Heavy quark potential and quarkonia dissociation rates

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Abstract. Quenched lattice data for the $Q\bar{Q}$ interaction (in terms of heavy quark free energies) in the color singlet channel at finite temperatures are fitted and used within the nonrelativistic Schrödinger equation formalism to obtain binding energies and scattering phase shifts for the lowest eigenstates in the charmonium and bottomonium systems in a hot gluon plasma. The partial dissociation rate due to the Bhanot-Peskin process is calculated using different assumptions for the gluon distribution function, including free massless gluons, massive gluons, and massive damped gluons. It is demonstrated that a temperature dependent gluon mass has an essential influence on the heavy quarkonia dissociation, but that this process alone is insufficient to describe the heavy quarkonia dissociation rates.

1 Introduction

Heavy quarkonia have been suggested as hard probes of the quark gluon plasma [1] since the modification of static interactions at finite temperature eventually implies a dissolution of heavy quarkonia bound states into the continuum of scattering states (Mott effect). This effect results in a suppression of heavy quarkonia production in heavy-ion collisions as an observable signal. Since the Mott temperatures for J/ψ, Υ and Υ' as obtained by solving the Schrödinger equation for a screened Cornell-type potential lie well above the critical temperature T_c for deconfinement [2], it has soon been realized that a kinetic theory is necessary for the description of heavy quarkonia dissociation [3], see [4,5] for recent formulations. Solutions of the Schrödinger equation provide the basis for the evaluation of cross sections and rates for the Bhanot-Peskin process [6] of heavy quarkonia dissociation by gluon impact [7]. In this contribution we present a new fit to the singlet free energies from quenched lattice QCD simulations [8,9] and show that binding energies and cross sections deviate from those obtained for Debye potential fits [10,11,12].

2 Heavy quark potential

The main source of our knowledge of the quark-antiquark static interaction at high temperature originates from calculations of the free energy for a quark-antiquark system

from the Polyakov-loop correlator from lattice QCD. The color averaged correlator receives contributions from the color singlet and color octet channels

$$\langle\text{Tr}[L(0)]\text{Tr}[L^\dagger(r)]\rangle = \frac{1}{9}\exp\left(-\frac{F_1(r)}{T}\right) + \frac{8}{9}\exp\left(-\frac{F_8(r)}{T}\right).\tag{1}$$

The color-singlet part F_1 is extracted from the equation

$$\langle\text{Tr}[L(0)L^\dagger(r)]\rangle = \exp\left(-\frac{F_1(r)}{T}\right)\tag{2}$$

and the color octet part is found by subtraction. The free energy of a quark-antiquark system thus obtained is interpreted hereafter as the effective static interaction potential for a quark-antiquark pair surrounded by gluons.

2.1 Zero temperature

As there exist no color octet mesons in the vacuum, we will use the color singlet free energies as a potential in the Schrödinger equation description of the heavy quarkonia spectrum. Two additional parameters are to be defined in fitting the spectra: the heavy quark masses and a constant shift of the whole potential. The behavior of the quark-antiquark interaction in the color singlet channel was investigated in Ref. [13] within the combined lattice and perturbative QCD approach. We fit these data points implementing a χ^2 minimization to the Ansatz

