UM-TH-95-23
hep-ph/9509375
September 1995

ψ' Polarization as a test of colour octet quarkonium production

M. Beneke[1] and I.Z. Rothstein[2]

Randall Laboratory of Physics,
University of Michigan, Ann Arbor, MI 48109-1120, U.S.A.

Abstract

We calculate the α_s corrections to the transverse polarization fraction of ψ''s produced at the Tevatron. If the 'ψ'-anomaly' is explained by gluon fragmentation into a colour octet Fock component of the ψ', the ψ''s should be 100% transversely polarized at leading order in α_s, up to spin symmetry breaking long-distance corrections. We find that the short-distance correction to the transverse polarization fraction is a few percent, so that a polarization measurement would provide a reliable test of the colour octet mechanism.

[1] Address after Oct, 1, 1995: SLAC, P.O. Box 4349, Stanford, CA 94309, U.S.A.
[2] Address after Oct, 1, 1995: University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093, U.S.A.

1. Introduction

The topic of quarkonium production has received renewed attention as of late due to a plethora of data and new theoretical developments. While it has been known for quite some time that quarkonia calculations are tractable as a consequence of the fact that the non-perturbative dynamics may be swept into non-relativistic wave functions, an organizational principle was still lacking. This shortcoming manifested itself in the fact that the theory of P wave decays was plagued by infrared divergences that could not be factorized. This problem was resolved [1] once the calculation was organized using non-relativistic QCD (NRQCD) [2]. Within the framework of this effective field theory, the scales involved in quarkonia calculations are cleanly separated and factorization becomes explicit.

NRQCD has been widely used in calculations of quarkonium production rates. For transverse momentum p_T of the quarkonium larger than the quarkonium mass, the dominant production mechanism will be fragmentation. This is essentially a resonance effect resulting from the fact that the off-shellness of the fragmenting parton is small ($\sim 4m_c^2$) compared to the transverse momentum squared. The rate for this process is enhanced by a factor of $p_T^2/(4m_c^2)$, which can overcome the suppression due to any additional powers of α_s present in the fragmentation amplitudes. The fragmentation functions for quarkonia are partially calculable in perturbation theory [3].

If one does not include the fragmentation contribution, the prediction for the prompt production cross sections for both J/ψ and ψ' at the Tevatron are off by several orders of magnitude. The inclusion of fragmentation into color singlet quark-antiquark pairs brings the J/ψ production rate into agreement with experiment within theoretical errors [4, 5, 6], but still leaves the theory an order of magnitude short for ψ' production [5]. A possible explanation [7] for this discrepancy is that the dominant contribution to ψ' production results from fragmentation into two charmed quarks in a relative color octet state.[1] This is easily seen once we consider the process within NRQCD. In this formalism the full Fock state decomposition of the ψ',

$$|\psi'\rangle = |c\bar{c}[^3S_1^{(1)}]\rangle + O(v)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + O(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + O(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \ldots, \qquad (1)$$

is taken into account by the set of all possible production matrix elements [10]. The nota-

[1] Recent measurement of the J/ψ production fraction from χ_c decays indicates that colour octet production is also important to account for the fraction of J/ψ's not coming from B and χ_c decays [8, 9].

tion here is spectroscopic ${}^{2S+1}L_J$, with an additional superscript in parenthesis to denote the colour state. Each Fock component is weighted by some power of the heavy quark velocity. This weight is determined by the velocity scaling rules which will be discussed below. Although the transition of a $c\bar{c}$ state in a relative colour octet state into a ψ' is suppressed by $v^4 \sim (0.3)^2$ relative to the transition from a colour singlet pair, this suppression is off-set by an $(\pi/\alpha_s(2m_c))^2$-enhancement for the production rate of $c\bar{c}$ pairs in a ${}^3S_1^{(8)}$ state. Additional numerical factors ensure that the value of the matrix element (associated with the hadronization of the colour octet pair) required for agreement with experiment is indeed consistent with the velocity scaling rules [7].

As noted by Cho and Wise [11], the colour octet production mechanism could be checked by measuring the polarization of the ψ', as they should be strongly transversely polarized at leading order in α_s. The reason for this asymmetry is that the fragmenting gluons are nearly on-shell, so that the $c\bar{c}$ pair inherits the gluon's transverse polarization up to corrections of order $(4m_c^2)/q_0^2$, where q_0 is the gluon energy in the lab frame. Due to spin symmetry of the leading order NRQCD Lagrangian, the polarization of the quark pair stays intact in the subsequent (nonperturbative) evolution into a ψ' state up to spin symmetry breaking corrections of order $v^4/3 \sim (3-4)\%$. In this paper we address the question whether this signature persists after inclusion of radiative corrections to the short-distance production process and calculate the order α_s corrections to the prediction of 100% polarization (neglecting spin symmetry breaking). We find that these corrections are small, and thus the colour octet proposal can be easily tested since the polarization will not be destroyed by radiative corrections. Other deviations from pure transverse polarization from (spin-symmetry breaking) long-distance effects or higher twist effects will most likely be smaller than the short-distance correction, except, possibly, at small p_T, where higher twist effects can become large.

