Polarization of Prompt J/ψ at the Tevatron

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The polarization of prompt J/ψ at the Fermilab Tevatron is calculated within the nonrelativistic QCD factorization framework. The contribution from radiative decays of P-wave charmonium states decreases, but does not eliminate, the transverse polarization at large transverse momentum. The angular distribution parameter α for leptonic decays of the J/ψ is predicted to increase from near 0 at $p_T = 5$ GeV to about 0.5 at $p_T = 20$ GeV. The prediction is consistent with measurements by the CDF Collaboration at intermediate values of p_T, but disagrees by about 3 standard deviations at the largest values of p_T measured.

PACS numbers: 13.85.t, 13.85.Ni, 14.40.Gx

The production of charmonium and bottomonium states in high-energy collisions probes both the hard-scattering parton processes that create heavy quark-antiquark ($Q\overline{Q}$) pairs and the hadronization process that transforms them into color-singlet bound states. One particularly sensitive probe is the polarization of the $J^{PC} = 1^{--}$ quarkonium states. The nonrelativistic QCD (NRQCD) factorization approach to inclusive quarkonium production [1] makes the remarkable prediction that in hadron collisions these states should be transversely polarized at sufficiently large transverse momentum (p_T) [2]. Recent measurements at the Tevatron by the CDF Collaboration seem to be in dramatic contradiction with this prediction [3].

As first pointed out by Cho and Wise [2], the prediction of transverse polarization for 1^{--} states at large p_T follows from three simple features of the dynamics of massless partons and heavy quarks. First, the inclusive production of quarkonium (or any other hadron) at sufficiently large p_T is dominated by *fragmentation.* In $p\bar{p}$ collisions at the Tevatron, the dominant contribution to the charmonium production rate at large p_T comes from gluon fragmentation [4]. The gluon is almost on shell and thus predominantly transversely polarized. Second, a $Q\overline{Q}$ pair with small relative momentum created by the virtual gluon is, at leading order in α_s, in a color-octet $^3S_1$ state [5] with the same transverse polarization as the gluon. Third, the spin symmetry of nonrelativistic heavy quarks implies the suppression of spin-flip transitions in the binding of the $Q\overline{Q}$ pair into quarkonium. Thus, 1^{--} states should have a large transverse polarization at sufficiently large p_T. A convenient measure of the polarization is the variable $\alpha = (\sigma_T - 2\sigma_L)/(\sigma_T + 2\sigma_L)$, where σ_T and σ_L are the transverse and longitudinal components of the cross section, respectively. Beneke and Rothstein studied the dominant fragmentation mechanisms for σ_L [6], and concluded that, at sufficiently large p_T, α should be in the range 0.5 – 0.8.

For charmonium production at the Tevatron, fragmentation does not yet dominate for most of the p_T range that is experimentally accessible. In order to study the onset of the polarization effect, it is necessary to take into account the *fusion* contributions from parton processes $ij \to c\bar{c} + k$. Quantitative calculations of the polarization variable α for *direct* ψ' mesons (*i.e.* those that do not come from decays) at the Tevatron have been carried out by Beneke and Krämer [7] and by Leibovich [8]. They predicted that α should be small for $p_T \lesssim 5$ GeV, but then should rise dramatically to 0.77 ± 0.08 at $p_T = 20$ GeV, according to Beneke and Krämer, and to 0.90 ± 0.04, according to Leibovich. The CDF Collaboration has measured the polarization of direct ψ' [3], but the error bars are too large to draw any definitive conclusions.

The CDF Collaboration has also measured the polarization of *prompt* J/ψ mesons [3] (*i.e.* those that do not come from the decay of B hadrons). The number of J/ψ events is larger than for ψ' by a factor of about 100, allowing α to be measured more precisely and in more p_T bins. They find that α has a positive value 0.32 ± 0.10 in the p_T bin from 8 to 10 GeV. However, instead of increasing at larger p_T, α decreases to -0.29 ± 0.23 in the highest p_T bin from 15 to 20 GeV. Theoretical predictions of the polarization of prompt J/ψ are complicated by the fact that the prompt signal includes J/ψ mesons that come from decays of the higher charmonium states χ_{c1}, χ_{c2}, and ψ'. They account for about 15%, 15%, and 10% of the prompt J/ψ signal, respectively [9]. The polarization of J/ψ from ψ' not via χ_{cJ} is straightforward to calculate, since the spin is unchanged by the transition. The polarization of J/ψ from χ_{cJ} and of J/ψ from ψ' via χ_{cJ} is more complicated, because the χ_{cJ} mesons are produced in various spin states and they decay into J/ψ through radiative transitions.