$$F_1(r) = \begin{cases} F_{1,\text{short}}(r), & r < r_0, \\ F_{1,\text{long}}(r), & r \geq r_0, \end{cases}\tag{3}$$
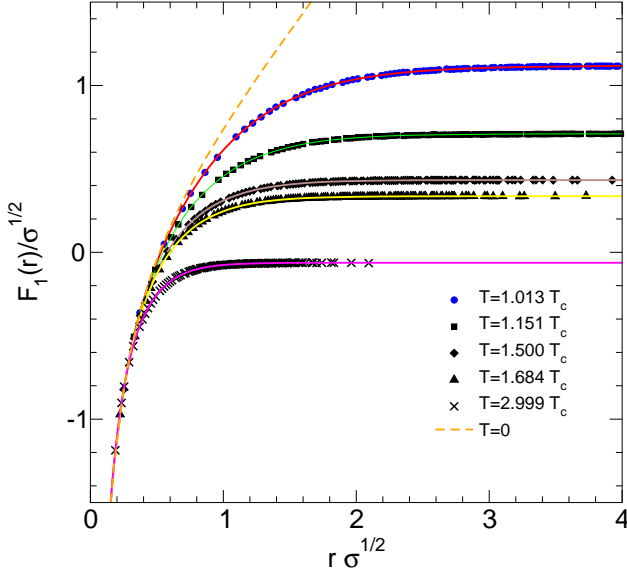

Fig. 1. Lattice data for the singlet free energy [9] at different temperatures, see legend. Solid lines are obtained with the fit on the basis of the Dixit formula, dashed line is the $T = 0$ potential.

where $F_{1,\text{short}}(r)$ describes the interaction at short distances whereas $F_{1,\text{long}}(r)$ is responsible for the long-distance forces. Both expressions are matched at $r = r_0$ that is defined below. This point, as we shall see, lies in the domain of perturbative QCD. We use the combined linear and Coulomb potential to describe the long-distance interaction and the Coulomb interaction with the r-dependent coupling constant $\alpha(r)$ for short distances

$$F_{1,\text{long}}(r) = \sigma r - \frac{\pi}{12r}, \tag{4}$$

$$F_{1,\text{short}}(r) = -\frac{4}{3}\frac{\alpha(r)}{r}, \tag{5}$$

$$\alpha(r) = \frac{16\pi}{33}\left(\frac{1}{\ln(r^2/c^2)} - \frac{r^2}{r^2 - c^2}\right). \tag{6}$$

The formula for $\alpha(r)$ is obtained by solving the one-loop renormalization group equation for the running coupling constant in QCD followed by the pole subtraction [14]. The constant $c\sqrt{\sigma} \approx 1.816$ and the point $r_0\sqrt{\sigma} \approx 0.031$ are determined in units of the string tension $\sqrt{\sigma} = 0.42$ GeV from the condition that the potential is a smooth function at $r = r_0$. The result is given by the dashed line in Fig. 1.

2.2 Singlet free energy at high temperature

For the singlet free energy of a static quark-antiquark system at high temperature we use the Ansatz

$$F_1(r,T) = \begin{cases} F_{1,\text{short}}(r), & r < r_0, \\ F_{1,\text{long}}(r,T), & r \geq r_0, \end{cases} \tag{7}$$

where for the short-range interaction governed by pQCD the zero-temperature form of the previous subsection is assumed.

The long-distance interaction $F_{1,\text{long}}(r,T)$ requires theoretical assumptions about its shape. Instead of the frequently used screened Coulomb potential, we follow Dixit [15] and assume the potential behavior at large r as $V \sim \exp(-\mu r^2)/\sqrt{r}$. Above T_c, the expression for the potential without the Coulomb interaction was deduced in [15]

$$F_{1,\text{long}}^{(0)}(r, T > T_c) = -\frac{q^2}{2^{3/4}\Gamma(3/4)}\sqrt{\frac{r}{\mu}}K_{1/4}\left[(\mu r)^2\right]$$
$$+q^2\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)\mu}, \tag{8}$$

and contains two parameters: q and μ. A similar parameterization was used by Digal et al. [16]. We add to this term a Coulomb attraction at short distances and obtain

$$F_{1,\text{long}}(r, T > T_c) = F_{1,\text{long}}^{(0)}(r, T > T_c) + F_{1,\text{short}}e^{-(\mu r)^2}. \tag{9}$$

The parameters q and μ are determined from a fit to lattice data [9], the results are shown in Fig. 1.

