QCD corrections to J/ψ production via color-octet states at the Tevatron and LHC

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Abstract

The Next-To-Leading-Order (NLO) QCD corrections to J/ψ production via S-wave color-octet states at the Tevatron and LHC are calculated. There are only slight changes to the transverse momentum p_t distributions of J/ψ production and polarization. By fitting

B-factories, $J/\psi[{}^{3}P_{J}^{(8)}]$ production suggested by the authors of Ref. [8] has not been observed. Even more seriously, the LO NRQCD calculation predicts a sizable transverse polarization rate for large $p_t J/\psi$ [9] whereas the Tevatron measurement at Fermilab [10] displays a slight longitudinal polarization at large

On the other hand, obvious discrepancies between LO predictions [11, 12] and experimental results [13, 14] for single and double charmonia productions at B-factories drew close attention of theorists. Further studies indicate that they may be resolved by including higher order corrections: both NLO QCD and relativistic corrections [11, 15, 16], or at least the

is employed. However, it is noted that $J/\psi[^{3}P_{I}^{(8)}]$ is not included since the part for dealing with P-wave loop processes in FDC is not completed yet.

> According to the NRQCD factorization formalism, the inclusive cross section for direct J/ψ production in hadron-hadron collision is expressed as

$$\sigma[pp \to J/\psi + X] = \sum_{i,j,n} \int dx_1 dx_2 G_{i/p} G_{j/p}$$
$$\times \hat{\sigma}[i+j \to (c\bar{c})_n + X] \langle O_n^H \rangle, \tag{1}$$

where p is either a proton or an antiproton, the indices i, j run over all partonic species and n denotes the color, spin and angular momentum states of the intermediate $c\bar{c}$ pair. The short-

Preprint submitted to Elsevier September 22, 2010 distance contribution $\hat{\sigma}$ can be perturbatively calculated order by order in α_s . The hadronic matrix elements $\langle O_n^H \rangle$ are related to the hadronization probabilities from the state $(c\bar{c})_n$ into J/ψ which are fully governed by the non-perturbative QCD effects. In the following, $\hat{\sigma}$ represents the corresponding partonic cross section.

At LO, there are three partonic processes:

L1:
$$gg \to J/\psi[\overset{(8)}{5}_{0}^{(8)}, \overset{3}{5}_{1}^{(8)}]g$$
, L2: $gq \to J/\psi[\overset{(8)}{5}_{0}^{(8)}, \overset{3}{5}_{1}^{(8)}]q$, L3: $q\overline{q} \to J/\psi[\overset{(8)}{5}_{0}^{(8)}, \overset{3}{5}_{1}^{(8)}]g$.

where q runs over all possible light quarks or anti-quarks: $u, d, s, \overline{u}, \overline{d}, \overline{s}$.

The NLO corrections include virtual and real corrections. There exist UV, IR and Coulomb singularities in the calculation of the virtual corrections. The UV-divergences from self-energy and triangle diagrams are removed by the renormalization procedure. Here we adopt the dimensional renormalization scheme and technique used in Ref. [22] without performing an explicit matching between the cross sections calculated in perturbative QCD and perturbative NRQCD. The renormalization constants Z_m , Z_2 , Z_{2l} and Z_3 which correspond to charm quark mass m_c , charm-field ψ_c , light quark field ψ_q and gluon field A^a_μ are defined in the on-mass-shell(OS) scheme while Z_g for the QCD gauge coupling α_s is defined in the modified-minimal-subtraction($\overline{\rm MS}$) scheme:

$$\delta Z_{m}^{OS} = -3C_{F} \frac{\alpha_{s}}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \gamma_{E} + \ln \frac{4\pi\mu_{r}^{2}}{m_{c}^{2}} + \frac{4}{3} \right],$$

$$\delta Z_{2}^{OS} = -C_{F} \frac{\alpha_{s}}{4\pi} \left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_{E} + 3\ln \frac{4\pi\mu_{r}^{2}}{m_{c}^{2}} + 4 \right],$$

$$\delta Z_{2l}^{OS} = -C_{F} \frac{\alpha_{s}}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right],$$

$$\delta Z_{3}^{OS} = \frac{\alpha_{s}}{4\pi} \left[(\beta_{0} - 2C_{A}) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right) \right],$$

$$\delta Z_{g}^{\overline{MS}} = -\frac{\beta_{0}}{2} \frac{\alpha_{s}}{4\pi} \left[\frac{1}{\epsilon_{UV}} - \gamma_{E} + \ln(4\pi) \right],$$
(2)

where γ_E is the Euler constant, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ is the one-loop coefficient of the QCD beta function and $n_f = 3$ is the number of active quark flavors. μ_F is the renormalization scale.

