Numerical Simulation of Capillary Waves: Comparative Study of Volume of Fluid and Level Set Methods

Vineet Vishnu Mukim[1(✉)], Knut Erik Teigen Giljarhus[2], Rune Wiggo Time[1], and Andrianifaliana Herimonja Rabenjafimanantsoa[1]

[1] Department of Energy and Petroleum Engineering, University of Stavanger, Kjell Arholms gate 41, 4021 Stavanger, Norway
{vineet.v.mukim,rune.time,a.h.rabenja}@uis.no

[2] Department of Mechanical and Structural Engineering and Materials Science, University of Stavanger, Kjell Arholms gate 41, 4021 Stavanger, Norway
knut.e.giljarhus@uis.no

Abstract. Linear wave theory establishes a relationship between the interfacial tension, viscosity, and capillary wave parameters such as wavelength and decay coefficient. This makes capillary waves an excellent non-intrusive tool for measuring fundamental properties of liquids. But these waves exhibit distinct length scales for their amplitude and wavelength posing significant challenges for numerical simulation. This study investigates numerical simulation methods, Volume of Fluid (VoF) and Level Set (LS), in the context of capillary waves. Both methods are widely used to simulate multiphase flows and differ mainly in numerical implementation and approaches of handling the interface. Accuracy of measuring wave properties heavily relies on precise and reliable interface capturing prompting the need for a comparative investigation. For this study, interIsoFoam (OpenFOAM) and Level Set (COMSOL Multiphysics) solvers are considered. Numerical experiments conducted for a range of parameters show that VoF (average error = 1.09%) performs better than LS (average error = 3.92%). Additionally, VoF provides superior results in other aspects of the wave, including amplitude and sinusoidal appearance. This study also validates linear wave theory in real-world scenarios with finite-amplitude and finite-depth waves with reflections.

Keywords: capillary waves · Volume of Fluid (VoF) · Level Set (LS) · interface capturing · dynamic mesh motion

1 Introduction

Water waves are omnipresent and can be broadly categorized into two main types: gravity waves and capillary waves. Water waves are complex and this level

V. V. Mukim, K. E. T. Giljarhus, R. W. Time and A. H. Rabenjafimanantsoa—These authors contributed equally to this work.

A. C. Benim et al. (Eds.): ICCHMT 2023, LNME, pp. 704–714, 2024.
https://doi.org/10.1007/978-3-031-66609-4_65

of complexity primarily arises from the effects of dispersion and non-linearity, which defines the dynamics of water waves. Capillary waves, in particular, are distinguished by their minuscule amplitudes and high frequencies, with interfacial tension acting as the dominant force governing their behavior. The dispersion relation for water waves [1] is shown as below,

$$\omega^2 = k\left(g + \frac{k^2\sigma}{\rho}\right). \tag{1}$$

where,
$\omega = 2\pi n =$ angular frequency,
$g =$ acceleration due to gravity,
$k = \frac{2\pi}{\lambda} =$ wave number,
$\sigma =$ interfacial tension for liquid-air pair,
$\rho =$ density of liquid.

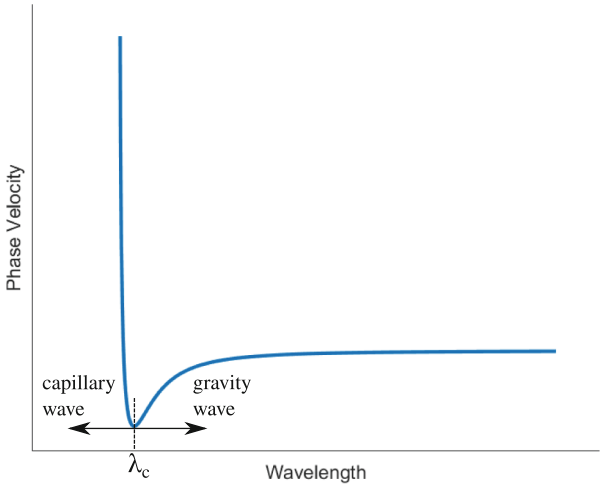

Fig. 1. Dispersion relation plot showing the effect of wavelength on phase velocity

The phase velocity is formally expressed as k/ω. As illustrated in Fig. 1, we designate λ_c as the critical wavelength, demarcating the boundary between the capillary and gravity wave domains.

