Portable Electric Winch Project

Transmission Design

1 Design specification on gear configuration and spool radius

1.1 Determining the optimal value of the specific ratio ρ

To determine the value of reduction ratio N and the corresponding spool radius r when motor operates at a condition that provides maximum power output, the first step is to check the value of optimal values of ratio ρ_{optimal}, which is the ratio of spool radius to reduction ratio. With the assumption that the motor is operating under steady-state condition, Equation (1) are used to calculate the value of under the optimal condition described above, which is T=0.5T_s.

$$\rho_{\text{optimal}} = \frac{r}{N} = \frac{0.5T_s}{(km + m_c)g\sin\theta} \tag{1}$$

m: the mass of shigley; m_c: the mass of container; θ: the angle of ramp;
k: the number of shigley; r: the spool radius.

For analysis, we make the following assumption:

m=2 kg; m_c=0.5 kg.

Then, the calculated results are shown in Table 1 below:

Condition	ρ_{optimal}
1	7.57×10^{-4}
2	2.88×10^{-4}
3	1.19×10^{-4}
4	7.12×10^{-5}

Table 1: The values of ρ_{optimal} under 4 conditions.

1.2 Determining the appropriate value of the reduction ratio N

The next step is to determine the gear reduction ratio N_i for each loading condition. Therefore, we take a four-stage compound gear so that there are four available gear reduction ratio.

The method used to determine the gear configuration is to assume that the spool radius for each stage are close enough so that Equation (2) can be satisfied:

$$(\rho_{\text{optimal}})_1\times N_1\approx(\rho_{\text{optimal}})_2\times N_2\approx(\rho_{\text{optimal}})_3\times N_3\approx(\rho_{\text{optimal}})_4\times N_4 \tag{2}$$

With the convention that N is usually an integer, we can have the relationship of reduction ratio at each stage with reference to the results in Table 1:

$$N_1 : N_2 : N_3 : N_4 = 1 : 3 : 7 : 15; \tag{3}$$

With the additional considerations on the availability of gears and distance between gear centers, the actual gear configuration, with reference to Equation (3), is made, as shown in Table 2 and Figure 1:

Stage	Tooth number of driving gear	Tooth number of driving gear
1	10	50
2	10	50
3	20	40
4	20	40

Table 2: The gear configuration at different stages.

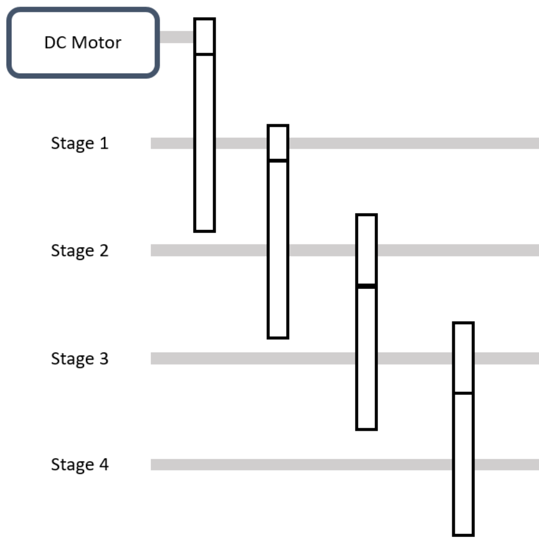

Figure 1: Conguration schematic.

With the configuration presented in Table 2 and Figure 1, Table 3 provides the location of spool used for each condition and the corresponding reduction ratio N_i for that condition:

Condition	Spool Location	Reduction Ration N_i provided
1	Stage 1	5
2	Stage 2	25
3	Stage 3	50
4	Stage 4	100

Table 3: Gear reduction ratio N_i provided when spool at stage i is used to haul loading.

As shown in Table 3, the reduction ratio have the similar ratio relationship with the one presented in Equation (3).

1.3 Determining the appropriate value of the spool radius r

To achieve the actual ratios ρ_i that are close to values mentioned in Table 1, the third step is to determine the desired spool radius at each stage. For easy manufacturing, all spool radius are determined using Equation (4): ($r_0 = 0.001$m)

$$r_i = k \times r_0;\ k \in \mathbb{Z}^+; \tag{4}$$

Therefore, with the value of ρ_{optimal} in Table 1 and Equation (4), the desired spool radius that can make ρ become closest to the optimal ρ_{optimal} are calculated and shown in Table 4:

Spool Location	k	r_i (m)
1	4	0.004
2	7	0.007
3	6	0.006
4	7	0.007

Table 4: Spool radius at stage i.

