1/ Introduction

In this lab, I will perform engineering computation on MATLAB to numerically integrate the differential equations generated from a bond graph model of a system. This process of integrating the time dependent equations is called "simulation" and it is a powerful method to analyze linear and non-linear systems. The system that is simulated in this lab is a simple model of a two degree of freedom quarter car suspension in which. The extra degree of freedom captures the simplified extension/compression of the tire between the wheel hub and the ground. The vehicle will be run through a pothole to capture and simulate the system responses. The model consists of 5 components: two masses, two springs, and one damper. The assumptions that are made in this analysis are: the spring between the unsprung mass and the ground represents the stiffness of the tire and the power flowing from the system to the ground is positive. Overall, the goal of this lab is to convert the given differential equations into a computer function that evaluates the equations at any given point in time and numerically integrate differential equations with MATLAB ode 45.

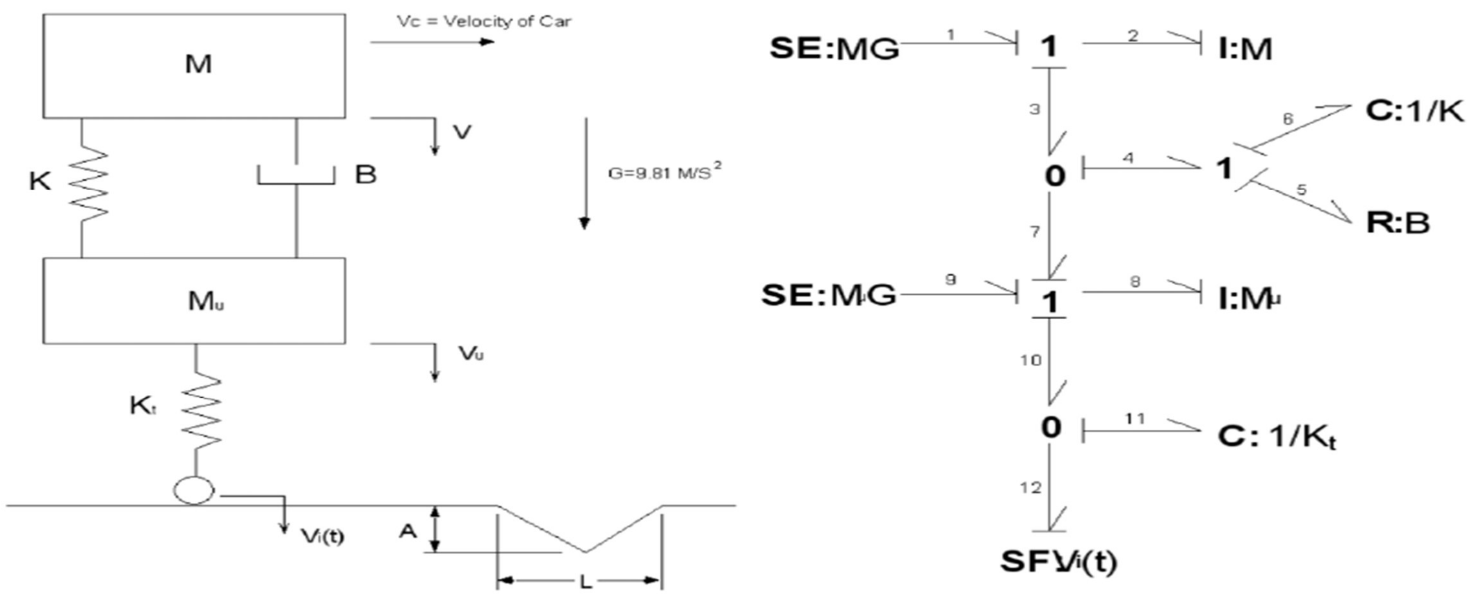


Fig 1: Mechanical Schematic of Two Degree of Freedom Quarter Car Model (left) and equivalent bond graph (right)

The state equations for the linearized system are:

$$\dot{p}_2 = -\frac{B}{M}p_2 - Kq_6 + \frac{B}{M_u}p_8 + Mg$$

$$\dot{q}_6 = \frac{1}{M}p_2 - \frac{1}{M_u}p_8$$

$$\dot{p}_8 = \frac{B}{M}p_2 + Kq_6 - \frac{B}{M_u}p_8 - K_t q_{11} + M_u g$$

$$\dot{q}_{11} = \frac{1}{M_u}p_8 - v_i$$

There are 4 first order ODEs to describe the dynamics of the system change with respect to time. These equations are given in explicit form.