2. Fragmentation functions

The fragmentation of a gluon (or any parton) into a quarkonium is a two-scale process. Over distances of order $1/m_c$ the gluon fragments into a $c\bar{c}$ pair with longitudinal momentum fraction z and small relative velocity v in the rest frame of the pair. The subsequent arrangement of the quark pair into a quarkonium bound state takes place over longer distances, of order $1/(m_c v)$. To the extent that $v^2 \ll 1$, the effects on both scales can be separated in

NRQCD and the longitudinal momentum fraction of the quarkonium identified with that of the quark pair, so that the z-dependence is perturbatively calculable. The fragmentation function, defined for instance as in [12], can then be written as

$$D_{g\to H_\lambda}(z,\mu) = \sum_n d_{n;ij}(z,\mu,\mu')\,\langle \mathcal{O}^{H_\lambda}_{n;ij}\rangle(\mu') \tag{2}$$

where H_λ denotes the quarkonium in a specific polarization state λ. The $d_{n;ij}$ are short-distance coefficients, independent of the quarkonium state and computable as expansions in α_s. All bound state information is summarized in the vacuum matrix elements of the operators $\mathcal{O}^{H_\lambda}_{n;ij}$. These parameters must be determined phenomenologically or from lattice calculations. Following the notation of [10], the generic form for the production operators is

$$\mathcal{O}^{H_\lambda}_{n;ij} = \chi^\dagger \kappa_{n;i}\,\psi\, a_H^{(\lambda)\dagger} a_H^{(\lambda)}\,\psi^\dagger \kappa'_{n;j}\,\chi, \tag{3}$$

where ψ, χ are nonrelativistic quark and antiquark fields and $a_H^{(\lambda)}$ destroys a quarkonium state H with polarization λ in the out-state. The matrix elements are supposed to be evaluated in the quarkonium rest frame. The kernels $\kappa_{n;i}$, $\kappa'_{n;j}$ contain colour matrices, spin matrices and derivatives. The colour and spin indices are suppressed. The operators $\mathcal{O}^{H_\lambda}_{n;ij}$ depend on two sets of three-vector indices i and j. If we consider unpolarized production and include summation over λ in the definition (3), the matrix elements $\langle \mathcal{O}^{H}_{n;ij}\rangle$ can not depend on any three-vector. A tensor decomposition then reduces the sum in (2) to a sum of a few scalar operators as listed in [10]. In the case of polarization this decomposition also includes the polarization tensor of the quarkonium. After decoupling of indices from the short-distance amplitude, the kernels κ_n contain projections on the spin and angular momentum orientation of the quark pair. The dependence on the factorization scale for NRQCD, $m_c v < \mu' < m_c$, drops out in the sum (2), while the dependence on μ, which separates the gluon production from the fragmentation process is governed by the usual Altarelli-Parisi evolution.

3. Colour octet production at leading order

Given the Fock state decomposition (1), the leading contribution in v^2 comes from gluon and charm fragmentation into the colour singlet ${}^3S_1$ state. Charm fragmentation is shown[2]

[2]This diagram as well as all others are understood to be calculated in axial gauge. Otherwise, diagrams with gluons attached to eikonal lines would also have to be considered, see [12].

in Fig. 1b. Everything prior to the shaded oval is considered as short-distance dominated, whatever is to the right is contained in the matrix element of an operator $\mathcal{O}_n^H$. In Fig. 1b, this is simply the non-relativistic wave-function (squared) at the origin. Using the velocity scaling rules derived in [13], colour singlet charm and gluon fragmentation yield contributions of order $\alpha_s^2 v^3$ and $\alpha_s^3 v^3$ respectively. The net contribution from singlet fragmentation falls short of the experimental data by a factor of about 30 [5]. However, since $v^2 \sim 0.3$ is not small, the contribution from operators, whose matrix elements are suppressed in v^2, may be important. This is especially so for $\mathcal{O}_8^{\psi'}({}^3S_1)$, because it is the only operator with a short-distance coefficient $d(z)$ at order α_s. A quark pair in a colour octet ${}^3S_1$ state has non-zero overlap with a Fock state component of the ψ' that contains two additional gluons, represented by the emission from the black boxes in Fig. 1a. The long-distance dynamics is accurately described by the NRQCD Lagrangian and the velocity scaling rules apply. The quark pair in the ${}^3S_1^{(8)}$ state can evolve into the ψ' by a double electric dipole transition, each of which contributes a factor of v to the amplitude as indicated in (1). Thus, the contribution from colour octet gluon fragmentation starts at order $\alpha_s v^7$. The short-distance coefficient was computed in [14] to leading order in α_s and the fragmentation function is given by

