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Quick Work
Page No.:
Date:
M T W T F S S

Solution = 1
Sample of size n is taken from Normal population
parameter = θ_1 mean
θ_2 var
Maximum likelihood Estimation of these two parameters.

Normal density is

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Joint density of $(X_1, X_2 \text{ ---- } X_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(x_i-\mu)^2}{2\sigma^2}}$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi(\theta_2)}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2(\theta_2)}}$$

$$\ln\left(L(\theta_1, \theta_2)\right) =$$

$$= \sum_{i}^{n} -n \ln\left(\sqrt{2\pi\theta_2}\right) - \frac{(x_i - \theta_1)^2}{2\theta_2}$$

$$= -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta_2) - \sum_{i=1}^{n}\left(\frac{(x_i-\theta_1)^2}{2\theta_2}\right.$$

$$= -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta_2) - \frac{\sum_{i=1}^{n}(x_i)^2}{2\theta_2} - \frac{n\theta_1^2}{2\theta_2} + \left(\frac{\theta_1}{\theta_2} \times \sum x_i\right)$$

Diff wrt θ_1

$$-\frac{2n\theta_1}{2\theta_2} + \frac{\sum x_i}{\theta_2} = 0$$

$$\frac{-n\theta_1 + \sum x_i}{\theta_2} = 0$$

$$-n\theta_1 + \sum x_i = 0$$

$$\theta_1 = \frac{\Sigma x_i}{n}$$

$$\left(\theta_1 = \bar{x}\right)$$

(Sample mean θ_1)

Diff wrt θ_2

$$\frac{-n}{2} \times \frac{\theta_2}{\theta_2^2} + \frac{\Sigma(x_i)^2}{2(\theta_2)^2} + \frac{n\theta_1^2}{2\theta_2^2} - \frac{2\theta_1 \Sigma x_i}{2\theta_2^2}$$

$$\frac{-n\theta_2 + \Sigma(x_i)^2 + n\theta_1^2 - 2\theta_1 \Sigma x_i}{2(\theta_2)^2} = 0$$

$$-n\theta_2 + \Sigma(x_i)^2 + n(\bar{x})^2 - 2\bar{x}\Sigma x_i = 0$$

$$n\theta_2 = 2\bar{x}\Sigma x_i - n(\bar{x})^2 - \Sigma(x_i)^2$$

$$\theta_2 = -\frac{2\bar{x}\Sigma x_i}{n} + (\bar{x})^2 + \frac{\Sigma(x_i)^2}{n}$$

or

$$\theta_2 = \frac{\Sigma(x_i)^2}{n} + \frac{n(\bar{x})^2}{n} - \frac{2\bar{x}\Sigma x_i}{n}$$

$$\theta_2 = E(x_i)^2 + E^2(x) - 2E^2(x)$$

$$= E(x_i)^2 - E^2(x)$$

$(\text{Var}) = \theta_2$

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Quick Work
Page No.:
Date:
M T W T F S S

Sol=2
Let $x_1, x_2, x_3 --- x_n$ be random Sample from $B(m, \theta)$
MLE of θ parameter.

PDF of Binomial =

$$f(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

according Given =>

$$f(k; n, \theta) = \binom{n}{k} \theta (1-\theta)^{n-k}$$

$$L(x_i, m, \theta) = \text{Joint pdf } (x_1 --- x_n, \theta)$$

$$= \prod_{i=1}^{n} {}^mC_{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

taking log on both sides

$$\ln(L(n, \theta)) = \sum_{i=1}^{n} \ln({}^nC_x) + x_i \ln\theta + (m-x_i) \ln(1-\theta)$$

Diff wrt Θ

$$= \sum_{i=1}^{n} \frac{x_i}{\theta} + \frac{nm - \Sigma x_i)}{(1-\theta)} = 0$$

$$= \frac{\Sigma x_i}{\theta} + \frac{nm - \Sigma x_i}{(1-\theta)} = 0$$

$$= \frac{\Sigma x_i}{nm} = \Theta$$

$$\boxed{\theta = \frac{\bar{x}}{m}}$$

$\bar{x}$ = mean of sample