Department of Physics

Verifying the Klein-Nishina prediction using Compton scattering of 661.7 keV photons in NaI scintillators

Vinh Q. Tran

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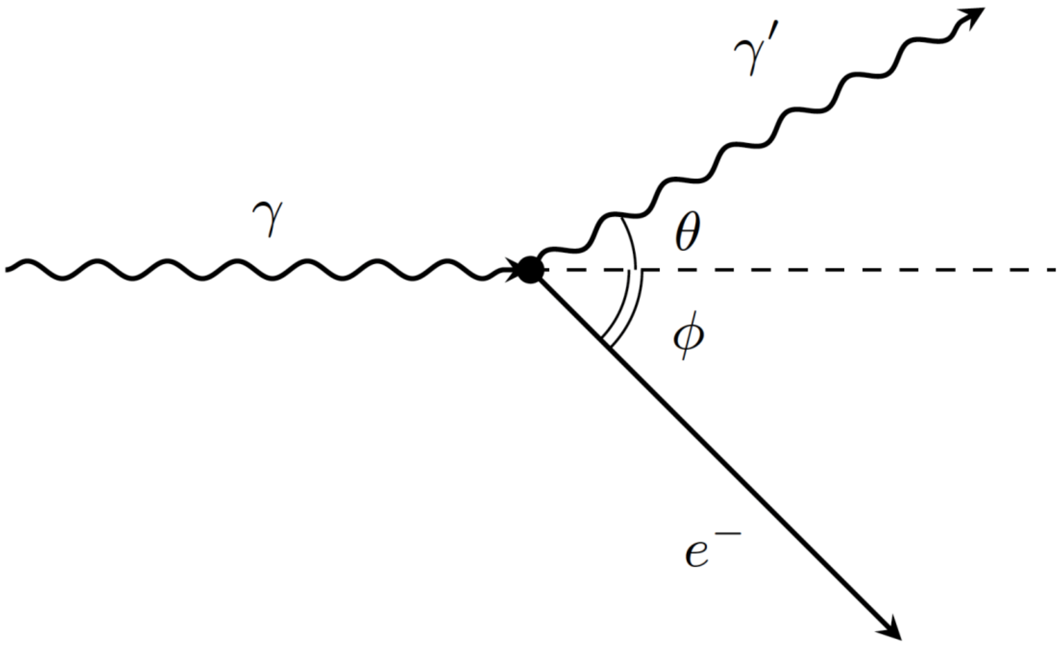
BSc Physics Candidate at MIT

April 7th, 2025

Kinematics of Compton scattering & Klein-Nishina differential cross section



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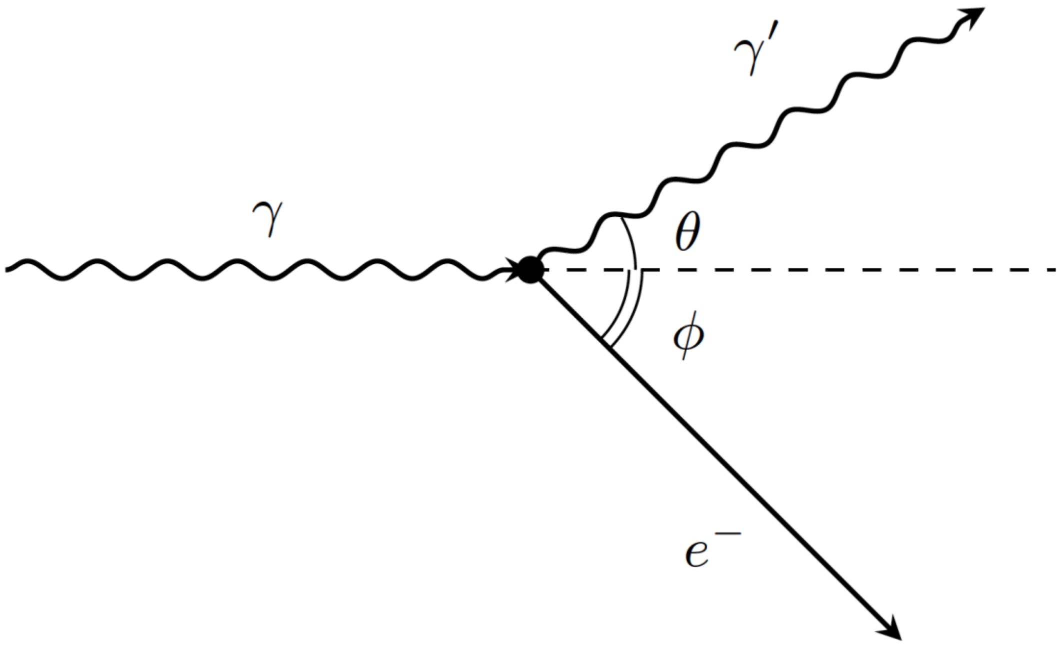

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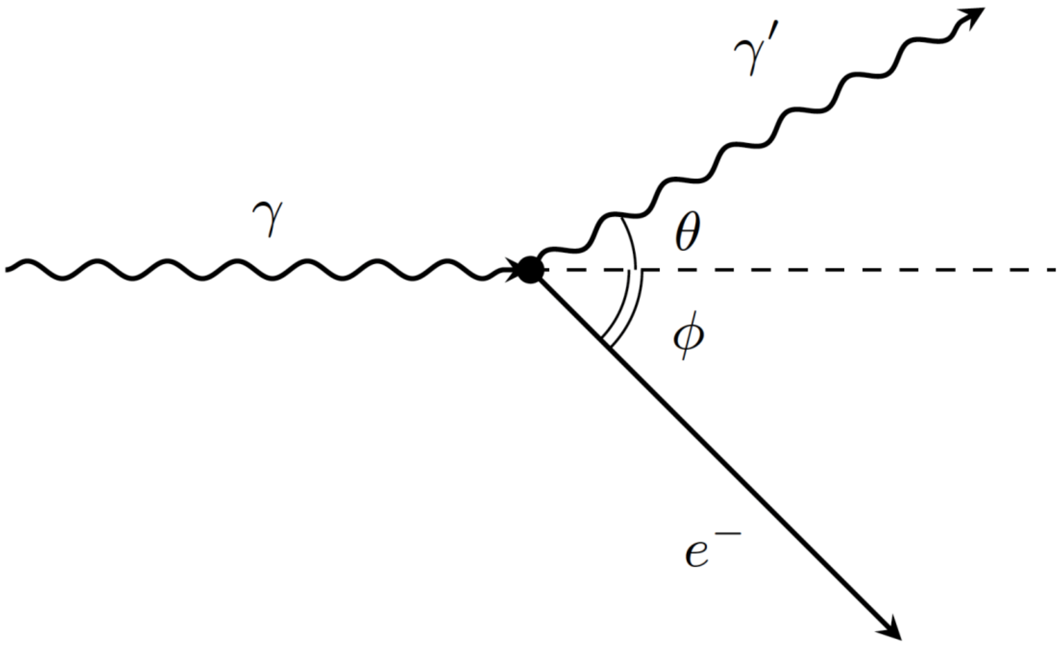

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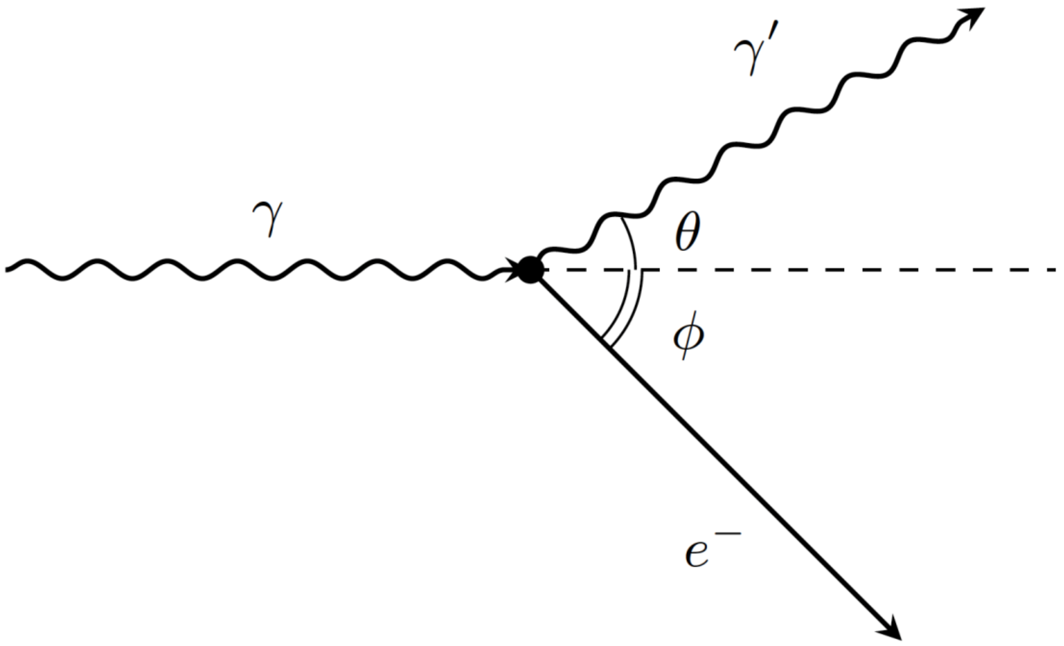
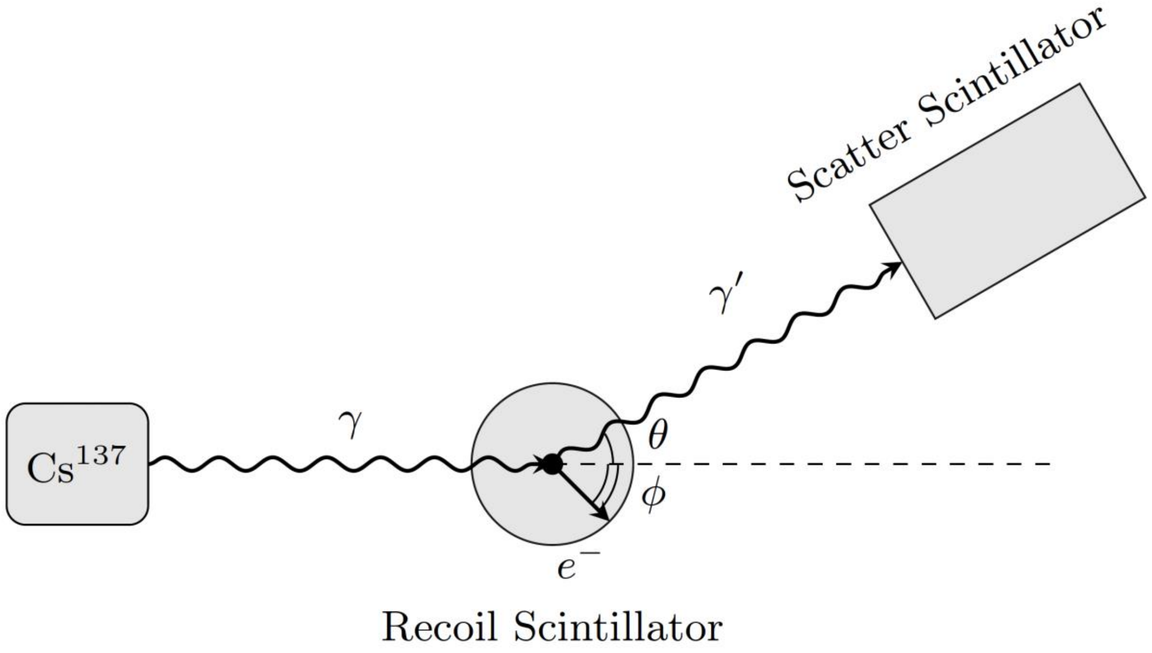