The foundational assumptions for deriving the dispersion equation involve infinitely small wave amplitudes, infinitely deep ($\tanh(kh) = 1$), and unbounded (reflection-free) liquid domain. The derivation also does not address the wave generation mechanism. This numerical study aims to compare two solvers using different interface capturing techniques and validate the linear wave theory (Eq. 1) in real-world scenarios. This analysis also aids our understanding of capillary wave behavior, benefiting future experimental validation efforts.

2 Theory

For our comparative study, we selected two widely used one-fluid formulation methods, as outlined in [2]: the Volume of Fluid (VoF) and Level Set (LS) techniques, based on their unique approaches (discussed later in this section) to interface capturing. The simulations were performed using OpenFOAM (version 2212) and COMSOL Multiphysics (version 6.1).

The study involves the simulation of immiscible, incompressible, laminar, transient, and isothermal two-phase flow. The governing equations primarily included the continuity and momentum (Navier-Stokes) equations, which can be expressed as follows,

$$\nabla \cdot \mathbf{u} = 0. \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = \frac{1}{\rho} \left(\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \rho \mathbf{g} + \mathbf{f}_{it} \right). \tag{3}$$

The interfacial tension force ($\mathbf{f}_{it}$) is modelled by Continuous Surface Force (CSF) method in both of the solvers as follows,

$$\mathbf{f_{it}} = \sigma \kappa \delta_s \mathbf{n}. \tag{4}$$

where,
κ = curvature of interface,
δ_s = surface Dirac delta function,
$\mathbf{n}$ = surface normal.

2.1 Volume of Fluid

VoF method uses a volume fraction variable to keep track of the interface. This variable is advected according to the local velocity field as follows,

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = 0. \tag{5}$$

This causes the smearing of the interface and hence it requires additional interface-capturing techniques.

$$\begin{aligned} \delta_s &= \nabla \alpha, \\ \kappa &= -\nabla \cdot \mathbf{n} = -\nabla \cdot \left(\frac{\nabla \alpha}{|\nabla \alpha|} \right). \end{aligned} \tag{6}$$

OpenFOAM version 2212 offers two VoF solvers, namely interFoam and interIsoFoam, which differ in the interface-capturing approaches. interFoam uses a so called algebraic VoF method which calculates flux algebraically at the faces without reconstructing the interface, as described in [3]. It employs the MULES (Multidimensional Universal Limiter with Explicit Solution) algorithm to introduce an artificial compression flux, mitigating interface blurring.

interIsoFoam, which is considered an improvement over interFoam, functions as a geometric VoF solver utilizing the isoAdvector method, operating through

a two-step process as detailed in references [4–6]. The first step involves interface reconstruction, where the interface is represented using a piece-wise linear (PLIC) approach. This method ensures the volume fraction values of the respective cells by employing a surface splitting them. In the subsequent interface advection step, the isosurface is initially identified, and the center and normal direction of this isosurface are computed. The entire interface is then advanced in the normal direction, utilizing interpolated center velocities.

Considering its enhancements, interIsoFoam is the chosen solver for this study. Generally, VoF methods exhibit superior mass conservation characteristics but encounter challenges in accurately calculating interface curvature, particularly in complex geometries and unstructured grids.

2.2 Level Set

In this method, the interface is represented by a signed distance function, ϕ. The function is smooth and transitions from one fluid to other over a very narrow interface region. Consequently, there is no need for any explicit interface capturing techniques.

