$J/\psi(\psi')$ production at the Tevatron and LHC at $\mathcal{O}(\alpha_s^4 v^4)$ in nonrelativistic QCD

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We present a complete evaluation for $J/\psi(\psi')$ prompt production at the Tevatron and LHC at next-to-leading order in nonrelativistic QCD, including color-singlet, color-octet, and higher charmonia feeddown contributions. The short-distance coefficients of ${}^{3}P_{J}^{[8]}$ at next-to-leading order are found to be larger than leading order by more than an order of magnitude but with a minus sign at high transverse momentum p_{T} . Two new linear combinations of color-octet matrix elements are obtained from the CDF data, and used to predict J/ψ production at the LHC, which agrees with the CMS data. The possibility of ${}^{1}S_{0}^{[8]}$ dominance and the J/ψ polarization puzzle are also discussed.

PACS numbers: 12.38.Bx, 13.60.Le, 14.40.Pq

Nearly 20 years ago, the CDF Collaboration found a surprisingly large production rate of ψ' at high $p_T[1]$. To solve the large discrepancy between data and theoretical predictions, the color-octet(CO) mechanism [2] was proposed based on nonrelativistic QCD (NRQCD) factorization[3]. With the CO mechanism, $Q\bar{Q}$ pairs can be produced at short distances in CO $({}^1\!S_0^{[8]}, {}^3\!S_1^{[8]}, {}^3\!P_J^{[8]})$ states and subsequently evolve into physical quarkonia by nonperturbative emission of soft gluons. It can be verified that the partonic differential cross sections at leading-order (LO) in α_s behave as $1/p_T^4$ for ${}^3\!S_1^{[8]}$, and $1/p_T^6$ for ${}^1\!S_0^{[8]}$ and ${}^3\!P_J^{[8]}$, all of which decrease at high p_T much slower than $1/p_T^8$ of the color-singlet (CS) state. The CO mechanism could give a natural explanation for the observed p_T distributions and large production rates of ψ' and J/ψ [4].

However, the CO mechanism seems to encounter difficulties in explaining the observed $J/\psi(\psi')$ polarizations. Dominated by gluon fragmentation to ${}^{3}S_{1}^{[8]}$, the LO NRQCD predicts transverse polarization for $J/\psi(\psi')$ at high $p_T[4]$ whereas measurements at the Fermilab Tevatron give almost unpolarized $J/\psi(\psi')$ [5]. To exploit the underlying physics, several efforts have been made, either by introducing new channels[6] or by proposing other mechanisms[7]. It is a significant step to work out the next-to-leading order (NLO) QCD correction for the CS channel, which enhances the differential cross section by about 2 orders of magnitude at high $p_T[8]$, and changes the polarization from being transverse at LO into longitudinal at NLO[9]. Although the CS NLO cross section still lies far below the experimental data, it implies that, compared to the α_s suppression, kinematic enhancement at high p_T is more important in the current issue. This observation is also supported by our recent work[10] for χ_c production, where we find the ratio of production rates of $\sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$ can be dramatically altered by the NLO contribution due to change of the p_T distribution from $1/p_T^6$ at LO to $1/p_T^4$ at NLO in the CS P-wave channels. So we

may conclude nothing definite until all important channels in $1/p_T$ expansion are presented. It means the CO channels ${}^{1}S_{0}^{[8]}$ and ${}^{3}P_{J}^{[8]}$ should be considered at NLO, while the CS channel ${}^3S_1^{[1]}$ at next-to-next-to-leading order (NNLO) in α_s . Among these corrections, the complete NNLO calculation for CS is beyond the state of the art, and the NNLO* method is instead proposed[11], in which only tree-level diagrams at this order are considered and an infrared cutoff is imposed to control soft and collinear divergences, and the NNLO* contributions are shown to be large. However, the only $1/p_T^4$ leading contribution at NNLO in CS is given by gluon fragmentation, which was found[12] to be negligible compared to the observed $J/\psi(\psi')$ production data. Other NNLO contributions may give a $1/p_T^6$ term. In a complete NNLO calculation with both real and virtual corrections, infrared and collinear divergences are removed and these NNLO $1/p_T^6$ contributions should be smaller than the NLO $1/p_T^6$ contribution due to α_s suppression. Therefore, to achieve a good description for $J/\psi(\psi')$ production a complete NLO calculation including both CS and CO seems to be necessary.