3 Quarkonia in a hot gluon plasma

3.1 Quarkonia at zero temperature

The masses of quarkonia in the vacuum are defined as

$$M = 2m_Q + E + v_0 \tag{10}$$

where m_Q is the quark mass, and the energy E is an eigenvalue of the Schrödinger equation

$$[-\nabla^2/m_Q + V(r)]\psi(r) = E\psi(r), \tag{11}$$

where the potential $V(r)$ is identified with the zero temperature free energy $F_1(r)$ of Eq. (3) up to an unknown constant v_0. Substituting the wave function $\psi_{n\ell m}(r, \theta, \phi) = r^{-1}R_{n\ell}(r)Y_{\ell m}(\theta, \phi)$ into (11), one obtains an equation for $R_{n\ell}(r)$. At large r, the potential is linear, and the solution of this equation behaves as the Airy function $R_{n\ell}(r) \sim \text{Ai}(\kappa r - \xi)$, where $\kappa^3 = m_Q\sigma$ and $\xi = m_Q E/\kappa^2$.

The masses of 1S and 4S states are used as input. For charmonium we obtain $m_c = 1.45$ GeV and the constant $v_0 = -302$ MeV. For the bottomonium we have $m_b = 4.785$ GeV with the same v_0. Once m_c, m_b, and v_0 are fixed, the remaining quarkonia spectrum is described [17].

3.2 Quarkonia at finite temperature

The Schrödinger equation for a bound state in the QGP has the form

$$[-\nabla^2 + m_Q V_{\text{eff}}(r,T)]\psi(r,T) = m_Q E_B(T)\psi(r,T), \tag{12}$$

where $E_B(T) > 0$ is the temperature dependent binding energy. The medium effects on the $Q\bar{Q}$ system are modeled using the singlet free energies as an effective potential
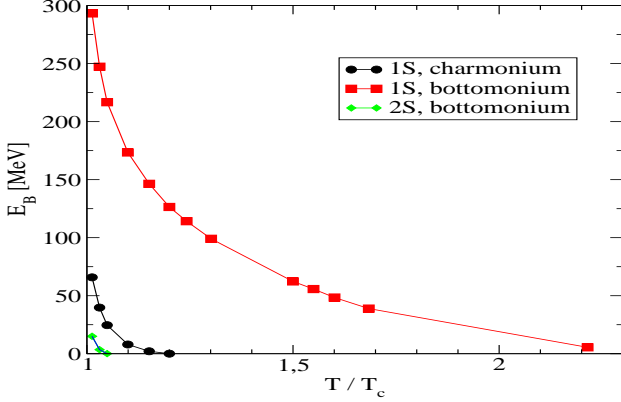
Fig. 2. Binding energies of heavy quarkonia states from solution of the Schrödinger equation for the T-dependent effective potential of Fig. 1.

$V_{\text{eff}}(r,T) = F_1(r,T) - F_\infty(T)$ with the continuum threshold $F_\infty(T) = \lim_{r\to\infty} F_1(r,T)$. The temperature dependent mass of a quarkonium bound state is defined as

$$M(T) = 2m_Q - E_B(T) + v_0 + F_\infty(T) . \qquad (13)$$

The solutions for the binding energy both for charmonium and bottomonium are shown in Fig. 2.

The wave function of an unbound quark-antiquark system can be calculated via the S-wave phase shift function $\delta_S(r)$ by solving the equation [18]

$$\frac{d\delta_S(k,r,T)}{dr} = -\frac{m_Q V_{\text{eff}}(r,T)}{k} \left[\sin(kr + \delta_S(k,r,T))\right]. \qquad (14)$$

The phase shift is defined as $\delta_S(k,T) \equiv \delta_S(k,\infty,T)$ and results are shown in Fig. 3 for charmonia and bottomonia states at different temperatures. In accordance with the Levinson theorem, the scattering phase shift at threshold changes by π once a bound state merges the continuum at its Mott temperature; $T^{\text{Mott}}/T_c = 1.05$, 1.20, 2.25 for Υ', J/ψ, Υ, respectively, see Figs. 2 and 3.

