

# Determining the jet transport coefficient $\hat{q}$ of the quark-gluon plasma using Bayesian parameter estimation

James Mulligan<sup>1,2</sup> on behalf of the JETSCAPE Collaboration

<sup>1</sup>*Physics Department, University of California, Berkeley, CA 94720, USA*

<sup>2</sup>*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

We present a new determination of  $\hat{q}$ , the jet transport coefficient of the quark-gluon plasma. Using the JETSCAPE framework, we use Bayesian parameter estimation to constrain the dependence of  $\hat{q}$  on the jet energy, virtuality, and medium temperature from experimental measurements of inclusive hadron suppression in Au-Au collisions at RHIC and Pb-Pb collisions at the LHC. These results are based on a multi-stage theoretical approach to in-medium jet evolution with the MATTER and LBT jet quenching models. The functional dependence of  $\hat{q}$  on jet energy, virtuality, and medium temperature is based on a perturbative picture of in-medium scattering, with components reflecting the different regimes of applicability of MATTER and LBT. The correlation of experimental systematic uncertainties is accounted for in the parameter extraction. These results provide state-of-the-art constraints on  $\hat{q}$  and lay the groundwork to extract additional properties of the quark-gluon plasma from jet measurements in heavy-ion collisions.

## 1 Introduction

At high temperatures, quantum chromodynamics (QCD) exhibits a deconfined state of matter known as the quark-gluon plasma (QGP).<sup>1,2</sup> The nature of the degrees of freedom of the QGP, however, remains largely unknown. By studying this state, we seek to understand how complex behaviors arise from QCD, including how strongly-coupled systems and their bulk properties emerge from quantum field theory.

Jets provide a compelling tool to pursue these questions. Depending on their transverse momentum ( $p_T$ ) and substructure, jets can probe from the smallest medium scales to the largest medium scales, and jet evolution can be computed from first principles. Accordingly, a major experimental and theoretical jet physics program has developed over the last two decades at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), where ultrarelativistic heavy-ion collisions create short-lived droplets of QGP.<sup>3,4</sup>

This jet quenching program faces two major challenges. Firstly, jet evolution in heavy-ion collisions involves multiple stages of physics that are not known from first principles, such as the initial state of the collision, the hydrodynamic evolution of the medium, and the hadronization phase. This can be addressed by global analyses that fit phenomenological models to a wide range of experimental data, and has recently been solved to a large extent.<sup>5,6,7,8,9</sup> A second challenge, however, is that jet evolution in the QGP, even for a perfectly characterized medium, involves numerous theoretical unknowns, from the strength of coupling of the jet-medium interaction to the role of factorization breaking and color coherence. No (known) golden observable exists to disentangle these open questions about the jet-medium interaction. This necessitates global analyses of multiple jet observables. Such global analyses provide a viable path not only to disentangle various theoretical approaches describing the jet-medium interaction but also to

precisely determine medium properties from experimental measurements – the ultimate goal of studying the QGP.

In this work, we focus on the medium property known as the jet transverse momentum diffusion coefficient,  $\hat{q}$ , which describes the average transverse momentum,  $\langle k_{\perp} \rangle$ , acquired by a parton as it traverses a given length  $L$  of QGP. This transport coefficient is agnostic to the microscopic interactions that generate the transverse diffusion, but rather characterizes the overall accumulated transverse momentum. The transport coefficient  $\hat{q}$  has been calculated under certain approximations,<sup>10,11,12,13,14,15,16,17</sup> but in general involves nonperturbative contributions that must be either computed on a lattice<sup>18,19</sup> or extracted from experimental measurements.<sup>20,21,22,23</sup>

We present a proof-of-principle determination of  $\hat{q}$  using Bayesian parameter estimation – the first such extraction using Bayesian techniques – laying the groundwork to extract additional properties of the quark-gluon plasma from jet measurements in heavy-ion collisions.<sup>24</sup>

## 2 Analysis

We perform an extraction of  $\hat{q}$  using a selection of inclusive hadron  $R_{AA}$  data at RHIC and the LHC.<sup>25,26,27</sup> We consider both central and semi-central data, shown in Fig. 1. In order to properly assess the uncertainty correlations within and between the experimental measurements, we decompose the overall experimental covariance matrix into several sources, according to the varying degree of information reported by the experiments.<sup>24</sup>

We model the jet evolution using the JETSCAPE event generator framework<sup>28,29</sup> with the MATTER<sup>30,31</sup> and LBT<sup>32</sup> jet quenching models. MATTER is hypothesized to be valid in the