At present, NRQCD factorization formalism with the CO mechanism is used to describe various processes in heavy quarkonium production and decay. While J/ψ production in two-photon collisions at CERN LEP2[13] and photoproduction at DESY HERA[14] are shown to favor the presence of CO contribution, the J/ψ production at B factories is described well using NLO CS model and leaves little room for CO contributions[15]. In order to further test the CO mechanism, it is necessary to study hadroproduction and extract CO long distance matrix elements (LDMEs) at NLO.

In view of the importance, here we present a complete NLO contribution to $J/\psi(\psi')$ production at the Tevatron and LHC, including all important CS and CO channels. According to the NRQCD factorization formalism, the inclusive cross section for direct J/ψ production in

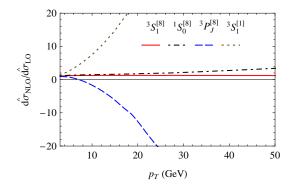


FIG. 1: Dependence of K factors (ratios of NLO to LO short-distance coefficients $\hat{d\sigma}$) on p_T in $J/\psi(\psi')$ direct production at the Tevatron.

hadron-hadron collisions is expressed as

$$d\sigma[pp \to J/\psi + X] = \sum_{n} \hat{d\sigma}[(c\bar{c})_{n}] \frac{\langle \mathcal{O}_{n}^{J/\psi} \rangle}{m_{c}^{2L_{n}}}$$
(1)
$$= \sum_{i,j,n} \int dx_{1} dx_{2} G_{i/p} G_{j/p} \times \hat{d\sigma}[i+j \to (c\bar{c})_{n} + X] \langle \mathcal{O}_{n}^{J/\psi} \rangle,$$

where p is either a proton or an antiproton, the indices i,j run over all the partonic species, and n denote the color, spin and angular momentum (L_n) of the intermediate $c\bar{c}$ states, including ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ in the present issue. Compared with the S-wave channel obtained in [8, 9, 16], the NLO treatment of ${}^3P_J^{[8]}$ is much more complicated. Fortunately, using the same method as in [10], we are able to get a compact expression for the virtual correction, which is both time-saving and numerically stable in the final state phase space integration. For technical details, we refer readers to Ref. [10].

For numerical results, we choose the same parameters as in [10] except that here we are restricted to $\sqrt{S}=1.96~{\rm TeV}$ and $|y_{J/\psi(\psi')}|<0.6$ with the Tevatron, while $\sqrt{S}=7~{\rm TeV}$ and $|y_{J/\psi(\psi')}|<2.4$ with the LHC.

Let us first have a glance at the overall correction behaviors as presented in Fig. 1. We find the K factor of short-distance coefficients $\hat{d\sigma}$ for ${}^3\!P_I^{[8]}$ channels (the sum over J=0,1,2 weighted with a factor of 2J+1 by spin symmetry in nonrelativistic limit) is large but negative at high p_T . As explained in [10], the negative value mainly originated from using the $\overline{\rm MS}$ scheme when choosing the renormalization scheme for S-wave spin-triplet NRQCD LDMEs, and does not affect the physical result. Another nontrivial phenomenon is that, differing from other channels, the K factor of ${}^{3}S_{1}^{[8]}$ channel is almost independent of p_T and not larger than 1.3. This can be understood since the α_s correction does not bring any new kinematically enhanced contributions for the ${}^{3}S_{1}^{[8]}$ channel, and it implies the expansion in α_s is under control once the leading p_T (scaling as $1/p_T^4$) channel is opened up. We also note that K factors of all other channels are just about 1

