

Induced gluon radiation in a QCD medium

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Abstract

Soft gluon radiation induced by multiple scattering of a fast quark or gluon propagating through QCD matter is discussed. After revisiting the Landau–Pomeranchuk–Migdal effect in QED we show that large formation times of bremsstrahlung quanta determine the QCD radiation intensity (in analogy to QED) and derive the gluon energy spectrum. Coherent suppression takes place as compared to the Bethe–Heitler situation of independent emissions. As a result the energy loss of fast partons in a QCD medium depends on the incident energy E similarly to QED, $-dE/dz \propto \sqrt{E}$.

1. Introduction

The radiative energy loss encountered by a charged fast particle which undergoes successive scatterings has been considered long ago in the framework of QED by Landau, Pomeranchuk and Migdal [1–4]. It is worth trying to reformulate the QED derivation with the aim of a generalization to QCD. Especially, this is of importance for understanding the energy loss mechanisms of quarks and gluons propagating through dense matter, such as a quark-gluon plasma [5,6].

To this end we adopt the model elaborated by Gyulassy and Wang [6] who have recently attempted the study of radiation induced by multiple scatterings of a colour charge (quark) in a QCD medium. This model disregards the collisional energy loss by intro-

ducing static scattering centres described by the Debye screened Coulomb potential resembling one gluon exchange. The assumption that the mean free path λ of the projectile is much larger than the screening radius, $\lambda \gg \mu^{-1}$, allows one to treat successive scatterings as *independent* and consider the induced radiation of *soft* gluons (photons) with energies $\omega \ll E$.

The soft approximation essentially simplifies the derivation and leads to an eikonal picture of classical propagation of a relativistic particle with energy $E \gg \mu$ which receives independent elastic kicks.

The QED emission amplitude that corresponds to a single scattering may be conveniently written in terms of a two-dimensional “angle” $u = k_{\perp}/\omega$ with k_{\perp} the transverse momentum of the photon with respect to the direction of the fast (massless) charge. This may be generalized to multiple scattering and the part of the full emission amplitude attached to the scattering centre $\#i$ reads

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$$J_i = \frac{u_i}{u_i^2} - \frac{u_{i-1}}{u_{i-1}^2}; \quad u_i = u_{i-1} - \frac{q_{\perp i}}{E}, \quad (1.1)$$

where $i-1$ and i mark the directions of incoming and outgoing charges respectively, and $q_{\perp i}$ stands for the momentum transfer.

An important role is played by the interference between the amplitudes (1.1). Thus the relative eikonal phase of the two amplitudes due to centres $\#i$ and $\#j$ reads in the relativistic kinematics

$$k^\mu (x_i - x_j)_\mu \approx \sum_{m=i}^{j-1} (z_m - z_{m+1}) / \tau_m(k);$$

$$\tau_m(k) = \frac{2\omega}{(k_{\perp m})^2} = \frac{2}{\omega u_m^2}, \quad (1.2)$$

with z_m the longitudinal coordinate position of the m th centre. $\tau(k)$ is usually referred to as a *formation time* of radiation.

For *small* formation times the phase is large and the centres act as independent sources of radiation. This is the Bethe–Heitler (BH) limit in which the radiation per unit length is maximal and proportional to the single scattering spectrum,

$$\omega \frac{dI}{d\omega dz} = \frac{1}{\lambda} \left(\omega \frac{dI}{d\omega} \right)_{(1)};$$

$$\left(\omega \frac{dI}{d\omega} \right)_{(1)} \propto \alpha, \quad (1.3)$$

with α the fine-structure constant. On the other hand, in the case of *large* formation times the phase vanishes and the amplitudes (1.1) with $i \leq m \leq j$ add up coherently which results in a *single* scattering amplitude

$$\sum_{m=i}^j J_m = \frac{u_j}{u_j^2} - \frac{u_{i-1}}{u_{i-1}^2}. \quad (1.4)$$

The radiation density gets suppressed by $(j-i)$. This coherent suppression ultimately leads to the so-called factorization limit.

Whether radiation follows the BH or the factorization pattern, depends on ω and properties of the medium. In the Migdal approach to the problem of electron propagation in matter, a Fokker–Planck type equation has been used in which the scattering properties of the medium are characterized by a single parameter

$$q = \langle \Theta_s^2 \rangle / \lambda; \quad \langle \Theta_s^2 \rangle = C \mu^2 / E^2, \quad (1.5)$$

with $\langle \Theta_s^2 \rangle$ the mean squared scattering angle due to a single kick. In agreement with qualitative arguments of [1], the radiation intensity which is found [2] exhibits suppression at small photon energies as compared to the BH regime (1.3):

$$\omega \frac{dI}{d\omega dz} \propto \alpha \sqrt{q \omega}. \quad (1.6)$$

This result may be easily illustrated qualitatively, as also discussed [3,7] using the notion of *coherence length*. To do that we may estimate the characteristic phase (1.2) as

$$\phi \equiv |k^\mu (x_i - x_j)_\mu| \approx \lambda \frac{\omega}{2} \sum_{m=0}^{j-i-1} u_{i+m}^2$$

$$\approx \frac{\lambda \omega}{2} \left\{ n u_i^2 + \frac{n(n-1)}{2} \left(\frac{\mu}{E} \right)^2 \right\}, \quad (1.7)$$

with $n \equiv (j-i)$. Here we treated the transverse movement of the charge as a random walk with a typical momentum transfer $\langle q_{\perp}^2 \rangle = \mu^2$: $u_{i+m}^2 = u_i^2 + m \mu^2 / E^2$.

