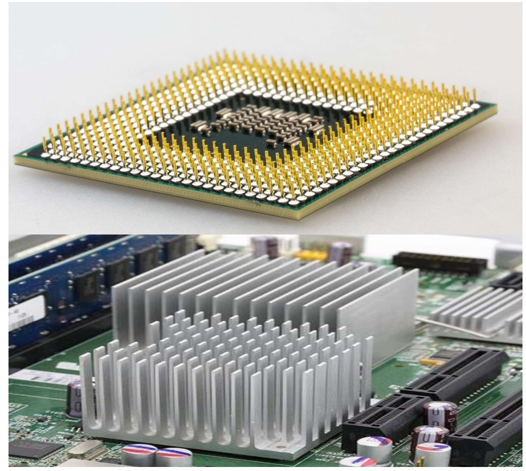
Thermal Analysis of Parallel Plate-Fin Heat Sink On Microprocessor



1/ Introduction:

The processors in electronic devices process data based on the movement of electronic impulses. When the temperature of the processor rises, the computing speed becomes slower. Improper heat dissipation can eventually make the chip crash and burn. With Moore's Law predicting that processor's speed for computers will double about every 18 months, proper cooling has become essential in the electronics industry.

2/ Case Study:

A **silicon chip is 3 mm** thick and is generating heat uniformly at the rate of **6 W/cm^3**. The experimental thermal conductivity of the chip is **K_{chip} = 100 W/m-K**. Assume that end effects can be neglected for the chip. Heat is being dissipated using rectangular straight fins that are located on one surface of the chip, and the other surface of the chip is insulated. There is a contact resistance of **R''_{tc} = 10-4 m^2-K/W** in between the silicon chip and the fins. The fins are **6 cm long and 1 cm thick** and span the entire depth. The spacing between the fins is **2 cm**. The freestream temperature is **20 °C** and the convective heat transfer coefficient is a constant at **50 W/m^2-K.** The fin thermal conductivity values of **K_{fin} = 100 W/m-K.**

<u>**Objective:**</u> Perform numerical conduction analysis on the system and examine the temperature distribution in the domain.

