

# Practice test

Lê Bá Quốc Việt

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## ● Question 1

☑ 0/1 pt ↺ 3 ↻ 19

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In  $\mathbb{R}^3$  given the linear mapping  $f$  be the orthogonal projection onto the plane  $3x + 1y + -1z = 0$ . Find the image of the triangle with vertices  $A(0, -4, 2)$ ,  $B(-5, -7, -1)$ ,  $C(3, 7, 5)$ .

The standard matrix of the  $f$  is

$$A_f =$$


(Input results as fraction)

Input the answer as the coordinates of image A'B'C'

$$A' =$$

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(Round the results to 4 decimal digits.)

$$B' =$$

--	--	--

( Round the results to 4 decimal digits.)

$$C' =$$

--	--	--

( Round the results to 4 decimal digits.)

**Question 2**

0/1 pt 3 19

Given the linear mapping be the rotation about the origin an angle  $2\frac{\pi}{3}$  by counterclockwise direction. Find the image of the vector  $\vec{u} = (-3, -1)$

$$f(\vec{u}) =$$

$$\left[ \begin{array}{cc} \boxed{\phantom{0000}} & \boxed{\phantom{0000}} \end{array} \right]$$

(Round the result to 4 decimal digits)

**Question 3**

0/1 pt 3 19

In  $\mathbb{R}^3$  given the linear mapping be the reflection through the plane  $9x + 9y + 5z = 0$ . Find the image of the triangle with vertices  $A(-2, 6, 9)$ ,  $B(0, -4, 1)$ ,  $C(-7, 10, -10)$ . Round the results to 4 decimal digits.

Input the answer as the coordinates of image A'B'C'

A' =

[	<input type="text"/>	<input type="text"/>	<input type="text"/>	]
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B' =

[	<input type="text"/>	<input type="text"/>	<input type="text"/>	]
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C' =

[	<input type="text"/>	<input type="text"/>	<input type="text"/>	]
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● Question 4

✓ 0/1 pt ↺ 3 ↻ 19

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In  $\mathbb{R}^2$  given the linear mapping be the reflection through the line  $5x + 7y = 0$ . Find the image of the triangle with vertices  $A(10, -6)$ ,  $B(-7, -3)$ ,  $C(-5, 6)$ .

The matrix representation  $A =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

Input the answer as the coordinates of image  $A'B'C'$ .

$A' =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$B' =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$C' =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

In  $\mathbb{R}^3$  given the linear mapping  $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  such that  $f(1, 0, 2) = (8, 4, -10)$ ,  $f(-1, 1, 1) = (4, -5, -1)$ ,  $f(0, 0, 1) = (2, -10, 3)$ .

Find the matrix representation in the basis  $E = \{(1, 1, 2), (1, 2, 1), (1, 1, 1)\}$ .