Since the QED radiation according to (1.1) vanishes for emission angles larger than the scattering angle, it is legitimate to assume $u_i^2 \lesssim \langle \Theta_s^2 \rangle = \langle q_{\perp}^2 \rangle / E^2 = \mu^2 / E^2$. Therefore for large distances, $n \gg 1$, the last term in (1.7) dominates³.

To keep the phase small, $\phi < 1$, one has to restrict the maximal distance between the scattering points that act coherently [3,7], namely

$$n \lesssim N_{\text{coh}}(\omega) \equiv 2 \sqrt{\frac{E^2}{\lambda \mu^2 \omega}}. \quad (1.8)$$

Treating, effectively, a group of centres with $(j-i) < N_{\text{coh}}$ as a single radiator, one arrives at the following estimate for the radiation intensity (up to log factors)

$$\omega \frac{dI}{d\omega dz} = \frac{1}{\lambda} \left(\omega \frac{dI}{d\omega} \right)_{(1)} \cdot \frac{1}{N_{\text{coh}}(\omega)}$$

$$\propto \frac{\alpha}{\lambda N_{\text{coh}}} \propto \alpha \sqrt{\frac{\mu^2}{\lambda E^2} \omega}, \quad (1.9)$$

³ Strictly speaking, this is not true for the Rutherford scattering (see below). In this case, however, the two terms of (1.7) are of the same order, and the qualitative estimate (1.9) still holds.

which expression coincides with the Migdal result (1.6).

However, in the pure Coulomb case, for which the potential is not screened at *short* distances, such that q becomes logarithmically divergent for large incident energies ($q \sim \log E$), the Migdal approach is not adequate. In view of the application to QCD, it is important to describe the energy spectrum of photons in the case of a potential that has pure Coulomb behaviour at large momentum transfer [8]. Below we present the main steps of the derivation and the result, which slightly differs from the canonical one [2] by an extra logarithmic energy dependent enhancement factor which is due to fluctuations with emission/scattering angles much larger than typical μ/E (see (2.14) below).

2. Revisiting the LPM spectrum in QED

The differential soft photon spectrum can be written as

$$\begin{aligned} \omega \frac{dI}{d\omega d^2u} &= \frac{\alpha}{\pi^2} \left\langle \left| \sum_{i=1}^N \mathbf{J}_i e^{ik_\mu x_i^\mu} \right|^2 \right\rangle \\ &= \frac{\alpha}{\pi^2} \left\langle 2 \operatorname{Re} \sum_{i=1}^N \sum_{j=i+1}^N \mathbf{J}_i \mathbf{J}_j \left[e^{ik_\mu (x_i - x_j)^\mu} - 1 \right] \right. \\ &\quad \left. + \left| \sum_{i=1}^N \mathbf{J}_i \right|^2 \right\rangle. \end{aligned} \quad (2.1)$$

The brackets $\langle \rangle$ indicate the averaging procedure discussed in [6] and hereafter. The differential energy distribution of photons radiated *per unit length* is given by

$$\begin{aligned} \omega \frac{dI}{d\omega dz} &= \lambda^{-1} \frac{\alpha}{\pi} \int \frac{d^2 U_0}{\pi} 2 \operatorname{Re} \sum_{n=0}^{\infty} \mathbf{J}_1 \mathbf{J}_{n+2} \\ &\quad \times \left[\exp \left(i\kappa \sum_{\ell=1}^{n+1} U_\ell^2 \frac{z_{\ell+1} - z_\ell}{\lambda} \right) - 1 \right]. \end{aligned} \quad (2.2)$$

Here we have expressed the relative phases in terms of the *rescaled* angular variable,

$$U \equiv \mathbf{u} \cdot \frac{\mathbf{E}}{\mu} \quad \left(\mathbf{u} \equiv \frac{\mathbf{k}_\perp}{\omega} \right), \quad (2.3)$$

which measures the relative photon angle in units of a typical scattering angle $\Theta_1 = \mu/E$. We have also introduced the characteristic parameter

$$\kappa = N_{\text{coh}}^{-2} = \frac{1}{2} \lambda \mu^2 \frac{\omega}{E^2} \ll 1. \quad (2.4)$$

In (2.2) we have taken advantage of the facts that 1) the last term in (2.1), corresponding to the factorization limit, remains finite and therefore does not contribute to the radiation *density*, and 2) the internal sum becomes i -independent for large N . The latter property allows one to suppress the index $\#i$ in (2.2) and keep only the number of kicks n between the scattering points 1 (former $\#i$) and $n+2$ ($\#j$).

It is worthwhile to note that only the expression integrated over photon angles has a well defined $N \rightarrow \infty$ limit. Meanwhile, the *angular spectrum* per unit length is meaningless since the direction of the charge inside the medium, which enters in the definition of U_0 cannot be precisely specified.