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \gamma \nabla \cdot \left(\epsilon \nabla \phi - \phi \left(1 - \phi\right) \frac{\nabla \phi}{|\nabla \phi|} \right). \tag{7}$$

$$\begin{aligned} \delta_s &= 6|\nabla \phi||\phi \left(\phi - 1\right)|, \\ \kappa &= -\nabla \cdot \mathbf{n} = -\nabla \cdot \left(\tfrac{\nabla \phi}{|\nabla \phi|} \right). \end{aligned} \tag{8}$$

In COMSOL Multiphysics [7], the level set variable is advected by Eq. 7. The right-hand side of Eq. 7 is non-zero in COMSOL Multiphysics to ensure numerical stability. Additionally, Eq. 8 presents a numerical approximation of the surface Dirac delta function (δ_s). Calculating curvature is straightforward due to the continuous nature of ϕ, unlike discrete α values in VoF. The LS method is expected to yield accurate values for interface curvature and surface normals, but at the expense of mass conservation. Nevertheless, COMSOL Multiphysics offers an advanced option known as the conservative LS method, which we have explored to assess its potential advantages before finalizing our simulation methodology.

3 Numerical Setup

Numerical simulations are conducted to investigate capillary wave generation in a rectangular tank. Planar symmetric numerical simulations are performed to expedite computations. The computational domain is shown in Fig. 2. The influence of contact angle is not accounted in this study. The use of a wall boundary condition in COMSOL Multiphysics also highlights this choice. This approach is grounded in the assumption that the effects of contact angle are

confined to the near-wall regions. The vibrating probe is continuously submerged in the liquid, undergoing sinusoidal excitation in vertical direction. The probe width and vibration amplitude are chosen to strike a balance, ensuring that the wave amplitude remains small enough for linear wave theory to apply, yet large enough to be accurately resolved by fine mesh near the interface.

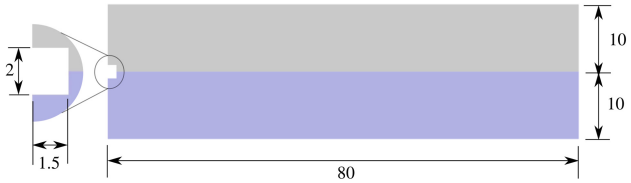

Fig. 2. Computational domain with probe dimensions highlighted on the left (all dimensions are in mm)

Detailed boundary definitions can be visualized in Fig. 3, while the specific boundary conditions for OpenFOAM and COMSOL Multiphysics are outlined in Tables 2 and 3, respectively. Some of the important boundary conditions used in this case are symmetry, open boundary, prescribed displacement and moving wall velocity. The top boundary remains open to the atmosphere, facilitating the free exchange of mass. Figure 4 shows probe location at different times representing the mesh motion with mesh finer at the interface.

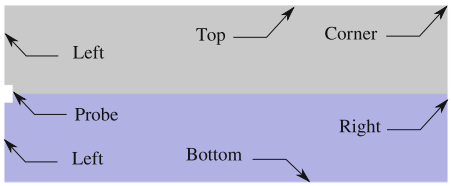

Fig. 3. Boundary definitions used in the simulations

Mesh in y-direction is graded to ensure very fine mesh at the interface to capture the tiny amplitude. Mesh size used in x-direction is uniform and always 5 times the size in y-direction. The dynamic mesh motion transfers the force to the liquid without simulating solid mechanics, constituting one-way Fluid-Structure Interaction (FSI) coupling. Figure 5 shows comparison of interfaces for different mesh sizes in y-direction at the interface for VoF method at 0.9 s.

The parametric study involves using six different excitation frequencies and interfacial tension values one at a time. Excitation frequencies are within the capillary wave region (Fig. 1) with 15 to 40 Hz, while interfacial tension values span from 20 to 70 mN/m, covering the range between alcohol-air and water-air pairs. Fluid properties used in the simulations are listed in Table 1.

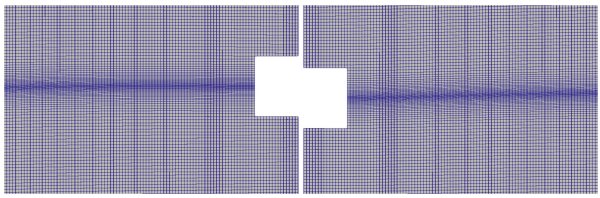

Fig. 4. Probe location at different times representing the mesh motion with mesh finer at the interface (image on left side is flipped to highlight and compare the motion of probe)

