Assignment for Matlab report

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Basic commands in Algebra

0.1 Complex numbers

- *real(z)*: real part of z
- *imag(z)*: imaginary part of z
- *conj(z)*: conjugate of z
- *abs(z), norm(z)*: modulus of z
- *angle(z)*: argument of z

0.2 Matrix

- *A=[1 2 3; 2 3 4]*: a 2×3 matrix
- *A*B*: AxB
- *Aˆ n*: A^n
- *eye(n)*: the n- identity matrix
- *zeros(n)*: the matrix 0
- *ones(n)*: the n-square matrix with entries 1
- *diag(v)*: the diagonal matrix, whose diagonal is the vector v.
- *A(i,j)*: the entry of A in i-th row, j-th col.
- *A(i,:)*: the i-th row
- *A(i:k,:)*: the i-th to k-th rows

- *A(:,j)*: the j-th column

- *A(:,j:k)*: the j-th to k-th columns

- *[m,n]=size(A)*: m: number of rows, m: number of col.

- *numel(A)*: number of entries of A

- *A=[]*: empty matrix

- *A(i,:)=[]*: delete the i-th row

- *A(:,j)=[]*: delete the j-th col.

- *rref(A)*: reduce to the row echelon form of A

- *rank(A)*: rank of A

- *det(A)*: determinant of A

- *A'*: transpose of A

- *trace(A)*: the sum of all entries on the diagonal of A

- *inv(A)*: A^{-1}

- $A\backslash\ b$: Solve the system $Ax = b$

- *null(A)*: Find the basis of the nullspace $Ax = 0$

- *null(A,'r')*: Find the basis of the nullspace $Ax = 0$ in rational numbers. Or you can use: *format rat*

- *[P,D]=eig(A)*: Find the eigenvalues, eigenvectors of A

- *null(A,'r')*: Find the eigenvalues, eigenvectors of A in rational numbers
- *max(X)*: the maximum element of X

- *min(X)*: the minimum element of X
- *dot(u,v)*: the dot product of u and v
- *syms x*: declare the symbolic variable x
- *f=x^2+2*x+2*: a function f(x)
- *solve(f)*: solve the equation $f(x) = 0$

0.3 An example program

Input: matrix A. Count the number of nonzero entries of A.

```
A = input('Enter the matrix A ')
[m, n] = size(A);
N = 0;
for i = 1 : m
    for j = 1 : n
        if A(i,j) ~= 0
            N = N+1;
        end
    end
end
disp('The number of nonzero entries of A ')
N
```

1 Exercises

Find the argument, module of

1. $z = \dfrac{1+i\sqrt{3}}{1+i}$.
2. $z = (1+i\sqrt{3})(1-i)$.
3. $z = \dfrac{-1+i\sqrt{3}}{1-i}$.

Solve the equation in the set of complex numbers

4. $z^2 = \bar{z}$.
5. $z^2 = z - \bar{z}$.
6. $A = \begin{pmatrix} 2 & -1 & 4 & 5 \\ 2 & 1 & 3 & -1 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 2 & 0 & -1 \\ -1 & 3 & 0 & -1 \end{pmatrix}$. Find $C = A^T B$, trace of C, rank of C, det of C.
7. $A = \begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \end{pmatrix}$. Prove that $rank(A) = rank(AA^T) = rank(A^T A)$.

8. $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 2 \\ 0 & 2 \\ -1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$. Find $2AC - (CB)^T$

9. $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 4 \\ -1 & 1 & 0 & 2 \\ 2 & 2 & 3 & m \end{pmatrix}$. Find m such that A is invertible

10. Find the inverse of $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$.

11. $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix}$. Find $f(A)$, with $f(x) = x^2 - 2x - 3$

12. $A = \begin{pmatrix} 3 & -2 & 6 \\ 5 & 1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 5 \\ 1 & -2 & m \end{pmatrix}$. Find m such that AB is invertible

13. $A = \begin{pmatrix} -1 & 3 & 2 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{pmatrix}$. Find P_A.

14. Find m such that $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & m \\ 3 & 2 & -1 \end{pmatrix} . \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 5 & 7 & 5 \end{pmatrix}$ is invertible.

15. $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \\ 5 & 3 & -1 \end{pmatrix}$. Find P_A.