For the averaging procedure, two steps have to be implemented in (2.2). First, one has to integrate over the longitudinal separation between successive scattering centres $\Delta_\ell = z_{\ell+1} - z_\ell$ with the normalized distribution probability

$$\prod_{\ell=1}^{n+1} \frac{d\Delta_\ell}{\lambda} \exp \left(-\frac{\Delta_\ell}{\lambda} \right). \quad (2.5)$$

This results in substituting for the square brackets in (2.2) the expression

$$\prod_{\ell=1}^{n+1} \psi(U_\ell^2) - 1; \quad \psi(U^2) = (1 - i\kappa U^2)^{-1}. \quad (2.6)$$

Secondly, an averaging over momentum transfers $\mathbf{q}_{\perp\ell}$ should be performed with the distribution corresponding to the screened Coulomb potential scattering:

$$\begin{aligned} \prod_{\ell=1}^{n+2} dV(\mathbf{q}_\ell); \quad dV(\mathbf{q}_\ell) &= \frac{\mu^2 d^2 q_\ell}{\pi(q_\ell^2 + \mu^2)^2}; \\ \int dV(\mathbf{q}_\ell) &= 1. \end{aligned} \quad (2.7)$$

Corresponding integrals may be evaluated analytically in the large angle approximation, $U^2 \gg 1$ (i.e., $u^2 \gg (\mu/E)^2$), which region a posteriori proves to give the main contribution to the energy spectrum [8].

The result may be written in the following form

$$\omega \frac{dI}{d\omega dz} = \lambda^{-1} \frac{\alpha}{\pi} \int_0^\infty \frac{dU^2}{U^2 + 1} \times 2 \operatorname{Re} \sum_{n=0}^\infty [f_n(U^2; 0) - f_n(U^2; \kappa)], \quad (2.8)$$

where the n -kick contributions satisfy approximately the recurrency relation

$$f_{n+1}(U^2) = \psi(U^2) f_n(U^2 + 1) \cdot \{1 + \mathcal{O}(U^{-4})\}, \quad (2.9a)$$

$$f_0(U^2) = \frac{\psi(U^2)}{U^2(U^2 + 1)}; \quad f_n(U^2) = \frac{\prod_{\ell=0}^n \psi(U^2 + \ell)}{(U^2 + n)(U^2 + n + 1)}. \quad (2.9b)$$

The dependence on κ enters implicitly via the ψ factors. The characteristic number of terms essential in (2.8) is estimated as

$$\langle n \rangle \sim U^2. \quad (2.10)$$

Finally, one arrives [8] at

$$\omega \frac{dI}{d\omega dz} = 2\lambda^{-1} \frac{\alpha}{\pi} \int_0^\infty \frac{dU^2}{U^2(U^2 + 1)} \Phi(\kappa U^4); \quad \Phi(x) = \int_0^\infty \frac{dt}{(t+1)^{3/2}} \sin^2 \frac{xt}{4}. \quad (2.11)$$

For large values of the argument $\kappa U^4 \sim \langle n \rangle^2 / N_{\text{coh}}^2 > 1$ the Φ factor tends to unity, which corresponds to the BH limit, which is in accord with the qualitative estimate given in the previous section. On the other hand, in the region $U^2 \ll \kappa^{-1/2}$ the BH law gets modified,

$$\Phi(x) \approx \frac{\sqrt{\pi}}{2} \sqrt{x} \quad (x \ll 1); \quad \omega \frac{dI}{d\omega dz} \approx \frac{\alpha}{\pi} \frac{\sqrt{\pi\kappa}}{\lambda} \int_0^{1/\sqrt{\kappa}} \frac{dU^2}{U^2 + 1}. \quad (2.12)$$

As a result, the main contribution originates from a broad logarithmic region

$$1 \ll U^2 \ll \kappa^{-1/2}. \quad (2.13)$$

The resulting radiation density reads

$$\omega \frac{dI}{d\omega dz} \approx \frac{\alpha}{\pi} \frac{\sqrt{\pi\kappa}}{2\lambda} \ln(\kappa^{-1}) = \frac{\alpha}{2\lambda} \sqrt{\frac{\lambda\mu^2 \omega}{2\pi E^2}} \ln \frac{2E^2}{\lambda\mu^2 \omega}. \quad (2.14)$$

The above derivation has been based on the approximation $1 \ll \langle n \rangle \sim U^2 < N_{\text{coh}} = \kappa^{-1/2}$. This implies that the photon spectrum is suppressed for energies below a specific value given by:

$$\kappa < 1 \iff \omega < \omega_{\text{BH}} = \frac{2E^2}{\mu^2 \lambda} \equiv E \left(\frac{E}{E_{\text{LPM}}} \right), \quad E_{\text{LPM}} = \frac{1}{2} \mu^2 \lambda. \quad (2.15a)$$

Photons with $\omega > \omega_{\text{BH}}$ are radiated according to the BH law.