In this letter, we present a quantitative analysis of the polarization of prompt J/ψ using the NRQCD factorization formalism. We reanalyze the CDF data on the p_T distributions for J/ψ, χ_c, and ψ' to determine the relevant color-octet NRQCD matrix elements (ME's). The cross sections for the spin states of J/ψ, χ_{cJ}, and ψ' are calculated using these ME's and the appropriate parton cross sections. The cross sections for the spin states of the χ_{cJ} required the calculation of new parton cross sections, which will be published elsewhere [10]. The variable α is then obtained by combining these cross sections with the appropriate branching ratios into longitudinally polarized J/ψ (ψ_L).

The *NRQCD factorization formula* for the differential cross section for the inclusive production of a charmonium state H of momentum P and spin quantum number λ has the schematic form

$$d\sigma^{H_\lambda(P)} \;=\; d\sigma^{c\bar{c}_n(P)} \; \langle O_n^{H_\lambda(P)} \rangle, \tag{1}$$

where the summation index n runs over all the color and angular momentum states of the $c\bar{c}$ pair. The $c\bar{c}$ cross sections $d\sigma^{c\bar{c}_n}$ can be calculated using perturbative QCD. All dependence on the state H is contained within the nonperturbative ME's $\langle O_n^{H_\lambda(P)}\rangle$. In general, they are Lorentz tensors that depend on the momentum P and the polarization tensor of H_λ. The Lorentz indices are contracted with those of $d\sigma^{c\bar{c}_n}$ to give a scalar cross section. The symmetries of NRQCD can be used to reduce the tensor ME's $\langle O_n^{H_\lambda(P)}\rangle$ to scalar ME's $\langle O_n^H\rangle$ that are independent of P and λ. Thus one may calculate the cross section for polarized quarkonium once the relevant scalar ME's are known. A nonperturbative analysis of NRQCD reveals how the various ME's scale with the typical relative velocity of the heavy quarks. It also gives exact and approximate symmetry relations that can be used to simplify the ME's. The most important ME's for the production of $J/\psi = \psi(1S)$ or $\psi' = \psi(2S)$ can be reduced to one color-singlet parameter $\langle O_1^{\psi(nS)}(^3S_1)\rangle$ and three color-octet parameters $\langle O_8^{\psi(nS)}(^3S_1)\rangle$, $\langle O_8^{\psi(nS)}(^1S_0)\rangle$, and $\langle O_8^{\psi(nS)}(^3P_0)\rangle$. The most important ME's for χ_{cJ} production can be reduced to a color-singlet parameter $\langle O_1^{\chi_{c0}}(^3P_0)\rangle$ and a single color-octet parameter $\langle O_8^{\chi_{c0}}(^3S_1)\rangle$. The ME's enumerated above should be sufficient for a calculation of the polarization of prompt J/ψ.

In $p\bar{p}$ collisions, the parton processes that dominate the $c\bar{c}$ cross section depend on p_T. If p_T is of order m_c, those which dominate are *fusion* processes, whose contributions can be expressed as

$$d\sigma_{\rm fu}^{H_\lambda(P)} = f_{i/p} \otimes f_{j/\bar{p}} \otimes d\hat{\sigma}_{ij}^{c\bar{c}_n(P)} \; \langle O_n^{H_\lambda(P)}\rangle, \tag{2}$$

