

## Practice test

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## Question 1

0/1 pt 999 998

In  $\mathbb{R}^3$  given the linear mapping  $f$  be the orthogonal projection onto the plane  $1x + 6y + 2z = 0$ . Find the image of the triangle with vertices  $A(-9, 4, 0)$ ,  $B(-3, 9, -8)$ ,  $C(-10, 6, 5)$ .

The standard matrix of the  $f$  is

$$A_f =$$


(Input results as fraction)

Input the answer as the coordinates of image A'B'C'

$$A' =$$

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(Round the results to 4 decimal digits.)

$$B' =$$

--	--	--

(Round the results to 4 decimal digits.)

$$C' =$$

--	--	--

(Round the results to 4 decimal digits.)

## Question 2

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Given the linear mapping in  $\mathbb{R}^2$  be the rotation about the origin an angle  $\frac{\pi}{3}$  by counterclockwise direction. Find the image of the vector  $\vec{u} = (-1, -1)$

$$f(\vec{u}) =$$

$$\left[ \begin{array}{cc} \boxed{\phantom{0000}} & \boxed{\phantom{0000}} \end{array} \right]$$

(Round the result to 4 decimal digits)

### Question 3

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In  $\mathbb{R}^3$  given the linear mapping be the reflection through the plane  $8x + 1y + 2z = 0$ . Find the image of the triangle with vertices  $A(-4, 9, 10)$ ,  $B(-6, -1, 4)$ ,  $C(5, 2, -5)$ . Round the results to 4 decimal digits.

Input the answer as the coordinates of image A'B'C'

$$A' =$$

$$\left[ \begin{array}{ccc} \boxed{\phantom{0000}} & \boxed{\phantom{0000}} & \boxed{\phantom{0000}} \end{array} \right]$$

$$B' =$$

$$\left[ \begin{array}{ccc} \boxed{\phantom{0000}} & \boxed{\phantom{0000}} & \boxed{\phantom{0000}} \end{array} \right]$$

$$C' =$$

$$\left[ \begin{array}{ccc} \boxed{\phantom{0000}} & \boxed{\phantom{0000}} & \boxed{\phantom{0000}} \end{array} \right]$$

### Question 4

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In  $\mathbb{R}^2$  given the linear mapping be the reflection through the line  $5x + -1y = 0$ . Find the image of the triangle with vertices  $A(-7, -8)$ ,  $B(-6, 8)$ ,  $C(-2, -10)$ .

The matrix representation  $A=$


Input the answer as the coordinates of image  $A'B'C'$ .

$A'=$

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$B'=$

--	--

$C'=$

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● Question 5

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In  $\mathbb{R}^3$  given the linear mapping  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  such that  $f(1, -2, 0) = (-3, -10, 2)$ ,  
 $f(0, 1, 0) = (10, -9, -3)$ ,  $f(-3, 2, 1) = (9, 7, 6)$ .

Find the matrix representation in the basis  $E = \{(1, 1, 2), (1, 2, 1), (1, 1, 1)\}$ .