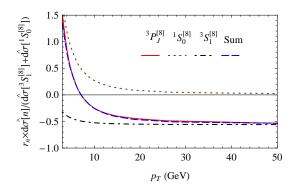


FIG. 2: NLO short-distance coefficients $\hat{d\sigma}[{}^{3}P_{J}^{[8]}]$, $r_{0}\hat{d\sigma}[{}^{1}S_{0}^{[8]}]$, $r_{1}\hat{d\sigma}[{}^{3}S_{1}^{[8]}]$, and $Sum = r_{0}\hat{d\sigma}[{}^{1}S_{0}^{[8]}] + r_{1}\hat{d\sigma}[{}^{3}S_{1}^{[8]}]$ as functions of p_{T} at the Tevatron, where $r_{0} = 3.9$, $r_{1} = -0.56$ and each contribution is divided by $\hat{d\sigma}[{}^{1}S_{0}^{[8]}] + \hat{d\sigma}[{}^{3}S_{1}^{[8]}]$.

when $p_T \approx 3$ GeV, which can be seen in Fig. 1. All the large corrections can be attributed to the enhancement in $1/p_T$ expansion.

Since we find ${}^{3}P_{J}^{[8]}$ channels can give a $1/p_{T}^{4}$ term and have a large K factor, the ${}^{3}S_{1}^{[8]}$ channel is no longer the unique source for high p_{T} contributions. In fact, for the short-distance coefficients defined in Eq. (1) the following decomposition holds within an error of a few percent

$$\hat{d\sigma}[^{3}P_{J}^{[8]}] = r_{0} \hat{d\sigma}[^{1}S_{0}^{[8]}] + r_{1} \hat{d\sigma}[^{3}S_{1}^{[8]}], \tag{2}$$

where we find $r_0 = 3.9$ and $r_1 = -0.56$ for the Tevatron, and $r_0 = 4.1$ and $r_1 = -0.56$ for the LHC. This decomposition in direct $J/\psi(\psi')$ production at the Tevatron is shown in Fig.2, where each contribution is divided by $d\hat{\sigma}[^1S_0^{[8]}] + d\hat{\sigma}[^3S_1^{[8]}]$ to make it easy to read. As a result, it is convenient to use two linearly combined LDMEs

$$M_{0,r_0}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^{1}S_{0}^{[8]})\rangle + \frac{r_0}{m_c^2} \langle \mathcal{O}^{J/\psi}(^{3}P_{0}^{[8]})\rangle,$$

$$M_{1,r_1}^{J/\psi} = \langle \mathcal{O}^{J/\psi}(^{3}S_{1}^{[8]})\rangle + \frac{r_1}{m_c^2} \langle \mathcal{O}^{J/\psi}(^{3}P_{0}^{[8]})\rangle, \qquad (3)$$

when comparing theoretical predictions with experimental data for production rates at the Tevatron and LHC.

We note that, although both $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ and $\hat{d\sigma}[^3P_J^{[8]}]$ depend on the renormalization scheme and the factorization scale μ_{Λ} , $M_{1,r_1}^{J/\psi}$ does not. The reason is that the dependence of $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ is canceled by that of r_1 , which is originated from decomposing $\hat{d\sigma}[^3P_J^{[8]}]$ at high p_T with all information for the dependence (here we ignore the contribution of $^3S_1^{[1]}$, which decreases quickly at high p_T in LO). So r_1 should be viewed as $r_1(\overline{\mathrm{MS}}, \mu_{\Lambda})$ but for simplicity we suppress these variables in the expression.

By fitting the p_T distributions of prompt ψ' and J/ψ production measured at Tevatron[17] in Fig. 3 and Fig. 4, the CO LDMEs are determined as showing in Table I, while the CS LDMEs are estimated using a potential

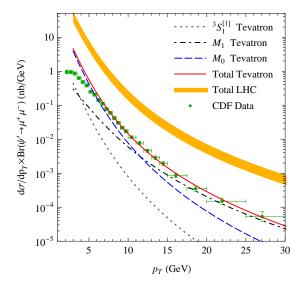


FIG. 3: Transverse momentum distributions of prompt ψ' production at the Tevatron and LHC. CDF data are taken from Ref.[17]. The yellow bands indicate the uncertainty due to CO LDMEs.