The apparatus arrangement



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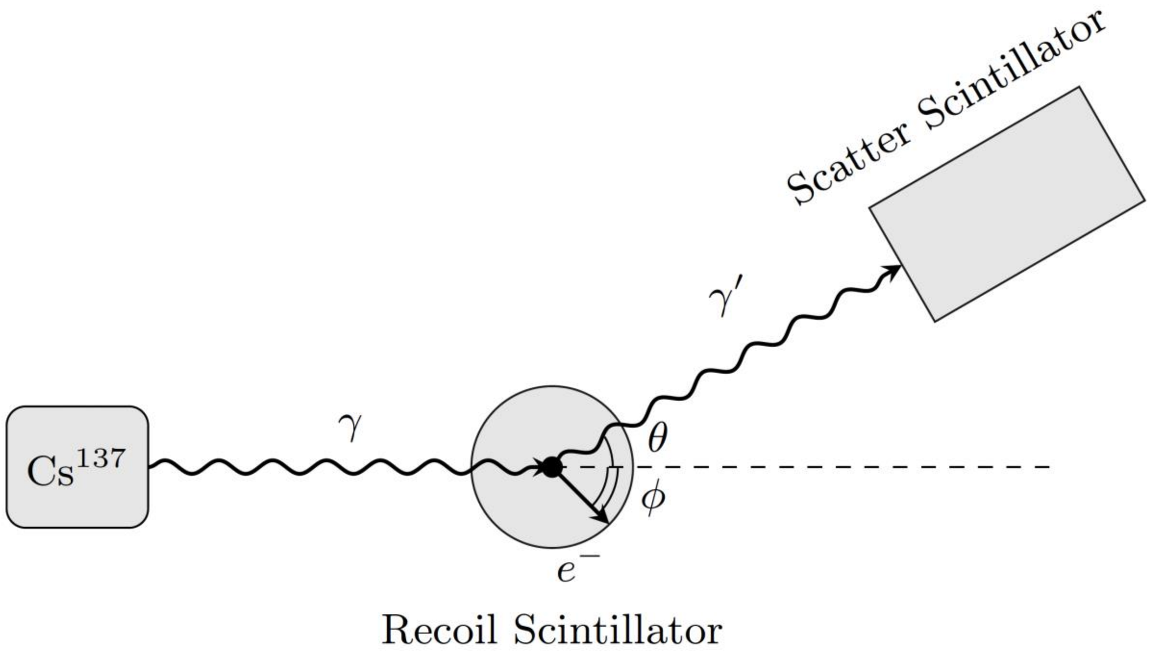

$\theta \in [0^\circ, 135^\circ]$ (at 15° intervals)

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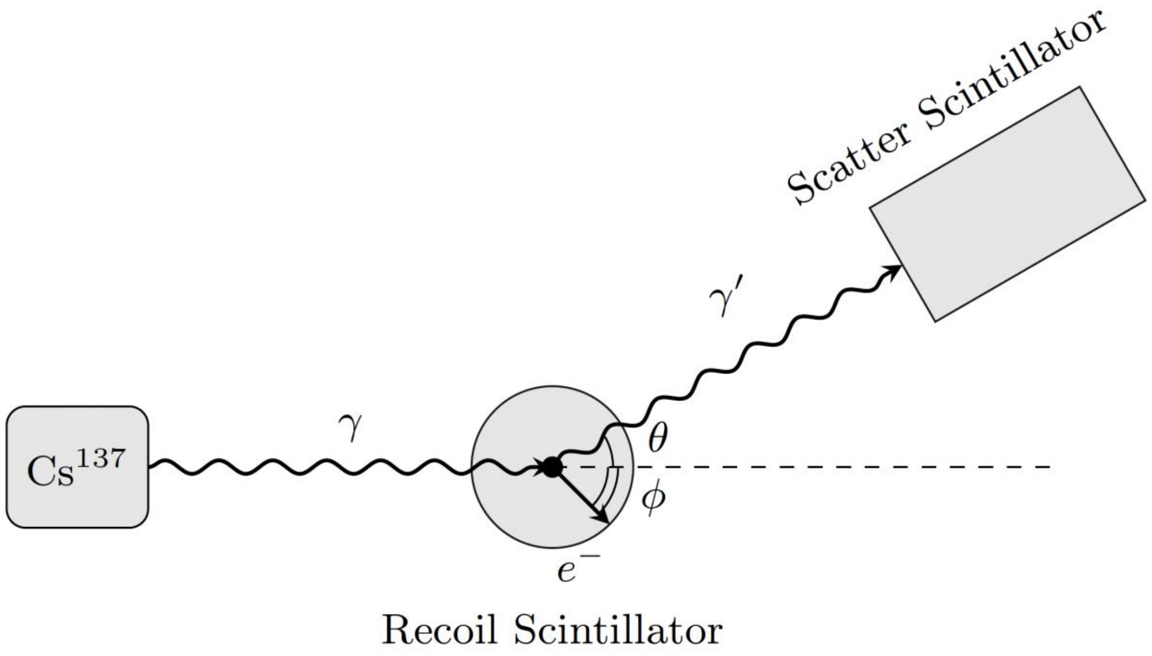

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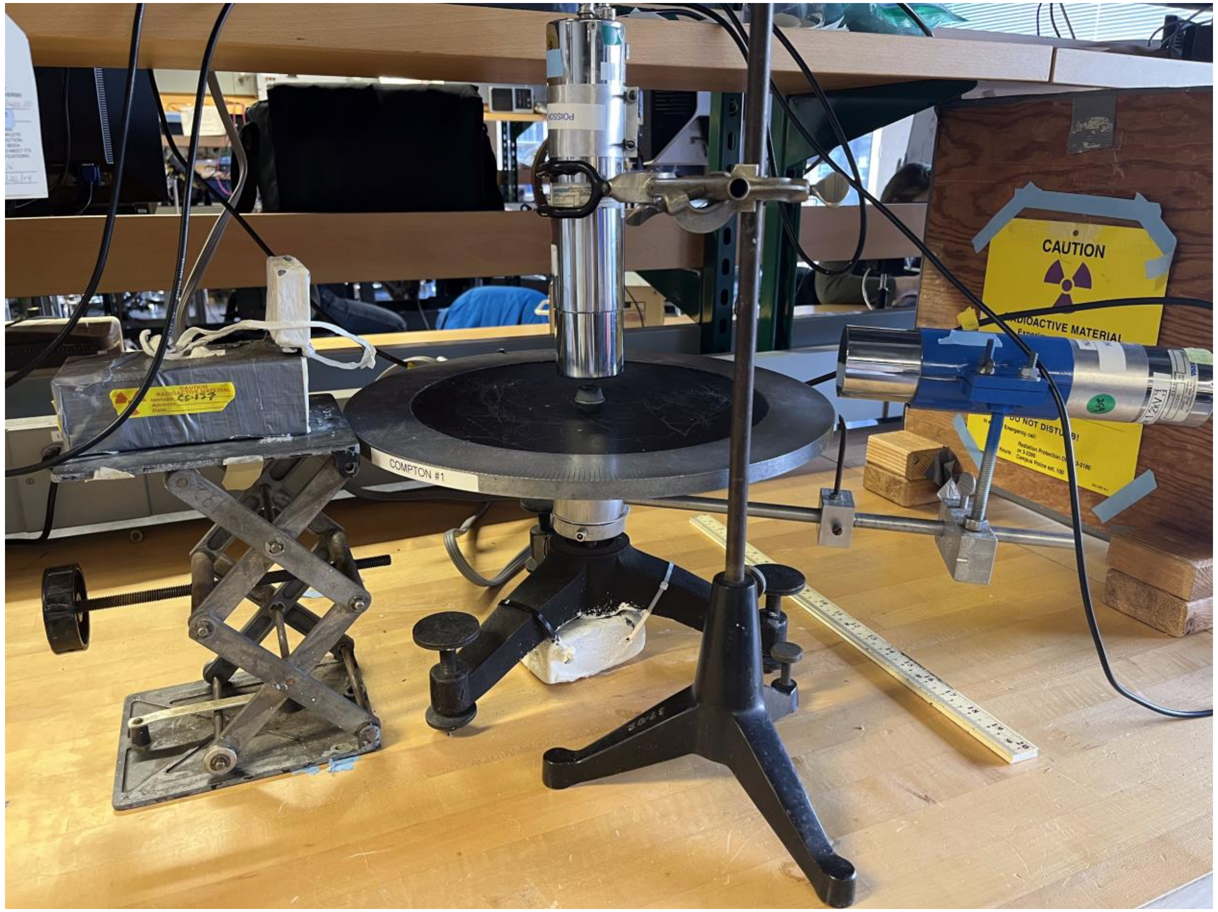
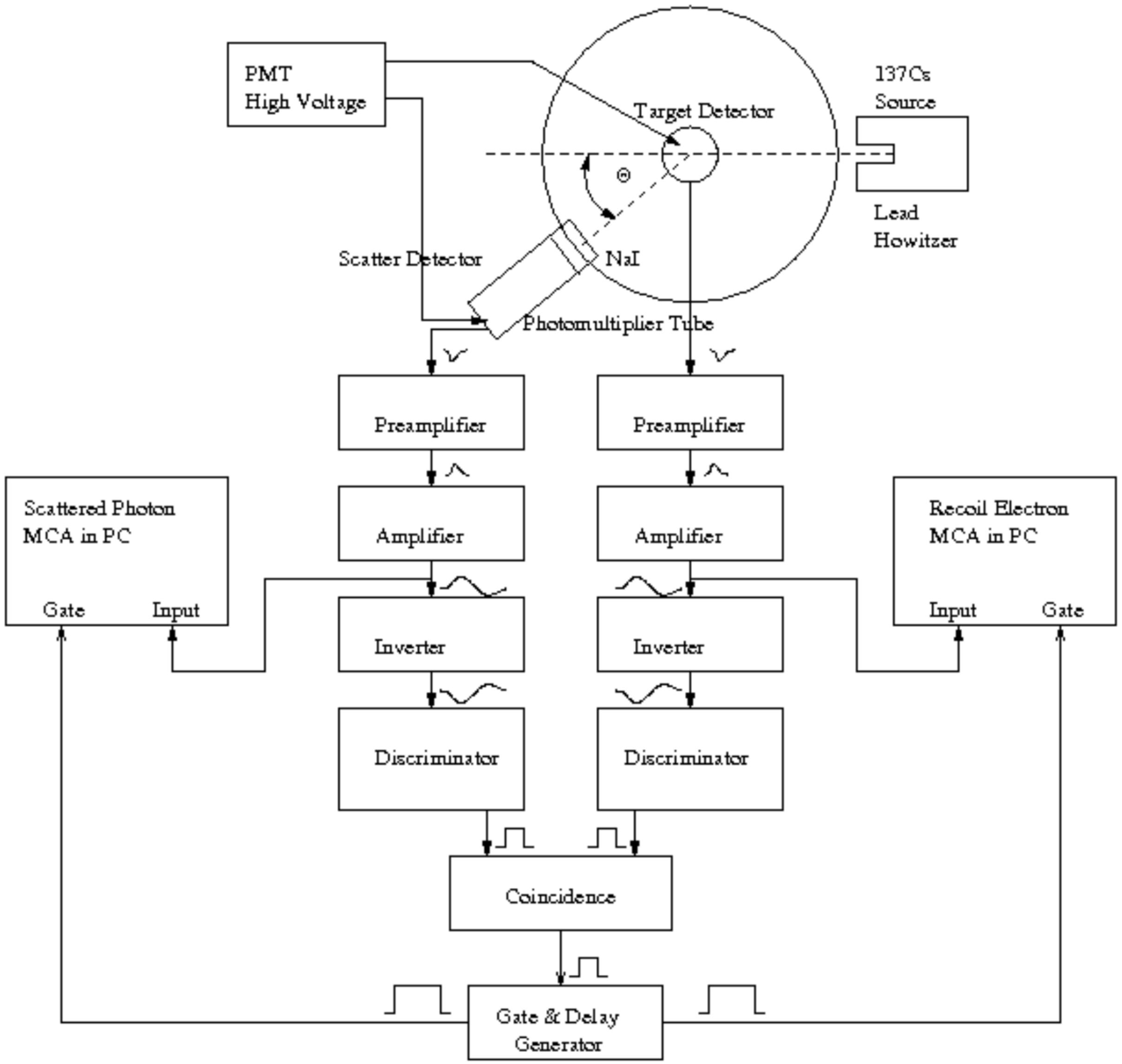
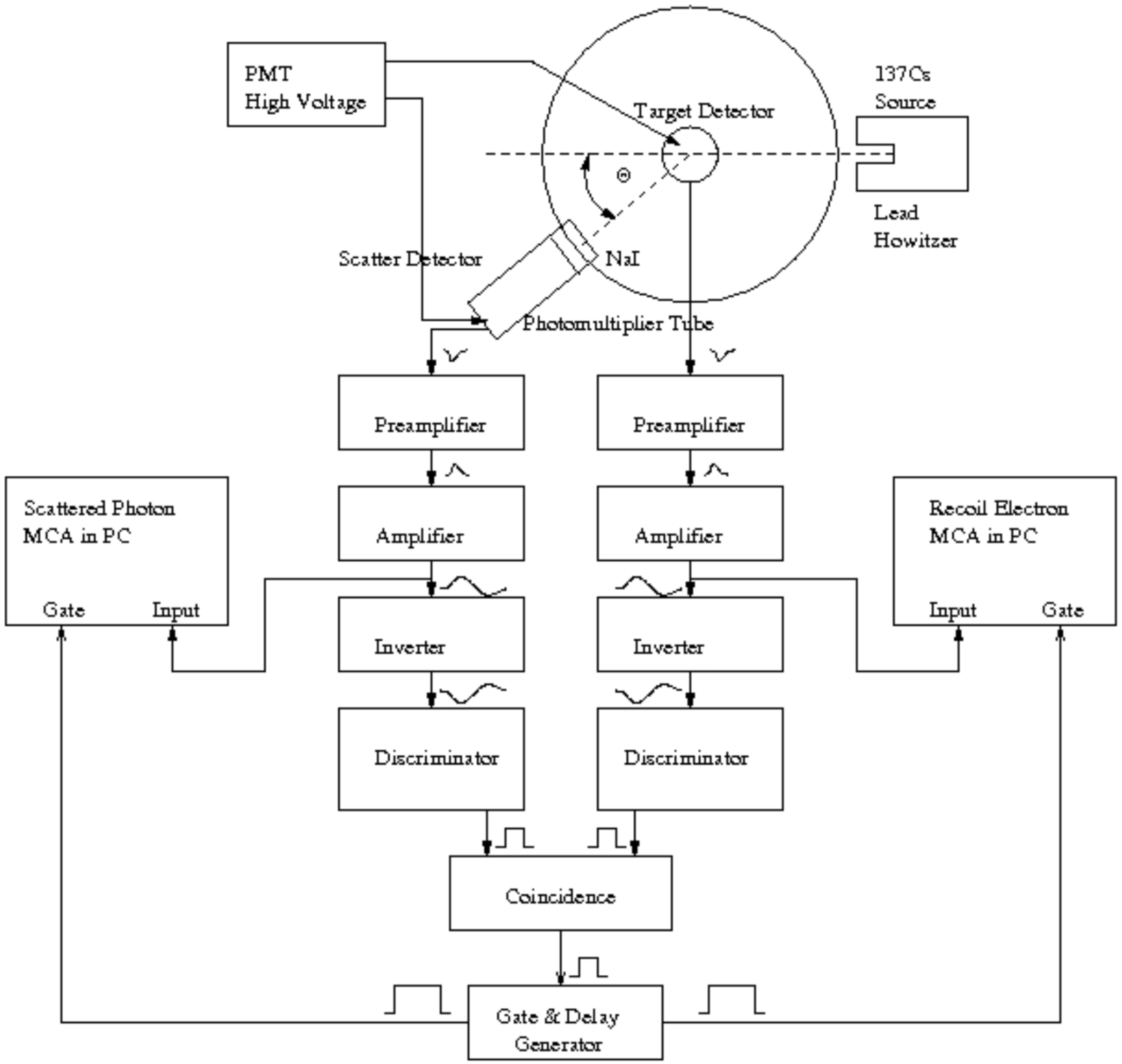


