Homework 2

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Due date: Thursday, February 7

1. (2 points) **Exercises 1.30:** If the NCAA has applications from six universities for hosting its intercollegiate tennis championships in two consecutive years, in how many ways can they select the hosts for these championships

(a) if they are not both to be held at the same university;

The number of permutations of n distinct objects taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

For the first year, $n = 6$ and $r = 1$. For the next year, $n = 5$ and $r = 1$. The total number of permutations is equal to the product of the number of permutations for both years.

$$_6P_1 \times \, _5P_1 \;=\; \frac{6!}{(6-1)!} \times \frac{5!}{(5-1)!} \;=\; 6 \times 5 \;=\; 30$$

(b) if they may both be held at the same university?

For both the first and second year, $n = 6$ and $r = 1$.

$$_6P_1 \times \, _6P_1 \;=\; \frac{6!}{(6-1)!} \times \frac{6!}{(6-1)!} \;=\; 6 \times 6 \;=\; 36$$

2. (2 points) **Exercises 1.43:**

(a) How many distinct permutations are there of the letters in the word "statistics"?

The permutations of n objects of k different kinds is given by

$$\binom{n}{n_1,\; n_2,\; \cdots\;,\; n_k} \;=\; \frac{n!}{n_1! \times n_2! \times \cdots \times n_k!}$$

In the word "statistics" there are $n = 10$ letters in total and $k = 5$ unique letters: s, t, a, i, and c. The respective count for each letter is 3, 3, 1, 2, and 1.

$$\binom{10}{3,\; 3,\; 1,\; 2,\; 1} \;=\; \frac{10!}{3! \times 3! \times 1! \times 2! \times 1!} \;=\; 50,400$$

```
> factorial(10)/(factorial(3)*factorial(3)*factorial(2))
[1] 50400
```

(b) How many of these begin and end with the letter `s`?

If two of the s letters are fixed, there are $n = 8$ total letters remaining that can be rearranged. Of these, the $k = 5$ unique letters s, t, a, i, and c have respective counts of 1, 3, 1, 2, and 1.

$$\binom{8}{1,\; 3,\; 1,\; 2,\; 1} \;=\; \frac{8!}{1! \times \; 3! \times 1! \times 2! \times 1!} \;=\; 3,360$$

```
> factorial(8)/(factorial(3)*factorial(2))
[1] 3360
```

3. (2 points) **Exercises 1.44:** A college team plays 10 football games during a season. In how many ways can it end the season with five wins, four losses, and one tie?

Here there are $n = 10$ events, and $k = 3$ unique types of events (win, loss, and tie).

$$\binom{10}{5,\ 4,\ 1} = \frac{10!}{5! \times 4! \times 1!} = 1,260$$

```
> factorial(10)/(factorial(5)*factorial(4))
[1] 1260
```

4. (2 points) The following codes gives a vector of -1's and 1's.

```
> c(-1, 1, -1, 1, -1, 1, -1, 1, -1, 1)
 [1] -1  1 -1  1 -1  1 -1  1 -1  1
> rep(c(-1, 1), 5)
 [1] -1  1 -1  1 -1  1 -1  1 -1  1
```

Give two more approaches that return the same vector of -1's and 1's.

```
> round(cos(9:0 * pi))
 [1] -1  1 -1  1 -1  1 -1  1 -1  1
> (-1)^(seq(from = 1, to = 10))
 [1] -1  1 -1  1 -1  1 -1  1 -1  1
```

5. (2 points) In **exercise 1.5** (page 16), the problem uses the formula

$$2\left[\binom{m-1}{m-1} + \binom{m}{m-1} + \cdots + \binom{2m-2}{m-1}\right]$$

to find the total number of different outcomes in a two-team basketball play-off that ends when a team wins m games. Use ***R*** and this formula to find the total number of different outcomes when $m = 20$.

The combinations formula for r objects taken from n distinct objects without regard to order of selection is

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

```
> m = 20
> r = m - 1
> n = 2*r
>
> s = seq(from = r, to = n)
> s
 [1] 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
>
> 2*sum(choose(s,r))
[1] 137846528820
```

$Total = 137,846,528,820$