16. Reduce the matrix $\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 6 & 9 \end{pmatrix}$ to the row echelon form

17. Solve the equation $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} .X = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

18. Solve the equation $\begin{pmatrix} 0 & -8 & 3 \\ 1 & -5 & 9 \\ 2 & 3 & 8 \end{pmatrix} X = \begin{pmatrix} 23 & -30 \\ -2 & -26 \\ -16 & 7 \end{pmatrix}$

19. Find the NUMBER of solutions of this system $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 &= 7 \\ 2x_1 + x_2 + 2x_3 + 3x_4 &= 6 \\ 3x_1 + 2x_2 + x_3 + 2x_4 &= 7 \\ 4x_1 + 3x_2 + 0x_3 + x_4 &= 8 \end{cases}$

20. Find the NUMBER of solutions of this system $\begin{cases} x_1 + 2x_2 - 3x_3 + 5x_4 &= 1 \\ x_1 + 3x_2 - 13x_3 + 22x_4 &= -1 \\ 3x_1 + 5x_2 + x_3 - 2x_4 &= 5 \\ 2x_1 + 3x_2 + 4x_3 - 7x_4 &= 4 \end{cases}$

21. Find the NUMBER of solutions of this system $\begin{cases} x_1 & -2x_2 & +3x_3 & -4x_4 & = & 2 \\ 3x_1 & +3x_2 & -5x_3 & +x_4 & = & -3 \\ -2x_1 & +x_2 & +2x_3 & -3x_4 & = & 5 \\ 3x_1 & & +3x_3 & -10x_4 & = & 8 \end{cases}$

22. Find m such that the following system has a unique solution $\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ 2x_1 + x_2 + 3x_3 - x_4 = 2 \\ 3x_1 + 4x_2 + 2x_3 = 6 \\ -2x_1 - x_2 + mx_4 = m - 1 \end{cases}$

23. Solve $\begin{cases} x_1 + 3x_2 + 3x_3 + 2x_4 + 4x_5 & = & 0 \\ x_1 + 4x_2 + 5x_3 + 3x_4 + 7x_5 & = & 0 \\ 2x_1 + 5x_2 + 4x_3 + x_4 + 5x_5 & = & 0 \\ x_1 + 5x_2 + 7x_3 + 6x_4 + 10x_5 & = & 0 \end{cases}$

24. Find the rank of the set $M = \{(1; 1; 1; 0), (1; 2; 1; 1)(2; 0; m; -1)\}$ associated with m.

25. Find the dimension and one basis of the span $V =< (1; 2; 1; -1), (3; 1; 0; 5), (0; 5; 3; -8) >$

26. Given $V =< (1; 2; 1; 1), (2; -1; 1; 3), (5; 5; 4; m) >$. Find m such that $\dim(V)$ is max. Find a basis of V.

27. Find the dimension and one basis of the nullspace

$$V = \{(x_1; x_2; x_3; x_4) \in R_4 : x_1 + x_2 - x_3 = 0 \land 2x_1 - x_3 - x_4 = 0\}$$

28. In R_3 given a basis $E = \{(1; 1; 1), (1; 1; 2), (1; 2; 1)\}$ and $[x]_E = (1; -3; 2)^T$. Find x.

29. In R_3 and a basis $E = \{(1; 1; 1), (1; 1; 0), (1; 0; 1)\}$. Find the coordinates of $x = (1; 2; -1)$ in E.

30. Find m such that $M = \{(1; 2; -1), (2; 1; 3), (-1; 2; m)\}$ is a spanning set for R_3.

31. Find m such that $M = \{(1; -2; 1), (3; 1; -1), (m; 0; 1)\}$ is a basis of R_3.

32. Find m such that $\{mx^2 + x + 1, 2x^2 + x + 1, x^2 + 2x + 2\}$ is a basis of $P_2[x]$.

33. In R^3, given 2 bases $E = \{(1; 0; 1), (1; 1; 1), (1; 1; 0)\}$ and $E' = \{(1; 1; 2), (1; 2; 1), (1; 1; 1)\}$. Find the change of basis matrix from E to E'.

34. Find m such that $x = (1; 0; m)$ is a linear combination of $M = \{(1; 1; 1), (2; 3; 1)\}$.

35. In R_4, given 2 subspaces

$$F = \left\{ x \in R^4 | \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}, G =< (2; -1; 0; m) > .$$

Find m such that $G \subset F$.

36. InR_4 given a subspace

$$V = \{(x_1; x_2; x_3; x_4) \in R_4 | x_1 - x_2 + x_3 = 0 \wedge x_2 + x_3 + x_4 = 0\}.$$

Find the dimension, one basis of V.

37. In R_4, given 2 subspaces

$$V_1 =< (8; -6; 1; 0), (-7; 5; 0; 1) >, V_2 =< (1; 0; -8; 7), (0; 1; 6; -5) > .$$

Check if $V_1 \perp V_2$ or not?

