

# Hadronic Production of $\chi_c$ -mesons at LHC

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## Abstract

Hadronic production of  $P$ -wave charmonium states  $\chi_{cJ}$  in hadronic interaction is considered. Using experimental results of CDF and LHCb collaborations we show, that contributions of color-singlet components are dominant. As for color-octet mechanism, we show, that contributions from  $P$ -wave states can also be observed, while  $S$ -wave states can be neglected. The best experimental observables that give information about relative importance of color-singlet and color-octet components are the ratios  $\chi_{c2}/\chi_{c1}$  and  $\chi_{c0}/\chi_{c1}$ .

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## 1. Introduction

Heavy quarkonia production in hadronic experiments is an extremely interesting task for theoretical and experimental investigation. It is well known, that at high energies the dominant mechanism for these processes is gluon-gluon fusion, but at leading order approximation of perturbation theory such approach cannot describe observed experimentally production of  $\chi_{c1}$  meson and distributions over the transverse momentum of final quarkonium. This problem can be solved by considering higher order processes and in the following we show, that in high  $p_T$  region is sufficient to study only NLO approximation and consider subprocesses  $gg \rightarrow \chi_{cJ}g$ .

The other interesting topic in heavy quarkonia production is the influence of color octet (CO) components. According to NonRelativistic Quantum Chromo-

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Dynamics (NRQCD) [1] the quark-antiquark pair  $c\bar{c}$  in charmonium should not necessary be in the color-singlet (CS) state. There are also CO components in  $\chi_{c,J}$  mesons accomplished with additional gluons. Differential cross sections of production of such states depends on their quantum numbers, so from analysis of experimental distributions over different kinematical variables (e.g. transverse momentum  $p_T$ ) one can determine the relative contributions of different states into total and differential cross sections.

The rest of the paper is organized as follows. In the next section used in our paper partonic subprocesses are briefly discussed. In section III we present the analysis of experimental data obtained by CDF and LHCb collaborations and determine contributions of CS and CO components into  $\chi_{c1,2}$  production cross sections, Theoretical predictions for  $\chi_{c0}$  production cross sections and  $p_T$  dependence of the ratios  $\chi_{c0}/\chi_{c1}$ ,  $\chi_{c0}/\chi_{c2}$  are also given in sec.III. Brief analysis of our results is given in the conclusion.

## 2. Partonic Processes

Our article is devoted to charmonia production in high energy hadronic experiments at Tevatron and LHC (preliminary discussion of this topic can be found for example in our previous paper [2]). It is well known, that main contribution at these conditions comes from gluon-gluon fusion

$$gg \rightarrow (c\bar{c}), \quad (1)$$

where quark-antiquark pair hadronizes into color-singlet charmonium meson  $\mathcal{Q}$ . The cross section of its hadronic production can be written in the following form

$$\sigma_{\text{LO}} = \int_0^1 dx_1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}_{\text{LO}}(gg \rightarrow \mathcal{Q}), \quad (2)$$

where  $x_{1,2}$  are momentum fractions of the incoming partons,  $f_g(x_{1,2})$  are distribution functions of these partons in initial hadrons, and  $\hat{\sigma}(gg \rightarrow \mathcal{Q})$  is the cross sections of the hard subprocess (1).

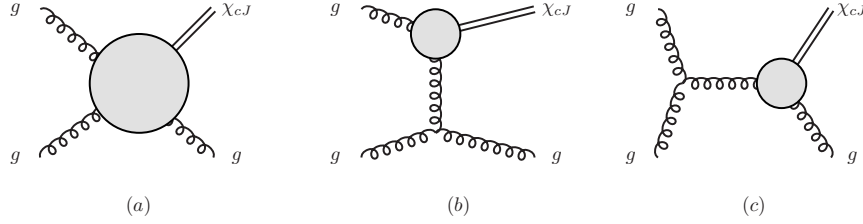


Figure 1: Diagrams of the color-singlet  $gg \rightarrow \chi_{cJ}g$  subprocess

There are however some drawbacks in such approach. First of all, in collinear approximation, when transverse motion of initial gluons is neglected, expression (2) cannot describe the distributions over the transverse momentum of final charmonium. Moreover, in CS approximation only mesons with positive charge parity ( $\eta_c$ ,  $\chi_{cJ}$ ) can be produced in reaction (1) and the case of axial charmonium is excluded by Landau-Yang theorem [3, 4], that forbids its production in  $gg \rightarrow \chi_{c1}$  with two massless gluons interaction. It is clear, that these results contradict dramatically existing data, since both transverse momentum distributions and  $\chi_{c1}$  meson production were observed experimentally. The cross section of the latter process is even larger than the cross section of tensor charmonium production.

