Nonparametric Bandits Leveraging Informational Externalities to Learn the Demand Curve

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  - How can we incorporate (simple) economic theory?
  - What is the benefit?

The Problem - Learning Unknown Demand Curves

- In 2016, the Atlanta Falcons dramatically slashed concessions prices to improve brand equity. When asked how they projected the volume of sales to change, the CEO of ownership, Steve Cannon replied, "It could be a **10% bump**, it could be a **30% bump**, who knows." The next season sales volume for food **rose 50%**.

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Solution?

Experimentation

But... Companies are reluctant to experiment with price

Column: Why Businesses Don't Experiment

A debate ensued among the group: Are we willing to sacrifice some customers "just" to learn how the new pricing approaches work?

They hedged. They asked me what *I* thought the best approach was. I told them that I was willing to share my intuition but that intuition is a remarkably bad thing to rely on. Only an experiment gives you the evidence you need. In the end, it wasn't enough to convince them, and they called off the project.

Issues

- Potential short term losses for uncertain long term gains
- May confuse or alter consumers' expectations

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Can incorporating theory improve learning with an **unknown** demand curve?

Desired Features For Price Experimentation

Goal

Maximizing earning while learning

- Maximize long run profits while minimizing costs of experimentation

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Desired Features

- Be able to test many prices
- No parametric form for demand curve assumed
- Adaptively set price based on incoming experimentation data

Roadmap

Current Approach

- Multi-armed Bandits or Reinforcement Learning

Experimentation Setting

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- Combine bandits with human domain knowledge about pricing (i.e. incorporate basic economic theory)

Reinforcement Learning

Definition of RL

Reinforcement learning (RL) is a type of machine learning where an "agent" learns optimal behavior through interaction with its environment. Rather than relying on explicit programming or labeled datasets, this agent learns by trial and error, receiving feedback in the form of rewards or penalties for its actions. – Google on Reinforcement Learning

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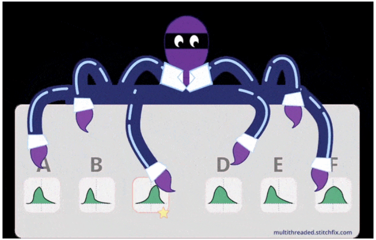
Introducing... Multi-Armed Bandits

- Each arm represents a choice



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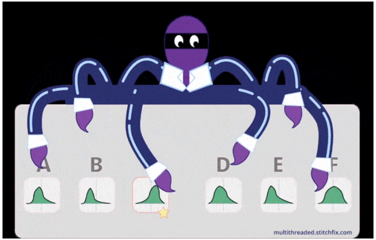

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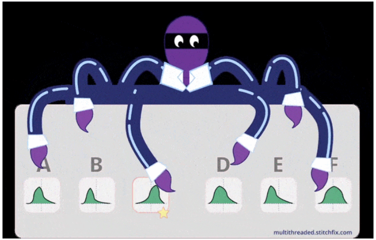

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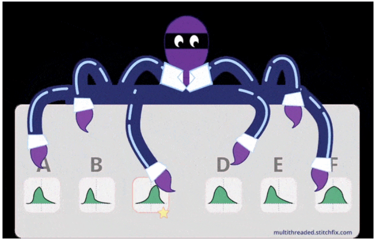

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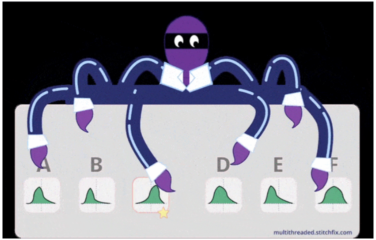

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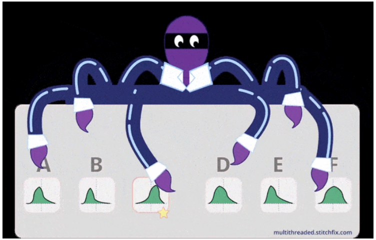
- Belongs to class of reinforcement learning problems



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Multi-Armed Bandits in Marketing

Use Case	Objective (maximize)	Examples of Arms	Arm A	Arm B	Arm C
Advertisements	Conversions	Ads	Emotional	Informative	Funny
Recommendation Systems	Purchases	Movies	E.T.	Indiana Jones	Rain Man
Pricing	**Profits**	**Prices**	**\$1**	**\$5**	**\$20**

Extra-Terrestrial? Economic Theory?

Why is Pricing Different?

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Goal

Leverage informational externalities within MAB framework to improve learning

Contributions and Features

Contribution

Develop new principled approach that incorporates informational externalities derived from theory with nonparametric bandits

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Features

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- Provides an automatic approach with no human judgement required
- Runs in real time

Multi-armed

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Baseline: Start with standard multi-armed Bandit methods (**UCB** and **TS**)

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  - We propose **new sampling method** to obtain monotonic draws from the GP