On the other hand, we treated the medium as being large enough in the longitudinal direction to embody N_{coh} successive scatterings. For a medium of finite size L this condition limits photon energies from below,

$$\lambda N_{\text{coh}} < L \iff \omega > \omega_{\text{fact}} = \frac{\lambda E^2}{L^2 \mu^2} \equiv E \left(\frac{L_{\text{cr}}}{L} \right)^2, \quad L_{\text{cr}} = \sqrt{\frac{\lambda E}{\mu^2}}. \quad (2.15b)$$

Photon radiation with $\omega < \omega_{\text{fact}}$ corresponds to the factorization limit.

The LPM effect may be observed in the photon energy range

$$\frac{L_{\text{cr}}^2}{L^2} < \frac{\omega}{E} < \frac{E}{E_{\text{LPM}}}, \quad (2.16)$$

which obviously demands the size of the medium to be big enough, $L > \max \left\{ \lambda, \sqrt{\lambda E / \mu^2} \right\}$.

3. LPM effect and energy dependent energy loss in a QCD medium

In this section we consider gluon radiation in a QCD medium, induced by multiple scattering of an energetic quark due to one gluon exchange with a static centre. An essential feature of the QCD model is that

the accompanying radiation does not vanish in the $E \rightarrow \infty$ limit as it was the case for QED scattering. The reason is simple: gluon emission does not vanish in the case of forward scattering, $\Theta_s \rightarrow 0$, due to “re-painting” of the incident quark via colour exchange.

This makes it possible to simplify the treatment and consider the quark as moving along the z axis. In such an approximation the QED-like part of the induced radiation gets suppressed and only the dominant specific non-Abelian contribution survives. The radiation amplitude for the latter is proportional to the commutator of colour generators, $[T^a, T^b] = if_{abc}T^c$, with a, b the colour indices of the emitted gluon and of the octet potential, respectively. In the soft approximation the basic amplitude reads

$$J_i = \frac{\mathbf{k}_\perp}{k_\perp^2} - \frac{\mathbf{k}_\perp - \mathbf{q}_{\perp i}}{(\mathbf{k}_\perp - \mathbf{q}_{\perp i})^2}. \quad (3.1)$$

It combines the three contributions with the same eikonal phase $\exp(i z_i k_\perp^2 / 2\omega)$ adjusted to the position of the centre $\#i$, which originate from Feynman diagrams for the gluon emission off incoming and outgoing quark lines as well as from the exchanged gluon line (via 3-gluon coupling).

Hereafter we depict the effective eikonal current (3.1) by a blob attached to the gluon line (as shown in Fig. 1 below) which convention is chosen to stress the if_{abc} colour structure of the current.

Gauge invariance can be enforced, as discussed in detail in [[9]]. To this end we remark here that the only way to construct a simple self-consistent first order QCD model that embodies the screened gluon potential is to ascribe to the gluon field a finite mass μ which acts as screening parameter for space-like exchange and as particle mass for the real emission. In the following, for the sake of simplification, we shall be using, however, the unmodified current (3.1) which a posteriori will be justified by the fact that the resulting expression for the induced radiation density proves to be collinear safe and dominated by $k_\perp^2 \gg \mu^2$.

3.1. Gyulassy–Wang treatment of induced gluon radiation

As a first step, we focus on quark rescattering processes in the spirit of Gyulassy–Wang’s approach [6].

The derivation basically follows that of the QED case, with the only essential difference coming from the fact that the contributions associated to successive quark rescatterings between $\#i$ and $\#j$ centres acquire colour factors $(-(2N_c C_F)^{-1})^{j-i-1} = (-N_c^2 + 1)^{-(j-i-1)}$. This leads to colour suppression as implied by the nonplanar nature of the corresponding diagrams.

As long as it is not the angle but the transverse momentum which is relevant for the emission amplitude (cf. (1.1) and (3.1)), we express the eikonal phase difference (1.2) in terms of $(\mathbf{k}_{\perp m})^2 = k_\perp^2$ which remains m -independent within the adopted approximation ($E \gg \mu$). After averaging over the position of scattering centres with the weight (2.5) where λ now stands for the quark mean free path, the ψ factors emerge

$$\psi(k_\perp^2) = (1 - i\kappa k_\perp^2 / \mu^2)^{-1} \equiv (1 - i\kappa U^2)^{-1}. \quad (3.2)$$

Here we have introduced the characteristic QCD parameter

$$\kappa = \frac{1}{2} \lambda \mu^2 / \omega \ll 1. \quad (3.3)$$

The two-dimensional vector U now stands for the gluon transverse momentum measured in units of μ .

To integrate the product of the currents $J_i J_j$ over transferred momenta $\mathbf{q}_i, \mathbf{q}_j$ one uses

$$\int \frac{d^2 q}{\pi} \frac{\mathbf{k}_\perp - \mathbf{q}}{(\mathbf{k}_\perp - \mathbf{q})^2} f(q^2) = \frac{\mathbf{k}_\perp}{k_\perp^2} \int_0^{k_\perp^2} dq^2 f(q^2), \quad (3.4)$$

and arrives at the radiation density

$$\begin{aligned} \omega \frac{dI}{d\omega dz} &= \lambda^{-1} \frac{N_c \alpha_s}{\pi} \int_0^\infty \frac{dU^2}{U^2 (U^2 + 1)^2} \\ &\times \text{Re} [g(1) - g(\psi(U^2))] ; \\ g(\psi) &\equiv \frac{N_c}{2C_F} \psi \sum_{n=0}^\infty \left(-\frac{\psi}{2N_c C_F} \right)^n \\ &= \frac{\psi}{1 + (\psi - 1)/N_c^2} \quad (n = j - i - 1). \end{aligned} \quad (3.5)$$