where $f_{i/p}(x,\mu)$ and $f_{j/\bar{p}}(x,\mu)$ are parton distribution functions (PDF's) and a sum over the partons i,j is implied. The leading-order parton cross sections $d\hat{\sigma}$ are proportional to $\alpha_s^3(\mu)$. These cross sections are given in Refs. [8] and [11] for all the relevant $c\bar{c}$ spin states with the exception of color-singlet ^3P_J states, which required a new calculation. For $p_T \gg m_c$ the parton cross sections are dominated by *fragmentation* processes with the scaling behavior $d\hat{\sigma}/dp_T^2 \sim 1/p_T^4$. These contributions can be expressed as

$$d\sigma_{\rm fr}^{H_\lambda(P)} = f_{i/p} \otimes f_{j/\bar{p}} \otimes d\hat{\sigma}_{ij}^{k(P/z)} \otimes D_k^{c\bar{c}_n} \; \langle O_n^{H_\lambda(P)}\rangle, \tag{3}$$

where $D_k^{c\bar{c}_n}(z,\mu_{\rm fr})$ is a fragmentation function (FF). We use a common renormalization and factorization scale μ for $f_{i/p}$, $f_{j/\bar{p}}$, and $d\hat{\sigma}$, but we allow $\mu_{\rm fr}$ to be different. The momentum k of the fragmenting parton is denoted by P/z in (3). However, it is inconsistent to set $k^\mu = P^\mu/z$, since the parton is massless while $P^2 = 4m_c^2$. We choose k^μ so that z is the fraction of the light-cone momentum of the parton k that is carried by the $c\bar{c}$ pair in the parton CM frame. The covariant expression is $k^\mu = \left[(\Delta + K\cdot P)P^\mu - P^2 K^\mu\right]/(2z\Delta)$, where K^μ is the total momentum of the colliding partons i and j, and $\Delta = [(K\cdot P)^2 - K^2P^2]^{1/2}$.

In order to predict the polarization of prompt J/ψ at the Tevatron, we need values for the scalar ME's. The color-singlet ME's $\langle O_1^{\psi(nS)}(^3S_1)\rangle$ and $\langle O_1^{\chi_{c0}}(^3P_0)\rangle$ can be determined phenomenologically from the decay rates for $\psi(nS) \to \ell^+\ell^-$ and $\chi_{c2} \to \gamma\gamma$ [12]. Using the vacuum saturation approximation and spin symmetry in the NRQCD factorization formulae and including NLO QCD radiative corrections [13], we obtain the values in Table I. The errors come from the experimental errors in the decay rates only.

The color-octet ME's are phenomenological parameters that must be determined from production data. To predict the polarization at the Tevatron, it is preferable to use ME's extracted directly from Tevatron data in order to cancel theoretical errors associated with soft gluon radiation. There have been several previous extractions of the color-octet ME's [7,14–16] from the CDF data on the p_T distributions of J/ψ, χ_c, and ψ' [9]. We carry out an updated analysis largely following the strategy used in Ref. [16]. In the fusion cross section (2), we include the parton processes $ij \to c\bar{c} + k$, with $i,j = g,q,\bar{q}$ and $q = u,d,s$. In the fragmentation cross section (3), we include only the $g \to c\bar{c}_8(^3S_1)$ term, since this is the only fragmentation process for which $D_k^{c\bar{c}_n}$ is of order α_s. The FF $D_g^{c\bar{c}_8(^3S_1)}$ is evolved in $\mu_{\rm fr}$ using the standard homogeneous timelike evolution equation. The effects of the violation of the phase-space constraint $\mu_{\rm fr} > 4m_c^2/z$ are negligible at the Tevatron due to the rapid fall-off of the p_T distribution [17].