1.4 Determining the actual value of ρ_i and estimating the run time t_i

The actual values of ρ_i are calculated based on section 1.2 and 1.3. The results are presented in Table 5:

Condition	ρ_i
1	8.00×10^{-4}
2	2.80×10^{-4}
3	1.20×10^{-4}
4	7.00×10^{-5}

Table 5: The values of ρ_i under 4 conditions.

After determining the actual values of ρ_i, we need to estimate the run time for each loading condition. Steady-state analysis is used as a foundation of calculating time. However, we need the determine whether we can use steady-state assumption. The first step is to use Equation (5) to determine $\rho_{\max}$:

$$\rho_{\max} = \frac{T_s}{(km + m_c)g\sin\theta}; \tag{5}$$

Small deviations are relatively inconsequential if the ratio f, which is equal to the ratio of ρ_i to $\rho_{\max}$, is between 0.4 and 0.6. Therefore, the values of f are determined, as shown in Table 6:

Condition	f
1	0.53
2	0.49
3	0.51
4	0.49

Table 6: The values of f under 4 conditions.

From the results presented in Table 6, it is reasonable to use steady-state assumption. Equation (6) can be used to estimate the run time based on steady-state analysis: (L: total ramp length.)

$$t_i = \frac{LT_s}{\omega_0 \rho_i [T_s - (km + m_c)g\rho_i \sin\theta]}; \tag{6}$$

ω_0: no-load angular speed of the motor.

To determine the no-load angular velocity, we can use equation (7) provided below:

$$\omega_0 = \frac{2\pi n_0}{60}; \tag{7}$$

n_0: the number of turns in one minute under no-load condition.

The specification of the motor indicates that n_0 is 9200 rpm. Take L=0.7 m, the results of estimated run time is presented in Table 7:

Condition	Estimate run time t_i (s)
1	1.93
2	5.05
3	12.27
4	20.42

Table 7: The values of t_i under 4 conditions.

A time limit $t_{\max}$= 40 seconds is imposed for each run. Therefore, from the results in Table 7, the gear and spool configuration can satisfy the requirement under theoretical analysis.

2 Design specification on case and shaft

2.1 Determining the distance between the center of each stage.

It is important to put each pair of gears in an appropriate distance to ensure they are meshing. We first define some symbols used for part 2.1 and 2.3 in Table 8:

Symbols	Meaning
d_u	pitch diameter
P	diametral pitch
$\mathcal{N}_u$	tooth number
D_u	dedendum circle diameter
$\Delta x_{i\to j}$	distance between stage i and stage j

Table 8: List of symbols used for analysis in part 2.1 and 2.3.

Therefore, the distance between each stage can be expressed as:

$$\begin{cases} \delta_1 = \Delta x_{\mathrm{M}\to 1} = \Delta x_{1\to 2} = \dfrac{d_{10}}{2} + \dfrac{d_{50}}{2}; \\ \delta_2 = \Delta x_{2\to 3} = \Delta x_{3\to 4} = \dfrac{d_{20}}{2} + \dfrac{d_{40}}{2}; \end{cases} \tag{8}$$

Also, the diametral pitch can be expressed as:

$$P = \frac{\mathcal{N}_u}{d_u}; \tag{9}$$

Because P specifies the tooth spacing along the pitch circle of each gear, P must be the same for the gears to work together. Therefore, from Equations (8) and (9), we can rewrite the simultaneous equations as system of equations (10):

$$\begin{cases} \delta_1 P = \dfrac{\mathcal{N}_{10}}{2} + \dfrac{\mathcal{N}_{50}}{2}; \\ \delta_2 P' = \dfrac{\mathcal{N}_{20}}{2} + \dfrac{\mathcal{N}_{40}}{2}; \end{cases} \tag{10}$$

Based on the tooth number provided for the available gears, we can get:

$$\delta_1 P = \delta_2 P = 30; \tag{11}$$

Therefore, we can get:

$$\Delta x = \Delta x_{\mathrm{M}\to 1} = \Delta x_{1\to 2} = \Delta x_{2\to 3} = \Delta x_{3\to 4}; \tag{12}$$