$$D_{g\to\psi'}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{24m_c^3}\delta(1-z)\,\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle \tag{4}$$

with $\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle$ defined as in [10]. Braaten and Fleming found [7] that the prompt ψ' production cross section at the Tevatron and its p_T-dependence can be fitted by $\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle = 0.0042\,\mathrm{GeV}^3$, a value consistent with a v^4-suppression relative to $\langle\mathcal{O}_1^{\psi'}({}^3S_1)\rangle = 0.11\,\mathrm{GeV}^3$.

Since the $1/p_T^4$-dependence of the production cross section is rather generic for any fragmentation mechanism, additional information is necessary to establish the colour octet mechanism. One important signature of the colour octet mechanism is the ψ' polarization. The polarization of the quarkonium is calculable to a certain degree, because the non-relativistic dynamics that governs the long-distance evolution conserves the heavy quark spin up to corrections of order v^2 that enter through the magnetic coupling $\sigma\cdot B$ in the NRQCD Lagrangian. To implement the implications of heavy quark spin symmetry on the tensor decomposition of matrix elements, it is useful to represent the quarkonium state as a product of a spin and orbital angular momentum part, similar to the decomposition in heavy and light degrees of

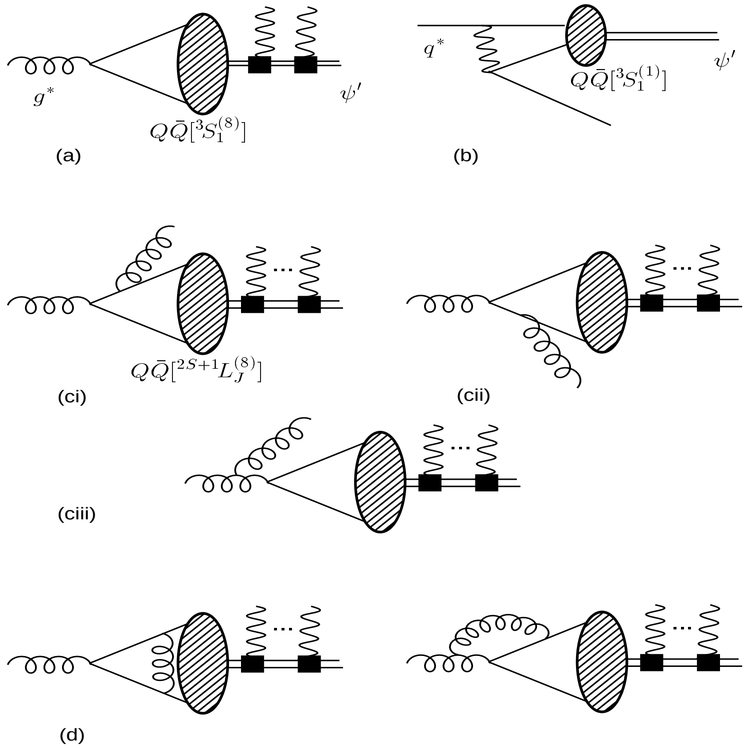

Figure 1: (a) Leading order gluon fragmentation into $c\bar{c}[^3S_1^{(8)}]$. (b) Charm fragmentation. (c), (d) Real and virtual fragmentation diagrams at order α_s^2. The black boxes denote nonperturbative evolution of the $c\bar{c}$-pair into a ψ' through soft gluon emission.