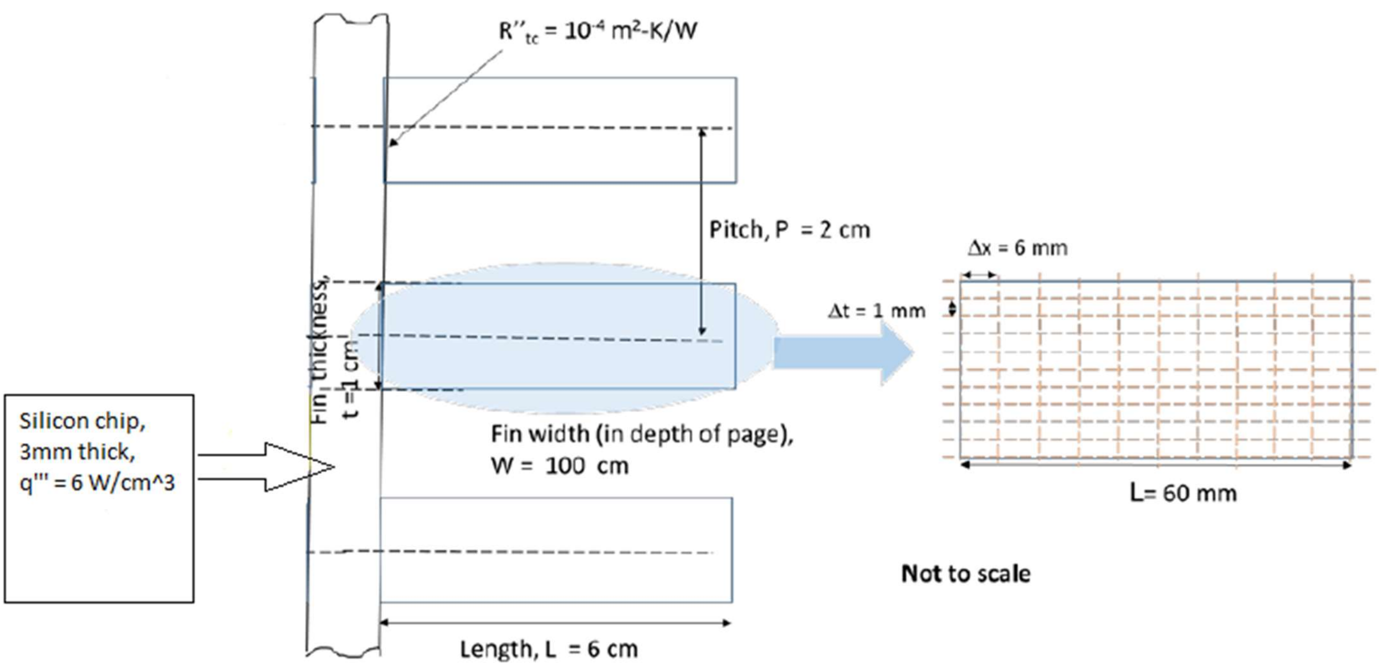


Figure 1: Simplified diagram of processor and heat sink setup.

3/ Result:

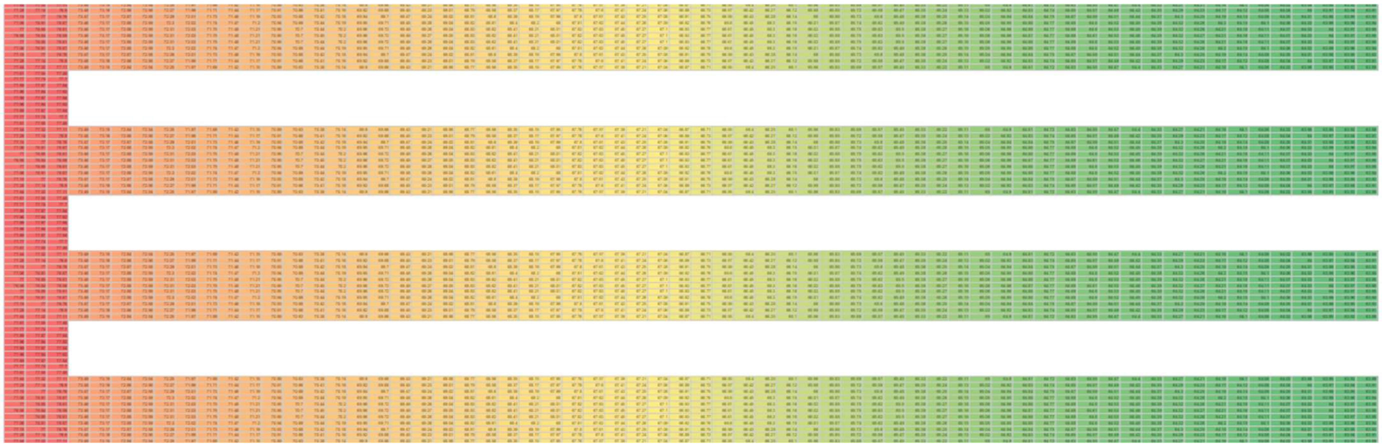
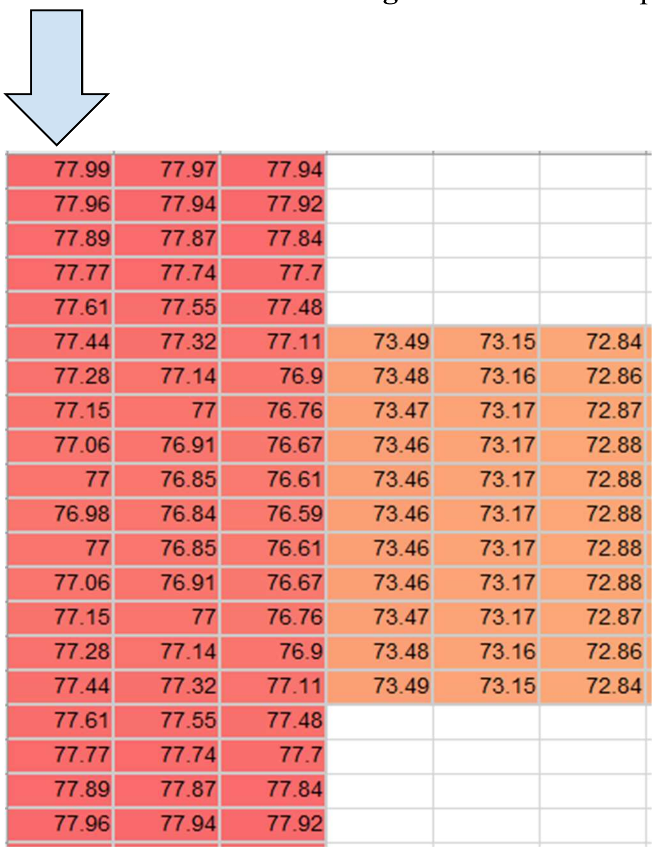
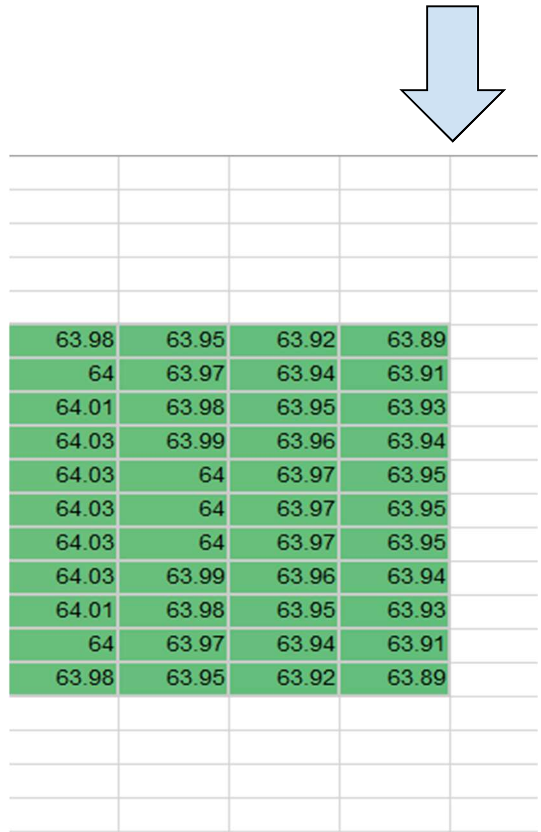


