

Generative Interpretable Visual Design

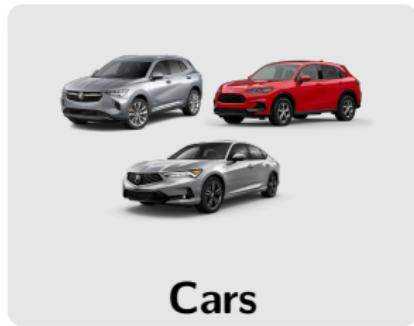
Application to Visual Conjoint

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Yale School of Management

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Visual (or aesthetic) design matters across many product categories . . .



Cars

Visual (or aesthetic) design matters across many product categories . . .



Cars



Fashion

Visual (or aesthetic) design matters across many product categories . . .



Cars



Fashion



Furniture

...even for mundane categories like yogurt



"We worked hard to get the packaging right ... American yogurt has always been sold in containers with relatively narrow openings. In Europe yogurt containers are wider and squatter, and that's what I wanted for Chobani."

—Hamdi Ulukaya, Founder & CEO, Chobani

Visual design matters



Visual design matters



“Exterior look/design is the top reason shoppers avoid a particular vehicle (30%), followed by cost (17%).”

—JD Power Avoider Study 2015

What this paper seeks to do

Research Goals

Our research aims to obtain **interpretable** visual characteristics (not surprising / outlier) directly from unstructured product images

- *automatically discover (extract) characteristics*

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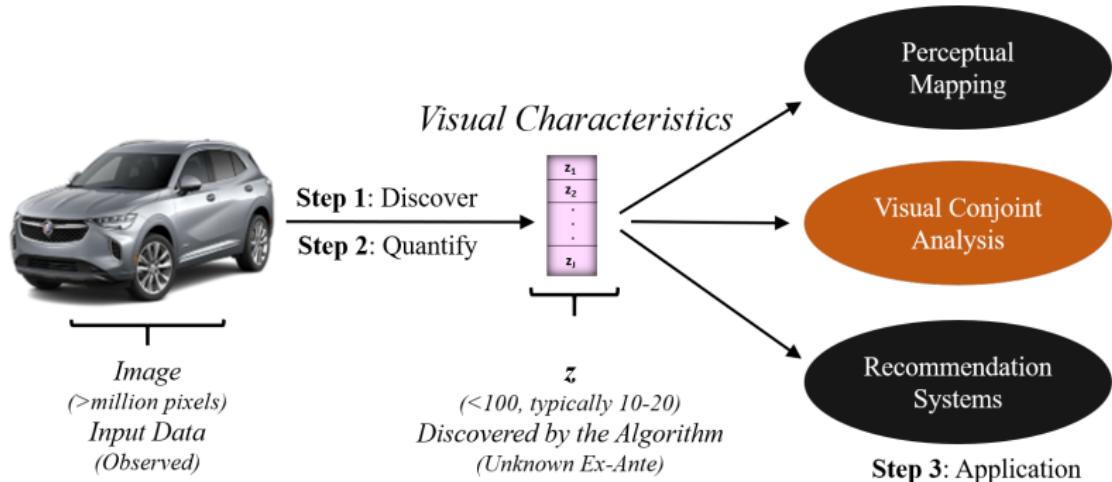
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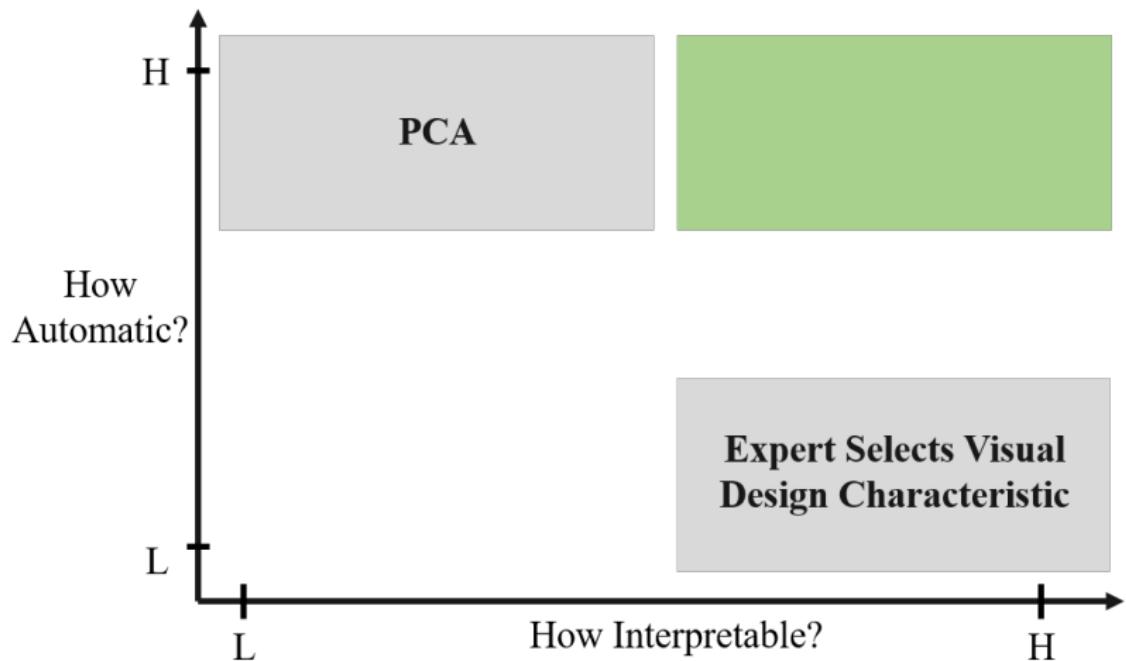


Hyundai: (3, 8, 5, 9) compared to BMW: (1, 3, 10, 1)

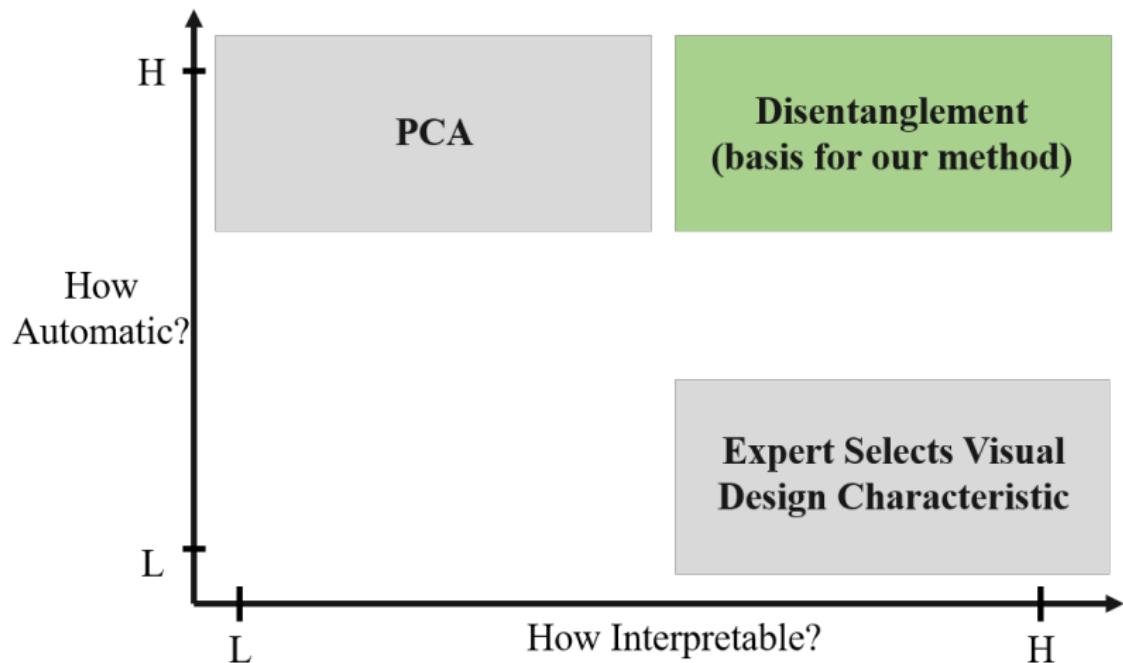
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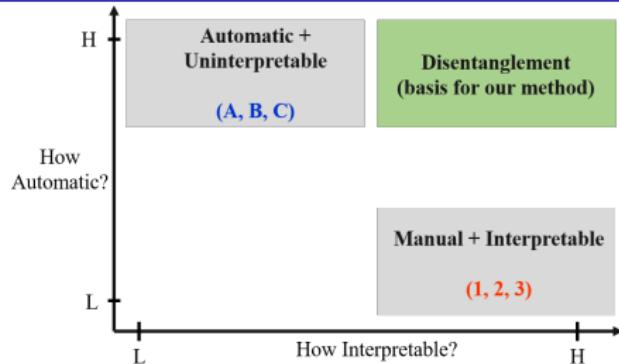
Modeling Visual Characteristics: A comparison of methods