From Equation (12), we can know that spacing between each stage must be equal to ensure the proper operation of gear system. Also, standard tooth system uses $1.25/P$ as the depth of teeth below pitch circle. Therefore, we can acquire the following relationship:

$$d_u = 2 \times \frac{1.25}{P} + D_u; \tag{13}$$

Also, we can use the following geometric configuration to express d_i in terms of Δx:

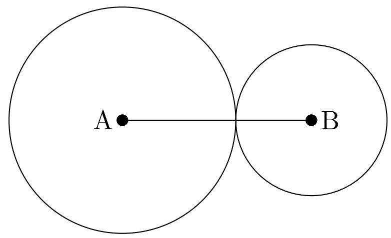

Figure 2: Geometric configuration for pitch diameter. (Not in scale, $|AB| = \Delta x$)

From Equation (8) and Figure 2, with same diametral pitch P, we can express the d_{50} as:

$$d_{50} = \frac{5}{3}\Delta x; \tag{14}$$

With equation (9), (13) and (14), we can have the following expression for a 50 tooth number gear:

$$\frac{5}{3}\Delta x = 0.05(\frac{5}{3}\Delta x) + D_{50}; \tag{15}$$

From measurement, we can determine that D_{50} is equal to 1.85 inch. Hence, the value of Δx should be:

$$\underline{\underline{\Delta x = \frac{111}{95}\,\text{inch} \approx 1.1684\,\text{inch};}}$$

For easy manufacturing, we use 1.17 ± 0.005 inch as the distance between each two stages.

2.2 Determining the specification and configuration related to shafts.

For efficiency in manufacturing, two standard size shafts are chosen to both install gears and spools and transmit power. The properties of shafts and the usage of shafts are provided in Table 9:

Shaft diameter q (in)	Applied Stage(s)	Material	Young's Modulus E (ksi)	Yield Strength S_y (ksi)
$\frac{3}{16}$	1	AISI 4130 Steel	29700	63.1
$\frac{1}{4}$	2/3/4			

Table 9: The properties of available shafts.

Then, in order to avoid substantial deflection at the location of gear installed at each stage, we just try to make the shafts in an appropriate length that is not too long. Therefore, we choose 7 inch as the total effective length $\mathcal{L}$ (the length from the center of left support to the center of right support) and 7.5 inch as the total actual length for each shaft. The other configuration related to shafts are listed below:
(i) the case fixes the shaft at left end, right end and the point 4.375 inches away from the left end;
(ii) the distance between two adjacent pairs of meshing gears is set to be 0.875 inch;
(iii) The center of spool at each stage is 5.6875 inches away from the center of left end;

In the next subsection, the stresses and deflection analysis are made to determine if this configuration is statically proper.

2.3 Determining if the design in part 2.2 is appropriate.

It is important to determine if our case can provide enough support to avoid too much stresses and deflections on the shaft. Therefore, we determine the value of maximum shear stress $\tau_{\max}$ under MSS criteria and the value of deflection δy at the location of gear installed. Our design is aiming to:
(a) The deflection δy at the location of gear installed for each stage should be less than 0.005 inch;
(b) Meet a safety factor of yield strength n_y for at least 3 and a safety factor of deflection n_d for at least 2;

Before the analysis, the following assumptions are made:
(i) The torque on the driving gear is entirely transmitted to the spool; (ii) Friction can be neglected;
(iii) Cable around the spool is tight; (iv) Weight of shafts, gears, fixtures can be ignored during analysis;

Therefore, the free-body diagram for each stage will be:

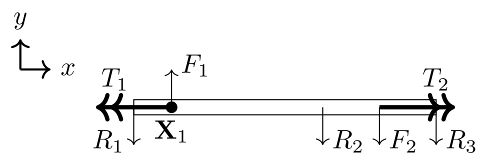

Figure 3: Free body diagram for the shaft at stage 1. ($\mathbf{X}_1$ is the location of gear with $\mathcal{N} = 50$)

