



82-9-9/a

FERMILAB-Pub-82/44-THY
July, 1982

Highly Relativistic Nucleus-Nucleus Collisions: The Central Rapidity Region

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ABSTRACT

The space-time evolution of the hadronic matter produced in the central rapidity region in extreme relativistic nucleus-nucleus collisions is described. We find, in agreement with previous studies, that quark-gluon plasma is produced at a temperature $\gtrsim 200\text{--}300$ MeV, and that it should survive over a time scale $\gtrsim (5 \text{ fermi})/c$. Our description relies on existence of a flat central plateau and on the applicability of hydrodynamics.



I. INTRODUCTION

Collisions of highly relativistic nuclei offer the possibility of producing quasimacroscopic systems of dense nucleonic and/or quark-gluon matter at relatively high temperature. In principle this seems to be an interesting way to explore the question of phase transitions between ordinary (confined) matter and (unconfined) quark-gluon plasma. It is also of interest to historians of the early universe. At some early epoch, of order 10^{-6} seconds after the big bang, the conditions in the universe were probably rather similar.

On the other hand, interpretation of these complex collisions poses a major problem. What are the clean experimental signatures and how can one deduce what is going on? Is there information which unambiguously teaches us about the state of the matter formed during and immediately after the collision?

All these problems are under active investigation nowadays.¹ There seems to be a consensus that enough initial kinetic energy is converted into heat so that quark-gluon plasma is created. Less understood is the question of how the system evolves. Furthermore most (but not all) of the work has concentrated on the system of leading particles which carries the baryon-number of the incident nucleus. This system is especially interesting since it is essentially compressed nuclear matter and carries with it not only a heritage of

nuclear physics but also of nuclear astrophysics; e.g. the question of neutron-star composition. However, the remaining phase-space, i.e. the so-called central rapidity region, is of interest in its own right.² And it will be the case that this region of phase space is perhaps easier to study experimentally.

It is our purpose in this note to sketch out a picture of the space-time evolution³ of the collision process in this "central" region of phase-space. We shall treat the problem in the context of the Landau hydrodynamic model,⁴ but with a different initial boundary condition. We shall assume that at sufficiently high energy there is a "central-plateau" structure for the particle production as function of the rapidity variable, be it in nucleon-nucleon, nucleon-nucleus, or nucleus-nucleus collisions. The essence of this assumption is the assertion that the space-time evolution of the system looks essentially the same in all center-of-mass-like frames, i.e. in all frames where the emergent excited nuclei are, shortly after the collision, highly Lorentz-contracted pancakes receding in opposite directions from the collision point at the speed of light.

This assertion implies a symmetry property of the system. We will impose this symmetry as an initial condition. However, the hydrodynamic equations respect the symmetry as well. This leads to simple solutions of the hydrodynamic equations.⁵ In particular, for central collisions of large nuclei, the fluid

expansion near the collision axis is longitudinal and homogeneous. The fluid midway between the receding pancakes remains at rest, while the fluid a longitudinal distance z from that midpoint moves with longitudinal velocity z/t , where t is the time elapsed since the pancakes collided. This picture is modified at large transverse distances, comparable to the nuclear radii. In that region there will a rarefaction front moving inward at the velocity of sound of the fluid. For transverse distances larger than that rarefaction front, fluid will expand radially outward, cooling more rapidly than the fluid in the interior.

The initial energy density produced in the collision is very roughly estimated to be $\sqrt{3} \text{ GeV}/f^3$, with an uncertainty of at least a factor 3 in either direction. The estimate is based simply on the energy-release per unit of rapidity in nucleon-nucleon collisions. This energy density (and consequent entropy or particle density) is sufficiently high to make it very likely that the system rapidly comes into local thermal equilibrium. It is also, as we already mentioned, sufficiently high to make it likely that the plasma is in the deconfined quark-gluon phase. However, the initial temperature is not expected to be high; we estimate $\sqrt{200-300} \text{ MeV}$.²

During the expansion the energy density drops (in its local rest frame) as t^{-Y} with $1 \leq Y \leq 4/3$, while the temperature drops as t^{Y-1} . The entropy density falls as t^{-1} . This implies that the entropy per unit rapidity is conserved, a result which

depends only upon the boost symmetry of our boundary conditions and not upon details of the equation of state. This result implies that the particle production per unit rapidity (which is proportional to the entropy) in turn does not depend on the details of the hydrodynamic evolution, but only on the initial energy (hence, entropy) deposition in the early stage of the collision itself.

As the system evolves, the amount of fluid undergoing homogeneous longitudinal expansion decreases. When the separation of the outgoing pancakes exceeds their diameter, the fluid enclosed between them will undergo 3-dimensional radial expansion and should rapidly cool. Already at the onset of this part of the evolution, we estimate that any phase transition will have been traversed, and that the system is one of dense hadronic matter, with temperature $\sqrt{150-200} \text{ MeV}$.

In the next section we discuss our proposed space-time picture of the collision. In Section IV, we briefly consider the question of equation of state, and whether it has an effect on the picture. Section IV is devoted to miscellaneous comments and conclusions.

II. SPACE-TIME EVOLUTION

In order to motivate our starting point for ion-ion collisions, we begin by describing the assumptions we shall make for the simpler cases of hadron-hadron and hadron-nucleus

collisions.

In the case of hadron-hadron collisions we shall assume (1) there exists a "central-plateau" structure in the inclusive particle productions as function of the rapidity variable. This is reasonably well borne out by SPS collider data.⁶ It is true that the plateau height is energy-dependent, but that will not affect our considerations very much. The existence of the plateau implies that the particle distribution at large angles, as seen in a typical center-of-mass frame, does not depend upon the particular frame which is chosen. For example, at SPS energies the 90° particle production in a 250 + 250 GeV $p\bar{p}$ collision appears to be not dissimilar to the 90° particle production in a 10 GeV + 6 TeV $p\bar{p}$ collision. This apparent symmetry will be a central theme in the discussion to follow.

Our second assumption is similar: (2) For nucleon-nucleus collisions, there also exists a "central-plateau" structure in the inclusive particle production as function of the rapidity variable, with plateau height about the same as for a nucleon-nucleon collision. $p - \alpha$ collisions at the ISR⁷ lend some support for this behavior, although it would be reassuring to have better data on nucleon collisions with heavier nuclei.

The final assumption is: (3) there exists a "leading-baryon" effect. That is, the net baryon number of a projectile is found in fragments of comparable momentum (more precisely, of rapidity within $\sqrt{2}$ -3 units of the rapidity of the source). Likewise, the net baryon number from a target baryon

originally at rest is found in those produced hadrons of relatively low momentum. This assumption is again consistent with what is seen in pp , $p\alpha$ and $\alpha\alpha$ collisions⁷ at the CERN ISR.

Given these hypotheses, we may now consider the real case of ion-ion collisions. First, let us consider the collision in the rest frame of one of the nuclei. As the highly Lorentz contracted pancake passes through this nucleus, it is reasonable that each nucleon in that nucleus is struck. It is also reasonable - and we shall assume its correctness - that the secondary nucleon from each collision possesses a momentum distribution similar to what it would possess were it in isolation and not bound in nuclear matter. This means it recoils semirelativistically, with a typical momentum of several hundred MeV. The result, as very thoroughly and well-described by Anishetty, Koehler, and McFerran,⁸ is that the nuclear matter in the target nucleus is found (in its original rest frame) in a distinct ellipsoidal region (Figure 1) moving with a γ^2 , and lagging behind the highly contracted projectile pancake.

The fact that the γ of this system of baryons is expected to be finite and not too large implies that in ion-ion collisions the baryon number should be found in (or near) the projectile fragmentation regions. We shall sharpen this statement somewhat later on.