4 Dissociation of quarkonia by gluon impact

We calculate the cross section for the quarkonium dissociation after a gluon impact similarly to the calculation of the deuteron decay via the photon absorption [11,19]

$$\sigma_{(Q\bar Q)g}(\omega) = \frac{4\pi\alpha_{gQ}}{3} \frac{(k^2 + k_0^2)}{k} \left(\int_0^\infty u_{1P}(r)u_{1S}(r)\, r\, dr\right)^2, \qquad (15)$$

$$\int_0^\infty |u_{1S}(r)|^2 dr = 1, \ \ \alpha_{gQ} = \alpha_s/6, \ \ k_0^2 = m_Q E_B(T), \quad (16)$$

where we used $R_{n\ell}(r) = u_{n\ell}(r)e^{-k_0 r}$. For the 1P state, one can use the wave function of a free quark-antiquark
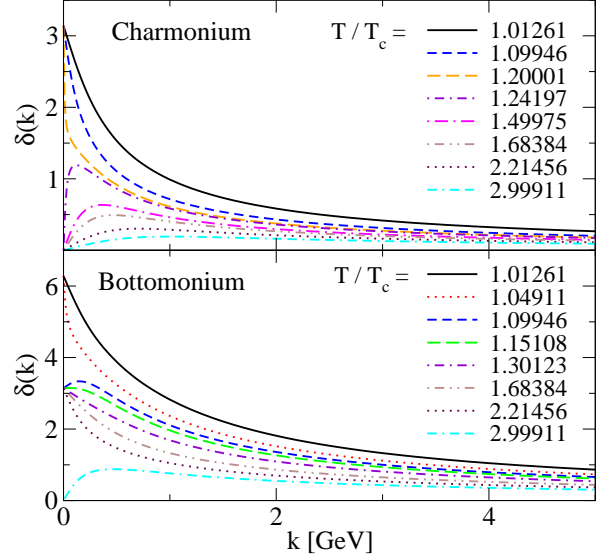

Fig. 3. Scattering phase shifts for heavy charmonia and bottomonia states from solution of the Schrödinger equation for the T-dependent potential of Fig. 1.

system:

$$u_{1P}(r) = \frac{\sin kr}{kr} - \cos kr, \quad k^2 = m_Q(\omega - E_B(T)). \quad (17)$$

For the constant α_s in (16), we take an average over the low energy region below 1 GeV, which gives $\alpha_s \approx 0.48$. As a result, we obtain the cross sections shown in Fig. 4. Their peak values correspond to the geometrical ones $(\pi R^2(T))$ with the T-dependent radius $R(T)$ of the quarkonia wave function [17].

Now we can estimate the dissociation rate of the charmonium and bottomonium by gluon impact according to

$$\Gamma_{(\bar Q Q)g}(T) = \int_0^\infty ds A(s) \int_s^\infty \frac{d\omega^2}{4\pi^2} \sqrt{\omega^2 - s}\, \sigma_{(Q\bar Q)g}(\omega)n_g(\omega), \qquad (18)$$

where $A(s)$ is the normalized gluon spectral function

$$A(s) = \frac{1}{\pi} \frac{\sqrt{s}\gamma}{(s - m_g^2)^2 + s\gamma^2}, \qquad \int_0^\infty ds A(s) = 1, \quad (19)$$

and the thermal gluon distribution function is given by $n_g(\omega) = 2(N_c^2 - 1)[\exp(\omega/T) - 1]^{-1}$. The temperature dependent gluon mass m_g and damping width γ are taken from a recent fit to lattice QCD data for the entropy in pure gauge [20],

$$m_g^2 = 2\pi\bar\alpha T^2, \quad \gamma = 3\bar\alpha T \ln(2.67/\bar\alpha), \qquad (20)$$