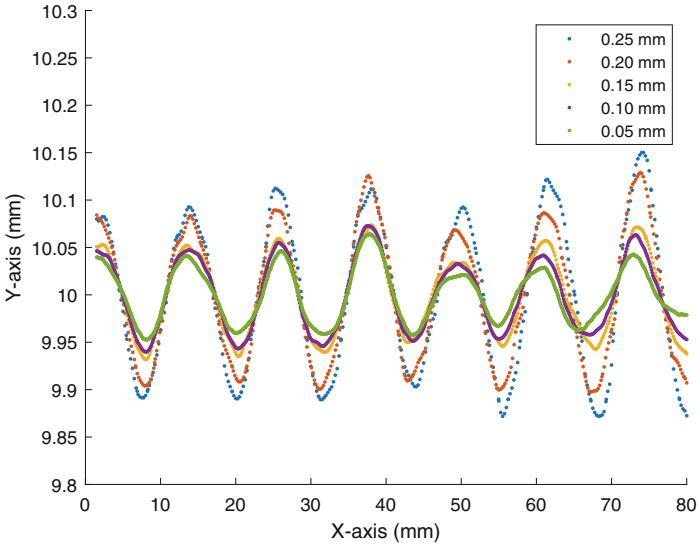

Fig. 5. Comparison of interfaces using VoF method at different mesh sizes in y-direction at excitation frequency of 20 Hz and simulated time of 0.9 s

Table 1. Fluid properties

Fluid	Density (kg/m^3)	Dynamic Viscosity (Pa.s)
Water	1000	1e−3
Air	1	1.48e−5

4 Results and Discussion

Parametric simulations are performed followed by the extraction of the interfaces for subsequent post-processing. Analytical wavelength values are calculated using Eq. 1 for specified frequency and interfacial tension, serving as a benchmark for comparison. As mentioned in Sect. 2.2, both conservative and non-conservative options are explored. Figure 6 shows a comparison of interfaces obtained with three different simulation methods after a simulated time of 1 s. As anticipated, the conservative LS method exhibited significant improvements in maintaining liquid mass conservation. Both conservative and non-conservative approaches yielded similar wavelengths but displayed notably higher wave amplitudes compared to VoF outcomes. However, an anomaly is spotted in the non-conservative LS results, which exhibits an unusual drop in the interface level near the wall. The reason for this behavior is not investigated further.

Table 2. Boundary conditions OpenFOAM

Boundary	U	p_rgh	alpha	pointDisplacement
Left	symmetryPlane	symmetryPlane	symmetryPlane	symmetryPlane
Bottom	noSlip	fixedFluxPressure	zeroGradient	fixedValue
Right	noSlip	fixedFluxPressure	zeroGradient	fixedValue
Top	pressureInletOutletVelocity	totalPressure	inletOutlet	fixedValue
Probe	movingWallVelocity	fixedFluxPressure	zeroGradient	oscillatingDisplacement

Table 3. Boundary conditions COMSOL

Boundary	Laminar Flow	Level Set	Moving Mesh
Left	Symmetry	Symmetry	Symmetry
Bottom	Wall	No Flow	-
Right	Wall	No Flow	-
Top	Outlet	Outlet	-
Probe	Wall	No Flow	Prescribed Displacement
Corner	Pressure Point Constraint	-	-

The temporal evolution of the interfaces obtained from VoF method shown in Fig. 7, reveals the motion of waves. Wave fronts are observed propagating outward from the oscillating probe, subsequently reflecting off the wall, ultimately forming a standing wave pattern. To ensure consistency in post-processing, we selected a specific time point before which the reflected wave returns to the oscillating probe. Crests/troughs are identified manually with a reasonable distance from the probe and the wall (around 10 mm) and then average wavelength

