HCMC UNIVERSITY OF TECHNOLOGY
Faculty of Applied Science
Department of Applied mathematics

FINAL EXAM - SEMESTER 1
Subject: Linear Algebra (TT)
Duration: 90 minutes Date: June 12^{th}, 2019

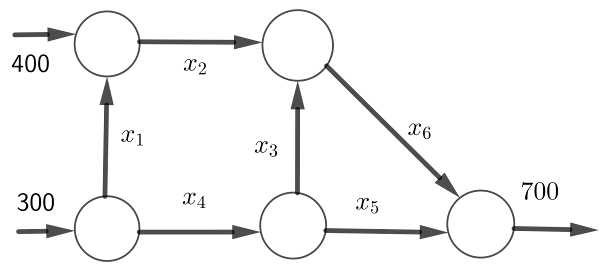
- *This is a closed book exam. Only your calculator is allowed. Total available score:* 10.
- *You MUST fill in your full name and student ID on this question sheet. There are 6 questions on 1 page.*
- *You MUST submit your answer sheets and this question sheet.* Otherwise, your score will be ZERO.

Full name: .... Student ID number: ....

Question 1. (1.5 points) To encode a message, we convert the text message into a stream of numerals by associating each letter with its position in the alphabet: A is 1, B is 2, ..., Z is 26, and assigning the number 27 to a space between two words. After that, we break the enumerated message above into a sequence of 3×1 column vectors and place them into a matrix A. Multiplying this matrix A by the Encoder K (the key), we obtain the encrypted message $B = KA$.

Decode the message, where $K = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 67 & 67 & 57 \\ 82 & 94 & 60 \\ 52 & 55 & 46 \end{bmatrix}$.

Question 2. (1.5 points) The figure shows the flow of traffic of a network. The diagram indicates the average number of vehicles per hour entering and leaving the intersections and the arrows indicate the direction of traffic flows. All streets are one-way. Setup and solve a system of linear equations to find the possible flows x_1, x_2, x_3, x_4, x_5, and x_6.



Question 3. In $\mathbb{R}_3$ with the inner product $(x, y) = 2x_1y_1 + 2x_1y_2 + 2x_2y_1 + 3x_2y_2 + 4x_3y_3$, let $F = \{x \in \mathbb{R}_3 | x_1 + x_2 + 3x_3 = 0\}$ be a subspace.

a. (1 point) Find the dimension and a basis for $F^{\perp}$.

b. (1 point) Find the vector projection of $u = (3, -2, 1)$ onto F.

Question 4. Let $f : \mathbb{R}_3 \to \mathbb{R}_3$ be a linear transformation satisfying:
$f(1, 1, 2) = (1, 2, 3), f(1, 2, 2) = (-2, 1, -4), f(1, 2, 3) = (1, 7, 5)$ and $E = \{(1, 1, m), (1, 2, -1), (1, 0, 1)\}$ be a vector set of $\mathbb{R}_3$.

a. (1 point) Find all real numbers m such that E is linearly independent.

b. (1 point) Find the dimension and a basis for $\ker(f)$.

c. (1 point) Prove that with $m = 2$, E is a basis of $\mathbb{R}_3$, and then find the transformation matrix A_E in the basis E.

Question 5. Two competing companies offer mobile phone service to a city with 100 000 households. Suppose that each citizen uses one of these services. Every month, 15% of the subscriber of company A changes to use the service of company B, and 10% of company B's subscribers changes to use the service of A. Company A now has 60 000 subscribers and Company B has 40 000 subscribers.

a. (0.5 point) How many subscribers will each company have after 2 months?

b. (1.5 point) Using diagonalization method to find the numbers of subscribers after 10 years. Round each answer to the nearest integer.

Department of Applied Mathematics **Lecturer**

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