These problems can be solved in NLO approximation, when subprocesses

$$gg \rightarrow \chi_{cJ}g, \quad (3)$$

are considered. Typical diagrams for these reactions are shown in Fig.1. For the first time they were studied in paper [5], later a series of papers devoted to the same topic followed (see for example [6]). It is clear, that due to presence of final state gluon in (3) final charmonium has non-vanishing transverse momentum even in collinear approximation. As for  $\chi_{c1}$  meson production, it is allowed since one of the parent gluons is virtual (Fig.1b) and Landau-Yang theorem does not forbid the vertex  $gg^* \rightarrow \chi_{c1}$ .

In addition to CS states one should also take into account color octet con-

tributions. The physical  $\chi_{cJ}$  meson can be written as an infinite series [1]

$$|\chi_{cJ}\rangle \sim |R'(0)|^2 \left| c\bar{c} \left[ {}^3P_J^{[1]} \right] \right\rangle + \langle \mathcal{O}_S \rangle \left| c\bar{c} \left[ {}^3S_1^{[8]} \right] \right\rangle + \langle \mathcal{O}_P \rangle \left| c\bar{c} \left[ {}^1P_1^{[8]} \right] \right\rangle \quad (4)$$

where in parentheses quantum numbers of quark-antiquark state and its color charge are shown. The coefficients  $|R'(0)|^2$ ,  $\langle \mathcal{O}_{S,P} \rangle$  describe the probability for corresponding component to hadronize into experimentally observed meson and are usually taken as universal. According to NRQCD higher terms in expression (4) are suppressed by relative velocity of quarks in meson, so this series can be safely truncated. In the following we restrict ourselves to CS and  $S$ -,  $P$ -wave CO. In this approximation the cross section of  $\chi_{cJ}$  meson production in  $gg \rightarrow \chi_{cJ}g$  reaction is written in the form

$$\begin{aligned} \frac{d\hat{\sigma}(gg \rightarrow \chi_{cJ}g)}{d\hat{t}} &= |R'(0)|^2 \frac{d\hat{\sigma}(gg \rightarrow c\bar{c} [{}^3P_J^{[1]}]g)}{d\hat{t}} + \\ &\quad \frac{\pi}{6}(2J+1) \langle \mathcal{O}_S \rangle \frac{d\hat{\sigma}(gg \rightarrow c\bar{c} [{}^3S_1^{[8]}]g)}{d\hat{t}} + \\ &\quad \frac{\pi}{18}(2J+1) \langle \mathcal{O}_P \rangle \frac{d\hat{\sigma}(gg \rightarrow c\bar{c} [{}^1P_1^{[8]}]g)}{d\hat{t}}, \end{aligned} \quad (5)$$

where we show explicitly the dependence on wave function derivative at the origin (in CS case) and matrix elements of  $S$ -,  $P$ -wave octet states. The parameters  $|R'(0)|$ ,  $\langle \mathcal{O}_{S,P} \rangle$  are universal and do not depend on total spin of the final charmonium.

Hard subprocesses  $gg \rightarrow \mathcal{Q}g$  were already considered in a number of papers (see for example [5, 6, 7, 8]), so below we give only short qualitative analysis. In low  $p_T$  region some of these cross sections diverge (see second row of table 1). Such behavior is caused by  $t$ -channel gluon from diagram shown in Fig.1b, that in this region approaches the mass shell. In order to regularize this divergence one should consider higher order processes or perform a suitable cut. In our paper we consider only high  $p_T$  region, so this singularity is not crucial. It is interesting to note, however, that the cross section of  $\chi_{c1}$  production is finite over the whole  $p_T$  domain. The reason is mentioned above Landau-Yang theorem: in low  $p_T$  region the vertex  $gg \rightarrow \chi_{c1}$  vanishes and caused by gluon propagator

	$^3P_1^{[1]}$	$^3P_{0,2}^{[1]}$	$^1P_1^{[8]}$	$^3S_1^{[8]}$
$p_T \ll M$	$\sim p_T$	$\sim 1/p_T$	$\sim 1/p_T$	$\sim p_T$
$p_T \gg M$	$\sim 1/p_T^5$	$\sim 1/p_T^5$	$\sim 1/p_T^5$	$\sim 1/p_T^3$

Table 1: Behavior of partonic cross sections for different processes in low and high  $p_T$  regions

divergence is compensated.