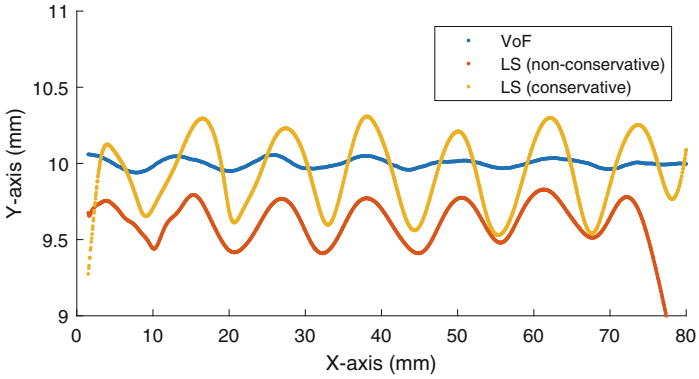

Fig. 6. Comparison of interfaces obtained from VoF method, conservative and non-conservative LS methods at excitation frequency of 20 Hz and simulated time of 1 s

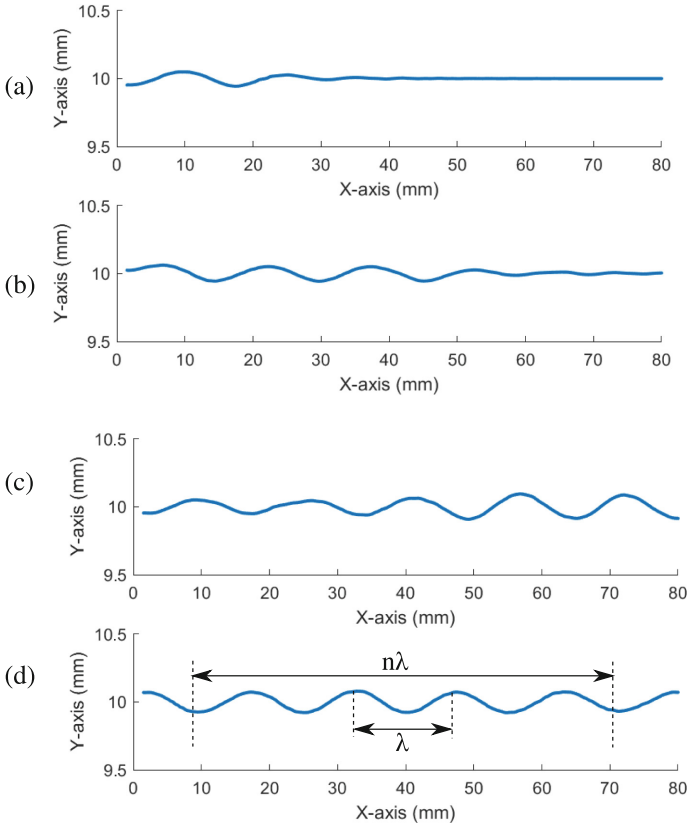

Fig. 7. Interface motion at time 0.1, 0.22, 0.5 and 1 s at 15 Hz and 70 mN/m using VoF method and calculation method for average wavelength shown in (d)

is calculated based on total number of crests/troughs lying between previously marked points. This value of average wavelength is used for the comparative analysis and it's calculation is explained in Fig. 7.

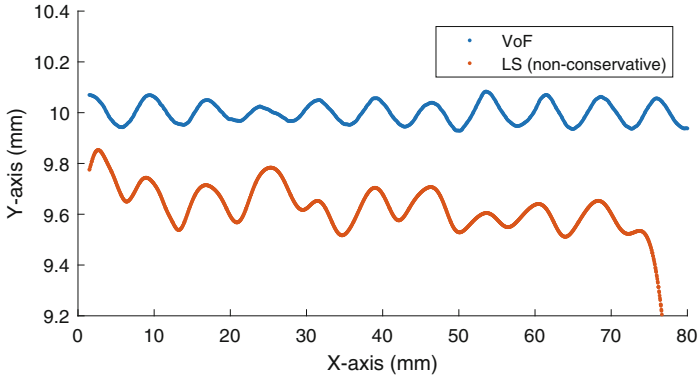

Fig. 8. Comparison of interface profiles for VoF and LS methods at 35 Hz showing a tricky case of average wavelength calculation which is different from interface profile than shown in Fig. 7

The interface shape depends on the specific time, especially when considering various excitation frequencies. We selected the time step closest to 1 s for all parameter combinations, where the crest and troughs of the interface are most clearly identified. Interface data at this specific time is used for the final analysis. To assess the agreement between analytical and simulated wavelengths, we conducted comparisons for both VoF and LS methods across various excitation frequencies and interfacial tension values. The results of these comparisons can be found in Tables 4 and 5. The simplification of ignoring contact angle is reasonable, considering that the near-wall region does not factor into the wavelength