Figure 2: 2D contour plot of the temperature distribution.



77.99	77.97	77.94			
77.96	77.94	77.92			
77.89	77.87	77.84			
77.77	77.74	77.7			
77.61	77.55	77.48			
77.44	77.32	77.11	73.49	73.15	72.84
77.28	77.14	76.9	73.48	73.16	72.86
77.15	77	76.76	73.47	73.17	72.87
77.06	76.91	76.67	73.46	73.17	72.88
77	76.85	76.61	73.46	73.17	72.88
76.98	76.84	76.59	73.46	73.17	72.88
77	76.85	76.61	73.46	73.17	72.88
77.06	76.91	76.67	73.46	73.17	72.88
77.15	77	76.76	73.47	73.17	72.87
77.28	77.14	76.9	73.48	73.16	72.86
77.44	77.32	77.11	73.49	73.15	72.84
77.61	77.55	77.48			
77.77	77.74	77.7			
77.89	77.87	77.84			
77.96	77.94	77.92			

Figure 3: zoom in temp distribution of the chip.



63.98	63.95	63.92	63.89
64	63.97	63.94	63.91
64.01	63.98	63.95	63.93
64.03	63.99	63.96	63.94
64.03	64	63.97	63.95
64.03	64	63.97	63.95
64.03	64	63.97	63.95
64.03	63.99	63.96	63.94
64.01	63.98	63.95	63.93
64	63.97	63.94	63.91
63.98	63.95	63.92	63.89

Figure 4: zoom in temp distribution at the fin tip.

The red temperature represents the hot region and the green represents the cool region of the system. The chip temperature is in the hottest region with $\mathbf{T_{max,chip} = 77.99\ °C}$. If there is no heat sink attached, the surface temperature of the chip will rise instantaneously to **380 °C** and burn the chip in a few seconds. This heat sink system is keeping the chip under **85 °C**, which is the safe range for many computer chips. The efficiency of the heat sink is **87.7 %.** The performance of the system can be improved by using material with higher conductivity and emissivity such as **Ceramic** material. Besides, cooling fans can be used to circulate airflow to remove more heat from the chip.