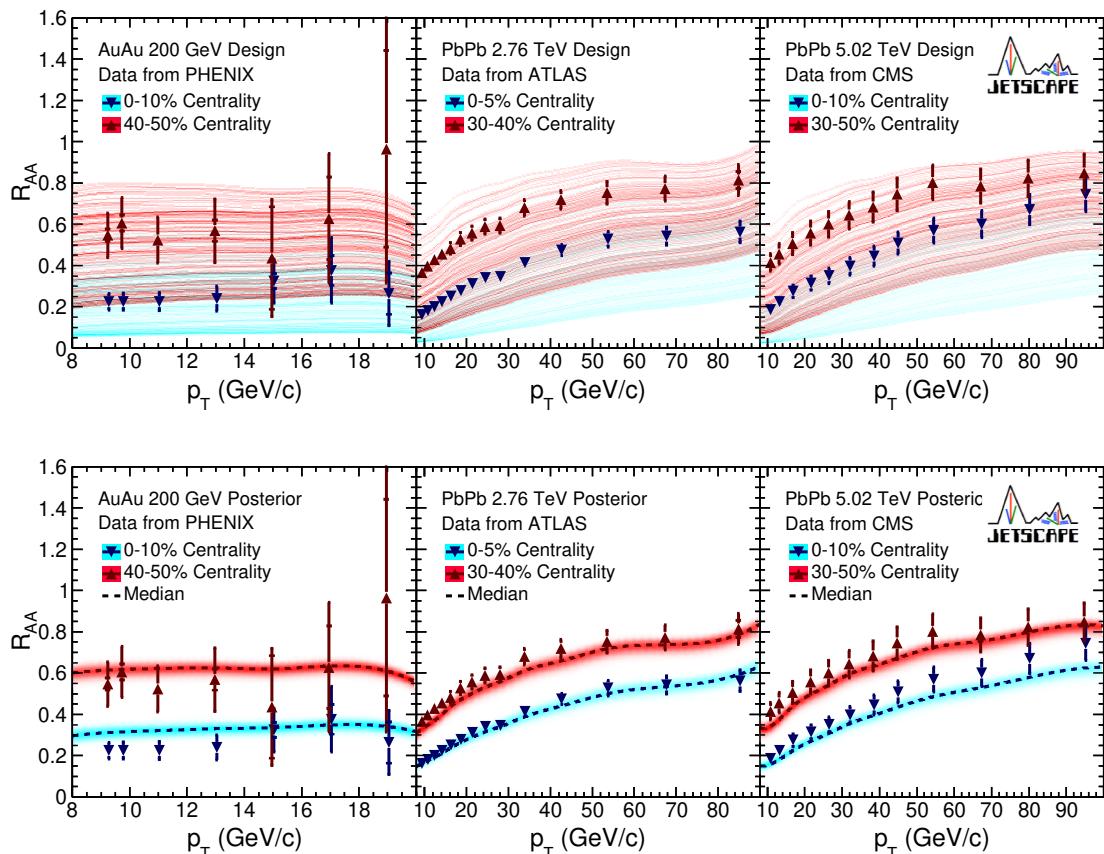


Figure 1 – Inclusive hadron  $R_{AA}$  for the three measured datasets,<sup>25,26,27</sup> together with prior calculations (top) and posterior distributions (bottom) from the LBT model. Inner error bars on experimental data points are statistical errors; outer error bars are the quadrature sum of statistical error and systematic uncertainty.<sup>24</sup>

high-virtuality, radiation-dominated regime, whereas LBT is hypothesized to be valid in the low-virtuality, scattering-dominated regime. We additionally consider a multi-stage model in which high-virtuality partons evolve according to MATTER, and low-virtuality partons evolve according to LBT, which we denote “MATTER+LBT”. Within these models, we parameterize  $\hat{q}$  as a function of the medium temperature,  $T$ , and parton energy,  $E$ , with an ansatz consisting of a sum of a high-virtuality,  $T$ -independent term, and a low-virtuality, elastic scattering term:

$$\frac{\hat{q}(E, T) |_{\theta=\{A,B,C,D\}}}{T^3} = 42C_R \frac{\zeta(3)}{\pi} \left(\frac{4\pi}{9}\right)^2 \left\{ \frac{A \left[ \ln\left(\frac{E}{\Lambda}\right) - \ln(B) \right]}{\left[\ln\left(\frac{E}{\Lambda}\right)\right]^2} + \frac{C \left[ \ln\left(\frac{E}{T}\right) - \ln(D) \right]}{\left[\ln\left(\frac{ET}{\Lambda^2}\right)\right]^2} \right\}, \quad (1)$$

where  $\theta \equiv \{A, B, C, D\}$  are parameters that will be determined from the experimental data using Bayesian parameter estimation.<sup>33</sup> For the multi-stage MATTER+LBT model, we consider two adaptations of this parameterization in which an additional parameter characterizing the virtuality switching scale between the two models is included.<sup>24</sup>