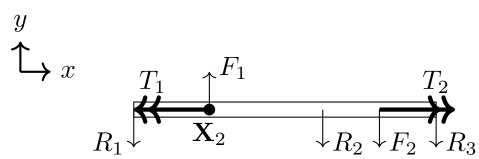

Figure 4: Free body diagram for the shaft at stage 2. ($\mathbf{X}_2$ is the location of gear with $\mathcal{N} = 50$)

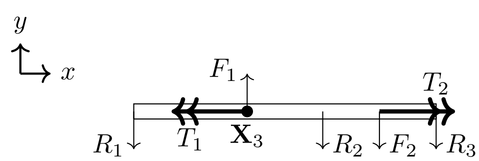

Figure 5: Free body diagram for the shaft at stage 3. ($\mathbf{X}_3$ is the location of gear with $\mathcal{N} = 40$)

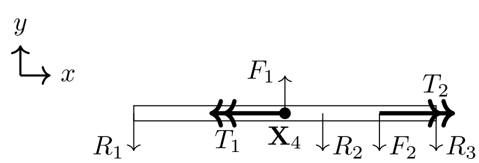

Figure 6: Free body diagram for the shaft at stage 4. ($\mathbf{X}_4$ is the location of gear with $\mathcal{N} = 40$)

The expression for F_1. F_2, T_1 and T_2 can be determined from the basic physics laws:

$$T_1 = T_2 = r_i(km + m_c)g\sin\theta; \tag{16}$$

$$F_1 = \frac{r_i(km + m_c)g\sin\theta}{d_u}; \tag{17}$$

$$F_2 = (km + m_c)g\sin\theta; \tag{18}$$

In order to determine the stresses and deflection, the reaction forces R_1, R_2 and R_3 should be determined first. The effect of combined loading on the shaft is resolved by superposition method. For each stage, the loading can be separated into three component parts, as shown in Figure 7:

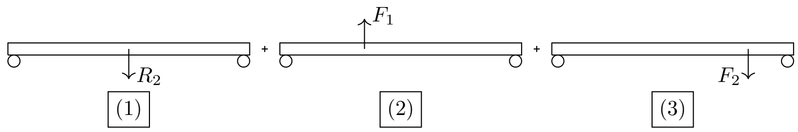

Figure 7: Three components under method of superposition. (Not in scale for the location of F_1)

Let the distance between the left end of the effective length of the shaft and the centerline of driving gear installed at stage i is λ_i and the deflection from each component in Figure 7 as ν. Therefore, we can acquire:

$$\nu_1(x) = -\frac{R_2 0.375\mathcal{L}}{6EI\mathcal{L}}(\mathcal{L}^2 - 0.375L^2 - x^2) = -\frac{R_2\mathcal{L}}{16EI\mathcal{L}}(\frac{55}{64}\mathcal{L}^2 - x^2); 0 < x \leq 0.625\mathcal{L}; \tag{19}$$

$$\nu_2(x) = \frac{F_1\lambda_i x}{6EI\mathcal{L}}(\mathcal{L}^2 - {\lambda_i}^2 - x^2);\ \lambda_i \leq x < \mathcal{L}; \tag{20}$$

$$\nu_3(x) = -\frac{0.1875F_2\mathcal{L}x}{6EI\mathcal{L}}(\mathcal{L}^2 - (0.1875L)^2 - x^2) = -\frac{F_2\mathcal{L}x}{32EI\mathcal{L}}(\frac{247}{256}\mathcal{L}^2 - x^2);\ 0 < x \leq 0.8125\mathcal{L}; \tag{21}$$

Because the shaft is supported by the frame structure at $x = \frac{5}{8}\mathcal{L}$, we can assume:

$$\nu_1(\frac{5}{8}\mathcal{L}) + \nu_2(\frac{5}{8}\mathcal{L}) + \nu_3(\frac{5}{8}\mathcal{L}) = 0; \tag{22}$$

Therefore, by using Equation from (19) to (22), we can get Table 10 with the values of R_2 if a load condition is applied at its corresponding stage i:

Condition	Spool Location	R_2 (lbf)
1	Stage 1	-1.0454
2	Stage 2	-2.0856
3	Stage 3	-4.0233
4	Stage 4	-4.9144

Table 10: The values of theoretical R_2.