Now let us look at the collision in the center-of-mass frame. From the arguments of the previous paragraph it is clear that at least the baryon content of the colliding pancakes interpenetrate, so that a short time (say $\sim 3f$) after the collision we will have two pancakes which recede from the collision point at the speed of light ($\gamma \gg 1$) and which contain the baryon number of the initial projectiles. Of course many of the other ultimate collision products will be contained in those pancakes and will only evolve into a distinguishable system at considerably later times. We shall concentrate on the system of quanta contained in the region between the two pancakes. Let us temporarily replace one of the projectiles by a single nucleon traveling at the same γ , and look at the central particle production. According to assumption (2) the isotropic portion of the particle production is approximately the same as in a nucleon-nucleon collision. At SPS collider energies, this means

$$\frac{dN_{\text{ch}}}{dy} \approx 3 \quad (1)$$

Guessing $\langle E \rangle \sim 400$ Mev and $N_{\text{neutral}}/N_{\text{ch}} \sim 0.5$, we would find, per colliding nucleon

$$\frac{d\langle E \rangle}{dy} \sim 3 \times 0.4 \times 1.5 \sim 1.8 \text{ GeV} \quad (2)$$

If the projectile, instead of a single nucleon, is a dilute gas of nucleons separated in impact parameter by mean distances λ lf., the energy production should be additive.

Let us now estimate for this case the initial energy density existing between the outward-moving pancakes. We concentrate on a thin slab, of thickness $2d$, centered between the pancakes (Figure 2). Ignoring collision between the produced hadrons, the energy contained within that slab is

$$E = N \frac{d\langle E \rangle}{dy} \Delta y = N \cdot \frac{d\langle E \rangle}{dy} \cdot \frac{1}{2} \left(\frac{2d}{c} \right) \quad (3)$$

It follows that the central energy density ϵ is

$$\epsilon \approx \frac{N}{\lambda} \cdot \frac{d\langle E \rangle}{dy} \cdot \frac{1}{2t} \quad (4)$$

In the case of real ion-ion collisions we must replace the number of incident nucleons per unit area N/A by some effective elementary area d_0^2

$$\frac{N}{A} \longleftrightarrow d_0^{-2} \quad (5)$$

If, for uranium, we assumed full additivity over the A nucleons we would get (with apologies for the execrably redundant notation)

$$\frac{N}{A} = \frac{A}{\pi (1.2A^{1/3} f.)^2} = \frac{A^{1/3}}{4.5 f.^2} = \frac{1}{d_0^2}$$

or

$$d_0 \approx 0.7 f. \quad (6)$$

We shall consider reasonable a range of values of d_0 from 0.3 to 1f.

$$0.3 f \leq d_0 \leq 1.0 f. \quad (7)$$

This leads to an estimate of

$$\epsilon \approx \frac{1 \text{ GeV}}{t d_0^2} \quad (8)$$

For an initial time t_0 of $\sim 1 f.$, this gives an initial energy density

$$\epsilon_0 \approx 1 - 10 \text{ GeV}/f^3 \quad (9)$$

It is not clear at this energy density what the produced quanta which carry this energy really are: constituent quarks? current quarks? gluons? hadrons? However, this uncertainty should not affect the estimated energy density provided the elementary collision processes which operate in nucleon-nucleon collisions are operative in nucleus-nucleus collisions. The quanta contained in our thin slab should collide; indeed we may anticipate that local thermal equilibrium will be established. With a mean energy density as given above, and with a mean energy per quantum of 400 MeV, this implies an initial density of quanta ρ_0 of $\sim 2-20 f^{-3}$. This in turn implies a collision mean free path λ_0

$$\lambda_0 \approx \left(\frac{10 \text{ mb}}{\sigma_{\text{int}}} \right) \times (0.05-0.5 f) \quad (10)$$

We shall be interested in a time scale of $\sim 5-10 f.$ for the evolution of the produced system. Thus an assumption of local thermal equilibrium, i.e. applicability of hydrodynamics, seems reasonable. Once thermal equilibrium is established and hydrodynamic expansion of the fluid commences, the t^{-1} time dependence of the energy density ϵ will not be valid (although we shall calculate a similar behavior $\epsilon \sim t^{-n}$ with $1 \leq n \leq 4/3$). Thus the time t appearing in the expression (8) for energy density

should be interpreted as an initial time for imposition of boundary conditions for hydrodynamic flow. While that initial time (τ_{lf}) is somewhat uncertain, the major uncertainty in imposition of initial conditions comes from lack of knowledge of the basic transverse scale factor d_0 .

Let us now make a modest Lorentz boost (say γ^3) and view the collision in another frame. Again we see a collision of two highly contracted incident nuclei followed by receding pancakes carrying the baryon-number. If, as before, we look at the nucleon-nucleus collision under these same circumstances, we will see the same large-angle particle production as before. This follows from the assumption of a central-plateau structure for the rapidity-distribution in nucleon-nucleus collisions. It is therefore very reasonable that for nucleon-nucleus collision the initial conditions for the fluid of quanta produced between the receding pancakes are the same as existed in the other frame. This means in particular that the initial energy density is the same, and that the initial velocity is zero.

This is the basic feature of this description of the evolution of the system: throughout the "central-plateau" region the initial conditions - imposed a proper time τ_{lf} , after the collision time - are invariant with respect to Lorentz transformations. This will imply that the subsequent time evolution of the system should also possess this symmetry.

We shall describe this evolution of the system by assuming that the Landau hydrodynamical model⁴ is applicable. This means that we may define a local energy density $\epsilon(x)$, pressure $p(x)$, and temperature $T = \beta^{-1}(x)$ and 4-velocity of the fluid $u_\mu(x)$, with $u_\mu^2 = u_\mu u^\mu = 1$. Then the energy-momentum tensor

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu - g_{\mu\nu} p \quad (11)$$

is conserved:

$$\partial_\mu T^{\mu\nu} = 0 \quad (12)$$

(We shall - perhaps unjustifiably - neglect effects of viscosity and thermal conductivity).

The initial boundary conditions we have discussed may be displayed in a space-time diagram as shown in Fig. 3. We see that on the "proper-time" hyperbola $\tau = \sqrt{t^2 - z^2} = \text{constant}$ if., we have $\epsilon = \epsilon_0 = \text{constant}$ $\sim 10 \text{ GeV/f}^3$; hence $T_0 = \text{constant}$ as well.

Natural variables for describing the flow are therefore the rapidity y , defined as

$$y = 1/2 \ln \frac{t+z}{t-z} \quad (13)$$

and proper time τ

$$\tau = \sqrt{t^2 - z^2} \quad (14)$$

provided the flow may be considered one-dimensional, i.e. we have translational symmetry in transverse coordinates. This should be a good approximation for times small compared to the radius of the nucleus.

$$t \ll 1.2 A^{1/3} \approx 7 \text{ f. for Pb or U.} \quad (15)$$

Thereafter we must expect 3-dimensional expansion and a relatively short time evolution into the final system of produced hadrons. (We shall return to the description of the transverse flow later).