Circuit design & Coincidence mode



Circuit design & Coincidence mode

- Coincidence mode registers only events detected within $\tau = 2\ \mu s$.



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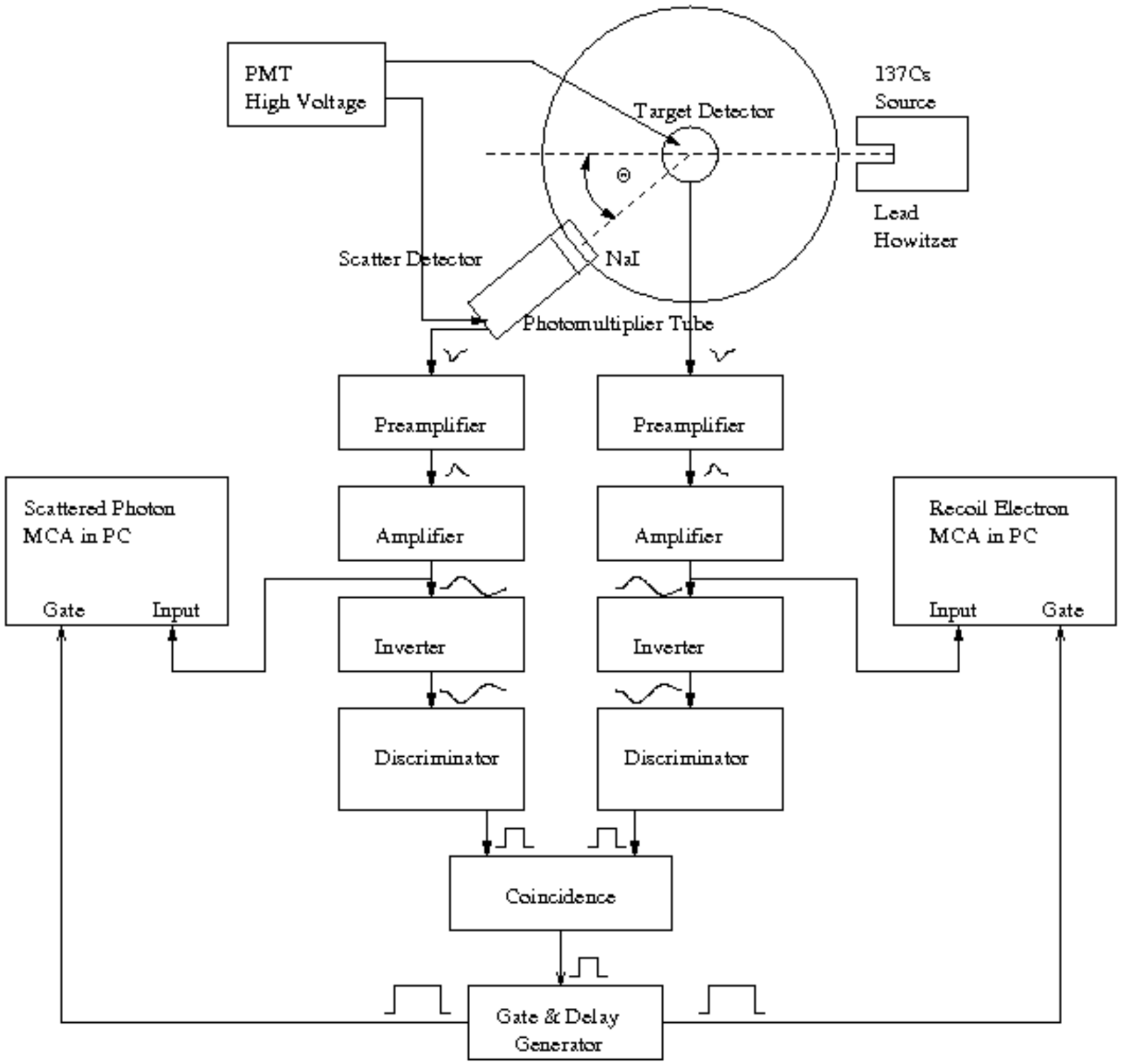

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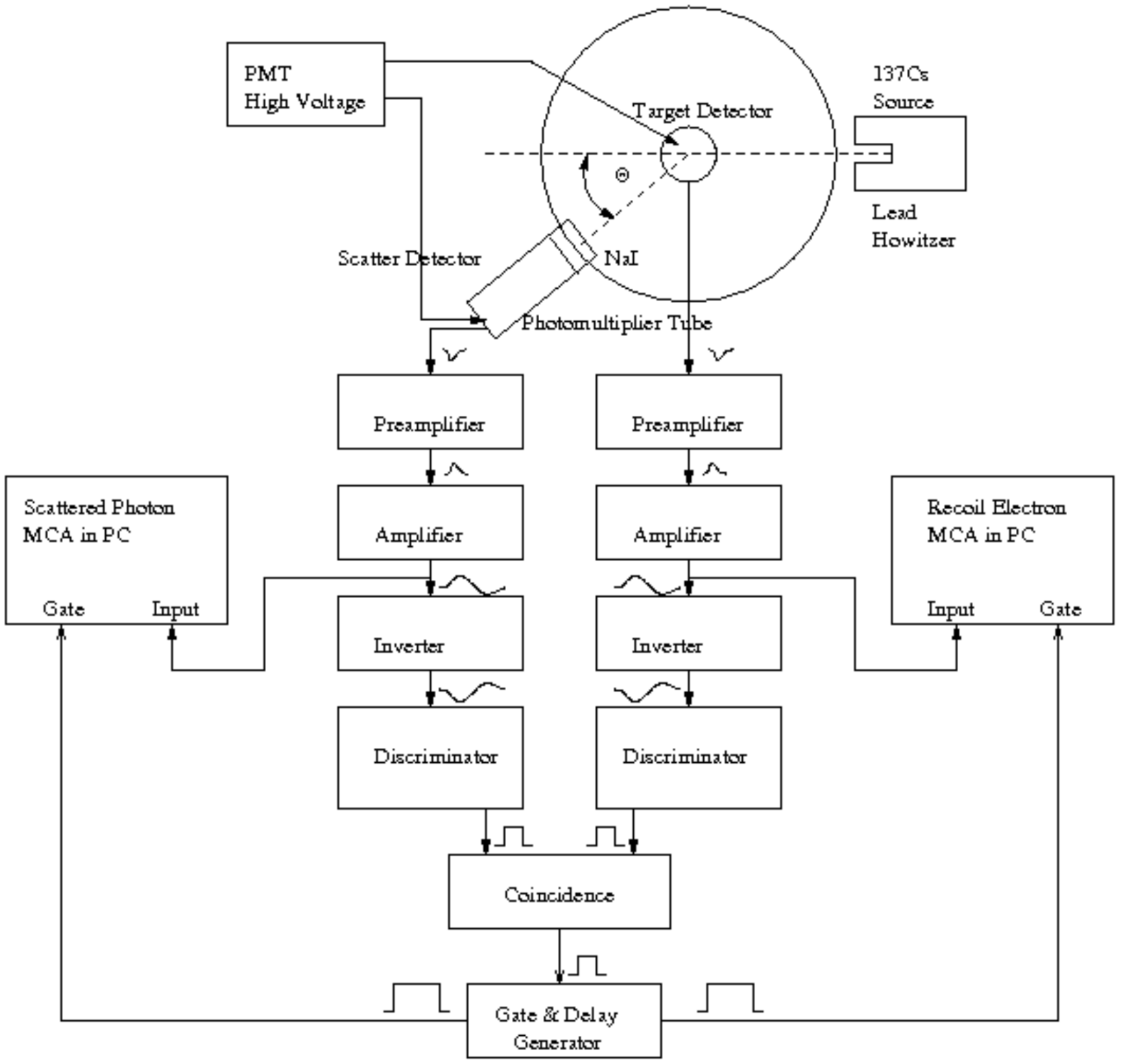
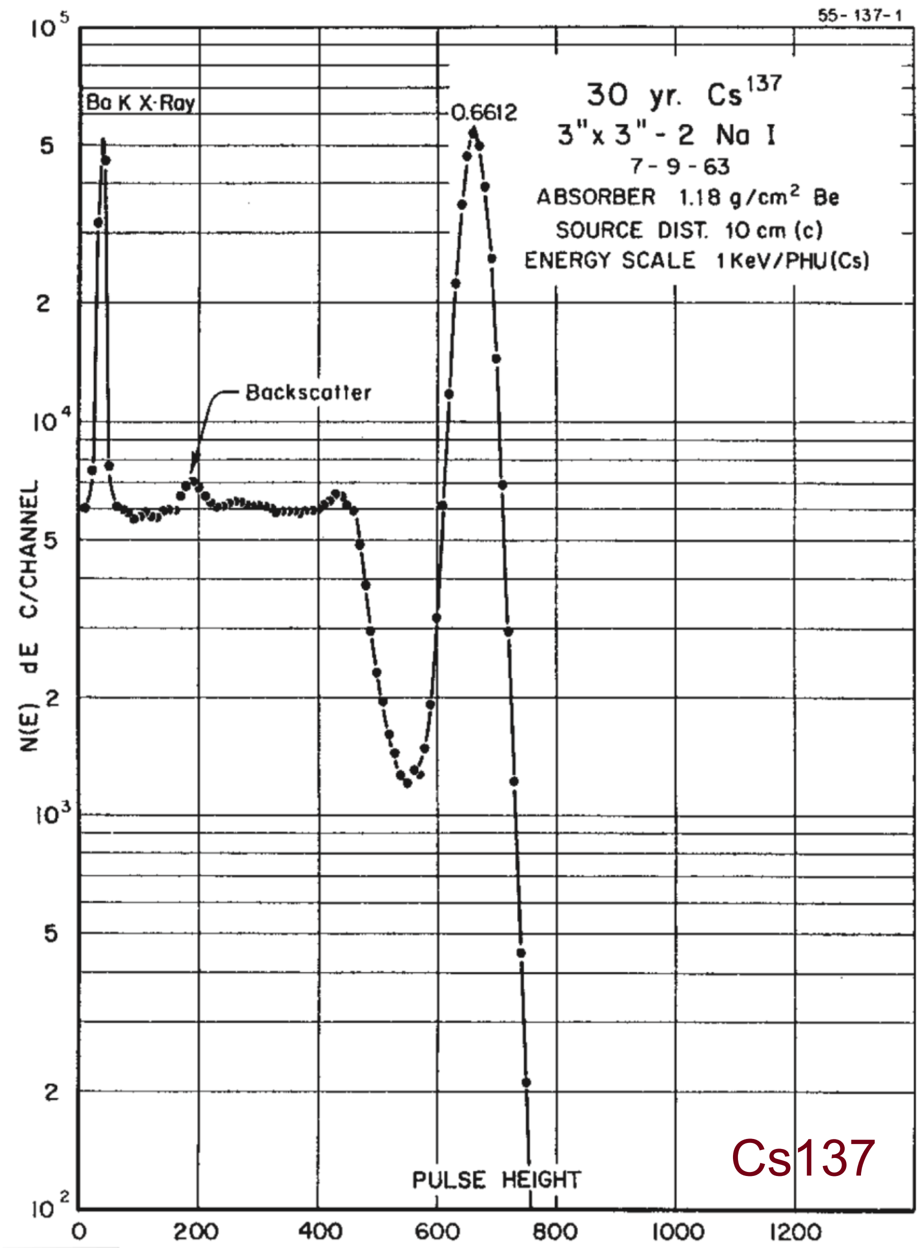
- Setting threshold for the discriminator at $60\ \mathrm{mV}$ help reduce background rate to $< 1\ \mathrm{Hz}$, while Compton scattering rates remain at $\sim 20\ \mathrm{Hz}$.