Then, for equilibrium, the forces and moments acting on each shaft should be balanced as:

$$\sum F = F_1 - F_2 - R_1 - R_2 - R_3 = 0; \tag{23}$$

$$\sum M_1 = F_1 \times \lambda_i - F_2 \times 0.8125\mathcal{L} - R_2 \times 0.625\mathcal{L} - R_3 \times \mathcal{L} = 0; \tag{24}$$

With Equation (23) and (24) and the values of R_2 in Table 10, we can get:

Condition	Spool Location	R_1 (lbf)	R_3 (lbf)
1	Stage 1	0.3050	-0.8408
2	Stage 2	0.9037	-2.3777
3	Stage 3	1.5302	-5.9076
4	Stage 4	1.6255	-9.6803

Table 11: The values of theoretical R_1 and R_3.

After finding the reaction forces, we can express the load $q_i(x)$ on the beam when load condition i is applied as:

$$q_1(x) = -R_1\langle x\rangle^{-1} + F_1\langle x-\lambda_1\rangle^{-1} - R_2\langle x-0.625\mathcal{L}\rangle^{-1} - F_2\langle x-0.8125\mathcal{L}\rangle^{-1}; \tag{25}$$

$$q_2(x) = -R_1\langle x\rangle^{-1} + F_1\langle x-\lambda_2\rangle^{-1} - R_2\langle x-0.625\mathcal{L}\rangle^{-1} - F_2\langle x-0.8125\mathcal{L}\rangle^{-1}; \tag{26}$$

$$q_3(x) = -R_1\langle x\rangle^{-1} + F_1\langle x-\lambda_3\rangle^{-1} - R_2\langle x-0.625\mathcal{L}\rangle^{-1} - F_2\langle x-0.8125\mathcal{L}\rangle^{-1}; \tag{27}$$

$$q_4(x) = -R_1\langle x\rangle^{-1} + F_1\langle x-\lambda_4\rangle^{-1} - R_2\langle x-0.625\mathcal{L}\rangle^{-1} - F_2\langle x-0.8125\mathcal{L}\rangle^{-1}; \tag{28}$$

By taking the integration, we can acquire the expression for $M(x)$ and $\delta y(x)$:

$$M_1(x) = -R_1\langle x\rangle^{1} + F_1\langle x-\lambda_1\rangle^{1} - R_1\langle x-0.625\mathcal{L}\rangle^{1} - F_2\langle x-0.8125\mathcal{L}\rangle^{1}; \tag{29}$$

$$M_2(x) = -R_1\langle x\rangle^{1} + F_1\langle x-\lambda_2\rangle^{1} - R_2\langle x-0.625\mathcal{L}\rangle^{1} - F_2\langle x-0.8125\mathcal{L}\rangle^{1}; \tag{30}$$

$$M_3(x) = -R_1\langle x\rangle^{1} + F_1\langle x-\lambda_3\rangle^{1} - R_2\langle x-0.625\mathcal{L}\rangle^{1} - F_2\langle x-0.8125\mathcal{L}\rangle^{1}; \tag{31}$$

$$M_4(x) = -R_1\langle x\rangle^{1} + F_1\langle x-\lambda_4\rangle^{1} - R_2\langle x-0.625\mathcal{L}\rangle^{1} - F_2\langle x-0.8125\mathcal{L}\rangle^{1}; \tag{32}$$

From the previous calculation, we can determine:
(1) the worst points and its corresponding maximum shear stresses;
(2) the deflection at the location of gear installed;
In order to determine maximum shear stress $\tau_{\max}$, we need to determine the worst location. First, by using Equation (33), we can find the expression for σ_{xx}:

$$\sigma_{xx}(x) = \frac{32M(x)}{\pi q^3}; \tag{33}$$

Also, due to the existence of torque in certain region of the shafts, torsional stress τ_{xy} should be be determined by using Equation (34):

$$\tau_{xy} = \frac{16T(x)}{\pi q^3}; \tag{34}$$

Therefore, we use the stresses vs. location diagram to determine the worst point, as shown from Figure 8 to 15:

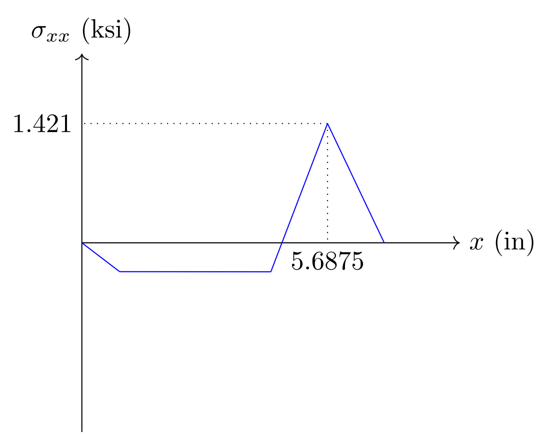

Figure 8: σ_{xx} vs. location diagram for condition 1.