In the following we shall assume one-dimensional flow. In general, this would imply that

$$\epsilon = \epsilon(\tau, y)$$

$$p = p(\tau, y)$$

$$T = T(\tau, y)$$

$$u_\mu(\tau, y) = (u_0(\tau, y), 0, 0, u_z(\tau, y)) \quad (16)$$

However, the initial condition

$$\epsilon(\tau_0, y) = \epsilon_0 \quad \text{etc.}$$

$$u_\mu(\tau_0, y) = \frac{1}{\tau_0} (t, 0, 0, z) \equiv \frac{\bar{x}_\mu}{t_0} \quad (17)$$

possess a symmetry with respect to Lorentz transformations which is preserved by the dynamics. Inasmuch as there is no dependence of initial properties on Lorentz boost angle (essentially the rapidity-variable y) there will be no such dependence at later times either. That is we may write

$$\epsilon = \epsilon(\tau)$$

$$p = p(\tau)$$

$$\beta = \tau^{-1} = \beta(\tau) \quad (18)$$

and, most importantly

$$u_\mu = \frac{\bar{x}_\mu}{\tau} \quad (19)$$

With use of the expressions

$$\frac{\partial \tau}{\partial x^\mu} = \frac{\bar{x}_\mu}{\tau} = u_\mu$$

$$\frac{\partial u_\mu}{\partial x^\nu} = \frac{1}{\tau} g_{\mu\nu} - \frac{\bar{x}_\mu \bar{x}_\nu}{\tau^3} = \frac{1}{\tau} (g_{\mu\nu} - u_\mu u_\nu), \quad (20)$$

where $g_{00} = g_{zz} = 1$ and all other elements zero, the hydrodynamic equation (12) simplifies to

$$\frac{d\epsilon}{d\tau} = - \frac{(\epsilon + p)}{\tau} \quad (21)$$

Even without an equation of state relating ϵ and p , we may go a little further. According to the assumptions of the Landau model, entropy is conserved during the expansion stage. Indeed, recall from thermodynamics that

$$TS = U + pV \quad (22)$$

(in the rest frame of the fluid) and therefore the entropy density is

$$s = \frac{S}{V} = s(\epsilon + p) \quad (23)$$

The entropy current four-vector is evidently

$$s_\mu = s(\epsilon + p)u_\mu \equiv su_\mu \quad (24)$$

The local conservation law

$$\frac{\partial s_\mu}{\partial x_\mu} = 0 \quad (25)$$

is a general consequence of the hydrodynamic equations (obtained by contraction of Eq. (12) with u^μ). In our case, this evidently implies (using Eq. (19))

$$\frac{ds}{d\tau} = - \frac{s}{\tau} \quad (26)$$

or

$$\frac{s(\tau)}{s(\tau_0)} = \frac{\tau_0}{\tau} \quad (27)$$

The meaning of this result is that the entropy content per unit of rapidity is a constant of the motion. To see this, note that, in a frame in which the fluid is at rest,

$$d^3x = d^2x_\perp (\tau dy)$$

Thus the entropy contained in interval dy around $y = 0$ is

$$dS = \int s_0 d^3x = \tau s \int d^2x_\perp dy \quad (28)$$

and

$$\frac{d}{d\tau} \left(\frac{dS}{dy} \right) = \frac{d}{d\tau} (\tau s) \int d^2x_\perp = 0 \quad (29)$$

This result allows a quite direct estimate of central multiplicity. If entropy is conserved throughout the hydrodynamic expansion, then the final pion multiplicity (per unit rapidity) should be in proportion to the entropy density. This in turn has been related via initial conditions to the pion multiplicity in nucleon-nucleon collisions. We phrased these initial conditions in terms of energy density, but could

have phrased them in terms of entropy density provided that in nucleon-nucleon collisions the hydrodynamic concepts still have some meaning. If that is the case, the previous argument becomes

$$\left(\frac{ds}{dy}\right)_{\text{Nucleus-nucleus}} = \frac{1}{d_0^2} \cdot \pi(1.2A)^{1/3} f_0^2 \cdot \left(\frac{ds}{dy}\right)_{pp} \quad (30)$$

where $1/d_0^2$ = number of effective independent nucleon-nucleon collisions (i.e. entropy producers) per unit area. Given entropy conservation, and assuming pion multiplicity to be in proportion to the entropy, we get

$$\frac{(dN_\pi/dy)_{A-A}}{(dN_\pi/dy)_{pp}} = \left(\frac{2f_0}{d_0}\right)^2 A^{2/3} \quad (31)$$

With our previous estimate $0.3\delta d_0$ c.f., the mindless extrapolation of this formula to $A=1$ fails by a factor ~ 4 to 40. However, if we apply it (equally mindlessly) to the ISR data⁷ on α - α collisions, we would find $d_0 \approx 0.7f_0$. In the long run the most profitable use of the above relation might be to use future multiplicity measurements to determine d_0 and thereby experimentally infer the initial conditions of the fluid.

Note that for U-U collisions, the multiplicity formula implies, for a central collision (at SPS-collider energies)

$$\begin{aligned} \frac{dN_\pi}{dy} &= 5 \times 40 \times 4 \times \left(\frac{1f_0}{d_0}\right)^2 \\ &\approx 800 \left(\frac{1f_0}{d_0}\right)^2 \end{aligned} \quad (32)$$

a large multiplicity indeed.

Other general features of the hydrodynamic expansion follow from the positivity condition on the trace of the energy-momentum tensor

$$T^\mu_\mu \geq 0 \quad (33)$$

which is true under quite general circumstances.⁹ This implies

$$\epsilon \geq 3p \quad (34)$$

and thus, from Eq. (21)

$$\left(\frac{\tau_0}{\tau}\right)^{4/3} \leq \frac{\epsilon(\tau)}{\epsilon(\tau_0)} \leq \left(\frac{\tau_0}{\tau}\right) \quad (35)$$

For an "ideal" relativistic fluid, $\epsilon=3p$ and the proper-time dependence of the energy-density and pressure is $\sim \tau^{-4/3}$. In this case $\epsilon\tau^{4/3}$ and hence the temperature drops rather slowly,

as $\tau^{-1/3}$. It is interesting to determine the differential equation for the time-dependence of the temperature of the fluid. From Eq. (21) we have, considering $c=c(p)$, and $p=p(T)$

$$\begin{aligned} \frac{dc}{dT} &= \frac{dc}{dp} \frac{dp}{dT} \frac{dT}{d\tau} \\ &= - \frac{(c+p)}{T} = - \frac{Ts}{T} \end{aligned} \quad (36)$$

But

$$p = - \frac{A}{V} \quad (37)$$

where A is the free energy. Furthermore the entropy S is

$$S = - \frac{\partial A}{\partial T} \bigg|_V \quad (38)$$

Hence in Eq. (36)

$$\frac{dp}{dT} = \frac{S}{V} = + s \quad (39)$$

In addition the sound velocity is

$$\frac{1}{v_s^2} = \frac{dc}{dp} \quad (40)$$

Putting all this together yields the interesting formula

$$\frac{1}{T} \frac{dT}{d\tau} = - \frac{v_s^2}{T} \quad (41)$$

For equations of state to be discussed in the next section,

$$v_s^2 \leq \frac{1}{3}, \quad (42)$$

so that the decrease in temperature during the one-dimensional expansion phase is slow.

Transverse Flow

Before consideration of the equation of state, let us review the picture of the collision process.

1. In the central rapidity region, the evolution looks the same no matter what the reference frame.
2. The longitudinal velocity of the fluid near the collision axis is proportional to the distance from the collision point; i.e., fluid a longitudinal

distance z from the collision point is moving at velocity z/t , where t is the time elapsed since the collision of the incident nuclear pancakes took place.

3. The baryon-number of the incident nuclei is found, in a reference frame appropriate to the central rapidity region, in thin pancakes receding at very near the speed of light from the collision point.
4. The entropy within a comoving volume between these pancakes is constant.
5. The temperature decreases as $\tau^{-1/3}$ or slower as long as there is one-dimensional flow; here τ is the "proper time" $\tau = \sqrt{t^2 - z^2}$.
6. The energy within a comoving volume also decreases as $\tau^{-1/3}$ or less as long as there is one-dimensional flow.

