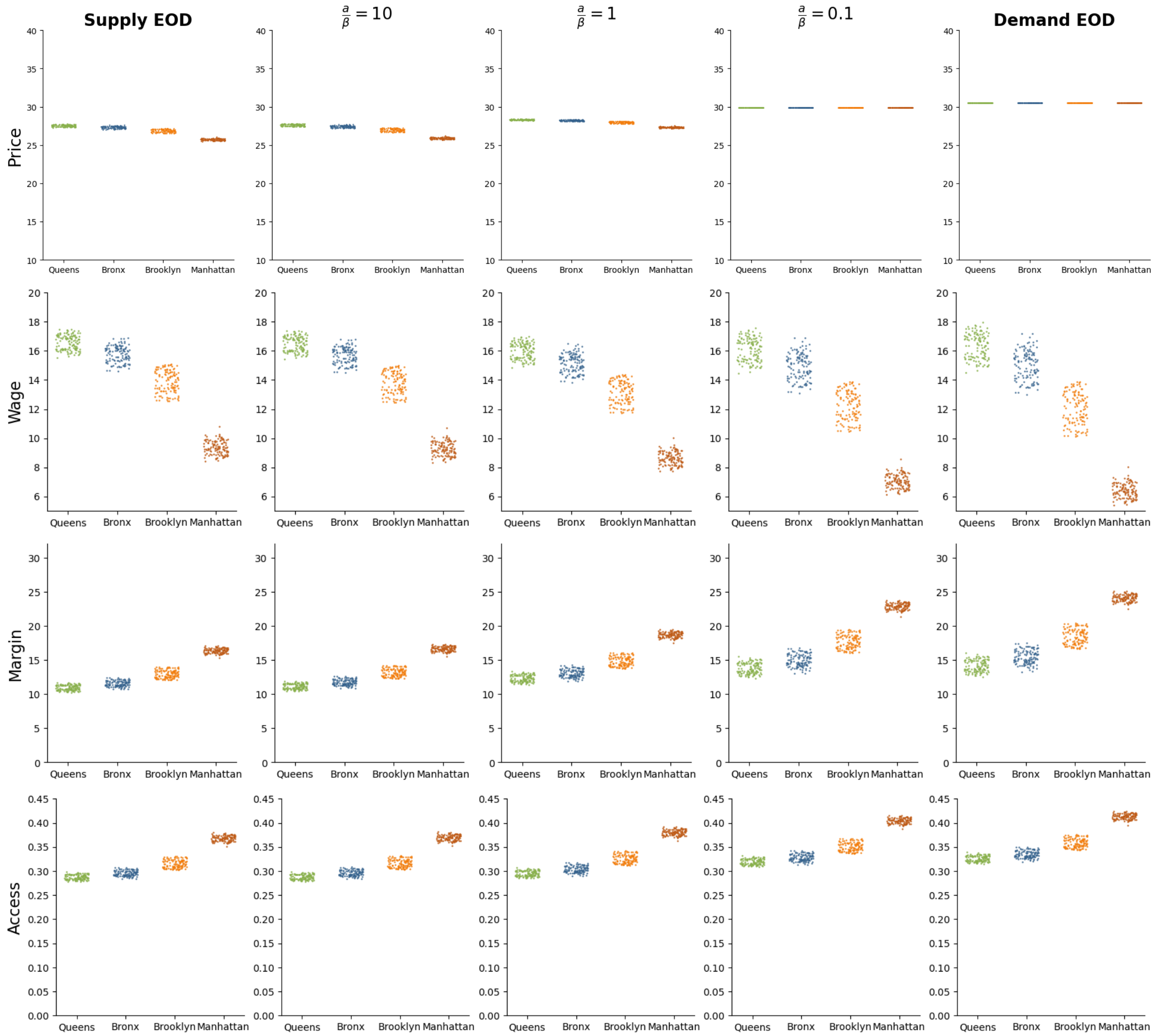
Table 20 presents the average (across days) potential demand density and access metrics based on all the models (including the comprehensive ones) in each region. Qualitatively, the patterns are consistent with those in Table 6 in that the access levels follow the same ordering as the demand density across regions. Besides, quantitatively, the numbers reported here are close to those in Table 6. Finally, we run the counterfactuals with the adjusted data and the corresponding model estimates. The implications are robust. For brevity, we present the results for the first counterfactual, market outcomes under optimal prices and wages, in Fig. 28. The patterns still hold. That is, denser regions receive lower prices, lower wages, higher margins, and higher access. Moreover, the numbers are quantitatively similar to those in Fig. 5. The robustness check here shows that the price variation leveraged in the main text is valid to identify the model parameters.