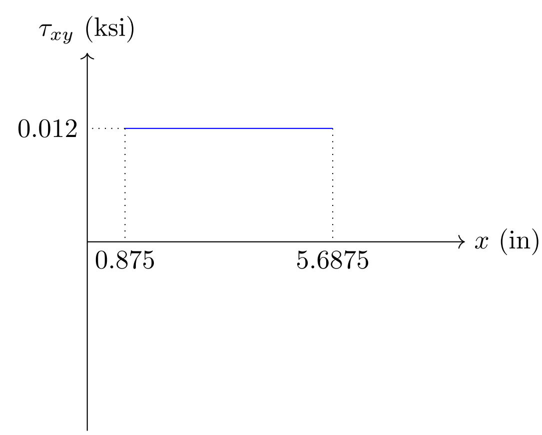

Figure 9: τ_{xy} vs. location diagram for condition 1.

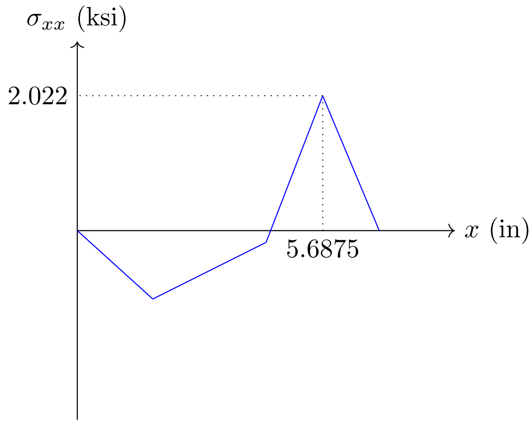

Figure 10: σ_{xx} vs. location diagram for condition 2.

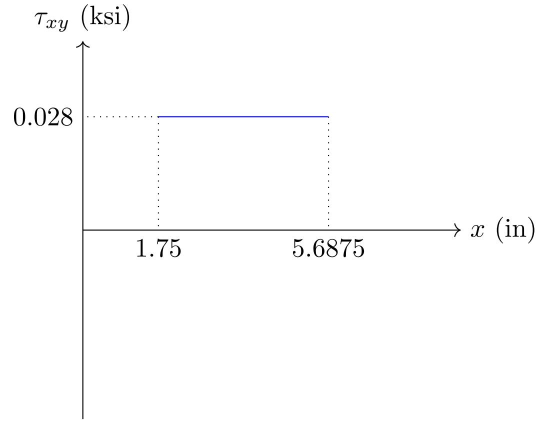

Figure 11: τ_{xy} vs. location diagram for condition 2.

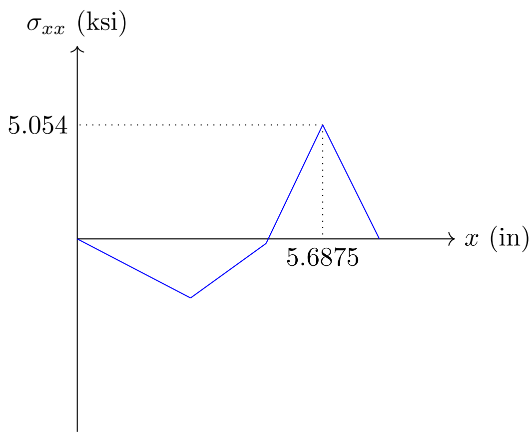

Figure 12: σ_{xx} vs. location diagram for condition 3.