$A_E =$

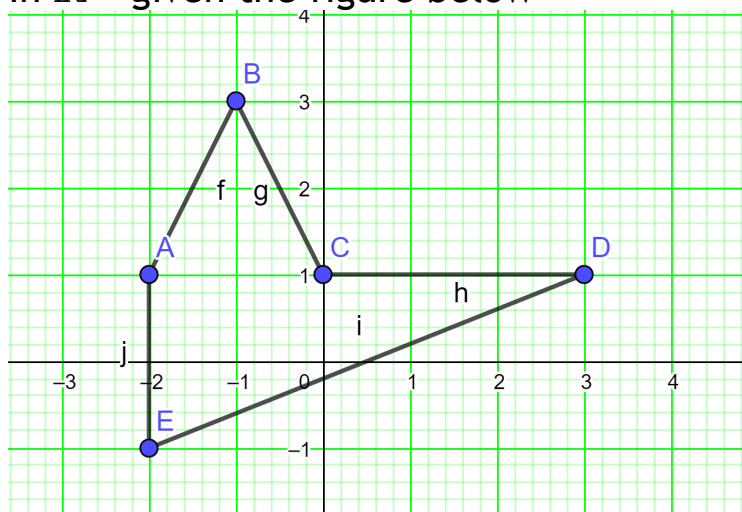

Given that  $x = (14, -1, 12)$ . Find  $[f(x)]_E$

$[f(x)]_E =$


Find the dimension of Kerf

dim Kerf=

In  $\mathbb{R}^2$  given the figure below



with  $A(-2, 1)$ ,  $B(-1, 3)$ ,  $C(0, 1)$ ,  $D(3, 1)$ ,  $E(-2, -1)$ . Find the image of this figure of the linear transformation  $f$  by performing two successive linear transformations  $f_1, f_2$ , where  $f_1$  is the rotation clockwise about origin angle  $\frac{\pi}{3}$  and  $f_2$  is the reflection across the line  $-8x - 9y = 0$ .

The standard matrix of  $f$  (round the result to 2 decimal digits)=


The matrix image =


### ● Question 7

✓ 0/1 pt ↺ 3 ↻ 19

Given  $E$  be a basis in  $\mathbb{R}^2$ , two vectors  $u_1, u_2$ ,  $[u_1]_E = [-4 \ 0]^T$ ,  $[u_2]_E = [-9 \ 2]^T$ . Let  $f$  be a linear mapping  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ , and  $[f(u_1)]_E = [4 \ 7]^T$ ,  $[f(u_2)]_E = [4 \ 7]^T$ .

a) Find the matrix representation of  $f$  with respect to the basis  $E$ .  
(Hint: use the formula  $[f(u)]_E = A_E[u]_E$ )

$A_E =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

b) If  $E = \{(4, -2)^T, (3, 10)^T\}$ , find the  $f\left(\begin{bmatrix} -2 \\ -1 \end{bmatrix}\right)$ .

$f\left(\begin{bmatrix} -2 \\ -1 \end{bmatrix}\right) =$

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

### ● Question 8

✓ 0/1 pt ↺ 3 ↻ 19



Suppose that the oldest age attained by the females in a certain animal population is 15 years and we divide the population into three age classes with equal durations of five

years. Let the Leslie matrix for this population be  $\begin{bmatrix} 0 & 4 & 3 \\ 0.39 & 0 & 0 \\ 0 & 0.46 & 0 \end{bmatrix}$ .

The vector of age distribution now is  $\begin{bmatrix} 21 \\ 24 \\ 22 \end{bmatrix}$ .

The probability for a 4-year-old animal to survive to the age of 8 is  (%).

The number of animals whose age from  $5 \leq \text{age} \leq 10$  after 2 years:



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● Question 9

✓ 0/1 pt ↺ 3 ↻ 19

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Given the equation  $AX + B^T = 5X + 3C$ , where

$$A = \begin{bmatrix} 6 & 5 & 1 \\ 0 & 6 & 0 \\ 0 & -1 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & 3 & 8 \\ -9 & -1 & -3 \\ -10 & 5 & 9 \end{bmatrix},$$

$$C = \begin{bmatrix} 10 & -2 & 8 \\ -7 & -9 & 1 \\ -1 & -5 & -3 \end{bmatrix}.$$

Find X.

X=


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● Question 10

✓ 0/1 pt ↺ 3 ⇌ 19

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Two competing companies(A,B) offer network service to a population of 103923 consumers. During any month, company A consumer has a 20% probability of switching to company B and a 5% probability of not using service of either company. A consumer of company B has a 15% probability of switching to company A and a 10% probability of not using service. A nonuser has a 15% probability of purchasing company B and 10% using service of A. How many people will be in each group after 3 months? Knowing that at this month, there are 37147 subscribers of A, 23948 subscribers of B.

Matrix transition=


$X^{(3)} =$


### ● Question 11

✓ 0/1 pt ↺ 3 ↻ 19

A machine can be either working or broken down on any given day. If it is working, it will break down in the next day with probability 7 %, and will continue working with probability 93 %. If it breaks down on a given day, it will be repaired and be working in the next day with probability 80%, and will continue to be broken down with probability 20%. Given that the probability that the machine works today is 85% .