$A_E =$

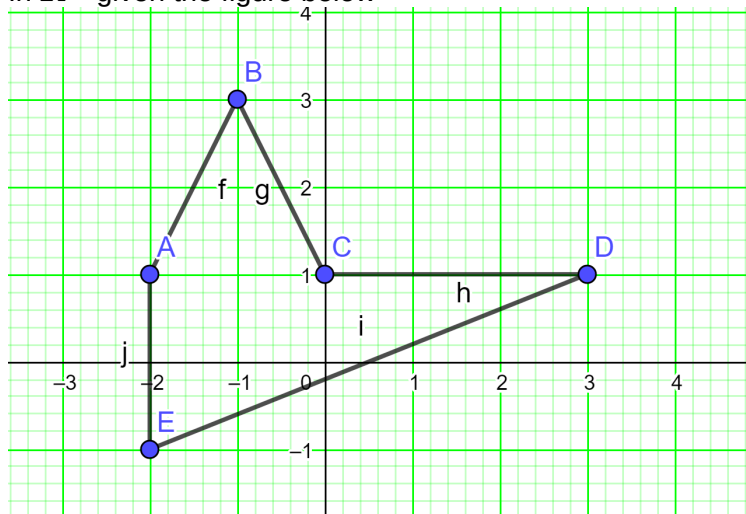

Given that  $x = (8, 2, -10)$ . Find  $[f(x)]_E$

$[f(x)]_E =$


Find the dimension of Kerf

dim Kerf=

In  $\mathbb{R}^2$  given the figure below



with  $A(-2, 1)$ ,  $B(-1, 3)$ ,  $C(0, 1)$ ,  $D(3, 1)$ ,  $E(-2, -1)$ . Find the image of this figure of the linear transformation  $f$  by performing two successive linear transformations  $f_1, f_2$ , where  $f_1$  is the rotation clockwise about origin angle  $5\frac{\pi}{6}$  and  $f_2$  is the reflection across the line  $-3x - 2y = 0$ .

The standard matrix of  $f$  (round the result to 2 decimal digits)=


The matrix image =


Given  $E$  be a basis in  $\mathbb{R}^2$ , two vectors  $u_1, u_2$ ,  $[u_1]_E = [-1 \ -5]^T$ ,  $[u_2]_E = [6 \ 6]^T$ . Let  $f$  be a linear mapping  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ , and  $[f(u_1)]_E = [-2 \ 2]^T$ ,  $[f(u_2)]_E = [-2 \ 2]^T$ .

a) Find the matrix representation of  $f$  with respect to the basis  $E$ . (Hint: use the formula  $[f(u)]_E = A_E[u]_E$ )

$A_E =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

b) If  $E = \{(1, 3)^T, (8, -3)^T\}$ , find the  $f\left(\begin{bmatrix} -2 \\ -6 \end{bmatrix}\right)$ .

$f\left(\begin{bmatrix} -2 \\ -6 \end{bmatrix}\right) =$

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

### Question 8

0/1 pt 999 998

Suppose that the oldest age attained by the females in a certain animal population is 15 years and we divide the population into three age classes with equal durations of five

years. Let the Leslie matrix for this population be  $\begin{bmatrix} 0 & 4 & 3 \\ 0.25 & 0 & 0 \\ 0 & 0.33 & 0 \end{bmatrix}$ . The vector of age distribution

now is  $\begin{bmatrix} 23 \\ 17 \\ 18 \end{bmatrix}$ .

The probability for a 4-year-old animal to survive to the age of 8 is  (%).

The number of animals whose age from  $5 \leq \text{age} \leq 10$  after 2 years:

## ● Question 9

✓ 0/1 pt ↻ 999 ⇌ 998

Given the equation  $AX + B^T = 5X + 3C$ , where  $A = \begin{bmatrix} 6 & 0 & 0 \\ -8 & 6 & 0 \\ 3 & 0 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 8 & 7 \\ 5 & -7 & 6 \\ -3 & 9 & 4 \end{bmatrix}$

$, C = \begin{bmatrix} 4 & -9 & 1 \\ -7 & 10 & 7 \\ 8 & -3 & -2 \end{bmatrix}$ .

Find X.

X=


## ● Question 10

✓ 0/1 pt ↻ 999 ⇌ 998

Two competing companies(A,B) offer network service to a population of 109557 consumers. During any month, company A consumer has a 20% probability of switching to company B and a 5% probability of not using service of either company. A consumer of company B has a 15% probability of switching to company A and a 10% probability of not using service. A nonuser has a 15% probability of purchasing company B and 10% using service of A. How many people will be in each group after 3 months? Knowing that at this month, there are 18827 subscribers of A, 28948 subscribers of B.