Now consider the region of the fluid near the edge of the disks. Near the periphery we may expect a few pions to be directly produced, which escape outward at the speed of light. At distances slightly less peripheral there should be a rarefaction front propagating inward at the velocity of sound v_s . At distances from the collision axis of less than $v_s t$ (or more accurately $\int_0^t v_s(t') dt'$) the news that the nuclei are of finite size and that a boundary exists will not yet have been received. The geometry of this rarefaction front is shown in Figure 4. At $z = 0$, the fluid is at rest and so at time t we have the estimate $\int_0^t v_s dt$ for the distance inward that the

rarefaction front has penetrated. For $z \neq 0$ the fluid is in motion and one must take into account time dilatation. It is the "proper time" $\tau = \sqrt{t^2 - z^2}$ which is relevant and the distance the front penetrates inward from the edge is

$$\int_0^\tau v_s(\tau') d\tau'.$$

To the extent that v_s is time independent, the equation for the rarefaction front is

$$\begin{aligned} \sqrt{t^2 - z^2} \\ \rho(t) = R - \int_0^\tau v_s(t') dt' \\ z = R - v_s \sqrt{t^2 - z^2} \end{aligned} \quad (44)$$

where R is the radius of the incident nuclei. Note that no more than 50% of the fluid moves one dimensionally after a proper time τ given by

$$\begin{aligned} v_s \tau \approx 0.3R \\ \text{or } \tau \approx 0.5R \end{aligned} \quad (45)$$

Even for U-U collisions, this does not amount to more than 3-4% of time.

The motion of the fluid at radii beyond the rarefaction front is more complex. No real attempt at an analysis will be made here.¹⁰ One must expect that the fluid will cool quite rapidly and drop out of equilibrium on a relatively short time-scale. As emphasized by T. D. Lee,¹¹ we may also expect a variety of unstable flows to develop ("volcanoes?"). To us it seems that the transverse flow is especially vulnerable to unstable behavior. But study of such questions is also beyond the scope of our considerations here.

III. EQUATION OF STATE

The details of the evolution of the produced fluid will depend upon its equation of state $\epsilon = \epsilon(p)$, or equivalently $p = p(\beta)$, with $\beta = T^{-1}$. Inasmuch as we are interested in the fluid produced in the central rapidity region, the net baryon number is zero and we need not introduce a chemical potential.

There are two limits in which the situation is simple, namely, extremely low and extremely high temperature. At very low temperature we expect to have a dilute pion gas. If, for simplicity, we neglect the pion mass, this leads to an equation of state

$$p = \frac{3\pi^2}{90} T^4 \quad T \gg \frac{M\pi}{2} \quad (46)$$

where the 3 in the numerator reminds us that there are three degrees of freedom in the gas (π^+ , π^0).

At extremely high temperature the situation is again simple; there should exist an ideal fluid of quarks and gluons.¹² In this case

$$p = \frac{\pi^2}{90} n(\beta) \beta^{-4} \quad (47)$$

where

$$\begin{aligned} n(\beta) &= 8 \times 2 && (\text{gluon color} \times \text{spin}) \\ &+ \frac{7}{8} (2 \times 3 \times 2 \times n_f) && (\text{quark and antiquark color, spin and flavor}) \\ &= 16 + \frac{21}{2} n_f && (48) \end{aligned}$$

with n_f (2 to 3) the effective number of quark flavors in the fluid. (The factor 7/8 takes into account the difference between Bose and Fermi statistics in the Boltzmann factor.) Thus we may take, for our purposes,

$$n(\beta) = \begin{cases} 3 & \beta \rightarrow \infty \\ 42 \pm 6 & \beta \rightarrow 0 \end{cases} \quad (49)$$

and write, at all temperatures,

$$p = \frac{\pi^2}{90} n(\beta) \beta^{-4} \quad (50)$$

We expect under quite general conditions that $n(\beta)$ is a monotone increasing function of temperature. What is needed to ensure this is the nonnegativity of the trace of the energy-momentum tensor T_μ^ν . If

$$T_\nu^\nu \geq 0 \quad (51)$$

it follows that

$$\epsilon \geq 3p \quad (52)$$

However, the pressure is related to the free-energy A by the relation

$$p = -\frac{A}{V} \quad (53)$$

while the energy is

$$E = \epsilon V = + \frac{\partial}{\partial \beta} \beta A$$

$$= -V \frac{\partial}{\partial \beta} \beta p$$

$$= -\frac{\pi^2}{90} V \left(3n(\beta) \beta^{-4} - \beta^{-3} \frac{\partial n}{\partial \beta} \right) \quad (54)$$

It follows that

$$\frac{\epsilon}{3p} = 1 - \frac{\beta}{3n} \frac{\partial n}{\partial \beta} \quad (55)$$

and the condition (52) implies

$$\frac{\partial n}{\partial \beta} \leq 0$$

We shall hereafter assume this condition is satisfied.

There is general agreement¹³ that the transition temperature between quark-gluon phase and pion phase is somewhere around 200 MeV. We shall, for the sake of (in) definiteness, take it to be 200 ± 50 MeV.

The question of the existence of a phase transition (or transitions?) and whether it (they?) is (are) first-order is more uncertain. We may note in this regard two recent results:

1) Montvay and Pietarinen,¹⁴ among others, have presented lattice Monte-Carlo calculations of the equation of state of a pure gluon gas. It shows an abrupt transition from glueball gas to gluon plasma. Immediately above the transition, the behavior is remarkably ideal, in contrast to what happens in perturbation theory, where the equation of state

$$n(\beta) = 16 \left[1 - 15 \left(\frac{\alpha_s(\beta)}{\pi} \right) + \frac{80}{\sqrt{3}} \left(\frac{3\alpha_s(\beta)}{\pi} \right)^{3/2} + \dots \right] \quad (57)$$

indicates, for any reasonable value of α_s , 0(1) deviations from ideal behavior (i.e. nonconvergence of the perturbation series). The nature of the gluon-gluon transition is not clear, but it might well be first order.

2) Kogut, et al. argue that there may be two phase transitions. The transition at highest temperature would be associated with spontaneous breaking of chiral symmetry and generation of a quark mass (plus "massless" Nambu-Goldstone pions). However, at this intermediate temperature there would be no color confinement. The lower-temperature transition would then be associated with the transitions to the color-confined hadron phase.

The nature of these phase transitions, however, is still somewhat obscure. We shall assume the behavior of $n(\beta)$ in the transition region to be quite abrupt, i.e. there is a quite narrow temperature interval in which the transition from quarks

and gluons to pions take place. We show some guesses in Fig. 5. In estimating energy and entropy densities, as well as sound velocity, we need also to evaluate

$$\Delta_1 = \frac{d(\ln n)}{d(\ln T)} = \frac{T}{n} \frac{dn}{dT} \quad (58)$$

and

$$\Delta_2 = \frac{d^2(\ln n)}{d(\ln T)^2} = \frac{T^2}{n} \frac{d^2 n}{dT^2} + \frac{T}{n} \frac{dn}{dT} \quad (59)$$

In terms of these quantities we have

$$\epsilon = p (3 + \Delta_1)$$

$$s = \beta p (4 + \Delta_1)$$

$$v_s^2 = \frac{1}{(3 + \Delta_1) + \Delta_2/(4 + \Delta_1)} \quad (60)$$

Sketches of Δ_1 and Δ_2 are given in Fig. 6. The peak values and shapes are sensitive to the width (in temperature) of the transition region. The maximum value of Δ_1 could well be ≥ 10 , leading to a small value of the sound velocity ($v_s^2 \leq 0.1$) and consequent very slow cooling of the fluid as it expands (cf. Eq. (41)). The maximum value of Δ_2 is large enough that it cannot be neglected in estimating v_s^2 . Below the transition, Δ_2 is positive, leading to a depressed value of the sound velocity, while above transition, v_s^2 should be closest to

its asymptotic value of $1/3$.