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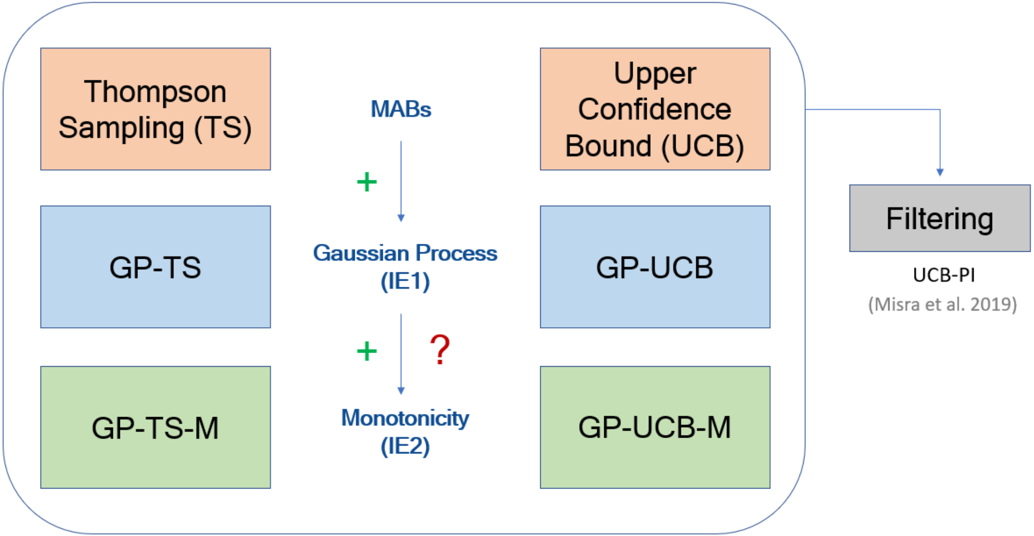
  - y variable: purchase probability
  - x variable: price
- Information from posterior GP scaled by price accordingly (GP-UCB / GP-TS)

Building Blocks

- Decision Rule for Picking Arm to Experiment: UCB, TS
- Gaussian Process can be used to flexibly model any nonparametric demand curve
    - Monotonicity restricts the set of functions

Overview of MABs and Informational Externalities



Overview of GPs

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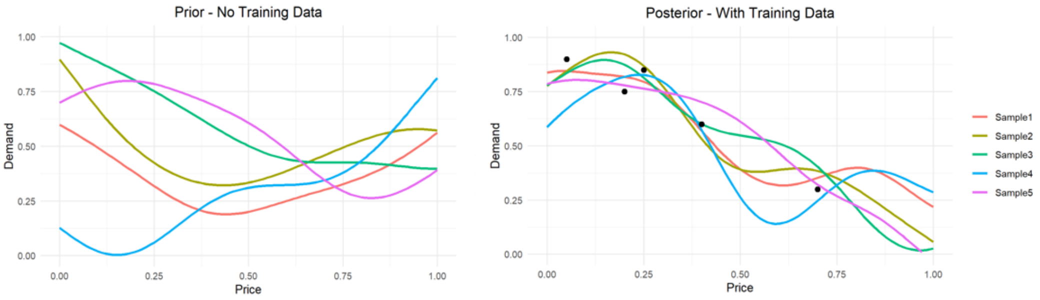

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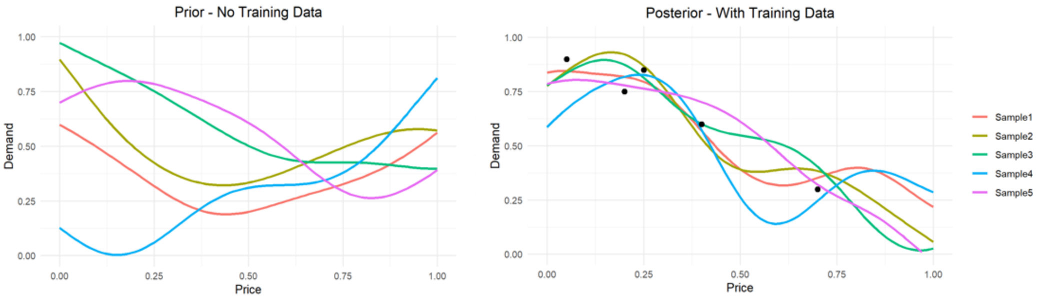

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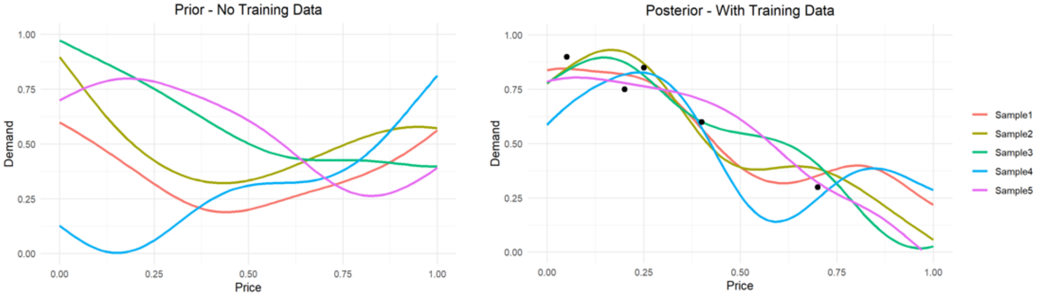
- Can **sample an entire demand curve** at once from GP