Matrix transition=

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$X^{(3)} =$

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

● Question 11

✓ 0/1 pt ↺ 999 ↻ 998



A machine can be either working or broken down on any given day. If it is working, it will break down in the next day with probability 5 %, and will continue working with probability 95 %. If it breaks down on a given day, it will be repaired and be working in the next day with probability 80%, and will continue to be broken down with probability 20%. Given that the probability that the machine works today is 81% .

1) The matrix transition in this model is:  $A =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

2) The probability of working state on tomorrow of the machine is:

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

( The result in percentage form)

3) The probability of working state after 4 days of the machine is

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

### Question 12

0/1 pt 999 998

Find the matrix  $X$  that satisfies the matrix equation  $XB + 3XA^T = C$ ,

where  $A = \begin{bmatrix} -1 & 1 \\ -3 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 7 & 12 \\ -6 & 7 \end{bmatrix}$ ,  $C = \begin{bmatrix} -2 & 3 \\ -1 & 5 \end{bmatrix}$ .

Answer:

$X =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

### Question 13

0/1 pt 999 998

In  $\mathbf{R}^3$  given inner product:  $\langle x, y \rangle = 4x_1y_1 + 5x_2y_2 + 4x_3y_3 - 2x_2y_3 - 2x_3y_2$  and a subspace  $F = \text{span}\{(1, 2, 1), (2, -1, 4), (-18, -6, -30)\}$ .

Find  $\langle x, y \rangle$  if  $x = (-6, -7, 4), y = (-5, -6, -9)$

Find the dimension and a basis of  $F^\perp$ .

$\langle x, y \rangle =$

Upload your hand writing solution for the second question here.

● Question 14

✓ 0/1 pt ↻ 999 ⇄ 998

In  $\mathbb{R}^3$ , given inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Given  $F = \text{span}\{(1, 3, 5), (2, 5, 7), (0, 1, 3), (3, 8, 12)\}$ . Find orthogonal projection of vector  $x = (-4, 4, 4)$  onto  $F$ . Round the result to 2 decimal digits.

$\text{Proj}_F x =$

$\begin{bmatrix} \text{ } \\ \text{ } \\ \text{ } \end{bmatrix}$

Find the dimension and a basis of  $F^\perp$  (Upload your hand writing solution).

● Question 15

✓ 0/1 pt ↻ 999 ⇄ 998

Given inner product in  $\mathbb{R}^3$

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Given the subspace  $F = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3, x_1 - x_2 + 3x_3 = 0\}$ .

Find orthogonal projection of vector  $x=(5,5,2)$  onto  $F$ . Round the result to 2 decimal digits.

$Proj_F x =$

$$\begin{bmatrix} \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \end{bmatrix}$$

● Question 16

✓ 0/1 pt ↻ 999 ⇌ 998

In  $\mathbb{R}^3$ , given the inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1.$$

Vector  $x = (6, 5, 9)$ ,  $y = (-5, -7, -6)$ . Find the distance between  $x, y$ . Round to 4 decimal digits.

$d(x, y) = \boxed{\phantom{00000000}}$

● Question 17

✓ 0/1 pt ↻ 999 ⇌ 998

In  $\mathbb{R}^3$ , given inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Vector  $x=(0,-6,-7)$ ,  $y=(2,-4,3)$

Find  $\langle x, y \rangle$ , the distance between  $x, y$  and the angle between  $x$  and  $y$ .

$\langle x, y \rangle = \boxed{\phantom{00000000}}.$

Distance =  $\boxed{\phantom{00000000}}$  (Round the result to 2 decimal digits)

Angle =  $\boxed{\phantom{00000000}}$  (Round the result to 2 decimal digits)

● Question 18

✓ 0/1 pt ↻ 999 ⇌ 998

Find the eigenvalues of the matrix  $\begin{bmatrix} 5 & -10 \\ 5 & -10 \end{bmatrix}$ .