For our analysis we need also the behavior of different partonic cross sections in high  $p_T$  region. Corresponding expressions can be found in the third row of table 1. It is clearly seen, that in this region cross sections of CS and  $P$ -wave CO cross sections are proportional, so it is not sufficient to study  $p_T$  distributions of different  $\chi_{cJ}$  mesons separately to determine relative contributions of CS and CO components. It is necessary to consider some combined quantity, for example the ratio

$$\hat{r}_{J_1 J_2} = \frac{d\hat{\sigma}(gg \rightarrow \chi_{cJ_1} g)/dp_T}{d\hat{\sigma}(gg \rightarrow \chi_{cJ_2} g)/dp_T}. \quad (6)$$

If the contribution of  $S$ -wave CO states is not negligible, they should dominate in high  $p_T$  region and the ration (6) takes the form

$$\hat{r}_{J_1 J_2}(p_T \gg M) \approx \frac{2J_1 + 1}{2J_2 + 1}. \quad (7)$$

In the opposite case the ratio for different values of final charmonia spins is equal to

$$\hat{r}_{2,1} = \frac{158\,184 |R'(0)|^2 + 295\,595\pi \langle \mathcal{O}_P \rangle}{474\,552 |R'(0)|^2 + 177\,357\pi \langle \mathcal{O}_P \rangle}, \quad (8)$$

$$\hat{r}_{0,1} = \frac{79\,092 |R'(0)|^2 + 59\,119\pi \langle \mathcal{O}_P \rangle}{474\,552 |R'(0)|^2 + 177\,357\pi \langle \mathcal{O}_P \rangle}, \quad (9)$$

$$\hat{r}_{0,2} = \frac{79\,092 |R'(0)|^2 + 59\,119\pi \langle \mathcal{O}_P \rangle}{158\,184 |R'(0)|^2 + 295\,595\pi \langle \mathcal{O}_P \rangle}. \quad (10)$$

From combined analysis of experimental  $p_T$  distributions of  $\chi_{cJ}$  mesons productions separately and their ratios one can determine contributions of CS and various CO states to these processes.

### 3. Hadronic Production and Fit of Matrix Elements

For comparison with experimental data considered in the previous section cross sections should be convoluted with gluon distribution functions in initial hadrons. Sometimes functions that depend explicitly on the transverse momentum of the parton are used (so called  $k_T$  factorization), that take into account multiple emission of soft gluons. It is clear, however, that in high  $p_T$  region such processes will be suppressed by small string coupling constant, so the emission of one hard gluon can be preferable. In our paper we use the latter approach.

The cross section of inclusive  $\chi_{cJ}$  production in hadronic interaction can be written [9] in the form similar to eq.(2):

$$\frac{d\sigma(pp \rightarrow \chi_{cJ} + X)}{dp_T} = \int \frac{d\hat{s}}{s} \frac{d\hat{\sigma}(gg \rightarrow \chi_{cJ}g)}{dp_T} \times \int dy f_g\left(\sqrt{\frac{\hat{s}}{s}}e^y\right) f_g\left(\sqrt{\frac{\hat{s}}{s}}e^{-y}\right), \quad (11)$$

$$\frac{d\hat{\sigma}}{dp_T} = \frac{2\hat{s}p_T}{\sqrt{(\hat{s} - M^2)^2 - 4\hat{s}p_T^2}} \left( \left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\hat{t}=\hat{t}_1} + \left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\hat{t}=\hat{t}_2} \right), \quad (12)$$

where  $\hat{s}, \hat{t}, \hat{u}$  are the Mandelstam variables of the partonic subprocess,

$$p_T = \sqrt{\frac{\hat{t}\hat{u}}{\hat{s}}} \quad (13)$$

is the transverse momentum of final charmonium (in collinear approximation this expression is valid both for partonic and hadronic reactions), and the following notations were introduced:

$$\hat{t}_{1,2}(p_T) = \frac{1}{2} \left\{ M^2 - \hat{s} \pm \sqrt{(\hat{s} - M^2)^2 - 4\hat{s}p_T^2} \right\}. \quad (14)$$