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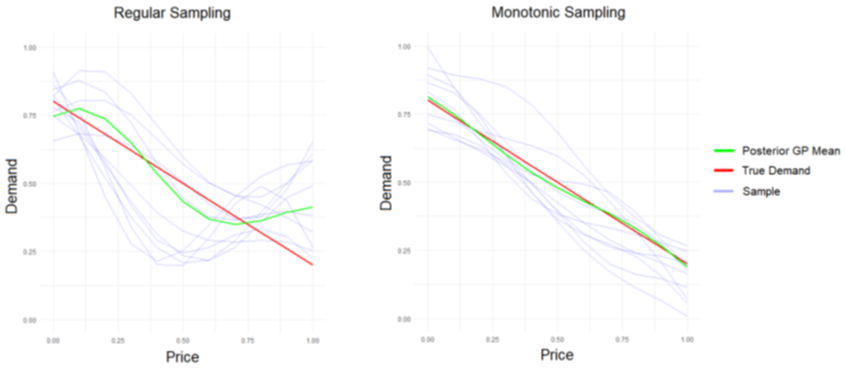
- Need way to obtain a random monotonically decreasing draw from the posterior GP
- Not entirely clear how to do so

Monotonic Sampling



How do we obtain a random monotonic draw from the GP in a principled way?

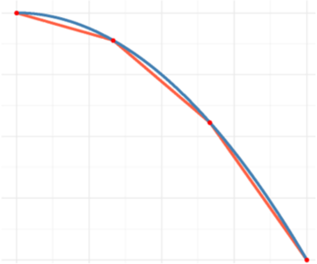
Proposed Sampling Approach (I)

Proposed Approach

Any function can be estimated by linearly interpolating between knot points

- Converges to true curve as number of knots approaches infinity

$$D(\cdot) \approx \sum_{j=0}^{N} D(\mu_j) h_j(\cdot)$$



Similarly, any function can be estimated by its intercept and first derivatives at knots

$$D(p) \approx \sum_{j=0}^{N} D'(\mu_j) \int_0^p h_j(t)dt$$

Proposed Sampling Approach (II)

Proposed Approach

[1]We use the *TruncatedNormal* package in R

Proposed Sampling Approach (II)

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- Demand function can be estimated in terms of its intercept and derivatives at various knots (prices)
- Possible because the sum and derivative of a GP is also a GP

Why would we want to do this?

- Transforms problem from one of obtaining a monotonic draw to one of obtaining negative draws

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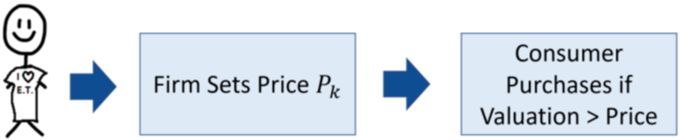
- Monotonically decreasing = all negative derivatives
- Easier sampling problem (that of sampling from a Truncated Normal[1])

[1] We use the *TruncatedNormal* package in R

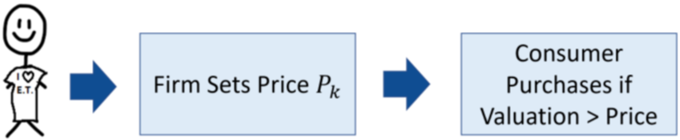
Algorithm Evaluation – Setup



- Underlying WTP distribution is chosen

[2] Follows setup from Misra et al. (2019)

Algorithm Evaluation – Setup



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- Prices are normalized between 0 and 1

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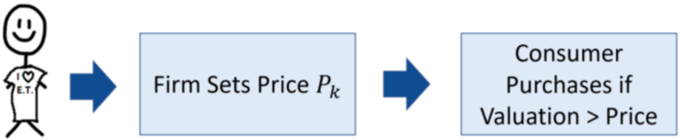

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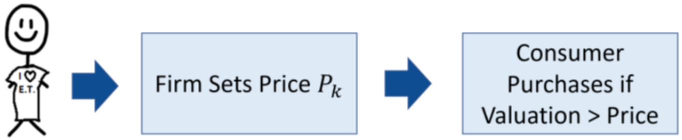

- Underlying WTP distribution is chosen
- Prices are normalized between 0 and 1
- 3 different price sets are tried
  - 5 arms: $\{0.1, 0.3, 0.5, 0.7, 0.9\}$

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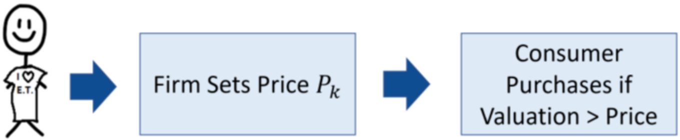
Algorithm Evaluation – Setup



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- Prices are normalized between 0 and 1
- 3 different price sets are tried
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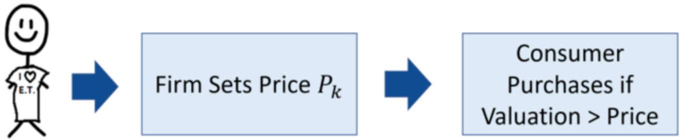
  - 10 arms: $\{0.1, 0.2, ..., 1\}$

[2] Follows setup from Misra et al. (2019)

Algorithm Evaluation – Setup



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  - 5 arms: $\{0.1, 0.3, 0.5, 0.7, 0.9\}$
  - 10 arms: $\{0.1, 0.2, ..., 1\}$
  - 100 arms: $\{0.01, 0.02, ..., 1\}$

