Z. Phys. C – Particles and Fields 53, 651–655 (1992)

Zeitschrift für Physik C Particles and Fields

Pionic parton distributions

M. Glück, E. Reya, A. Vogt

Institut für Physik, Universität Dortmund, W-4600 Dortmund 50, Federal Republic of Germany

Received 27 August 1991

Abstract. The gluon and sea distributions of the pion are uniquely determined by the requirement of a *valence*-like structure of the input parton distributions at some low resolution scale. These (dynamical) results are obtained with practically no free parameters, just using the experimentally determined pionic valence distribution combined with the constraints for the pionic gluon distribution provided by direct-γ data. Simple parametrizations of the resulting parton distributions are presented in the range $10^{-5} \lesssim x < 1$ and $0.3 \lesssim Q^2 \lesssim 10^8$ GeV^2 as obtained from the leading- and higher-order evolution equations.

In contrast to the nucleon, the parton distributions of the pion are poorly known even in the present experimentally accessible Bjorken-x region $x \gtrsim 0.2$. This is due to the fact that pionic distributions can only be inferred from purely hadronic πN reactions by assuming some specific set of nucleon distributions. Whereas Drell-Yan dilepton ($\mu^+\mu^-$) production experiments [1, 2–5] determine, for $x \gtrsim 0.2$, the shape of the pionic valence distribution $v_\pi(x, Q^2)$ rather well, they leave almost unconstrained the pionic sea and gluon distributions, $\bar{q}_\pi(x, Q^2)$ and $G_\pi(x, Q^2)$, respectively. Heavy quarkonia (J/ψ, Υ) production data [6, 7] are, under certain assumptions, more restrictive for extracting [6–8] $G_\pi(x, Q^2)$, which depends, however, on a specific bound state or semilocal duality model for describing heavy quark production. At present, probably the most reliable determination of $G_\pi(x, Q^2)$, for $x > 0.2$, results from an analysis [9] of measurements of direct photon production in $\pi^\pm p \to \gamma X$ [10].

All these analyses depend, however, on specific assumptions for the x-dependence of the pionic gluon distribution at a certain input scale $Q^2 = Q_0^2$, which become even more ambiguous for the experimentally much less constrained pionic sea distribution $\bar{q}_\pi(x, Q_0^2)$. Their behavior at $x < 0.2$ is totally unknown as in the case of the nucleon distributions for $x < 10^{-2}$. For the latter case we have developed a predictive scheme [11, 12] where gluon and sea distributions are radiatively generated, using 1- and 2-loops evolution equations, from some *valence*-like inputs (which vanish as $x \to 0$) at some low scale $Q^2 = \mu^2$. Thus the predicted very low-x behavior of gluon and sea distributions at $Q^2 > \mu^2$, experimentally not yet explored, are mainly due to the QCD dynamics and are independent of any ad hoc input ansatz for $x \to 0$.

In the present paper we apply this "dynamical" procedure to the pion relying, wherever possible, on the concepts, methods and notations introduced in [11, 12]. The main difference lies in the adopted input valence distribution at $Q_0^2 = 2$ GeV^2

$$xv_\pi(x, Q_0^2) = N_v x^{0.48}(1-x)^{0.85} \tag{1}$$

taken from the direct-γ analysis in [9] with $N_v = 0.681$ subject to the constraint

$$\int_0^1 v_\pi(x, Q^2)\,\mathrm{d}x = 1. \tag{2}$$

It should be noted that this valence density is consistent with all present πN experiments [1–4, 6, 7, 10] except the E615 one [5] which requires the valence distribution to be larger than the one in (1) by about 20%. (A similarly enhanced valence density has also been obtained in [13].) In view of such normalization ambiguities between different sets of data we will take the same valence distribution, as given in (1), as our input for the leading order (LO) as well as higher (HO) calculations. This is in addition supported by the fact that previous LO and HO analyses [3, 1] of the data did not result in any significant differences between the LO and HO pionic input valence distributions. Furthermore, the theoretical constraint (2) together with $xv_\pi(x, Q_0^2) \to 0$ as $x \to 0$ reduce the sensitivity of our predictions at small-x to the experimental uncertainties in $v_\pi(x, Q_0^2)$ at $x \gtrsim 0.2$.