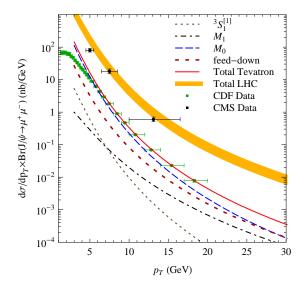


FIG. 4: The same as Fig. 3 but for J/ψ production. The preliminary CMS data, taken from Ref.[18], are compared with the theoretical prediction.

model result of the wave functions at the origin [19]. In the fit we introduce a p_T^{cut} and only use experimental data for the region $p_T \geq p_T^{cut}$. In Figs. 3and 4 and the following analysis, we prefer to use $p_T^{cut}=7~{\rm GeV}.$

We find the ratio $R = M_{1,r_1}^{J/\psi}/M_{0,r_0}^{J/\psi}$ is determined to be as small as 0.007. Based on this fit, we may conclude that the direct J/ψ production could be dominated by the ${}^{1}\!S_{0}^{[8]}$ channel in the chosen experimental p_T region. To achieve this conclusion, we emphasize the following

p_T^{cut} GeV	Н	$\langle \mathcal{O}^H \rangle$ GeV ³	M_{1,r_1}^H 10^{-2} GeV^3	10^{-2}GeV^3	$\chi^2/d.o.f.$
7	$J/\psi \ \psi'$	1.16 0.76	0.05 ± 0.02 0.12 ± 0.03	7.4 ± 1.9 2.0 ± 0.6	$0.33 \\ 0.56$
5	$J/\psi \ \psi'$	1.16 0.76	0.16 ± 0.05 0.17 ± 0.04	5.2 ± 1.3 1.1 ± 0.3	$\frac{3.5}{2.2}$

TABLE I: Fitted color-octet LDMEs in $J/\psi(\psi')$ production with chosen p_T^{cut} . Here $r_0=3.9,\ r_1=-0.56$ are determined from short-distance coefficient decomposition at the Tevatron. Errors are due to renormalization and factorization scale dependence only. Color-singlet $\binom{3}{5}\binom{1}{1}$ LDMEs $\langle \mathcal{O}^H \rangle$ are estimated using a potential model result[19].

points on the origination of the small R.

- (1) We find the fitted results are not good for data with $p_T < 7 \text{ GeV}$, while the data for $p_T \geq 7 \text{ GeV}$ can be fitted very well using the determined LDMEs for both J/ψ and ψ' . We perform a χ^2 analysis for comparing theoretical fit with experimental data with different p_T^{cut} . Values of $\chi^2/d.o.f.$ decrease rapidly as the cut increasing from 3 GeV to 7 GeV, and $\chi^2/d.o.f.$ becomes almost unchanged when p_T^{cut} is larger. This may be understood as factorization and perturbation expansion may not be reliable at low p_T . In Fig. 4 the curvature of observed cross section is positive at large p_T but negative at small p_T , with a turning point at $p_T \approx 6$ GeV. But the theoretical curvature is positive. This implies data below 7 GeV may not be well explained in this work (even in perturbative QCD) and needs further studying. Nevertheless, as an alternative choice, we also give the fitted result for $p_T^{cut}=5$ GeV, for which $M_{1,r_1}^{J/\psi}$ is increased by a factor of 3, while the price paid is $\chi^2/d.o.f.$ increases from 0.33 to 3.5. The results for both $p_T^{cut} = 7 \text{ GeV}$ and $p_T^{cut} = 5 \text{ GeV}$ are shown in Table I.
- (2) Feed-down contributions from ψ' and χ_{cJ} to J/ψ prompt production are properly considered. Because $m_{\psi'}$ and $m_{\chi_{cJ}}$ are larger than $m_{J/\psi}$ by only a few hundred MeV, J/ψ is almost motionless in the higher charmonium rest frame. So p_T of J/ψ can be expressed as $p_T \approx p_T' \times (m_{J/\psi}/m_H)$, where p_T' and m_H are the transverse momentum and mass of the directly produced higher charmonium H. LDMEs of ψ' are taken from Table I, while that of χ_{cJ} are chosen with relatively smaller values from Ref.[10]. From experimental data in Figs. 3 and 4; and Ref.[10], we see that the prompt production p_T distribution of J/ψ is steeper than that of ψ' and χ_{cJ} . This implies that the subtraction of more feeddown contributions will lead to a steeper J/ψ direct production distribution and hence a smaller R.
- (3) Errors come from other sources. Varying renormalization and factorization scales from $m_T/2$ to $2m_T$, where $m_T = \sqrt{4m_c^2 + p_T^2}$ typically changes both $M_{1,r_1}^{J/\psi}$ and $M_{0,r_0}^{J/\psi}$ by 30% (Table I). However, the ratio R is almost independent of changing scales, because the dependence between two LDMEs cancels each other. Vary-