Table 20: Model Outcomes (Regions Ordered by Demand Density)

Model Spec.	**Supply EOD**	**Supply & Demand EOD**			**Demand EOD**
$\frac{a}{\beta}$	∞	10	1	0.1	0
Demand Density ($\frac{\bar{\lambda}_i}{s_i}$)					
Queens	665.97	662.23	633.19	587.01	576.28
Bronx	746.23	741.64	707.70	655.74	643.80
Brooklyn	1010.73	1005.91	965.49	896.71	880.29
Manhattan	2716.03	2698.99	2574.31	2384.86	2341.42
Access ($\frac{r_i}{\lambda_i}$)					
Queens	0.46	0.46	0.49	0.52	0.53
Bronx	0.50	0.50	0.52	0.57	0.58
Brooklyn	0.53	0.53	0.56	0.60	0.61
Manhattan	0.56	0.57	0.60	0.64	0.65

Note: The results presented correspond to averages across days across the data time periods.

Figure 28: Prices, wages, margins and accesses under optimal platform strategy. In each plot, regions (boroughs) are arranged in ascending order by demand density from left to right. Given the estimated model parameters, we find the optimal prices and wages for each region on each day. Each dot represents the metric for a region of a specific day.



H Price Trend after 2019

In Section 6.2.1, our counterfactual analysis of platform optimal pricing predicts that overall the optimal price is about 50% higher than what Uber charged in 2019. One potential explanation could be the competition between Uber and other ridesharing platforms. Accordingly, it is expected that the prices increases as the competition is softened when Uber grows larger. This section shows that our prediction is in line with what Uber has done between 2019 and 2022.

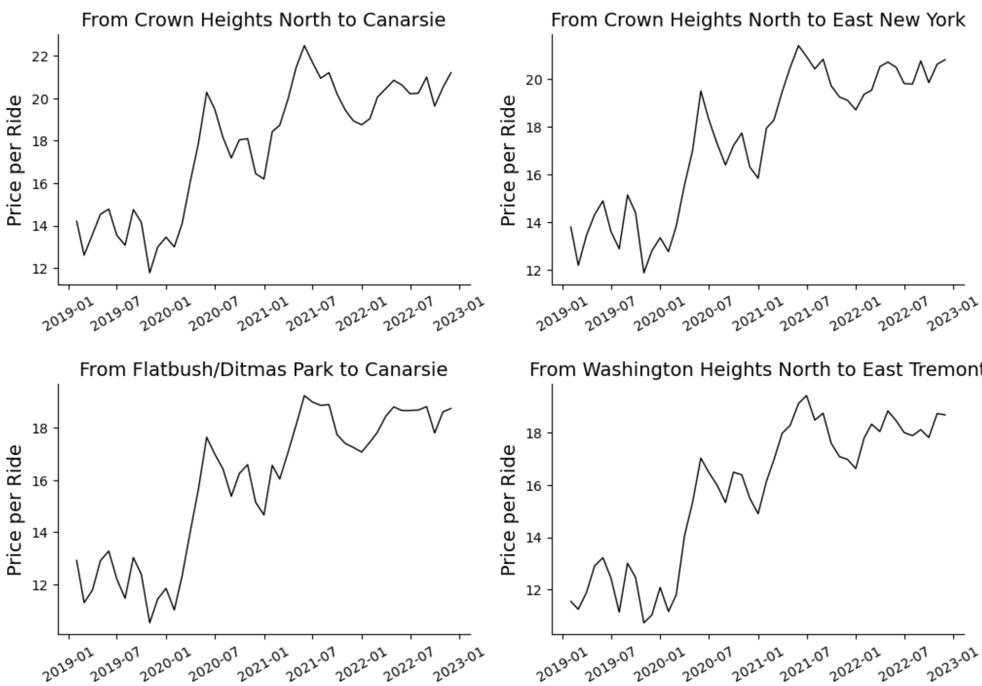
Table 21 lists the 25th percentile, median and 75th percentile of prices among all the rides taken in December 2019 and December 2022 respectively. Overall, the price has increased by about 50%.

Fig. 29 presents the monthly average price of sampled frequently requested routes between pairs of taxi zones from the beginning of 2019 to the end of 2022. The trend agrees with the prediction of the counterfactual.

Table 21: Summary Statistics of Price per Ride in December 2019 and December 2022

	25th percentille	median	75th percentile
December 2019	7.25	11.80	20.51
December 2022	11.24	18.32	29.43

Figure 29: Average price per ride by month from 2019 to 2022 for sampled routes (between pairs of taxi zones).