Generalizing previous purely dynamical approaches [14] where all sea and gluon distributions were radiatively generated just from the valence quarks in the pion, the only other input will be a *valence*-like gluon

$$G_\pi(x, \mu^2) = \kappa v_\pi(x, \mu^2) \tag{3}$$

at the low 'static' input scale $Q^2 = \mu^2$. Lacking any further experimental information on the small-x behavior of the pionic gluon and sea distributions, it is physically reasonable to assume that this scale where the valence structure of the hadron dominates is universal and we therefore utilize in the present calculation the previous values recently obtained for the nucleon [12],

$$\mu_{\mathrm{LO}}^2 = 0.25\,\mathrm{GeV}^2, \qquad \mu_{\mathrm{HO}}^2 = 0.3\,\mathrm{GeV}^2 \tag{4}$$

where $\alpha_s(\mu^2)/\pi$ is still safely small, i.e. about 0.29 and 0.22 in LO and HO, respectively. Furthermore, in view of the rather poor data mentioned above there is, in contrast to the nucleon distributions [12], no point in fixing a small non-vanishing input sea distribution at $Q^2 = \mu^2$ and we therefore adopt as a reasonable first approximation a vanishing input sea at $Q^2 = \mu^2$ which does not affect our main prediction, i.e. $G_\pi(x, Q^2)$ and $\bar{q}_\pi(x, Q^2)$ in the small-x region at $Q^2 > \mu^2$. The parameter κ in (3) is now uniquely fixed by the energy-momentum sum rule

$$\int_0^1 x[2v_\pi(x, \mu^2) + G_\pi(x, \mu^2)]\mathrm{d}x = 1 \tag{5}$$

which yields $\kappa_{\mathrm{LO}} = 1.460$ and $\kappa_{\mathrm{HO}} = 1.295$. We thus arrived at a model which allows us to calculate the gluon and sea distributions of the pion with essentially *no* free parameter, just using the experimentally determined input valence distribution in (1).

It should be noted that the momentum sum rule (5) together with the direct-γ data allow only slight modifications of our gluonic input $G_\pi(x, \mu^2)$ in (3) which turn out to be of marginal consequences for our predictions in the low-x region at $Q^2 > \mu^2$. Furthermore, the same constraints leave little room for a non-vanishing input sea distribution; any appreciable non-vanishing sea input $\bar{q}_\pi(x, \mu^2)$ worsens the agreement with the direct-γ data.

The explicit calculation of these distributions in LO and HO($\overline{\mathrm{MS}}$) proceeds now according to the methods described in detail in [11] with the Λ-values for $\alpha_s(Q^2)$ taken, for $f=4$ flavors, to be [12] $\Lambda_{\mathrm{LO}}^{(4)} = \Lambda_{\overline{\mathrm{MS}}}^{(4)} = 200$ MeV. Furthermore, the inclusion of heavy quarks ($h = c, b, t$) in the evolution equations is, as usual [11, 12], assumed to follow the same pattern as for the light u, d and s quarks: This implies the continuity of all parton distributions across the threshold $Q = m_h$ with the boundary condition $h_\pi(x, m_h^2) = \bar{h}_\pi(x, m_h^2) = 0$; the continuity of $\alpha_s(Q^2)$ in turn expresses $\Lambda^{(f+1)}$ in terms of $\Lambda^{(f)}$, where $f + 1$ denotes the number of relevant active flavors at $Q > m_h$,

$$\begin{aligned} &\Lambda_{\mathrm{LO}}^{(3,4,5,6)} = 232, 200, 153, 82\ \mathrm{MeV} \\ &\Lambda_{\overline{\mathrm{MS}}}^{(3,4,5,6)} = 248, 200, 131, 53\ \mathrm{MeV} \end{aligned} \tag{6}$$

for our choice $m_{c,b,t} = 1.5, 4.5, 100$ GeV where the precise value for m_t is of minor importance except for $t_\pi(x, Q^2)$. Convenient and simple parametrizations of the resulting LO and HO($\overline{\mathrm{MS}}$) parton distributions are presented in the Appendix.

