```
(*The 4x4 Jacobian matrix in Chapter 5- implemented into Mathematica*)
mat = {{(F*(k*M-(P-1)^2)-(P-1)^2*(k+gamma+2*P)+k*M*X)/(P-1)^2,
   (-k*(1-P-F-X))/(1-P), ((-k*(1-P-M))/(1-P))-P, ((-k*(1-P-M))/(1-P))},
  {-M/2, (1/2)*(-F-P), (1-M)/2, 0}, {-(F/2)-((k*M)/(1-P))+((k*M*(1-P-F-X))/(1-P)^2),
   k*(1-P-F-X)/(1-P), -1-F/2-gamma-delta-((k*M)/(1-P))+(1/2)*(-F-P),
   (-k*M)/(1-P)}, {-X/2, 0, -X/2, -gamma+(1/2)*(-F-P)}}
```

Out[1]=

$$\left\{\left\{\frac{F\left(k\,M-(-1+P)^2\right)-(-1+P)^2\,(\text{gamma}+k+2\,P)+k\,M\,X}{(-1+P)^2},-\frac{k\,(1-F-P-X)}{1-P},-\frac{k\,(1-M-P)}{1-P}-P,-\frac{k\,(1-M-P)}{1-P}\right\},\left\{-\frac{M}{2},\frac{1}{2}(-F-P),\frac{1-M}{2},0\right\},\left\{-\frac{F}{2}-\frac{k\,M}{1-P}+\frac{k\,M\,(1-F-P-X)}{(1-P)^2},\frac{k\,(1-F-P-X)}{1-P},-1-\text{delta}-\frac{F}{2}-\text{gamma}-\frac{k\,M}{1-P}+\frac{1}{2}(-F-P),-\frac{k\,M}{1-P}\right\},\left\{-\frac{X}{2},0,-\frac{X}{2},-\text{gamma}+\frac{1}{2}(-F-P)\right\}\right\}$$

```
M = 0
```

Out[2]= 0

```
F = 0
```

Out[3]= 0

```
(*The Jacobian at the pair bonding equilibrium point*)mat
```

Out[4]=

$$\left\{\left\{-\text{gamma}-k-2\,P,-\frac{k\,(1-P-X)}{1-P},-k-P,-k\right\},\left\{0,-\frac{P}{2},\frac{1}{2},0\right\},\left\{0,\frac{k\,(1-P-X)}{1-P},-1-\text{delta}-\text{gamma}-\frac{P}{2},0\right\},\left\{-\frac{X}{2},0,-\frac{X}{2},-\text{gamma}-\frac{P}{2}\right\}\right\}$$

```
(*Compute the eigenvalues in terms of the populations. In the paper,
we define some of these large expressions as r and s. *)
{n1, n2, n3, n4} = Eigenvalues[mat]
```

Out[5]=

$$\left\{\frac{4\,\text{gamma} + 2\,k + 5\,P - 4\,\text{gamma}\,P - 2\,k\,P - 5\,P^2 - (-1+P)\,\sqrt{4\,k^2 + 12\,k\,P + 9\,P^2 + 8\,k\,X}}{4\times(-1+P)},\right.$$

$$\frac{4\,\text{gamma} + 2\,k + 5\,P - 4\,\text{gamma}\,P - 2\,k\,P - 5\,P^2 + (-1+P)\,\sqrt{4\,k^2 + 12\,k\,P + 9\,P^2 + 8\,k\,X}}{4\times(-1+P)},$$

$$\frac{1}{4\times(-1+P)}\Big(2 + 2\,\text{delta} + 2\,\text{gamma} - 2\,\text{delta}\,P - 2\,\text{gamma}\,P - 2\,P^2 - \sqrt{\Big((-2 - 2\,\text{delta} - 2\,\text{gamma} + 2\,\text{delta}\,P + 2\,\text{gamma}\,P + 2\,P^2)^2 - 4\times(-2\,k + 2\,P + 2\,\text{delta}\,P + 2\,\text{gamma}\,P + 4\,k\,P - 3\,P^2 - 4\,\text{delta}\,P^2 - 4\,\text{gamma}\,P^2 - 2\,k\,P^2 + 2\,\text{delta}\,P^3 + 2\,\text{gamma}\,P^3 + P^4 + 2\,k\,X - 2\,k\,P\,X)\Big)}\Big),$$

$$\left.\frac{1}{4\times(-1+P)}\Big(2 + 2\,\text{delta} + 2\,\text{gamma} - 2\,\text{delta}\,P - 2\,\text{gamma}\,P - 2\,P^2 + \sqrt{\Big((-2 - 2\,\text{delta} - 2\,\text{gamma} + 2\,\text{delta}\,P + 2\,\text{gamma}\,P + 2\,P^2)^2 - 4\times(-2\,k + 2\,P + 2\,\text{delta}\,P + 2\,\text{gamma}\,P + 4\,k\,P - 3\,P^2 - 4\,\text{delta}\,P^2 - 4\,\text{gamma}\,P^2 - 2\,k\,P^2 + 2\,\text{delta}\,P^3 + 2\,\text{gamma}\,P^3 + P^4 + 2\,k\,X - 2\,k\,P\,X)\Big)}\Big)\right\}$$

```
(*Substitute equilibrium expressions to get the 3d plot. *)
P = (1/2) * ((k^2 - 2 * k * gamma + 4 * k + gamma^2)^(1/2) - (k + 3 * gamma))
```

Out[6]=

$$\frac{1}{2}\times\left(-3\,\text{gamma} - k + \sqrt{\text{gamma}^2 + 4\,k - 2\,\text{gamma}\,k + k^2}\right)$$

```
X = (2 * gamma) / (2 * gamma + P)
```

Out[7]=

$$\frac{2\,\text{gamma}}{2\,\text{gamma} + \frac{1}{2}\times\left(-3\,\text{gamma} - k + \sqrt{\text{gamma}^2 + 4\,k - 2\,\text{gamma}\,k + k^2}\right)}$$

```
(*Plot the 3d plot of all pair bonding equilibria *)
RegionPlot3D[k > 0 && gamma > 0 && delta > 0 && Re[n1] < 0 && Re[n2] < 0 && Re[n3] < 0 &&
  Re[n4] < 0 && 0 ≤ P ≤ 1 && 0 ≤ X ≤ 1, {k, 0, 20}, {gamma, 0, 1/2}, {delta, 10^-10, (1/2)},
 Mesh → All, MeshStyle → Automatic, AxesLabel → {kappa, gamma, delta}]
```

Out[9]=