Network Interventions Based on Inversity: Leveraging the Friendship Paradox in Unknown Network Structures

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Network intervention problems benefit from selecting a more connected node, which is more likely to result in stronger indirect effects. However, in many network contexts, the structure of the network is unknown. We derive and examine the mathematical properties of two distinct "informationally light" strategies, a global strategy and local strategy, that yield higher degree nodes in virtually any network structure. These strategies are based on the friendship paradox: "your friends have more friends that you do." We further identify a novel network property called Inversity, which connects the fundamental parameters of these two strategies. We prove that the sign of Inversity for any given network determines which of the two strategies will be most effective in that network. We assess the performance of these strategies across a wide range of generative network models and real networks, and we show how to leverage network structure through these strategies even when the network structure is unknown.

Network-based interventions are of crucial importance in any setting where an individual's choice or action has a multiplier impact on others, with a wide range of applications. Consider the following network intervention problems: (a) A new infectious disease is spreading through a large population. We want to minimize the number of infected individuals by inoculating using a new vaccine; however, we only have 1000 doses to administer. (b) We have a new product, and have a limited number of free samples to distribute to consumers, so they can share information through word of mouth. we would like to maximize the number of consumers who receive word of mouth. (c) We would like to identify virally spreading contagion (informational or biological) as quickly as possible by choosing individuals as observation stations.

Although seemingly distinct, these problems (a)-(c) represent a class of problems. In this class of network interventions (*1*), we benefit from identifying more central (or highly connected) individuals in the network. However, the challenge is that in many interventions, we do not have access to the relevant and current network structure. For example, in (a) having the Facebook network structure might not be useful, since the relevant network for this context would be the physical contact network. In the second problem (b), similarly if we are considering a professional product (say pharmaceutical samples to physicians), finding a high degree node using a physical contact network of everyone who interacts with a physician is unlikely to be useful.

We model and compare two *informationally light* network interventions (or strategies) which obtain higher degree nodes from a network by querying randomly chosen individuals, and which do not require access to the complete underlying network structure. The interventions are based on the *friendship paradox*, and result from theoretical network properties developed further here, *global mean* and *local mean* of friends of friends, used in the global and local strategy respectively. We also derive a structural network property called *inversity*, which determines the relative effectiveness of the local and global strategies.

The term friendship paradox was first used in (*2*) to express the idea "your friends have more friends than you do." A basic view of the friendship paradox is developed by plotting the average number of friends (degree) of individual nodes' "friends" on the vertical axis against the average degree (Fig. 1, Fig. S1). For example, in the Contact (In person Social) network, we see a deep blue region above and to the left of the $45°$ line. Although present across all networks, the pattern is most prominent in the WWW (Google) or Twitter (Online Social) network. Observe also that in the Road Network, only $\Delta = 37\%$ of nodes have a higher average number of friends of friends than their own degree. The friendship paradox has also been generalized to the idea that individual attributes and degree are correlated (*3*), e.g. an individual's co-authors are more likely to be cited (*4*), or that friends more active on social media (*5*).

First, we find that the individual-level friendship paradox cannot hold for all individuals in any given network structure (Theorem S1). In contrast to this individual view, the average number of friends of friends across the network was investigated earlier theoretically and found to be $\mu^{FOF} = \dfrac{\mu_D^2 + \sigma_D^2}{\mu_D}$ where μ_D and σ_D^2 are the mean and standard deviation of node degrees in a network (*6*) (Theorem S2). The idea of the friendship paradox can be used as a strategy ("random friend of an individual node") to obtain a higher degree node. However, using a "random friend" for an intervention generates an apparent new paradox with the above result from (*6*), which we term "The Paradox of Paradox of Friends." [1] In resolving this paradox, we characterize the two distinct network intervention strategies.

The Paradox of the Paradox of Friends Consider the network in Fig. 2. First, note that the variance in the number of friends is positive, e.g. node a has 2 friends whereas e has 1 friend.

[1] The term "friendship paradox" has been used as an approach for the above problems, e.g. (*7*) for (b), as well as a number of other related settings, including immunization of networks (*8–10*), sentinels for detection of infections (*11–14*) across social person-to-person networks, to surveillance of disease in animal movement networks (*14*), as well as detection of viral information sharing on large-scale online networks like Twitter (*12*). (*15*) extend the idea of leveraging the friendship paradox to asymmetric networks in the setting of an attack on a network, illustrating the increased effectiveness of removing an *upstream* neighbor.

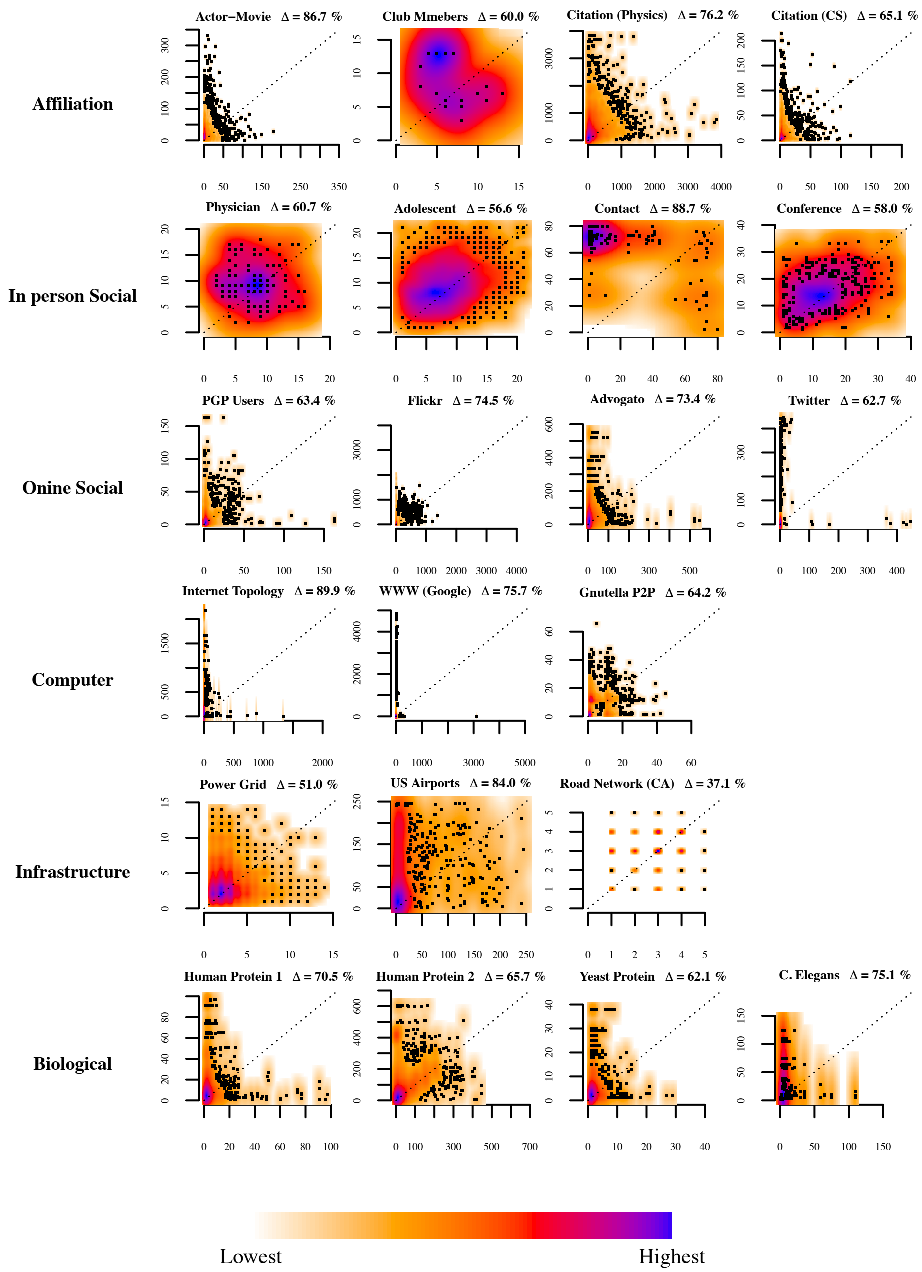

Figure 1: Friendship Paradox at Individual Level. Density plot of average number of friends of nodes compared to node degree in networks. Δ indictes the proportion of nodes that have a higher average number of friends of friends than their degree. Lowest density regions within each network are marked by white / orange, and highest density regions are marked in blue. For all networks, the highest density region lies above and to the left of the 45 degree line. For some networks like Adolescent Health or Road Network (CA), it is relatively more evenly distributed both above and below the 45 degree line, whereas for networks like Internet Topology or Twitter, the distribution is skewed above and to the left.

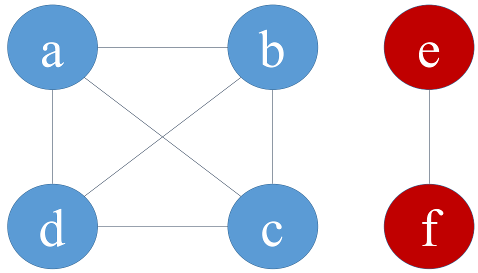

Figure 2: **Two Component Network**

Given $\mu^{FOF} = \dfrac{\mu_D^2 + \sigma_D^2}{\mu_D} > \mu_D$, the average number of friends of friends exceeds the average number of friends. However, everyone has exactly the same number of friends that their friends do. Therefore, choosing a random friend as suggested above cannot result in a higher degree individual, compared with choosing a random individual. We term this feature the Paradox of the Paradox of Friends.

The resolution of this paradox lies in the fact that there are two distinct ways to count these numbers of friends that our friends have, which in turn leads to two distinct network properties derived from the Friendship Paradox. One property — the local mean — has motivated the intervention strategy of choosing a random friend of a random individual. The other property — the global mean — connects to the mathematical foundations as formulated above. The two network properties are structurally distinct, and lead to a different outcome of the average number of friends of friends. Figure 3 shows us how the local and global mean are characterized for the network from Figure 2. Observe that the local mean for this network is identical to the average degree, implying no advantage from "choosing a random friend of a random individual."