Our input valence and valence-like gluon distributions at $Q^2 = \mu^2$ are shown in Fig. 1 and the Q^2-evolution of $v_\pi(x, Q^2)$ to $Q^2 > \mu^2$ is illustrated in Fig. 2. The curve for $Q^2 = 2\,\mathrm{GeV}^2$ in Fig. 2 corresponds to the experimental input in (1) which has been assumed to be the same for our LO and HO calculations. It is also instructive to follow the Q^2-evolutions of the momentum fractions $\int_0^1 xf_\pi(x, Q^2)\mathrm{d}x$ carried by quarks and gluons as shown in

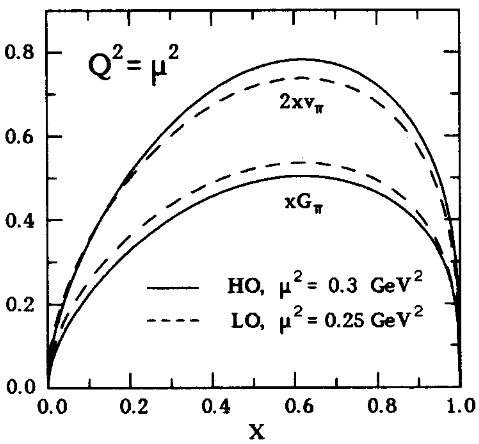

Fig. 1. The valence-like input distributions at $Q^2 = \mu^2$ for our LO and HO calculations. Note that the sea $\bar{q}_\pi$ vanishes at the input scale $Q^2 = \mu^2$

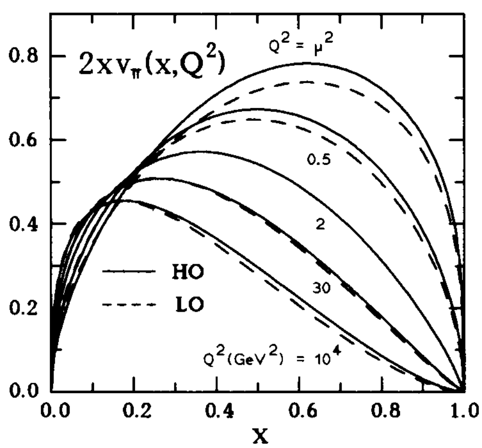

Fig. 2. The Q^2-evolution of the valence distribution. The curve at $Q^2 = 2\,\mathrm{GeV}^2$, being assumed to be the same in LO and HO, corresponds to (1)

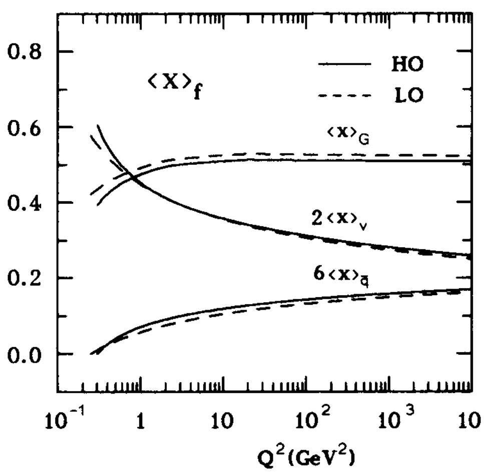

Fig. 3. The predicted Q^2-evolutions of LO and HO momentum fractions $\langle x \rangle_f \equiv \int_0^1 xf_\pi(x, Q^2)\mathrm{d}x$