Cs137 energy spectrum in NaI scintillator



(Heath 1964)

MCA calibration

- Channel-energy conversion relation

$$E_i = \alpha C_i + b$$

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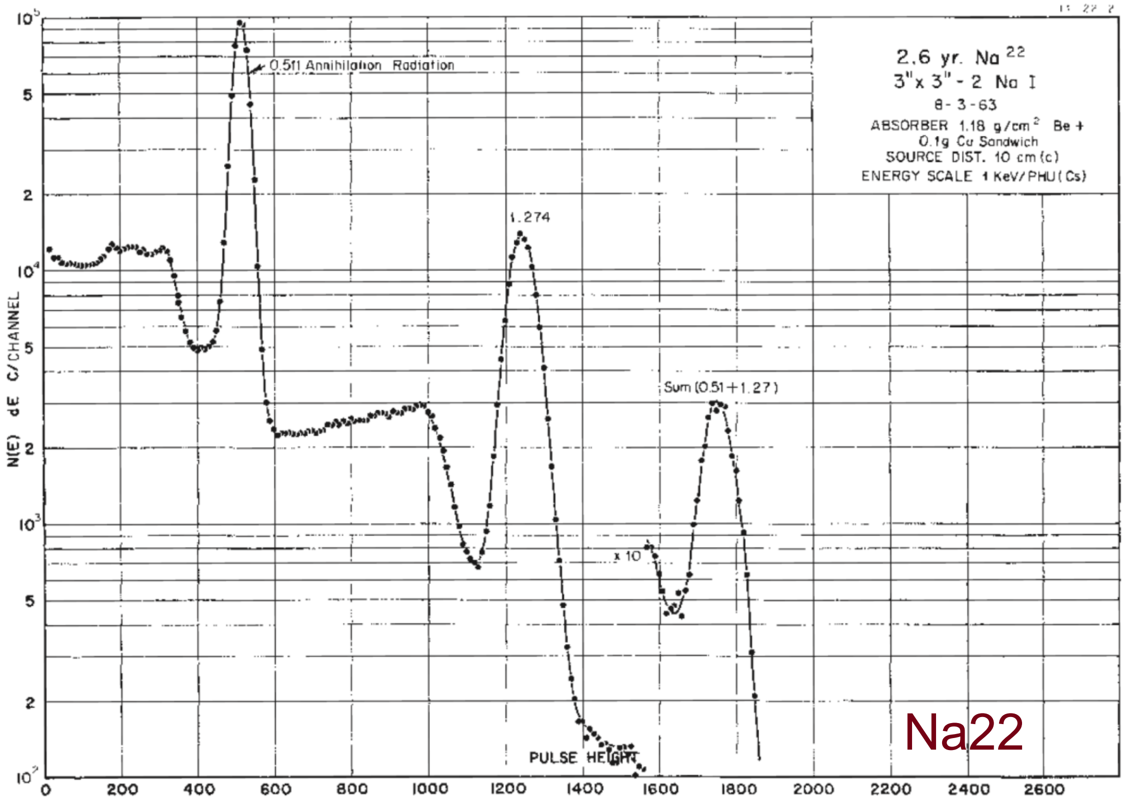
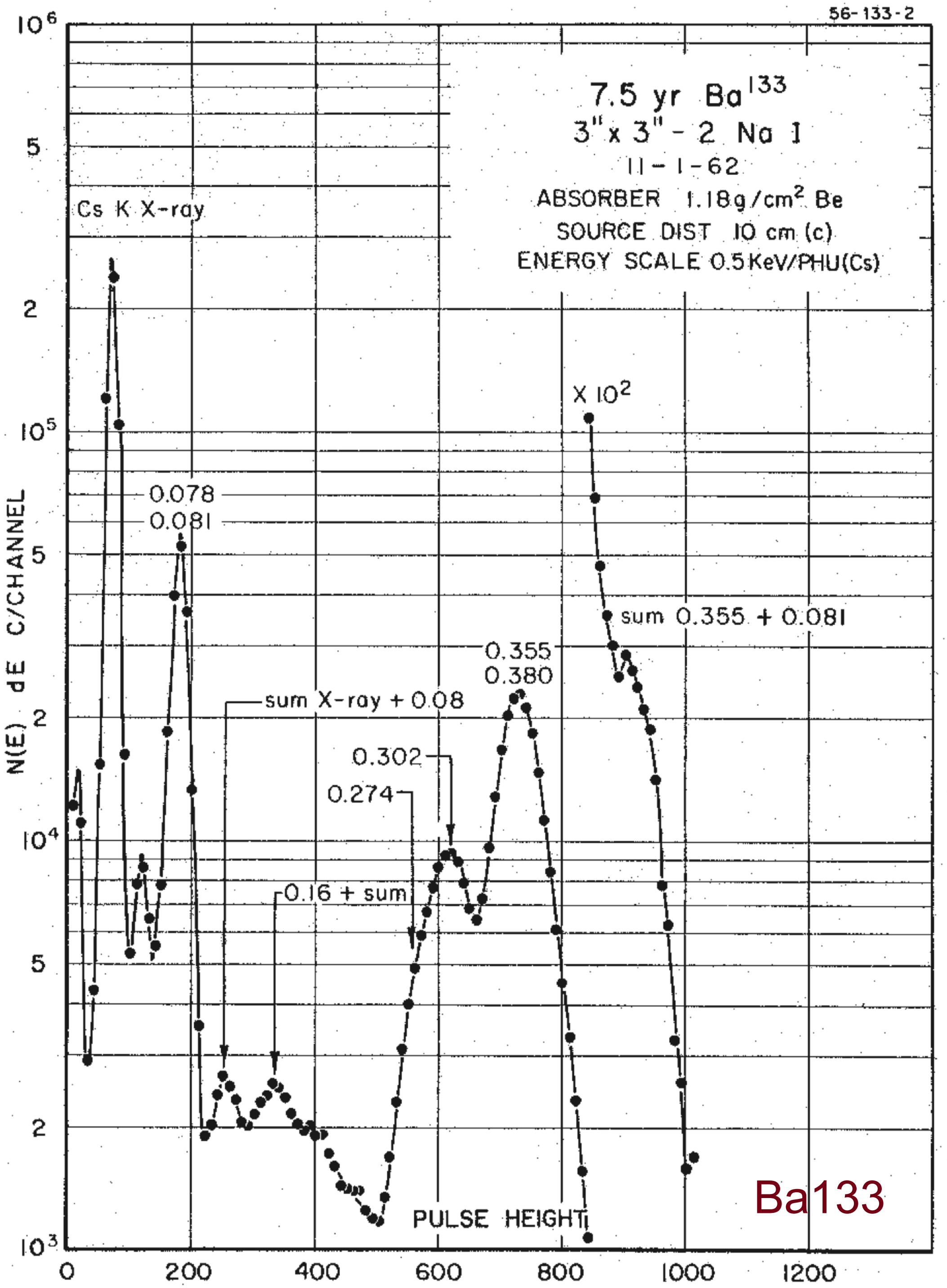
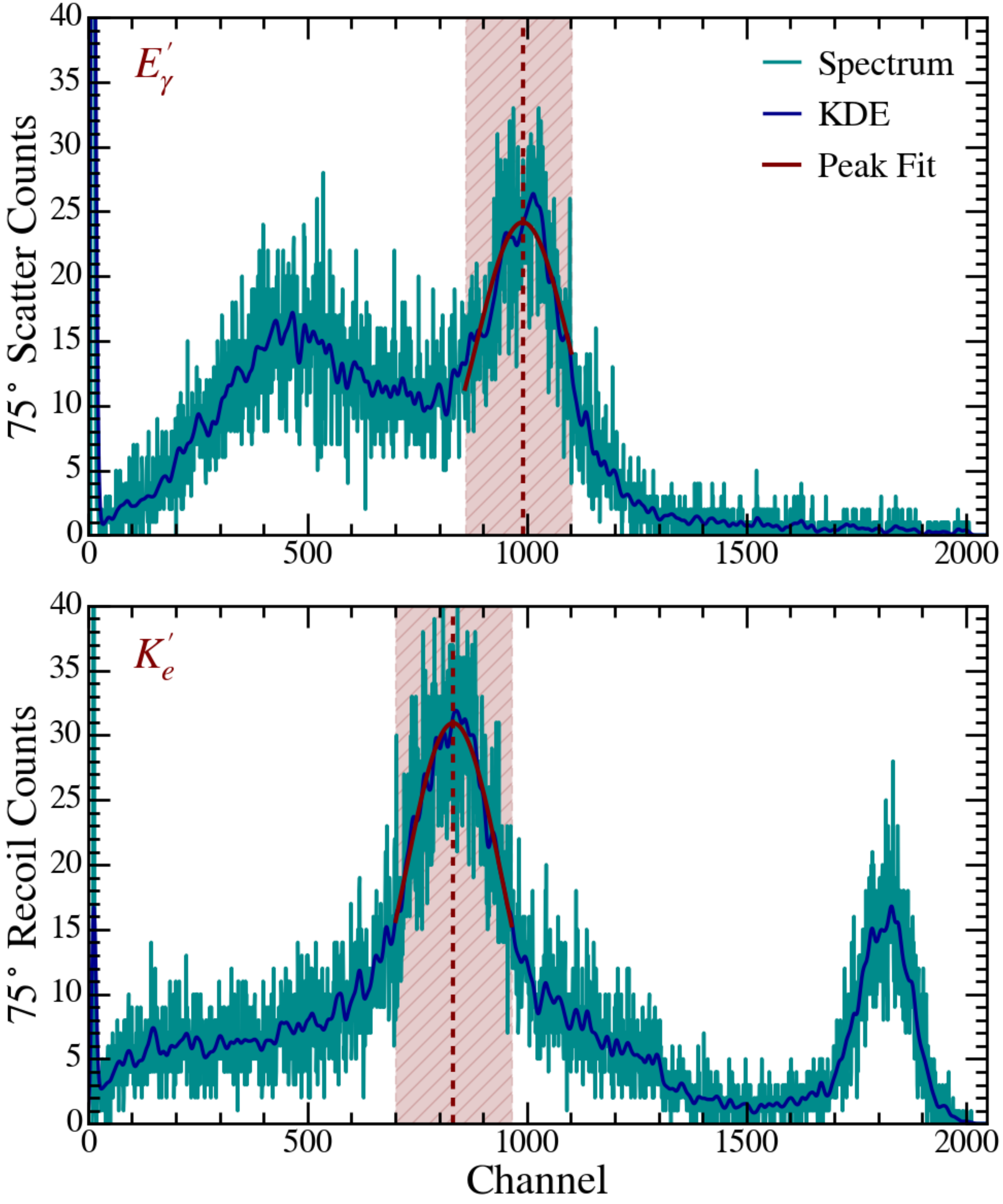