We consider two choices for the PDF's: MRST98LO as our default and CTEQ5L for comparison [18]. We evaluate α_s from the one-loop formula using the value of $\Lambda_{\rm QCD}$ appropriate for the PDF set [18]. We set $\mu = (4m_c^2 + p_T^2)^{1/2}$ and $m_c = 1.5$ GeV. The cross section for $\psi(nS)$ depends on the linear combination $M_r = \langle O_8(^1S_0)\rangle + r\langle O_8(^3P_0)\rangle/m_c^2$, where r varies from about 3.6 at $p_T = 5.5$ GeV to about 3.0 at $p_T = 18$ GeV, so we can only determine M_r at some optimal value of r. We determined $\langle O_8(^3S_1)\rangle$ and M_r for $\psi(nS)$ by fitting the p_T distributions from CDF following the strategy in [16]. We determined $\langle O_8^{\chi_{c0}}(^3S_1)\rangle$ by fitting the p_T distribution for χ_c together with the constraint from the preliminary CDF measurement of $\sigma_{\chi_{c1}}/\sigma_{\chi_{c2}}$ [9]. Our values for the color-octet ME's are summarized in Table I. The error bars take into account the statistical errors only. Our default ψ' color-octet ME's agree within errors with those of Ref. [14] used by Leibovich [8] and with those for 2 of the 3 PDF sets used by Beneke and Krämer [7]. Our default J/ψ color-octet ME's agree within errors with those of Ref. [14]. Our value for $\langle O_8^{J/\psi}(^3S_1)\rangle$ is about a factor of 3 smaller than in Ref. [7], while $M_r^{J/\psi}$ is about a factor of 2 larger.

We can calculate the cross sections for the polarized states H_λ using the scalar ME's in Table I. The cross section (1) can be reduced to an expression linear in the scalar ME's, with coefficients that involve the polarization tensor of H_λ. In the channel $c\bar{c}_8(^3S_1) \to \psi_\lambda(nS)$, we interpolate between the fusion cross section at low p_T and the fragmentation cross section at high p_T using the prescription

$$d\sigma^{H_\lambda} = d\sigma_{\rm fu}^{H_\lambda} \times \left(d\sigma_{\rm fr}^{H_\lambda}[\mu_{\rm fr} = \mu] / d\sigma_{\rm fr}^{H_\lambda}[\mu_{\rm fr} = 2m_c] \right). \quad (4)$$

We proceed to summarize our calculation of the errors in σ_L and σ_T. The errors in the ME's in Table I are taken into account. We take the central values of μ and m_c to be $\mu_T = (4m_c^2 + p_T^2)^{1/2}$ and 1.5 GeV and allow them to vary within the ranges $\frac{1}{2}\mu_T - 2\mu_T$ and 1.45 – 1.55 GeV, respectively. We take MRST98LO as our default PDF, and we treat the difference between it and CTEQ5L as an error. The cross section σ_L for $\psi(nS)$ is sensitive to a different linear combination of $\langle O_8(^1S_0)\rangle$ and $\langle O_8(^3P_0)\rangle$ than appears in M_r. We take this into account by expressing the cross section as a function of M_r and $x = \langle O_8(^1S_0)\rangle / M_r$, taking the central value of x to be $\frac{1}{2}$, and allowing x to vary between 0 and 1.

We first consider the polarization of direct ψ', since it is not complicated by feeddown from higher charmonium states. The polarization variable α measured by the CDF Collaboration [3] describes the angular distribution of leptons from the decay of the ψ' with respect to the ψ' momentum in the hadron CM frame. The covariant expression for the polarization vector of ψ'_L is $(P^2Q^\mu - P\cdot QP^\mu)/(\sqrt{P^2}\Delta)$, where $Q = p + \bar{p}$ is the total hadron momentum and $\Delta = [(P\cdot Q)^2 - P^2Q^2]^{1/2}$. In Fig. 1(a), we compare our result for α as a function of p_T with the CDF data [3] and with previous predictions from Refs. [7] and [8]. We present our result in the form of an error band obtained by combining in quadrature all the errors described above. The most important errors are those from the ψ' ME's, the PDF's, and x. The error bars in the CDF data are too large to draw any definitive conclusions. Our result for α is close to the prediction of Leibovich [8], and significantly larger than that of Beneke and Krämer [7]. Their calculations differ in the treatment of terms of order α_s^2 in the gluon FF [6]. Beneke and Krämer included these terms in σ_L but neglected them in σ_T, while Leibovich neglected them in both σ_L and σ_T. We have adopted the strategy of Beneke and Krämer, since these terms give a significant increase in σ_L at large p_T but have only a small effect on σ_T. Although this tends to decrease α, our smaller value of M_r tends to increase α, and the net result is close to the prediction of Leibovich.