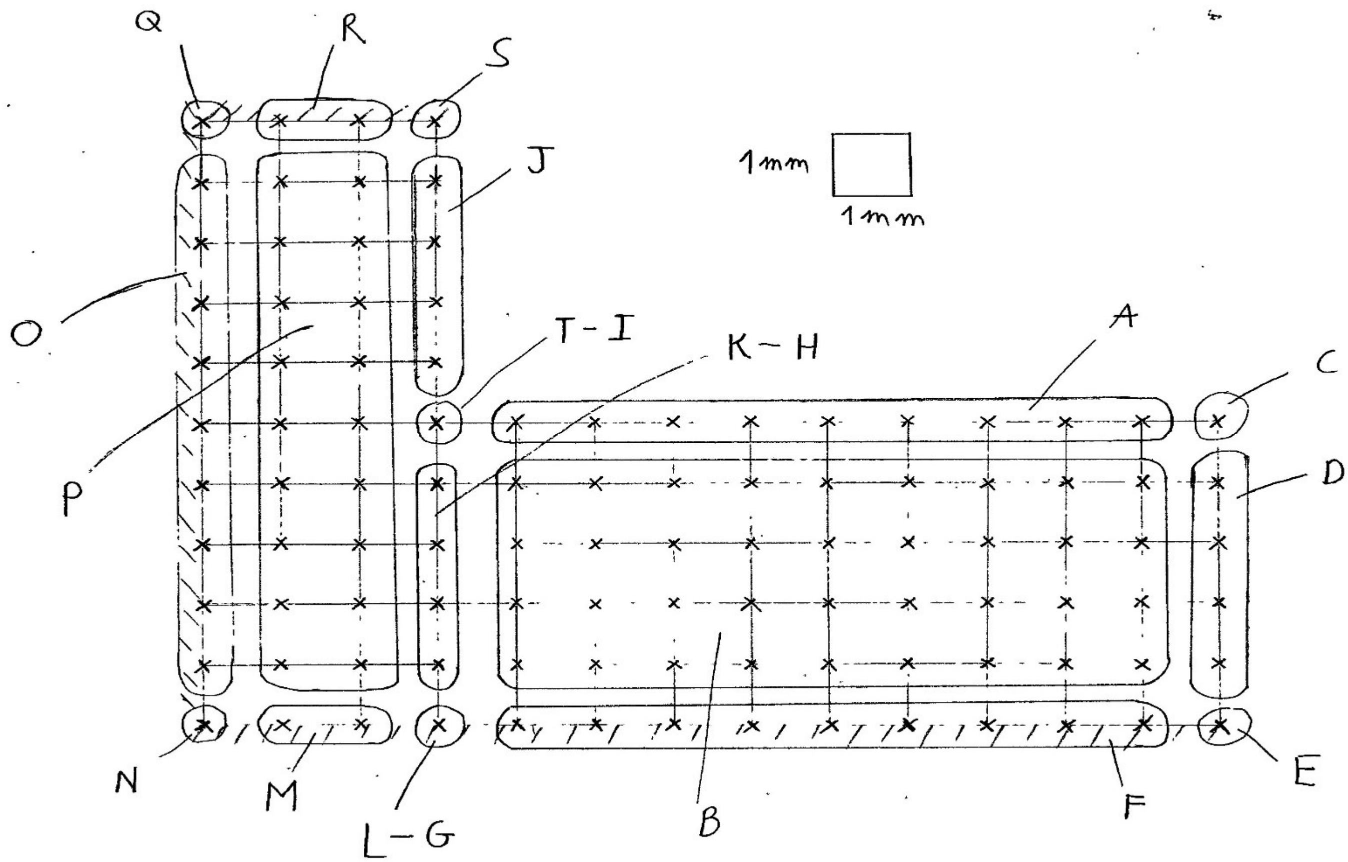


Figure 5: Nodal analysis diagram of the system.