number of active quark flavors. μ_r is the renormalization scale. There are 267 (for the ${}^{1}\!S_{0}^{(8)}$ state) and 413 (for the ${}^{3}\!S_{1}^{(8)}$ state) NLO diagrams for process (L1), including counter-term diagrams, while for both processes (L2) and (L3), there are 49 (for the ${}^{1}\!S_{0}^{(8)}$ state) and 111 (for the ${}^{3}\!S_{1}^{(8)}$ state) NLO diagrams altogether. Diagrams where a virtual gluon line connects the quark pair possess a Coulomb singularity, which can be isolated and attributed into the renormalization of the $c\bar{c}$ wave function.

For each process, by summing over contributions from all diagrams, the virtual corrections to the differential cross section can be expressed as

$$\frac{\mathrm{d}\hat{\sigma}_{i}^{V}}{\mathrm{d}t} \propto 2\mathrm{Re}(M_{i}^{B}M_{i}^{V*}),\tag{3}$$

where M_i^B is the amplitude of process (*i*) at LO, and M_i^V is the renormalized amplitude of corresponding process at NLO. M_i^V is UV and Coulomb finite, but still contains IR divergences.

It is noteworthy that to obtain a full cancelation of IR-singularities in the calculation, the three sub-processes (L1), (L2) and (L3) tangle together and must be considered simultaneously. In addition, there are eight tree processes involved in the real corrections:

where q,q' denote light quarks (anti-quarks) with different flavors. Phase space integrations of above processes generate IR singularities, which are either soft or collinear and can be conveniently isolated by slicing the phase space into different regions. Here we adopt the two-cutoff phase space slicing method [23] to deal with the problem. Then the real cross section can be written as

$$\sigma^{R} = \sigma^{S} + \sigma^{HC} + \sigma^{H\overline{C}} + \sigma^{HC}_{add}.$$
 (4)

It is observed that the IR singularities from one real process may be factorized into different parts and each of them should be added into the cross sections of different LO processes. This is the reason why we have to calculate the NLO corrections to the three LO processes together.

 $\hat{\sigma}^S$ from the soft region contains soft singularities and is calculated analytically under the soft approximation. One should notice that, unlike color-singlet case, the soft singularities caused by emitting a soft gluon from the charm quark pair in the S-wave color-octet exist and the factorized matrix element is the same as the case where a soft gluon is emitted from a gluon. σ^{HC} from the hard collinear region contains collinear singularities which are factorized out and the singularities are partly absorbed into redefinition of the parton distribution function (PDF) (usually called as mass factorization [24]). Here we adopt the scale dependent PDF using the $\overline{\rm MS}$ convention given in Ref. [23]. After redefining the PDF, an additional finite term σ^{HC}_{add} is separated out. The hard non-collinear part $\sigma^{H\overline{C}}$ is IR finite. Finally, all the IR singularities are canceled and $\hat{\sigma}^S + \hat{\sigma}^{HC} + \hat{\sigma}^V$ is IR finite.

To obtain the transverse momentum p_t distribution of J/ψ , a transformation of integration variables $(\mathrm{d}x_2\mathrm{d}t\to J\mathrm{d}p_t\mathrm{d}y)$ is needed. Then we have

$$\frac{d\sigma}{dp_t} = \sum \int J dx_1 dy G_{\alpha}(x_1, \mu_f) G_{\beta}(x_2, \mu_f) \frac{d\hat{\sigma}}{dt}, \tag{5}$$

where y is the rapidity of J/ψ in the laboratory frame and μ_f is the factorization scale. The polarization parameter α is defined as:

$$\alpha(p_t) = \frac{\mathrm{d}\sigma_T/\mathrm{d}p_t - 2\mathrm{d}\sigma_L/\mathrm{d}p_t}{\mathrm{d}\sigma_T/\mathrm{d}p_t + 2\mathrm{d}\sigma_L/\mathrm{d}p_t}.$$
 (6)

To evaluate $\alpha(p_t)$, the polarization of J/ψ must be explicitly retained in the calculation. The partonic differential cross section