Vertical momentum of the sprung mass: p_2 [kg m/s]

Displacement between the sprung and unsprung mass: q_6 [m]

Vertical momentum of the unsprung mass: p_8 [kg m/s]

Displacement between the unsprung mass and the ground: q_{11} [m]

Vertical displacement of the road: y [m] with $\dot{y}_i = v_i$

2/ Calculations

This system has five state variables, so there are five initial conditions. The initial momentum of the two masses were set to zero. The initial condition of the displacement q_6 and q_{11} reflected the equilibrium state of the springs. Initial condition for q_6 and q_{11} were solved by setting their momentum, time derivative, time derivatives of the displacements and the road velocity input to zero. After doing these steps the initial conditions were found:

$$\dot{p_2} = 0 = \frac{-B}{M}p_2 - Kq_6 + \frac{B}{M_u}p_8 + Mg\ (eqn\ 1)$$

$$\dot{q_6} = 0 = \frac{1}{M}p_2 - \frac{1}{M_u}p_8\ (eqn\ 2)$$

$$\dot{p_8} = 0 = \frac{B}{M}p_2 + Kq_6 - \frac{B}{M_u}p_8 - K_t q_{11} + M_u g\ (eqn\ 3)$$

$$\dot{q_{11}} = 0 = \frac{1}{M_u}p_8 - v_i\ (eqn\ 4)$$

When $v_i = 0,\ consequently\ p_8 = 0, then\ p_2 = 0.$ The substitute $p_8 = 0,\ p_2 = 0$ back into equation 1 and equation 3 we get the following initial condition:

$$p_2 = 0$$

$$p_8 = 0$$

$$q_6 = \frac{M * g}{K}$$

$$q_{11} = \frac{(M_u + M) * g}{K_t}$$

$$y = 0$$

3/ Systems Parameter

The following are constant parameters in the system:

- Sprung mass: $M = 250$ kg
- Ratio of the sprung and unsprung masses: $M/M_u = 5$
- Acceleration due to gravity: $g = 9.81$ m/s^2
- Natural frequency of the sprung mass: $f_n = 1$ Hz
- Damping ratio of the sprung mass: $\zeta = 0.3$
- Ratio of the tire and suspension spring stiffnesses: $K_t/K = 10$
- Forward speed of the car: $V_c = 10$ m/s
- Width of the pothole: L = 1.2 m
- Depth of the pothole: A = 0.08 m

4/ Time Parameter

Natural Frequency $\omega_n = \sqrt{\frac{K}{M}}$

Natural Frequency in Hz $f_n = \frac{\omega_n}{2\pi}$

Damping ratio $\xi = \frac{B}{2\sqrt{MK}}$

To find the time period, we chose the larger values between the natural frequency of the sprung mass and the unsprung mass which ever is larger. After that the time period is determined:

$$T_{period} = \max\left(\frac{2\pi}{\sqrt{\frac{K_t}{M_u}}}, \frac{2\pi}{\sqrt{\frac{K}{M}}}\right)$$

To show at least 10 data points per oscillation for shortest duration period we chose dt as shown below:

$$\mathrm{dt} = \frac{T_{period}}{30}$$

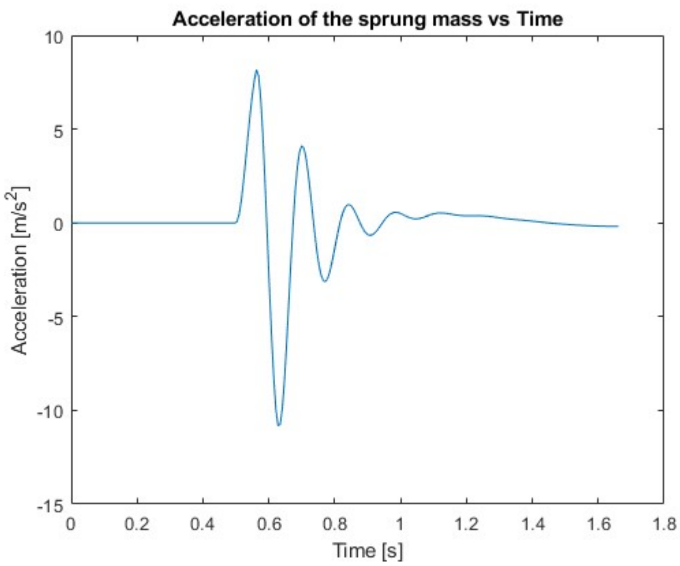
The time constant τ is derived from the formula given in the handout $e^{-\xi\omega_n t} = e^{-t/\tau}$. Therefore $\tau = 1/(\xi\omega_n)$

The final point in the time interval is $t_{final} = 0.6 + 2 * \tau$ to capture the whole plots in a single display window.

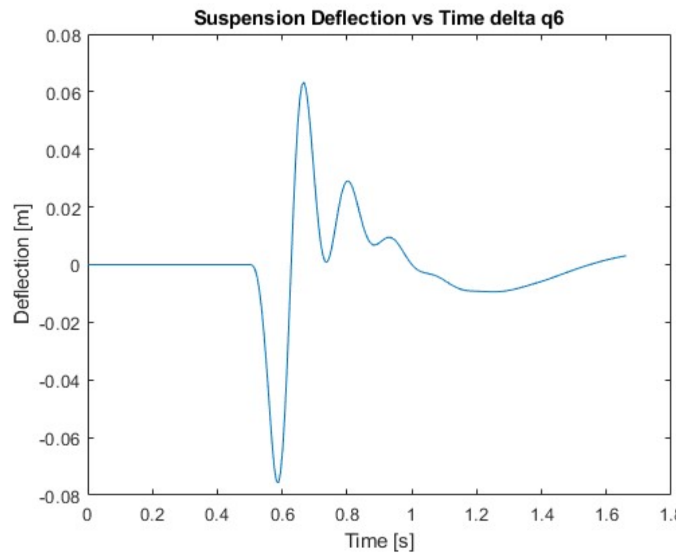
Finally, the number of steps is calculated as follow:

$$N_{step} = round\left(\frac{t_{final}}{dt}\right) + 1$$

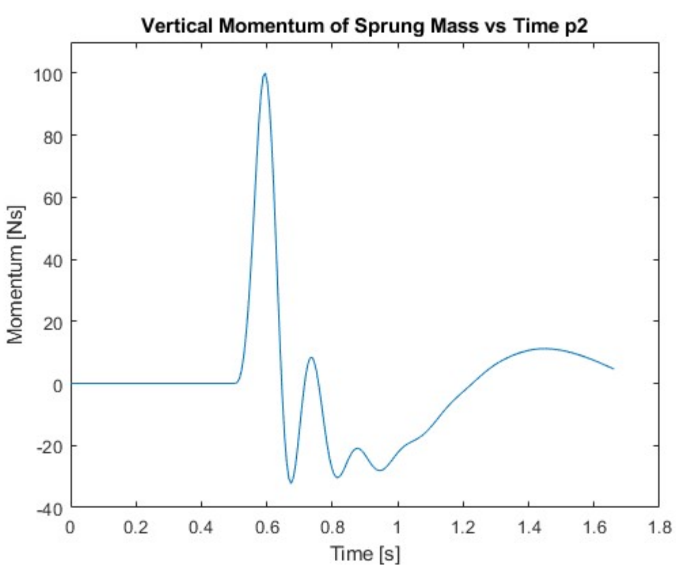
5/ Simulations



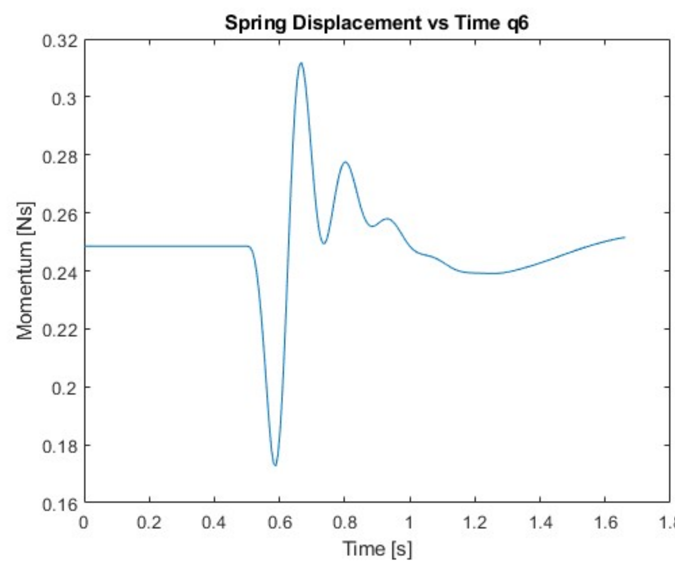
2a/ Acceleration vs time graph



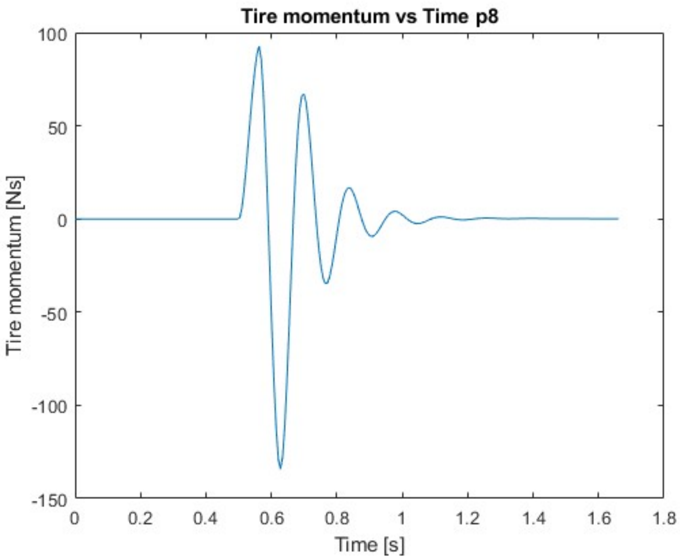
2b/ Suspension deflection vs time graph



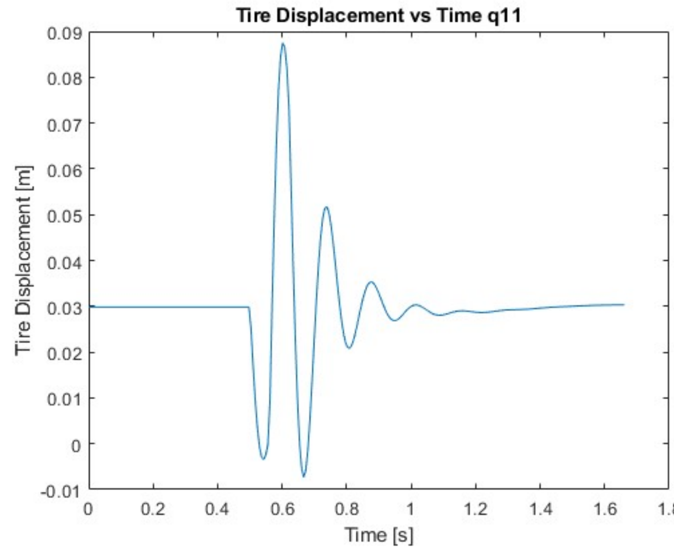
2c/ Sprung mass vertical Momentum vs time graph



