

Optical Interferometry

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The objective of this experiment is to observe the interference of light by combining coherent monochromatic light beams using a Michelson interferometer. You will determine the wavelength of the light source from your measured interference pattern.

PREPARATORY QUESTIONS

Please visit the Optical Interferometry chapter on the 8.13x website at mitx.mit.edu to review the background material for this experiment. Answer all questions found in the chapter. Work out the solutions in your laboratory notebook; submit your answers on the web site.

SAFETY

CAUTION: TAKE EXTREME CARE WHEN HANDLING OPTICAL COMPONENTS. Do not touch mirrors, lenses, beam splitters, or any other optical surfaces with your bare hands. The oils on your fingers will significantly degrade the performance of these optics, severely compromising your data.

WARNING: TAKE CARE WHEN ACTIVATING A LASER. The human eye does not take kindly to laser light. Take whatever precautions are necessary to ensure that no spurious or stray beams leave your experimental area where they may be absorbed by your, your partner's, or some other unsuspecting individual's retina.

I. INTERFERENCE

When we superimpose two waves of the same or nearly the same frequency traveling in a medium, we can observe interference. Interference is, in fact, one of the cornerstones of wave phenomena. Young's double slit experiment, which showed interference fringes (bright and dark bands) for light passing through two slits, was an early indicator of the wave nature of light. Since then, many variants have been used in thought experiments and real experiments that address the wave-particle duality that is a building block of quantum mechanics.

The superposition principle for waves states that when two or more waves interact in a region of space, the net amplitude at each point is the sum of the amplitudes of the individual waves. If the waves sum to a larger amplitude than each individual wave, then we have *constructive interference*. Likewise, if the waves add up to be smaller than the amplitude of the individual waves, then we have *destructive interference*. If two waves of the same frequency and amplitude are in phase, *i.e.* crests line up with crests and troughs line up with troughs, then we

have perfectly constructive interference. Similarly, two waves of equal amplitude and frequency but completely out of phase with one another, *i.e.* crests of one wave line up with troughs of another wave and vice versa, will result in completely destructive interference.

We can derive a more general equation for interference between two superimposed waves with the same frequency, but each with arbitrary amplitude and phase. Since this experiment is about optical interference with Michelson interferometers, we will talk specifically about waves in the electric field, but the following discussion applies to waves in general.

Suppose we have one wave of the form $E_1 e^{i(\phi_1 - \omega t)}$ and another wave $E_2 e^{i(\phi_2 - \omega t)}$ where E_1 and E_2 are the magnitudes of their respective fields, ϕ is the phase, ω is the angular frequency, and t is time. Superimposing both waves gives

$$E_T = E_1 e^{i(\phi_1 - \omega t)} + E_2 e^{i(\phi_2 - \omega t)}. \quad (1)$$

If $\phi_1 - \phi_2 = 2n\pi$ where $n = \{0, 1, 2, \dots\}$, then the waves are perfectly in phase and we recover the case of completely constructive interference. Likewise, if $\phi_1 - \phi_2 = (2n + 1)\pi$, then the two waves are out of phase and we recover the case of completely destructive interference. This is illustrated in Figure 1.

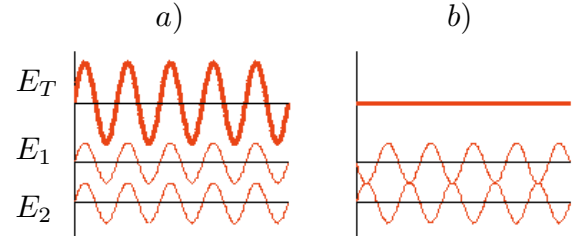


FIG. 1. An illustration of how superimposing two waves results in an interference pattern. Figure 1(a) represents completely constructive interference, whereas Figure 1(b) represents completely destructive interference.

However, we often measure intensity of the field rather than the field itself. The intensity is proportional to the square of the field,

$$I \propto \langle E_T^* E_T \rangle = E_1^2 + E_2^2 + 2E_1 E_2 \cos(\phi_1 - \phi_2), \quad (2)$$

where $\langle \cdot \rangle$ indicates time averaging. We can also write I in terms of the intensities of the individual fields, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_1 - \phi_2)$.