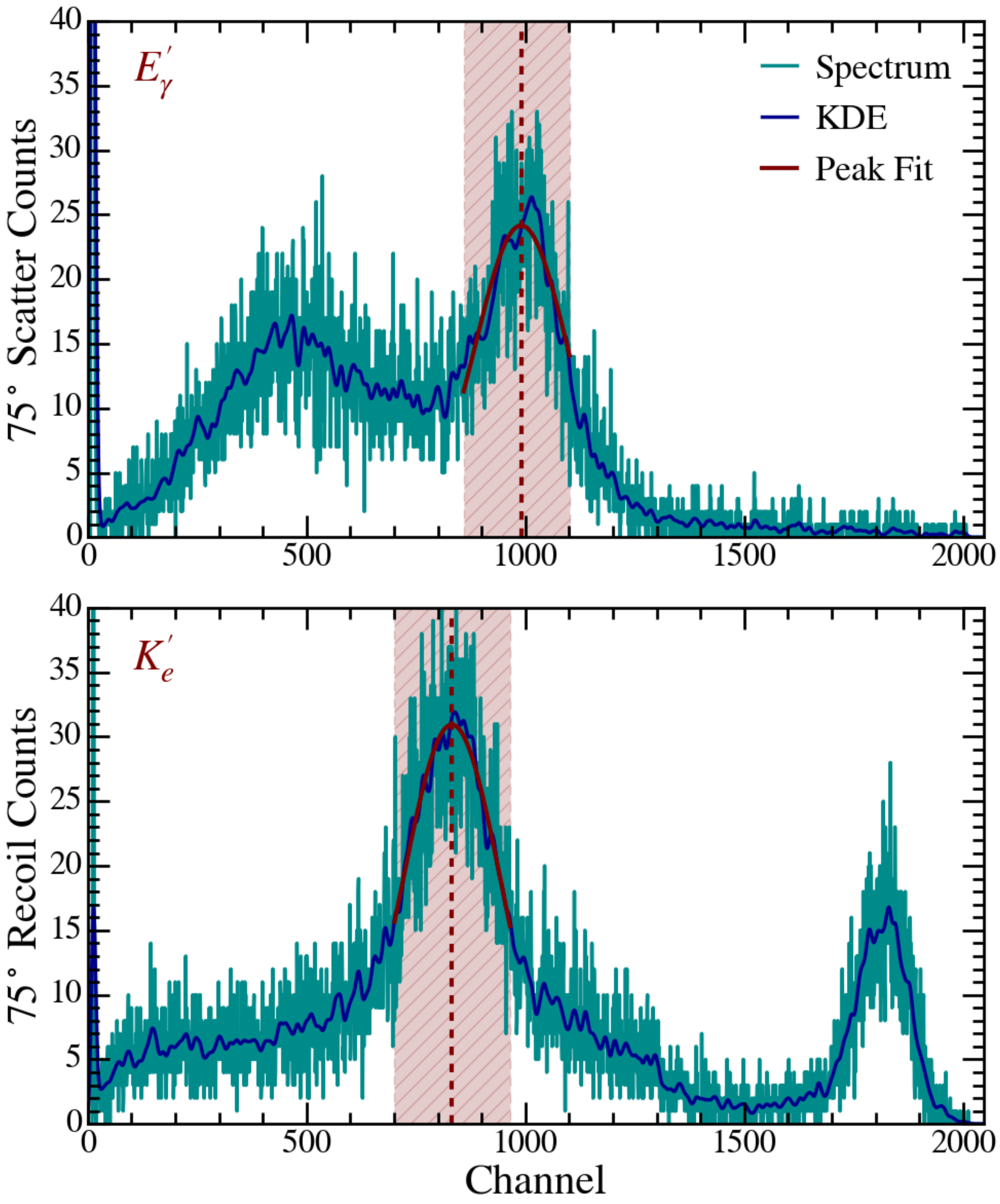
(Heath 1964)

Spectrum analysis



Spectrum analysis

- Apply a Gaussian filter ($\sigma = 5$ channels) to smoothen the data.



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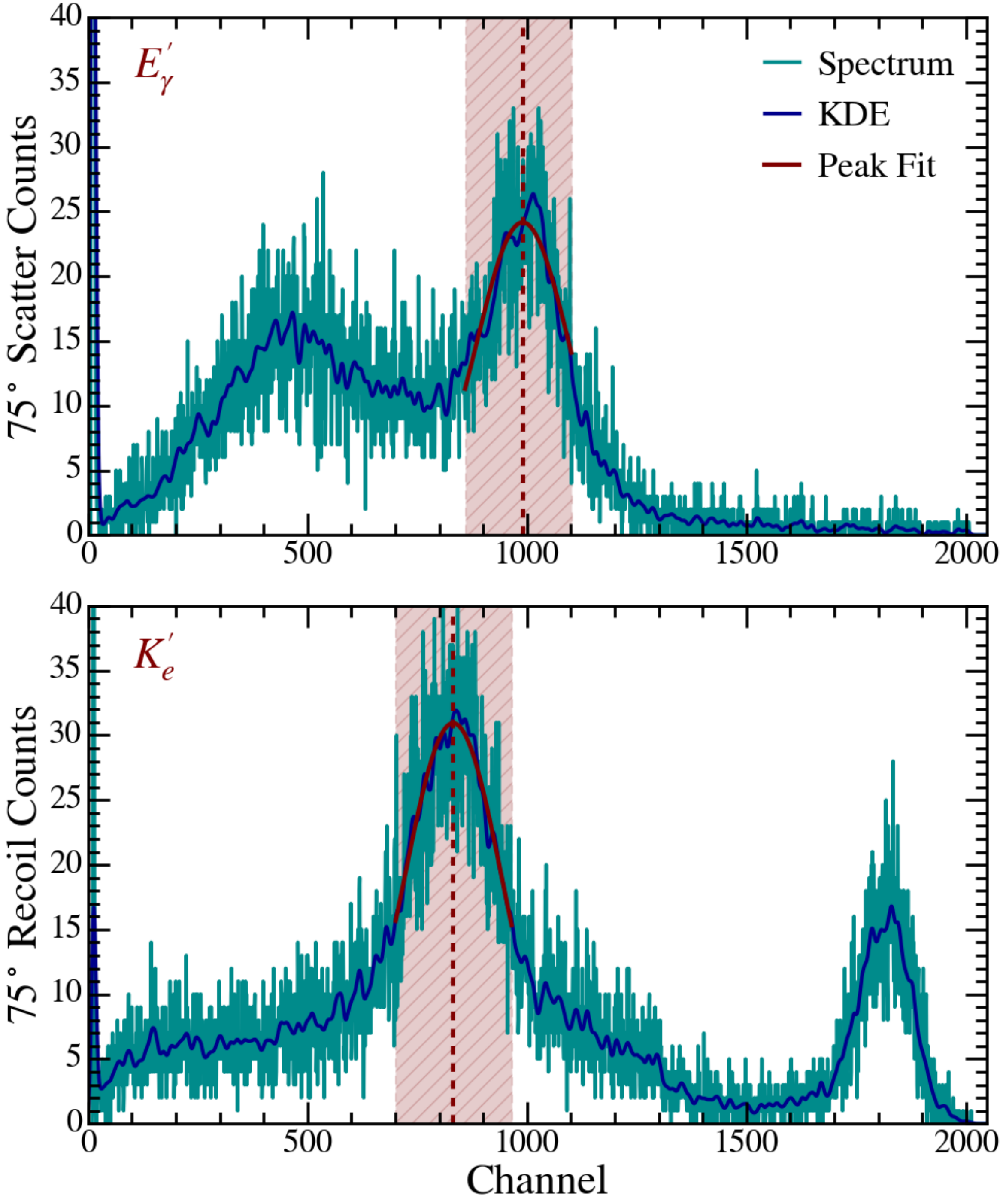

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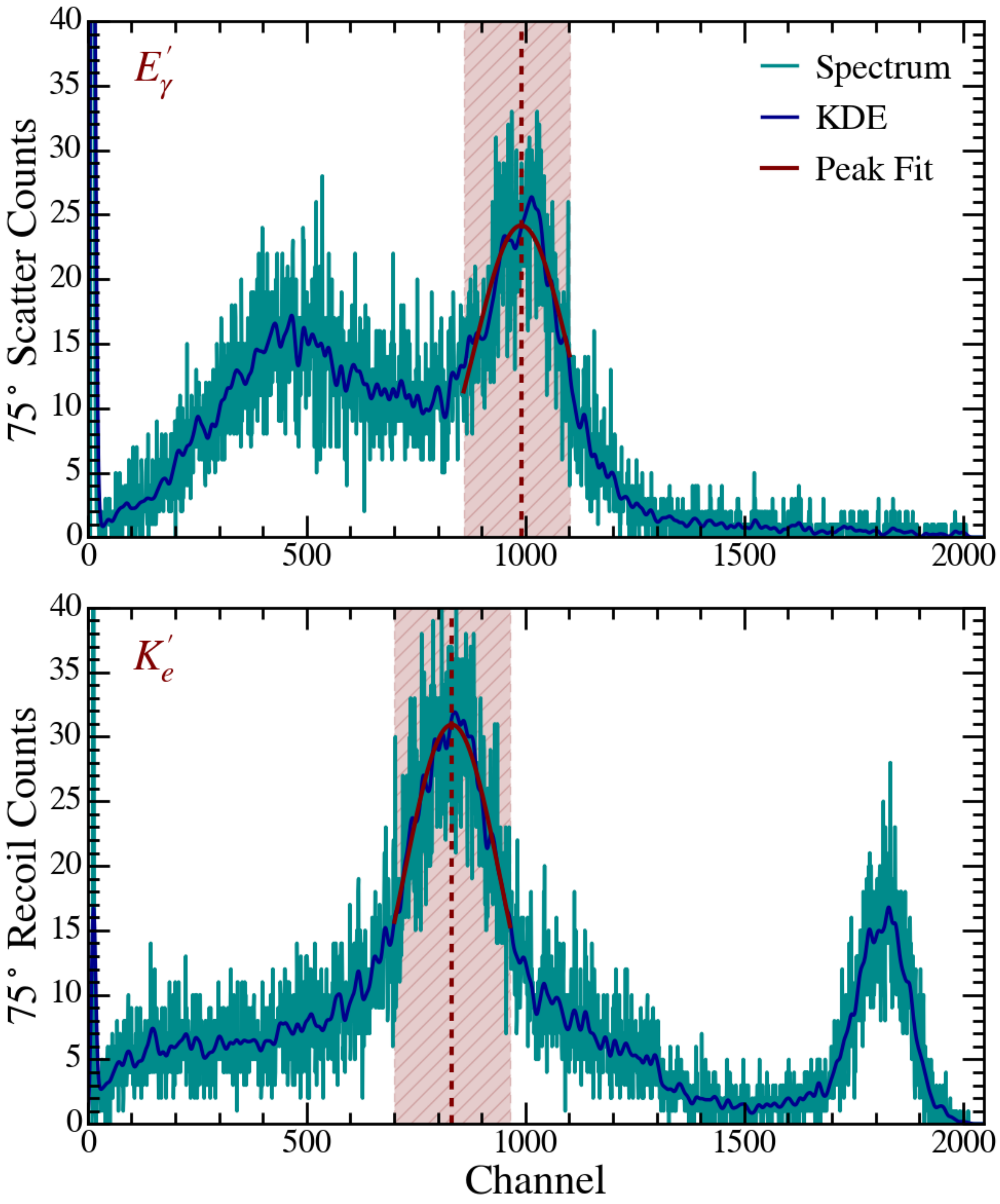