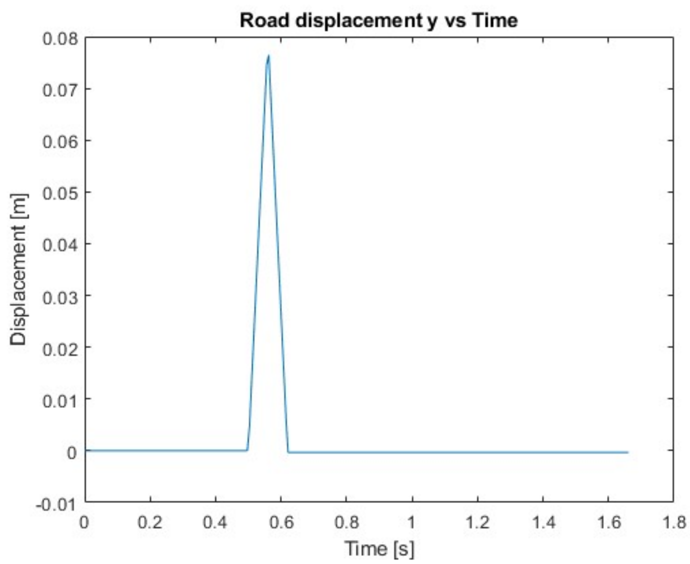
2d/ Spring displacement vs time graph



2e/ Tire momentum vs time graph

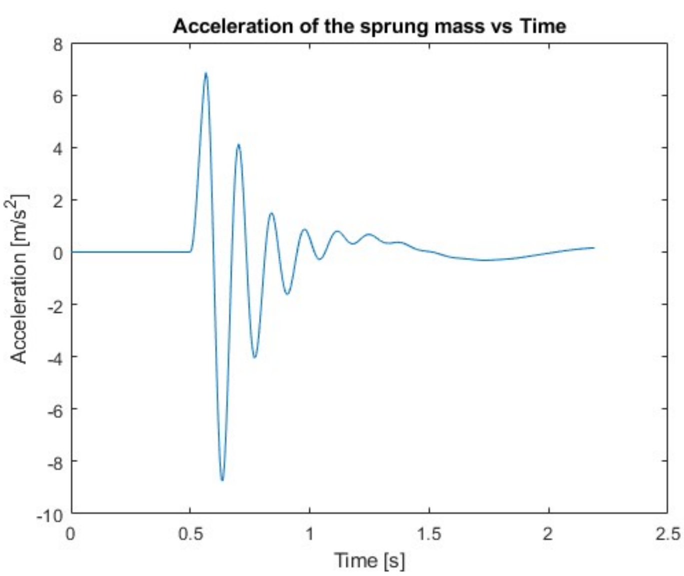


2f/ Tire displacement vs time graph

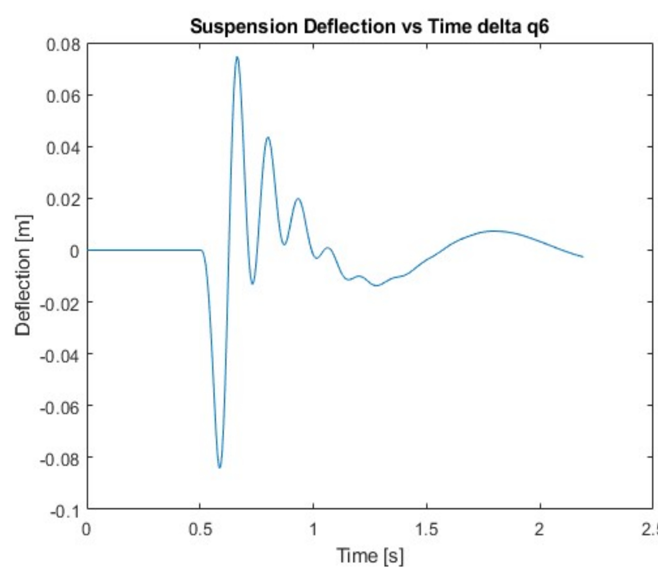


2g/ Road displacement vs time graph

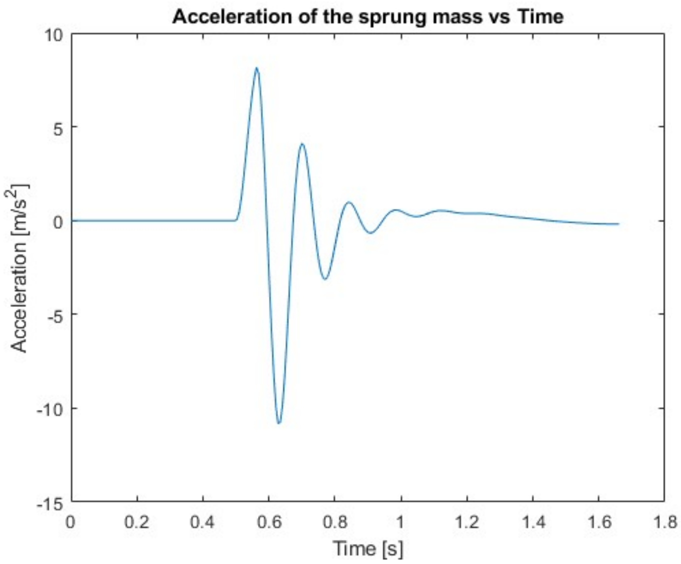