4/ Method and Calculation

Chip temperature without fin

q'''*V = h*A*(T_chip - T_inf)

0.003*6*10^6 = 50*(T_chip - 20)

T_chip = 380 Celcius

System properties

$dx = \dfrac{1}{1000}$ *m*

$dy = \dfrac{1}{1000}$ *m*

$dm = \dfrac{1}{1000}$ *m*

$dn = \dfrac{1}{1000}$ *m*

$R2p_{tc} = 10^{-4}$ *m²-k/W*

$h = 50$ *W/m²-K*

$Tinf = 20$ *C*

$Kchip = 100$ *W/m-K*

$q3p = 6 \cdot 10^{6}$ *w/m³*

$Kfin = 100$ *W/m-k*

Thermal Contact Resistance Calculation

$$R_{tc} = \frac{R2p_{tc}}{dy}$$

$$R_{condchip} = \frac{\frac{dx}{2}}{Kchip \cdot dy}$$

$$R_{condfin} = \frac{\frac{dx}{2}}{Kfin \cdot dy}$$

$$R_{t1} = R_{tc} + R_{condchip} + R_{condfin}$$

$$R_{t2} = R_{tc} + \frac{R_{condchip}}{2} + \frac{R_{condfin}}{2}$$

Fin Temperature Calculations

Node A - Upper Fin Nodes w/convection

$$T_{i,6} \cdot \left[Kfin \cdot \frac{dy}{dx} + Kfin \cdot \frac{dx}{dy} + h \cdot dx \right] = \frac{Kfin}{2} \cdot \frac{dy}{dx} \cdot \left(T_{i-1,6} + T_{i+1,6} \right) + Kfin \cdot \frac{dx}{dy} \cdot T_{i,5} + h \cdot dx \cdot Tinf$$

(for i = 6 to 64)

Node B - Interior Fin Node

$$T_{i,j} \cdot \left[2 \cdot \frac{dy}{dx} + 2 \cdot \frac{dx}{dy} \right] = \frac{dy}{dx} \cdot \left(T_{i-1,j} + T_{i+1,j} \right) + \frac{dx}{dy} \cdot \left(T_{i,j+1} + T_{i,j-1} \right) \quad \text{(for } i = 6 \text{ to } 64), \ j = 2 \text{ to } 5)$$

Node C - Outer Corner Node

$$T_{65,6} \cdot \left[\frac{Kfin}{2} \cdot \left[\frac{dx}{dy} + \frac{dy}{dx} \right] + \frac{h}{2} \cdot \left(dx + dy \right) \right] = \frac{Kfin}{2} \cdot \left[\frac{dy}{dx} \cdot T_{64,6} + \frac{dx}{dy} \cdot T_{65,5} \right] + \frac{h}{2} \cdot Tinf \cdot \left(dy + dx \right)$$

Node D - Outer Fin Convection Nodes

$$T_{65,j} \cdot \left[Kfin \cdot \left[\frac{dx}{dy} + \frac{dy}{dx} \right] + h \cdot dy \right] = \frac{Kfin}{2} \cdot \frac{dx}{dy} \cdot \left(T_{65,j+1} + T_{65,j-1} \right) + Kfin \cdot \frac{dy}{dx} \cdot T_{64,j} + h \cdot dy \cdot Tinf$$

(for j = 2 to 5)

Node E - Bottom Outer Fin Convection Node w/Insulation

$$T_{65,1} \cdot \left[\frac{Kfin}{2} \cdot \frac{dy}{dx} + \frac{Kfin}{2} \cdot \frac{dx}{dy} + h \cdot \frac{dy}{2} \right] = \frac{Kfin}{2} \cdot \frac{dy}{dx} \cdot T_{64,1} + \frac{Kfin}{2} \cdot \frac{dx}{dy} \cdot T_{65,2} + \frac{h}{2} \cdot dy \cdot Tinf$$

Node F - Interior Node w/Bottom Insulation

$$T_{i,1} \cdot Kfin \cdot \left[\frac{dy}{dx} + \frac{dx}{dy} \right] = \frac{Kfin}{2} \cdot \frac{dy}{dx} \cdot \left(T_{i-1,1} + T_{i+1,1} \right) + Kfin \cdot \frac{dx}{dy} \cdot T_{i,2} \quad \text{(for } i = 6 \text{ to } 64)$$