It is clear that in the large N_c limit the answer is determined by the interference between the nearest neighbours, $j = i + 1$. Straightforward algebra leads to

$$\begin{aligned} \frac{\omega dI}{d\omega dz} &= \frac{N_c \alpha_s}{\pi \lambda} \\ &\times \int_0^\infty \frac{U^2 dU^2}{(U^2 + 1)^2} \left[U^4 + \left(\frac{N_c \omega}{C_F \lambda \mu^2} \right)^2 \right]^{-1} \\ &= \frac{N_c \alpha_s}{\pi \lambda} \int_0^\infty \frac{\mu^4 dk_\perp^2}{k_\perp^2 (k_\perp^2 + \mu^2)^2} \left[1 + \left(\frac{N_c \tau}{2C_F \lambda} \right)^2 \right]^{-1} \end{aligned} \quad (3.6)$$

Except in the case $\lambda \mu^2 \gg E$ where one formally recovers the BH limit, (3.6) leads to a sharply falling gluon energy distribution $\omega dI/d\omega \propto \omega^{-2}$ and, therefore, to finite energy losses

$$-dE/dz \sim \text{const} \cdot \alpha_s \mu^2. \quad (3.7)$$

This qualitatively agrees, up to $\log E$ factors, with the conclusions of [6].

The origin of this result is quite simple: in the quark rescattering model the gluon radiation with formation time τ (cf. (1.2)) exceeding λ is negligible, which severely limits gluon energies:

$$\omega = \frac{1}{2} \tau k_\perp^2 \lesssim \lambda q_\perp^2 \sim \lambda \mu^2.$$

The fact that the quark after having emitted a gluon at point $\#i$ effectively stops interacting with the medium looks as if it had lost its “colour charge”. The truth is, however, that the colour charge has not disappeared but has been transferred to the radiated gluon.

3.2. Radiated gluon rescattering in the large N_c limit and the analogy with QED

The diagrams for the product of the emission currents $J_i J_j$ which are displayed in Fig. 1 dominate in the large N_c limit.

Let us consider the Feynman diagrams with gluon self-interaction as shown in Fig. 2. Integration over the position of the quark-gluon interaction vertex t between the successive scattering centres, $z_{i-1} \leq t \leq z_i$, gives rise to two contributions with the phase factors attached to z_i and z_{i-1} . The first term in the rhs of Fig.

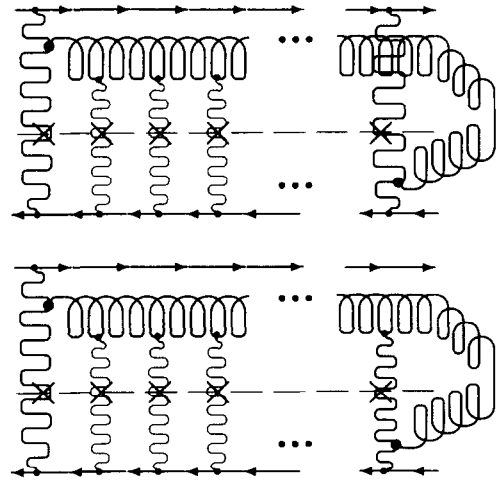


Fig. 1. Eikonal graphs with gluon rescattering dominate in the large N_c limit. The part of the diagram below the dashed line corresponds to the conjugate emission amplitude. Crosses mark the scattering centres.

2 corresponds to an instantaneous interaction mediated by the *virtual* (Coulomb) gluon ($k - q_i$) and is responsible for the second term of the basic scattering amplitude (3.1). The second one corresponds to the production of the *real* gluon ($k - q_i$) in the previous interaction point, which then rescatters at the centre $\#i$.

The sum of these two terms is proportional to the difference of the two phase factors. Therefore when phases are set to zero (factorization limit), gluon production *inside* the medium cancels. The only contributions which survive correspond to the gluon radiation at the very first and the very last interaction vertices. As a result, the subtraction term analogous to $|\sum J_i|^2$ of (2.1) remains N -independent and does not contribute to the radiation *density*. In the QCD case this statement is less transparent than in QED, since the initially produced gluon is still subject to multiple rescattering in the medium. The latter, however, does not affect the *energy* distribution of radiation.

A similar argument applies to the final state interaction after the point $\#j$ which has been neglected in Fig. 1: Coulomb interaction of the quark-gluon pair with the medium does affect the transverse momentum distribution of the gluon but does not change its energy spectrum, the only quantity which concerns us here.