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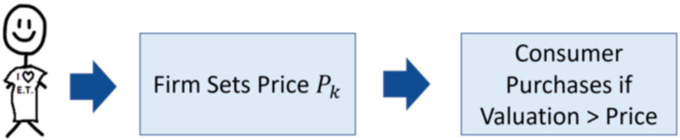
Algorithm Evaluation – Setup



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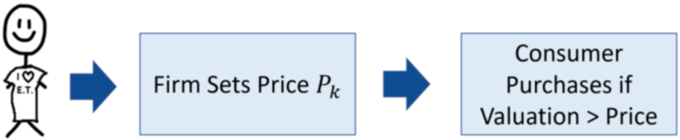
Algorithm Evaluation – Setup



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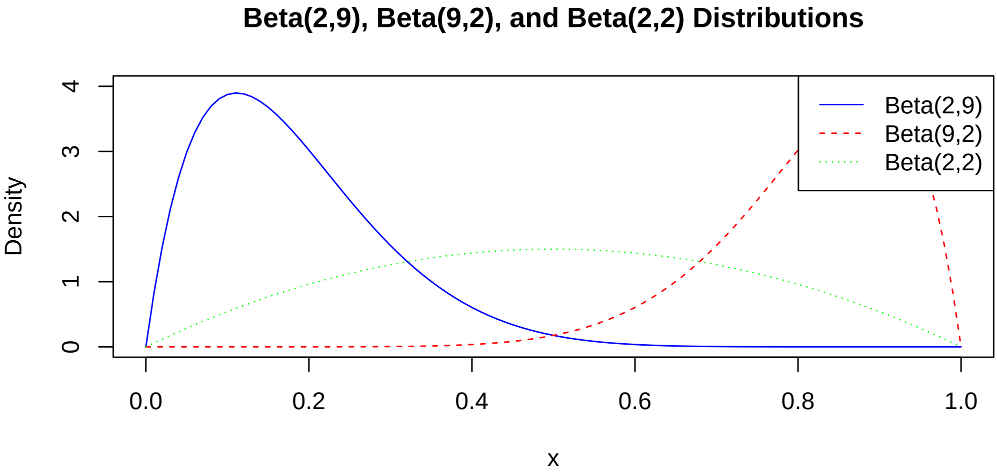
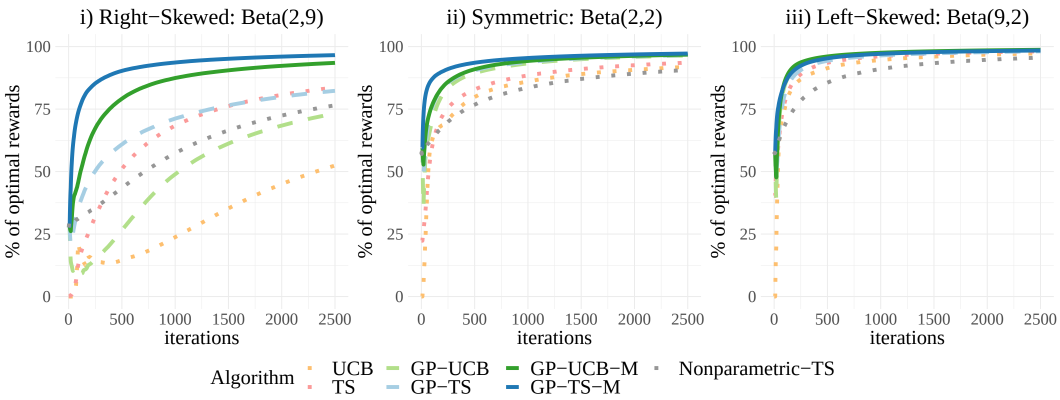
- Algorithm is updated with purchase decision every 10 customers[2]
- Results are averaged across 1000 separate runs

[2] Follows setup from Misra et al. (2019)