freedom for heavy-light mesons [15]. For the $^3S_1^{(8)}$-operators one obtains

$$\langle \chi^\dagger \sigma^a T^A \, \psi \, a_H^{(\lambda)\dagger} a_H^{(\lambda)} \, \psi^\dagger \sigma^b T^A \, \chi \rangle = \begin{cases} \frac{1}{3} \, \langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle \, \epsilon^{a*}(\lambda) \epsilon^b(\lambda) \\ \langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle \, \sum_{\lambda'} \epsilon^{a*}(\lambda') \epsilon^b(\lambda') \, |\langle 1(\lambda-\lambda'); 1\lambda' | J\lambda \rangle|^2 \end{cases} \tag{5}$$

where $\epsilon^a(\lambda)$ is a spin-1 polarization vector in the quarkonium rest frame. The first line holds for $H = \psi'$ and the second for $H = \chi_{cJ}$. When inserted in (2), these equations determine the required projections of the short-distance amplitude on the spin (and, in general, orbital angular momentum) of the heavy quark pair with appropriate Clebsch-Gordon coefficients. They are equivalent to and reproduce the spin symmetry constraints investigated in [11]. For a ψ' with polarization λ, one must project on a quark pair with equal polarization, as expected, since the two E1 transitions do not change the spin. Since the quark pair in Fig. 1a is transversely polarized, the projection with $\epsilon^a(0)$ vanishes, and, at leading order in α_s, we get

$$D_{g\to\psi'_L}(z,\mu) = 0 \qquad D_{g\to\psi'_T}(z,\mu) = D_{g\to\psi'}(z,\mu) \tag{6}$$

for the fragmentation functions into longitudinally and transversely polarized ψ''s. Because spin-symmetry is not exact, long-distance corrections of order v^4 arise, if the quark pair in the octet state undergoes a double magnetic dipole transition.

4. Short-distance corrections to transverse polarization

Corrections to pure transverse polarization also arise, when the polarization of the quark pair prior to nonperturbative evolution is modified by the emission of hard gluons. The corresponding real and (some of the) virtual corrections to gluon fragmentation are shown in Fig. 1c and d. Let us define

$$\xi \equiv 1 - \frac{L}{T+L} \equiv 1 - \delta\xi \tag{7}$$

as the ratio of the (prompt) production cross section for transversely and unpolarized ψ''s. Since L vanishes at leading order, $\delta\xi$ is obtained from the fragmentation function into longitudinally polarized ψ''s at order α_s^2, while the total production rate $L+T$ in the denominator can be evaluated in leading order approximation.

For the same reason as in leading order, the virtual corrections do not yield longitudinally polarized ψ''s. This results in considerable simplification, since along with the virtual corrections, radiative corrections to the matrix elements in NRQCD would also have to be calculated. The full matching calculation for the next-to-leading short-distance coefficient of $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ has been performed by Ma [16] for unpolarized quarkonia. When combined with the result for the fragmentation function into ψ'_L in eq. (10) below, the production rates for longitudinally and transversely polarized ψ''s through a quark pair in a $^3S_1^{(8)}$ state could be obtained separately at next-to-leading order in α_s. Here we consider only the ratio ξ, so that the total rate can be evaluated in leading order.

After hard gluon emission the quark pair that enters the shaded ovals in Fig. 1c can be in various spin and angular momentum states. At order $\alpha_s^2 v^7$, the three distinct possibilities are $^3S_1^{(8)}$,$^3P_J^{(8)}$ and $^1S_0^{(8)}$. The power v^7 arises as combination of factors of v associated with the probability for binding a quark pair in a certain angular momentum state (v^3 for S-waves and v^5 for P-waves) and additional factors associated with the amplitude of the relevant Fock state in (1). For $^3S_1^{(8)}$ and $^1S_0^{(8)}$ a factor v^4 is obtained in the amplitude squared for the double E1 and single magnetic dipole transition, respectively. In case of $^3P_J^{(8)}$ a single dipole transition provides v^2.

Since there is no interference between S-wave and P-wave amplitudes and spin-0 and spin-1 amplitudes, the contributions from the three different intermediate quark pair states can be determined separately. Let us turn first to the ${}^3S_1^{(8)}$ intermediate state. This is the only one to which diagram (ciii) contributes. The projections onto the short-distance amplitude are determined by (5). In the actual calculation it is useful to boost into a frame where the '+-component' of the quarkonium momentum is large and to use the covariant projection operators of [17]. We then follow the method of [3, 14] for computing fragmentation functions. The result for the integral of the fragmentation function can be expressed as

$$\int_0^1 dz\, d(z,\mu,\mu') = \int_{s_{\min}(\mu')}^{\mu^2} \frac{ds}{s^2} \int_{4m_c^2/s}^{1} dz\, A(s,z), \tag{8}$$

with $A(s,z)$ a certain projection of the amplitude squared. After interchange of integrations, one can read off the fragmentation function. In general, the above expression is divergent. Divergences come from the upper limit of integration over invariant mass s of the fragmenting gluon (cut off by the fragmentation scale μ above) due to the collinear approximation implicit in the definition of the fragmentation function. Furthermore, the integral over z may diverge due to soft gluon emission.

