HCMC UNIVERSITY OF TECHNOLOGY
Faculty of Applied Science
Department of Applied mathematics

FINAL EXAM - SEMESTER 1
Subject: Linear Algebra (CC)
Duration: 90 minutes Date: June 13th, 2019

- *This is a closed book exam. Only your calculator is allowed. Total available score:* 10.
- *You MUST fill in your full name and student ID on this question sheet. There are 5 questions on 1 page.*
- *You MUST submit your answer sheets and this question sheet.* Otherwise, your score will be ZERO.

Full name: .... Student ID number: ....

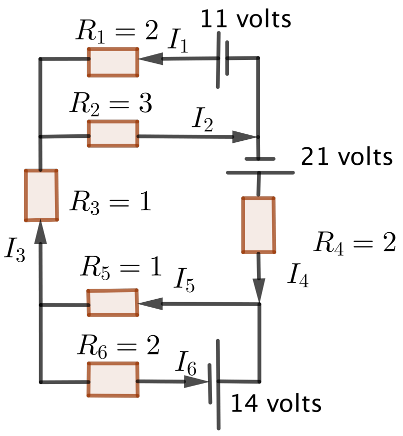
Question 1. Let $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 1 & -1 & m \end{bmatrix}$,
$B = \begin{bmatrix} 0 & 1 & 3 \\ -2 & 0 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 8 & 5 & 7 \\ -1 & 13 & 19 \\ -13 & -24 & 6 \end{bmatrix}$ be matrices, where m is a real number.

a. (1 point) Given that $m = 2$. Find a matrix X that satisfies $AX - 3A = 3B^T X + C$.

b. (1 point) Find all real numbers m such that 1 is an eigenvalue of A.

c. (0.5 point) Given that $m = 0$, find $\det(2A^{2019}B^{-1})$.

Question 2. (1.5 points) Determine the currents I_1, I_2, I_3, I_4, I_5 and I_6 for the electrical network shown in the following figure:



Question 3. Let $F = \{(2, -1, 3), (1, 1, 2), (3, 0, 5), (-1, -4, -3)\}$ be a subspace of $\mathbb{R}_3$ with the inner product $(x, y) = 3x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 + 4x_3y_3$.

a. (1 point) Find the dimension and a basis for $F^{\perp}$.

b. (1 point) Find the vector projection of $w = (3, -2, 1)$ onto F.

Question 4. Let $f : \mathbb{R}_3 \to \mathbb{R}_3$ be a linear transformation satisfying: $f(1, -1, 2) = (2, 1, -2)$, $f(1, 1, 3) = (1, 1, -3)$, $f(2, 2, 5) = (4, 3, -8)$ and $E = \{(1, 1, 2), (1, 2, 1), (1, 1, 1)\}$ be a basis of $\mathbb{R}_3$.

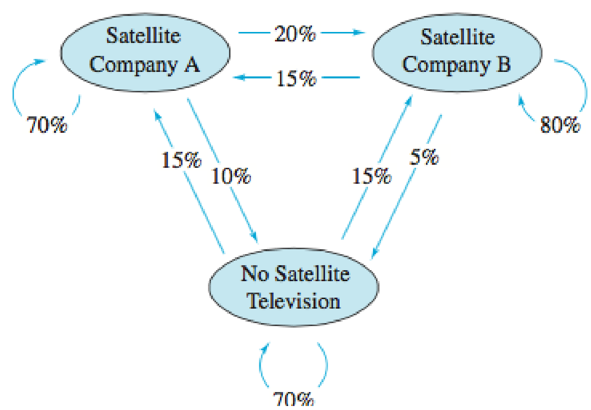
a. (1 point) Find the dimension and a basis for $\ker(f)$.

b. (1 point) Given that the coordinate vector of a vector u with respect to E is $[u]_E = (-2, 1, 3)^T$, find $f(u)$.

Question 5. Two competing companies offer satellite television service to a city with 100 000 households. The transition matrix is $\begin{bmatrix} 0.7 & 0.15 & 0.15 \\ 0.2 & 0.8 & 0.15 \\ 0.1 & 0.05 & 0.7 \end{bmatrix}$ (see the figure below for the changes in satellite subscriptions each year). Company A now has 10 000 subscribers and Company B has 15 000 subscribers.

a. (0.5 point) How many subscribers will each company have after 1 year?

b. (1.5 point) Using the method of diagonalizing the transition matrix to find the numbers of subscribers after 50 years. Round each answer to the nearest integer.



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