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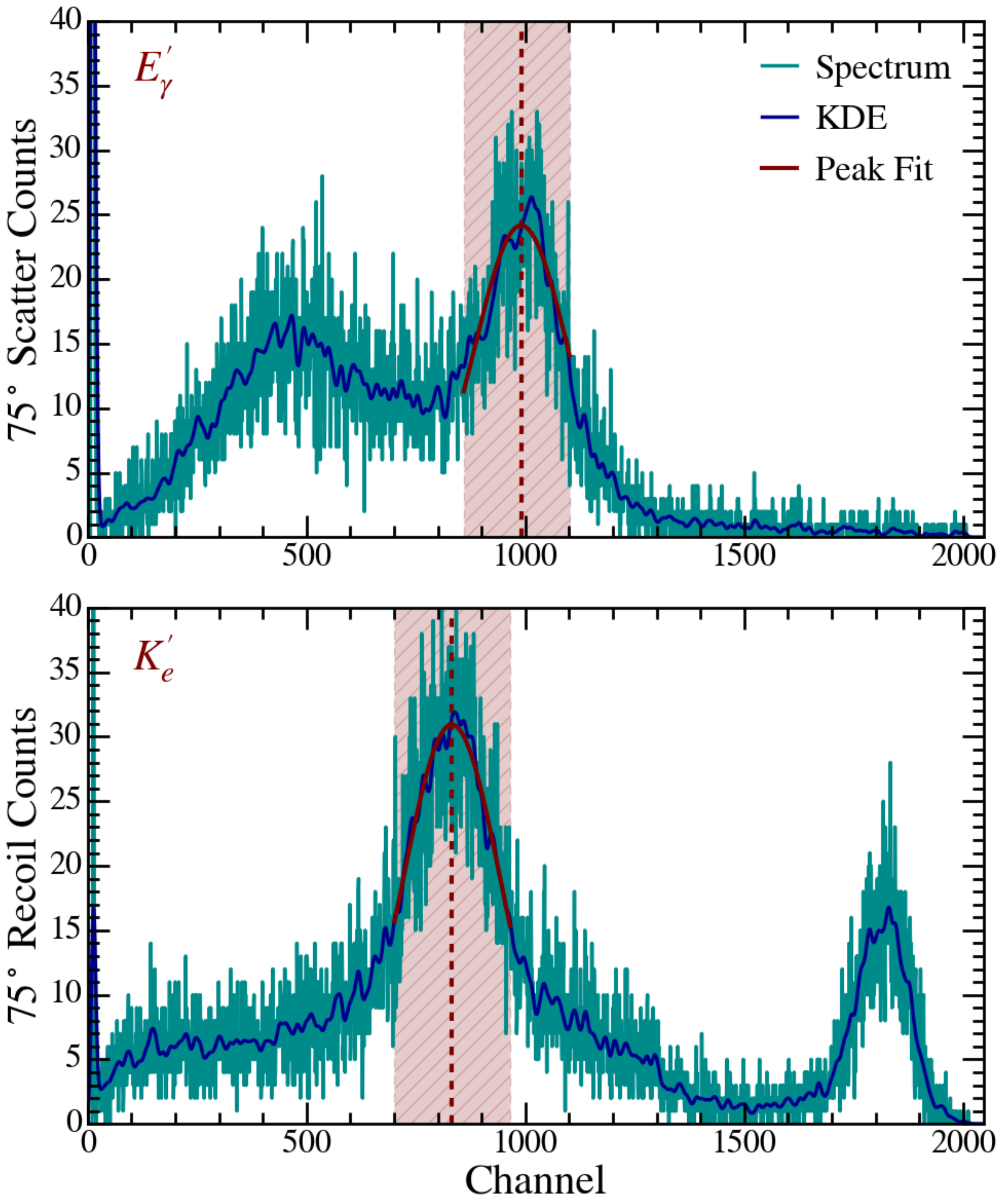
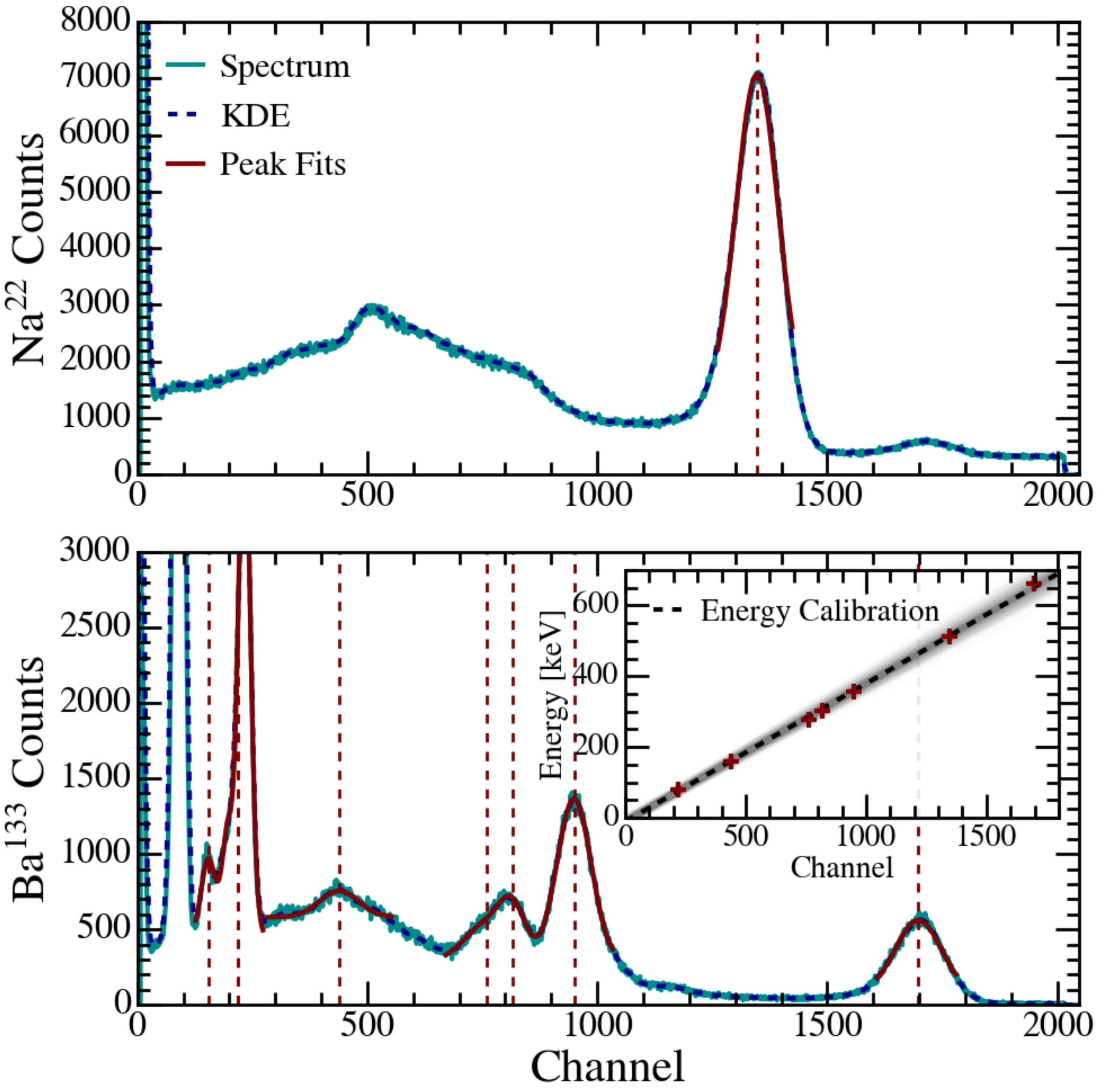
- For Compton scattering data, use Poisson-based likelihood approach

$$\chi^2 = -\sum 2 \log \frac{\hat{y}_i^{y_i} e^{-\hat{y}_i}}{y_i!} + \log 2\pi y_i$$



Calibration result

- Example: Recoil scintillator, March 4th 2025



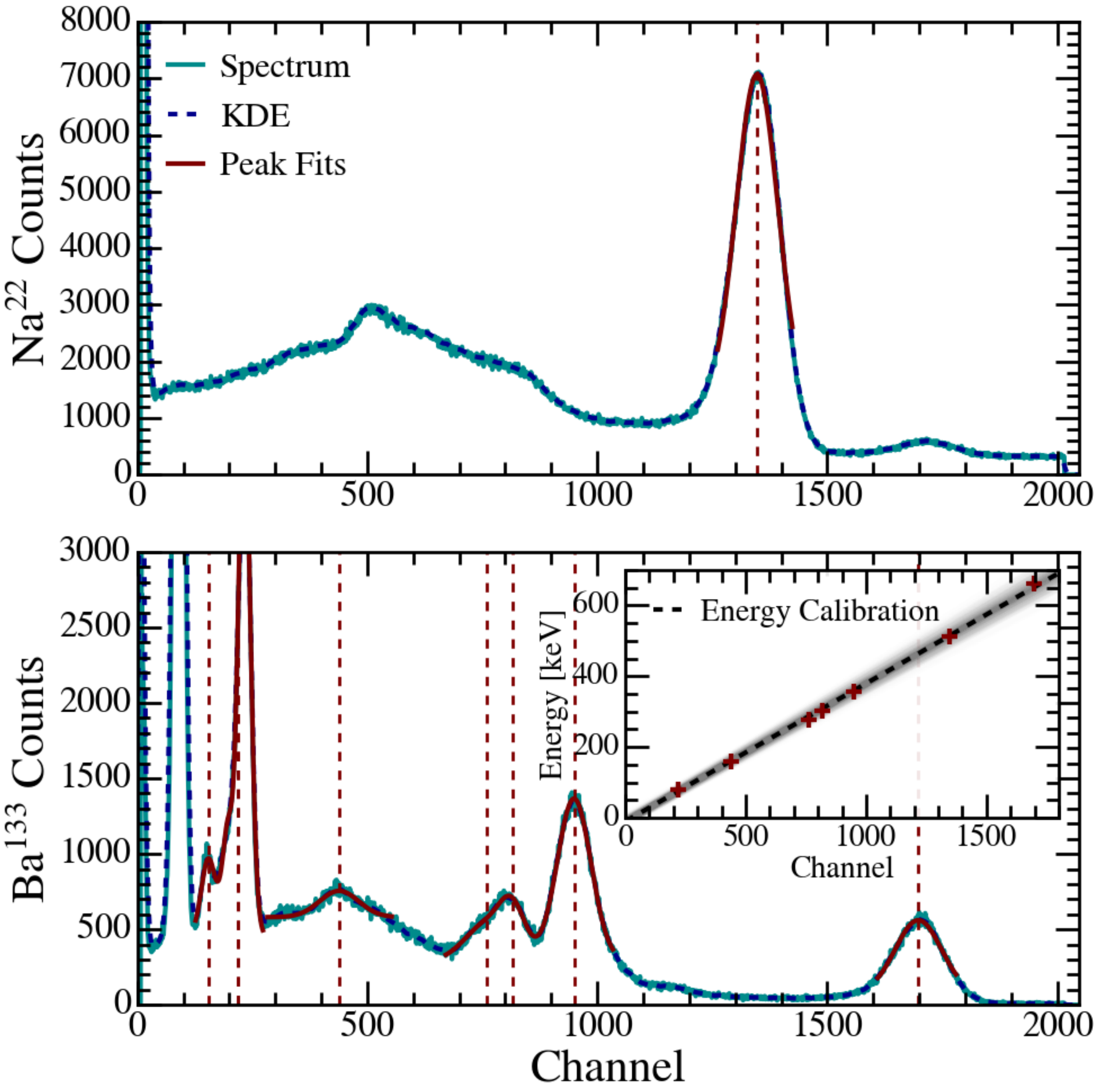
Calibration result

- Example: Recoil scintillator, March 4th 2025
- Channel-energy conversion relation