We next consider the polarization variable α for prompt J/ψ. The prompt cross section $\sigma_T + \sigma_L$ is the sum of the direct cross section for J/ψ and the cross sections for χ_{cJ} and ψ' weighted by the branching fractions $B_{H\to J/\psi}$. The prompt longitudinal cross section σ_L is the sum of the direct cross section for ψ_L and the cross sections for each of the spin states $\chi_{cJ(\lambda)}$ and ψ'_λ weighted by $B_{H\to J/\psi}$ and by the probability $P_{H_\lambda\to\psi_L}$ for the polarized state to decay into ψ_L. The observed transitions of ψ' to J/ψ involve no spin flips, so that $P_{\psi'_\lambda\to\psi_L}$ is 1 for ψ'_0 and 0 for $\psi'_{\pm1}$. For the radiative decay of $\chi_{cJ(\lambda)}$ into J/ψ, the probability $P_{\chi_{cJ(\lambda)}\to\psi_L}$ is $\frac{1}{3}$ for χ_{c0}, $\frac{1}{2}$ for $\chi_{c1(\pm1)}$, $\frac{2}{3}$ for $\chi_{c2(0)}$, $\frac{1}{2}$ for $\chi_{c2(\pm1)}$, and 0 for the other spin states [19]. In Fig. 1(b), we compare our result for α as a function of p_T with the CDF data [3]. The shaded area indicates the error band obtained by adding the errors in quadrature. The most important errors are those from the PDF's, the J/ψ ME's, and x. Our result for α is small around $p_T = 5$ GeV, but it increases with p_T to a value around 0.5 at $p_T = 20$ GeV. Our result is in good agreement with the CDF measurement at intermediate values of p_T, but it disagrees by about 3 standard deviations in the highest p_T bin. The three solid lines in Fig. 1(b) are the central curves of α for direct J/ψ, J/ψ from χ_{cJ}, and J/ψ from ψ'. The α for direct J/ψ is smaller than that for direct ψ', because $\langle O_8(^3S_1)\rangle$ is comparable for J/ψ and ψ', while M_r is significantly larger for J/ψ. In the moderate-p_T region, the contributions from ψ' and from χ_c add to give an increase in the transverse polarization of prompt J/ψ compared to direct J/ψ. In the high-p_T region, the contributions from ψ' and χ_c tend to cancel. The prediction of Beneke and Krämer for α for direct J/ψ is identical to their prediction for direct ψ' in Fig. 1(a). At $p_T = 20$ GeV, it is significantly larger than our prediction for direct J/ψ. The difference comes from our smaller value for $\langle O_8^{J/\psi}(^3S_1)\rangle$ and our larger value for $M_r^{J/\psi}$. Beneke and Krämer's prediction for α for J/ψ from ψ' would be significantly lower than our result in Fig. 1(b), but it would have a small effect on the value of α for prompt J/ψ. The discrepancies between their predictions and ours could be eliminated by more accurate data on the J/ψ and ψ' cross sections, which would decrease some of the ambiguities in the analysis.

The CDF measurement of the polarization of prompt J/ψ presents a serious challenge to the NRQCD factorization formalism for inclusive quarkonium production. There are many effects that could change our quantitative prediction for α, such as next-to-leading order radiative corrections, but the qualitative prediction that α should increase at large p_T seems inescapable. In Run II