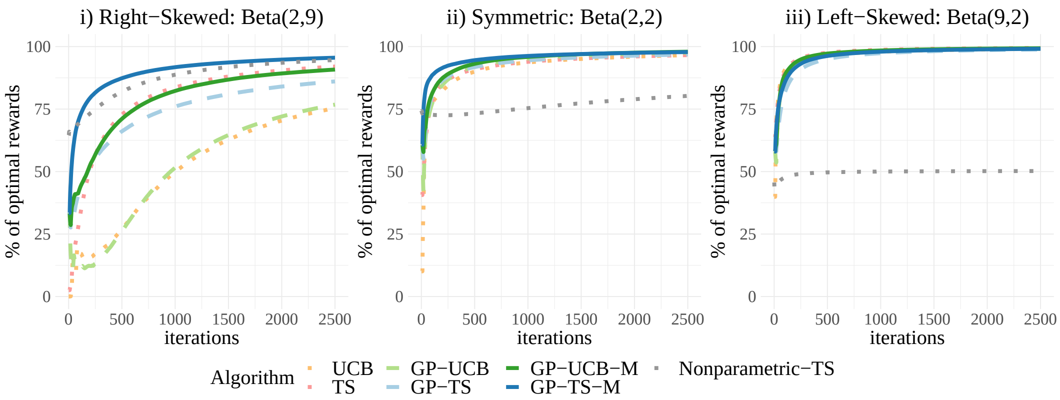
Distributions



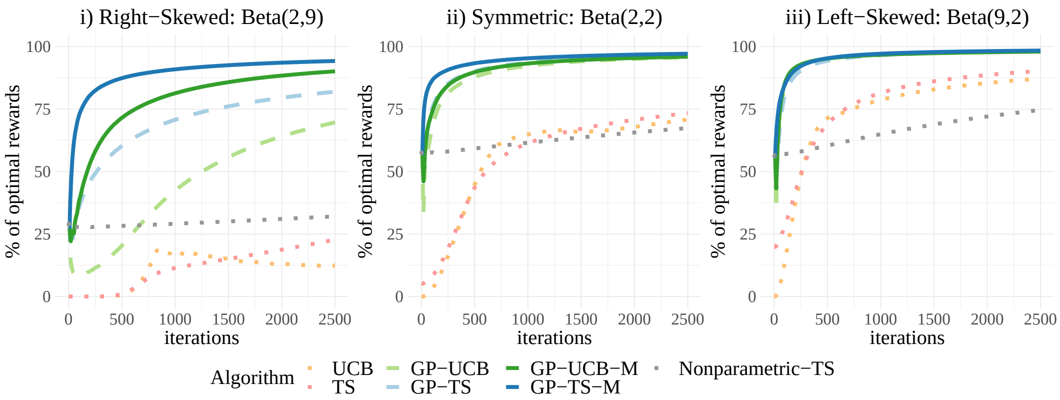
Rewards - 10 Arms



Rewards - 5 Arms



Rewards - 100 Arms



Performance Uplift from Informational Externalities

2500 consumers

After 2500 Consumers

		TS			UCB		
		5 Arms	10 Arms	100 Arms	5 Arms	10 Arms	100 Arms
Uplift from 1st externality (GP compared to base algos)	**B(2,9)**	-6.4% (-6.5, -6.2)	-1.6% (-1.9, -1.3)	262% (260, 264)	2.2% (1.9, 2.4)	40.4% (39.7, 41.1)	464% (462, 467)
	B(2,2)	0.3% (0.1, 0.4)	3.4% (3.2, 3.5)	31.4% (31.2, 31.6)	1.0% (0.9, 1.1)	5.0% (4.8, 5.1)	35.2% (35.0, 35.4)
	B(9,2)	-0.5% (-0.6, -0.5)	0.0% (0.0, 0.1)	8.8% (8.7, 8.9)	0.1% (0.0, 0.1)	1.3% (1.3, 1.4)	12.5% (12.4, 12.6)
	Mean	**-2.1% (-2.2, -2.0)**	**0.6% (0.5, 0.8)**	**48.4% (48.2, 48.6)**	**1.0% (0.9, 1.1)**	**11.1% (11.0, 11.3)**	**54.7% (54.5, 54.9)**
Uplift from 2nd externality (GP-M compared to GP algos)	**B(2,9)**	10.9% (10.6, 11.1)	17.4% (17.1, 17.8)	14.9% (14.6, 15.2)	18.0% (17.6, 18.5)	27.6% (27.1, 28.1)	29.4% (28.9, 30.0)
	B(2,2)	1.0% (0.9, 1.1)	0.7% (0.5, 0.8)	0.9% (0.7, 1.0)	0.4% (0.3, 0.5)	0.3% (0.2, 0.5)	0.3% (0.1, 0.4)
	B(9,2)	0.2% (0.2, 0.3)	0.3% (0.2, 0.3)	0.3% (0.2, 0.3)	0.1% (0.1, 0.2)	0.2% (0.1, 0.2)	-0.1% (-0.1, 0.0)
	Mean	**3.7% (3.6, 3.8)**	**5.5% (5.4, 5.6)**	**4.8% (4.7, 4.9)**	**5.3% (5.1, 5.4)**	**7.7% (7.5, 7.8)**	**7.8% (7.7, 7.9)**
Uplift from both externalities (GP-M compared to base algos)	**B(2,9)**	3.8% (3.6, 4.0)	15.4% (15.2, 15.6)	315% (313, 318)	20.5% (20.0, 21.0)	78.9% (78.1, 79.7)	629% (627, 632)
	B(2,2)	1.3% (1.1, 1.4)	4.0% (3.9, 4.2)	32.5% (32.3, 32.7)	1.4% (1.3, 1.6)	5.3% (5.1, 5.5)	35.6% (35.4, 35.8)
	B(9,2)	-0.3% (-0.4, -0.3)	0.3% (0.2, 0.4)	9.0% (8.9, 9.1)	0.2% (0.2, 0.3)	1.5% (1.4, 1.5)	12.4% (12.3, 12.5)
	Mean	**1.5% (1.4, 1.6)**	**6.2% (6.1, 6.3)**	**55.5% (55.4, 55.7)**	**6.3% (6.2, 6.4)**	**19.6% (19.5, 19.8)**	**66.8% (66.6, 66.9)**

1st Externality

- Uplift increases with number of arms
- Magnitude of uplifts largest for right-skewed

2nd Externality

- Consistent uplifts regardless of number of arms
- Large uplift for right-skewed, small for symmetric and left-skewed