I Comparison of Data Range to Rosaia (2023)

This appendix compares the geographical locations we include in our analysis to those included in Rosaia (2023). Fig. 30 shows the map of regions included in Rosaia (2023). Fig. 31 presents the variation of net incoming trips across the locations included in our data and that across the locations included in Rosaia (2023) data. Fig. 32 compares the variation of net outgoing trips and Fig. 33 displays the population density. For all three measures, the geographical span of the data we use encompasses additional sparser regions compared to Rosaia (2023) and thus is more amenable to capture economies of density.

Figure 30: Taxi Zones included in Rosaia (2023); copied from Rosaia (2020) Figure 1

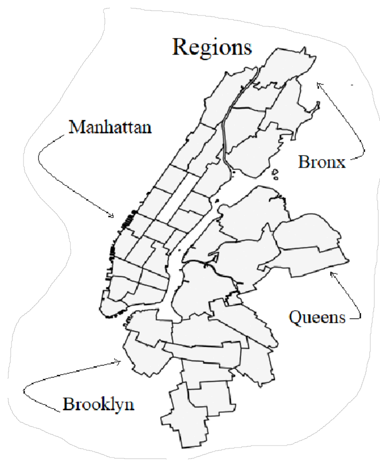


Figure 31: Total Number of Net Incoming Trips in June 2019 by Taxi Zones

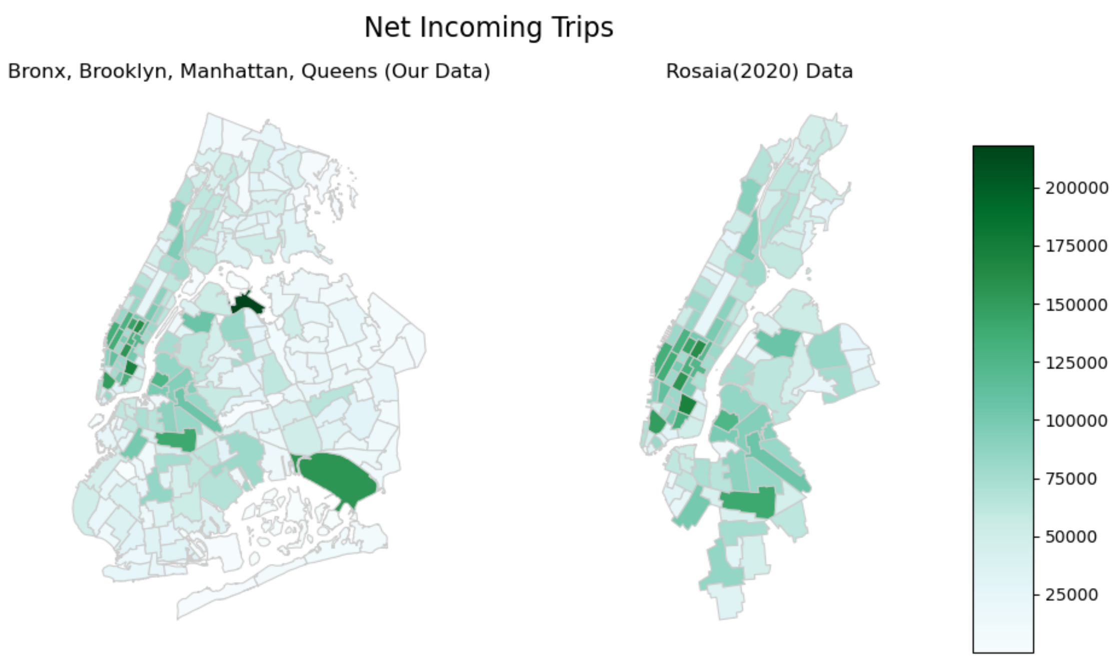


Figure 32: Total Number of Net Outgoing Trips in June 2019 by Taxi Zones

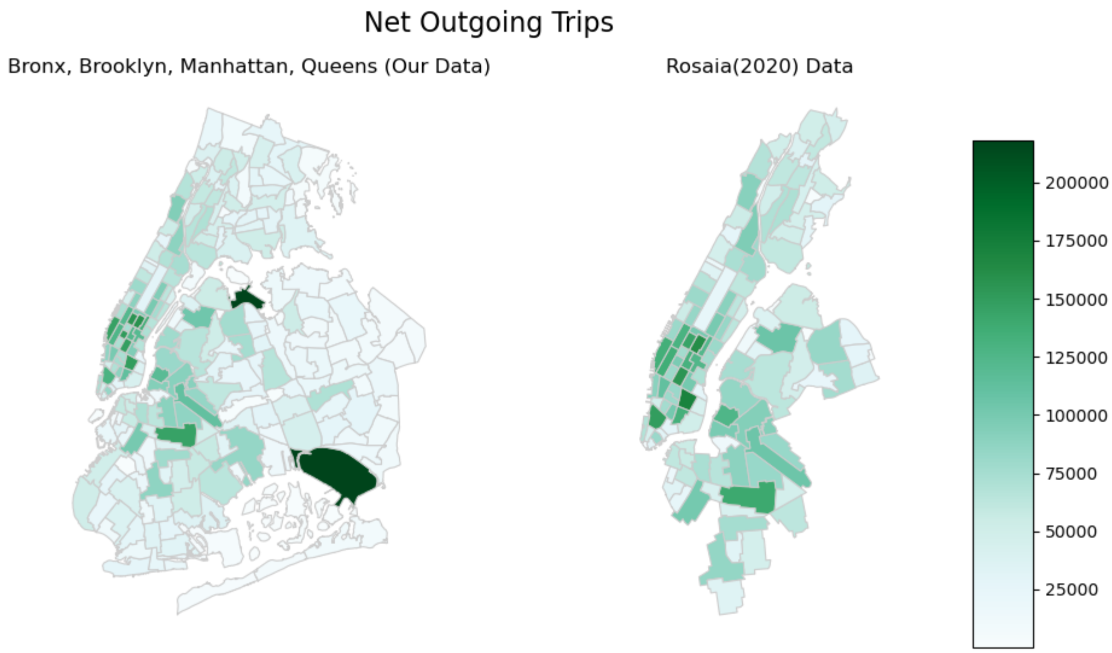
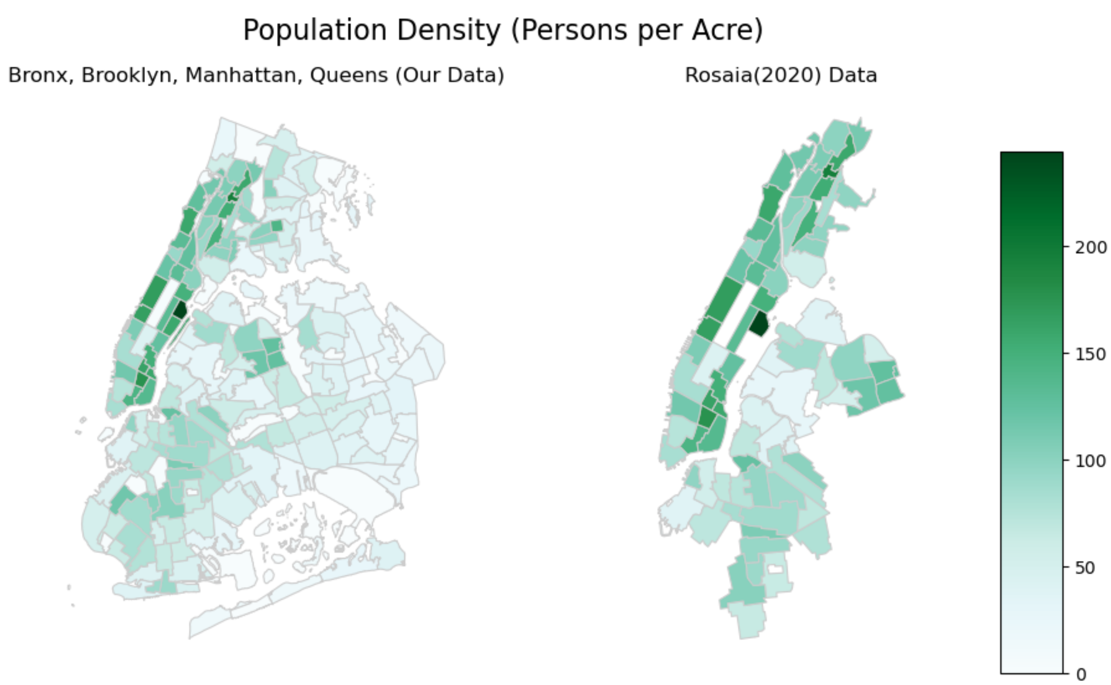


Figure 33: Population Density by Neighborhood. Note that the neighborhoods are not divided as the same as the taxi zones, so the areas on the right hand side do not exactly coincide with Rosaia's.



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