For the unpolarized ψ' fragmentation function the logarithmically divergent part of the integration over s gives

$$D_{g\to\psi'}^{{}^3S_1^{(8)}}(z,\mu,\mu') = \frac{\pi\alpha_s}{24}\,\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle \cdot \frac{\alpha_s}{2\pi} P_{g\to gg}(z) \ln\frac{\mu^2}{4m_c^2}, \tag{9}$$

where $P_{g\to gg}(z)$ is the Altarelli-Parisi gluon-gluon splitting function. The logarithmic divergence occurs as part of the production amplitude, when the fragmenting gluon splits into two collinear gluons of low invariant mass. Since both gluons are parallel and transverse, so is the final quark pair. Thus, the s-integral is finite for longitudinally polarized ψ''s.

For a quark pair in a ${}^3S_1^{(8)}$ state the integral over z is finite. A divergence can potentially arise for P-waves, when the amplitude is expanded in the relative velocity. However, soft gluon emission with energy less than μ', the NRQCD factorization scale, should be considered as part of a NRQCD matrix element. For longitudinal polarization there is no contribution at order α_s, into which an infrared logarithm could be absorbed and all longitudinal fragmentation functions must be finite as $z \to 1$. The integral above can then be evaluated taking μ to infinity and μ' to zero in all cases considered in this paper.

The result for the fragmentation function into longitudinally polarized ψ''s through a colour octet ${}^3S_1$ quark pair is

$$D_{g\to\psi'_L}^{{}^3S_1^{(8)}}(z,2m_c) = \frac{\alpha_s^2(2m_c)}{24} N_c \, \frac{1-z}{z} \, \frac{\langle \mathcal{O}_8^{\psi'}({}^3S_1)\rangle}{m_c^3}, \tag{10}$$

where $N_c = 3$ is the number of colours.

For a ψ' produced through an intermediate quark pair with orbital angular momentum one, we need the matrix elements

$$\langle \chi^\dagger \sigma^a T^A \left(-\frac{i}{2}\overleftrightarrow{D}_i\right) \psi \, a_H^{(\lambda)\dagger} a_H^{(\lambda)} \, \psi^\dagger \sigma^b T^A \left(-\frac{i}{2}\overleftrightarrow{D}_j\right) \chi\rangle = \langle \mathcal{O}_8^{\psi'}({}^3P_0)\rangle \, \delta_{ij} \, \epsilon^{a*}(\lambda)\epsilon^b(\lambda). \tag{11}$$

The right hand side incorporates the constraints from spin symmetry and is accurate up to v^2-corrections. Thus, we have to project the short-distance amplitude on a longitudinally polarized quark pair and sum over all orbital angular momentum states. Note that the total angular momentum J of the quark pair is not specified. The resulting fragmentation function is

$$\begin{aligned} D_{g\to\psi'_L}^{{}^3P_J^{(8)}}(z,2m_c) &= \frac{\alpha_s^2(2m_c)}{16} \frac{N_c^2-4}{N_c} \left[\frac{z^2+8z-12}{z^2}(1-z)\ln(1-z) \right. \\ & \left. \quad - \frac{2z^3+z^2-28z+24}{2z} \right] \frac{\langle \mathcal{O}_8^{\psi'}({}^3P_0)\rangle}{m_c^5}. \end{aligned} \tag{12}$$

Because of the decomposition of the matrix element (11), which would differ for χ_{cJ} states, this fragmentation function can not be obtained as a linear combination of the short-distance part of the fragmentation functions into polarized χ_{cJ} states [18]. Note also that it is finite as $z \to 1$, as expected from the considerations above.

Finally we have to account for the quark pair in a spin-0 state. While in the previous two cases, the nonperturbative evolution could be considered as spin-conserving at order v^7, a non-zero value for $\langle \mathcal{O}_8^{\psi'}({}^1S_0)\rangle$ arises through a magnetic dipole transition at order v^7. Since the initial quark pair evolves into a ψ' with a probability independent of the polarization of ψ', the fragmentation function for a longitudinally polarized ψ' is given by dividing the total fragmentation probability by three. The result is

$$D_{g\to\psi'_L}^{{}^1S_0^{(8)}}(z,2m_c) = \frac{\alpha_s^2(2m_c)}{864} \frac{N_c^2-4}{N_c} \left[3z - 2z^2 + 2(1-z)\ln(1-z)\right] \frac{\langle \mathcal{O}_8^{\psi'}({}^1S_0)\rangle}{m_c^3}. \tag{13}$$

The functional dependence is the same as for the colour singlet fragmentation function for η_c in [3].