Node G - Chip Boundary Corner w/insulation

$$T_{5,1} \cdot \left[Kfin \cdot \left[\frac{dx}{dy} + \frac{dy}{dx} \right] + \frac{1}{R_{t2}} \right] = Kfin \cdot \frac{dy}{dx} \cdot T_{6,1} + Kfin \cdot \frac{dx}{dy} \cdot T_{5,2} + \frac{T_{4,1}}{R_{t2}}$$

Node H - Chip Boundary Interior Nodes

$$T_{5,j} \cdot \left[Kfin \cdot \frac{dy}{dx} + Kfin \cdot \frac{dx}{dy} + \frac{1}{R_{t1}} \right] = \frac{Kfin}{2} \cdot \frac{dx}{dy} \cdot \left(T_{5,j+1} + t_{5,j-1} \right) + Kfin \cdot \frac{dy}{dx} \cdot T_{6,j} + \frac{T_{4,j}}{R_{t1}} \quad \text{(for } j = 2 \text{ to } 5)$$

Node I - Chip Boundary w/Convection

$$T_{5,6} \cdot \left[Kfin \cdot \frac{dy}{dx} + Kfin \cdot \frac{dx}{dy} + \frac{1}{R_{t2}} \right] = Kfin \cdot \frac{dy}{dx} \cdot T_{6,6} + Kfin \cdot \frac{dx}{dy} \cdot T_{5,5} + h \cdot dx \cdot Tinf + \frac{T_{4,6}}{R_{t2}}$$

Chip Temperature Calculations

Node J - Chip Exterior Convection Nodes

$$T_{4,j} \cdot \left(2 \cdot Kchip + h \cdot dm \right) = Kchip \cdot T_{3,j} + \frac{Kchip}{2} \cdot \left(T_{4,j+1} + T_{4,j-1} \right) + h \cdot dm \cdot Tinf + 1 / 2 \cdot q3p \cdot dm$$

$$\cdot \ dn \quad \text{(for } j = 7 \text{ to } 10)$$

Node K - Chip/Fin Boundary Nodes

$$T_{4,j} \cdot \left[2 \cdot Kchip + \frac{1}{R_{t1}} \right] = 1/2 \cdot q3p \cdot dm \cdot dn + \frac{T_{5,j}}{R_{t1}} + \frac{Kchip}{2} \cdot \left(T_{4,j+1} + T_{4,j-1} \right) + Kchip \cdot T_{3,j}$$

(for j = 2 to 6)

Node L - Chip/Fin Boundary Corner Node w/Insulation

$$T_{4,1} \cdot \left[2 \cdot Kchip + \frac{1}{R_{t2}} \right] = Kchip \cdot T_{3,1} + Kchip \cdot T_{4,2} + \frac{T_{5,1}}{R_{t2}} + 1/2 \cdot q3p \cdot dm \cdot dn$$

Node M - Lower Interior Chip Nodes w/Insulation

$$T_{i,1} \cdot 2 \cdot Kchip = \frac{Kchip}{2} \cdot \left(T_{i-1,1} + T_{i+1,1} \right) + Kchip \cdot T_{i,2} + 1/2 \cdot q3p \cdot dm \cdot dn \qquad \text{(for } i = 2 \text{ to } 3)$$

Node N - Lower Interior Corner Chip Node (Double Insulation)

$$T_{1,1} \cdot Kchip = \frac{Kchip}{2} \cdot \left(T_{2,1} + T_{1,2} \right) + 1/4 \cdot q3p \cdot dm \cdot dn$$

Node O - Interior Side Chip Node (Insulation)

$$T_{1,j} \cdot 2 \cdot Kchip = \frac{Kchip}{2} \cdot \left(T_{1,j+1} + T_{1,j-1} \right) + Kchip \cdot T_{2,j} + 1/2 \cdot q3p \cdot dm \cdot dn \qquad \text{(for } j = 2 \text{ to } 10)$$

Node P - Interior Chip Node

$$T_{i,j} \cdot 4 \cdot Kchip = Kchip \cdot \left(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} \right) + q3p \cdot dm \cdot dn \qquad \text{(for } i = 2 \text{ to } 3), j = 2 \text{ to } 10)$$