Performance Uplift from Informational Externalities

500 consumers

		After 500 Consumers					
		TS			UCB		
		5 Arms	10 Arms	100 Arms	5 Arms	10 Arms	100 Arms
Uplift from 1st externality (GP compared to base algos)	B(2,9)	-8.0% (-8.6, -7.3)	20.6% (19.2, 22.1)	5940% (5840, 6040)	2.0% (0.0, 4.1)	92.1% (86.6, 97.6)	2720% (2650, 2790)
	B(2,2)	1.0% (0.6, 1.3)	9.4% (9.0, 9.8)	108% (107, 109)	2.4% (2.0, 2.7)	12.6% (12.1, 13.1)	96.6% (96.0, 97.2)
	B(9,2)	-2.4% (-2.5, -2.2)	0.8% (0.6, 1.0)	36.8% (36.6, 37.1)	0.1% (0.0, 0.3)	4.6% (4.5, 4.8)	32.7% (32.6, 32.8)
	Mean	**-2.8% (-3.1, -2.6)**	**8.2% (7.8, 8.5)**	**116% (115, 117)**	**0.9% (0.7, 1.2)**	**14.3% (13.9, 14.7)**	**73.9% (73.4, 74.4)**
Uplift from 2nd externality (GP-M compared to GP algos)	B(2,9)	31.5% (30.5, 32.5)	50.1% (48.4, 51.8)	45.5% (44.1, 47.0)	176% (170, 182)	237% (222, 252)	296% (281, 310)
	B(2,2)	2.9% (2.6, 3.2)	3.5% (3.2, 3.9)	3.4% (3.1, 3.8)	1.3% (1.0, 1.6)	1.4% (1.0, 1.8)	2.1% (1.6, 2.5)
	B(9,2)	1.0% (0.9, 1.2)	1.1% (1.0, 1.2)	1.1% (0.9, 1.2)	0.6% (0.5, 0.7)	0.5% (0.4, 0.6)	0.5% (0.4, 0.5)
	Mean	**9.6% (9.3, 9.8)**	**13.7% (13.3, 14.0)**	**12.6% (12.3, 12.9)**	**21.5% (21.1, 21.9)**	**25.6% (25.1, 26.0)**	**26.3% (25.9, 26.7)**
Uplift from both externalities (GP-M compared to base algos)	B(2,9)	20.4% (19.6, 21.3)	78.5% (76.9, 80.0)	8610% (8470, 8740)	174% (168, 179)	468% (457, 479)	975% (970, 980)
	B(2,2)	3.8% (3.5,4.1)	13.2% (12.7, 13.6)	115% (114,116)	3.6% (3.2, 3.9)	14.1% (13.6, 14.5)	100% (100, 101)
	B(9,2)	-1.4% (-1.5, -1.2)	1.9% (1.7, 2.1)	38.3% (38.0, 38.6)	0.8% (0.7, 0.9)	5.2% (5.0, 5.3)	33.3% (33.2, 33.4)
	Mean	**6.4% (6.1, 6.6)**	**22.9% (22.5, 23.2)**	**143% (143, 144)**	**22.6% (22.1, 23.0)**	**43.3% (42.9, 43.7)**	**119% (119, 120)**

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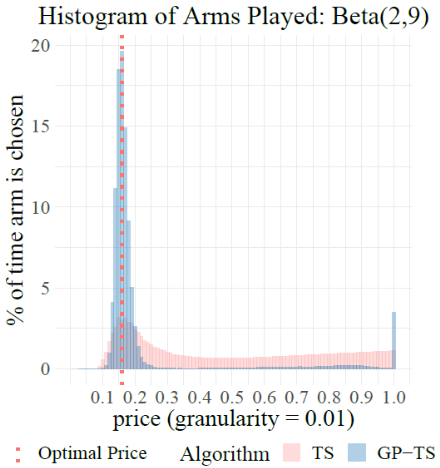
Explanation - 1st Externality

The first externality becomes more important as the number of arms increases

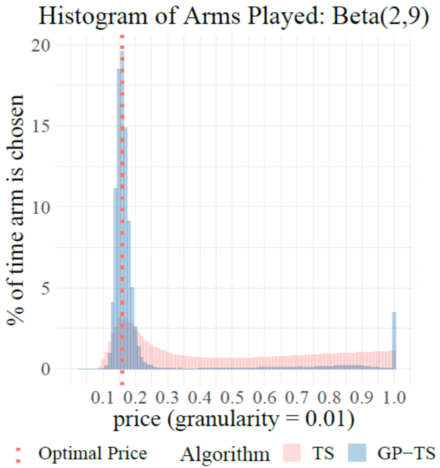
- Without considering correlation, an algorithm has to test each arm individually



Explanation - 1st Externality

The first externality becomes more important as the number of arms increases

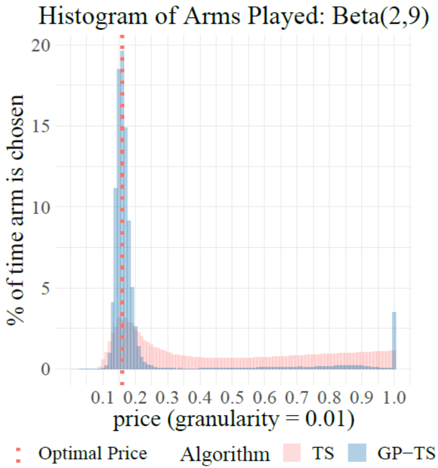
- Without considering correlation, an algorithm has to test each arm individually
- As the number of arms increases, learning is spread too thin