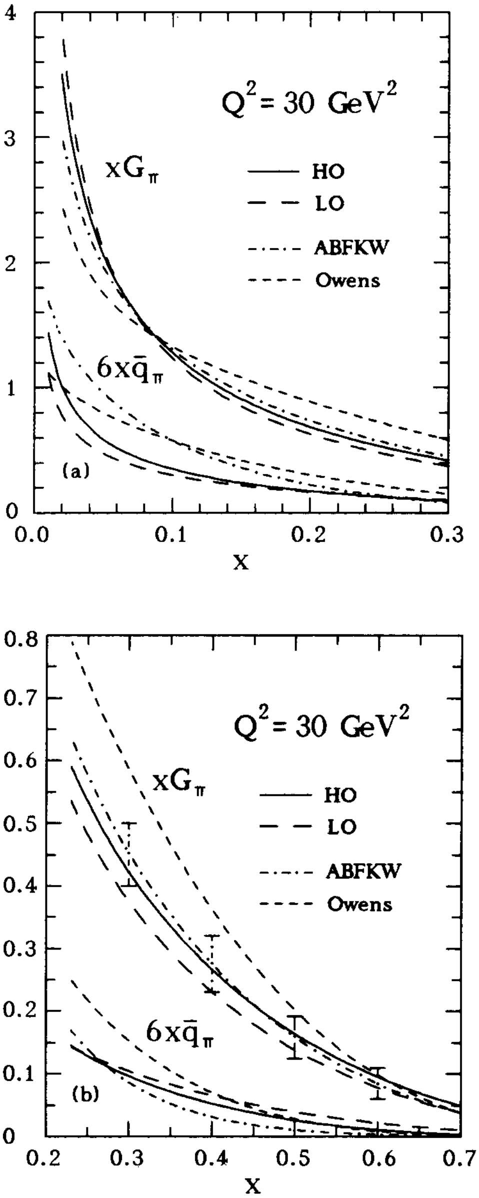

Fig. 4a,b. Comparison of our LO and HO pionic gluon and sea distributions at $Q^2 = 30\ \mathrm{GeV}^2$ with the results derived from direct-γ data (ABFKW) [9] and with an older fit by Owens [8]. Measurements exist only for $x \gtrsim 0.2$ and the vertical bars in **b** indicate the allowed range of the ABFKW result for G_π due to the experimental statistical and systematic errors added in quadrature

Fig. 3. The perturbative stability, i.e. the compatibility of the LO and $\mathrm{HO}(\overline{\mathrm{MS}})$ results down to $Q^2 \simeq 0.3\ \mathrm{GeV}^2$ is remarkable. Note that the non-vanishing sea $\bar{q}_\pi$ contribution is mainly due to the QCD dynamics since it has been generated only from a finite valence and gluon input but from a vanishing sea input at $Q^2 = \mu^2$.

Our LO and $\mathrm{HO}(\overline{\mathrm{MS}})$ gluon and sea distributions at $Q^2 = 30\ \mathrm{GeV}^2$ are shown in Fig. 4 and compared with an older fit (and a $\bar{q}_\pi$ suggestion) by Owens [8] and with a recent one by ABFKW [9]. The agreement of our radiatively generated parameter-free gluon distribution in Fig. 4b with the one obtained by the latter ABFKW fit, which was based on direct-γ production where the pionic gluon distribution is sensitively tested for $x > 0.2$, is particularly encouraging since it strengthens the reliability of our low-x predictions* for $x < 0.2$ where no experimental information is available. These low-x predictions of the pionic gluon and sea distributions, mainly due to the QCD dynamics, are shown in Figs. 5 and 6 at some typical fixed values of Q^2. A comparison with the assumed low-x behavior of the ABFKW analysis [9] shows that our distributions are significantly steeper for $x < 10^{-2}$ and that this difference gradually disappears for sufficiently large values of Q^2.

To summarize, the pionic gluon and sea distributions have been radiatively generated with essentially *no* free parameter, using 1- and 2-loop evolution equations, by assuming a purely valence-like input structure for the pion at a sufficiently low resolution scale where the latter is taken to coincide with the one obtained for the nucleon. The resulting predictions for the pionic parton distributions agree with the results obtained from fit-analyses of various πN data ($x \gtrsim 0.2$). Our predictions for the experimentally not yet accessible x-region, $x < 0.2$, are mainly due to the QCD dynamics and are rather insensitive to the experimental uncertainties of the input distributions of the pion. Simple parametrizations of our pionic distributions predicted in LO and HO are given in the Appendix.

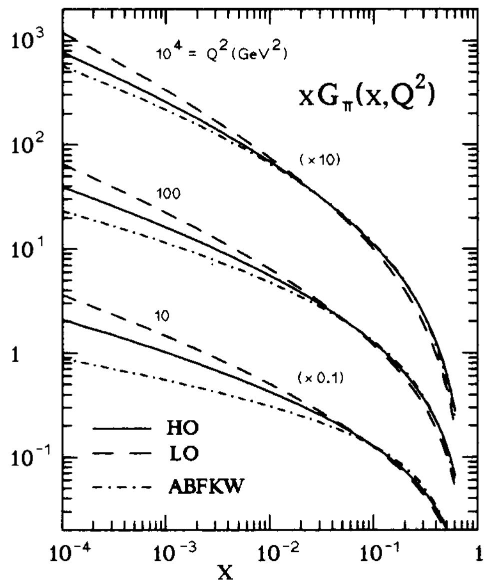

Fig. 5. The detailed small-x behavior of our radiatively generated gluon distributions in LO and HO at fixed values of Q^2, compared with the ABFKW fits [9] which have been extrapolated into the experimentally not established region $x < 0.2$

* This is particularly important, via VMD, for the small-x hadronic structure of the photon relevant, for example, in high energy cosmic photon collisions as well as for future QCD studies of photonic parton distributions at HERA and LEP. [See, for example, M. Drees, F. Halzen: Phys. Rev. Lett. 61 (1988) 275; J.C. Collins, G.A. Ladinsky: Pennsylvania State Univ. report PSU/TH71/HT-HEP-67 (1990); M. Drees, R.M. Godbole: Phys. Rev. Lett. 61 (1988) 682, Phys. Rev. D39 (1989) 169; DESY 90–143]