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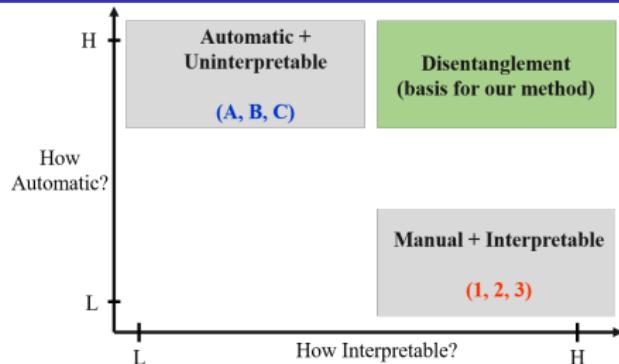
Modeling Visual Characteristics: A comparison of methods



Automatic + Uninterpretable

- A - Bajari, P. L. et al. (2021) : Hedonic prices and quality adjusted price indices powered by AI, *CENMAP working paper*
- B - Law, S., et al. (2019) : Take a look around: using street view and satellite images to estimate house prices. *ACM Transactions on Intelligent Systems and Technology (TIST)*
- C - Aubry, S., et al. (2019) : Machine learning, human experts, and the valuation of real assets. *CFS Working Paper Series*

Modeling Visual Characteristics: A comparison of methods



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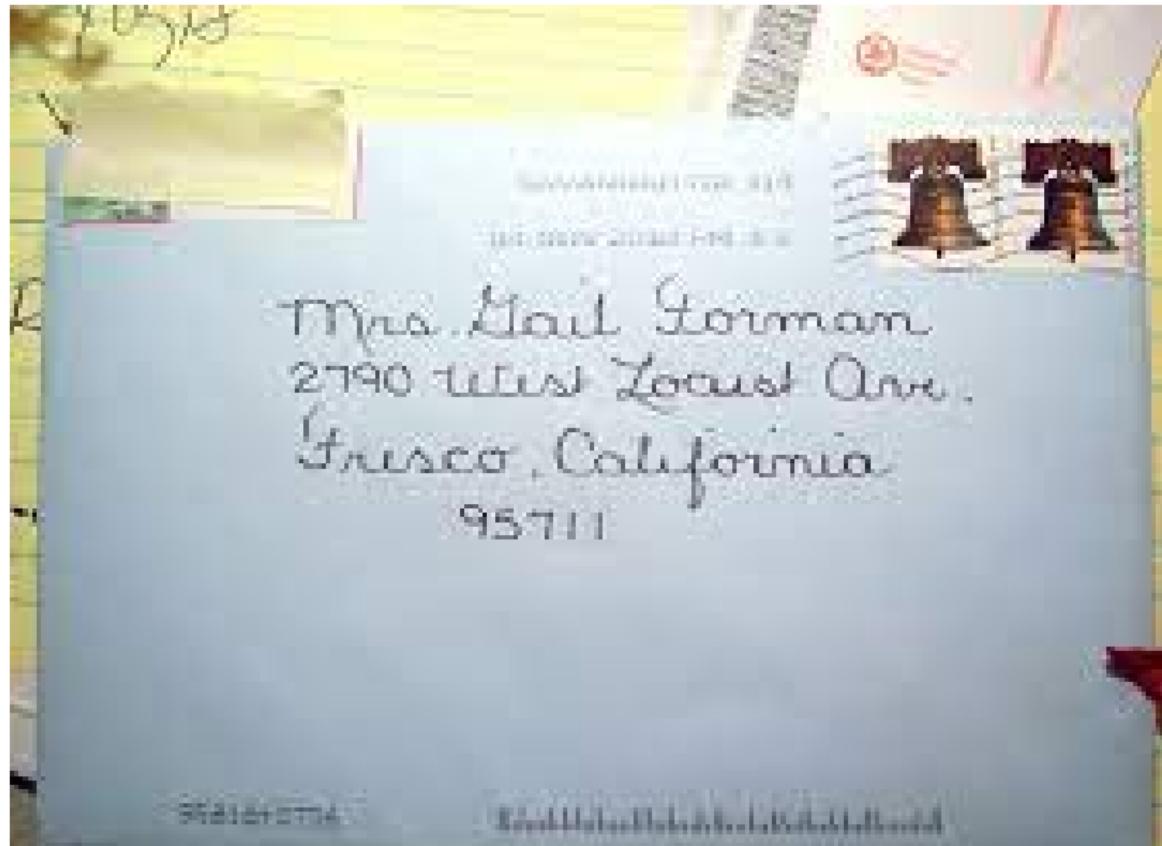
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Manual + Interpretable

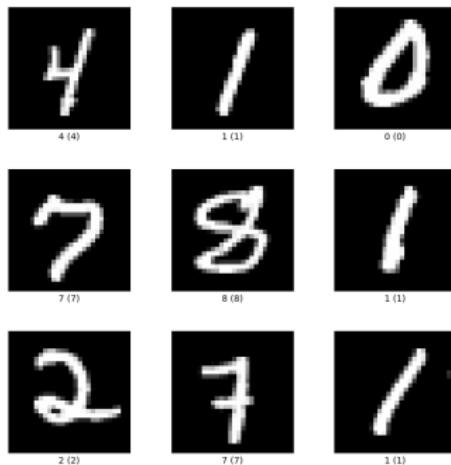
- 1 - Zhang, M. et al. (2022) : Can consumer-posted photos serve as a leading indicator of restaurant survival? Evidence from yelp. *Management Science*
- 2 - Liu, Y., et al. (2017) : The effects of products' aesthetic design on demand and marketing-mix effectiveness: The role of segment prototypicality and brand consistency. *Journal of Marketing*
- 3 - Zhang, S., et al. (2021) : What makes a good image? Airbnb demand analytics leveraging interpretable image features. *Management Science*



Is Human Interpretability always necessary?



Is Human Interpretability always necessary?



What is disentanglement?