Node Q - Upper Interior Corner Chip Node (Double Insulation)

$$T_{1,11} \cdot Kchip = \frac{Kchip}{2} \cdot \left(T_{2,11} + T_{1,10} \right) + 1/4 \cdot q3p \cdot dm \cdot dn$$

Node R - Upper Chip Nodes (Insulated)

$$T_{i,11} \cdot 2 \cdot Kchip = \frac{Kchip}{2} \cdot \left(T_{i-1,11} + T_{i+1,11} \right) + Kchip \cdot T_{i,10} + 1/2 \cdot q3p \cdot dm \cdot dn \qquad \text{(for } i = 2 \text{ to } 3)$$

Node S - Upper Chip Exterior Node (Convection and Insulation)

$$T_{4,11} \cdot \left(Kchip + h \cdot dm \right) = \frac{Kchip}{2} \cdot \left(T_{3,11} + T_{4,10} \right) + h \cdot \frac{dm}{2} \cdot Tinf + 1/2 \cdot q3p \cdot dm \cdot dn$$

Part B - Fin Efficiency Calculation

Heat Calculation

$$q_{i,6} = h \cdot dx \cdot \left(T_{i,6} - Tinf \right) \qquad \text{(for } i = 6 \text{ to } 64)$$

$$q_{65,j} = h \cdot dy \cdot \left(T_{65,j} - Tinf \right) \qquad \text{(for } j = 1 \text{ to } 5)$$

$$q_{5,6} = h \cdot 1/2 \cdot dx \cdot \left(T_{5,6} - Tinf \right)$$

$$q_{65,6} = h \cdot 1/2 \cdot dx \cdot (T_{65,6} - Tinf) + h \cdot 1/2 \cdot dy \cdot (T_{65,6} - Tinf)$$

$$q_{fin} = 2 \cdot \left[q_{65,6} + q_{5,6} + \sum_{i=6}^{64} (q_{i,6}) + \sum_{j=1}^{5} (q_{65,j}) \right]$$

$$T_{base} = \frac{T_{5,1} + T_{5,2} + T_{5,3} + T_{5,4} + T_{5,5} + T_{5,6}}{6}$$

$$A_{fin} = 2 \cdot 0.06 \cdot 0.01 + 2 \cdot 0.06 + 0.01 \cdot 1$$

$$q_{max} = h \cdot A_{fin} \cdot (T_{base} - Tinf)$$

Efficeincy Value

$$n = \frac{q_{fin}}{q_{max}}$$

SOLUTION

Unit Settings: SI C kPa kJ mass deg

A_{fin} = 0.1312	dm = 0.001	dn = 0.001
dx = 0.001	dy = 0.001	h = 50
Kchip = 100	Kfin = 100	n = 0.8772
q3p = 6.000E+06	q_{fin} = 307.7	q_{max} = 350.8
$R2p_{tc}$ = 0.0001	$R_{condchip}$ = 0.005	$R_{condfin}$ = 0.005
R_{t1} = 0.11	R_{t2} = 0.105	R_{tc} = 0.1
Tinf = 20	T_{base} = 73.47	

No unit problems were detected.

Arrays Table: Main

	$T_{i,1}$	$T_{i,2}$	$T_{i,3}$	$T_{i,4}$	$T_{i,5}$	$T_{i,6}$	$T_{i,7}$	$T_{i,8}$	$T_{i,9}$	$T_{i,10}$	$T_{i,11}$	$q_{i,1}$
1	77.03	77.05	77.1	77.19	77.32	77.47	77.64	77.78	77.9	77.97	77.99	
2	76.98	77	77.06	77.15	77.28	77.44	77.61	77.77	77.89	77.96	77.99	
3	76.84	76.85	76.91	77	77.14	77.32	77.55	77.74	77.87	77.94	77.97	
4	76.59	76.61	76.67	76.76	76.9	77.11	77.48	77.7	77.84	77.92	77.94	
5	73.46	73.46	73.46	73.47	73.48	73.49						
6	73.17	73.17	73.17	73.17	73.16	73.15						
7	72.88	72.88	72.88	72.87	72.86	72.84						
8	72.59	72.59	72.59	72.58	72.56	72.54						
9	72.31	72.31	72.3	72.29	72.27	72.25						
10	72.03	72.03	72.02	72.01	71.99	71.97						
11	71.75	71.75	71.74	71.73	71.71	71.69						
12	71.48	71.48	71.47	71.46	71.44	71.42						
13	71.21	71.21	71.2	71.19	71.17	71.15						
14	70.95	70.95	70.94	70.93	70.91	70.89						
15	70.7	70.7	70.69	70.68	70.66	70.63						
16	70.45	70.44	70.44	70.42	70.41	70.38						
17	70.2	70.2	70.19	70.18	70.16	70.14						
18	69.96	69.96	69.95	69.94	69.92	69.9						
19	69.72	69.72	69.71	69.7	69.68	69.66						
20	69.49	69.49	69.48	69.47	69.45	69.43						