How well two waves interfere with each other will depend on their coherence. Mathematically, coherence is a measure of how well correlated the waves are, as quantified by the cross-correlation function. Physically, coherence is a measure of how constant the phase remains between two waves. For this experiment, we will concentrate on temporal coherence: the correlation between the wave and a time-delayed version of itself. In essence, temporal coherence tells us how monochromatic a wave is. *Interferometric visibility* \mathcal{V} , also known as contrast or modulation, is an experimentally observable measure of how coherent two waves are. Visibility is defined as

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}. \quad (3)$$

An example of an interferogram with I_{min} and I_{max} measurements can be seen in Figure 2. Since the visibility depends on the coherence of the two waves, any dissimilarities between them will result in decreased visibility.

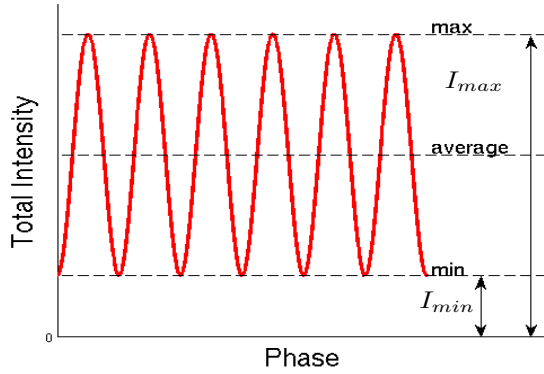


FIG. 2. Visibility is a measure of how well two waves interfere. Higher visibility means more range between the maximum and minimum intensity.

II. MICHELSON INTERFEROMETERS

The Michelson interferometer is the most common configuration for optical interferometry. Michelson interferometers can be used for astronomical interferometry, for optical coherence tomography (a medical imaging technique), and for gravitational wave detection [1]. The Michelson interferometer is best known for its use in the Michelson-Morley experiment, which disproved the existence of the luminiferous ether and paved the way to the development of the theory of relativity [2].

Shown in Figure 3, a Michelson interferometer uses optical interference to detect the length difference between two paths ℓ_1 and ℓ_2 , however small they may be. It does so by splitting a laser beam into two perpendicular paths using a half-silvered mirror acting as a 50 % beam splitter. The beams then bounce off mirrors at the end of their respective paths and are returned to the beam

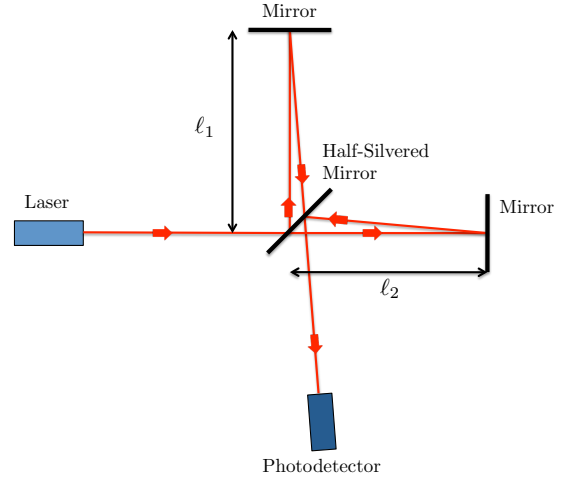


FIG. 3. Michelson interferometer configuration.

splitter. The light beams from the two paths are aligned to impinge on the same point on the photodetector where they interfere and are detected. The amplitude of the detected light is determined by the total phase difference between the combined beams.