Bengio et al (2013)

*"A disentangled representation can be defined as one where **single latent units** are sensitive to changes in **single generative factors**, while being relatively invariant to changes in other factors"*

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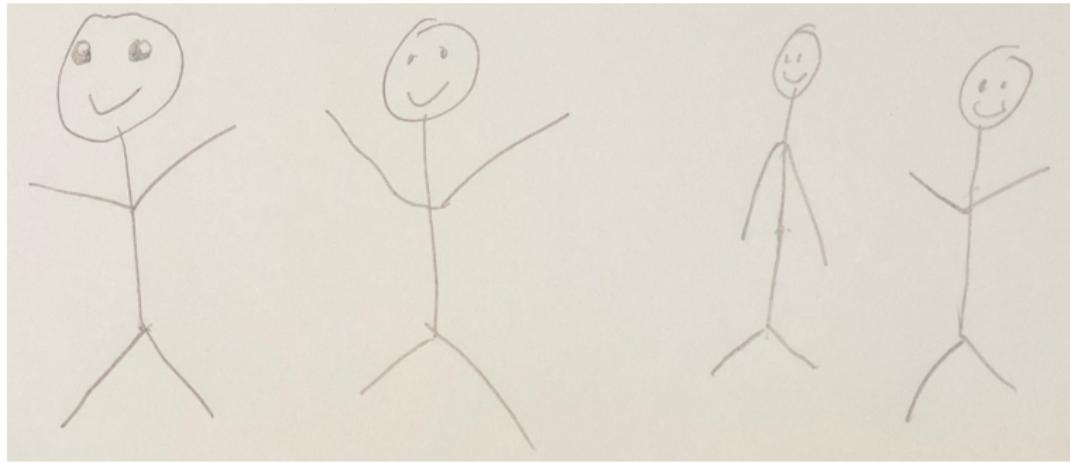
- Latent Units (\mathbf{z}): Dimensions in the model's latent space
- Generative factors (\mathbf{c}): Human-interpretable true characteristics

What is disentanglement?

Stick

What is disentanglement?

Stick



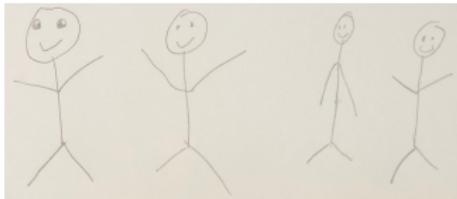
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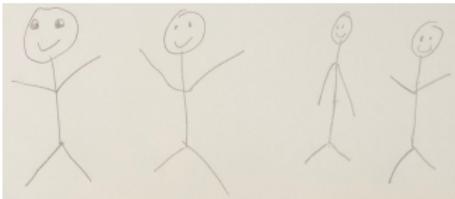


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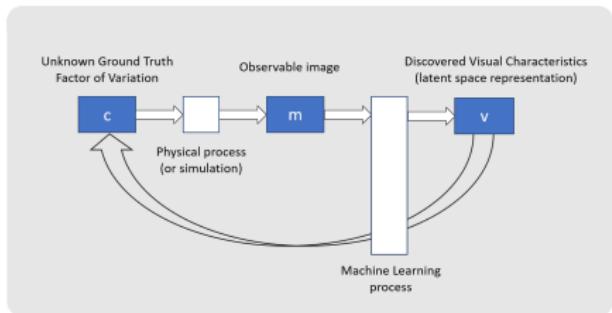
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Goal: One to one mapping between z and c

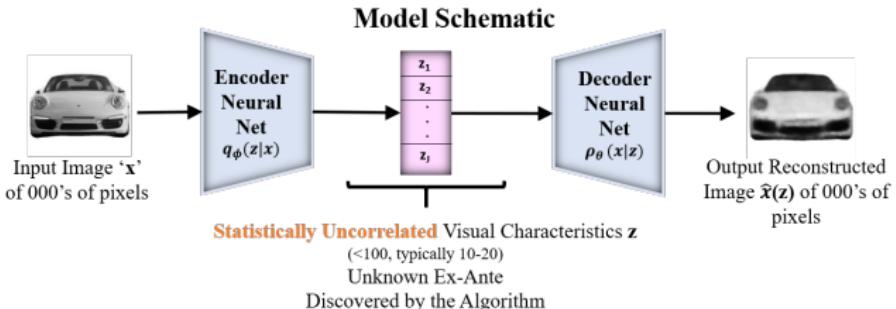
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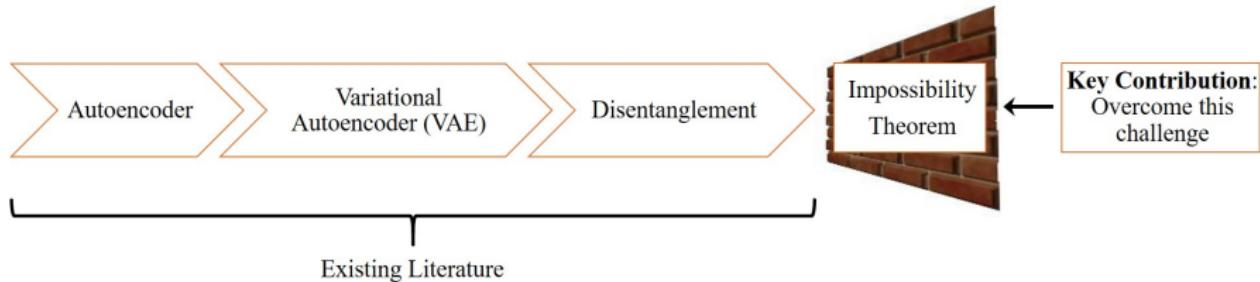


Models in Existing Literature



Model	Goal
Autoencoder (AE)	Reconstruction accuracy
Variational Autoencoder (VAE)	...+ structured latent space
Disentanglement	...+ ...+ statistically independent latent space

Roadmap of Our Approach



Contribution

We aim to overcome this impossibility theorem with a simple approach of using structured product characteristics.

Disentangled and Entangled Representations

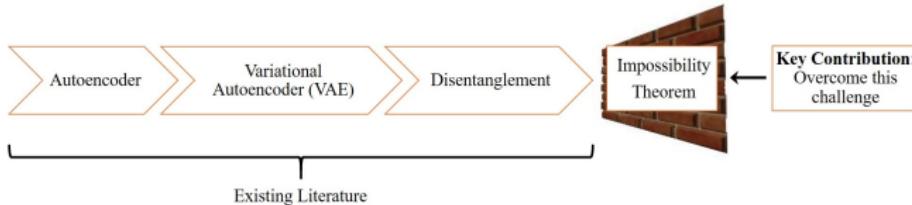
Example of *Entangled* Visual Characteristics



Example of *Disentangled* Visual Characteristics



Impossibility Theorem



Impossibility Theorem

Unsupervised (*i.e. only images*) learning of disentangled representations is *fundamentally impossible* except under certain restrictive conditions.^a

^aLocatello, Francesco, et al. "Challenging common assumptions in the unsupervised learning of disentangled representations." ICML. PMLR, 2019.

Implication: Every disentangled representation can have other *infinite* equivalent entangled representations.

Impossibility Theorem – Implications



z_1
z_2
.
.
.
z_j

predicts →

A horizontal arrow pointing from the learned characteristics table to the ground truth characteristics table.

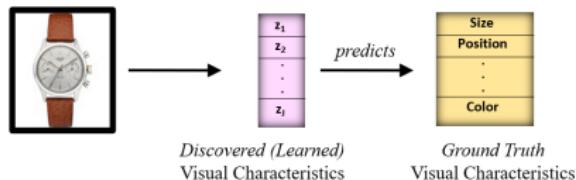
Size
Position
.
.
.
Color

Discovered (Learned)
Visual Characteristics

Ground Truth
Visual Characteristics

Impossibility Theorem – Implications

Common approach to ground truth in ML is to get humans to label¹



What's the Problem?