1) The matrix transition in this model is:  $A =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

2) The probability of working state on tomorrow of the machine is:

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

( The result in percentage form)

3) The probability of working state after 4 days of the machine is

$$\begin{bmatrix} \boxed{\phantom{00}} \\ \boxed{\phantom{00}} \end{bmatrix}$$

Find the matrix  $X$  that satisfies the matrix equation  $XB + 2XA^T = C$ ,

where  $A = \begin{bmatrix} 4 & 0 \\ -2 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} -4 & 13 \\ -1 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 6 & 3 \\ -4 & 4 \end{bmatrix}$ .

Answer:

$X =$


### Question 13

0/1 pt 3 19

In  $\mathbb{R}^3$  given inner product:  $\langle x, y \rangle = 4x_1y_1 + 5x_2y_2 + 4x_3y_3 - 4x_2y_3 - 4x_3y_2$  and a subspace  $F = \text{span}\{(1, 2, 1), (2, -1, 4), (-26, -7, -44)\}$ .

Find  $\langle x, y \rangle$  if  $x = (8, 5, -2)$ ,  $y = (6, 5, 9)$

Find the dimension and a basis of  $F^\perp$ .

$\langle x, y \rangle =$

Upload your hand writing solution for the second question here.

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## Question 14

0/1 pt 3 19

In  $\mathbb{R}^3$ , given inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Given  $F = \text{span}\{(1, 3, 5), (2, 5, 7), (0, 1, 3), (3, 8, 12)\}$ . Find orthogonal projection of vector  $x = (-4, -2, -2)$  onto  $F$ . Round the result to 2 decimal digits.

$\text{Proj}_F x =$

$$\begin{bmatrix} \boxed{\phantom{000000}} \\ \boxed{\phantom{000000}} \\ \boxed{\phantom{000000}} \end{bmatrix}$$

Find the dimension and a basis of  $F^\perp$  (Upload your hand writing solution).

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## Question 15

0/1 pt 3 19

Given inner product in  $\mathbb{R}^3$

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Given the subspace  $F = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3, x_1 - x_2 + 3x_3 = 0\}$

.

Find orthogonal projection of vector  $x = (7, -2, -9)$  onto  $F$ . Round the result to 2 decimal digits.

$\text{Proj}_F x =$

$$\begin{bmatrix} \boxed{\phantom{000000}} \\ \boxed{\phantom{000000}} \\ \boxed{\phantom{000000}} \end{bmatrix}$$

### Question 16

0/1 pt 3 19

In  $\mathbb{R}^3$ , given the inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Vector  $x = (0, -7, 9), y = (-4, 8, -5)$ . Find the distance between  $x, y$ . Round to 4 decimal digits.

$d(x, y) =$

### Question 17

0/1 pt 3 19

In  $\mathbb{R}^3$ , given inner product:

$$\langle x, y \rangle = 4x_1y_1 + 2x_2y_2 + 8x_3y_3 - x_1y_3 - x_3y_1$$

Vector  $x = (-3, 8, 6), y = (-6, -8, 7)$

Find  $\langle x, y \rangle$ , the distance between  $x, y$  and the angle between  $x$  and  $y$ .

$\langle x, y \rangle =$   .

Distance =  (Round the result to 2 decimal digits)

Angle =  (Round the result to 2 decimal digits)

● Question 18

✓ 0/1 pt ↻ 3 ⇌ 19

Find the eigenvalues of the matrix  $\begin{bmatrix} -5 & 10 \\ -5 & 10 \end{bmatrix}$ .

The eigenvalues are  $\lambda =$   (enter the eigenvalues, separated by commas)

Question Help: [Video](#) [Written Example](#)

● Question 19

✓ 0/1 pt ↻ 3 ⇌ 19



In a city , there are 3 supermarkets A; B and C. In this month there are `21902` householders going to A , `40614` going to B and `62260` going to C. The transition after each month as:

`A -gt B: 7.5 % , A -gt C: 2.5 % , B -gt A: 1.5 % , B-gtC: 5 % , C-gtA:10 % , C-gtB: 7.5 %` .