We now derive a general case for Michelson interference in a similar manner to that of Section I. We aim monochromatic light of the form $E_0 e^{i\omega t}$ on to the beam splitter. Two beams of equal frequency and amplitude come out of the beam splitter. (Realistically, the amplitudes will not be exactly equal, but they will be very close.) The beam intensity — not the electric field — gets reduced by a factor of 2. Since $I \propto E^2$, we divide the resulting electric fields by $\sqrt{2}$ rather than by 2. The beam that transmits through will have the form $E_1 = \frac{E_0}{\sqrt{2}} e^{i\omega t}$, whereas the reflected beam will obtain a π phase shift and will have the form $E_2 = \frac{E_0}{\sqrt{2}} e^{i(\omega t + \pi)} = -\frac{E_0}{\sqrt{2}} e^{i\omega t}$. Both waves will be reflected by the mirrors (both beams pick up another π phase shift, but this fact is not important because we care about relative phase shift) and return to the beam splitter with a phase shift that depends on the length of the interferometer arms. The superimposed electric field will therefore be $E_T = \frac{E_0}{2} e^{i\omega t} (e^{2ik\ell_1} - e^{2ik\ell_2})$, where $k = \frac{2\pi}{\lambda}$ is the wavenumber and λ is the wavelength of the laser.

In practice, it is more useful to derive the equation for intensity since that is what the photodetector will actually measure:

$$\begin{aligned} I &\propto E_T^* E_T \\ &= \frac{E_0^2}{4} |(e^{2ik\ell_1} - e^{2ik\ell_2})|^2 \\ &= \frac{E_0^2}{2} [1 - \cos(2k(\ell_1 - \ell_2))]. \end{aligned} \quad (4)$$

Using this, we can readily solve for the conditions of maximum and minimum intensity which should correspond to maximally constructive and destructive interference.

If $2(\ell_1 - \ell_2) = n\lambda$, where ℓ_1 and ℓ_2 are the different path lengths, λ is the beam wavelength, and $n = \{0, 1, 2, \dots\}$, then the two beams are perfectly out of phase, resulting in completely destructive interference and a minimum beam power reaching the photodetector. Conversely, if $2(\ell_1 - \ell_2) = \lambda(2n + 1)/2$, the two beams are perfectly in phase, resulting in completely constructive interference. *N.b.:* If you are trying to convince yourself that these relationships are true, keep in mind that the half-silvered beam splitter has a preferential direction and only imparts a π phase change on reflected laser light when incident from one direction. Also keep in mind that the waves traverse the arms twice, so the difference between the lengths of the arms is doubled. Further discussion can also be found in [3].

III. EXPERIMENTAL SETUP

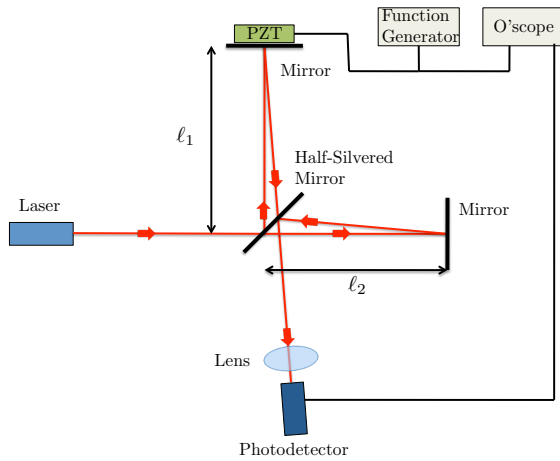


FIG. 4. A block diagram of the experimental setup. Recall that the arm lengths ℓ_1 and ℓ_2 should be approximately equal.

A block diagram of the setup for this introductory experiment can be seen in Figure 4 and is very similar to what was discussed in the previous section. We specify the light source as a laser, which emits a light of a single unknown wavelength. You will derive this wavelength in lab. We also specify the photodetector to be an amplifying photodiode, which translates the light incident on it into a voltage. The photodiode output voltage V_{PD} is proportional to the intensity of the light incident on the detector: $V_{PD} \propto I$. Most notable in the setup is the piezoelectric transducer (PZT) attached to one of the mirrors. The PZT will displace the mirror by an amount depending on the voltage applied to it. The calibration factor $\Delta\ell/\Delta V$ between mirror displacement and applied voltage has been determined for each PZT available in the lab to within an uncertainty of a few percent. Each device is labeled with its measured value, for example $41.6 \pm 1.3 \text{ nm/V}$. That means that if 1 V is applied to this example PZT using the function generator, then the

mirror will translate by $41.6 \pm 1.3 \text{ nm}$. You may or may not find it useful to add a lens to focus the incident beam on the photodiode. Finally, the overall setup is kept as compact as possible. The laser only remains coherent over a length scale of a couple tens of centimeters.