Explanation - 1st Externality

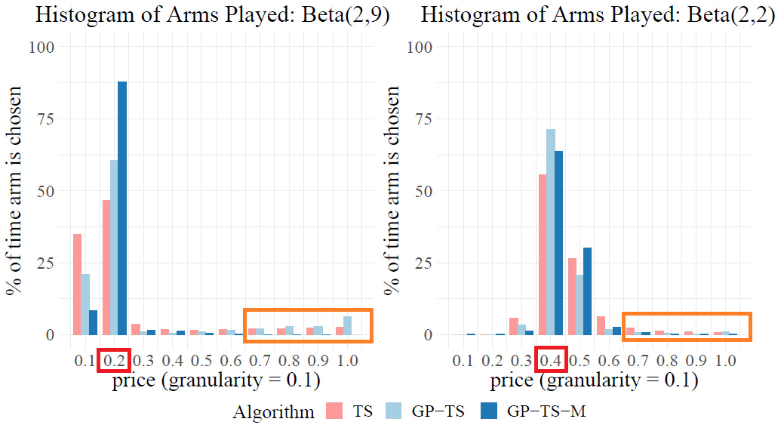
The first externality becomes more important as the number of arms increases

- Without considering correlation, an algorithm has to test each arm individually
- As the number of arms increases, learning is spread too thin
- Learning across arms reduces this problem allowing algorithm to narrow in on best arms more quickly



Explanation - 2nd Externality

- Generally, bandit algorithms tend to over-explore higher prices

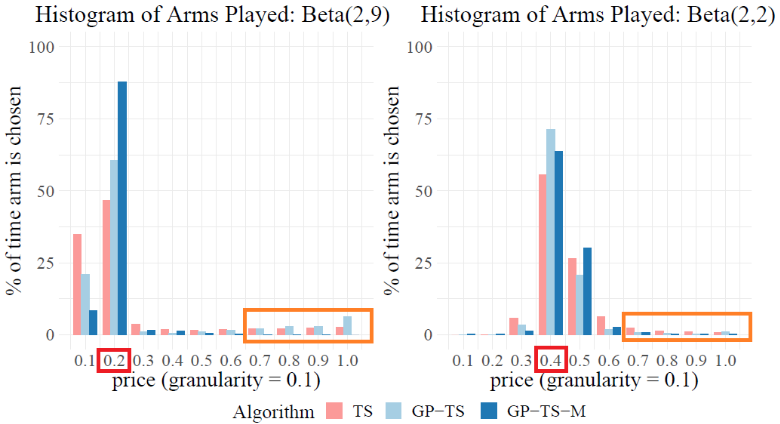


- For Beta(2,9), including monotonicity helps remove the choice of many low performing high prices (orange box)

- For Beta(2,2), the effect is largely mitigated because other algorithms are already learning quickly to move on from these low performing high prices (orange box)

Explanation - 2nd Externality

- Generally, bandit algorithms tend to over-explore higher prices
- When the optimal price is low, including monotonicity allows many 'exploitations' from erroneous upward sloping curves to be eliminated



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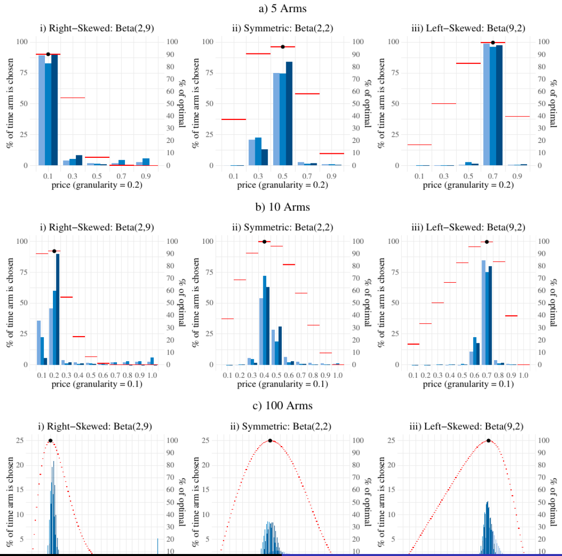
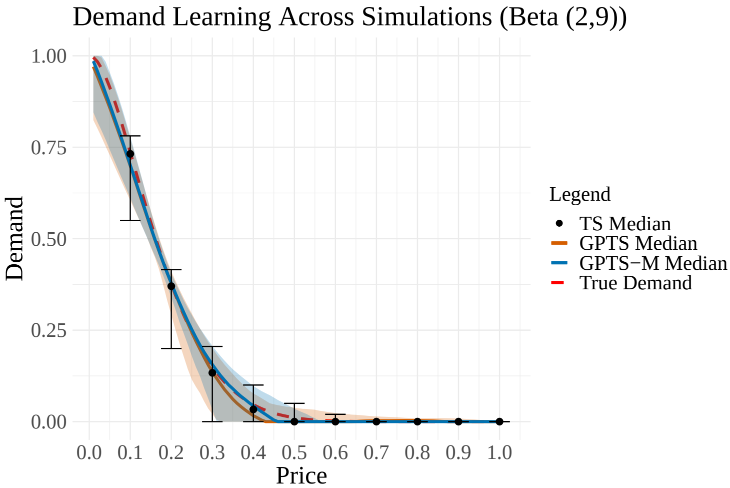
Mechanism and Explanation