Global and Local Mean of Friends of Friends

We formally show the two distinct but related network properties deriving from the friendship paradox relating to the "average number of friends of friends." Denoting a network (see Table S1 for full notation) as an undirected graph $G = (V, E)$ with V the set of nodes and E the set of edges ($e_{ij} \in \{0, 1\}$ denoting absence or presence of a connection between i and j), and using D_i to refer to the degree of node i, we specify the local mean as:

$$\mu_L = \frac{1}{N} \sum_{i \in V} \left[\frac{1}{D_i} \sum_{j \in N(i)} D_j \right] \tag{1}$$

The global mean consistent with (*6*) is defined as the ratio of the total number of friends of friends to the total number of friends in the network:

$$\mu_G = \frac{1}{\sum_{i \in V} D_i} \sum_{i \in V} D_i \left[\frac{1}{D_i} \sum_{j \in N(i)} D_j \right] \tag{2}$$

where we see that the means arise from differently weighting the average degree across friends.

Both means above are consistent with the "average number of friends of friends," although they are distinct network properties (Fig. 3). The local mean of the example network (Fig. 3A), is computed by taking the average number of friends of friends for each node, and then averaging this quantity across the nodes in the network (Fig. 3B). The global mean is computed differently, by taking all the friends of friends of any node in the network (Fig. 3C). For this network, the local mean is higher than the global mean, and both of these means are greater than the mean degree.

We term these means local or global since we can prove that the local mean depends on the local structure, whereas the global mean only depends on the global network structure (degree distribution). Both means are greater than the mean degree for *all* undirected networks (Theorem S2, S3). All results and proofs are in Supplement).

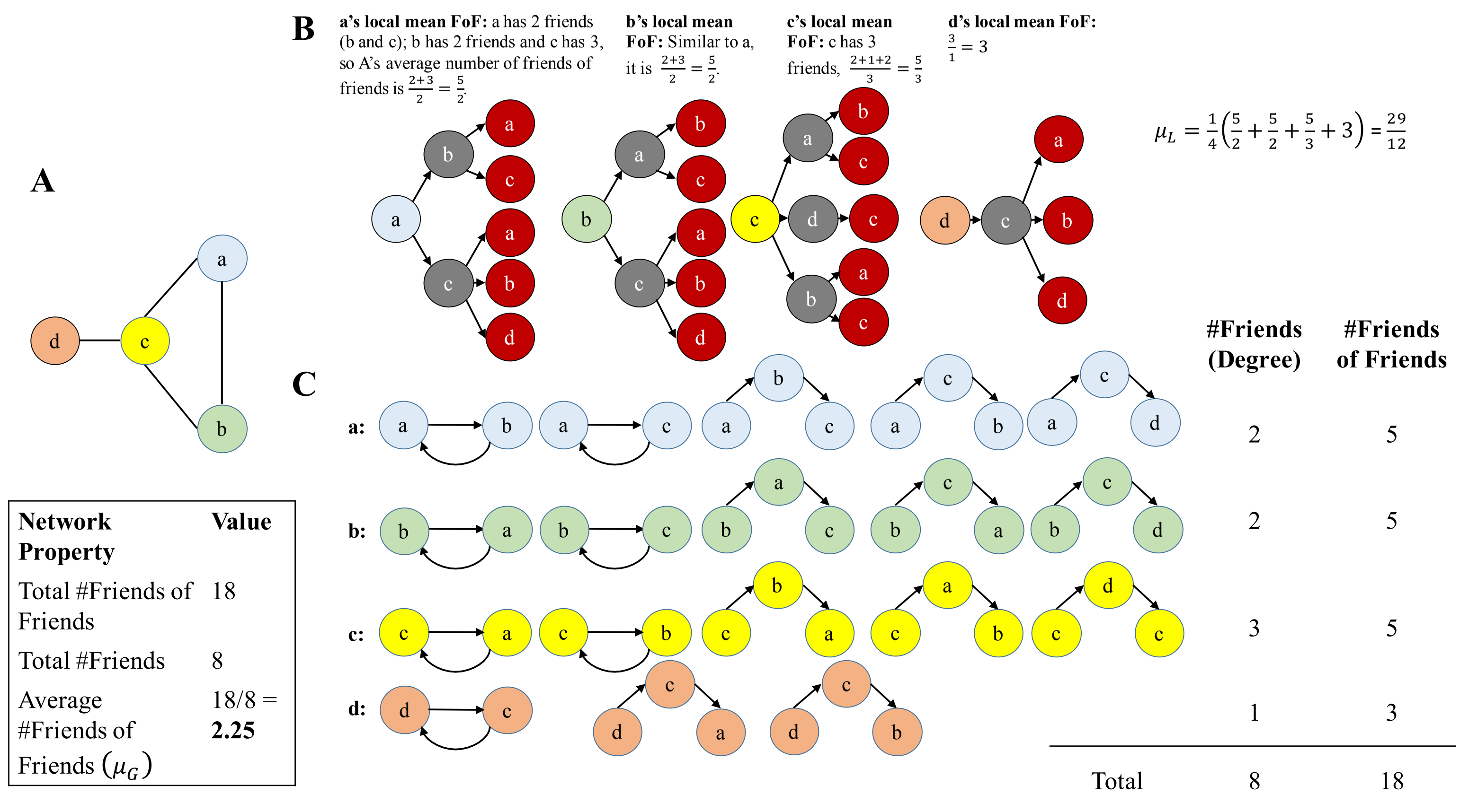

Figure 3: **(A) Network.** Example network with 4 nodes a,b,c and d. **(B) Local Mean.** a has 2 friends, b and c. a has total number of friends of friends in a's network is 5: (a's friend b has 2 friends, a and c; a's friend c has 3 friends, a, b and d). So average number of friends of friends for a is the ratio of the number of nodes in black to the number of nodes in red, i.e. $\frac{2+3}{2} = \frac{5}{2}$. Similarly, for the other nodes, we have $b : \frac{2+3}{2} = \frac{5}{2}$, $c : \frac{2+1+2}{3} = \frac{5}{3}$, $d : \frac{3}{1} = 3$. The local mean of the network is the mean of these local average friends of friends, so $\mu_L = \frac{1}{4}\left(\frac{5}{2} + \frac{5}{3} + \frac{1}{4} + \frac{3}{1}\right) = 2.42$. **(C) Global Mean.** Person a has 2 friends, b and c. The total number of friends of friends in a's network is 5: as above. Similarly, b has 2 friends (a and c), and the total number of friends of friends that these two friends has is also 5 (a has 2 friends and c has 3). c has three friends (a, b and d). The total number of friends of friends in c's network is also 5 (a has 2, b has 2, and d has 1). d has one friend, c. The total number of friends of friends in d's network is 3 (c has 3 friends). Thus, across the whole network, there are a total count of 18 friends of friends. The total number of friendships is 8. Thus, on the average these 8 friendships extend to 18 friends of friends, so the global mean is $\mu_G = \frac{18}{8} = 2.25$ mean number of friends of friends per friend.

We illustrate the practical impact of the distinction between the two means, with the following questions : (a) Is the Local Mean always greater (or smaller) than the Global Mean? (b) Can both means be relatively high (or low)? (c) What network (sub)structures result in a high Local Mean or Global Mean? We examine four illustrative network structures (Fig. 4) to answer these questions and to understand the differences between the two means. We find that both local and global mean can be much greater than the mean degree, and between these two means, either of them can be greater than the other. Especially noteworthy is the difference between the Local and Global panels for network (Fig. 4C): the Local mean is equivalent to the mean degree, because each node is equally weighted in terms of w_i^L. However, the Global mean is higher for this network since it assigns a higher weight w_i^G to higher degree nodes.

Impact of Local Network Structure

Degree distribution is commonly used to characterize network structure (*16*) For the global mean, the degree distribution (mean and variance) provides a complete characterization. In contrast, the local mean depends strongly on the local network structure. We identify properties of the local structure that maximize the local mean. First, we examine how rewiring the network while maintaining the same degree distribution affects the local mean. We find that rewiring the network by creating a connection between nodes with more extremal degrees (high or low) while simultaneously removing a connection between less extremal nodes, will increase the local mean (Theorem S6). Next, we examine how the local mean can be maximized, and demonstrate that the highest and lowest degree nodes must be connected for this to happen (Theorem S7). Second, if we now have an unrestricted degree distribution, for what network structure is the local leverage maximized? We find that of all the possible network structures, no network structure can improve upon the star (or hub and spoke) network.