It is worthwhile to mention that in the QCD problem the very notion of scattering cross section (and, thus, of the mean free path λ) becomes elusive: the

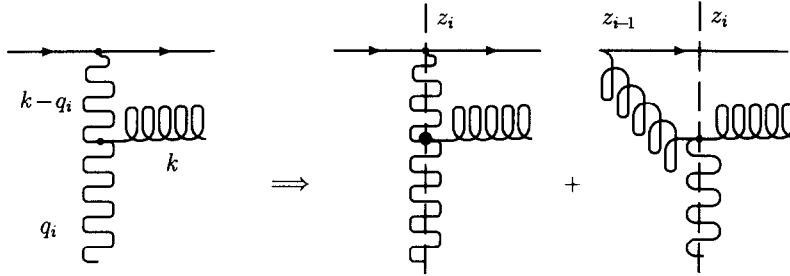


Fig. 2. Feynman 3-gluon diagram produces two eikonal graphs: it participates in the gluon production current (second term of the amplitude (3.1)) and gives rise to Coulomb rescattering.

scattering cross section of the quark-gluon pair depends on the transverse size of the 2-particle system and therefore is z -dependent. As we shall see below, the radiation intensity of gluons with $\omega \gg \lambda\mu^2$ is due to comparatively *large* formation times $\tau \gg \lambda$ but not large enough for the quark and the gluon to separate in the impact parameter space. Such a pair as a whole acts as if it were a single quark propagating through the medium. Therefore the mean free path of the quark, $\lambda \propto C_F^{-1}$, may be effectively used for the eikonal averaging in the spirit of (2.5). Better formalized considerations will be given elsewhere [9].

The emission current (3.1) and the structure of the diffusion in the variable u prove to be identical for the QCD and QED problems. The only but essential difference is that now u has to be related to the *transverse momentum* instead of the *angle* of the gluon:

$$\begin{aligned} \text{QED: } u_i &= \frac{k_{\perp i}}{\omega}, \quad U_i = u_i \frac{E}{\mu}; \quad \kappa = \frac{\lambda\mu^2}{2} \frac{\omega}{E^2} \ll 1 \\ \Rightarrow \\ \text{QCD: } u_i &= k_{\perp i}, \quad U_i = \frac{u_i}{\mu}; \quad \kappa = \frac{\lambda\mu^2}{2\omega} \ll 1. \end{aligned} \quad (3.8)$$

Correspondingly, the κ parameter gets modified according to the QCD expression (3.3).

Given this analogy, the QCD derivation follows that of the previous section. The graph of Fig. 1b is twice the graph of Fig. 1a due to colour factors. One finds the gluon energy distribution (cf. (2.11))

$$\begin{aligned} \omega \frac{dI}{d\omega dz} &= \frac{3N_c\alpha_s}{\pi\lambda} \int_0^\infty \frac{dU^2}{U^2(U^2+1)} \Phi(\kappa U^4) \\ &\approx \frac{3N_c\alpha_s}{4\lambda} \sqrt{\frac{\lambda\mu^2}{2\pi\omega}} \ln \frac{2\omega}{\lambda\mu^2}. \end{aligned} \quad (3.9)$$

3.3. Rescattering of the incident parton and the complete spectrum

Now we are in a position to complete the analysis of induced QCD radiation by taking into account subdominant quark rescattering contributions.

Each gluon scattering provides the phase factor $\psi(U^2)$ accompanied by the colour factor $N_c/2$ which one normalizes by the quark elastic scattering factor $C_F^{-1} \propto \lambda_q$. Each quark scattering graph supplies the colour factor $(-1/2N_c) \cdot C_F^{-1} = (C_F - N_c/2)C_F^{-1}$ and the same ψ factor (since the gluon momentum remains unaffected, and the change in the quark direction is negligible). Accounting for an arbitrary number of quark rescatterings in-between two successive gluon rescatterings results in a modified phase factor:

$$\begin{aligned} \frac{N_c}{2C_F} \psi \cdot \sum_{m=0}^\infty \psi^m \left(1 - \frac{N_c}{2C_F}\right)^m \\ = \left(1 + \frac{2C_F}{N_c} [\psi^{-1} - 1]\right)^{-1} \equiv \tilde{\psi}; \end{aligned} \quad (3.10a)$$

$$\begin{aligned} \tilde{\psi}(U^2) &= (1 - i\tilde{\kappa}U^2)^{-1}; \\ \tilde{\kappa} &\equiv \frac{\tilde{\lambda}\mu^2}{2\omega}, \quad C_F\lambda_q \equiv \frac{N_c}{2} \tilde{\lambda}. \end{aligned} \quad (3.10b)$$

It is worthwhile to note that the modified effective mean free path $\tilde{\lambda}$ does not depend on the nature of the colour representation R of the initial particle. For example, in the case of the *gluon* substituted for the initial quark, Coulomb rescattering of both projectile and radiated gluons supplies the colour factor $N_c/2$, while the elastic cross section provides the normalization $N_c^{-1} \propto \lambda_g$. Thus, since $N_c\lambda_g = C_F\lambda_q$, the same

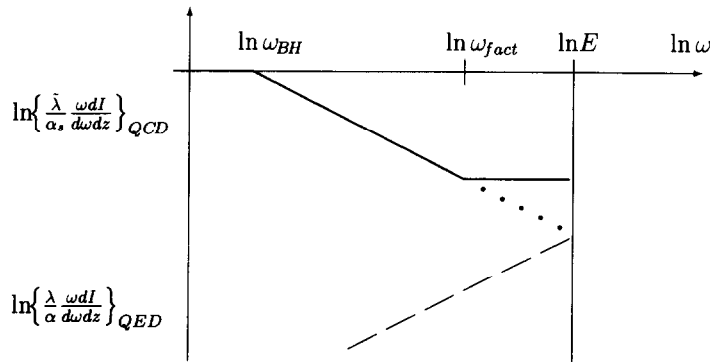


Fig. 3. The normalized QCD radiation density (solid line) for a finite size medium ($L < L_{cr}$). The QED LPM spectrum ($E > E_{LPM}$) is shown by the dashed line.