Fig 2 a-g: Suspension system going over a pothole bump in the road



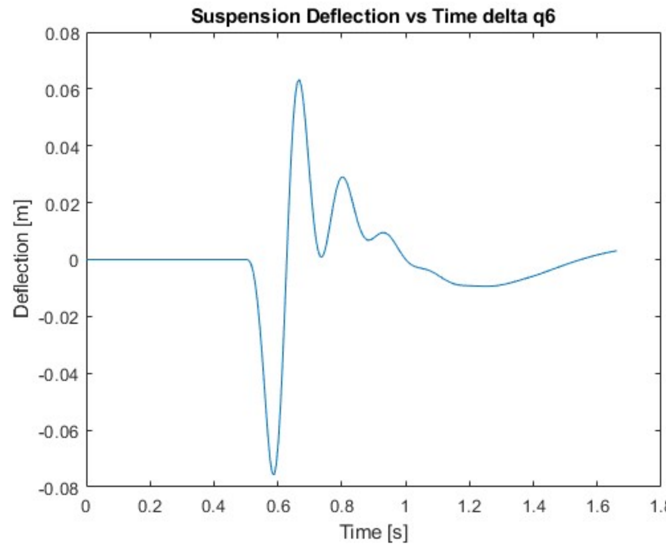
3a/ Acceleration of the sprung mass with $\zeta = 0.2$



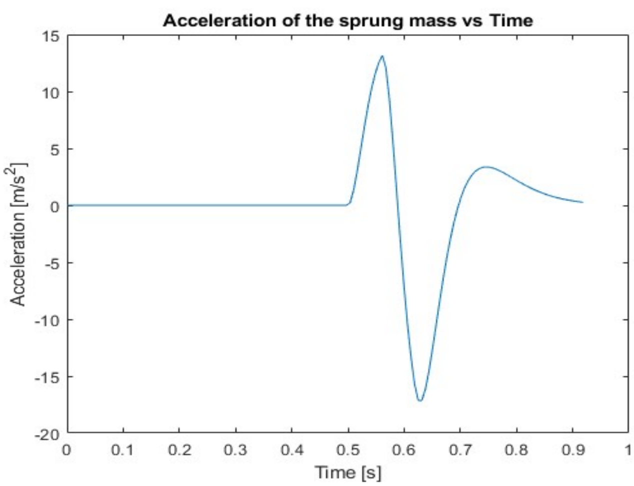
3b/ Suspension deflection with $\zeta = 0.2$



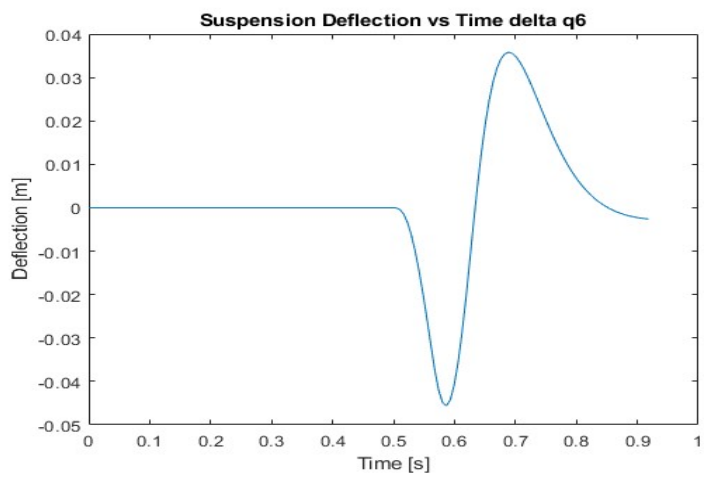
3c/ Acceleration of the sprung mass with ζ = 0.3