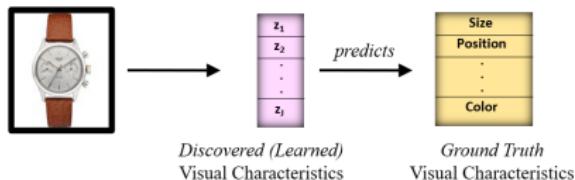
- Ground truth on visual characteristics is *unknown*. In fact, these are precisely what we want to find.

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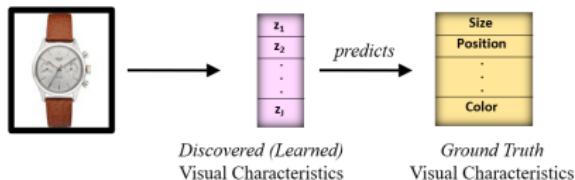
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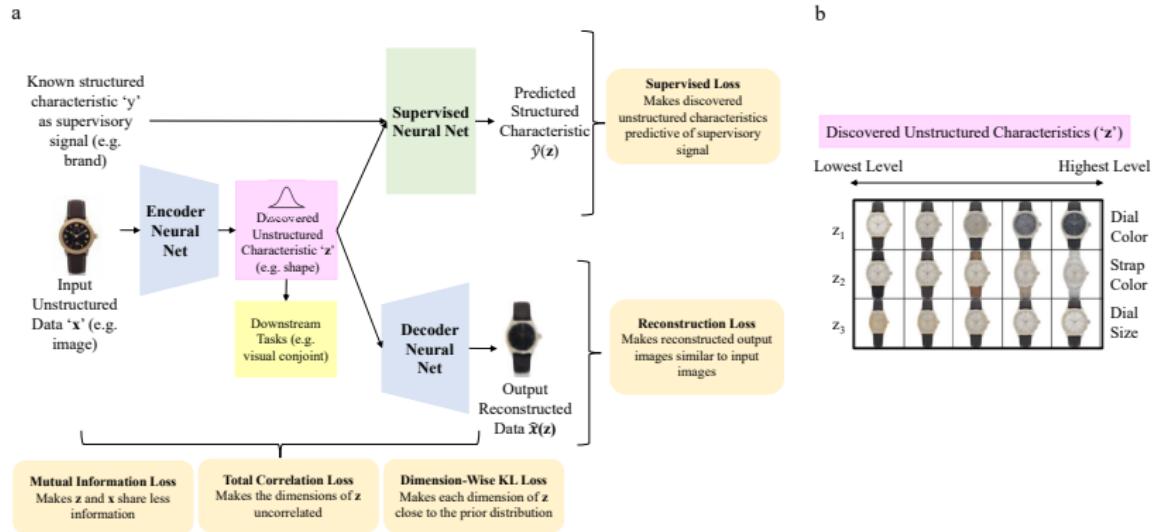


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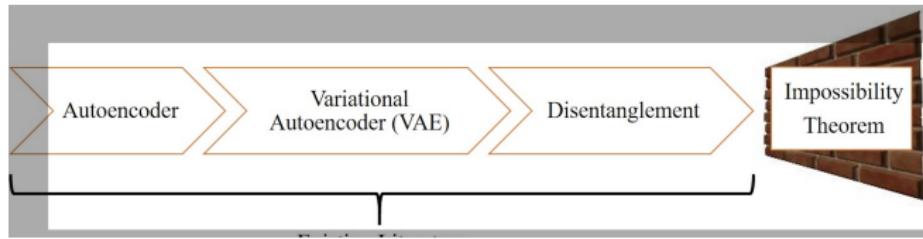
- Ground truth on visual characteristics is *unknown*. In fact, these are precisely what we want to find.
- Researcher needs to determine what are the *true characteristics* to focus on \Rightarrow not Automatic
- Need to ensure humans understand what these labels are and *how to quantify them* for each image

¹Locatello, Francesco, et al. "Disentangling factors of variation using few labels." ICLR. 2020.

Schematic of Proposed Approach



Contribution



- **Solution** without ground truth on visual characteristics:
- Leverage **structured product characteristics** to provide a supervisory signal for disentanglement

Model

- Learn model parameters by minimizing loss $L(\theta, \phi; \mathbf{x}, \mathbf{z})$ of integrated model
- θ and ϕ are encoder and decoder parameters; \mathbf{x} are images

$$\underbrace{L(\theta, \phi, \mathbf{w}; \mathbf{x}, \mathbf{z})}_{\text{Total Loss}} = \underbrace{\mathbf{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} + \alpha \underbrace{I_q(\mathbf{z}, \mathbf{x})}_{\text{Mutual Information Loss}} + \beta \underbrace{KL \left[q(\mathbf{z}) || \prod_{j=1}^J q(z_j) \right]}_{\text{Total Correlation Loss}} \\ + \gamma \underbrace{\sum_{j=1}^J KL \left[q(z_j) || p(z_j) \right]}_{\text{Dimension-Wise KL Divergence Loss}} + \delta \underbrace{P(\hat{\mathbf{y}}(\mathbf{z}), \mathbf{y})}_{\text{Supervised Loss}}$$

Loss Term	Why is this term included?
Reconstruction	Promotes accurate reconstruction of images
Mutual Information	Minimizes redundant information
Total Correlation	Promotes statistical independence between visual characteristics
Dimension-Wise KL	Penalizes deviations from a prior
Supervised	Provides a signal to address the impossibility theorem

Model – Role of Supervised Loss

$$\underbrace{L(\theta, \phi, \mathbf{w}); \mathbf{x}, \mathbf{z})}_{\text{Total Loss}} = \underbrace{\mathbf{E}_{q_\phi(\mathbf{z}|\mathbf{x})} [\log p_\theta(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} + \alpha \underbrace{I_q(\mathbf{z}, \mathbf{x})}_{\text{Mutual Information Loss}} + \beta \underbrace{KL \left[q(\mathbf{z}) || \prod_{j=1}^J q(z_j) \right]}_{\text{Total Correlation Loss}} + \gamma \underbrace{\sum_{j=1}^J KL \left[q(z_j) || p(z_j) \right]}_{\text{Dimension-Wise KL Divergence Loss}} + \delta \underbrace{P(\hat{\mathbf{y}}(\mathbf{z}), \mathbf{y})}_{\text{Supervised Loss}}$$

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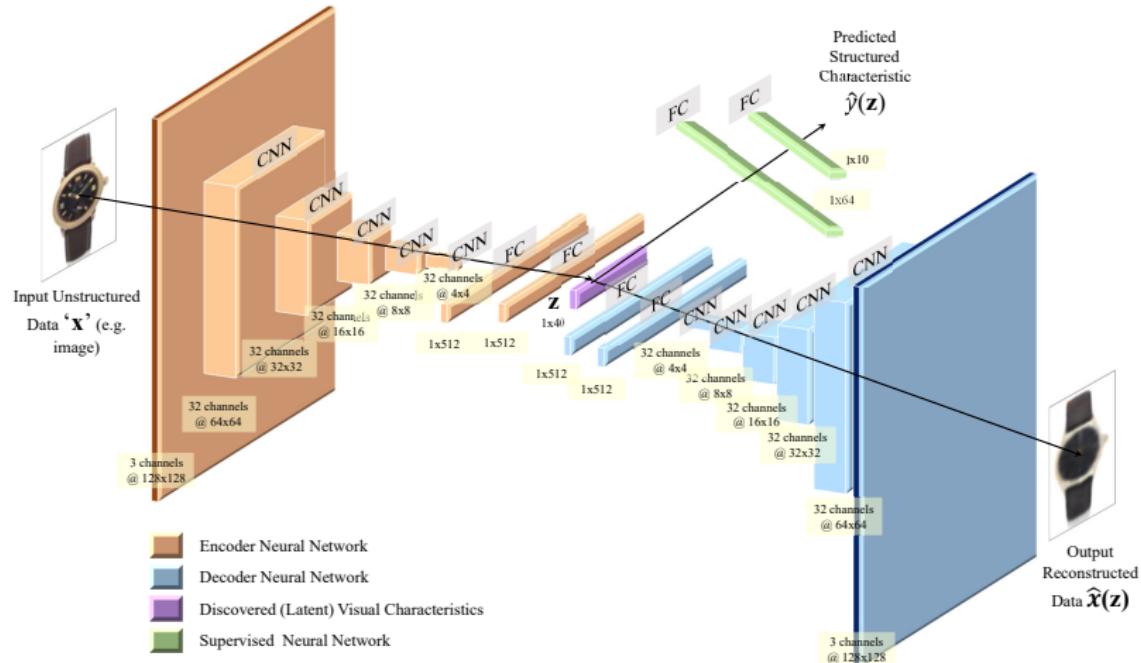
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Idea to Overcome Impossibility Theorem