The eigenvalues are  $\lambda =$   (enter the eigenvalues, separated by commas)

Question Help: [Video](#) [Written Example](#)

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● Question 19

✓ 0/1 pt ↻ 999 ↺ 998

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In a city, there are 3 supermarkets A; B and C. In this month there are `52743` householders going to A, `46900` going to B and `30355` going to C. The transition after each month as:

`A -gt B: 7.5 % , A -gt C: 2.5 % , B -gt A: 1.5 % , B -gt C: 5 % , C -gt A: 10 % , C -gt B: 7.5 %` .

What is the matrix transition P?

P=

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

How many householders that go to A, B and C after 1 month?

The number of householders going to A,B,C (Round the answer to the nearest integer)=

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

Find the number of householders in each group after 1 year?

The eigenvalues of P (input the answer from biggest to smallest value of eigenvalues)=

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

The number of householders A,B,C after 1 year (Round the answer to the nearest integer)=

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

Given the matrix  $A = \begin{bmatrix} -9 & 2 & -1 \\ -6 & -2 & 9 \\ -4 & 4 & m \end{bmatrix}$ . Find  $m$  such that  $\lambda = 8$  is an eigenvalue of  $A$ .

● Question 21

✓ 0/1 pt ↻ 999 ⇌ 998

Given matrix  $A = \begin{bmatrix} -23 & 66 & 30 \\ -24 & 67 & 30 \\ 34 & -94 & m \end{bmatrix}$ , find  $m$  such that vector  $X = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$  is an eigenvector of  $A$ .

$m =$

● Question 22

✓ 0/1 pt ↻ 999 ⇌ 998

Given two square matrices size  $2 \times 2$   $A$  and  $B$  such that  $|A| = -4$ ,  $|B| = 3$ . Applying 2 successive row operations on  $A$  to obtain  $A_1$  as the following:  $r_1 \leftrightarrow r_2$ , (interchanging  $r_1$  and  $r_2$ ),  $r_2 \rightarrow -10r_2 + 3r_3$  (replacing  $r_2$  by multiplying row 2 by  $-10$  and adding to row 3 time 3). Compute  $|2A_1B^{-1}|$ .

Answer= \_\_\_\_\_.

● Question 23

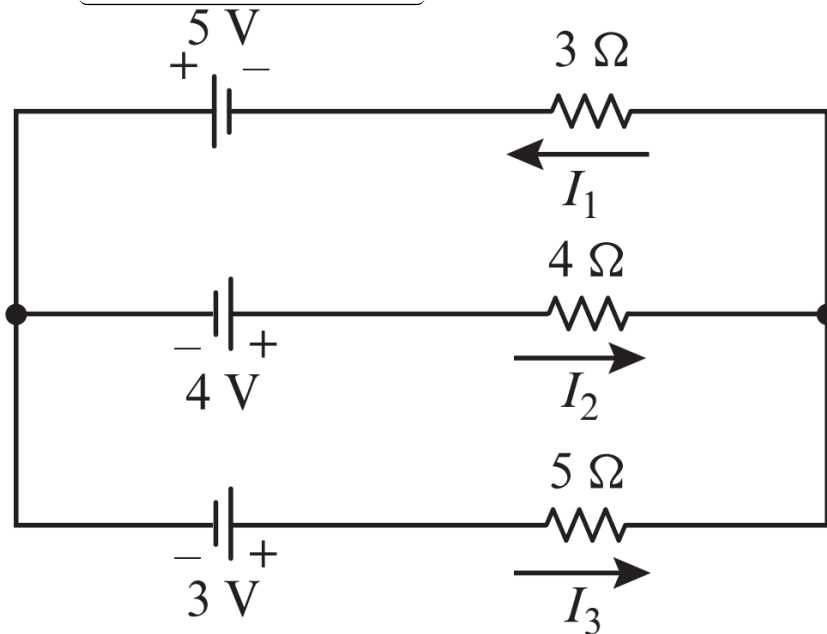
✓ 0/1 pt ↻ 999 ⇌ 998

Find all the unknown current on the circuit.(Round the result to 2 decimal digits).