result follows:

$$\frac{N_c}{2} N_c^{-1} \psi \cdot \sum_{m=0}^{\infty} \left(\frac{N_c}{2} N_c^{-1} \psi \right)^m = \tilde{\psi}. \quad (3.11)$$

In general, for an arbitrary colour state R one has to replace C_F in (3.10a) by a proper Casimir operator C_R which dependence then cancels against the C_R^{-1} factor that enters the expression for the mean free path of a particle R :

$$C_F \lambda_q = N_c \lambda_g = C_R \lambda_R = \frac{1}{2} N_c \tilde{\lambda}.$$

Thus, (3.9) holds provided one replaces κ by the modified $\tilde{\kappa}$ according to (3.10b). To factor out the dependence on the type of the projectile one may express the answer in terms of the gluon mean free path $\lambda_g = \frac{1}{2} \tilde{\lambda}$:

$$\begin{aligned} \omega \frac{dI}{d\omega dz} &= 3 \cdot \frac{C_R \alpha_s}{\pi \lambda_g} \int_0^{\infty} \frac{dU^2}{U^2(U^2 + 1)} \Phi(\tilde{\kappa} U^4) \\ &\approx \frac{3 C_R \alpha_s}{4 \lambda_g} \sqrt{\frac{\lambda_g \mu^2}{\pi \omega}} \left(\ln \frac{\omega}{\lambda_g \mu^2} + \mathcal{O}(1) \right). \end{aligned} \quad (3.12)$$

This is our final result for the QCD induced radiation spectrum. It has been derived with logarithmic accuracy in the energy region

$$\omega / \lambda_g \mu^2 = \tilde{\kappa}^{-1} \gg 1. \quad (3.13)$$

It is this condition which, a posteriori, justifies the use of the quark scattering cross section for the quark-gluon system. Indeed, the random walk estimate for

the transverse separation between q and g in course of $\langle n \rangle \lesssim 1/\sqrt{\kappa}$ kicks gives

$$\begin{aligned} \Delta \rho_{\perp} &\approx \lambda \sum_{m=1}^{\langle n \rangle} k_{\perp m} / \omega : \\ (\Delta \rho_{\perp})^2 &\sim \frac{\lambda^2 \mu^2}{\omega^2} \langle n \rangle^2 \lesssim \kappa \mu^{-2} \ll \mu^{-2}. \end{aligned} \quad (3.14)$$

Given the separation which is much *smaller* than the radius of the potential, the mean free path of the qg system in the medium coincides with that of a single quark.

A comment concerning the factorization and BH regime limits of the spectrum is in order and proceeds along the same lines as in QED (see (2.15)). For a medium of finite size $L < L_{cr} = \sqrt{\lambda_g E / \mu^2}$ the ω -independent factorization limit holds for energetic gluons with

$$\begin{aligned} \omega &\gtrsim \omega_{fact} = E (L / L_{cr})^2 \\ (N_{coh} = \tilde{\kappa}^{-1/2} &\gtrsim L / \lambda_g). \end{aligned} \quad (3.15a)$$

The BH regime corresponds formally to small gluon energies⁴,

$$\omega \lesssim \omega_{BH} = \lambda_g \mu^2 \quad (\tilde{\kappa} \gtrsim 1). \quad (3.15b)$$

⁴ Note that for $\kappa \gg 1$ the separation between the colour charges according to (3.14) becomes much *larger* than $1/\mu$. In these circumstances the normalization cross section tends to the sum of independent q and g contributions, $C_F \Rightarrow C_F + N_c \approx 3C_F$. The factor 3 in (3.12) disappears, leading to the standard BH expression.

The spectrum (3.12) is depicted in Fig. 3 together with the QED spectrum. When it is integrated over ω up to E , it leads to energy losses

$$-\frac{dE}{dz} \propto C_R \alpha_s \sqrt{E \frac{\mu^2}{\lambda_g}} \ln \frac{E}{\lambda_g \mu^2} \quad (\text{for } L > L_{\text{cr}}). \quad (3.16)$$

4. Discussion and concluding remarks

In this letter we have considered induced soft gluon radiation off a fast colour charge propagating through the medium composed of static QCD Coulomb centres.

Due to the colour nature of the scattering potential, the specific non-Abelian contributions to the gluon yield dominate over the QED-like radiation for all gluon energies (up to $\omega \lesssim E$). They have been singled out here by choosing the $E \rightarrow \infty$ limit in which the Abelian radiation vanishes.