IV. PROCEDURE

1. Begin by assembling the experiment as shown in Figure 4. Remember to keep the length of each arm of the interferometer approximately the same length. **CAUTION: TAKE EXTREME CARE WHEN HANDLING OPTICAL COMPONENTS.** Do not touch mirrors, lenses, beam splitters, or any other optical surfaces with your bare hands. The oils on your fingers will significantly degrade the performance of these optics, severely compromising your data. (*N.b.:* The grid spacing of the optical breadboard is 1".)
2. Make sure that the output of the photodiode is connected to CHANNEL 1 of the oscilloscope and the output of the function generator is connected to CHANNEL 2. Also make sure that the output of the function generator is connected to the PZT.
3. **WARNING: YOU ARE ABOUT TO TURN ON A LASER.** The human eye does not take kindly to laser light. Take whatever precautions are necessary to ensure that no spurious or stray beams leave your experimental area where they may be absorbed by your, your partner's, or some other unsuspecting individual's retina. Turn on the laser. Align the optics and follow the paths of each beam to make sure that they both hit the photodiode at the same location. *You may find it extremely helpful to align each piece of the optical setup individually rather than the entire system at once.* Consider the difficulty of minimizing a system of so many degrees of freedom simultaneously!
4. Set the function generator¹ to output a 10 Hz sinusoid or triangle waveform with a peak-to-peak voltage of 10 V. (Your function generator may indicate peak-to-peak voltage by V_{pp} .) The output of the function generator will drive the PZT. Recall that the amount that the mirror will translate

¹ As with all suggested apparatus settings in Junior Lab lab manuals, the given value will be near a value which gives good signal, but you may find that a different value works better. In general, do not use a value simply because the lab manual says so. Instead, try to understand what experimental condition would cause one to prefer one value over another, and then optimize your choice of settings for that condition.

is determined by the measured $\Delta\ell/\Delta V$ as labeled on your device, which will be several tens of nm/V.

5. You should see some sort of fringing pattern from the photodiode output on the oscilloscope. Tweak your alignment to obtain the largest possible signal. Using the “cursors” or the grid on the oscilloscope, measure² the minimum and maximum voltage coming from the photodiode. You will use these numbers to calculate the visibility. You may find it useful to “RUN/STOP” or “SINGLE mode” the display on the oscilloscope while making your measurement.
6. Measure the driving voltages from the function generator which correspond to the minima and maxima in the photodiode output. From these you will use the relationships $\frac{E_o^2}{2}[1 - \cos(2k(\ell_1 - \ell_2))]$ and the value of $\Delta\ell/\Delta V$ to derive the wavelength of the laser.
7. Review your measurements as they are recorded in your notebook. Be sure that you have recorded enough data to allow evaluation of the statistical (random) error in your measurements. Be sure that

you have recorded enough information in narrative description and sketches to allow evaluation of systematic errors.

V. ANALYSIS

As with all experiments, you should perform at least a rough analysis before you leave the lab to confirm that you have the necessary data to perform the analysis and that the data you have obtained are of sufficient quality. *Never assume that an experiment has gone as planned unless you have first taken action to assure that it has done so, and then followed up with action to confirm success. Successful experiments require good planning: plans which depend on everything going as planned are bad plans!*

1. Calculate the visibility $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$. Explain why the visibility is less than ideal.
2. From the calibration of the driving voltage to mirror displacement and from your measurements from in Step 6 of the previous section, calculate the wavelength λ of the laser beam.

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- [1] B. P. A. et al, Reports on Progress in Physics **72**, 076901 (2009), <http://stacks.iop.org/0034-4885/72/i=7/a=076901>
 - [2] A. A. Michelson and E. W. Morley, American Journal of Science **34**, 333 (1887)
 - [3] E. Hecht, *Optics* (Addison-Wesley, 1998)

² Whenever experimental conditions feasibly allow, it is desirable to use your own mind and hands to record data, rather than an automated data logger, as this process more readily engages the senses and other mental pathways, bringing about a more active awareness and understanding of the experiment.