3d/ Suspension deflection with ζ = 0.3



3e/ Acceleration of the sprung mass with ζ = 1



3f/ Suspension deflection with ζ = 1

Fig 3 a-f: Acceleration and Suspension Deflection of the system with different damping ratio

Explanation increasing ratio: When increasing the damping ratio from 0.3 to 1, the system is expected to return to equilibrium after passing the pothole as quickly as possible without oscillating. This case is also called “critically damp” where the damping ratio ζ = 1. In addition, the period of the sprung mass acceleration and the period of the suspension deflection also increased when the damping ratio increases. However, the magnitude of the sprung mass acceleration and magnitude of suspension deflection decreases when damping ratio increases.

Explanation decreasing damping ratio: When decreasing the damping ratio from 0.3 to 0.2, the system oscillates sinusoidally more intensively after passing the pothole. In addition, the period of the sprung mass acceleration and the period of the suspension deflection also decrease when the damping ratio decrease. However, the magnitude of the sprung mass acceleration and magnitude of suspension deflection increases when damping ratio decreases.

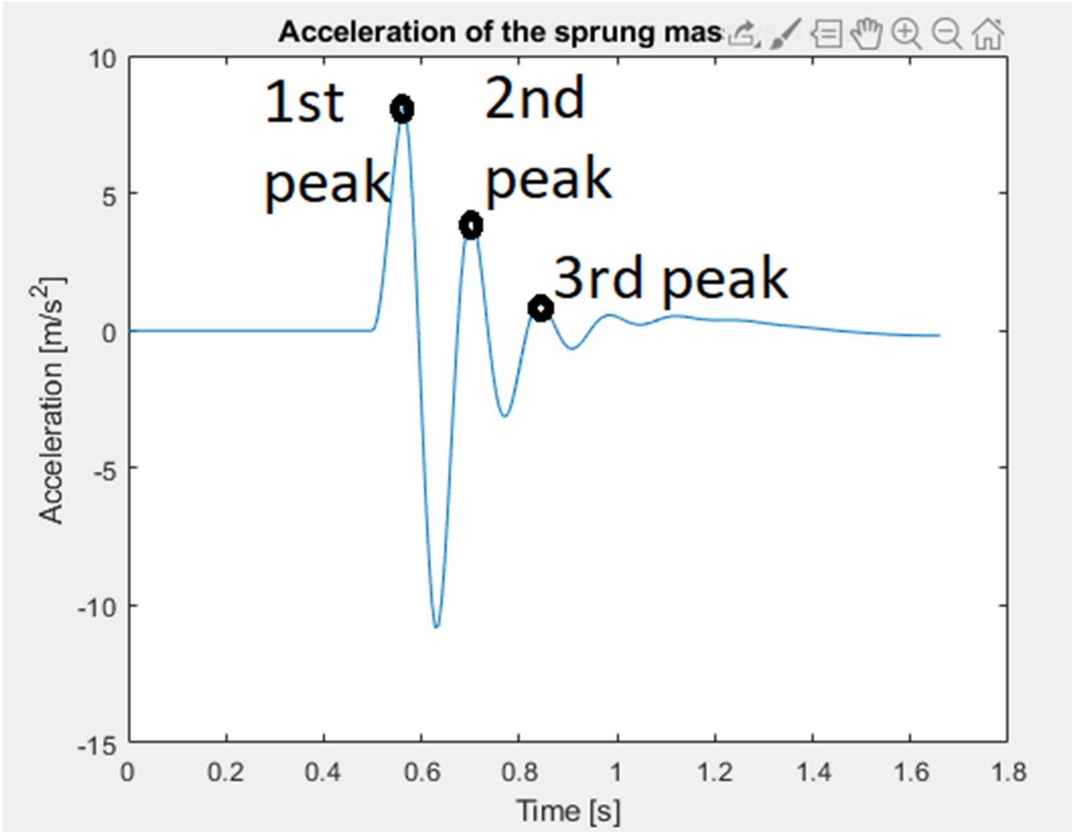


Fig 4: The peak values of the acceleration vs time plot

From the plot of the acceleration sprung mass, we identify three peaks as marked in figure 2 and find their corresponding coordinates using matlab cursor tool. The coordinates found was peak1 (0.564, 8.162), peak2 (0.6997, 4.115), peak3 (0.8436, 0.9793). From the coordinates of the peaks, the system vibration period and vibration frequency of the oscillation were found to be Vibration Period = 0.1398 (s) and Vibration Frequency = 7.153 (HZ). The detail calculating steps were shown below.

$$1st\ Period = 0.6997 - 0.564 = 0.1357\ (s)$$
$$2nd\ Period = 0.8436 - 0.6997 = 0.1439\ (s)$$
$$Average\ Vibration\ Period = (0.1357 + \ 0.1439) * 0.5 = 0.1398\ (s)$$
$$Average\ Vibration\ Frequency = 1/(0.1398) = 7.153\ (Hz)$$

6/ Contributions

Team members were Vinh and Koosha. Both members worked together to ensure the lab report was completed effectively and on time. The report was done by both members. Koosha worked on the system states, system parameters, time parameters, and initial conditions subsections of the report. Vinh completed the rest of the report, including explanation of the results and the plots from Matlab. Both