Model Architecture



Disentanglement Evaluation Metric

Unsupervised Disentanglement Ranking (UDR) measures disentanglement

- **Why UDR?**: “There are no labels available for many real-life applications and for some data, generative factors of interest are hard or impossible for humans to annotate.”²

² Estermann, B., Marks, M., & Yanik, M. F. (2020). Robust Disentanglement of a Few Factors at a Time using rPVAE. *Advances in Neural Information Processing Systems*, 33, 13387-13398.

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- *“All happy families are alike; each unhappy family is unhappy in its own way.” — Leo Tolstoy , Anna Karenina*

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- “*All happy families are alike; each unhappy family is unhappy in its own way.*” — Leo Tolstoy , Anna Karenina
- **Key Idea:** “Models that disentangle well are more likely to be similar to each other than the ones that do not disentangle”³

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Starting from the image on the left, what part of the watch changes the **most** as you go from left to right? Carefully check both large and small visual aspects. Go through each part of the watch one by one before selecting any option. Refer to the above image to see parts of the watch.



Note: Images are low-quality on purpose

- Bezel
- Crown
- Date Window
- Dial
- Hands
- Hour Marker
- Lug
- Strap

How is that part of the watch changing?

Human Interpretable Characteristics?

Can we get human interpretable characteristics from the model without any domain knowledge about the product category?

Which pair of watches in your judgment are more similar in terms of dial color than the other pair? (ignore all the other features of the watches)



Left



Right

Human Interpretable Characteristics?

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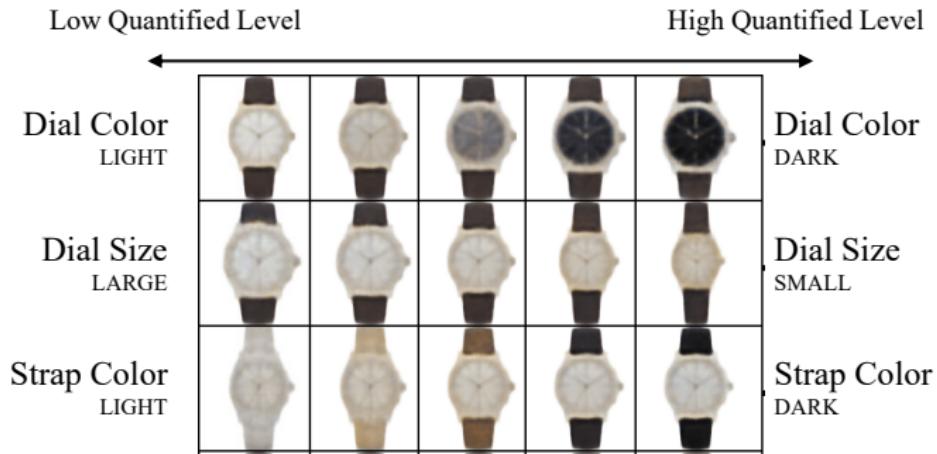
Visual characteristic	Interpretability Survey	Quantification Survey
Dial Size	76%	83%
Dial Color	80%	92%
Strap Color	88%	92%
Rim (Bezel) Color	79%	88%
Dial Shape	87%	68%
Knob (Crown) Size	70%	85%