$I_1 =$  \_\_\_\_\_

$I_2 =$  \_\_\_\_\_

$I_3 =$  \_\_\_\_\_



● Question 24

0/1 pt 999 998

In  $\mathbb{R}^2$  given the linear mapping  $f$  be the reflection through the line  $-6x + 8y = 0$  and the triangle  $ABC$ . We know that  $f(ABC) = A'B'C'$ , where  $A(6, 10)$ ,  $B(-3, -6)$ ,  $C(-6, 3)$ . Find the coordinates of the points  $A, B, C$ .

The standard matrix representation  $A$  of  $f$  is =

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

Input the answer as the coordinates of  $A, B, C$ .

$A =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$B =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$C =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

### Question 25

0/1 pt 999 998

A batch of 134 computers has 3 types of products: 8 millions VND computers, 6 millions VND computers and 4 millions VND computers. If the value of the batch is 744 millions VND and the total value of the 8 millions VND computers and 6 millions VND computers is twice of the value of 4 millions VND computers, then how many of each type of computers are in the batch?

Answer:

The number of 8 millions VND computers is \_\_\_\_\_

The number of 6 millions VND computers is \_\_\_\_\_

The number of 4 millions VND computers is \_\_\_\_\_



## Question 26

0/1 pt 999 998

Find  $m$  such that the following system has a unique solution  $\{(6y+z, -2x+6y+2z, 7x+2y+mz)\}$

Answer:  $m =$

## Question 27

0/1 pt 999 998

Find all **values** of  $m \in \mathbb{R}$  such that the system of vectors  $\{(-2, (10), (1)), ((7), (m), (1)), ((5), (7), (2))\}$  is linearly dependent.

Answer:  $m =$  .

## Question 28

0/1 pt 999 998

In  $\mathbb{R}^3$ , given 3 bases  $S_1, S_2, S_3$  such that the matrix change of bases are:  $P_{S_1 \rightarrow S_2} = [(1, -3, 0), (-2, 1, 2), (6, -11, -3)]$ ,  $P_{S_3 \rightarrow S_2} = [(1, 0, 0), (0, 1, -3), (6, -1, 4)]$ , knowing that

$[u]_{S_1} = [(-2), (4), (6)]$ . Find  $[u]_{S_3}$ .

$[u]_{S_3} =$

$$\begin{bmatrix} \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \end{bmatrix}$$

## Question 29

0/1 pt 999 998

Given the system of vectors  $S: \{(2, 1, 3), (2, 1, 1), (x-1, 2x, 3x)\}$ . Find  $x \in \mathbb{R}$  such that  $S$  is a spanning set of a two dimensional space.

$x =$   .s

## Question 30

0/1 pt 999 998

Given the set of vectors :  $S = \{ (-2, 1, 6), (-2, -10, -3) \}$  and let  $F$  be a subspace spanned by  $S$ , find all values  $m \in \mathbb{R}$  such that  $M = \{ (-8, -7, 15), (3, 5, m) \} \in F$ .

Answer:

$m$

### Question 31

0/1 pt 999 998

Given the equation  $AX + B^T = 7X + 5C$ , where  $A = [(8, 0, 2), (4, 8, 5), (-4, -1, 3)]$ ,  $B = [(-7, 10, 4), (6, -10, 2), (9, -3, -4)]$ ,  $C = [(-8, 9, 2), (6, -9, 7), (-6, 4, 10)]$ .

1) Find  $X$ .

2) Compute  $E = \|2A^2 C^{-1}\|$

1)  $X =$

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<input type="text"/>	<input type="text"/>	<input type="text"/>
<input type="text"/>	<input type="text"/>	<input type="text"/>

2)  $E =$   (Let the result in decimal number with 2 decimal digits rounded)