Starting with broad prior distributions of  $\theta$ , we use Bayesian parameter estimation to produce posterior probability distributions of  $\theta$ , and thereby  $\hat{q}(E, T)$ . To do so, we employ Gaussian Process Emulators to interpolate our computationally expensive event generator across  $\theta$ -space, and at each explored  $\theta$  evaluate the likelihood to observe the experimental data given the model results at that particular  $\theta$ . The posterior distributions are sampled using Markov Chain Monte Carlo. By using Bayesian parameter estimation, this procedure improves upon previous extractions of  $\hat{q}$  due to its rigorous statistical approach and quantification of uncertainties on the extracted  $\hat{q}$ . Further details can be found in a recently submitted article.<sup>24</sup>

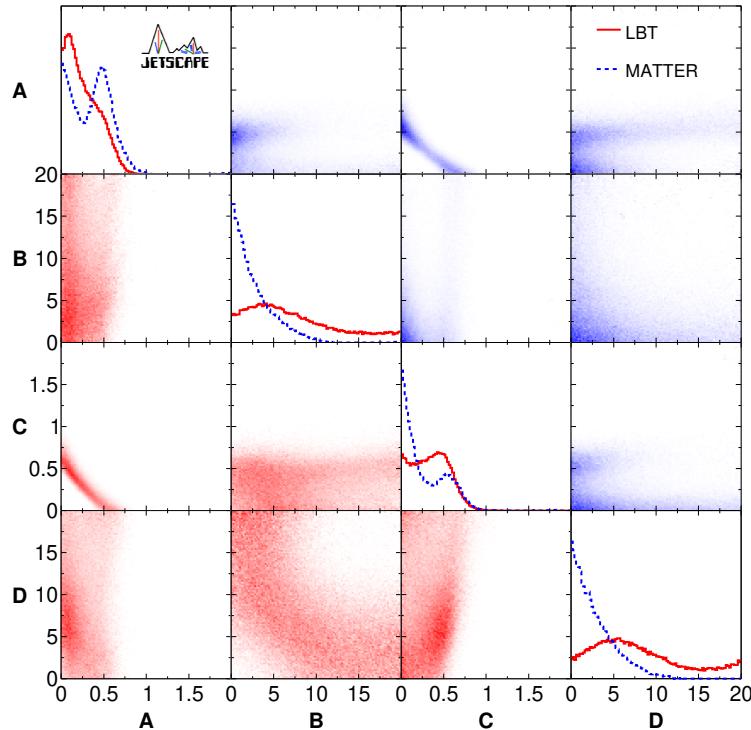


Figure 2 – Posterior distributions of the  $\theta \equiv \{A, B, C, D\}$  space for  $\hat{q}$  when MATTER and LBT are applied separately. Off-diagonal panels show correlations of posterior distributions for LBT (lower left, red) and MATTER (upper right, blue).<sup>24</sup>

### 3 Results

We perform separate extractions of  $\hat{q}$  using either MATTER, LBT, or MATTER+LBT. Figure 1 shows an example of the explored  $R_{AA}$  values before (top) and after (bottom) constraining the model to the experimental data. We find that LBT describes the data reasonably well, with some small systematic deviations, as does MATTER, albeit with slightly larger deviations. For MATTER+LBT, we find no evidence that the multi-stage model improves the description of the experimental data, suggesting that further theoretical work is needed.

The posterior distributions of  $\theta \equiv \{A, B, C, D\}$  are shown in Fig. 2 for both LBT and MATTER. We find that the extracted parameters are substantially different for LBT compared to MATTER. In particular, LBT exhibits a preference for smaller values of  $A$  and larger values of  $C$ , whereas MATTER exhibits a preference for larger values of  $A$  and smaller values of  $C$ . This is in fact consistent with the original motivation of the  $\hat{q}$  parameterization ansatz in Eq. 1: The first additive term, associated with high-virtuality physics, has overall coefficient  $A$  – and is preferred by the radiation-dominated MATTER model – while the second additive term, associated with elastic scattering off of a thermal medium, has overall coefficient  $C$  – and is preferred by the scattering-dominated LBT model.