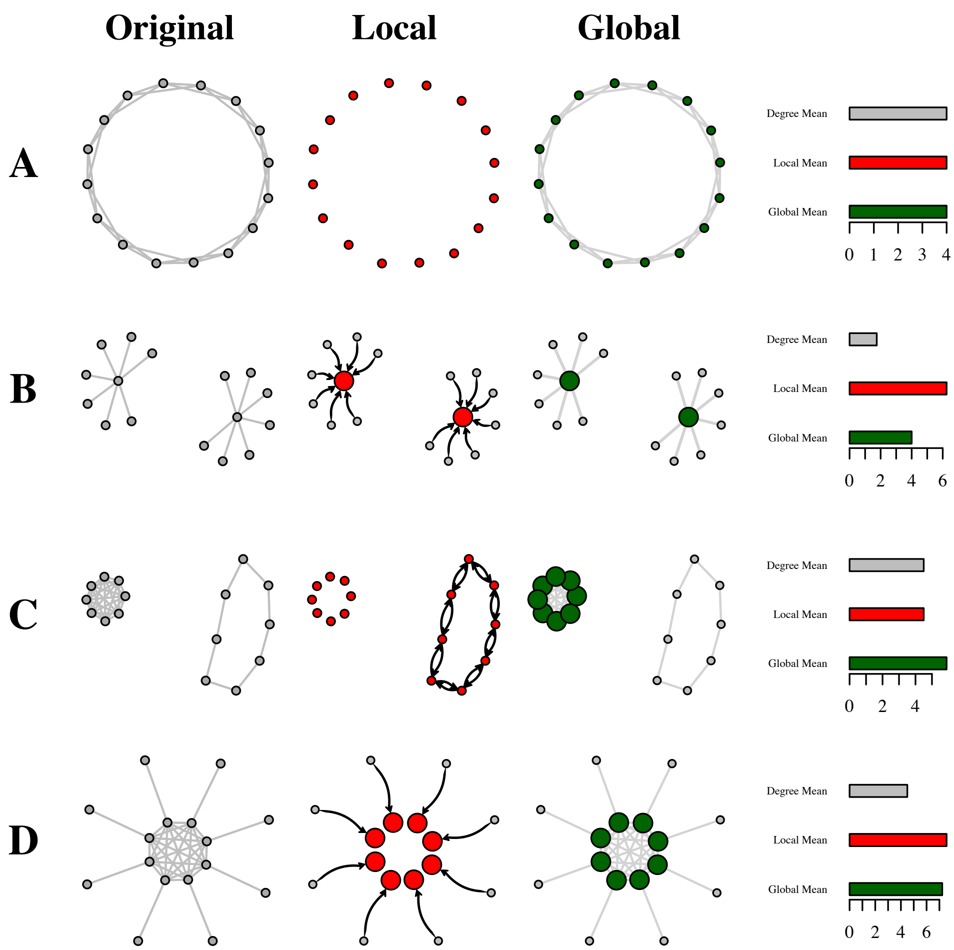

Figure 4: **Four Illustrative Networks with Varying Local and Global Means.**
Each network in (A)-(D) has the original network plot (left), local weighted network (middle) and global weighted network (right). On the right is a barplot indicating the mean degree, local mean and global mean for each of the networks. *Local Panel (Red)*: In the local weighted network plot (middle), nodes are sized proportional to their weight $\left(w_i^L\right)$ in contributing to the local mean. Edges that receive a *higher than median* weight in computing the local mean are in black color. Otherwise, the edges are not plotted in the middle panel. Note that although the original networks are undirected, the selected edges are illustrated as *directed* since the weights are directed. *Global Panel (Green)*: Nodes are sized proportional to their weight $\left(w_i^G\right)$ in contributing to the global mean. Edges are all weighted equally in the global weighted network. (A) *Small World Ring:* Each node has four friends, and local and global mean are both equal to avert age degree (4). None of the edges are shown in the middle panel since all edges have identical weight in computing the local mean. All nodes in both local and global means have the same weight, and size in the middle and right panel. (B) *Two Central Hubs with Spokes:* Each central hub is connected to 7 nodes. The mean degree is lowest in this network. However, local mean is substantially higher than the global mean, and is higher than the mean degree across all networks (a)-(d). In local panel, we see that the weight of central hubs has increased, whereas the corresponding weight for the low degree "spoke" nodes has decreased. In the global panel, the node weights are proportional to degree. (C) *Heavy Core with Attached Cycle:* The global mean is substantially higher than the local mean (and mean degree). Here, we see in the local panel that the weight of each of the nodes has not changed, and all nodes have the same weight. However, in the global panel, we see that the high degree nodes in the complete graph has higher weight compared to the original network, whereas the weights for the nodes in the 2-cycle are lower than in the original network. (D) *Heavy Core with Pendants:* Both the local and global mean are substantially higher than mean degree. In the local panel, the edges connecting core nodes to other nodes (both core and pendant) have a relatively low weight, and are not displayed.

Random: Node f is chosen at random as initial node and also used as seed node.

Local: Node n is chosen at random as initial node and is asked for **one** random friend, and ***g*** is selected through the edge (n-g), and used as the seed node.

Global: Node s is chosen at random as the initial node, and is asked to choose **each** friend with fixed probability p. Therefore, nodes **m**, **k**, **t**, **a** and **b** are each chosen with probability p.

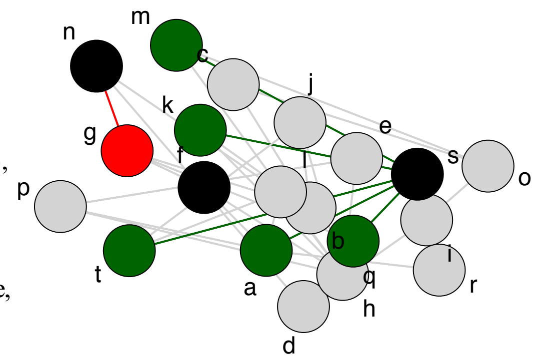

Figure 5: **Strategies:** Random, Local and Global Strategies with example network. Note that whereas the Local strategy results in one node chosen as seed, the number of nodes chosen under the Global strategy is variable. For the Global strategy, any arbitrary probability $p \in (0, 1]$ can be chosen in advance.

Intervention Strategies

Our formulation and distinction between local and global mean suggests distinct intervention strategies. We illustrate random, local and global strategies (Fig. 5) to choose a "seed" node in the network beginning with an initial randomly chosen node. Observe that with the local strategy, the number of seed nodes is fixed, whereas it is probabilistic under the global strategy. For the local strategy, the proof is by construction of the local mean. For the global strategy, we prove that the expected degree of chosen nodes is equal to the global mean (Theorem S5).

Though the local and global strategies appear to be similar in the sense that we are choosing friends of randomly chosen individuals, the crucial distinction lies in whether we are choosing *one* random friend with certainty or whether we are choosing among *each* random friend with a specified probability.

Strategies in Generated Networks

Networks generated from a number of commonly used generative mechanisms are used to assess a number of structural features with regard to the friendship paradox. We are interested in structural leverage, as defined by the ratio of the local (or global) mean to the mean degree, i.e. $\lambda_L = \frac{\mu_L}{\mu_D}$ and $\lambda_G = \frac{\mu_G}{\mu_D}$. We examine 3 generative mechanisms: (a) Erdos-Renyi (ER), (b) Scale Free (SF) and (c) Small World (SW) networks (Fig. 6) (*17–19*).

We find that for ER networks, at very low density (edge probability), the leverage is very low because most edges connect nodes that have a degree of 1. As density increases, we obtain more variation in degrees, and local leverage increases. However, beyond an edge probability of $p = 0.05$, leverage decreases as the density of the network decreases. Local leverage thus forms a non-monotonic pattern with ER networks. For SF networks, rather than density or edge probability, we initially examine leverage as the network becomes more centralized (as γ increases above 1, very high degree nodes have a lower probability of occurring). We find that as γ increases from 1 to 2, the leverage increases, but then decreases beyond 2. For WS networks, unlike in the ER and SF networks, leverage is monotonically decreasing with number of neighbors (or density), and is monotonically increasing with rewiring probability.

Strategies in Real Networks

We investigate a wide variety of real networks to determine how much leverage the local or global strategies provide. The networks are selected across several categories (Affiliation, Face-to-face Social, Online Social, Computer, Infrastructure and Biological networks), and span a wide range in network characteristics like size and density (Table S2). First, observing the local strategy (Fig. 7A), we find that for all networks, as expected, the local strategy is at least as good as the random strategy. Second, for networks like Twitter (OS4) or Internet Topology (C1), the local leverage λ_L can be as high as 100. Thus, obtaining a friend of a random node will provide