What is the matrix transition P?

P=


How many householders that go to A, B and C after 1 month?

The number of houslders going to A,B,C (Round the answer to the nearest integer)=


.

Find the number of houseloders in each group after 1 year?

The eigenvalues of P (input the answer from biggest to smallest value of eigenvalues)=

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$$\begin{bmatrix} \boxed{\phantom{0000}} \\ \boxed{\phantom{0000}} \end{bmatrix}$$

The number of housholders A,B,C (Round the answer to the nearest integer)=

$$\begin{bmatrix} \boxed{\phantom{0000}} \\ \boxed{\phantom{0000}} \\ \boxed{\phantom{0000}} \end{bmatrix}$$

### ● Question 20

✓ 0/1 pt ↻ 3 ⇌ 19

Given the matrix  $A = \begin{bmatrix} 7 & -8 & -9 \\ 6 & 5 & 1 \\ -4 & 0 & m \end{bmatrix}$ . Find  $m$  such that  $\lambda = 4$  is an eigenvalue of  $A$ .

### ● Question 21

✓ 0/1 pt ↻ 3 ⇌ 19

Given matrix  $A = \begin{bmatrix} -25 & 10 & 8 \\ -46 & 18 & 16 \\ -26 & 10 & m \end{bmatrix}$ , find  $m$  such that vector  $X = \begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix}$  is an eigenvector of  $A$ .

$m =$

### ● Question 22

✓ 0/1 pt ↻ 3 ⇌ 19

Given two square matrices size `2` `A` and `B` such that `|A|=2`, `|B|=-8`. Applying 2 successive row operations on `A` to obtain `A\_1` as the following: `r\_1 \leftrightarrow r\_2`, (interchanging `r\_1` and `r\_2`), `r\_2 \rightarrow -7r\_2+3r\_3` (replacing `r\_2` by multiplying row 2 by `-7` and adding to row 3 time 3). Compute `|3A\_1B^{-1}|`.

Answer= \_\_\_\_\_.

### Question 23

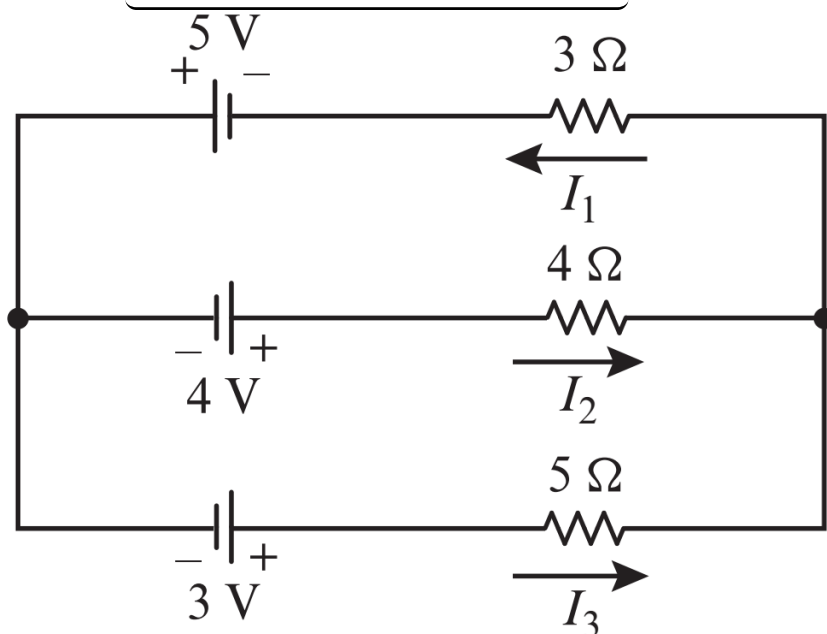
0/1 pt 3 19

Find all the unknown current on the circuit. (Round the result to 2 decimal digits).