Partonic cross sections and distribution functions that enter these expressions depend strongly on factorization scale  $\mu^2$ . In order to study the dependence of final results on the choice of this scale we use the following values:  $\mu^2 = M^2$ ,  $\mu^2 = m_T^2 = p_T^2 + M^2$ ,  $\mu^2 = 2m_T^2$  and  $\mu^2 = m_T^2/2$ . Gluon distribution functions were taken in CTEQ6 parameterization.

For determination of CS and CO parameters  $|R'(0)|^2$ ,  $\langle \mathcal{O}_{S,P} \rangle$  we use experimental data obtained by CDF [10, 11] and LHCb [12] collaborations. In paper

$\mu^2$	$ R'(0) ^2, \text{ GeV}^5$	$\langle \mathcal{O}_S \rangle, \text{ GeV}^3$	$\langle \mathcal{O}_P \rangle, \text{ GeV}^5$
$M^2$	0.22	$1.9 \times 10^{-9}$	0.029
$m_T^2/2$	0.20	0	0.026
$m_T^2$	0.19	$4.8 \times 10^{-11}$	0.022
$2m_T^2$	0.19	$4.7 \times 10^{-9}$	0.019

Table 2: CS and CO model parameters for different values of the scale  $\mu^2$

[10] transverse momentum distribution of  $J/\psi$ -meson production cross section in radiative decays  $\chi_{cJ} \rightarrow J/\psi\gamma$  at  $\sqrt{s} = 1.8 \text{ TeV}$ ,  $|\eta| < 0.6$  is presented:

$$\frac{d\sigma(pp \rightarrow J/\psi + X)}{dp_T} = \sum_{J=0}^2 \text{Br}[\chi_{cJ} \rightarrow J/\psi\gamma] \frac{d\sigma(pp \rightarrow \chi_{cJ} + X)}{dp_T} \quad (15)$$

Using discussed above theoretical predictions of CS and CO  $\chi_{cJ}$  meson differential cross sections and experimental values of radiative decay branching fractions [13] one can determine parameters  $|R'(0)|^2$ ,  $\langle \mathcal{O}_S \rangle$ ,  $\langle \mathcal{O}_P \rangle$  from eq.(4). Due to small value of the branching fraction of the decay  $\chi_{c0} \rightarrow J/\psi\gamma$  the scalar charmonium can be excluded from expression (15).

We have already stressed above that in high  $p_T$  region up to constant factor differential of CS and  $P$ -wave CO cross sections coincide, so one should use some other variable to separate these components. One of such observables is the ratio

$$r_{J_1 J_2}(p_T) = \frac{d\sigma(pp \rightarrow \chi_{cJ_1} + X)/dp_T}{d\sigma(pp \rightarrow \chi_{cJ_2} + X)/dp_T}. \quad (16)$$

In high  $p_T$  region, where hard cross sections of  $gg \rightarrow \chi_{cJ_1}g$  and  $gg \rightarrow \chi_{cJ_2}g$  are almost proportional to each other, partonic distribution functions in this ratio cancel and it becomes equal to the ratio of hard cross sections (6). It should be noted, that this cancellation is universal and does not depend on experimental cutoffs.

The results of our fit for different values of the scale  $\mu^2$  are presented in table 2 and  $p_T$  distribution of  $J/\psi$  production cross section in comparison with experimental data is shown in fig.2. It is clear, that results of our model are in