From these parameter posterior distributions, we plot the extracted  $\hat{q}/T^3$  as a function of the medium temperature  $T$  and parton momentum  $p$  in Fig. 3. We plot the prior distributions of  $\hat{q}/T^3$  in the insets of Fig. 3, which demonstrate that the data provide considerable constraints on the value of  $\hat{q}$ . We find a generally weak dependence on both  $T$  and  $p$ , consistent with earlier work by the JET Collaboration.<sup>20</sup> The values of  $\hat{q}$  are similar between the different models considered, although with notably smaller central values in the multi-stage MATTER+LBT model, due to the fact that quenching is performed over a wider range of parton virtualities in the multi-stage model than in MATTER or LBT alone.

Bayesian parameter estimation can also be used to study the impact of particular observables on the precision of the extracted quantities. Figure 4 shows the impact of RHIC vs. LHC data by separately extracting  $\hat{q}$  with each collider dataset. We find that the posterior distributions are dominated by the LHC data. This can be partially attributed to the impressive precision and scope of the LHC measurements, but we also note that it is impacted by our choice of input data, which for this analysis we limited to  $p_T > 8$  GeV/c, thus intrinsically disfavoring RHIC data. This calls for future study, and highlights the important role that Bayesian parameter estimation can play in guiding experimental measurements.

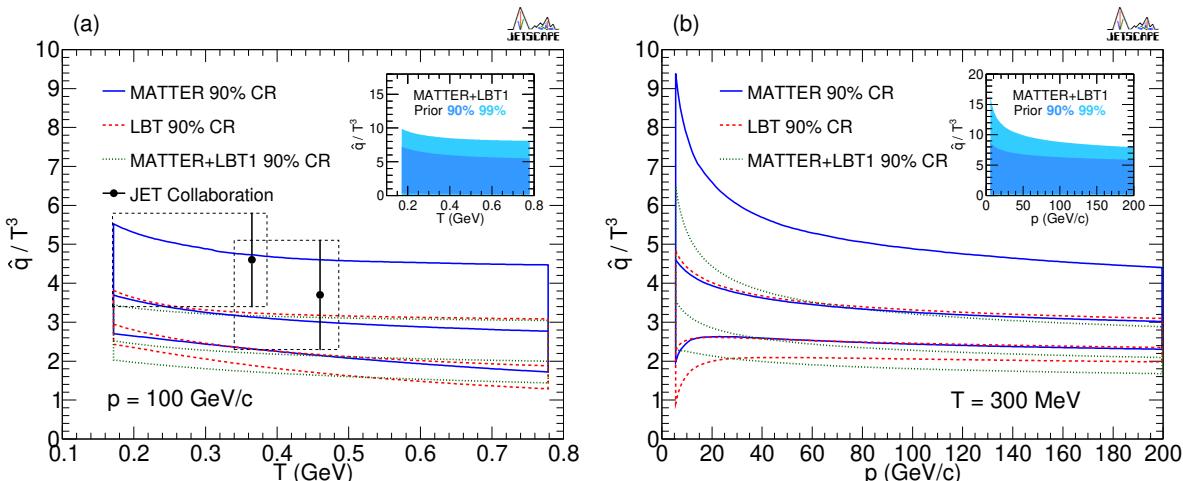


Figure 3 – The (quark) jet transport coefficient  $\hat{q}$  from Bayesian parameter extraction using MATTER and LBT separately as well as MATTER+LBT: (a) as function of the medium temperature, and (b) as function of quark momentum. The lines at the center of the bands indicate their median values.<sup>24</sup>