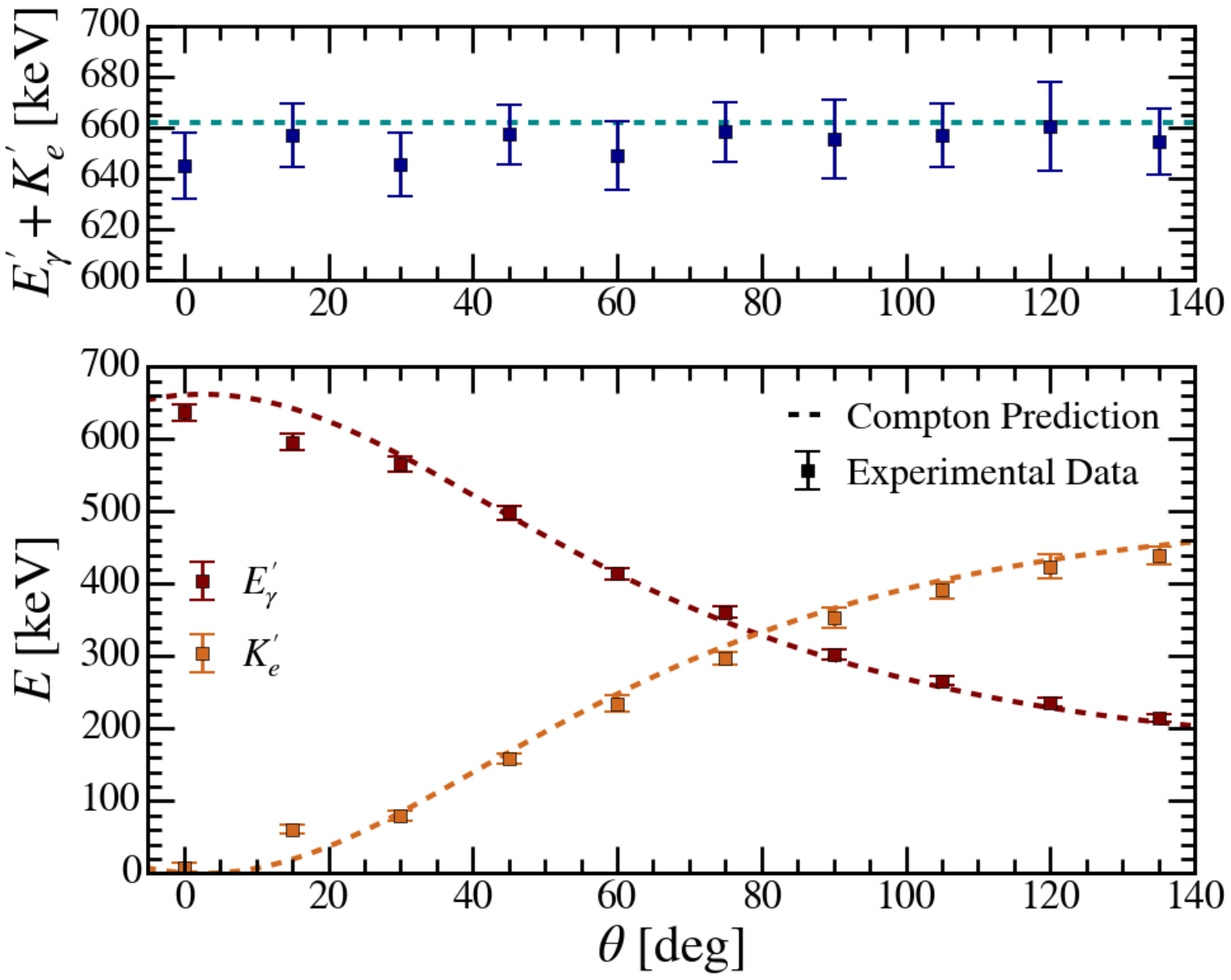
$$E_i = \alpha C_i + b$$

$\alpha = 0.390 \pm 0.014\ \mathrm{keV}$

$b = -10.5 \pm 6.2\ \mathrm{keV}$

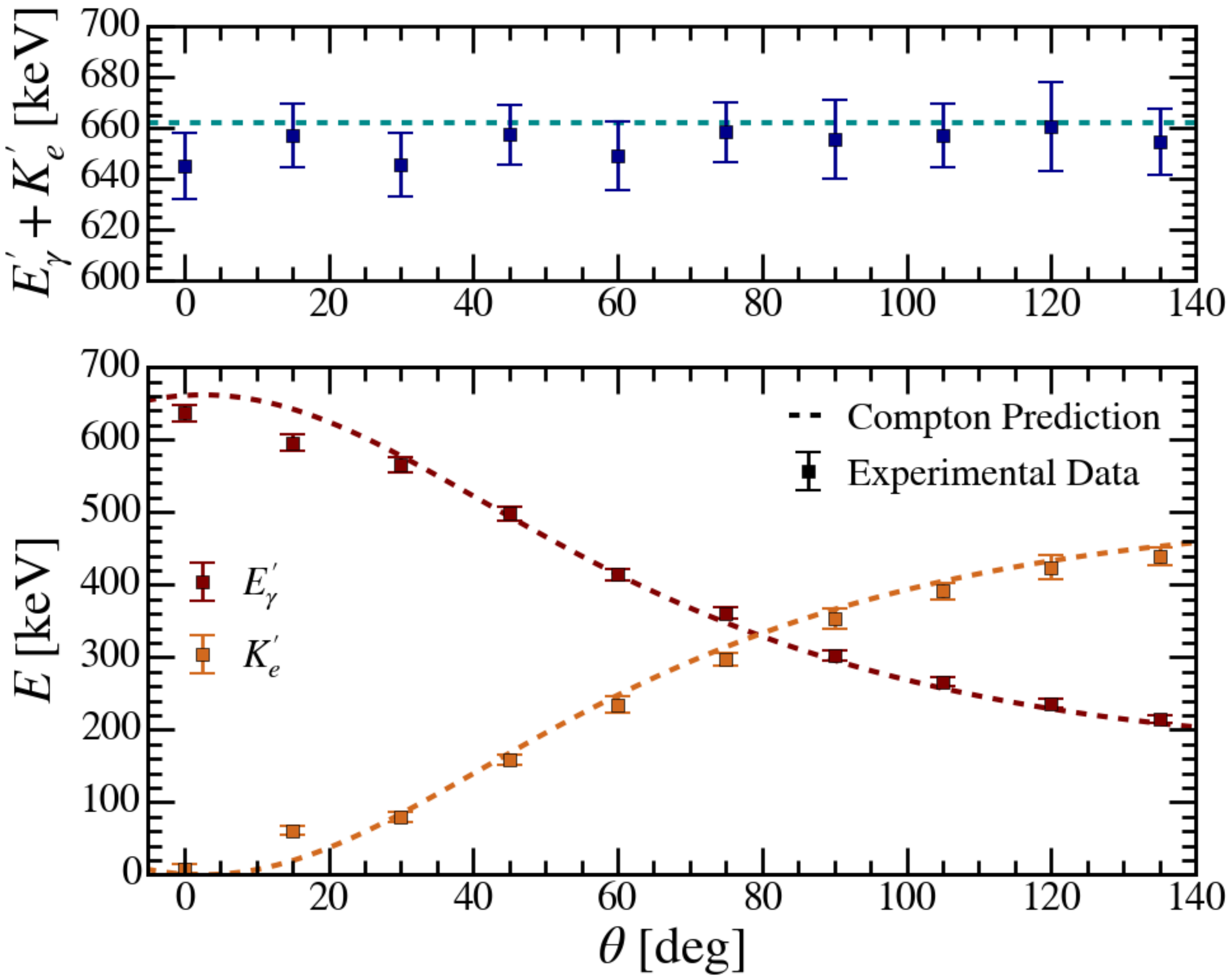


Energy-angle dependency



Energy-angle dependency

- The energies of the incident photons are well-reconstructed. Systematic errors of $\sigma_{E_\gamma}^{\text{sys}} \sim 10\ \text{keV}$.



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- The energies of the scattered photons and recoil electrons follow as expected from the Compton theory

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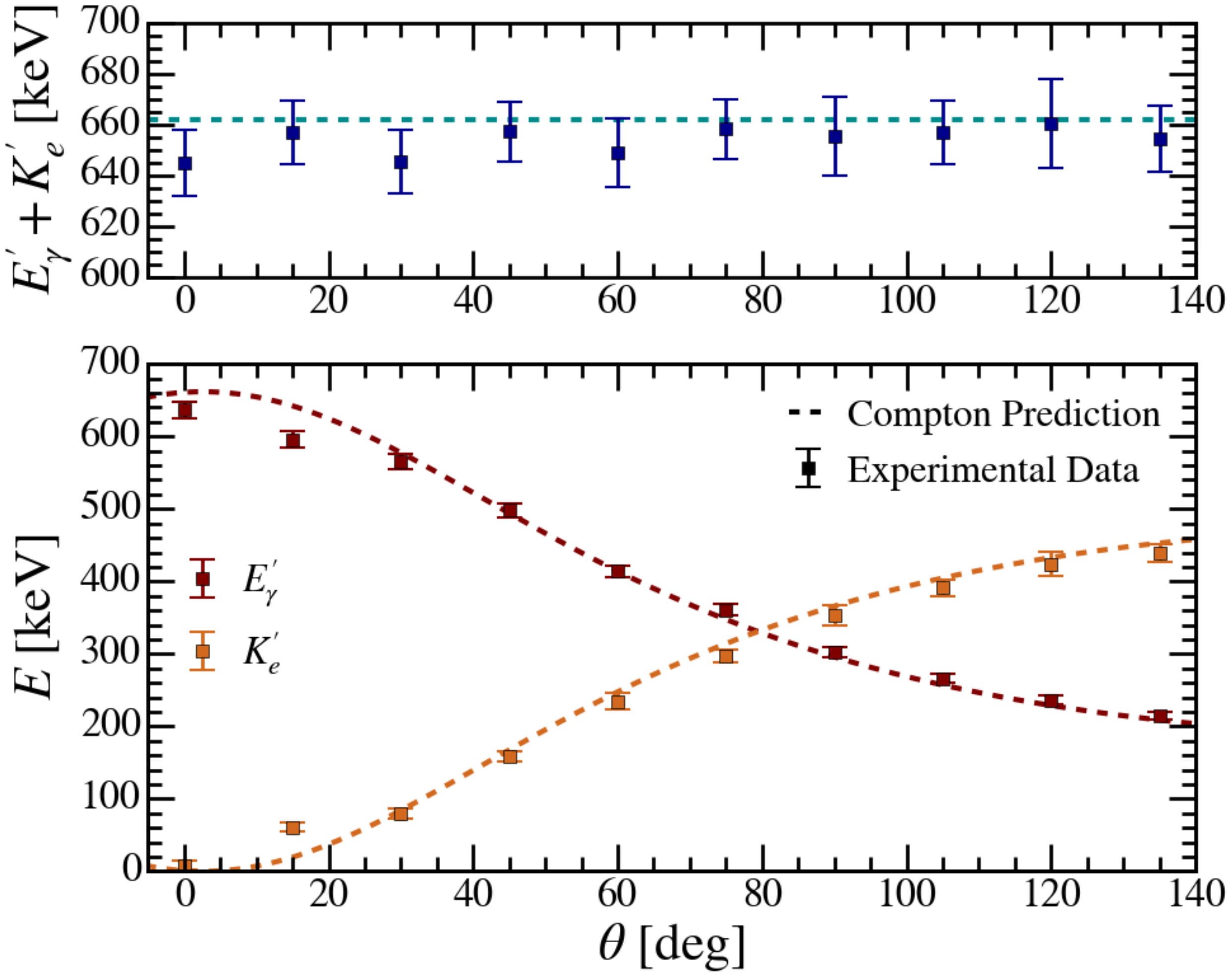
$$K'_e = E_\gamma - E'_\gamma$$



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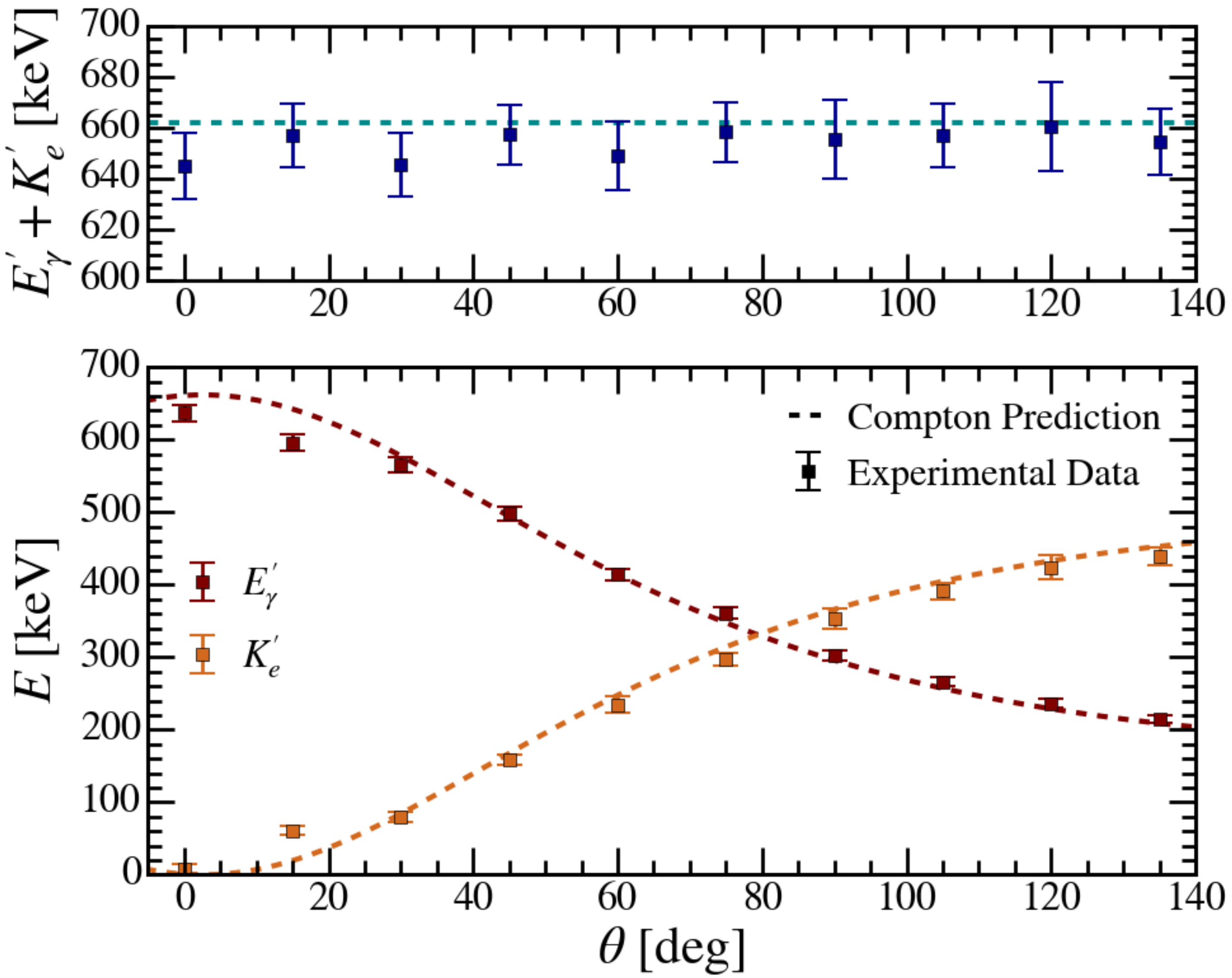
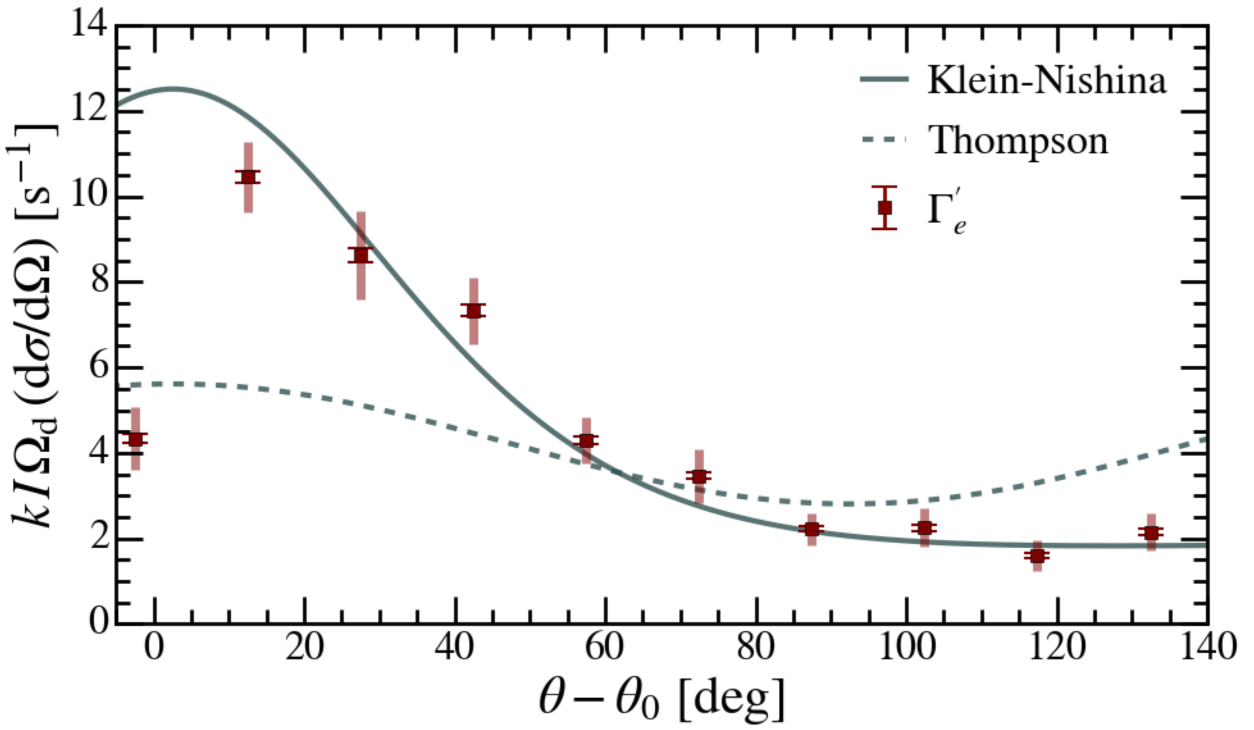
- Initial angle offset, $\cos\theta \rightarrow \cos(\theta - \theta_0)$