A sketch of v_s^2 is shown in Fig. 7.

If there is a first order phase transition, Δ_1 will have a discontinuity, with evidently a larger value above the critical temperature T_c than below.

By examining Fig. 6, it is evident that the value of Δ_1 at the critical temperature, approached from above, which we call $\Delta_1(qg)$ should be 25 ; we take 7 ± 2 . The value of Δ_1 as approached from below, which we call $\Delta_1(\pi)$ must be less; let us say 3 ± 2 just to get an estimate. Then, the ratio of entropy density of pion plasma to that of quark-gluon plasma is

$$r = \frac{s(\pi)(T_c)}{s(qg)(T_c)} = \frac{4 + (3 \pm 2)}{4 + (7 \pm 2)} = 0.7 \pm 0.2 \quad (61)$$

This ratio controls the length of time the plasma remains in a mixed phase. Let $f(qg)(\tau)$ be the fraction of the fluid which is in the quark-gluon phase, i.e. at any time τ we have

$$s = f(\tau)_{qg} s(T_c)_{qg} + (1-f(\tau)_{qg}) s(T_c)_{\pi} \quad (62)$$

Inasmuch as $s = (\text{const}) \tau^{-1}$, it follows from a simple calculation that

$$f(\tau)_{qg} = \frac{\tau_0 - \tau}{\tau(1-\tau)} \quad (63)$$

where τ_0 is the proper time at which the mixed phase first appears. The final time $\tau_f = \tau^{-1} \tau_0$ occurs at a time anywhere from roughly $1.1\tau_0$ to $2\tau_0$. Uncertainties in estimates of initial energy density as well as in the value of $n(T_c)$ make an estimate of τ_0 , and consequently of the duration of the mixed phase, quite uncertain. The mixed phase could last anywhere from a proper time interval of $1f$ to $5f$.

We cannot expect one-dimensional expansion to persist much longer than a time $c\tau \sim 5-10f$. At the end of this period we have an energy density $\sim 100-300 \text{ MeV}/f^3$, with a temperature very near the critical temperature T_c (this endpoint can be located in Fig. 8). With a mean energy per quantum of $\sqrt{2}T_c$, say $400 \pm 100 \text{ MeV}$, this gives a density of quanta of, say, $\sim 0.4 \pm 0.2 f^{-3}$. This is two or three times nuclear matter density. It may not be too bad an approximation to consider it to already be distinguishable pion matter. If this is the case, it is also reasonable to presume that the physics of the subsequent 3-dimensional expansion to be relatively unremarkable. There is, in any case, not much more time available before the density is decreased enough for this to be manifestly the case.

IV. COMMENTS & CONCLUSIONS

Evidently many important questions have not been addressed in this note. We have not calculated the transverse motion of the fluid. And we have not addressed the final stage of three-dimensional radial expansion of the fluid.

Another area of concern is the question of the stability of the solutions we have found. In the case of longitudinal flow, a preliminary look suggests that the solution we have found is stable and that small perturbations or irregularities in the initial conditions will not grow. An indication of this stability can be seen in the situation regarding the fluid in the leading-particle regions which carry the baryon-number. It follows from causality alone that the baryon-number cannot diffuse very far from the fragmentation regions toward the central rapidity region. A simple calculation based on the geometry in Fig. 3 shows that in going from initial proper time τ_i to final proper time τ_f , the baryon-number diffusion is limited to a rapidity interval $|\Delta y| \leq \log \tau_f / \tau_i$.

The case of transverse flow may be more complex; hot fluid in the interior region is encased in a cooler exterior region, and supersonic unstable flows might be contemplated. Furthermore, there are certain to be local "hot spots" at the initial time for onset of hydrodynamic flow. These are associated with high $-p_T$ jets produced by hard scattering of quarks and/or gluons in the incident projectiles. These "hot

spots" carry both high energy and (transverse) momentum density. They should be an interesting initial perturbation to add to the smooth initial configuration we have assumed.

We have also neglected the effects of viscosity and heat conductivity. The importance of such effects is measured by the ratio of mean free path to the natural scale of variation of the parameters (ϵ , T , etc.) of the system. For the longitudinal motion, we estimated, at the initial time when the natural scale of variation is $\sqrt{s}/2$, that the mean free path is (cf. Eq. 10) probably small in comparison. The uncertainty in this conclusion is very large. However, we may remark that during longitudinal expansion both the mean-free path (inversely proportional to entropy density) and natural scale increase in proportion to the elapsed time. Hence, if it is justifiable to neglect viscosity and heat conductivity in the initial stages, it should continue to be the case throughout the longitudinal expansion stage. No such simple statement holds, however, for the transverse expansion.

The estimate of initial energy density is quite uncertain. In addition to the uncertainty in choice of transverse scale d_0^2 , there is uncertainty in the methodology itself. For example, the Landau boundary conditions of total arrest and equilibrium of the fluid in the initial collision leads, in the case of nucleon-nucleon collisions, to a final rapidity distribution of produced hadrons not dissimilar to what is experimentally seen. The presence of leading baryons in

nucleon-nucleus collisions in our opinion argues against assuming Landau-like boundary conditions. However, it is possible that a large fraction of the incident gluon fields and/or quanta in the projectiles does become initially equilibrated. If this should be the case the initial energy density would be considerably higher. However, it would require a very large increase to increase the temperature by a large factor owing to the T^4 dependence of energy density implied by the Stefan-Boltzmann behavior of the quark-gluon plasma.

Finally, we have not addressed questions of experimental observables and signatures, other than commenting that in this model the final pion multiplicity should not depend upon details of the equation of state or how the system evolves in time but only upon the entropy density imposed in the initial boundary conditions. We here make a few brief remarks on some of the proposed signatures.

1) Direct Photons:

We have not addressed the calculation of the direct photon flux. The mean photon energy will not be large (~400-500 MeV) and thus the observed distribution may be rather unremarkable. There may also be considerable uncertainty in estimating the production rate near and during the phase transition.

2) Direct Dileptons:

Many of the above comments again apply. In particular the mean mass of the dileptons is low, and again there may be a lot of uncertainty in calculating their rate of production. The calculation must go well beyond Drell-Yan style perturbation theory,¹⁶ which only seems to be applicable for dilepton masses in excess of 3-4 GeV.

3) Enhanced K/ π Ratio:

In this model, the temperature is rather low. The strange-quark mass exceeds the non-strange quark mass by about 150 MeV. The Boltzmann-factor will suppress the strange quarks by a factor $e^{\Delta m/T} \sim 2$. This is the level of suppression already seen in hadron-hadron collisions,¹⁷ once the effects of resonance production (which enhances pion, but not kaon production) is taken into account. The effects of enhanced strangeness production need not be dramatic.

4) Unusual Event Structure (e.g. "volcanoes"):

T. D. Lee has suggested¹¹ that hydrodynamic instability may lead to existence of patchy regions in the phase space of produced particles, within which one would find increased mean energy, multiplicity, and K/ π ratio. These may exist in this model, and would be interesting to

investigate further.

5) Production of Exotic Metastable Structures:

The "Centauroid" events reported in cosmic-ray emulsion experiments led L. McLerran and this author to speculate on the existence of metastable globs of dense quark matter.¹⁸ Since then, the same experimental group has reported¹⁹ other curious "Chiron" events which might be interpreted in terms of similar structures. Ideas along these lines are clearly extremely speculative. The suggested structures also are characterized by nonvanishing baryon-number density. Could such structures be produced in nucleus-nucleus collisions? The initial density in the central region is high, and if there is a first-order phase transition, there should be enough inhomogeneity present to encourage the production of such objects.