ing the charm quark mass m_c can change the values of both LDMEs and R, and the dependence of R on m_c is approximately $R \propto m_c^2$. Thus choosing $m_c = 1.5 \pm 0.1$ may cause an error of 20% for R.

So, using the Tevatron data of J/ψ prompt production for $p_T \geq 7$ GeV or even $p_T \geq 5$ GeV, we find very small values for R, or equivalently, $M_{1,r_1}^{J/\psi} \ll M_{0,r_0}^{J/\psi}$ (see Table I). If we make a simple assumption that the smallness of $M_{1,r_1}^{J/\psi}$ is not due to accidental cancellation between $\langle \mathcal{O}^{J/\psi}(^3\!S_1^{[8]})\rangle$ and $\langle \mathcal{O}^{J/\psi}(^3\!P_0^{[8]})\rangle$, we would have an order of magnitude estimate for the three LDMEs

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle \approx \langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2 \ll \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle.$$

This would lead to a nontrivial result that J/ψ direct production is dominated by the ${}^{1}S_{0}^{[8]}$ channel, hence J/ψ is mainly unpolarized, which agrees with the polarization measurement [5].

We have also compared our prediction for prompt J/ψ production with the CMS data in Fig. 4 and a good agreement is achieved.

As for ψ' , since the difference between two LDMEs is not as large as that of J/ψ , $M_{1,r_1}^{\psi'}$ may be dominant at not too high p_T ; hence, ψ' may be transversely polarized in this region. However, it should be noted that $M_{1,r_1}^{\psi'}$ is always a combination of $\langle \mathcal{O}^{\psi'}({}^3\!S_1^{[8]})\rangle$ and $\langle \mathcal{O}^{\psi'}({}^3\!P_0^{[8]})\rangle$ at NLO; thus, whether ψ' is transversely polarized at high p_T is unclear and needs further studying.

In summary, we calculate $J/\psi(\psi')$ prompt production at the Tevatron and LHC at $\mathcal{O}(\alpha_s^4 v^4)$, including all CS, CO, and feeddown contributions. A large K factor of P-wave CO channels at high p_T results in two linearly combined LDMEs $M_{0,r_0}^{J/\psi(\psi')}$ and $M_{1,r_1}^{J/\psi(\psi')}$, which can be extracted at NLO from the Tevatron data. Because of the steep shape of experimental J/ψ prompt production data, we get a very small $M_{1,r_1}^{J/\psi}$, which might indicate the possibility that CO $^1S_0^{[8]}$ dominates J/ψ direct production. If this is the case, J/ψ will be mainly unpolarized, which may provide a possible solution to the long-standing J/ψ polarization puzzle.

We thank C. Meng and Y.J. Zhang for helpful discussions, and B. Gong and J.X. Wang for useful communications. This work was supported by the National Natural Science Foundation of China (No.10721063, No. 11021092, No.11075002) and the Ministry of Science and Technology of China (No.2009CB825200).

Note added. Soon after this work was submitted for publication, a similar study appeared[20], and for all color-singlet and octet channels in J/ψ direct hadroproduction their short-distance coefficients are consistent with ours.

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