Having established a close analogy between the angular structure of the QED problem and the transverse momentum structure of the QCD problem, one can qualitatively obtain the gluon energy density spectrum from the known QED result by the substitution $\omega/E^2 \rightarrow 1/\omega$. The spectrum of gluon radiation is E -independent and, analogously to the Landau-Pomeranchuk-Migdal effect in QED, acquires coherent suppression as compared to the Bethe-Heitler regime of independent emissions:

$$\omega \frac{dI}{d\omega dz} \propto \lambda_g^{-1} \alpha_s \cdot \sqrt{\frac{\lambda_g \mu^2}{\omega}} \ln \frac{\omega}{\lambda_g \mu^2} \quad \left(\frac{\lambda_g \mu^2}{\omega} \equiv \tilde{\kappa} = N_{\text{coh}}^{-2} \ll 1 \right). \quad (4.1)$$

As a result, the radiative energy loss per unit length amounts to

$$-dE/dz \propto \alpha_s \sqrt{E \mu^2 / \lambda_g} \ln(E / \lambda_g \mu^2). \quad (4.2)$$

This is in apparent contradiction with the statements existing in the literature on the subject. In particular, it contradicts the recent conclusion presented in [6] about the *finite* density of energy losses.

As discussed above, the latter statement is due to a sharply falling energy spectrum $\omega dI/d\omega dz \propto \omega^{-2}$

which one finds within the approach disregarding Coulomb rescattering of the radiated gluon. Only gluon radiation with restricted formation time $\tau \lesssim \lambda$ survives such a treatment, while (4.1) is dominated by large formation times $\lambda \ll \tau \ll \lambda N_{\text{coh}}$.

It is interesting to note that in spite of the fact that large momentum transfers do not essentially contribute to the scattering cross section, $d\sigma/dq_{\perp}^2 \propto q_{\perp}^{-4}$ for $q_{\perp}^2 \gg \mu^2$, the answer is actually related to *hard* scatterings. The logarithmic enhancement factor in (4.1) originates from interference between two radiation amplitudes: one is related to a “typical” scattering with $q_n^2 \sim \mu^2$, while the other corresponds to a hard fluctuation with a very large momentum transfer up to $q_1^2 \sim \mu^2 N_{\text{coh}} \gg \mu^2$. These amplitudes get a chance to interfere because of the random walk in the gluon transverse momentum in a course of $n \sim k_{\perp}^2 / \mu^2 \lesssim N_{\text{coh}}$ rescatterings with typical $q_m^2 \sim \mu^2$. Such a fluctuation actually is not rare: the probability that at least one of n scatterings will supply the momentum transfer q_{\perp}^2 exceeding $n\mu^2$ amounts to $1 - (1 - 1/n)^n \sim 1$.

Our results should be directly applicable to “hot” (deconfined) plasma (with $\lambda \gg 1/\mu$) in which case the model potential (2.7) with μ as a screening parameter could be taken at its face value. The structure of the spectrum (4.1) is such that one has to know, strictly speaking, the mean free path λ separately from the screening mass μ at high temperature. This is in contrast to the Migdal approach extended to QCD [9], in which the rhs of (4.1) is given by $\alpha_s \sqrt{q/\omega}$, i.e. the plasma properties only enter via the transport coefficient $q \equiv \langle q_{\perp}^2 \rangle / \lambda$.

At the same time, one might think of applying the above consideration to the propagation of a fast quark/gluon through “cold” nuclear matter. In such a case the parameter μ has to be introduced by hand as a lower bound for transverse momenta which one may treat perturbatively. Given the ill-defined nature of μ it is worthwhile to note that the inverse size of the potential and the mean free path enter (4.1) in the combination $\mu^2 / \lambda_g = \mu^2 \rho \sigma_g$, with σ_g the gluon scattering cross section. This makes it possible to express the results in terms of the physical density of scattering centers ρ and the dimensionless parameter $\theta = \sigma_g \mu^2 \propto N_c \alpha_s^2$ which measures the strength of gluon interaction with the medium and should be much less sensitive to a finite uncertainty in μ .

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References

- [1] L.D. Landau and I.Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 92 (1953) 535, 735.
- [2] A.B. Migdal, Phys. Rev. 103 (1956) 1811, and references therein.
- [3] E.L. Feinberg and I.Ya. Pomeranchuk, Nuovo Cimento Suppl. III (1956) 652.
- [4] See, e.g.: M.L. Ter-Mikaelian, High Energy Electromagnetic Processes in Condensed Media (John Wiley & Sons, 1972); A.I. Akhiezer and N.F. Shul’ga, Sov. Phys. Usp. 30 (1987) 197, and references therein.
- [5] J.D. Bjorken, Fermilab report PUB-82/59-THY, 1982 (unpublished);
M. Gyulassy et al., Nucl. Phys. A 538 (1992) 37c, and references therein.
- [6] M. Gyulassy and X.-N. Wang, Nucl. Phys. B 420 (1994) 583;
see also X.-N. Wang, M. Gyulassy and M. Plümer, preprint LBL-35980, August 1994.
- [7] V.M. Galitsky and I.I. Gurevich, Nuovo Cimento 32 (1964) 396;
A.H. Sørensen, Z. Phys. C 53 (1992) 595.
- [8] Yu.L. Dokshitzer, in preparation.
- [9] R. Baier, Yu.L. Dokshitzer, S. Peigné and D. Schiff, in preparation.