On the other hand, production of Centauros and "Chirons" are associated with leading-particle effects, and this lies outside the scope of our considerations. Also, the fluid in the central region has no net baryon number, so that there would need to be a spontaneous generation of net baryon density to make these objects. Thus, while the extension of our picture to the fragmentation regions might allow such speculation, it does not seem a very promising idea to entertain with respect to central production.

Nevertheless, such speculation reminds us that the possibility of totally unexpected phenomena may be the most compelling reason to consider relativistic nucleus-nucleus collisions. It is regrettable that it is so hard to estimate the odds for this to happen.

ACKNOWLEDGEMENTS

The author thanks his colleagues at Fermilab for many helpful discussions. He also especially wishes to thank G. Baym, L. McLerran, A. Mueller, R. Weiner, and E. Friedlander for teaching him some of the lore of this subject. Finally he thanks W. Czyz for especially helpful criticisms and for rectifying errors in the manuscript.

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FIGURE CAPTIONS

- Fig. 1. Schematic of the evolution of compressed "baryon fireball" in nucleus-nucleus collisions, according to the mechanism of Anishetty, Koehler, and McLerran.⁸
- Fig. 2. Geometry for initial state of centrally produced plasma in nucleus-nucleus collisions.
- Fig. 3. Space-time diagram of longitudinal evolution of the quark-gluon plasma.
- Fig. 4. Geometry of fluid expansion near the edge of the nuclei.
- Fig. 5. Effective number of plasma degrees of freedom versus temperature. Solid curve: first order phase transition at $T=200$ MeV. Dashed curve: no prominent phase transition.
- Fig. 6. Sketches of the parameters Δ_1 and Δ_2 .
- Fig. 7. Crude estimate of sound velocity versus temperature.
- Fig. 8. Energy density versus temperature.