The fragmentation function $D_{g\to\psi'_L}(z)$ for longitudinally polarized ψ''s is given by the sum of (10), (12) and (13) at leading non-vanishing order in α_s. There is also a contribution from charm fragmentation, which is formally of the same order $\alpha_s^2 v^7$, where the quark pair in Fig. 1b is in an octet state. The corresponding fragmentation functions are identical up to a colour factor to the colour singlet charm fragmentation functions, which contribute at order $\alpha_s^2 v^3$. Since colour singlet fragmentation (including charm fragmentation) can account only for 1/30 of all ψ''s, colour octet charm fragmentation contributes at most a tiny fraction to the total production rate. It can be neglected without affecting our conclusions.

5. Results

The production cross section for longitudinally polarized ψ''s at the Tevatron is obtained by convoluting the fragmentation function with the gluon production cross section in $p\bar{p}$ collisions with center-of mass-energy $\sqrt{s} = 1.8\,\mathrm{TeV}$,

$$\frac{d\sigma}{dp_T}(p\bar{p}\to\psi'_L + X) = \int\limits_{2p_T/\sqrt{s}}^{1} dz\, \frac{d\sigma}{dp_T}(p\bar{p}\to g(P_T/z) + X)\, D_{g\to\psi'_L}(z,\mu). \tag{14}$$

The longitudinally polarized production fraction $\delta\xi$ is given by dividing this expression by a similar expression with $D_{g\to\psi'_L}(z)$ replaced by $D_{g\to\psi'}(z)$ given in (4). We have used the CTEQ3L parton distributions to evaluate the gluon production cross section. The dominant production channel is gluon-gluon fusion, while the $q\bar{q}$ channel is negligible. The gluon fragmentation functions are evolved from the initial scale $2m_c = 3\,\mathrm{GeV}$ to μ of order p_T by the usual Altarelli-Parisi equations. To very good approximation, mixing can be neglected in solving the AP equations. As input we have chosen $\alpha_s(2m_c) = 0.27$. In addition we have implemented the pseudo-rapidity cut $|\eta| > 0.6$ as in the latest CDF analysis [19].

The final result for $\delta\xi$ can be represented as the sum of the three contributions to longitudinal fragmentation discussed earlier:

$$\delta\xi_{SD}(p_T,\mu) = r_1(p_T,\mu) + r_2(p_T,\mu)\frac{1}{m_c^2}\frac{\langle\mathcal{O}_8^{\psi'}({}^3P_0)\rangle}{\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle} + r_3(p_T,\mu)\frac{\langle\mathcal{O}_8^{\psi'}({}^1S_0)\rangle}{\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle}. \tag{15}$$

We have added a subscript 'SD' to indicate that spin symmetry breaking long distance

p_T/GeV	r_1	r_2	r_3	$\delta\xi_{SD}$
6	0.027	0.035	0.001	0.062
9	0.022	0.030	0.001	0.052
12	0.020	0.027	0.001	0.047
15	0.018	0.025	0.001	0.043
18	0.017	0.024	0.001	0.041
21	0.015	0.024	0.001	0.039
24	0.015	0.023	0.001	0.038

Table 1: The coefficients r_i as a function of p_T with $\mu = p_T$ and $|\eta| > 0.6$. The last column gives the deviation from pure transverse polarization, $\delta\xi_{SD} = r_1 + r_2$, for $\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle / \langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle = 1$ and neglecting the contribution from $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$.

corrections of order $\alpha_s v^{11}$ are not included here. The coefficients r_i are listed in Table 1 for $\mu = p_T$.

As can be seen from the Table, the contribution from r_3 is small and can be dropped for any reasonably expected ratio of the matrix elements that multiplies r_3. This leaves us with one ratio of matrix elements which is not very well constrained at present. The velocity scaling rules tell us that this ratio should be of order unity. A slightly more definite, but crude estimate can be obtained by the following argument borrowed from [1]: The operators $\mathcal{O}_8^{\psi'}(^3P_0)$ and $\mathcal{O}_8^{\psi'}(^3S_1)$ mix under renormalization [10]. The solution to the renormalization group equation in the one-loop approximation is given by

$$\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle(\mu') = \langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle(\mu_0') + \frac{6(N_c^2-4)}{N_c\beta_0 m_c^2} \ln\frac{\alpha_s(\mu_0')}{\alpha_s(\mu')} \, \langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle, \tag{16}$$

where we used $\langle \mathcal{O}_8^{\psi'}(^3P_J)\rangle = (2J+1)\,\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle$, as follows from (11), and $\beta_0 = 11N_c/6 - N_f/3$. The second term is formally enhanced by a logarithm if μ_0' and μ' are very different. Neglecting the first term on the right hand side and choosing the initial scale of order of the typical non-relativistic momenta $\mu_0' \approx m_c v$ and $\mu' \approx m_c$, we get, with $\alpha_s(1.5\,\mathrm{GeV}) = 0.35$, the estimate

$$\frac{1}{m_c^2}\frac{\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle}{\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle} \approx 1.25. \tag{17}$$