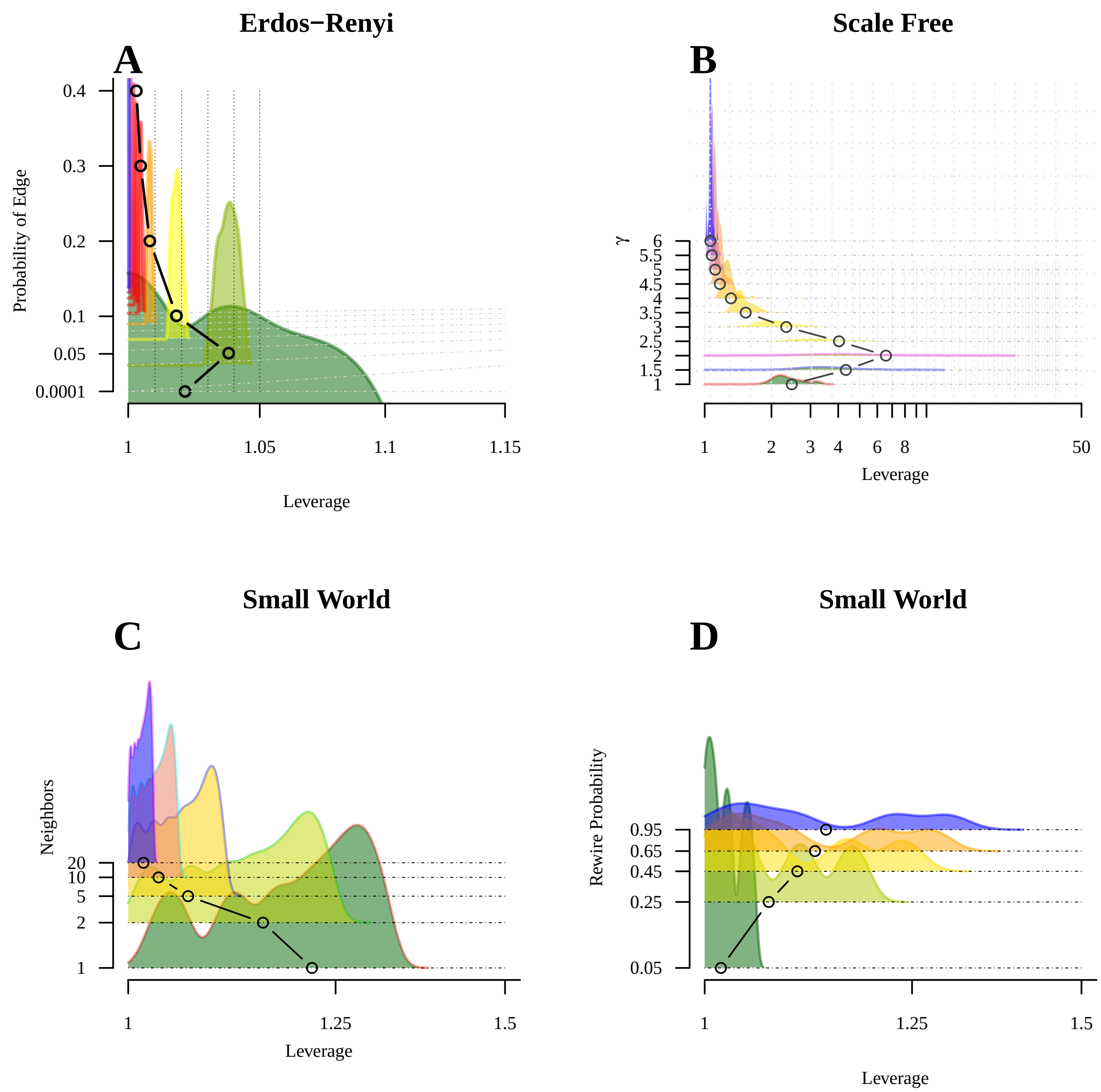

Figure 6: Local Leverage Density in Generated Networks from three different generative models, and spans the parameter space. A sample of 1,000 networks was used for each of the models. (A) Erdos-Renyi (ER) networks generated with edge probabilities, $p \in [0.05, 0.95]$, and size ranging from N=50 to N=1000 nodes. We find that local leverage is highest for the lowest edge probabilities, and leverage converges to 1 as the networks become more dense. (B) Static Scale Free (BA or Barabasi Albert) networks with scale-free parameter $\gamma \in [1, 6]$. For these networks, observe that the leverage spans a wider range, e.g. for $\gamma = 2$, the samples range from leverage of 1 to over 40. The mean leverage is non-monotonic in terms of γ, increasing when $\gamma < 2$ and decreasing for $\gamma > 2$. The distribution of leverage across the samples also displays decreasing variance when $\gamma > 2$. At very high levels of $\gamma \approx 6$, the local mean converges to the mean degree. With small world (Watts-Strogatz) networks, we have two parameters. First is the number of neighbors each node is connected to initially, n. The edges are then rewired with a specified probability, p_r. First, in panel (C), we find that with a small number of neighbors, the leverage distribution is quite spread out, and there is a substantial leverage effect. However, as we begin to create very dense networks, both the mean and the variance of the leverage distribution leverage diminish substantially. Second, we examine the impact of rewiring probability on the leverage distribution in panel (D). We find that with lower rewiring probabilities, say $p_r = 0.05$, the leverage distribution is closer to 1, whereas with a higher rewiring probabilities, the distributions feature increased variance as well as higher mean leverage.

a 100-fold increase in the expected degree of a chosen node. Third, we observe that both local and global leverage (Fig. 7B) are higher for nodes when average degree is intermediate, i.e. not too low or high. Some networks like the CA Roads network (I3) have very little degree variation and local and global strategies are relatively less effective. Finally, we want to examine when local and global strategies make a relative difference (Fig. 7C). We find that the highest ratio of local to global mean is for Twitter network (OS4), whereas the lowest ratio (indicating that global strategy has a higher expected mean degree) is shown by Flickr (OS2), both of which belong to the same category of online social networks. Citation networks tend to have higher global mean, whereas for Infrastructure networks, both strategies seem to work just as well.

Finally, we show that the aggregation can cause network properties to be significantly altered. For example, in a group of 75 village networks in India (S2.1), we find that the local mean is higher than global mean for most villages when we consider individual-level networks. However, when we examine household-level networks, the conclusion is the opposite and global mean is higher for most networks.

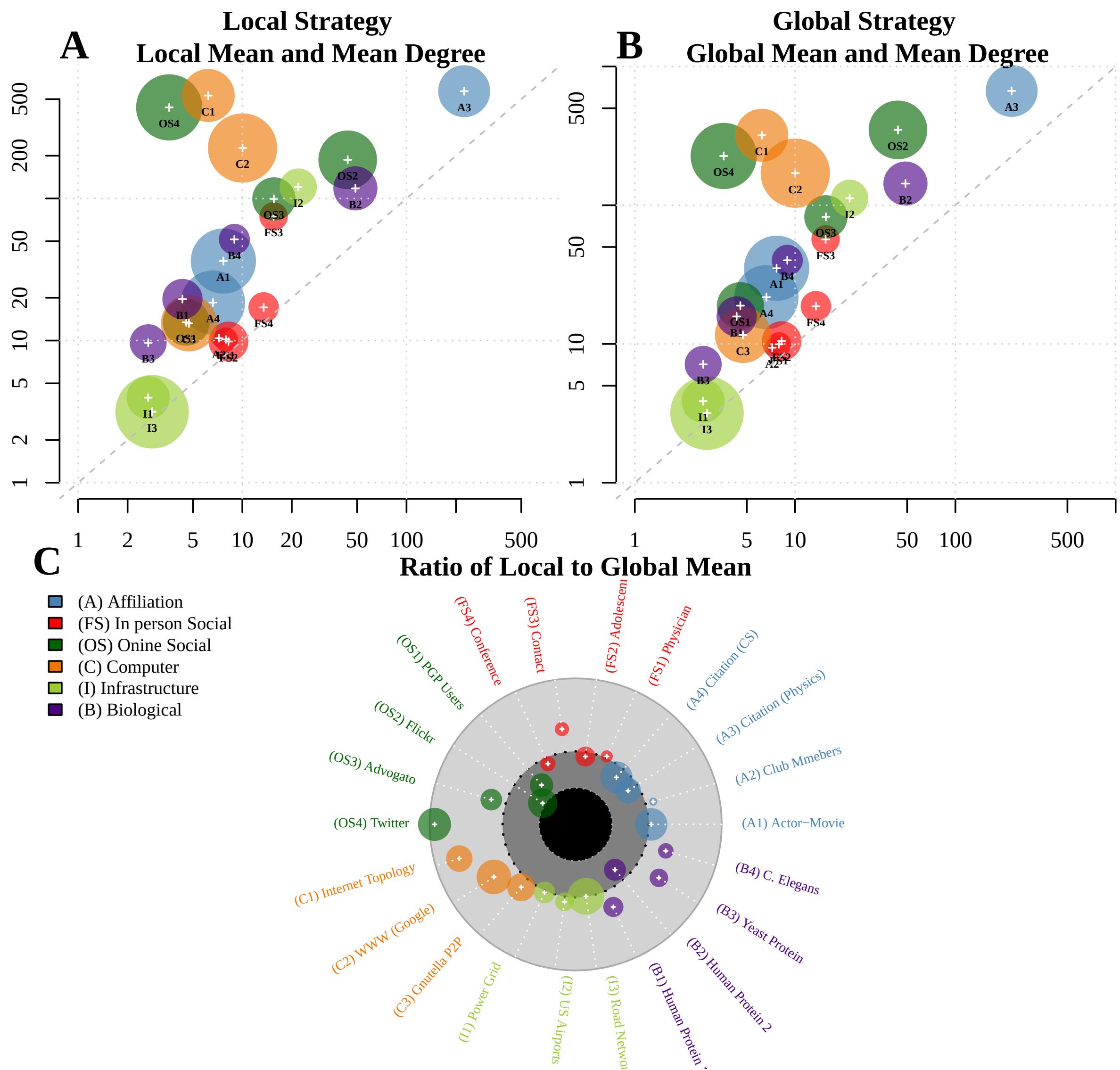

Figure 7: Global and Local Leverage in Real Networks Local and Global Means across Networks. Area of circles indicates size of networks (number of nodes) in log scale. Color of circle indicates network category. (A) **Local Mean** is higher tha mean degree in all real networks, with the highest differences occurring in online social networks and computer networks. Most large networks also tend to show a higher ratio of local mean to mean degree. For in person or face to face networks, the pattern is more variable with (B) **Global Mean** also demonstrates a value higher than mean degree for all networks. (C) **Comparison**: Ratio of Local to Global Mean. The ratio of local to global mean $\frac{\mu_L}{\mu_G}$ is represented as follows ($< \frac{1}{2}$ in black circle, $\frac{1}{2} < \frac{\mu_L}{\mu_G} < 1$ in dark gray circle and $1 < \frac{\mu_L}{\mu_G} < 2$ in light gray circle. For example, in the Twitter network, local mean is almost twice the global mean, whereas in the Flickr network, global mean is almost twice the local mean. Computer networks have higher values of the ratio, whereas Infrastructure networks have similar values of local and global means.

Inversity: Connecting Local and Global Means

To characterize when the local mean is greater than the global mean (or vice versa), we identify a novel network property: *Inversity*. This property captures *all* local network information related to the local mean and is scale-invariant, i.e. independent of the size or density of the network. We find that the sign of inversity helps us determine which of the local mean or the global mean is higher for any given network.

Inversity is a correlation-based metric that allows us to connect directly the global and local means for any network. First, define the following edge-based distributions to examine the relationship between the means. The *origin* degree (**O**), $D^O(e)$, *destination* degree (**D**), $D^D(e)$, and *inverse destination* degree (**ID**) distribution, $D^{ID}(e)$, are defined across directed edges $e \in \hat{E}$ as: $D^O(e_{jk}) = D_j, D^D(e_{jk}) = D_k, D^{ID}(e_{jk}) = \frac{1}{D_k}$. We define the *inversity* across the edge distribution as the Pearson correlation across the origin and inverse degree distributions.

$$\rho = Corr\left(D^{\mathbf{O}}, D^{\mathbf{ID}}\right) \tag{3}$$

We can then connect (see Theorem S4) the local and global means with *inversity* and the degree distribution $\left(\kappa_m = \sum_{i \in V} D_i^m\right)$ as:

$$\mu_L = \mu_G + \rho\, \Psi(\kappa_{-1}, \kappa_1, \kappa_2, \kappa_3) \tag{4}$$

where Ψ is a positive function of the degree distribution.