38. In R_4 given 2 subspaces

$$V_1 =< (-2; 0; -6; 5), (1; 1; -1; 0) >, V_2 =< (2; -1; 1; 2), (-1; 3; 2; m) > .$$

Find m such that $V_1 \perp V_2$.

39. In R^3 with the standard inner product, given $u = (1; 1; 2), v = (2; 1; -1)$. Find $\cos(u, v)$.

40. In R^3 with the dot product, given $u = (1; 1; 2), v = (2; 1; -1)$. Find $d(u, v)$ and find a vector w orthogonal to u, v.
In R^3, given an inner product

$$(x, y) = 2x_1y_1 - 3x_1y_2 - 3x_2y_1 + 5x_2y_2 - x_2y_3 - x_3y_2 + 4x_3y_3$$

(*For the questions from 41 to 43*)

41. Find the distance between 2 vectors $u = (1; 2; 1)$ and $v = (-1; 1; 2)$.

42. Find $\cos(u, v)$, with $u = (1; 2; 1)$ and $v = (-1; 1; 2)$.

43. Given $F =< 1; 2; 1 >$. Find a basis of $F^{\perp}$.

44. Find the dimension, one basis of Kerf:
$f(x_1; x_2; x_3) = (2x_1 + x_2 - 3x_3; x_1 - 4x_2)$.

45. Find the dimension, one basis of Imf:
$f(x_1; x_2; x_3) = (x_1 + x_2; x_2 + x_3; x_1 - x_3)$.

46. Given $f : R^3 \longrightarrow R^2$ satisfying $f(1; 1; 0) = (2; -1)$, $f(1; 1; 1) = (1; 2)$, $f(1; 0; 1) = (-1; 1)$. Find $f(2; 0; 3)$.

47. Given $f : R^3 \longrightarrow R^2$ and the matrix representation of f in
$E = \{(1; 1; 1), (1; 0; 1), (1; 1; 0)\}$, $F = \{(1; 1), (2; 1)\}$ is $A_{E,F} = \begin{pmatrix} 2 & 1 & -3 \\ 0 & 3 & 4 \end{pmatrix}$.
Find $f(1; 2; 3)$.

48. Given $f(x_1; x_2; x_3) = (x_1 + x_2; x_2 + x_3; x_3 + x_1)$. Find x such that $f(x) = (1; 2; 3)$.

49. Given $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$ and $u = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, v = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Which vectors are eigenvectors of A?.

50. Given $A = \begin{pmatrix} 3 & 4 \\ 6 & 5 \end{pmatrix}, \lambda_1 = -1, \lambda_2 = 3$. Which number is an eigenvalue of A?

51. Given $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}$. Find all eigenvalues and the associated eigenvectors of A.

52. Given $A = \begin{pmatrix} 0 & -8 & 6 \\ -1 & -8 & 7 \\ 1 & -14 & m \end{pmatrix}$. Find m such that A has an eigenvalue $\lambda = 2$. Find all eigenvalues and the associated A with m found.

53. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 7 & 8 \end{pmatrix}$. Find the inverse of A using the elementary operations.

54. In R_3, given $M = \{(1;2;-1),(3;2;-1),(0;2;-1)\}$. Find m such that $(3;8;m)$ is a linear combination of M.

55. In R_3, given $V =< (1;2;-1),(3;2;-1),(0;2;-1) >$. Find m such that $(-3;5;m) \in V$.

56. In R^4, given $U = \langle(1,2,1,1);(2,1,0,-2)\rangle$ and $V = \langle(1,5,3,5);(3,0,-1,m)\rangle$. Find m such that $U \equiv V$.

57. In R^4, V is the nullspace

$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ 2x_1 + 2x_2 + x_3 + x_4 = 0 \\ x_1 + x_2 + 2x_3 + mx_4 = 0 \end{cases}$$

Find m such that $\dim(V)$ is max. Find the dimension and a basis of V with m in the question a.

58. In R^4, given $U = \langle(1,2,1,0);(2,-1,1,1)\rangle$ $V = \langle(1,1,-2,1);(2,0,4,m)\rangle$. Find m such that $\dim(U+V)$ is min. Find the dimension, a basis of $U+V$.

59. In R^4, given 2 nullspaces
$U : \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 0 \\ -1 & 1 & -1 & 2 & 0 \end{array}\right]$, $\quad V : \left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & 0 \\ -1 & 0 & -1 & m & 0 \end{array}\right]$.
Find m such that $\dim(U \cap V)$ is max. Find the dimension, a basis of $U \cap V$.

60. In R_4, given a nullspace

$$V = \{(x_1;x_2;x_3;x_4) \in R_4 | x_1 - x_2 + x_3 = 0 \wedge x_2 + x_3 + x_4 = 0\}.$$

Find a basis of V.