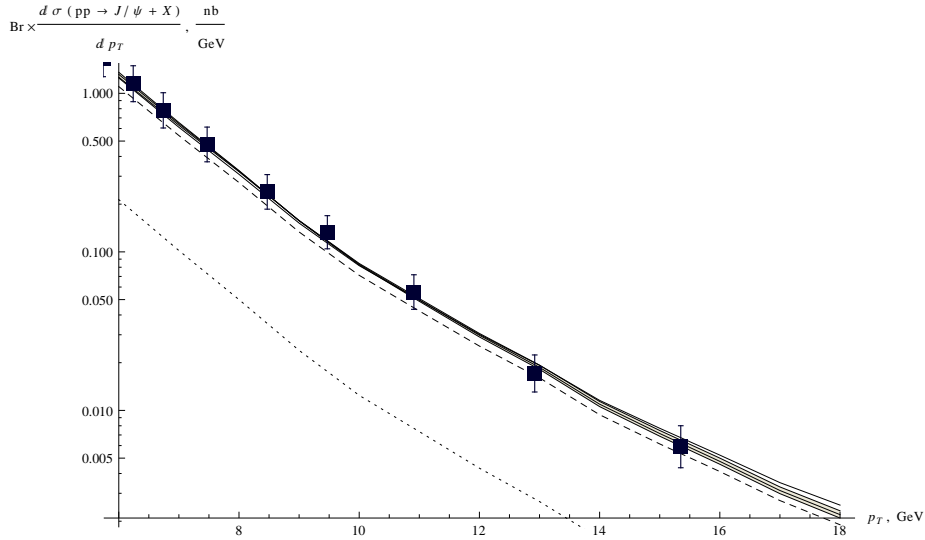


Figure 2: Transverse momentum distribution of  $J/\psi$  production in radiative  $\chi_{cJ}$  decays at CDF in comparison with experimental data [10]. Solid, dashes and dotted lines correspond to total  $pp \rightarrow \chi_{cJ} + X \rightarrow J/\psi + X$  cross section, CS contributions and  $P$ -wave CO contributions respectively.

excellent agreement with experiment and contribution of CS states dominate. As for CO states, our results show, that contribution of  $S$ -wave components can be safely neglected, while  $P$ -wave CO contributions are small but visible. This conclusion contradicts NRQCD scaling rules, that state that CS and  $S$ -wave CO states should give similar contributions, while  $P$ -wave CO states should be suppressed.

In figure 3 we compare theoretical results for  $\chi_{c2}/\chi_{c1}$  ratio (solid lines) with experimental data from [12, 11] (dots with error bars). Earlier we have said that this ratio can be used to separate contributions from CS and  $P$ -wave CO components. Dashed and dotted lines in this figure show theoretical predictions of this ratio with only CS or  $P$ -wave CO contributions taken into account. It can be clearly seen, that CS mechanism is dominant, but some CO contribution is also required.

It should be noted, that presented in table 2 values of color-singlet param-



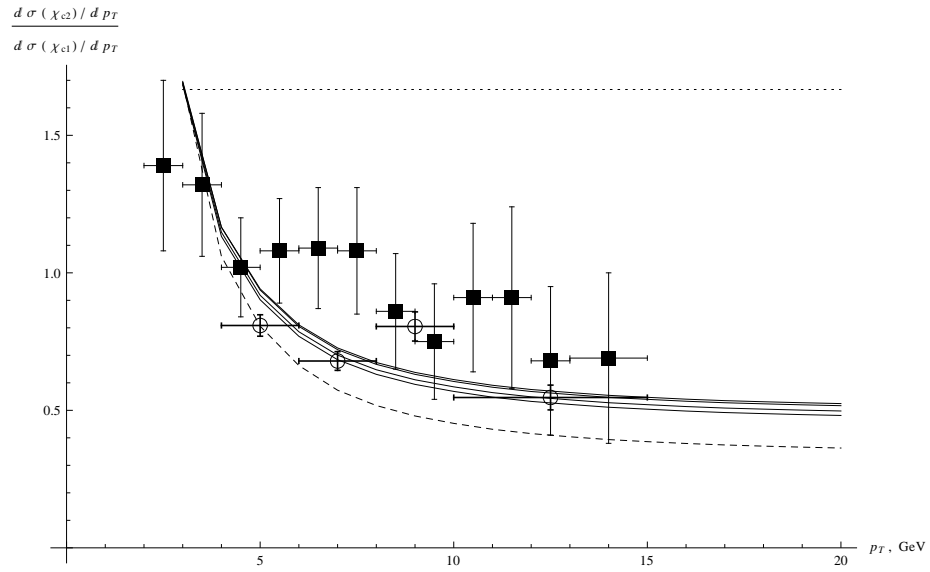


Figure 3: Theoretical results of the ratio  $\chi_{c2}/\chi_{c1}$  in comparison with experimental data from [11] ( $\circ$ ) and [12] ( $\blacksquare$ ). Solid lines correspond to parameter values presented in table 2, while dashed and dotted lines show predictions of our model with only CS and CO contributions taken into account.