Table 4. Comparison of average wavelengths at $\sigma = 70$ mN/m

Frequency	Analytical	VoF		LS (non-conservative)	
	λ	λ	Error	λ	Error
(Hz)	(mm)	(mm)	(%)	(mm)	(%)
15	15.30	15.45	0.98	14.92	2.48
20	11.80	11.82	0.15	11.52	2.37
25	9.81	9.55	2.65	9.36	4.62
30	8.50	8.44	0.71	8.14	4.27
35	7.56	7.45	1.49	7.24	4.20
40	6.84	6.79	0.74	6.43	5.95

Table 5. Comparison of average wavelengths at frequency = 20 Hz

Interfacial Tension	Analytical	VoF		LS (non-conservative)	
	λ	λ	Error	λ	Error
(mN/m)	(mm)	(mm)	(%)	(mm)	(%)
20	8.38	8.23	1.85	8.01	4.36
30	9.32	9.17	1.59	8.95	4.01
40	10.08	9.93	1.46	9.67	4.10
50	10.73	10.67	0.53	11.00	2.52
60	11.29	11.20	0.80	10.64	5.76
70	11.80	11.82	0.15	11.52	2.37

calculations, as depicted in Fig. 7. The interface doesn't consistently exhibit a neatly sinusoidal appearance, as illustrated in Fig. 8, which can pose challenges in calculating the average wavelength. This figure also highlights the drop in the liquid level near the walls, mentioned earlier. Nevertheless, reasonably accurate results are obtained through straightforward manual calculations.

5 Conclusion

Capillary waves serve as a valuable tool for non-intrusive measurements of fundamental liquid properties. This is achieved through the dispersion relation, which establishes a connection between wave properties, interfacial tension, and viscosity. However, this relationship is based on certain assumptions, necessitating numerical simulations to assess its applicability in real-world scenarios.

In the present study, two different interface capturing methods, VoF and LS, are compared. While the conservative LS method enhances mass conservation, it leads to notably higher amplitudes compared to other approaches. The anomalous behavior observed in the near-wall region in the non-conservative LS method is not investigated further. Consequently, comparison of average wavelength values obtained from VoF and the non-conservative LS method with analytical results is performed for six different values each for excitation frequency and interfacial tension.

VoF consistently demonstrated superior accuracy with an average error rate of 1.09%, outperforming LS, which exhibits a higher average error rate of 3.92%. Furthermore, VoF provides superior results in other aspects of the wave, including amplitude and sinusoidal appearance. Lower error values indicate the applicability of linear wave theory for real-world case of finite-amplitude and finite-width waves with reflections. Because of obtaining very good accuracy with reflected waves in both parametric sweeps with VoF method, the average wavelength before the reflections is not studied. In conclusion, VoF method is selected for further numerical analysis.

References

1. Kundu, P.K.: Fluid Mechanics. Academic Press, London (1990)
2. Mirjalili, S., Jain, S.S., Dodd, M.: Interface-capturing methods for two-phase flows: an overview and recent developments. Center Turbulence Res. Ann. Res. Briefs **2017**(117–135), 13 (2017)
3. Deshpande, S.S., Anumolu, L., Trujillo, M.F.: Evaluating the performance of the two-phase flow solver interfoam. Comput. Sci. Discov. **5**(1), 014016 (2012)
4. Roenby, J., Bredmose, H., Jasak, H.: A computational method for sharp interface advection. Roy. Soc. Open Sci. **3**(11), 160405 (2016)
5. Olsson, E., Nilsson, H.: A description of isoadvector-a numerical method for improved suface sharpness in two-phase flows. Edited by Nilsson, H (2017)
6. Farsoiya, P.K.: Free and forced, linearised interfacial waves. Ph.D. thesis, Indian Institute of Technology, Bombay (2019)
7. CFD module user's guide. COMSOL Multiphysics v. 6.0 (2022)