Although the logarithm is in fact not large, a comparable estimate for χ_c states as in [1] is only off by a factor of two from the phenomenologically known ratio in that case. If we assume that this difference arises from an inaccurate choice of the initial scale and adjust the logarithm to the known ratio for matrix elements in the χ_c case, the estimate above changes

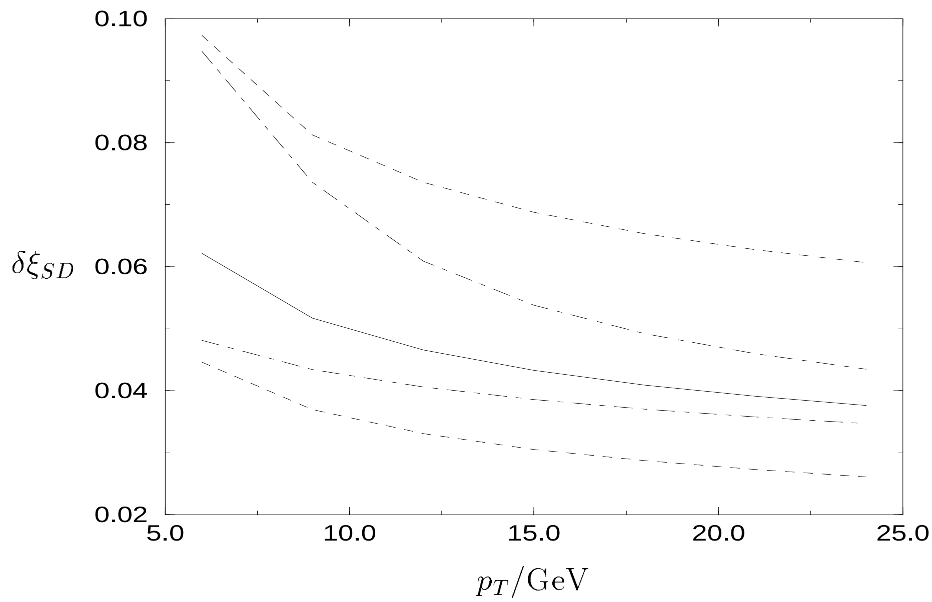

Figure 2: The short-distance correction $\delta\xi_{SD}$ as function of p_T. The solid line shows the prediction for $\mu = p_T$ and $\langle\mathcal{O}_8^{\psi'}({}^3P_0)\rangle/\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle = 1$. The two dash-dotted lines are for $\mu = p_T/2$ (upper curve) and $\mu = 2p_T$ (lower curve) at fixed $\langle\mathcal{O}_8^{\psi'}({}^3P_0)\rangle/\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle = 1$. The two dashed lines correspond to variation of $\langle\mathcal{O}_8^{\psi'}({}^3P_0)\rangle/\langle\mathcal{O}_8^{\psi'}({}^3S_1)\rangle$ between 0.5 and 2 at fixed $\mu = p_T$.

from 1.25 to 0.65. In view of this we consider a ratio of one as our best guess. With this choice the prediction for $\delta\xi_{SD}$ is shown in the last column of the Table and as solid line in Fig. 2. The Figure also shows the prediction if we modify the above ratio of matrix elements to 2 or to 1/2. This uncertainty constitutes the dominant source of theoretical error until $\langle\mathcal{O}_8^{\psi'}({}^3P_0)\rangle$ is determined from another process. For comparison, we have plotted the variation of $\delta\xi$, when the renormalization scale is set to $\mu = p_T/2$ or $\mu = 2p_T$. This uncertainty is about $\pm 20\%$, except at small p_T, where the choice $p_T/2$ is not very well motivated. It should also be mentioned that by nature of the fragmentation approximation, the prediction is more reliable at large p_T. If one integrates the production cross section over p_T starting from some $p_{T,\mathrm{min}}$, the corresponding integrated $\delta\xi$ is roughly equal to $\delta\xi$ at $p_{T,\mathrm{min}}$, because the absolute production cross section decreases rapidly with p_T.