Arrays Table: Main

	$T_{i,1}$	$T_{i,2}$	$T_{i,3}$	$T_{i,4}$	$T_{i,5}$	$T_{i,6}$	$T_{i,7}$	$T_{i,8}$	$T_{i,9}$	$T_{i,10}$	$T_{i,11}$	$q_{i,1}$
21	69.27	69.26	69.26	69.24	69.23	69.21						
22	69.05	69.04	69.04	69.02	69.01	68.98						
23	68.83	68.83	68.82	68.81	68.79	68.77						
24	68.62	68.62	68.61	68.6	68.58	68.56						
25	68.41	68.41	68.4	68.39	68.37	68.35						
26	68.21	68.21	68.2	68.19	68.17	68.15						
27	68.01	68.01	68	67.99	67.97	67.95						
28	67.82	67.82	67.81	67.8	67.78	67.76						
29	67.63	67.63	67.62	67.61	67.6	67.57						
30	67.45	67.45	67.44	67.43	67.41	67.39						
31	67.27	67.27	67.26	67.25	67.24	67.21						
32	67.1	67.1	67.09	67.08	67.06	67.04						
33	66.93	66.93	66.92	66.91	66.89	66.87						
34	66.77	66.77	66.76	66.75	66.73	66.71						
35	66.61	66.61	66.6	66.59	66.57	66.55						
36	66.45	66.45	66.45	66.43	66.42	66.4						
37	66.3	66.3	66.3	66.28	66.27	66.25						
38	66.16	66.16	66.15	66.14	66.12	66.1						
39	66.02	66.02	66.01	66	65.98	65.96						
40	65.88	65.88	65.87	65.86	65.85	65.83						
41	65.75	65.75	65.74	65.73	65.72	65.69						
42	65.63	65.62	65.62	65.6	65.59	65.57						
43	65.5	65.5	65.49	65.48	65.47	65.45						
44	65.39	65.38	65.38	65.36	65.35	65.33						
45	65.27	65.27	65.26	65.25	65.24	65.22						
46	65.16	65.16	65.15	65.14	65.13	65.11						
47	65.06	65.06	65.05	65.04	65.02	65						
48	64.96	64.96	64.95	64.94	64.92	64.9						
49	64.87	64.86	64.86	64.84	64.83	64.81						
50	64.77	64.77	64.77	64.75	64.74	64.72						
51	64.69	64.69	64.68	64.67	64.65	64.63						
52	64.61	64.6	64.6	64.59	64.57	64.55						
53	64.53	64.53	64.52	64.51	64.49	64.47						
54	64.46	64.45	64.45	64.44	64.42	64.4						
55	64.39	64.39	64.38	64.37	64.35	64.33						
56	64.32	64.32	64.32	64.3	64.29	64.27						
57	64.27	64.26	64.26	64.25	64.23	64.21						
58	64.21	64.21	64.2	64.19	64.17	64.16						
59	64.16	64.16	64.15	64.14	64.12	64.1						
60	64.11	64.11	64.1	64.09	64.08	64.06						
61	64.07	64.07	64.06	64.05	64.04	64.02						
62	64.03	64.03	64.03	64.01	64	63.98						
63	64	64	63.99	63.98	63.97	63.95						
64	63.97	63.97	63.96	63.95	63.94	63.92						
65	63.95	63.95	63.94	63.93	63.91	63.89						