Therefore, if inversity is known, we don't need the entire degree distribution to obtain the local mean. Rather, *four* moments of the degree distribution are sufficient for that purpose. Inversity ρ captures the local information regarding the network structure, i.e. which nodes are connected to which other nodes, whereas the moments of the degree distribution represent global information about the network. ρ has a critical role in determining whether the local or global mean is larger for a network; specifically, $\rho < 0$ indicates the global mean is higher than the local mean, whereas $\rho > 0$ indicates the reverse.

Is Inversity Different from Assortativity?

A natural question is whether inversity captures the same information (with opposite sign) as assortativity, which is a well known network property $\rho_a = Corr\left(D^{\mathbf{O}}, D^{\mathbf{D}}\right)$ capturing the correlation in degree across all edges in the network (*20–22*). To examine this question, we generate $1,000$ networks using different generative methods as above. We find that assortativity and inversity are not guaranteed to have opposite signs (Fig. 8A). Therefore, the sign of assortativity cannot be used to determine whether the local or global mean is greater for a network, unlike with inversity. All 3 network generating processes create networks with the same sign for both metrics (detail in Fig. 8B). Example networks for the case of same sign assortativity and inversity are illustrated (Fig. 8C, 8D), showing that it is not obvious to predict inversity of a network if we know its assortativity.

We also examine the relative degree to which inversity (ρ) and the degree distribution (Ψ) contribute to the ratio of local to global mean (Fig. S2), and find that for some networks, the degree distribution plays a relatively greater role. The WWW - Google (C2) and Yeast Protein (B3) networks both have similar ratio of means, but the Protein network has a relatively higher inversity but lower degree variation compared to the WWW network.

Discussion and Conclusions

We show that with unknown networks, the friendship paradox can be leveraged to obtain such individuals with minimal informational requirements. We identify intervention strategies (local and global) have theoretical guarantees on obtaining a highly connected individual. Using both generated random network models as well as real networks, our results show the value of using the local and global strategies to obtain highly connected nodes. In the vast majority of networks, we obtain at least double the average degree, and in select networks, using the local and global strategies provides several hundred-fold increase in the node degree.

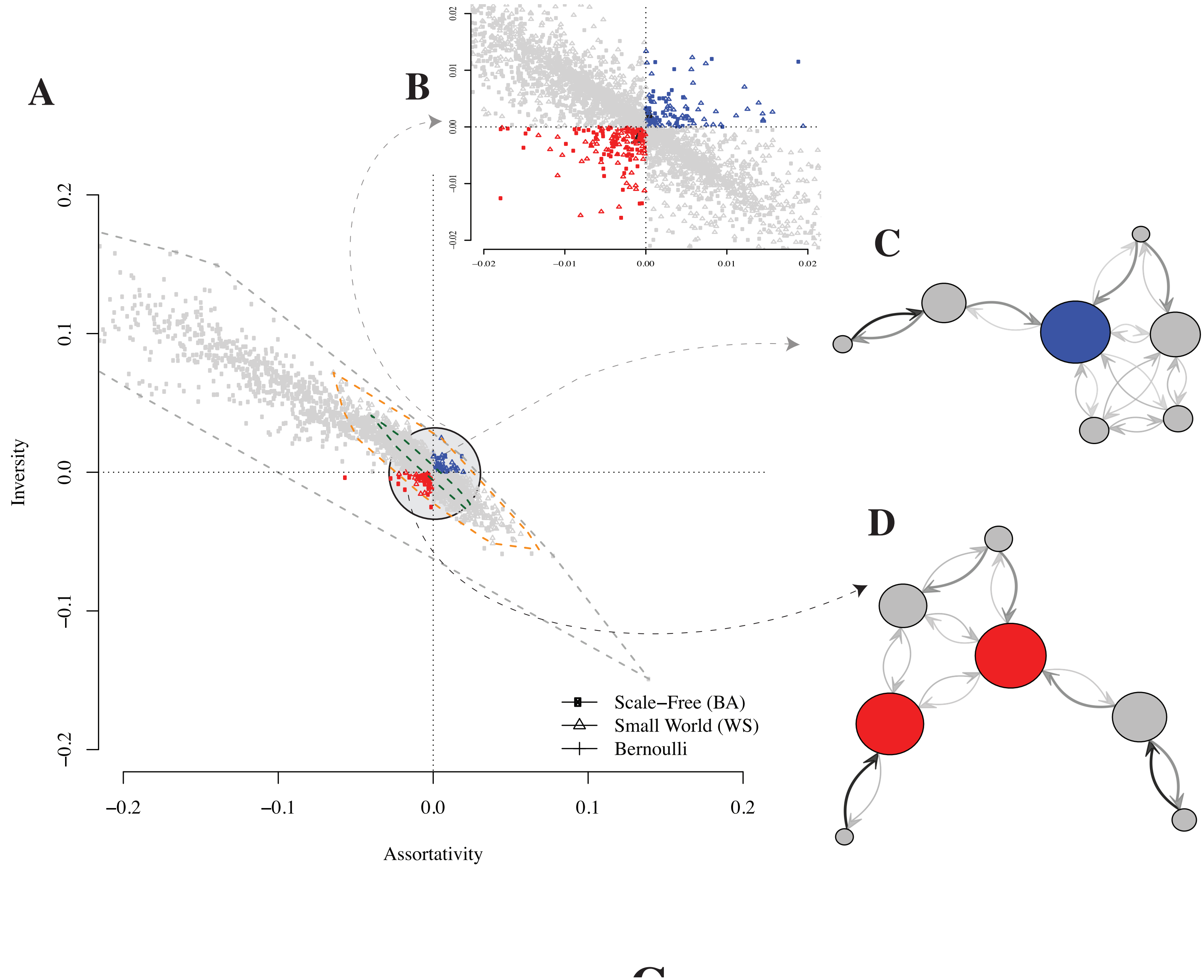

Figure 8: Assortativity and Inversity. N=1000 networks are generated from three classes of networks. (A) Erdos-Renyi (ER), Scale Free (BA or Barabasi Albert) and Small World (WS or Watts Strogatz), parameters detailed in legend of Figure 5. Observe the regions in red and blue, where networks have the same sign of assortativity and inversity. (B) Detailed view of region around (0,0) showing all three network types can produce networks with same sign for assortativity and inversity. (C) Example network with N=7 nodes where assortativity and inversity are both positive. (D) Similar example where both measures are negative. Overall, it demonstrates that inversity and assortativity are not equivalent measures, e.g. using assortativity in place of inversity could result in using a global strategy when local may be more appropriate.

Inversity captures the local network structure (i.e. who is connected to who) as distinct from global network properties like the degree distribution, which do not distinguish local network structure. The sign of inversity guarantees which strategy will perform best for any given network structure, but network category or the degree distribution is not predictive of which strategy is relatively more advantageous (e.g. local is relatively better than global for the Twitter network, but the opposite is true for Flickr). We also show how Inversity is different from the commonly used assortativity property, which is insufficient to capture the local network structure.

We fully expect these findings to be useful in developing further strategies for network interventions when the *relevant* network structure is not known, as well as settings in which the complete network structure is evolving and not obtainable.

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Supplementary Material

S1 Mathematical Appendix

Formally, the network graph $G = (V, E)$ is comprised of a set of N individual nodes and a set of undirected edges E. Each element of E is a pair of nodes, (i, j) indicates an edge (connection) with $e_{ij} \in \{0, 1\}$. We also define the directed edge set $\hat{E}$ including both (i, j) and (j, i) as distinct elements of $\hat{E}$ corresponding to an undirected edge $i \leftrightarrow j$. We detail the table of notation in Table S1.

Table S1: Table of Notation

Symbol	Term	Definition
G,V,E	Network	Network Graph of Nodes V and Edges E
$\hat{E}$	Directed Edge Set	Each edge in E is replaced by two directed edges
D_i	Degree	Number of friends of i
$\mathscr{N}(i)$	Neighbors	*Set* of Friends of i
F_i	Average degree of friends of i	$\frac{1}{D_i} \sum_{j \in \mathscr{N}(i)} D_j$
μ_D, σ_D^2	Mean and variance of Degrees	$\frac{1}{N} \sum_i D_i,\ \frac{1}{N} \sum_i (D_i - \mu_D)^2$
μ_L	Local Mean	$\frac{1}{N} \sum_i F_i$
μ_G	Global Mean	$\frac{\sum_i D_i F_i}{\sum_i D_i}$
ρ	Inversity	$Corr\left(D_i, \frac{1}{D_j}\right) \forall (i, j) \in \hat{E}$

The basic idea of the friendship paradox can be expressed as ”your friends have more friends than you." We examine the degree to which the friendship paradox holds for individual nodes, or the individual friendship paradox. We find in the result below that it cannot hold for all nodes, but can hold for an arbitrarily high proportion (< 1) of nodes.

Theorem S1. *The friendship paradox statement that "your friends have more friends than you" cannot hold for all nodes in a network. Also, the statement can hold for all nodes, except one.*

Proof. Consider a connected network where not all degrees are identical (if all are identical, the statement cannot hold). There must be at least one node that has the highest degree D_{max} and which is connected to at least one node with a lower degree. If not, then the connected network is comprised entirely of highest (identical) degree nodes, thus contradicting the initial statement. If the highest degree node is connected to a lower degree node, then the average friends of friends of the highest degree node must be lower than D_{max}. Thus the statement cannot hold for *all* nodes. To show the second part that it can hold for all nodes except one, consider the star (hub and spoke) network, where all of the nodes except the central node have fewer friends than their friends do. □

Theorem S2. *[Feld 1991] For a network $G = (V, E)$ with degree mean μ_D and variance σ_D^2, the global mean of friends of friends is*

$$\mu_G = \left(\mu_D + \frac{\sigma_D^2}{\mu_D}\right) \tag{5}$$

Proof. *(as given in Feld, 1991).* $\mu_G = \dfrac{\sum_i \sum_j e_{ij} D_j}{\sum_i D_i} = \dfrac{\sum_i D_i^2}{\sum_i D_i} = \dfrac{\mu_D^2 + \sigma_D^2}{\mu_D}$ □

Theorem S3. *For any general network $G = (V, E)$ with mean degree μ_D, the local mean of friends is given by*

$$\mu_L = \mu_D + \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{(D_i - D_j)^2}{D_i D_j} \right] \tag{6}$$

where D_i is the degree of node i, and $e_{ij} \in \{0, 1\}$ indicates a connection between i and j.