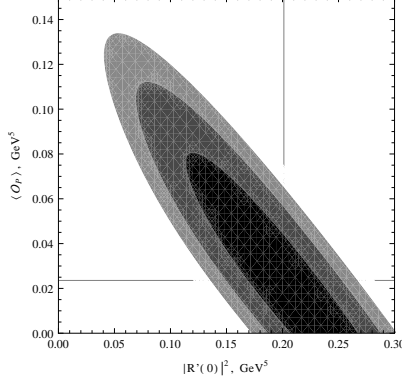


Figure 4: Allowed region of parameters  $|R'(0)|^2$  and  $\langle \mathcal{O}_P \rangle$

eter  $|R'(0)|^2$  are higher than phenomenological value  $|R'(0)|^2 \approx 0.08 \text{ GeV}^5$ , determined from experimental hadronic width of  $\chi_{c2}$  meson [14]

$$\Gamma(\chi_{c2} \rightarrow \text{hadrons}) \approx \Gamma(\chi_{c2} \rightarrow 2g) = \frac{128}{5} \frac{\alpha_s^2}{M^4} |R'(0)|^2$$

and predictions of different potential models [15, 16, 17, 18, 19, 20]. One should take into account, that presented in table 2 results are strongly correlated. It can be seen clearly from figure 4, where we show the allowed region of parameters  $|R'(0)|^2, \langle \mathcal{O}_P \rangle$ , where the error  $\chi^2/DOF$  is increased by one unit maximum. The very use of potential model predictions and  $\chi_{c2}$  decay width for charmonium production at high energies is also rather questionable. From double charmonia production in exclusive electron-positron annihilation [21, 22, 23] we know, the with the increase of the interaction energy the width of the momentum distributions of heavy quarks in quarkonia also increases. In coordinate space it corresponds to the increase of the charmonium wave function and its derivative at the origin. From Fig.4 it is clear, that such modification of  $|R'(0)|^2$  leads to decrease of the parameter  $\langle \mathcal{O}_P \rangle$  and the contributions from color octet states.

To remove this error one can measure with better accuracy cross sections of  $\chi_{c1,2}$  mesons production and their ratios in various experimental conditions. The other experiment, that can shed light onto this question is the observation

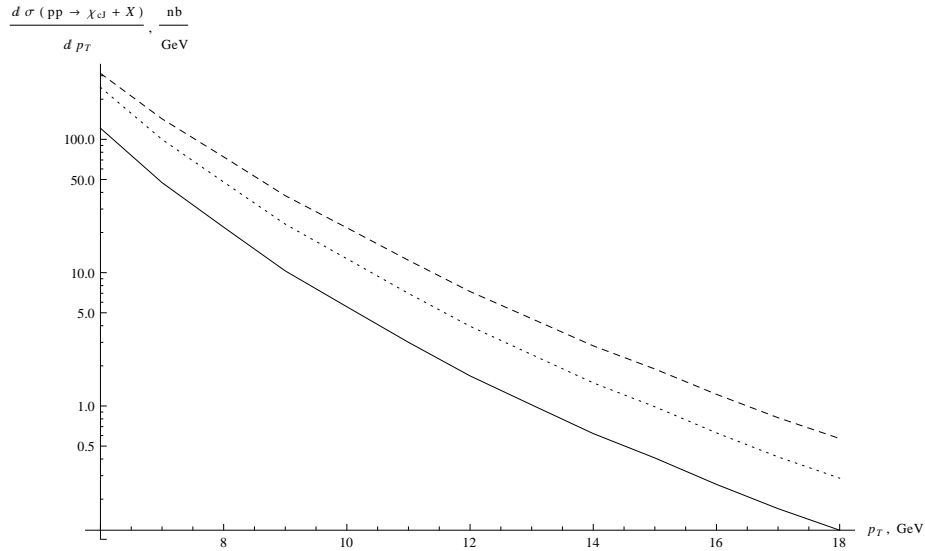


Figure 5: Transverse momentum dependence of  $\chi_{c0}$ ,  $\chi_{c1}$ , and  $\chi_{c2}$  mesons at LHCb (solid, dashed and dotted lines respectively)

of  $\chi_{c0}$  meson. The branching fraction of its radiative decay is small, so this task looks very difficult, but nevertheless possible. In Figs.5, 6 we show theoretical predictions of  $\chi_{c0,1,2}$  meson production cross sections and their ratios at LHCb.