$\theta_0 = 2.56 \pm 0.71°$

$\sigma_{\theta_0} \sim \sigma_\theta^{sys} \sim 0.5°$



Scattering rate



Scattering rate

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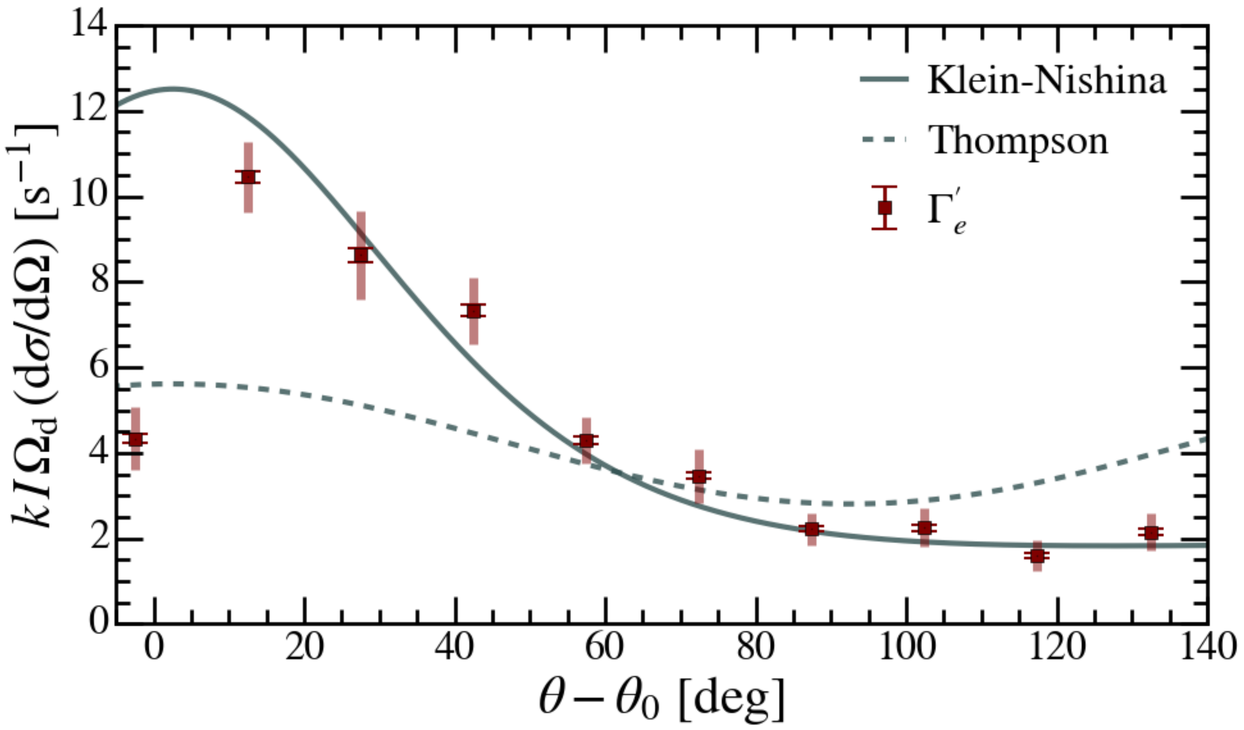

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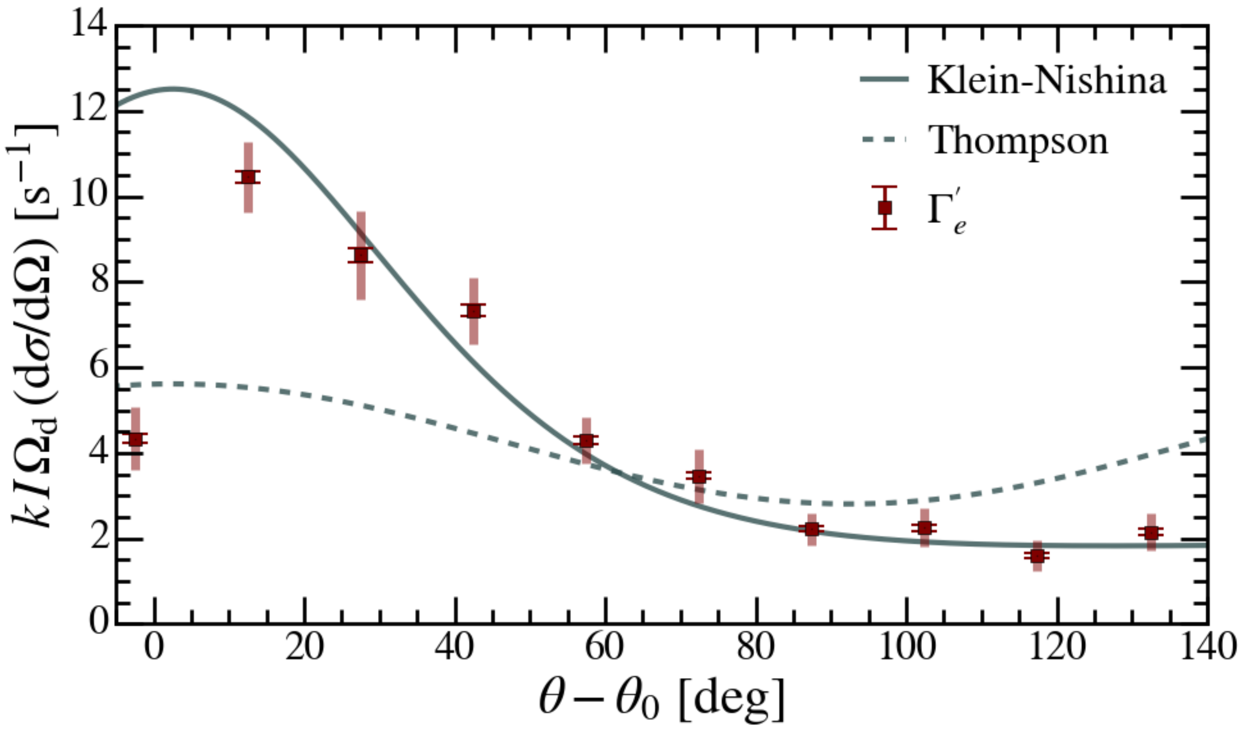

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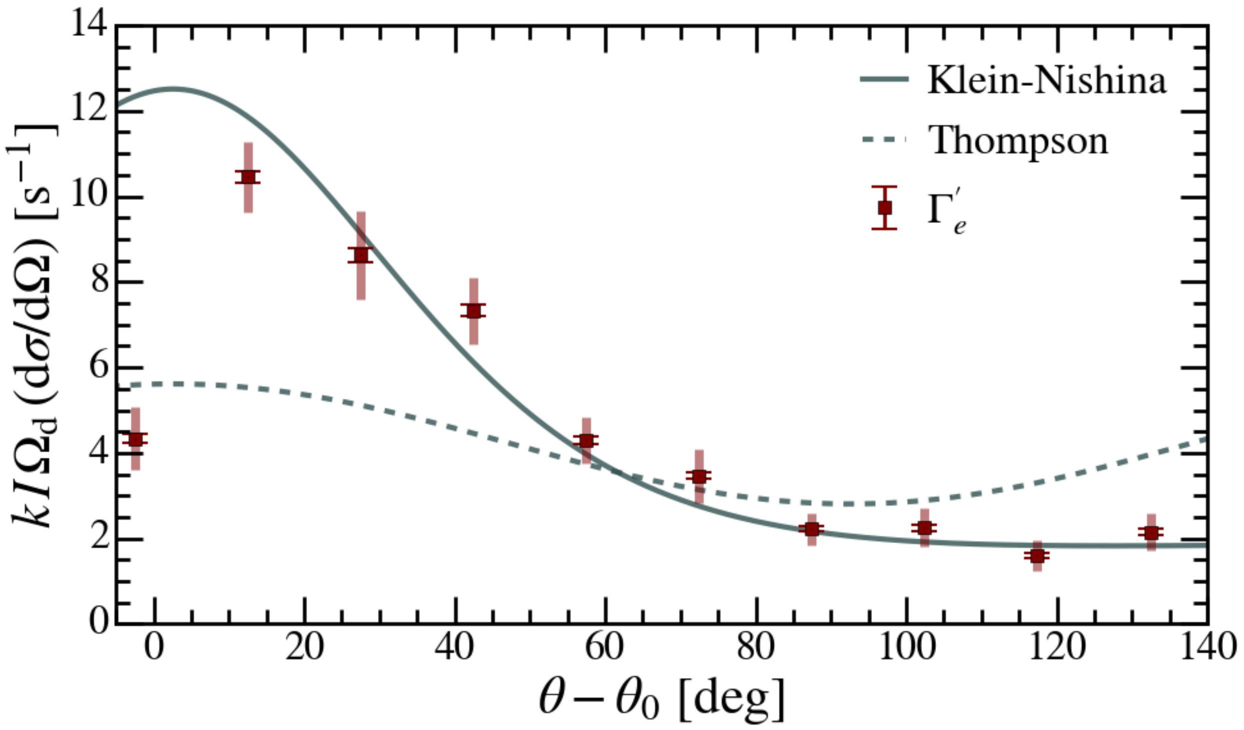
- Quantum relativistic treatment is necessary for the energy regime of $E_\gamma \sim 1\ \mathrm{MeV}$.



Conclusion

- Compton kinematics is confirmed!
- The quantum relativistic treatment of the Klein-Nishina prediction is necessary for the measured energy regime $E_\gamma \sim 1\ \mathrm{MeV}$.