Comparison of Learning Across Simulations



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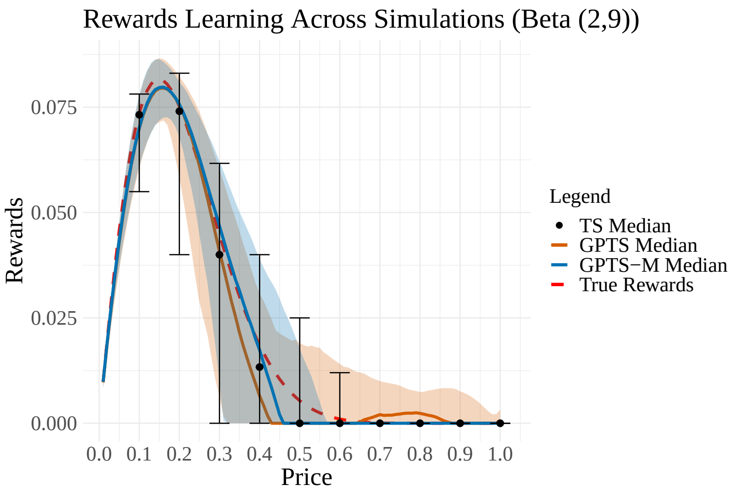

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- Incorporate "Demand Curves are continuous and downward sloping" into Reinforcement Learning (MAB) model
    - Want to retain flexibility of ML models (Nonparametric, incredible flexible forms)
- Theory $\implies$ **Informational externalities** across arms
- Incorporating Theory into ML models requires careful modeling but has significant benefits in improving ML
- Learns much more *efficiently* $\implies$ Experimentation ($\Downarrow$) $\implies$ Practical value

Thank You!

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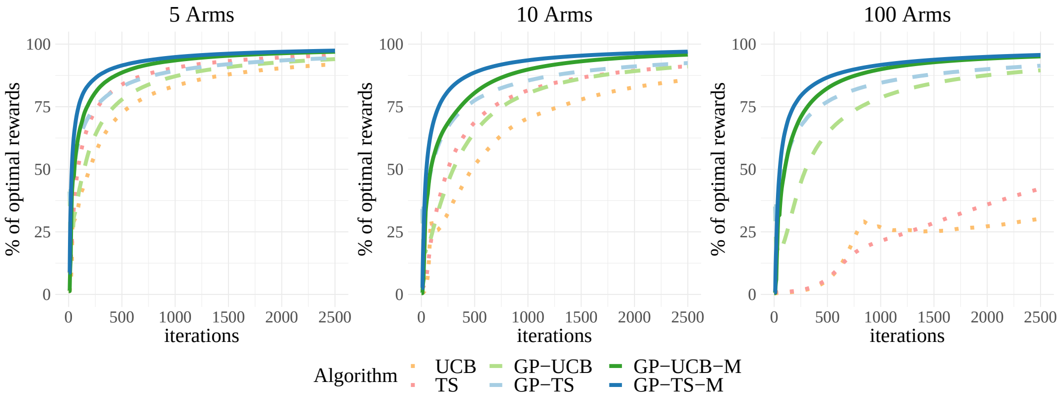
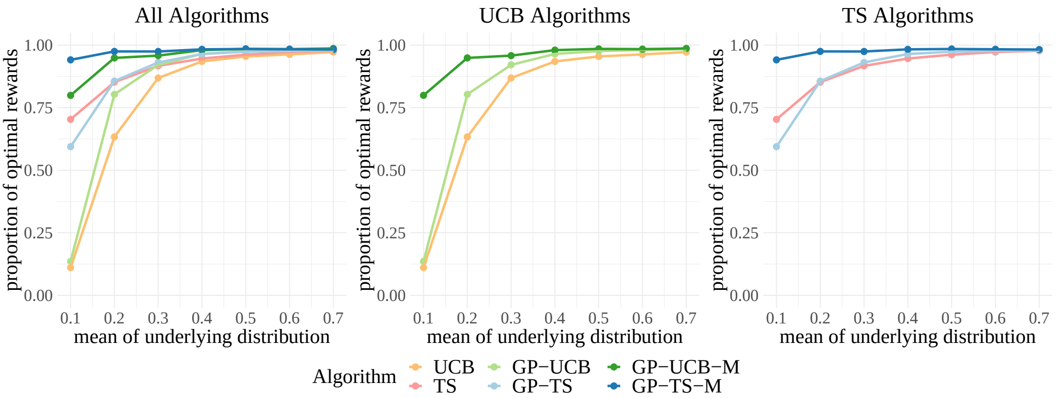
ADDITIONAL

Field Data - Replication



Fixed Variance - Altering the Mean of Underlying Distribution



The difference in performance shrinks as the mean gets higher