Discovered Visual characteristics

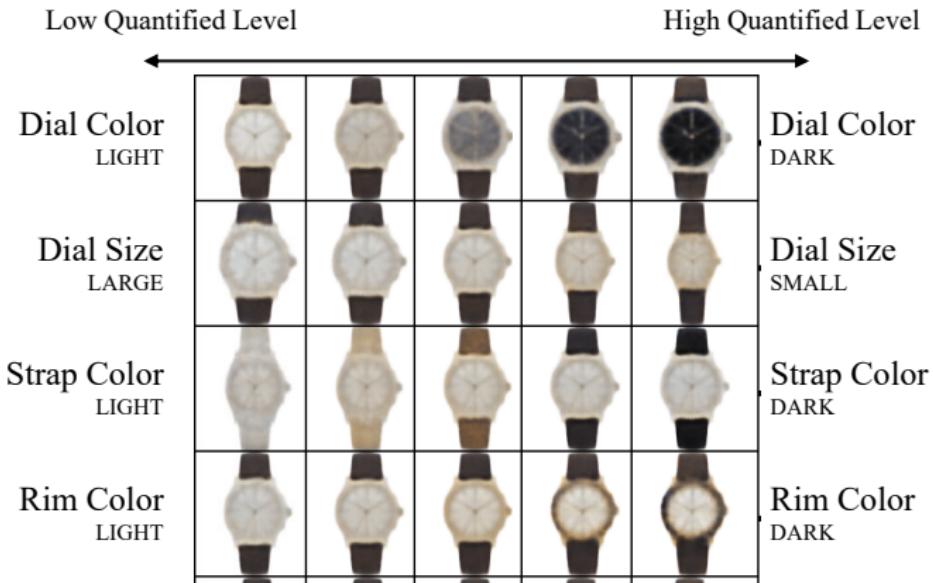
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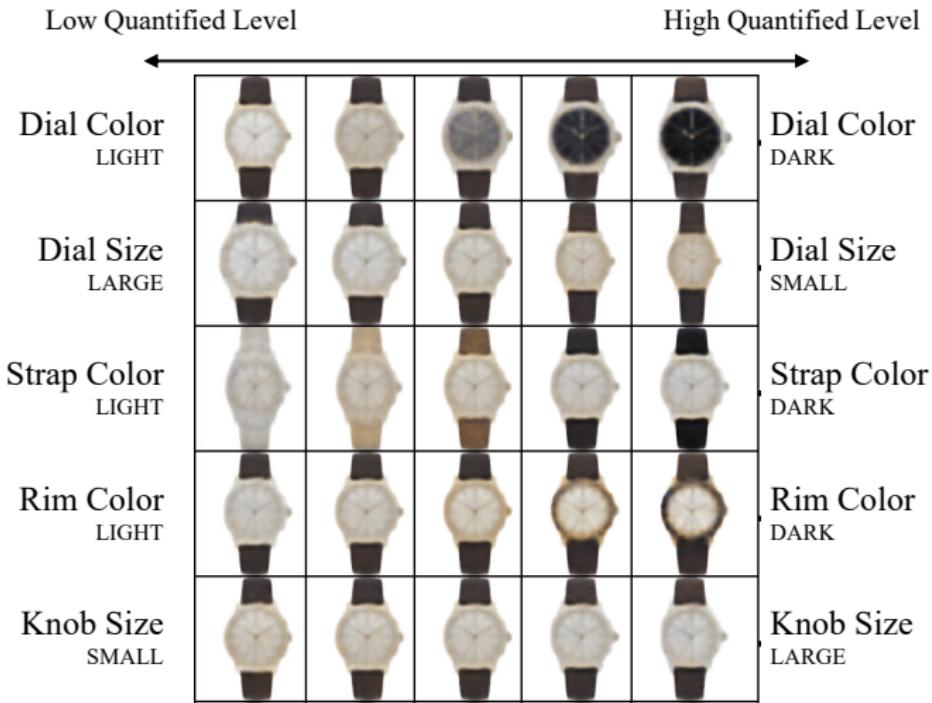
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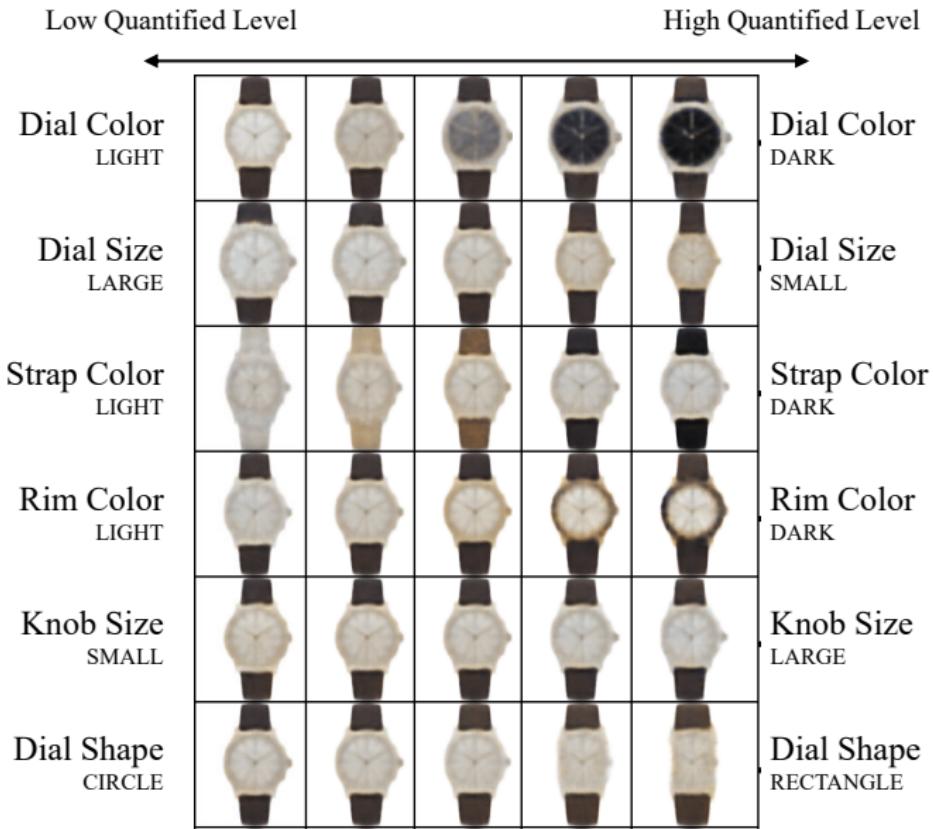
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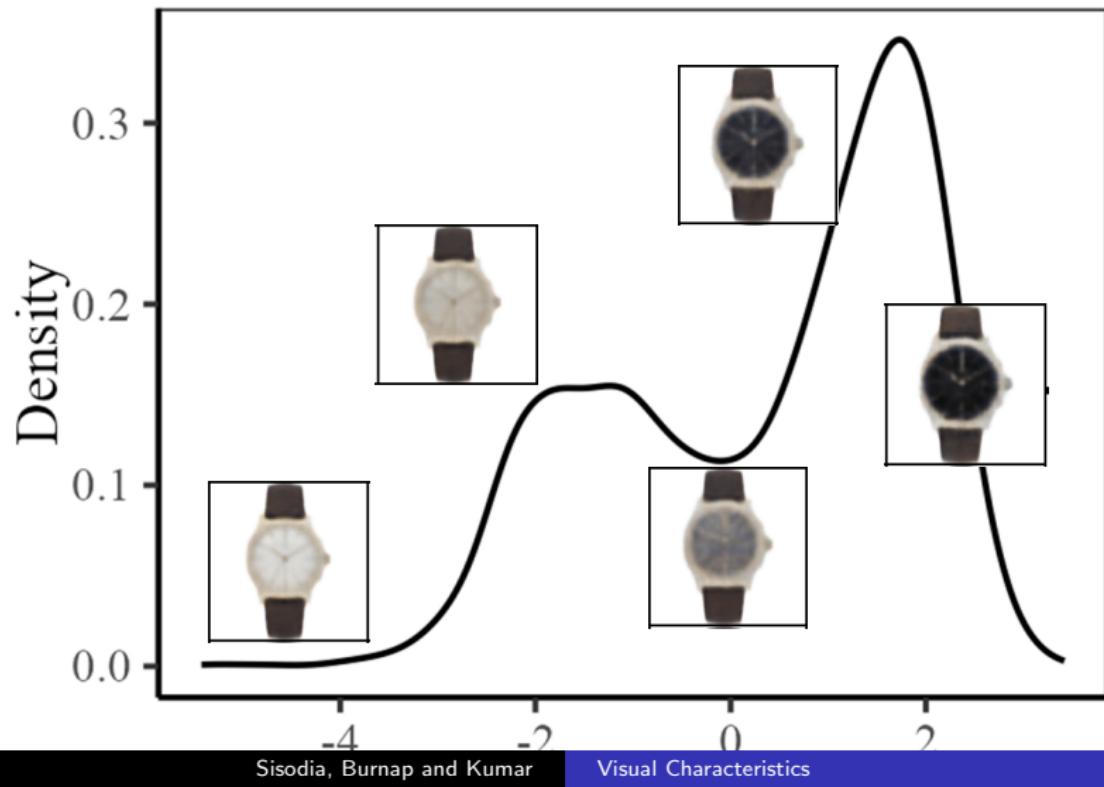
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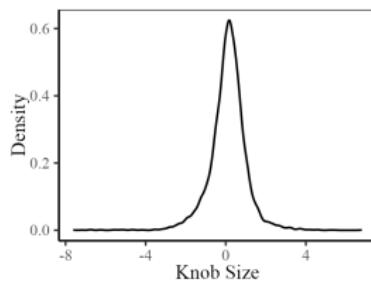
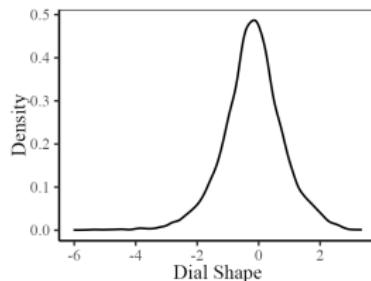
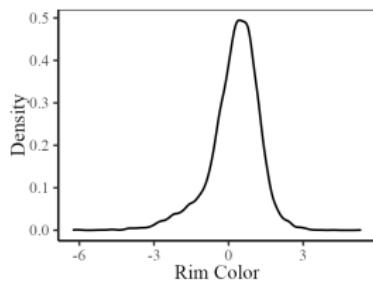
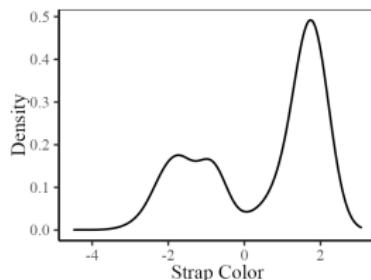
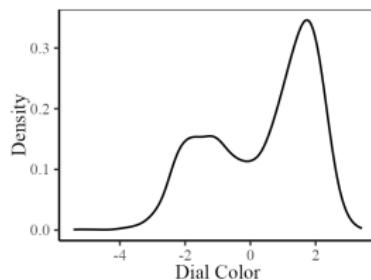
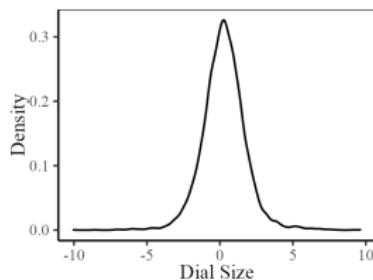
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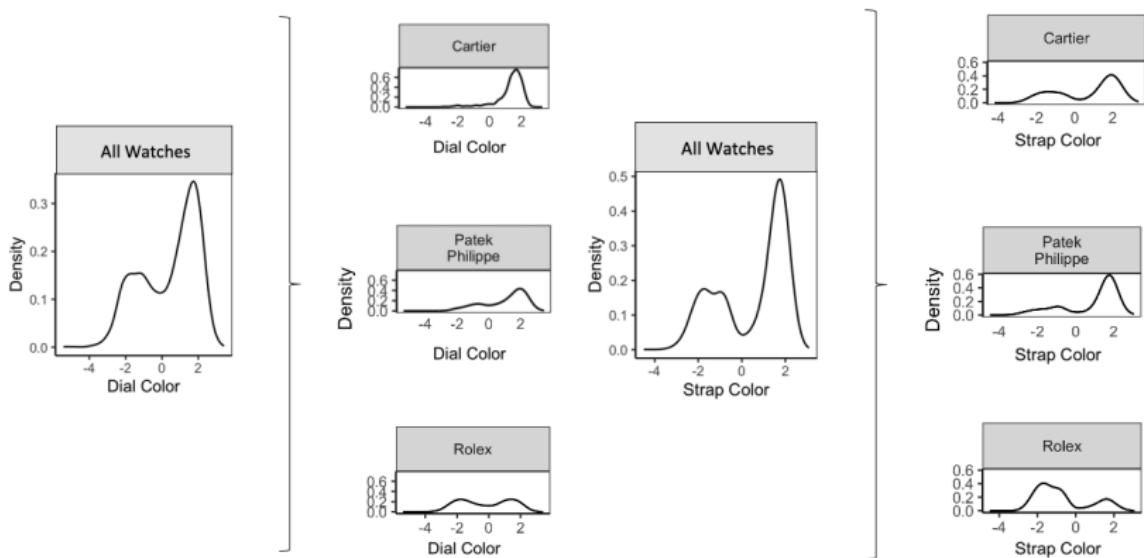
Density of Discovered Visual characteristics (from 'Brand+Material' Signal)