#### 4. Conclusion

The paper is devoted to inclusive  $P$ -wave charmonia production in high energy hadronic experiments.

Using existing experimental data presented by collaborations CDF [10, 21] and LHCb [12] we determined the cross sections of color singlet and color octet  $\chi_{cJ}$  mesons. Our analysis show, that contributions of color singlet components are dominant, while  $P$ -wave color octet components are strongly suppressed. As for  $S$ -wave color octet components, we found, that their contributions can safely be neglected completely. We also present theoretical predictions for  $\chi_{c1,2}$  production cross sections and transverse momentum distributions at LHCb and discuss in details processes of scalar charmonium production  $\chi_{c0}$ .

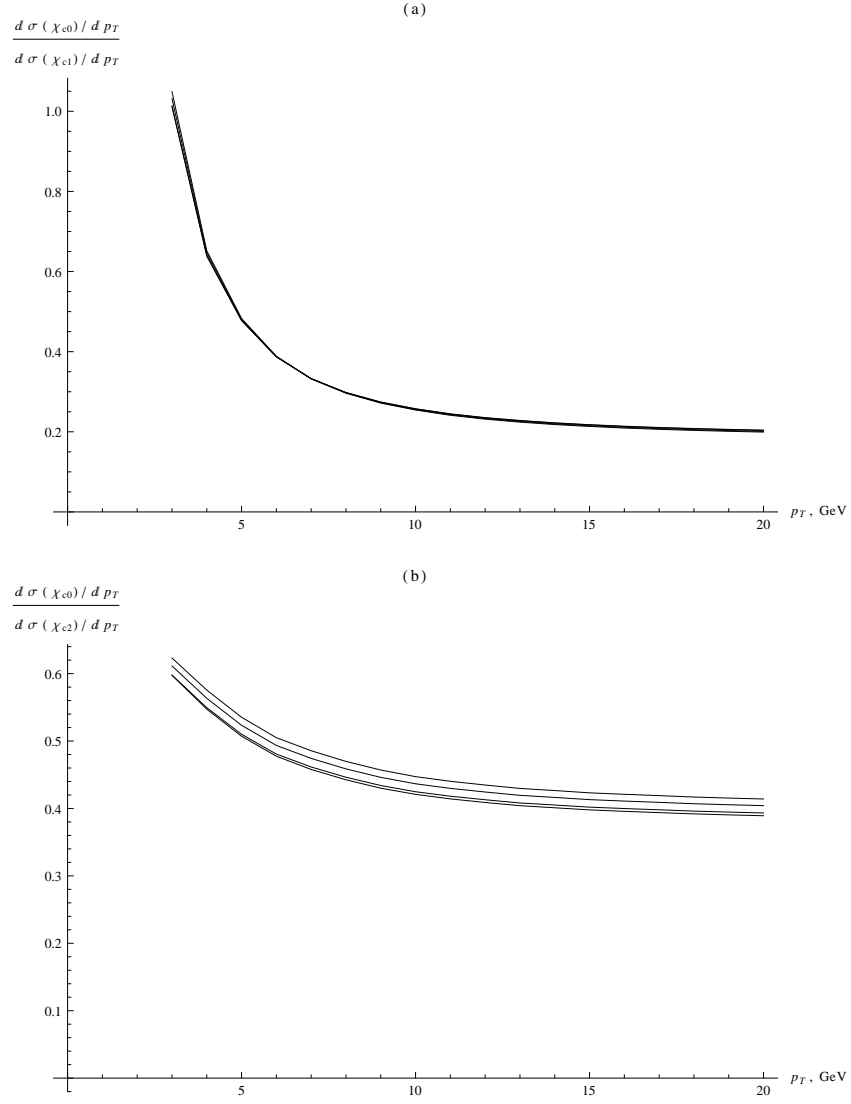


Figure 6: The ratios  $\chi_{c0}/\chi_{c1}$  (top panel) and  $\chi_{c0}/\chi_{c2}$  (bottom panel) at LHCb

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