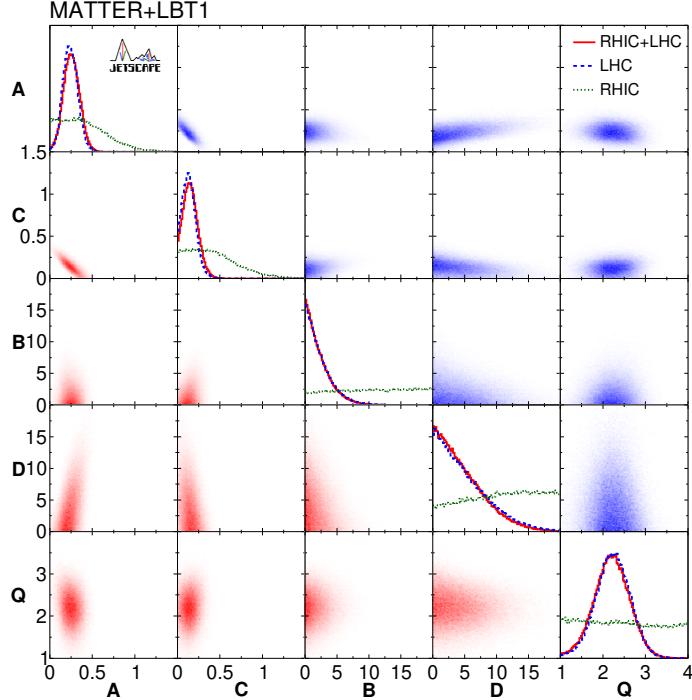


Figure 4 – Posterior distribution of  $\theta \equiv \{A, B, C, D\}$  with the multi-stage MATTER+LBT model, for separate extractions using either RHIC, LHC, or RHIC+LHC data. Off-diagonal panels show correlations of posterior distributions for RHIC+LHC (lower left, red) and LHC only (upper right, blue).<sup>24</sup>

## 4 Summary

We extracted the jet transverse diffusion coefficient,  $\hat{q}$ , of the quark-gluon plasma as a continuous function of the medium temperature and parton momentum. By using Bayesian parameter estimation, we improved upon previous extractions of  $\hat{q}$  with rigorous statistical methods and quantification of uncertainties. We explored several models within the JETSCAPE event generator framework: MATTER, LBT, and a multi-stage model MATTER+LBT. We used a set of hadron  $R_{AA}$  measurements at RHIC and the LHC, and found that the data provides significant constraints on the prior distributions, with results consistent with previous extractions.

Global analysis will be key to uncovering the nature of deconfined QCD matter. This study serves as a proof-of-principle that can be systematically extended to include additional observables, such as fully reconstructed jets, and extraction of additional medium properties, such as the path-length dependence of jet quenching, and eventually the nature of the constituents of deconfined QCD.

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## References

1. Wit Busza, Krishna Rajagopal, and Wilke van der Schee. Heavy Ion Collisions: The Big Picture, and the Big Questions. *Ann. Rev. Nucl. Part. Sci.*, 68:339–376, 2018.
2. Edward Shuryak. Strongly coupled quark-gluon plasma in heavy ion collisions. *Rev. Mod. Phys.*, 89:035001, 2017.
3. Guang-You Qin and Xin-Nian Wang. Jet quenching in high-energy heavy-ion collisions. *Int. J. Mod. Phys.*, E24(11):1530014, 2015.

