```
(*These are just some tests to gain intuition about which eigenvalue is
 changing its sign for the mutltiple mating case in Chapter 5. *)P = 0
M =
 1
```

Out[3]= 0

Out[4]= 1

In[5]:=
```
mat = {{(F * (k * M - (P - 1) ^ 2) - (P - 1) ^ 2 * (k + gamma + 2 * P) + k * M * X) / (P - 1) ^ 2,
    (-k * (1 - P - F - X)) / (1 - P), ((-k * (1 - P - M)) / (1 - P)) - P, ((-k * (1 - P - M)) / (1 - P))},
   {-M / 2, (1 / 2) * (-F - P), (1 - M) / 2, 0}, {-(F / 2) - ((k * M) / (1 - P)) + ((k * M * (1 - P - F - X)) / (1 - P) ^ 2),
    k * (1 - P - F - X) / (1 - P), -1 - F / 2 - gamma - delta - ((k * M) / (1 - P)) + (1 / 2) * (-F - P),
    (-k * M) / (1 - P)}, {-X / 2, 0, -X / 2, -gamma + (1 / 2) * (-F - P)}}
```

Out[5]=
$$\left\{\{-\text{gamma} + F\,(-1+k) - k + k\,X,\ -k\,(1-F-X),\ 0,\ 0\},\ \left\{-\frac{1}{2},\ -\frac{F}{2},\ 0,\ 0\right\},\right.$$
$$\left.\left\{-\frac{F}{2} - k + k\,(1-F-X),\ k\,(1-F-X),\ -1-\text{delta}-F-\text{gamma}-k,\ -k\right\},\ \left\{-\frac{X}{2},\ 0,\ -\frac{X}{2},\ -\frac{F}{2}-\text{gamma}\right\}\right\}$$

In[6]:=
```
{l1, l2, l3, l4} = Eigenvalues [mat]
```

Out[6]=
$$\left\{\frac{1}{4}\times\left(-2-2\,\text{delta}-3\,F-4\,\text{gamma}-2\,k-\sqrt{4+8\,\text{delta}+4\,\text{delta}^2+4\,F+4\,\text{delta}\,F+F^2+8\,k+8\,\text{delta}\,k+4\,F\,k+4\,k^2+8\,k\,X}\right),\right.$$
$$\frac{1}{4}\times\left(-2-2\,\text{delta}-3\,F-4\,\text{gamma}-2\,k+\sqrt{4+8\,\text{delta}+4\,\text{delta}^2+4\,F+4\,\text{delta}\,F+F^2+8\,k+8\,\text{delta}\,k+4\,F\,k+4\,k^2+8\,k\,X}\right),$$
$$\frac{1}{4}\times\left(-3\,F-2\,\text{gamma}-2\,k+2\,F\,k+2\,k\,X-\sqrt{(3\,F+2\,\text{gamma}+2\,k-2\,F\,k-2\,k\,X)^2-4\times\left(2\,F^2+2\,F\,\text{gamma}-2\,k+4\,F\,k-2\,F^2\,k+2\,k\,X-2\,F\,k\,X\right)}\right),$$
$$\left.\frac{1}{4}\times\left(-3\,F-2\,\text{gamma}-2\,k+2\,F\,k+2\,k\,X+\sqrt{(3\,F+2\,\text{gamma}+2\,k-2\,F\,k-2\,k\,X)^2-4\times\left(2\,F^2+2\,F\,\text{gamma}-2\,k+4\,F\,k-2\,F^2\,k+2\,k\,X-2\,F\,k\,X\right)}\right)\right\}$$

```
(*Compute the numerical value of the real
 parts as a test to check what is happening*)k = 12
gamma = 2 / 15
delta = 0.40
F = -1 - delta - 2 * gamma - k + (1 + 2 * delta + delta ^ 2 + 4 * k + 2 * delta * k + k ^ 2) ^ (1 / 2)
X = (2 * gamma) / (2 * gamma + F)
N[l2]
N[l3]
N[l4]
```

Out[93]= 12

Out[94]= $\frac{2}{15}$

Out[95]= 0.4

Out[96]= 0.600779

Out[97]= 0.307416

Out[98]= -0.300389

Out[99]= -2.13587

Out[100]= -0.000288471