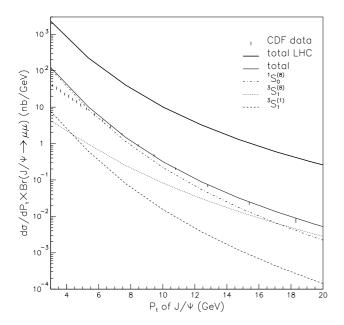


Figure 1: Transverse momentum distribution of prompt J/ψ production at Tevatron. CDF data is from ref [26]. The center-of-mass energy are 1.96 TeV at Tevatron and 14 TeV at LHC.

for a polarized J/ψ is expressed as:

$$\frac{\mathrm{d}\hat{\sigma}_{\lambda}}{\mathrm{d}t} = a \; \epsilon(\lambda) \cdot \epsilon^*(\lambda) + \sum_{i,j=1,2} a_{ij} \; p_i \cdot \epsilon(\lambda) \; p_j \cdot \epsilon^*(\lambda), \quad (7)$$

where $\lambda = T_1, T_2, L$. $\epsilon(T_1)$, $\epsilon(T_2)$, $\epsilon(L)$ are the two transverse and longitudinal polarization vectors of J/ψ respectively, and the polarizations of all the other particles are summed over in n-dimension. One can find that a and a_{ij} are finite when the virtual corrections and real corrections are properly handled as aforementioned. The gauge invariance is explicitly checked that the amplitude is exactly zero as the gluon polarization vector being replaced by its 4-momentum in the final numerical calculation.

In our numerical computations, the CTEQ6L1 and CTEQ6M PDFs [25], and the corresponding fitted value $\alpha_s(M_Z) = 0.130$ and $\alpha_s(M_Z) = 0.118$ are used for LO and NLO calculations respectively. The charm quark mass is set as 1.5 GeV. The two phase space cutoffs $\delta_s = 10^{-3}$ and $\delta_c = \delta_s/50$ are chosen, and the invariance for different values of δ_s and δ_c is obviously observed within the error tolerance. All the results in this paper are restricted to the NRQCD applicable domain $p_t > 3$ GeV, and $|y_{J/\psi}| < 3$ for LHC, $|y_{J/\psi}| < 0.6$ for Tevatron respectively.

By fitting the p_t distribution of prompt J/ψ production measured at Tevatron [26], the NRQCD matrix elements $\langle O_n^H \rangle$ are determined as $\langle O_8^{\psi}(^{1}S_1) \rangle = 0.0021 \,\text{GeV}^3$ and $\langle O_8^{\psi}(^{1}S_0) \rangle = 0.075 \,\text{GeV}^3$, and the results are shown in Fig. 1. In the fitting procedure, contributions from both color singlet at NLO[18] and octet states at NLO are included. However, it is worth noticing that we have to abandon the experimental data with $p_t < 6 \,\text{GeV}$, since it is impossible to obtain a satisfactory p_t distribution in terms of a unique $\langle O_n^H \rangle$ value. In addition, one should consider an additional contribution of the feeddown from ψ' which may bring up an extra factor $B(\psi' \rightarrow J/\psi + X) \times \langle O_n^{\psi'} \rangle / \langle O_n^{\psi} \rangle$, a short calculation determines it as 1.29

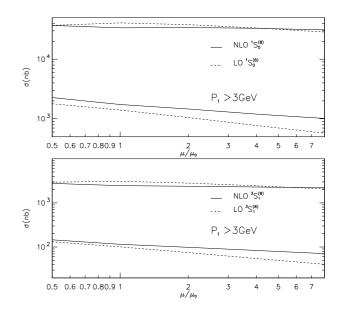


Figure 2: The partial cross section (with cut conditions) of J/ψ hadronproduction at LHC (upper curves) and Tevatron (lower curves), as a function of μ with $\mu_r = \mu_f = \mu$ and $\mu_0 = \sqrt{(2m_c)^2 + p_t^2}$.