6. Discussion

From the above we conclude that short-distance corrections to the fragmentation functions decrease the transverse polarization of 100% at leading order in α_s by approximately 5% and hardly more than 10%, even if the ${}^3P_0$-matrix element is large. We have ignored the colour singlet production mechanisms, since they account for only 3% of all ψ''s. If we assume that the singlet mechanisms produce randomly polarized ψ''s, they would contribute only an additional percent to the longitudinal polarization fraction. A potential correction of order $4m_c^2/p_T^2$ comes from the fragmentation approximation. However, the octet production processes at leading order in α_s yield basically pure transverse polarization even without the fragmentation approximation to the matrix elements due to a particular structure of the gluon-gluon fusion amplitude [8]. Thus the higher twist correction is suppressed by $(\alpha_s/\pi)\, 4m_c^2/p_T^2$, which is small for sufficiently large $p_T \sim 10\,\mathrm{GeV}$.

Deviations from pure transverse polarization arise from spin symmetry breaking already at leading order in α_s, when the quark pair in Fig. 1a undergoes spin-changing nonperturbative evolution. Because of parity and charge conjugation symmetry these corrections scale with v^4. Since random polarization would give $\delta\xi = 1/3$, a natural estimate of the long-distance correction is $\delta\xi_{LD} = \mathcal{O}(v^4/3) \approx (3-4)\%$. Despite their smallness, the short distance corrections constitute the dominant source of depolarization. We estimate, conservatively, that if the colour octet mechanism does account for the observed prompt ψ' production cross section, then the ψ''s should be $(85 - 95)\%$ transversely polarized.

This prediction can be tested rather simply by measuring the lepton angular distribution in the $\psi' \to l^+l^-$ decay channel,

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \frac{1 - 3\delta\xi}{1 + \delta\xi}\, \cos^2\theta, \tag{18}$$

where θ denotes the angle between the lepton three momentum in the ψ' rest frame and the ψ' three momentum in the lab frame. As statistics improves, a measurement of the angular distribution will become feasible. Time will tell whether such a measurement, together with other signatures of the colour octet mechanism, such as those in e^+e^- annihilation [20], provides a consistent picture of quarkonium production that includes the relativistic structure of the bound state.

Acknowledgements. We benefitted from conversations with Wolfgang Kilian and Mark Wise. M. B. is supported by the Alexander von Humboldt foundation.

References

[1] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D46** (1992) R1914, *ibid.* **D46** (1992) R3703

[2] W.E. Caswell and G.P. Lepage, Phys. Lett. **B167** (1986) 437

[3] E. Braaten and T.C. Yuan, Phys. Rev. Lett. **71** (1993) 1673

[4] M. Cacciari and M. Greco, Phys. Rev. Lett. **73** (1994) 1586

[5] E. Braaten, M. Doncheski, S. Fleming and M. Mangano, Phys. Lett. **B333** (1994) 548

[6] D.P. Roy and K. Sridhar, Phys. Lett. **B339** (1994) 141

[7] E. Braaten and S. Fleming, Phys. Rev. Lett. **74** (1995) 3327

[8] P. Cho and A. Leibovich, CalTech preprint CALT-68-1988 [hep-ph/9505239]

[9] M. Cacciari, M. Greco, M.L. Mangano and A. Petrelli, Phys. Lett. **B356** (1995) 553

[10] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51** (1995) 1125

[11] P. Cho and M.B. Wise, Phys. Lett. **B346** (1995) 129

[12] J.C. Collins and D.E. Soper, Nucl. Phys. **B194** (1982) 445

[13] G. P. Lepage, L. Magnea, C. Nakleh, U. Magnea and K. Hornbostel, Phys. Rev. **D46** (1992) 4052

[14] E. Braaten and T.C. Yuan, Phys. Rev. **D50** (1994) 3176

[15] A.F. Falk, Nucl. Phys. **B378** (1992) 79

[16] J.P. Ma, Nucl. Phys. **B447** (1995) 405

[17] B. Guberina, J.H. Kühn, R.D. Peccei and R. Rückl, Nucl. Phys. **B174** (1980) 317

[18] P. Cho, M.B. Wise and S. Trivedi, Phys. Rev. **D51** (1995) 2039

[19] The CDF collaboration, Fermilab preprint Fermilab-Conf-94/136-E, contribution to the 27th International Conference on High Energy Physics, Glasgow, July 20-27, 1994

[20] E. Braaten and Y.-Q. Chen, Northwestern University preprint NUHEP-TH-95-9 [hep-ph/9508373]; K. Cheung, W.-Y. Keung and T.C. Yuan, Femilab preprint FERMILAB-PUB-95/300-T [hep-ph/9509308]