61. In R_4, given a nullspace

$$V = \{(x_1;x_2;x_3;x_4) \in R_4 | x_1 + x_2 + x_3 = 0 \wedge -x_1 + x_2 + x_4 = 0\}.$$

find a basis of $V^\perp$.

62. In R_4, given a span $V =< (2;-1;1;0),(-2;1;0;1) >$ and a vector $x = (1;1;0;1)$. Find $Pr_V(x)$.

63. In R_3, given 2 subspaces

$$V_1 =< (1; 2; 1), (-1; 0; 1) >, V_2 = \{(x_1; x_2; x_3) \in R_3 | x_1 - x_2 + mx_3 = 0\}$$

Find m such that $V_1 \equiv V_2$.

64. In R^3 with the standard basis, given $F =< (1; 1; 2), (2; 1; -1) >$ and a vector $x = (1; 2; 3)$. Find the projection of x onto F.

65. In R^3, given an inner product $(x, y) = x_1y_1 + 2x_2y_2 + 3x_3y_3 - x_1y_3 - x_3y_1$. Find the angle and the distance between $u = (1; 1; 2)$ and $v = (2; 1; -1)$.

66. In R^3, given an inner product $(x, y) = x_1y_1 + 2x_2y_2 + 5x_3y_3 - 2x_1y_3 - 2x_3y_1$. Find the orthogonal complement of $F =< (1; 2; 3) >$.

67. Given a linear transformation $f : R^3 \longrightarrow R^2$ satisfying $f(1; 1; 0) = (2; -1), \quad f(1; 1; 1) = (1; 2), \quad f(1; 0; 1) = (-1; 1)$. Find $f(x_1; x_2; x_3)$.

68. Given a linear transformation $f : R^3 \longrightarrow R^2$, $f(x_1; x_2; x_3) = (x_1 + 2x_2 - 3x_3; 2x_1 + x_3)$.
Find the matrix representation of f in $E = \{(1; 1; 1), (1; 0; 1), (1; 1; 0)\}$, $F = \{(1; 3), (2; 5)\}$.

69. Given a linear transformation $f : R^3 \longrightarrow R^3$ satisfying
$f(1; 1; 1) = (1; 2; 1), \quad f(1; 1; 2) = (2; 1; -1), \quad f(1; 2; 1) = (5; 4; -1)$..
Find the matrix of f in $E = \{(1; 1; 0), (0; 1; 1), (1; 1; 1)\}$.

70. Given a linear transformation $f : R^3 \longrightarrow R^2$, and the matrix of f in
$E = \{(1; 1; 1), (1; 0; 1), (1; 1; 0)\}$, $F = \{(1; 1), (2; 1)\}$ là $A_{E,F} = \begin{pmatrix} 2 & 1 & -3 \\ 0 & 3 & 4 \end{pmatrix}$.
Find the matrix of f in the standard bases.

71. Given a linear transformation $f : R^3 \longrightarrow R^3$ with the matrix in the basis $E = \{(1; 2; 1), (1; 1; 2), (1; 1; 1)\}$ is

$$A_E = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{pmatrix}.$$

Find the matrix of f in the standard bases.

72. Given a linear transformation $f : R^3 \longrightarrow R^3$ with the matrix in the basis $E = \{(1; 2; 1), (1; 1; 2), (1; 1; 1)\}$ is

$$A_E = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{pmatrix}.$$

Find the matrix of f in $E' = \{(1; 2; 3), (2; 3; 5), (5; 8; 4)\}$.

73. Given a linear transformation $f : R^3 \longrightarrow R^3$ with the matrix in the basis $E = \{(1; 1; 2), (1; 1; 1), (1; 2; 1)\}$ is

$$A_E = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 3 \\ 1 & 2 & 4 \end{pmatrix}.$$

Find the dimension, a basis of Imf.

74. Given a linear transformation $f : R^3 \longrightarrow R^3$ with the matrix in the basis $E = \{(1;0;1),(1;1;1),(1;1;0)\}$ is

$$A_E = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 3 \\ 1 & 2 & 4 \end{pmatrix}.$$

Find the dimension, a basis of $\ker f$.

Write a program to find:

75. Input: matrix A. Count the number of even entries of A
76. Input: matrix A. Find the maximum elements in each row of A
77. Input: matrix A. Find the maximum positive entry in A.
78. Input: matrix A. Find the sum of all odd entries in A.
79. Input: matrix A. Find the product of all odd entries in A.
80. Input: matrix A. Check if A is square and symmetric.
81. Input: matrix A. Check if A is orthogonal.