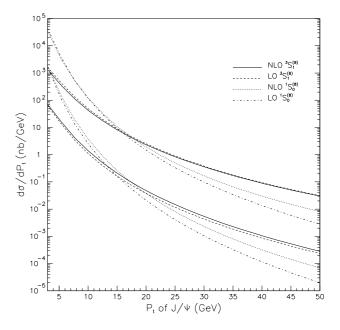


Figure 3: Transverse momentum distribution of J/ψ production with $\mu_r = \mu_f = \mu_0$ at LHC (upper curves) and Tevatron (lower curves).

for color-singlet part, and the values of our fitted $\langle O_8^{\psi}({}^{\mbox{\it Y}}_{1}) \rangle$ and $\langle O_8^{\psi}({}^{\mbox{\it Y}}_{0}) \rangle$ include the contributions of the feed-down for color-octet part. However, the feed-down from $\psi'[{}^{3}\!P_{J}^{(8)}]$ and χ_{cJ} is not considered in our fitting since at NLO it cannot be properly calculated so far and this omission should be treated as an approximation.

The dependence of the total cross section on the renormalization scale μ_r and factorization scale μ_f are shown in Fig. 2. It is obvious that the NLO QCD corrections make such dependence milder. The p_t distributions of J/ψ production are presented in Fig. 3 where only slight change appears when the NLO QCD corrections is included. $J/\psi[{}^{l}S_{0}^{(8)}]$ produces unpolarized J/ψ for both LO and NLO. The p_t distributions of J/ψ polarization parameter α for $J/\psi[{}^{3}S_{1}^{(8)}]$ are shown in Fig. 4 and there is a slight change when the NLO corrections are taken into account.

As a summary, in this work, we have calculated the NLO QCD corrections to J/ψ production via color-octet states $J/\psi[{}^{1}S_{0}^{(8)}, {}^{3}S_{1}^{(8)}]$ at Tevatron and LHC. With $\mu_{r} = \mu_{f} = \mu_{0} =$ $\sqrt{(2m_c)^2 + p_t^2}$, transverse momentum cut $p_t > 3$ GeV and rapidity cut |y| < 0.6 (Tevatron) and |y| < 3(LHC) for J/ψ , the K factors of total cross section (ratio of NLO to LO) are 1.235 and 1.139 for $J/\psi[{}^{1}S_{0}^{(8)}]$ and $J/\psi[{}^{3}S_{1}^{(8)}]$ at Tevatron, while at LHC they are 0.826 and 0.800 respectively. Unlike for the colorsinglet case, there are only slight changes to the transverse momentum distributions of J/ψ production rate and the J/ψ polarization when the NLO QCD corrections are taken into account. The results imply that the perturbative QCD expansion quickly converges for J/ψ production via the S-wave color-octet state, in contrast with that via color-singlet, where the NLO contributions are too large to hint a convergence at the NNLO. As shown in Fig. 4, an obvious gap between the theoretical results for J/ψ polarization calculated up to NLO and the experimental measurements at Tevatron is observed, even though both color singlet and octet are included. In the well established theoretical framework of NRQCD there still remains a narrow window which might make up the gap, namely one needs to investigate the NLO corrections to J/ψ production via P-wave color octet state and J/ψ production by feed-down from χ_{cJ} . It is unclear how the situation would be when contributions from these two sources at NLO are taken into account, as we know that NLO QCD corrections to P-wave state in $e^+ + e^- \rightarrow J/\psi + \chi_{c0}$ evaluated in Ref. [27] are very large. However, a careful analysis indicates that it is not really the case because as aforementioned, the P-wave color-octet was not observed at B-factory experiments. Even though the P-wave color-octets do contribute, in analog to the case for the S-wave color-octet, it is reasonable to assume that the NLO QCD correction to the P-wave color-octet is not too large due to the same power counting of the p_t distribution behavior. Then the p_t distribution of the P-wave coloroctets will be almost the same as the color-octet ${}^{1}S_{0}$ at NLO. It means that the fitting at NLO, while including the color-octet ${}^{1}S_{0}$ state, can be thought as the P-wave color-octet part is also included. Then a definite conclusion is drawn that the huge discrepancy of J/ψ polarization between theoretical predication and the experimental measurement cannot be solved by just including NLO QCD corrections within NRQCD framework and

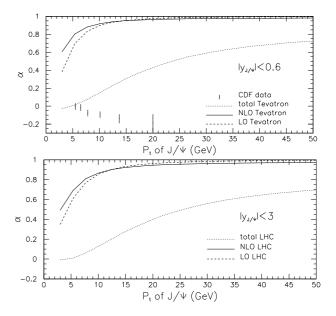


Figure 4: Transverse momentum distribution of polarization α for prompt J/ψ production, CDF data is from ref [10].

one should explore real solution along other lines.

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