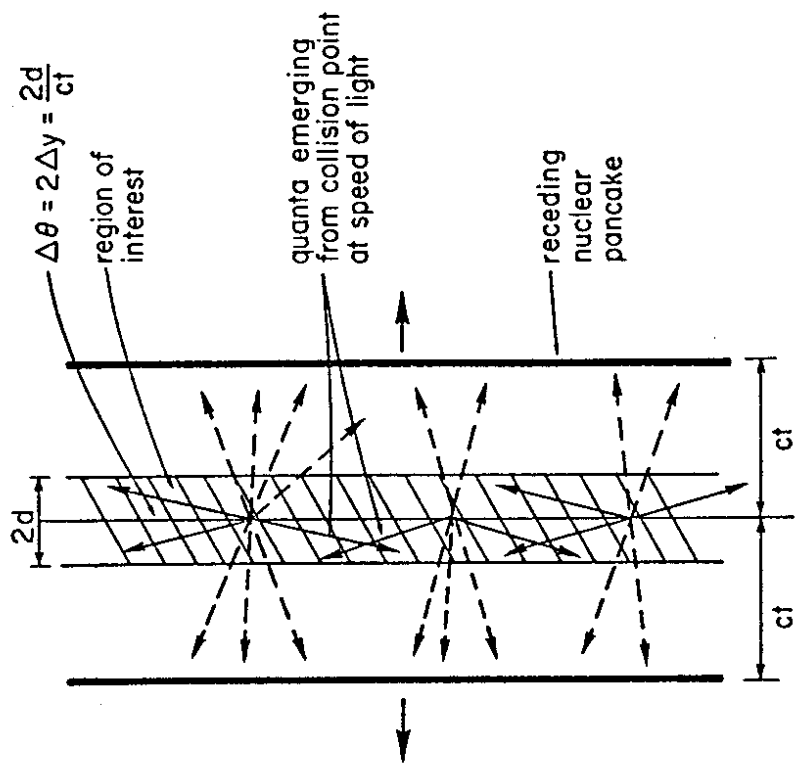


Fig. 2

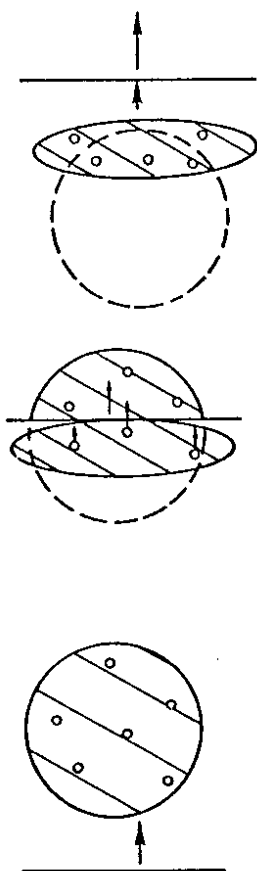


Fig. 1

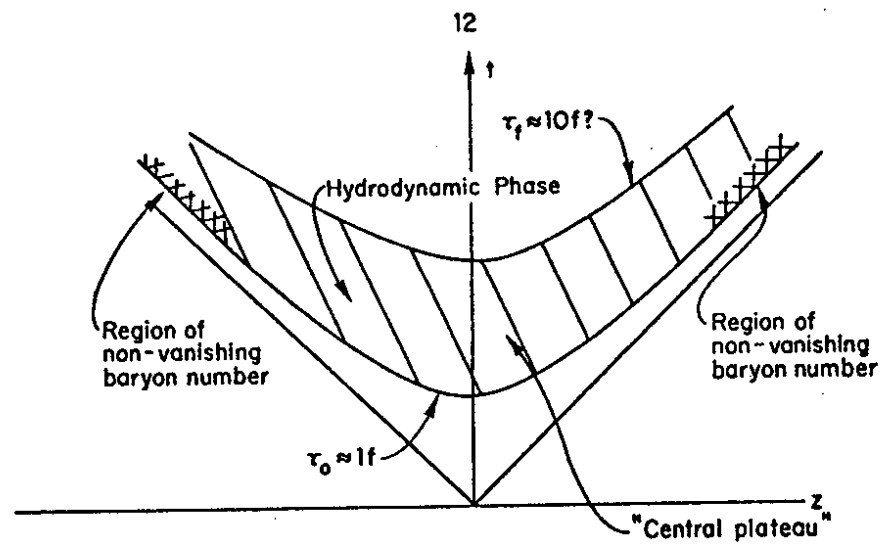


Fig. 3

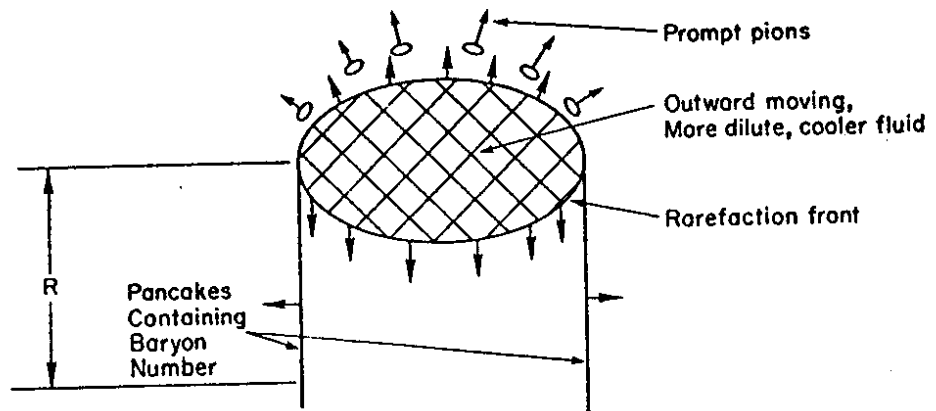


Fig. 4

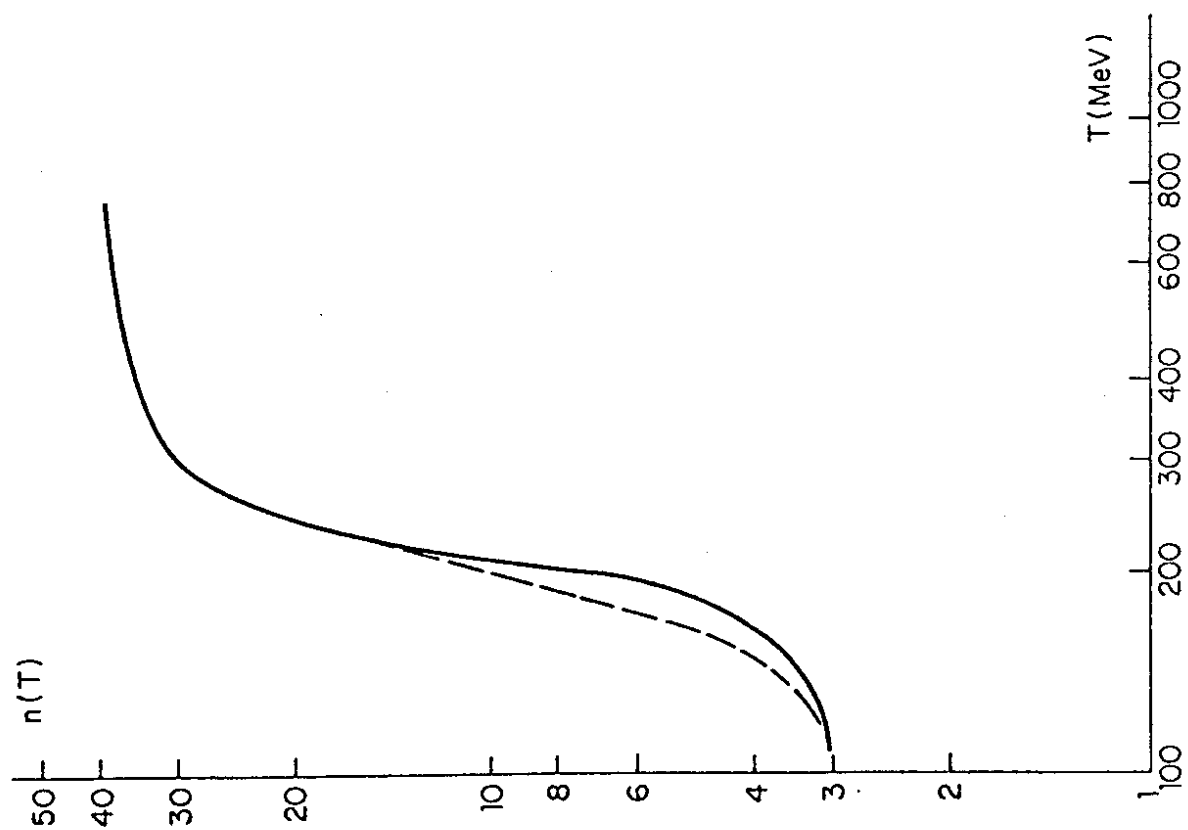


