HCMC UNIVERSITY OF TECHNOLOGY
Faculty of Applied Science
Department of Applied mathematics

FINAL EXAM - SEMESTER 3
Subject: Linear Algebra
Duration: 90 minutes Date: August 10^{th}, 2019

- *This is a closed book exam. Only your calculator is allowed. Total available score:* 10.
- *You MUST fill in your full name and student ID on this question sheet. There are 5 questions on 1 page.*
- *You MUST submit your answer sheets and this question sheet.* Otherwise, your score will be ZERO.

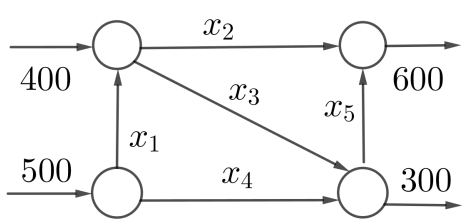
Full name: .... Student ID number: ....

Question 1. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 0 & -1 \\ 2 & -3 & m \end{bmatrix}$,
$B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & -4 \\ -3 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} -37 & 41 & -22 \\ -4 & -5 & 7 \\ 0 & 19 & 7 \end{bmatrix}$ be matrices, where m is a real number.

a. (1 point) Given that $m = 1$. Find a matrix X that satisfies $XA - 3XB^T = 2X + C$.

b. (1 point) Find all real numbers m such that $(2, -1,\ 3)^T$ is an eigenvector of A.

Question 2. (1.5 points) The figure shows the flow of traffic of a network. The diagram indicates the average number of vehicles per hour entering and leaving the intersections and the arrows indicate the direction of traffic flows. All streets are one-way. Setup and solve a system of linear equations to find the possible flows x_1, x_2, x_3, x_4 and x_5.



Question 3. Let
$F = \{x \in \mathbb{R}_4 | x_1 + 3x_2 - x_3 = 0, x_1 - x_2 - x_3 - x_4 = 0\}$ be a subspace of $\mathbb{R}_4$ with the inner product
$(x, y) = 4x_1y_1 + x_2y_2 + 3x_3y_3 + 2x_4y_4$.

a. (1 point) Find the dimension and one basis for F.

b. (1 point) Find the vector projection of $w = (1, -1, 1, 2)$ onto F.

Question 4. Let $f : \mathbb{R}_3 \to \mathbb{R}_3$ be a linear transformation satisfying:
$f(1, 1, 1) = (-1, 2, 0), f(1, 1, 3) = (1, 1, 2), f(0, 2, 1) = (3, 6, 8)$ and $E = \{(1, 1, 2), (1, 2, 1), (1, 1, 1)\}$ be a basis of $\mathbb{R}_3$.

a. (1 point) Find the transformation matrix A_E of f in the basis E.

b. (0.5 point) Given that $x = (3, 4, -5)$, find $[f(x)]_E$.

c. (1 point) Find the dimension and one basis for $Ker(f)$.

Question 5. A machine can be either working or broken down on any given day. If it is working, it will break down in the next day with probability 5%, and will continue working with probability 95%. If it breaks down on a given day, it will be repaired and be working in the next day with probability 80%, and will continue to be broken down with probability 20%. Given that the probability that the machine works today is 80% (then, the probability that the machine breaks down is 20%).

a. (0.5 point) Find the probability that the machine will work tomorrow.

b. (1.5 point) Using the method of diagonalizing the transition matrix to find the probability that the machine will work after 1 year (365 days).

Department of Applied Mathematics **Lecturer**

Dr. Le Xuan Dai **Phan Thi Khanh Van**