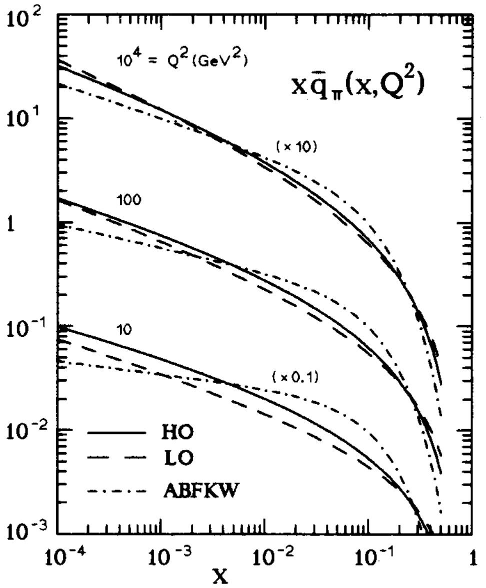

Fig. 6. As Fig. 5, but for the pionic sea distribution

Acknowledgement. This work has been supported in part by the "Bundesministerium für Forschung und Technologie", Bonn.

Appendix

A. Parametrization of LO parton distributions

Defining

$$s \equiv \ln \frac{\ln [Q^2/(0.232\ \mathrm{GeV})^2]}{\ln [\mu_{\mathrm{LO}}^2/(0.232\ \mathrm{GeV})^2]}, \tag{A.1}$$

to be evaluated for $\mu_{\mathrm{LO}}^2 = 0.25\ \mathrm{GeV}^2$, all our resulting pionic parton distributions can be expressed by the following simple parametrizations, valid for $\mu_{\mathrm{LO}}^2 \leqslant Q^2 \lesssim 10^8\ \mathrm{GeV}^2$ (i.e. $0 \leq s \lesssim 2.7$) and $10^{-5} \lesssim x < 1$. For the valence distributions we take

$$xv_\pi(x, Q^2) = Nx^a(1 + A\sqrt{x})(1-x)^D \tag{A.2}$$

with

$$\begin{aligned} N &= 0.519 + 0.180\,s - 0.011\,s^2 \\ a &= 0.499 - 0.027\,s \\ A &= 0.381 - 0.419\,s \\ D &= 0.367 + 0.563\,s. \end{aligned} \tag{A.3}$$

The gluon distribution is parametrized as

$$xG_\pi(x, Q^2) = \left[x^a(A + B\sqrt{x} + Cx)\left(\ln\frac{1}{x}\right)^b + s^\alpha \exp\left(-E + \sqrt{E' s^\beta \ln\frac{1}{x}}\right)\right](1-x)^D \tag{A.4}$$

with

$$\begin{aligned} &\alpha = 0.599 &&, \beta = 1.263 \\ &a = 0.482 + 0.341\sqrt{s} &&, b = 0 \\ &A = 0.678 + 0.877\,s - 0.175\,s^2 &&, B = 0.338 - 1.597\,s \\ &C = -0.233\,s + 0.406\,s^2 &&, D = 0.390 + 1.053\,s \\ &E = 0.618 + 2.070\,s &&, E' = 3.676. \end{aligned} \tag{A.5}$$

The sea quark distributions are parametrized as

$$xw_\pi(x, Q^2) = \frac{(s - s_w)^\alpha}{\left(\ln\frac{1}{x}\right)^a}(1 + A\sqrt{x} + Bx)(1-x)^D \times \exp\left(-E + \sqrt{E' s^\beta \ln\frac{1}{x}}\right). \tag{A.6}$$