Proof. Let D_i denote the number of connections of individual i, i.e. $D_i = \big|\{k \in V : \quad (i,k) \in E\}\big|$. Denote the set of neighbors of i by $\mathscr{N}(i) = \{k \in V : \quad (i,k) \in E\}$. Define $F_i = \frac{1}{D_i} \sum_{j\in N(i)} D_j$ as the mean number of friends for friends of i. The local mean is defined as:

$$\mu_L = \frac{1}{|V|} \sum_i F_i = \sum_{i\in V} \left[\frac{1}{D_i} \left(\sum_{j\in\mathscr{N}(i)} D_j \right) \right]$$

Rewriting the expression for μ_L in terms of the connections (edges) between individuals, we obtain:

$$\begin{aligned}
\mu_L &= \frac{1}{|V|} \sum_{i\in V} \left[\frac{1}{D_i} \left(\sum_{j\in V} e_{ij} D_j \right) \right] = \frac{1}{|V|} \sum_{i\in V} \sum_{j\in V} \left[e_{ij} \frac{1}{D_i} (D_j) \right] \\
&= \frac{1}{2|V|} \sum_{(i,j)\in V\times V} \left[e_{ij} \left(\frac{D_j}{D_i} \right) + e_{ji} \left(\frac{D_i}{D_j} \right) \right] = \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{D_j}{D_i} + \frac{D_i}{D_j} \right] \\
&= \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{D_j^2 + D_i^2}{D_i D_j} \right] = \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{(D_i - D_j)^2 + 2D_i D_j}{D_i D_j} \right] \\
&= \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{(D_i - D_j)^2}{D_i D_j} \right] + \frac{1}{2|V|} (4|E|) \\
&= \mu_D + \frac{1}{2|V|} \sum_{(i,j)\in V\times V} e_{ij} \left[\frac{(D_i - D_j)^2}{D_i D_j} \right] \quad \square
\end{aligned}$$

$\square$

Note that what we characterize as the local mean defined as above was examined by others including (*6*) etc. and was independently shown to be greater than the mean degree by others (including in an online comment to an article by (*23*), and by (*24*)). However, the properties of the local mean have not been formally examined and characterized.

Theorem S4. *Define the m-th moment of the degree distribution by* $\kappa_m = \frac{1}{N}\sum_{i \in V} D_i^m$*. The local and global means are connected by the following relationship involving the inversity* ρ *and the -1,1,2, and 3rd moments of the degree distribution as follows:*

$$\mu_L = \mu_G + \rho\sqrt{\left(\frac{\kappa_1\kappa_3 - \kappa_2^2}{\kappa_1}\right)\left[\kappa_{-1} - (\kappa_1)^{-1}\right]} \tag{7}$$

Proof. Define the moments of the degree distribution as: $\kappa_m = \frac{1}{N}\sum_i D_i^m$. Since we defined ρ the measure of inversity as the correlation of two distributions that we specify as the origin degree (**O**) and inverse desitnation degree (**ID**) distributions. The **O** distribution consists of the degree of nodes corresponding to edges, and **ID** distribution consists of the inverse degree of nodes corresponding to edges. Thus, each connection (edge) contributes *two* entries to *each* distribution. For example, if there is a connection between i and j, i.e. $e_{ij} = 1$, we would have $\left(D_i, \frac{1}{D_j}\right)$ and $\left(D_j, \frac{1}{D_i}\right)$. Observe that each individual appears in both distributions multiple times based on degree.

Next, we detail the mean and variance of the distributions. First, we consider the means. The mean of the origin distribution is $\mu_O = \frac{1}{2|E|}\sum_i D_i^2 = \frac{\mu_D^2+\sigma_D^2}{\mu_D} = \mu_G = \frac{\kappa_2}{\kappa_1}$. Similarly, the **ID** mean is $\mu_{ID} = \frac{1}{2|E|}\sum_i D_i\left(\frac{1}{D_i}\right) = \frac{1}{\mu_D}$. Next, consider the variances. The variance of the origin distribution (**O**) is computed as:

$$\begin{aligned}
\sigma_O^2 &= \frac{1}{2|E|}\sum_{(i,j)\in E}(D_i-\mu_O)^2 = \frac{1}{2|E|}\sum_{i\in V} D_i\,(D_i-\mu_O)^2 \\
&= \frac{1}{N\mu_D}\sum_{i\in V}\left[D_i^3 - 2\mu_O D_i^2 + (\mu_O)^2 D_i\right] = \frac{\kappa_3}{\kappa_1} - \left(\frac{\kappa_2}{\kappa_1}\right)^2
\end{aligned}$$

Next, we express the corresponding variance of the inverse destination degree distribution (**ID**), σ_{ID}^2. Again, recall that $\frac{1}{D_i}$ does not appear just once, but D_i times. Therefore, we have:

$$\begin{aligned}
\sigma_{ID}^2 &= \frac{1}{2|E|}\sum_{(i,j)\in E}\left[\left(\frac{1}{D_j}-\frac{1}{\mu_D}\right)^2\right] = \frac{1}{2|E|}\sum_{(i,j)\in E}\left(\frac{1}{D_j^2}+\frac{1}{\mu_D^2}-\frac{2}{\mu_D D_j}\right) \\
&= \frac{1}{2|E|}\left[\sum_{(i,j)\in E}\frac{1}{D_j^2}+\frac{1}{\mu_D^2}\left(\sum_{(i,j)\in E}1\right)-\frac{2}{\mu_D}\sum_{(i,j)\in E}\frac{1}{D_j}\right] = \frac{1}{2|E|}\left[\sum_{j\in V}\frac{1}{D_j}+\frac{1}{\mu_D^2}2|E|-\frac{2}{\mu_D}N\right] \\
&= \frac{1}{\mu_D N}\left[\sum_{j\in V}\frac{1}{D_j}\right]-\frac{1}{\mu_D^2} \qquad = (\kappa_1)^{-1}\left[\kappa_{-1}-(\kappa_1)^{-1}\right]
\end{aligned}$$

We next turn to the inversity and based on the definition we connect it to the local and global means and the degree distribution.

$$\begin{aligned}
\rho &= \left(\frac{1}{2|E|\sigma_O\sigma_{ID}}\right)\sum_{(i,j)\in E} e_{ij}\left[(D_i-\mu_O)\left(\frac{1}{D_j}-\frac{1}{\mu_D}\right)\right] \\
(N\mu_D\sigma_O\sigma_{ID})\,\rho &= \left[\sum_{(i,j)\in E} e_{ij}\left(\frac{D_i}{D_j}\right)-\mu_O\left(\sum_{(i,j)\in E}\frac{1}{D_j}\right)-\frac{1}{\mu_D}\sum_{(i,j)\in E}D_i+\sum_{(i,j)\in E}e_{ij}\left(\frac{\mu_O}{\mu_D}\right)\right] \\
&= \left[N(\mu_L)-\mu_O\cdot N-\frac{1}{\mu}\sum_{(i,j)\in E}D_i+\sum_{(i,j)\in E}e_{ij}\left(\frac{\mu_O}{\mu_D}\right)\right] \\
&= \left[(N\mu_L)-N\mu_O-\frac{1}{\mu}\sum_{(i,j)\in E}D_i+2|E|\left(\frac{\mu_O}{\mu_D}\right)\right] \\
\Longrightarrow \mu_L &= \mu_G+\rho\mu_D\,\sigma_O\sigma_{ID}
\end{aligned}$$

Finally, substituting the expressions for the variances, we obtain:

$$\mu_L = \mu_G + \rho\sqrt{\left(\frac{\kappa_1\kappa_3 - \kappa_2^2}{\kappa_1}\right)\left[\kappa_{-1} - (\kappa_1)^{-1}\right]} \tag{8}$$

□

Theorem S5. *The expected degree of nodes chosen by global strategy is the global mean.*

Proof. To determine the expected degree of a node chosen by the global strategy: Choose $M = 1$ node initially, (say X). With probability q, choose each neighbor of X. For a node k with degree D_k, the probability of being chosen by this process is the first step when any of k's friends is chosen as the initial node, and the second step is k being chosen with probability q. This probability is $p_k = \frac{1}{N}\ D_k \times q = \frac{qD_k}{N}$. The expected degree of a chosen "seed" node is then the degree-weighted probability:

$$\frac{\sum_{k\in V}\ p_k D_k}{\sum_{k\in V} p_k} = \frac{\sum_{k\in V} \frac{1}{N} q\ D_k^2}{\sum_{k\in V} \frac{1}{N} q\ D_k} = \frac{\frac{1}{N}\sum_{k\in V} D_k^2}{\frac{1}{N}\sum_{k\in V} D_k} = \frac{\mu_D^2 + \sigma_D^2}{\mu_D} = \mu_G$$

□

Similar logic applies if we choose any arbitrary initial sample of size M as long as the network is large, i.e. $N \gg M$.

Denote an undirected tie (a, b) as a connection between nodes a and b.

Observation. *For any network with a given distribution of degrees, the distribution of degrees is unchanged if any two ties* (a, b) *and* (c, d) *are rewired to either (i)* (a, c),(b, d) *or (ii)* (a, d),(b, c).

Each of these nodes loses one tie and gains another and therefore the degrees are unchanged.