where $\bar\alpha = 12\pi/[33\ln(3.7(T - T_s)/T_c)^2]$ with $T_s = 0.46$ GeV. The results are shown in Fig. 5 and compared with the cases when the damping width and also the gluon mass are neglected.
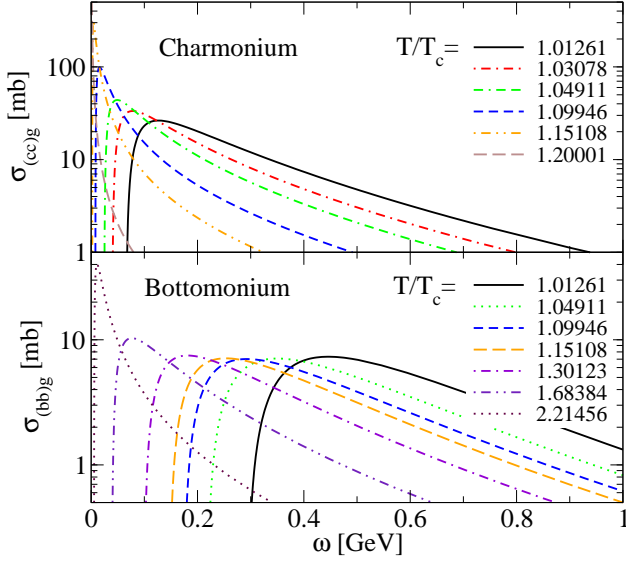
Fig. 4. Cross sections for J/ψ and Υ dissociation by the Bhanot-Peskin process.
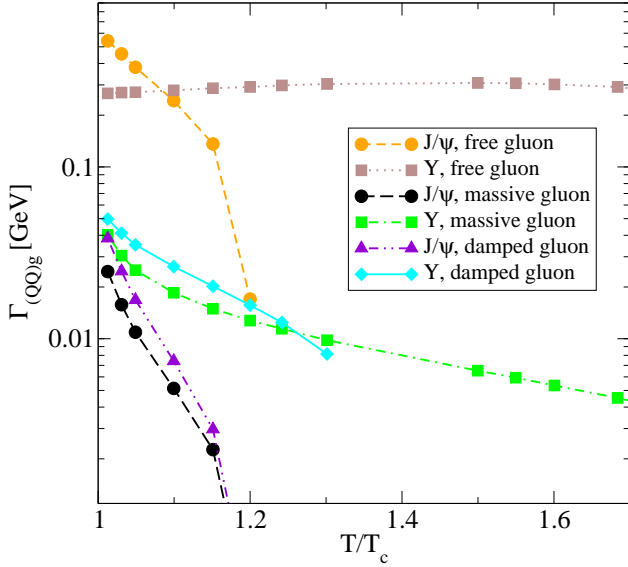

Fig. 5. Dissociation rates of heavy quarkonia in a hot gluon plasma obtained with massive damped, massive undamped and massless (free) gluon distributions in Eq. (18), respectively.

5 Conclusions

We have used a new fit to recent quenched lattice data for the $Q\bar{Q}$ singlet free energies at finite temperatures to obtain binding energies and scattering phase shifts for the lowest eigenstates in the charmonium and bottomonium systems within the Schrödinger equation formalism. In contrast to results on the basis of a Debye potential fit, we obtain much smaller finite temperature quarkonia bind-

ing energies, entailing large dissociation cross sections for the Bhanot-Peskin process. The corresponding dissociation rates have been evaluated using different assumptions for the gluon distribution function, including free massless gluons, massive gluons, and massive damped gluons. We have demonstrated that that a temperature dependent gluon mass has an essential influence on the heavy quarkonia dissociation. However, the Bhanot-Peskin process alone is insufficient to describe the quarkonium dissociation process [5,7]. On the basis of the spectrum and wave functions obtained here we will study next the $Q\bar{Q}$ spectral functions above the Mott temperature and compare the results with corresponding lattice studies [21].

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