`I\_1` = \_\_\_\_\_

`I\_2` = \_\_\_\_\_

`I\_3` = \_\_\_\_\_



### Question 24

0/1 pt 3 19

In  $\mathbb{R}^2$  given the linear mapping  $f$  be the reflection through the line  $-7x - 2y = 0$  and the triangle  $ABC$ . We know that  $f(ABC) = A'B'C'$ , where  $A'(-5, 3)$ ,  $B'(4, 10)$ ,  $C'(10, 0)$ . Find the coordinates of the points  $A, B, C$ .

The standard matrix representation  $A$  of  $f$  is =

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \\ \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

Input the answer as the coordinates of  $A, B, C$ .

$A =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$B =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

$C =$

$$\begin{bmatrix} \boxed{\phantom{00}} & \boxed{\phantom{00}} \end{bmatrix}$$

A batch of computers has 3 types of products: millions VND computers, millions VND computers and millions VND computers. If the value of the batch is millions VND and the total value of the millions VND computers and millions VND computers is twice of the value of millions VND computers, then how many of each type of computers are in the batch?

Answer:

The number of millions VND computers is \_\_\_\_\_

The number of millions VND computers is \_\_\_\_\_

The number of millions VND computers is \_\_\_\_\_

● Question 26

✓ 0/1 pt ↺ 3 ↻ 19

Find  $m$  such that the following system has a unique solution  
 $\{(7y, +, 7x, +, z, =, 300), (, , 2x, +, 2z, =, 150), (-5y, -, 2x, +, mz, =, 250):\}$

Answer: m  \_\_\_\_\_

● Question 27

✓ 0/1 pt ↺ 3 ↻ 19

Find all values of  $m \in \mathbb{R}$  such that the system of vectors  $[(4), (2), (m)], [(2), (m), (1)], [(2), (1), (2)]$  is linearly dependent.

Answer:  $m =$   .

(If you can not find  $m$ , you input the answer as "there is no  $m$ ", otherwise, you round your answer to 2 decimal digits)

● Question 28

✓ 0/1 pt ↺ 3 ↻ 19

In  $\mathbb{R}^3$ , given 3 bases  $S_1, S_2, S_3$  such that the matrix change of bases are :  $P_{\{S_1 \rightarrow S_2\}} = [(-3, 6, 1), (0, 1, 0), (2, -4, -1)]$ ,  $P_{\{S_3 \rightarrow S_2\}} = [(1, 2, 7), (0, -2, -7), (3, 7, 24)]$ , knowing that

$[u]_{S_1} = [(-4), (6), (5)]$ . Find  $[u]_{S_3}$ .

$[u]_{S_3} =$

$$\begin{bmatrix} \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \\ \boxed{\phantom{000}} \end{bmatrix}$$

.

### Question 29

0/1 pt 3 19

Given the system of vectors  $S: [(9, 1, 3), (2, 1, 1), (x-1, 2x, 3x)]$ . Find  $x \in \mathbb{R}$  such that  $S$  is a spanning set of a two dimensional space.

$x = \boxed{\phantom{000}}$ .  $S$

### Question 30

0/1 pt 3 19

Given the set of vectors :  $S = \{ (8, -6, -7), (4, -6, -10) \}$

\_\_\_\_\_

### Question 31

0/1 pt 3 19

Given the equation  $AX + B^T = 5X + 2C$ , where  $A = \begin{bmatrix} 9 & 13 & -1 \\ -3 & -11 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} -2 & 10 & -4 \\ 7 & -7 & -5 \\ -1 & -10 & 4 \end{bmatrix}$ ,  $C = \begin{bmatrix} -10 & 10 & -4 \\ -8 & 8 & 3 \\ -7 & 2 & -3 \end{bmatrix}$ .

1) Find  $X$ .

2) Compute  $E = \|2A^2 C^{-1}\|$

1)  $X =$


2)  $E =$

(Let the result in decimal number

with 2 decimal digits rounded)