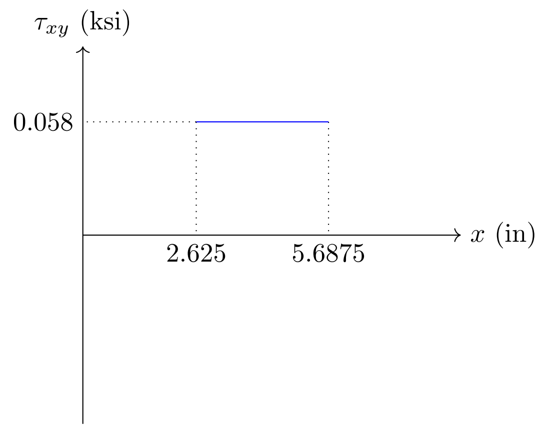

Figure 13: τ_{xy} vs. location diagram for condition 3.

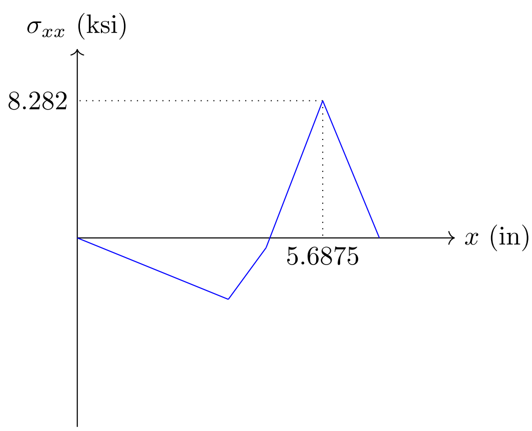

Figure 14: σ_{xx} vs. location diagram for condition 4.

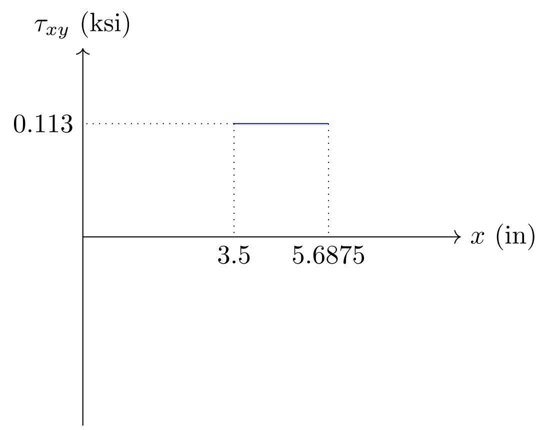

Figure 15: τ_{xy} vs. location diagram for condition 4.

Therefore, the worst points and the corresponding stresses at these points are as shown in Table 12:

Condition	The location of worst point (in)	σ_{xx} (ksi)	τ_{xy} (ksi)
1	5.6875^-	1.421	0.012
2	5.6875^-	2.022	0.028
3	5.6875^-	5.054	0.058
4	5.6875^-	8.282	0.113

Table 12: The maximum values of theoretical σ_{xx} and τ_{xy}.

The values of $\tau_{\max}$ can then be determined by using Equation (35) and (36). The results are shown in Table 13: ($\sigma_3 = 0$ due to plane stress)

$$\sigma_1, \sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{(\frac{\sigma_{xx} - \sigma_{yy}}{2})^2 + (\tau_{xy})^2}; \tag{35}$$

$$\tau_{\max} = \max[\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}]; \tag{36}$$

Condition	$\tau_{\max}$ (ksi)
1	1.422
2	2.023
3	5.056
4	8.285

Table 13: The values of theoretical $\tau_{\max}$.

Also, we can determine the value of δy by using Equation (37). The results are shown in Table 14:

$$\delta y = \nu_1(\lambda_i) + \nu_2(\lambda_i) + \nu_3(\lambda_i); \tag{37}$$

Condition	δy (in)
1	2.14×10^{-4}
2	4.47×10^{-4}
3	1.10×10^{-3}
4	1.03×10^{-3}

Table 14: The values of theoretical δy.

After acquire Table 13 and 14, we can finally calculate the values of n_y and n_d by using Equation (38) and (39):

$$n_y = \frac{S_y}{2\tau_{\max}}; \tag{38}$$

$$n_d = \frac{(\delta y)_{\text{limit}}}{(\delta y)_{\text{gear}}}; \tag{39}$$

The results are shown in Table 15:

Condition	n_y	n_d
1	22.19	23.36
2	15.60	11.19
3	6.24	4.55
4	3.81	4.85

Table 15: The values of n_y and n_d.

As shown in Table 15, our design satisfies out expectation on stresses and deflection.