members also worked on the code. Vinh wrote the main file. Koosha wrote the component files (state derivative, input, and output function files). However, due to miscommunication, both members wrote the component files so we ended up with similar codes. Vinh's code was included in the final report because he was working on the main file and it was easier for him to test and run the simulation.

7/ Code

eval_quarter_car_rhs.m – Evaluate the right hand side

```
function xdot = eval_quarter_car_rhs(t, x, w, c)
% EVAL_QUARTER_CAR_RHS - Returns the time derivative of the states, i.e.
% evaluates the right hand side of the explicit ordinary differential
% equations for the 2 DoF quarter car model.
%
% Syntax: xdot = eval_quarter_car_rhs(t, x, w, c)
%
% Inputs:
%   t - Scalar value of time, size 1x1.
%   x - State vector at time t, size 1x5
%   w - Anonymous function, w(t, x, c), that returns the input vector
%       at time t, size 1x1.
%   c - Constant parameter structure with 10 items: M,B,K,Mu,Kt,g,Vc,L,A,ti
% Outputs:
%   xdot - Time derivative of the states at time t, size 1x5.

% unpack the states into useful variable names
p2 = x(1);   % vertical momentum of sprung mass [kg m/s]
q6 = x(2);   % displacement betweeen the sprung and unsprung mass [m]
p8 = x(3);   % vertical momentum of the unsprung mass [kg m/s]
q11 = x(4);  % displacement between the unsprung mass and the ground [m]
y = x(5);    % vertical displacement of the road [m]

% unpack the parameters into useful variable names
M = c(1);    % sprung mass
B = c(2);    % damping coeff
K = c(3);    % spring constant N/m
Mu = c(4);   % unspring mass kg
Kt = c(5);   %tire spring constant N/m
g = c(6);    %gravitational acceleration m/s^2

% evaluate the input function
r = w(t, x, c);
vin = r(1);

% calculate the derivatives of the state variables
p2dot = (-B/M)*p2 - (K*q6) + (B/Mu)*p8 + (M*g);   % eqn 1
q6dot = (1/M)*p2 - (1/Mu)*p8;                     % eqn 2
p8dot = (B/M)*p2 + (K*q6) - (B/Mu)*p8 - (Kt*q11) + (Mu*g);  %eqn 3
q11dot = (1/Mu)*(p8) - vin;                         % eqn 4
ydot = vin;                                         % eqn 5

% pack the state derivatives into an 1x5 vector
xdot = [p2dot; q6dot; p8dot; q11dot; ydot];
end
```

eval_pothole_input.m – Generate the pothole in the road

```
function r = eval_pothole_input(t, x, c)
% EVAL_POTHOLE_INPUT - Returns the road pothole velocity at any given time.
%
% Syntax: r = eval_pothole_input(t, x, c)
%
```