PDF	$\langle O_1^{J/\psi}(^3S_1)\rangle$	$\langle O_8^{J/\psi}(^3S_1)\rangle$	$M_{3.4}^{J/\psi}$	$\langle O_1^{\psi'}(^3S_1)\rangle$	$\langle O_8^{\psi'}(^3S_1)\rangle$	$M_{3.5}^{\psi'}$	$\langle O_1^{\chi_{c0}}(^3P_0)\rangle$	$\langle O_8^{\chi_{c0}}(^3S_1)\rangle$
MRST98LO	1.3 ± 0.1	4.4 ± 0.7	8.7 ± 0.9	6.5 ± 0.6	4.2 ± 1.0	1.3 ± 0.5	8.9 ± 1.3	2.3 ± 0.3
CTEQ5L	1.4 ± 0.1	3.9 ± 0.7	6.6 ± 0.7	6.7 ± 0.7	3.7 ± 0.9	0.78 ± 0.36	9.1 ± 1.3	1.9 ± 0.2
unit	GeV^3	$10^{-3}\mathrm{GeV}^3$	$10^{-2}\mathrm{GeV}^3$	$10^{-1}\mathrm{GeV}^3$	$10^{-3}\mathrm{GeV}^3$	$10^{-2}\mathrm{GeV}^3$	$10^{-2}\mathrm{GeV}^5$	$10^{-3}\mathrm{GeV}^3$

TABLE I. NRQCD matrix elements. The error bars take into account the statistical errors only.

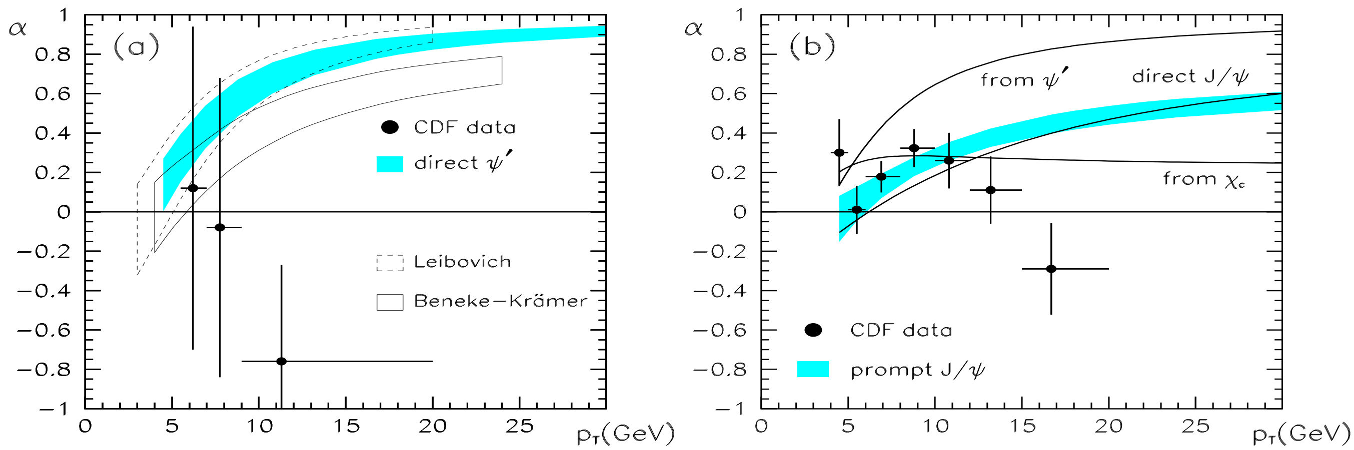

FIG. 1. Polarization variable α vs. p_T for (a) direct ψ' and (b) prompt J/ψ compared to CDF data.

of the Tevatron, the data sample for J/ψ should be more than one order of magnitude larger than in Run I, allowing the polarization to be measured with higher precision and out to larger values of p_T. If the result continues to disagree with the predictions of the NRQCD factorization approach, it would indicate a serious flaw in our understanding of inclusive charmonium production. The predictions of low-order perturbative QCD for the spin-dependence of $c\bar{c}$ cross sections could be wrong, or the use of NRQCD to understand the systematics of the formation of charmonium from the $c\bar{c}$ pair could be flawed, or m_c could simply be too small to apply the factorization approach to the charmonium system.

J.L. thanks Sungwon Lee for the graphics. This work was supported in part by DOE Grant No. DE-FG02-91-ER40690, by DFG Grant No. KN 365/1-1, by BMBF Grant No. 05 HT9GUA 3, through TMR Network No. ERBFMRX-CT98-0194, and by Humboldt Foundation.

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