Theorem S6. *[Rewiring Theorem] Let network $G = (V, E)$ with $N > 3$ nodes include nodes a, b, c, d with degrees ordered as: $D_a \leq D_b < D_c \leq D_d$. If G containing edges $(a, b), (c, d) \in E$, but $(a, d), (b, c) \notin E$ is rewired to network $G' = (V, E')$, containing edges $(a, d), (b, c) \in E'$, but $(a, b), (c, d) \notin E'$, then G' has higher local mean than G.*

Proof. First, observe that the degree distribution is unaffected by the change, and therefore the global mean (which only depends on mean and variance of the degree distribution) is also unaffected, i.e. $\mu_G(G) = \mu_G(G')$. Recall that the local mean is $\mu^L = \frac{1}{N}\sum_i \sum_j e_{ij}\left[\frac{D_i}{D_j} + \frac{D_j}{D_i}\right]$. Since between G and G' the degrees of all nodes are the same, and all edges are the same except the two rewired edges, we can write the difference between the local means the local means as:

$$\begin{aligned} \mu^L(G') - \mu^L(G) &= \frac{1}{N}\left[\left(\frac{D_a}{D_d} + \frac{D_d}{D_a} + \frac{D_b}{D_c} + \frac{D_c}{D_b}\right) - \left(\frac{D_a}{D_b} + \frac{D_b}{D_a} + \frac{D_c}{D_d} + \frac{D_d}{D_c}\right)\right] \\ &= \frac{1}{N}\left[(D_d - D_b)\left(\frac{1}{D_a} - \frac{1}{D_c}\right) + (D_c - D_a)\left(\frac{1}{D_b} - \frac{1}{D_d}\right)\right] > 0 \end{aligned}$$

The last inequality follows from the ordering of the node degrees. Note that we actually only require the conditions $D_b < D_d$ and $D_a < D_c$ to hold. $\square$

Theorem S7. *Given a connected network with $|V| = N > 3$ nodes and any non-degenerate degree distribution. To achieve maximum local mean among all networks satisfying the given degree distribution, the nodes with maximum and minimum degree must be connected to each other.*

Proof. We prove this by contradiction. Let the network $G = (V, E)$ have the maximum local mean for the specified degree distribution. Label a and z as the nodes with minimum and maximum degrees in our network. These degrees must be different ($D_a \neq D_z$) in a non-degenerate distribution. Assume a and z are not connected to each other.

There must be a highest degree node z connected to a node y that satisfies the following conditions: (1) y is not directly connected to a, i.e. $(a, y) \notin E$ and (2) $D_a < D_y < D_z$, Note that (1) must be satisfied since $D_z \geq D_a$, and (2) must be satisfied since a and z are lowest and highest degree nodes. Now, we can find a neighbor of a, say b with $D_b < D_z$. Choose a neighbor x of z that is not connected to b. x must exist, otherwise b and z would have the same degree, contradicting the assumption that lowest and highest degree nodes are not connected. Observe that we can increase the local mean by rewiring the network to G' by connecting (a, z) and (b, x) in place of (a, b) and (x, z) as in Theorem S6 above. Thus, network G that we started with could not have had the maximum local mean, and we have a contradiction. Thus, the statement of the theorem must hold. □

Theorem S8. *If the degree distribution is unconstrained, the star network maximizes the local leverage* $\lambda_L = \frac{\mu_L}{\mu_D}$.

Proof. Let δ and Δ be the minimum and maximum degree. Define $f(x, y) = \frac{x}{y} + \frac{y}{x}$. Without loss of generality, assume that $x \geq y$. First, observe that $f(x + 1, y - 1) > f(x, y)$. To prove this, we can express

$$\begin{aligned} f(x+1, y-1) - f(x, y) &= \frac{x+1}{y-1} + \frac{y-1}{x+1} - \left(\frac{x}{y} + \frac{y}{x}\right) \\ &= (x+y)\left[\frac{1}{y(y-1)} - \frac{1}{x(x+1)}\right] > 0 \end{aligned}$$

where the last inequality follows from the assumption $x > y$. Thus, the maximum value of

$f(x, y)$ when $\delta \leq x, y \leq \Delta$ is at $x = \Delta$, $y = \delta$ or $f(\Delta, \delta) = \frac{\Delta}{\delta} + \frac{\delta}{\Delta}$. Observe that the ratio of local mean to mean degree can be expressed as

$$\lambda_L = \frac{\mu_L}{\mu_D} \quad = \quad 1 + \frac{\sum_{i,j \in V} e_{ij} \left[\frac{D_i}{D_j} + \frac{D_j}{D_i} \right]}{\sum_{i,j} e_{ij}}$$

Thus, for each edge, the maximum value of $\left[\frac{D_i}{D_j} + \frac{D_j}{D_i} \right]$ from above is bounded by $\left(\frac{\Delta}{\delta} + \frac{\delta}{\Delta}\right)$, and the maximum local leverage is $\lambda_L^{\max} = 1 + \left(\frac{\Delta}{\delta} + \frac{\delta}{\Delta}\right)$. Observe that expression is maximum when the highest degree node is $\Delta = N - 1$ and is connected to a lowest degree node of degree $\delta = 1$, which implies a star network. Therefore, no network can have higher local leverage than the star network. $\square$

S2 Results from Real Networks

We have obtained a number of real networks to verify and illustrate the friendship paradox. These networks have been chosen to reflect a wide diversity of real-world networked settings, and range from human collaboration networks to biological protein networks to infrastructure networks. These networks also vary widely in terms of their size, from a low of 25 to networks with millions of nodes (e.g. Youtube). All network data was obtained from the Koblenz Network Collection (*25*).

We examine these real networks on a number of dimensions, the number of nodes, edges and the variation in the degree distribution.

Table S2: Real Network Characteristics

Label	Network.Name	Nodes	Edges	Min.Degree	Max.Degree
Collaboration					
A1	Actor-Movie	383640	1470338	1	655
A2	Club Mmebers	25	91	3	20
A3	Citation (Physics)	28045	3148413	1	4909
A4	Citation (CS)	317080	1049865	1	343
Face-toFace Interaction					
FS1	Physician	117	464	2	26
FS2	Adolescent	2539	10454	1	27
FS3	Contact	274	2124	1	101
FS4	Conference	410	2765	1	50
Online Social					
OS1	PGP Users	10679	24315	1	205
OS2	Flickr	105722	2316667	1	5425
OS3	Advogato	5042	40509	1	803
OS4	Twitter	465016	833539	1	677
Topology of Computer Networks					
C1	Internet Topology	34761	107719	1	2760
C2	WWW (Google)	855802	4291352	1	6332
C3	Gnutella P2P	62561	147877	1	95
Infrastructure					
I1	Power Grid	4941	6593	1	19
I2	US Airports	1572	17214	1	314
I3	CA Roads	1957027	2760387	1	12
Infrastructure					
B1	Human Protein 1	2783	6222	1	129
B2	Human Protein 2	5973	146385	1	855
B3	Yeast Protein	1458	1970	1	56
B4	C. Elegans	453	2033	1	237

S2.1 Individual Friendship Paradox

We illustrate this ”individual friendship paradox” using a scatterplot of the node degree versus the average friend degree in Figure S1. Nodes that have a higher degree than their average friends do are colored red, whereas nodes that have lower degree are colored blue. Across most real networks, we observe that the blues vastly outnumber the reds. Relatedly, there are several nodes with low degrees whose friends on average have a high degree.

Inversity & Intervention in Real Networks: Example

We examine data in the India villages networks from (*26*), who collected detailed full census data on the social networks of 75 villages in southern India. The social networks are captured at two different levels of aggregation, first at the level of individuals, so the basic network is comprised of individuals, with connections among them constructed from a wide variety of relationships (kinship, socialization, home visiting, borrowing and lending etc.). They also construct a network of connections across households, so the network nodes are households in the village instead of individuals. Below, we see the mean and standard deviation of the degree distribution across all the villages, for both individual and household networks (Fig. S3).

We find that networks can have either positive or negative inversity depending on how nodes and edges are defined. When nodes as defined as individuals, we find that the networks have positive inversity, whereas if the nodes are defined as households, the inversity of the resulting networks in mostly negative (Fig. S4). Thus, a household-based intervention might use the global strategy, and the individual-based intervention might use the local strategy, implying that the strategy leading to a higher degree node in expectation depends on the level of aggregation.

Thus, we find an illustration of how networks constructed from similar underlying relationships can demonstrate strong divergence in terms of inversity. We illustrate the inversity values across villages in Figure S4 by plotting histograms of the 75 inversity values for both individual networks and household networks.

We observe that for the network of individual ties, inversity for *all* villages is negative, whereas for the network of household ties, except for 1 village, all inversity values are positive. Thus, even networks obtained from similar underlying relationships can result in dramatically different inversity characteristics. Equally importantly, this finding suggests that a local strategy intervention as above, designed based on the individual level network could be altered to leverage the global strategy when dealing with household-based interventions.