4. A. Majumder and M. Van Leeuwen. The Theory and Phenomenology of Perturbative QCD Based Jet Quenching. *Prog. Part. Nucl. Phys.*, 66:41–92, 2011.
5. D. Everett et al. Multisystem Bayesian constraints on the transport coefficients of QCD matter. *Phys. Rev. C*, 103(5):054904, 2021.
6. Jonah E. Bernhard, J. Scott Moreland, Steffen A. Bass, Jia Liu, and Ulrich Heinz. Applying Bayesian parameter estimation to relativistic heavy-ion collisions: simultaneous characterization of the initial state and quark-gluon plasma medium. *Phys. Rev.*, C94(2):024907, 2016.
7. Jonah E. Bernhard, Peter W. Marcy, Christopher E. Coleman-Smith, Snehalata Huzurbazar, Robert L. Wolpert, and Steffen A. Bass. Quantifying properties of hot and dense QCD matter through systematic model-to-data comparison. *Phys. Rev. C*, 91(5):054910, 2015.
8. Scott Pratt, Evan Sangaline, Paul Sorensen, and Hui Wang. Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions. *Phys. Rev. Lett.*, 114:202301, 2015.
9. Govert Nijs, Wilke van der Schee, Umut Gürsoy, and Raimond Snellings. Bayesian analysis of heavy ion collisions with the heavy ion computational framework Trajectum. *Phys. Rev. C*, 103(5):054909, 2021.
10. R. Baier, Yuri L. Dokshitzer, Alfred H. Mueller, S. Peigne, and D. Schiff. Radiative energy loss and p(T) broadening of high-energy partons in nuclei. *Nucl. Phys. B*, 484:265–282, 1997.
11. B. G. Zakharov. Radiative energy loss of high-energy quarks in finite size nuclear matter and quark - gluon plasma. *JETP Lett.*, 65:615–620, 1997.
12. Tseh Liou, A. H. Mueller, and Bin Wu. Radiative  $p_\perp$ -broadening of high-energy quarks and gluons in QCD matter. *Nucl. Phys. A*, 916:102–125, 2013.
13. Jean-Paul Blaizot and Yacine Mehtar-Tani. Renormalization of the jet-quenching parameter. *Nucl. Phys. A*, 929:202–229, 2014.
14. Bin Wu. Radiative energy loss and radiative  $p_\perp$ -broadening of high-energy partons in QCD matter. *JHEP*, 12:081, 2014.
15. Peter Arnold, Tyler Gorda, and Shahin Iqbal. The LPM effect in sequential bremsstrahlung: nearly complete results for QCD. *JHEP*, 11:053, 2020.
16. Evan Bianchi, Jacob Elledge, Amit Kumar, Abhijit Majumder, Guang-You Qin, and Chun Shen. The  $x$  and  $Q^2$  dependence of  $\hat{q}$ , quasi-particles and the JET puzzle. 2 2017.
17. Hong Liu, Krishna Rajagopal, and Urs Achim Wiedemann. Calculating the jet quenching parameter from AdS/CFT. *Phys. Rev. Lett.*, 97:182301, 2006.
18. Abhijit Majumder. Calculating the jet quenching parameter  $\hat{q}$  in lattice gauge theory. *Phys. Rev. C*, 87:034905, 2013.
19. Amit Kumar, Abhijit Majumder, and Johannes Heinrich Weber. Jet transport coefficient  $\hat{q}$  in (2+1)-flavor lattice QCD. 10 2020.
20. Karen M. Burke et al. Extracting the jet transport coefficient from jet quenching in high-energy heavy-ion collisions. *Phys. Rev.*, C90(1):014909, 2014.
21. Carlota Andrés, Néstor Armesto, Matthew Luzum, Carlos A. Salgado, and Pía Zurita. Energy versus centrality dependence of the jet quenching parameter  $\hat{q}$  at RHIC and LHC: a new puzzle? *Eur. Phys. J. C*, 76(9):475, 2016.
22. Xiao-Fang Chen, Carsten Greiner, Enke Wang, Xin-Nian Wang, and Zhe Xu. Bulk matter evolution and extraction of jet transport parameter in heavy-ion collisions at RHIC. *Phys. Rev.*, C81:064908, 2010.
23. Weiyao Ke and Xin-Nian Wang. QGP modification to single inclusive jets in a calibrated transport model. *JHEP*, 05:041, 2021.
24. S. Cao et al. Determining the jet transport coefficient  $\hat{q}$  from inclusive hadron suppression measurements using Bayesian parameter estimation. 2 2021.
25. A. Adare et al. Neutral pion production with respect to centrality and reaction plane in Au+Au collisions at  $\sqrt{s_{NN}}=200$  GeV. *Phys. Rev. C*, 87(3):034911, 2013.
26. Serguei Chatrchyan et al. Study of high-pT charged particle suppression in PbPb compared to pp collisions at  $\sqrt{s_{NN}} = 2.76$  TeV. *Eur. Phys. J. C*, 72:1945, 2012.
27. Georges Aad et al. Measurement of charged-particle spectra in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV with the ATLAS detector at the LHC. *JHEP*, 09:050, 2015.
28. J. H. Putschke et al. The JETSCAPE framework. 3 2019.
29. S. Cao et al. Multistage Monte-Carlo simulation of jet modification in a static medium. *Phys. Rev.*, C96(2):024909, 2017.
30. Abhijit Majumder. Incorporating Space-Time Within Medium-Modified Jet Event Generators. *Phys. Rev.*, C88:014909, 2013.
31. Shanshan Cao and Abhijit Majumder. Nuclear modification of leading hadrons and jets within a virtuality ordered parton shower. *Phys. Rev. C*, 101(2):024903, 2020.
32. Shanshan Cao, Tan Luo, Guang-You Qin, and Xin-Nian Wang. Linearized Boltzmann transport model for jet propagation in the quark-gluon plasma: Heavy quark evolution. *Phys. Rev. C*, 94(1):014909, 2016.
33. Marc C. Kennedy and Anthony O'Hagan. Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3):425–464, aug 2001.