For the light $SU(3)$-symmetric sea distribution $w = \bar{q}$ we obtain

$$\begin{aligned} &s_{\bar{q}} = 0,\quad \alpha = 0.55, && \beta = 0.56 \\ &a = 2.538 - 0.763\,s, && A = -0.748 \\ &B = 0.313 + 0.935\,s, && D = 3.359 \\ &E = 4.433 + 1.301\,s, && E' = 9.30 - 0.887\,s, \end{aligned} \tag{A.7}$$

for $w = c = \bar{c}$

$$\begin{aligned} &s_c = 0.888,\ \alpha = 1.02, && \beta = 0.39 \\ &a = 0, && A = 0 \\ &B = 1.008, && D = 1.208 + 0.771\,s \\ &E = 4.40 + 1.493\,s, && E' = 2.032 + 1.901\,s, \end{aligned} \tag{A.8}$$

and for $w = b = \bar{b}$

$$\begin{aligned} &s_b = 1.351,\quad \alpha = 1.03, && \beta = 0.39 \\ &a = 0, && A = 0 \\ &B = 0, && D = 0.697 + 0.855\,s \\ &E = 4.51 + 1.490\,s, && E' = 3.056 + 1.694\,s. \end{aligned} \tag{A.9}$$

In view of the presently unknown m_t we have refrained from parametrizing the massless evoluted $t_\pi(x, Q^2)$ distribution which becomes relevant only at exceedingly large scales $Q^2 \gg m_t^2$ or sufficiently small values of x.

B. Parametrizations of HO parton distributions

Defining

$$s \equiv \ln \frac{\ln [Q^2/(0.248\ \mathrm{GeV})^2]}{\ln [\mu_{\mathrm{HO}}^2/(0.248\ \mathrm{GeV})^2]}, \tag{A.10}$$

to be evaluated for $\mu_{\mathrm{HO}}^2 = 0.3\ \mathrm{GeV}^2$, our HO predictions can be parametrized as the LO ones and are similarly valid for $\mu_{\mathrm{HO}}^2 \leq Q^2 \lesssim 10^8\ \mathrm{GeV}^2$ (i.e. $0 \leq s \lesssim 2.6$) and $10^{-5} \lesssim x < 1$. The valence distribution is given by (A.2) with

$$\begin{aligned} N &= 0.456 + 0.150\sqrt{s} + 0.112\,s - 0.019\,s^2 \\ a &= 0.505 - 0.033\,s \\ A &= 0.748 - 0.669\sqrt{s} - 0.133\,s \\ D &= 0.365 + 0.197\sqrt{s} + 0.394\,s. \end{aligned} \tag{A.11}$$

The gluon distribution is given by (A.4) with

$$\alpha = 1.096, \qquad \beta = 1.371$$

$$a = 0.437 - 0.689\sqrt{s}, \qquad b = -0.631$$
$$A = 1.324 - 0.441\sqrt{s} - 0.130\,s, \quad B = -0.955 + 0.259\,s$$
$$C = 1.075 - 0.302\,s, \qquad D = 1.158 + 1.229\,s$$
$$E = 2.510\,s, \qquad E' = 2.604 + 0.165\,s. \tag{A.12}$$

The sea distributions are parametrized by (A.6) where for the light sea quarks $w = \bar{q}$ we get

$$s_{\bar{q}}=0, \quad \alpha = 0.85, \qquad \beta = 0.96$$
$$a = -0.350 + 0.806\,s, \quad A = -1.663$$
$$B = 3.148, \qquad D = 2.273 + 1.438\,s$$
$$E = 3.214 + 1.545\,s, \quad E' = 1.341 + 1.938\,s, \tag{A.13}$$

whereas for $w = c = \bar{c}$

$$s_c = 0.820, \quad \alpha = 0.98, \qquad \beta = 0$$
$$a = -0.457\,s, \qquad A = 0$$
$$B = -1.00 + 1.40\,s, \qquad D = 1.318 + 0.584\,s$$
$$E = 4.45 + 1.235\,s, \qquad E' = 1.496 + 1.010\,s, \tag{A.14}$$

and for $w = b = \bar{b}$

$$s_b = 1.297, \quad \alpha = 0.99, \qquad \beta = 0$$
$$a = -0.172\,s, \qquad A = 0$$
$$B = 0, \qquad D = 1.447 + 0.485\,s$$
$$E = 4.79 + 1.164\,s, \qquad E' = 1.724 + 2.121\,s. \tag{A.15}$$

Let us recall that in the light quark sector $u^{\pi^+} = \bar{d}^{\pi^+} = \bar{u}^{\pi^-} = d^{\pi^-} \equiv v_\pi + \bar{q}_\pi$, $\bar{u}^{\pi^+} = d^{\pi^+} = u^{\pi^-} = \bar{d}^{\pi^-} = s^{\pi^\pm} = \bar{s}^{\pi^\pm} \equiv \bar{q}_\pi$ and that $q^{\pi^0} = \frac{1}{2}(q^{\pi^+} + q^{\pi^-})$.

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