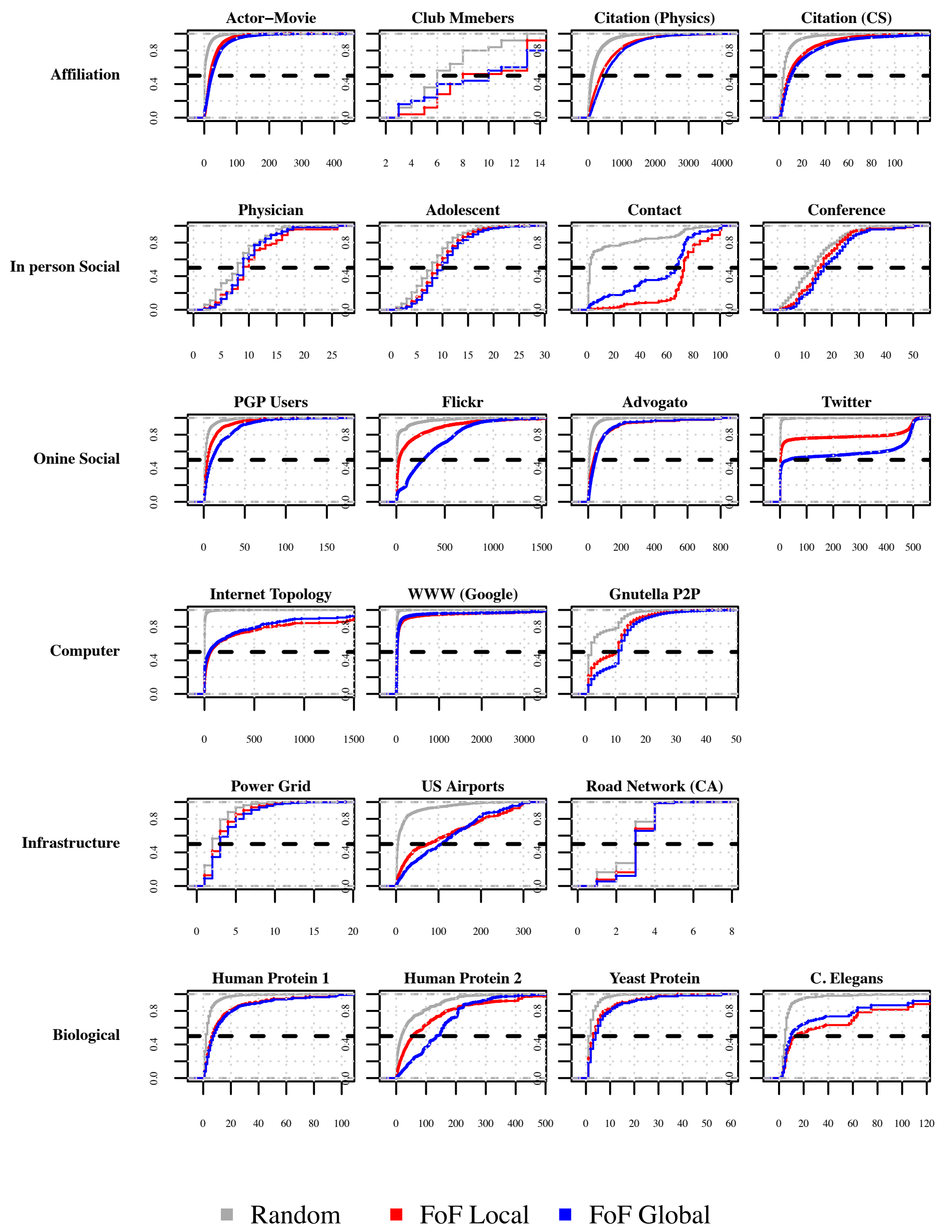

Figure S1: Individual Friendship Paradox. Empirical Cumulative Distribution Functions (CDF) of Real Networks. Panels show the CDF of 3 different network properties at the individual node level. For a specific node degree, the probability that a node with a lower (or identical) degree is chosen by the sampling strategy for random sampling (gray), local FoF sampling (red) and global FoF sampling (green). Across all networks, for lower degrees, the random sampling curve is to the left of the local and global FoF curves. In several networks, global FoF is to the left and higher than local FoF (e.g. Contact), whereas in others, it is to the right (e.g. Flickr).

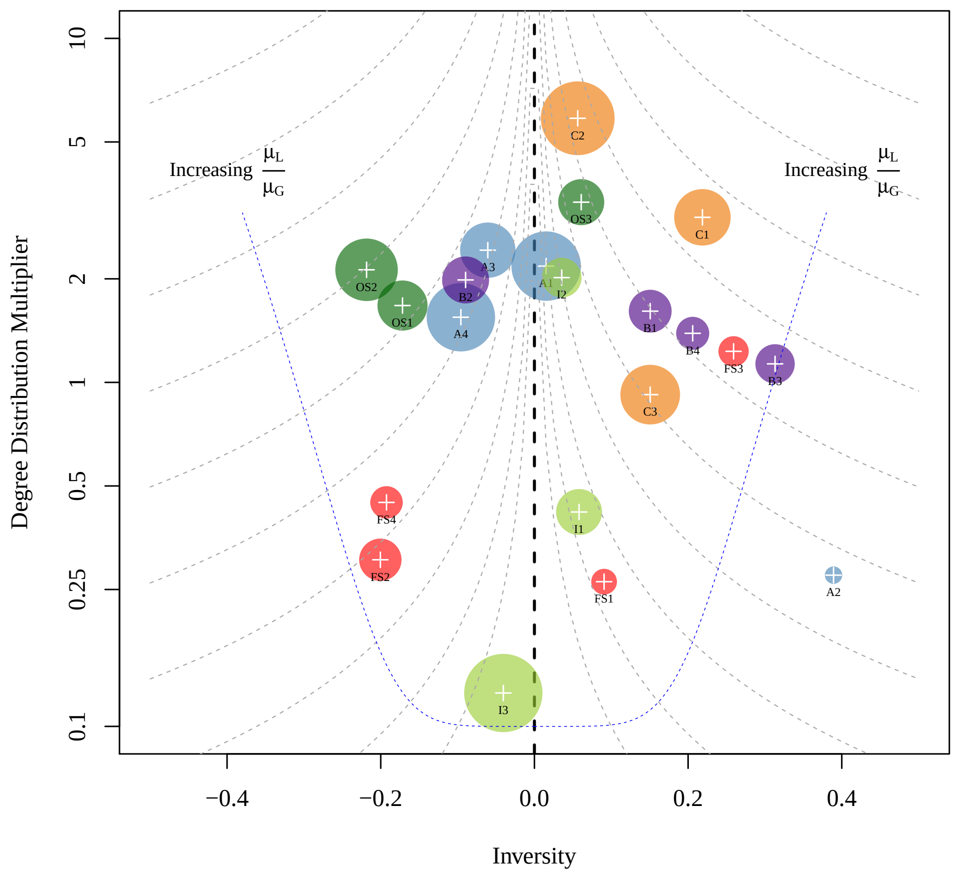

Figure S2: Source of Differences between Local and Global Mean Local and Global Means can be different due to both local structure and global degree distribution. Networks are represented by circles (color indicates category and area indicates size in log scale). Dashed gray lines are iso-curves indicating points with the same ratio of local to global mean. Differences between local and global mean increase when moving up and away from the center in the graph. Networks B4 and C2 have similar ratio of local to global means; however, for B4, this results from a higher local factor (inversity), whereas for C2, the local factor is lower in magnitude, but the impact is magnified by a high degree distribution multiplier.

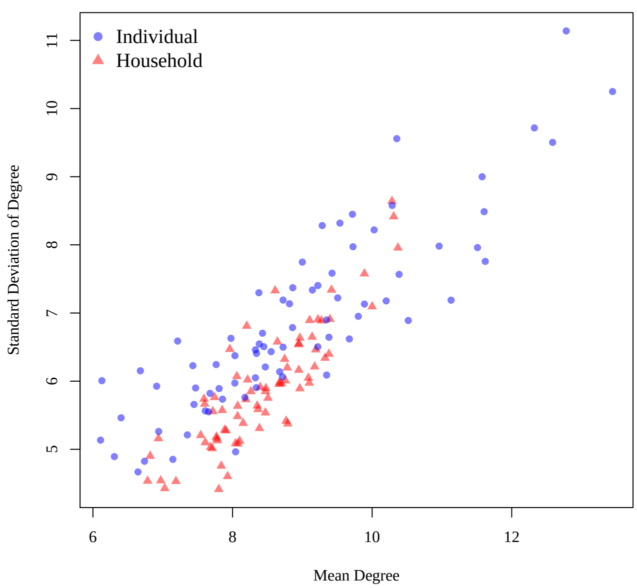

Figure S3: India Village Networks: Degree Distribution. For N=75 villages in South India, the degree mean and standard deviations show the characteristics of the village networks at two levels of aggregation: individual to individual connections and household to household connection. The degree distributions show some variation, but we see overlap between the two levels of aggregation.

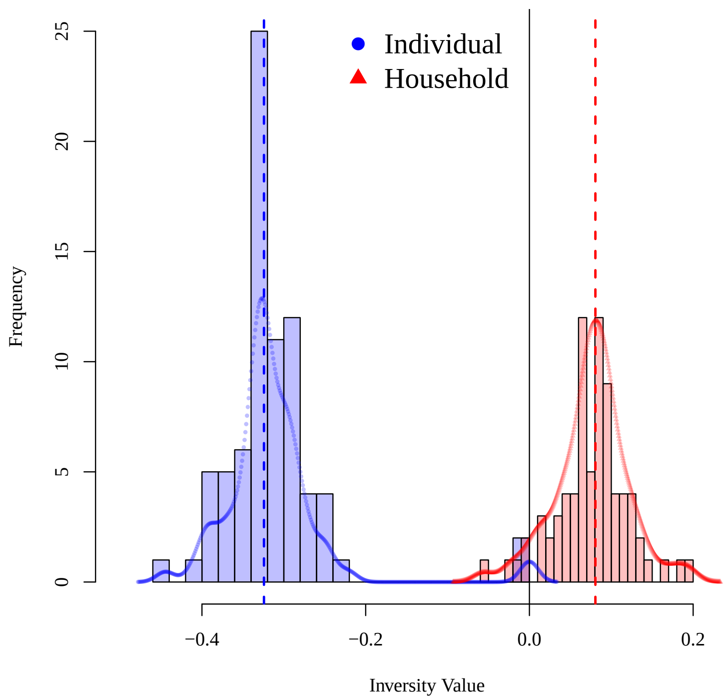

Figure S4: Inversity in India Village Networks. Inversity values depend strongly on how the network structure is aggregated. We observe positive inversity values across most of the networks when consider individual-to-individual ties, but negative inversity values when we consider household-to-household ties. Note that the household-level ties are aggregated from the individual-level ties.