Fig. 5

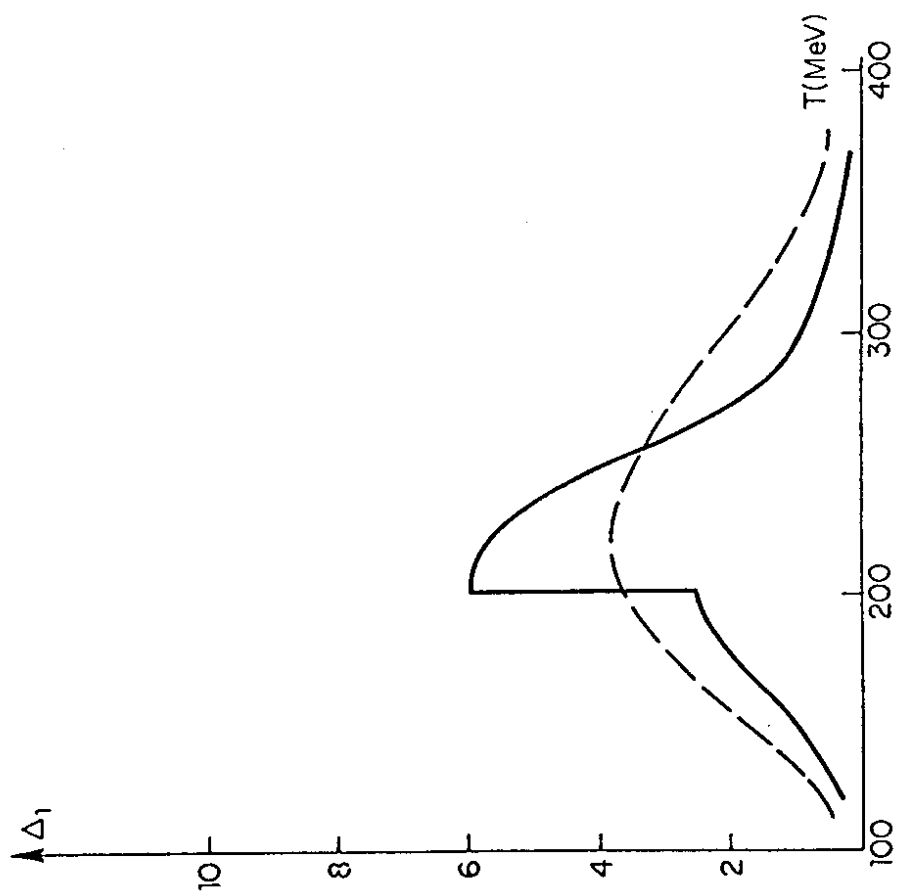


Fig. 6a

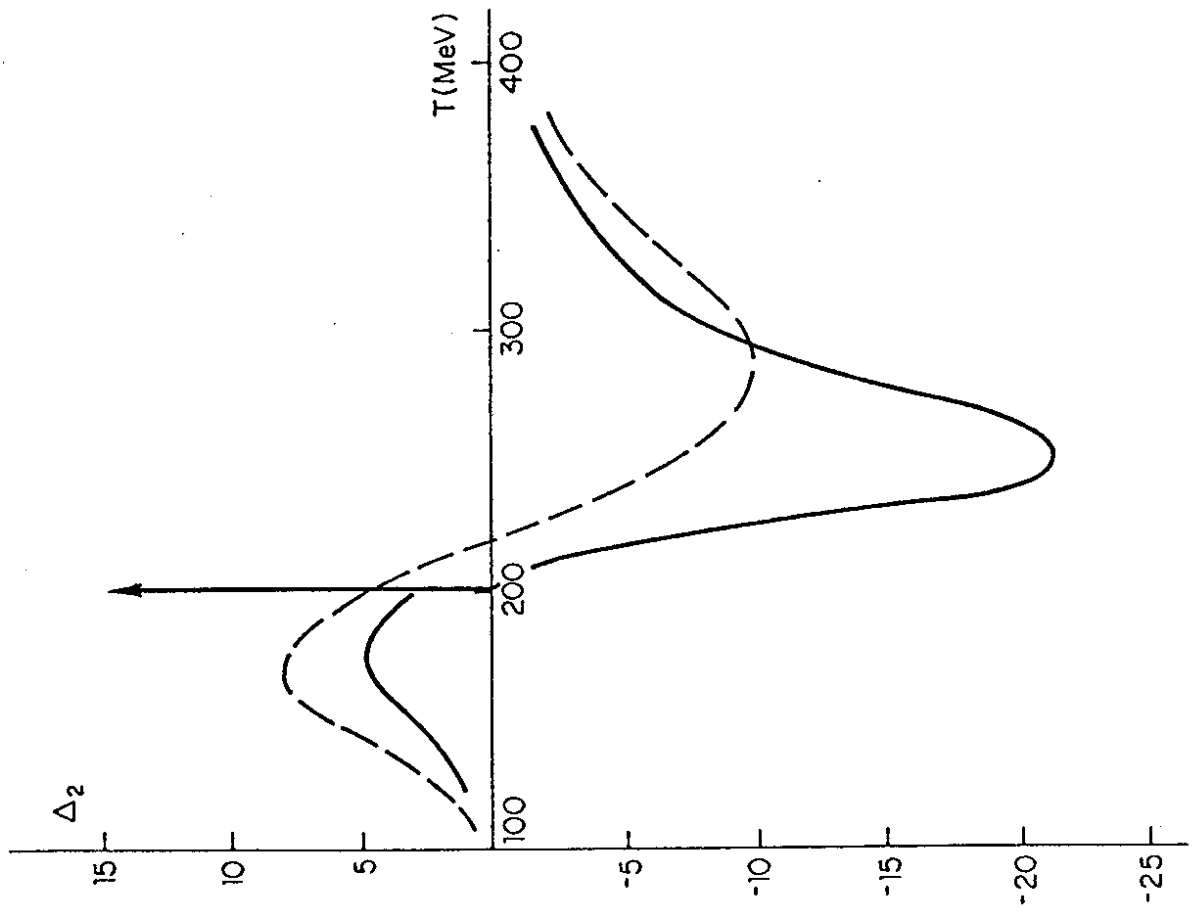


Fig. 6b

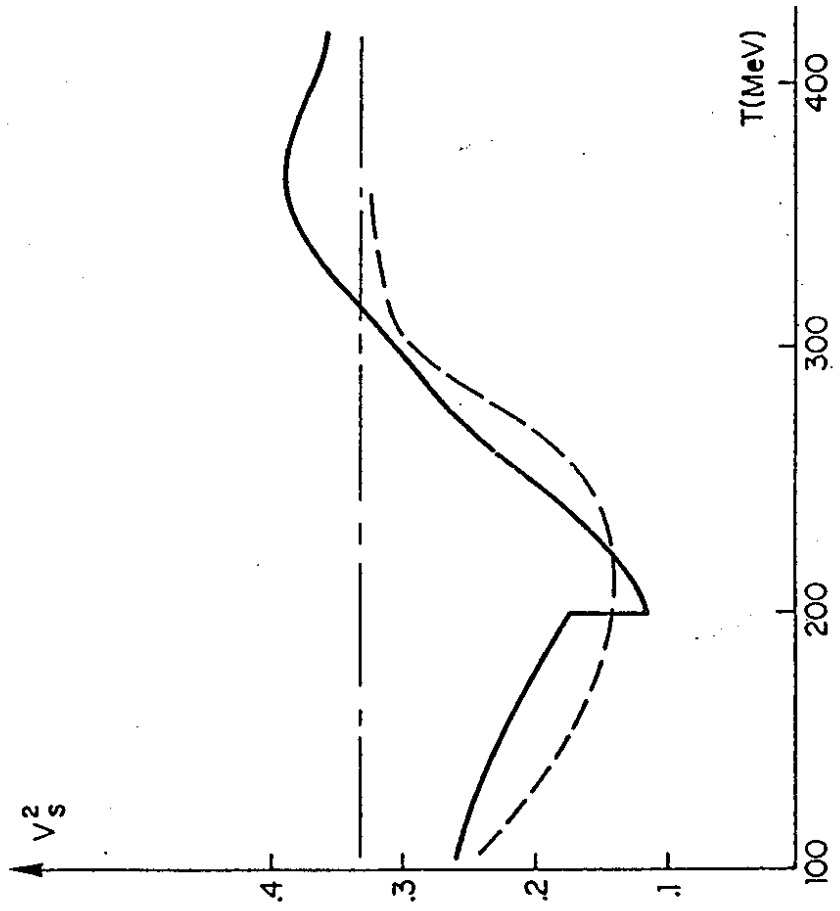


Fig. 7

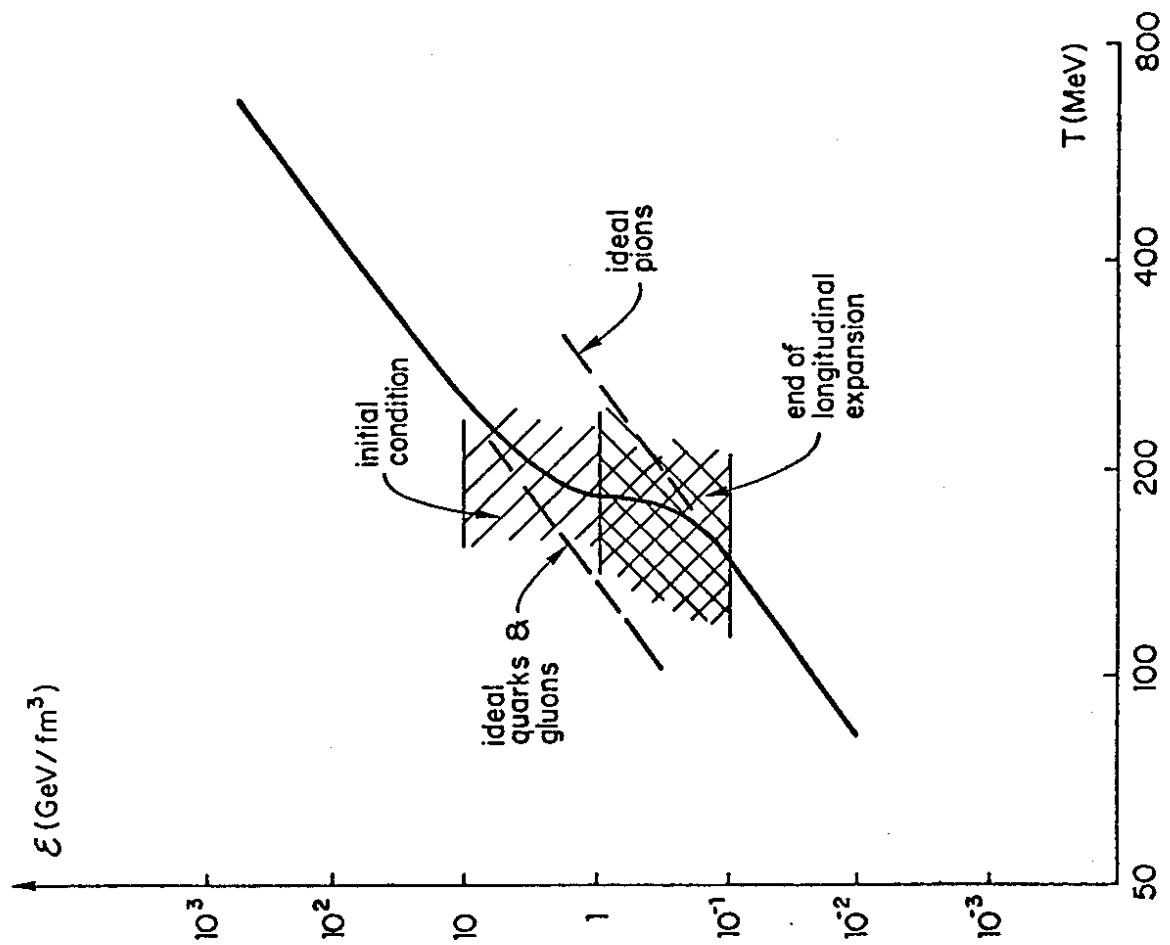


Fig. 8