Density of Discovered Visual characteristics (from 'Brand+Material' Signal)



Why does brand aid the disentanglement model?



Why does brand aid the disentanglement model?



Cartier



Patek Philippe



Rolex

Why does brand aid the disentanglement model?

Anecdotally:

- Patek has a strong focus on haute horlogerie (high-art of watchmaking), grand complications, and elegant dress watches. Patek is more conservative and classic on the design front, which is great since most people looking for a Patek are looking for a dress watch.

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- Rolex is much more well-known for its highly-functional and iconically-designed sports and tool watches . . . some of Rolex's most popular models are stainless steel sports watches.

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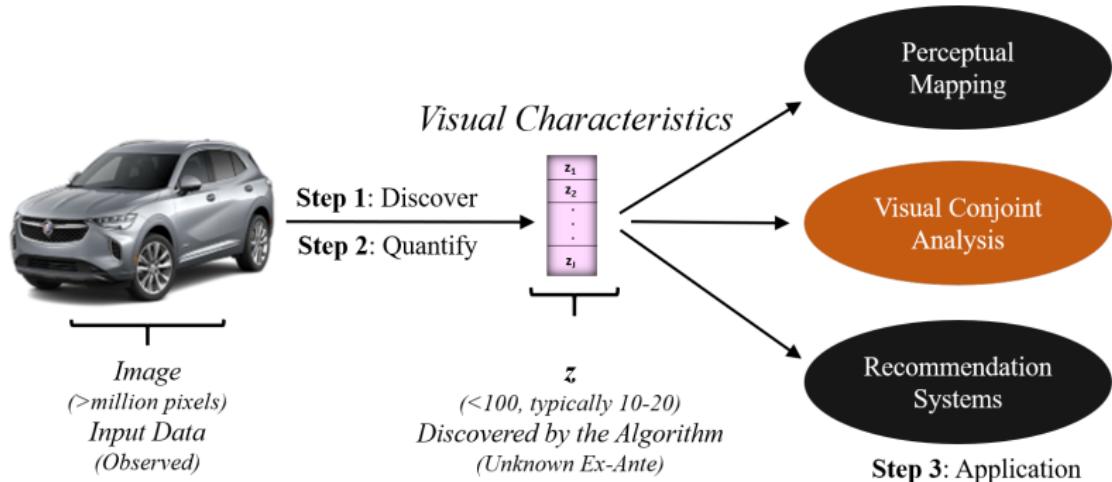
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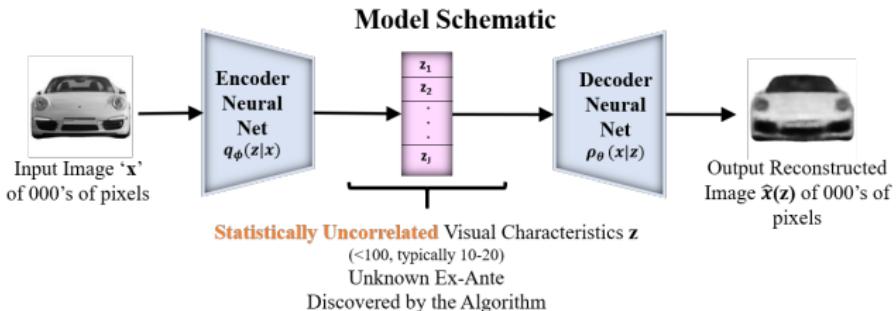
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- Almost all the modern Rolex watches that are round, whereas Cartier has many case shapes from round and oval to cushion-shaped, tonneau, and of course, the many square-shaped or rectangular-shaped Tank watches.

Research Goals



Visual Conjoint Analysis: Background



- Visual conjoint has been challenging to do because elements of visual space are correlated
- Designs have always been manually generated by product designers (prototypes)
- Our approach generates new never-seen visual designs (counterfactual)
- Can span the entire space of visual designs without being bound by the correlations in the data.

Example choice-based conjoint (CBC) question in conjoint survey.

Consider the two watches below that vary **only** on visual style. Of these two, which watch would you prefer more (for yourself)?



Select



Select

Next

Utility: Hierarchical Bayesian Model

$$u(\mathbf{z}; \beta_i) = \beta_1 z_1 + \dots + \beta_K z_K$$

$$\begin{aligned}\mu_\Theta &\sim \mathcal{N}(\mathbf{0}, \sigma_\Theta^2) \\ \Theta &\sim \mathcal{N}(\mu_\Theta, \Lambda_\Theta) \\ \Omega_\beta &\sim \text{LKJ}(\eta) \\ \Lambda_\beta &= \mathbf{D}(\sigma_\beta) \Omega_\beta \mathbf{D}(\sigma_\beta) \\ \beta_i &\sim \mathcal{N}(\Theta^T \mathbf{r}_i, \Lambda_\beta) \\ u_i^j &= z_j \beta_i + \epsilon_{ij} \\ y_i^{j,j'} &\sim \text{Bernoulli}(\omega_i(j, j')) \\ \text{where } \omega_i(j, j') &= \frac{\exp(u_i^j)}{\exp(u_i^j) + \exp(u_i^{j'})}\end{aligned}$$

where $\text{LKJ}(\eta)$ is a Cholesky factorization of the correlation matrix Ω_β of the individual "part-worth" preference vector over visual characteristics. $\mathbf{D}(\cdot)$ denotes a diagonal matrix, \mathbf{r}_i are consumer covariates, u_i^j is the utility customer i gets from watch design j , and ϵ_{ij} is a Gumbel random variable. The Bernoulli probability parameter $\omega_i(j, j')$ is specified by the logit function, and $\{j, j'\}$ denotes the set of all pairwise choice comparisons for watches $j, j' \in J$ that customer i chose over in the conjoint survey. Note that $\sigma_\Theta^2, \Lambda_\Theta, \eta$ are researcher-defined hyperparameters chosen via model selection using prediction accuracy on the validation data split as the evaluation metric.

Conjoint Model Accuracy

Generated Watches

Model	Out-of-Sample Hit Rate (SD)
Disentangled Embedding + Logit Model (-)	63.16% (2.34%)
Disentangled Embedding + Neural Net (-)	65.81% (2.22%)
Pretrained Deep Learning Model Embedding (O)	68.31% (1.54%)
Disentangled Embedding + Neural Net (O)	67.52% (0.92%)
Disentangled Embedding + Random Forest (O)	68.77% (0.90%)
Disentangled Embedding + XGBoost (O)	69.10% (0.41%)
Disentangled Embedding + HB Model (O + U)	71.61% (1.87%)
Disentangled Embedding + HB Model + Interactions (O + U)	70.68% (1.35%)

- Pretrained Deep learning model is trained on millions of images, and has millions of parameters
- Our HB model also has a small number parameters, and all predictions are based on only 6 visual characteristics

Ideal Point

- Marketing Literature has conceptualized the notion of ideal point (DeSarbo, Ramaswamy, and Cohen 1995).
- Optimal positioning of a product in the space of characteristics
 - In this study, visual characteristic space
- Can also do this across researcher-defined segments

Generated Ideal Point Watches for Two Segments

Ideal Point: Optimal positioning of a product in characteristic space based on preferences of a selected consumer segment.



Segment 1:
“Ideal Point” Watch Design



Segment 2:
“Ideal Point” Watch Design

Segment 1	Young moderately-affluent females
Segment 2	Older males

Conclusion

We obtain interpretable visual characteristics directly from unstructured product images

- *automatically discover (extract) characteristics*

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Applications

We then used the model to:

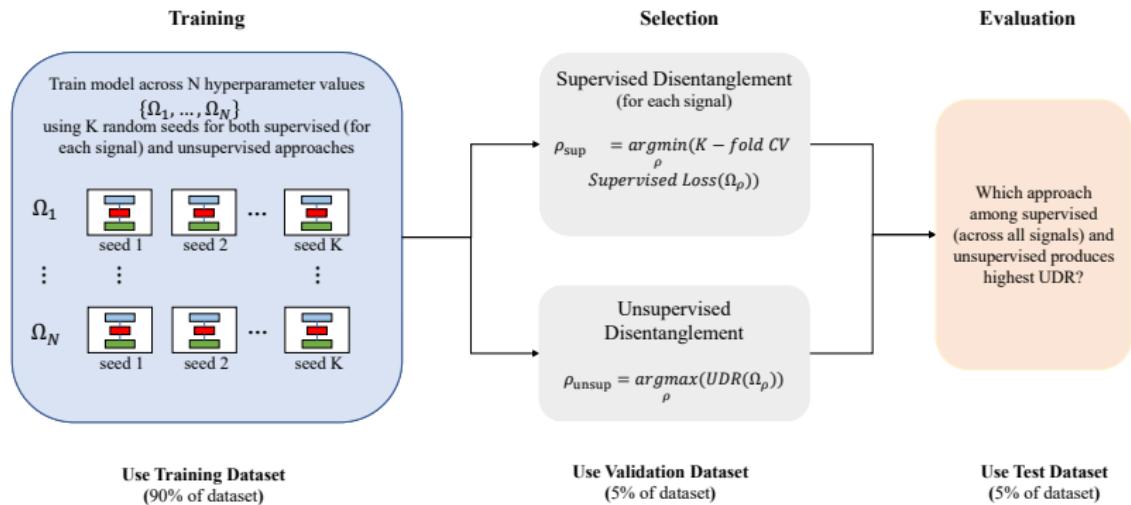
- generate new counterfactual designs to obtain consumer preferences over visual characteristics.
- obtain ideal point visual designs corresponding to different consumer segments

The End

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Additional Slides

Model Training, Selection, & Evaluation



Disentanglement Evaluation Metric

Esterman details the value of UDR, which we quote below:

“There are no labels available for many real-life applications and for some data, generative factors of interest are hard or impossible for humans to annotate.

Table of Notation

Symbol	Category	Meaning
\mathbf{x}	Input Data	Product image
\mathbf{y}	Input Data	Supervisory signal(s)
$\hat{\mathbf{x}}$	Output Data	Reconstructed image
$\hat{\mathbf{y}}$	Output Data	Predicted Supervisory Signal(s)
\mathbf{z}	Latent Space	Visual characteristic vector
\mathbf{z}_{inf}	Subset of Latent Space	Informative visual characteristic
$p(\mathbf{z})$	Model	Prior distribution
$p_{\theta}(\mathbf{x} \mathbf{z})$	Decoder Neural Net	Conditional Probability of Generating Image Data given Latent Space
$q_{\phi}(\mathbf{z} \mathbf{x})$	Encoder Neural Net	Conditional Probability of Latent Space given Image Data
$p_w(\mathbf{y} \mathbf{z})$	Supervisory Neural Net	Conditional Probability of Supervisory Signal given Latent Space
θ	Weights of Neural Net	Decoder's parameters
ϕ	Weights of Neural Net	Encoder's parameters
w	Weights of Neural Net	Supervisory Net's parameters
$\mathbf{E}_{q_{\phi}(\mathbf{z} \mathbf{x})} [\log p_{\theta}(\mathbf{x} \mathbf{z})]$	Loss Function	Reconstruction Loss
$I_q(\mathbf{z}, \mathbf{x})$	Loss Function	Mutual Information Loss
$KL \left[q(\mathbf{z}) \prod_{j=1}^J q(z_j) \right]$	Loss Function	Total Correlation Loss
$\sum_{j=1}^J KL [q(z_j) p(z_j)]$	Loss Function	Dimension KL Divergence Loss
$P(\hat{y}(\mathbf{z}), y)$	Loss Function	Supervised Loss
$\mathcal{L}(\theta, \phi, \beta; \mathbf{x}, \mathbf{z})$	Loss Function	Total Loss
J	Hyperparameter	Dimensionality of latent space
α	Hyperparameter	Weight on Mutual Information Loss
β	Hyperparameter	Weight on Total Correlation Loss
γ	Hyperparameter	Weight on Dimension KL Divergence Loss
δ	Hyperparameter	Weight on Supervised Loss

Why does brand aid the disentanglement model?

Average of pairwise JS Divergence Of a visual characteristic across different brands is much higher than across different price points.

Visual Characteristic	Brand	2 Discrete Prices	3 Discrete Prices	5 Discrete Prices	10 Discrete Prices
Dial Color	6.28	1.62	1.77	1.91	1.91
Dial Size	9.71	2.13	3.24	4.13	4.86
Strap Color	6.16	0.56	1.76	1.85	2.08
Rim Color	9.07	3.61	6.56	6.57	8.34
Dial Shape	5.97	0.29	0.66	0.75	3.16
Knob Size	8.20	6.02	5.76	13.08	13.46
Across All 6 Char	7.56	2.37	3.29	4.71	5.63