A.1. Distance Model

The distance model enables calculations of travel distances between pairs of pick locations in a generic retail warehouse layout with a s-shape routing policy. The generic warehouse layout consists of multiple parallel wide pick aisles, multiple perpendicular cross-aisles, complete racks in the middle of the warehouse and half-racks on both sides. Each rack is considered to be a unique pick location with a unique product. As the pick aisles are wide, picking can not be executed on both sides of the aisle at the same time. Consequently, additional distance is taken into account when traveling from one side of the aisle to the other.

The distance between two different pick location is computed by following the s-shape routing policy logic, where pick aisles may be traversed unidirectional and cross-aisles bidirectional. For a warehouse layout with unidirectional aisles, the distance matrix is asymmetrical. An order picker is able to use the front, middle or back cross-aisle to change aisles, which usually yield different distances. As the picking in retail warehousing is generally only on low-level, the high-level racks are not considered in the distance calculations. The presented model is based on the work of Zunic et al. (2018); Verhaert (2020), and adjusted for its purpose in this research.

In order to create a model for the distance matrix, several variables need to be introduced for the aisles, racks, and sides of the aisles. In addition, each SKU requires a unique location number. All distance model variables are defined in Table A.1, and visualized in Figure A.1. Note that in Table A.1, the aisle width $\gamma = 3.2\,\mathrm{m}$ is designed based on the minimum necessary aisle width of 3 m to use the considered AMRs (i.e., Autopilot lowlifter truck LAE250) from the solution provider Toyota Material Handling. These AMRs have a width of 930 mm; therefore, two AMRs can easily pass or overtake each other in a 3.2 m aisle.

The combination of the variables enables the calculation of the distance between every arbitrary pair of SKU i and j. For SKU i, x_i represents the aisle number and y_i its section number, where $y_i \in \{1, ..., S\}$ with S the total number of sections in an aisle. Since the pick aisle has two sides, a single section represents two pick locations, thus a pick aisle with for example 30 sections consists of 60 pick locations. The side at which SKU i is located in a section, is represented by z_i with $z_i \in \{1, 2\}$. In addition, the block number b_i can be retrieved directly from the section number, with $b_i \in \{1, 2\}$. When combining these variables, the location of SKU i can be portrayed as (x_i, y_i, z_i, b_i). Next

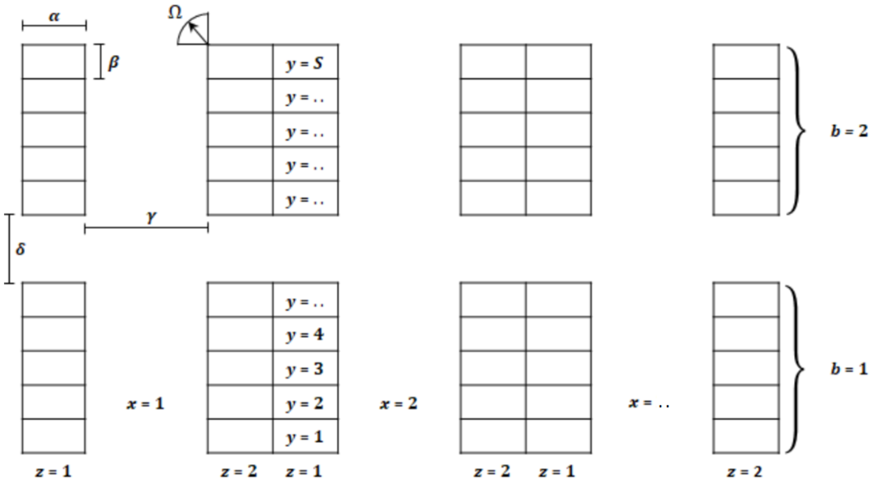

Figure A.1: Distance Model Warehouse Layout

to the notation for a SKU, the dimensions of the warehouse also need to be formulated. The width and depth of a rack, or combination of racks, are represented by β and α units of distance respectively. The width of a pick aisle is formulated as γ units of distance. Finally, the width of a cross-aisle is covered by δ units of distance. Furthermore, as every positional move from one aisle to another requires a turn around a corner, the turning radius is represented by Ω.

$D = (d_{i,j}) \in \mathbb{R}^{m \times m}$ with $i \in \{1, ..., m\} \wedge j \in \{1, ..., m\}$ with m number of unique location numbers. The definition of D is subject to two scenarios, namely if the two locations of SKU i and j are positioned in the same pick aisle or in different aisles. The first scenario is described in Equation A.1 below. Products i and j in the same aisle:

$$d_{i,j} = \begin{cases} |y_i - y_j|\beta + |b_i - b_j|\delta + |z_i - z_j|\gamma & \text{if } i < j \\ S\beta - |y_i - y_j|\beta + |b_i - b_j|(S\beta + \delta) + (4 - z_i - z_j)\gamma + 4\Omega + 4\alpha & \text{otherwise} \end{cases} \tag{A.1}$$

In Equation A.1, the first part of the equation represents a situation in which i and j are in located in the same pick aisle, where location number i is smaller than j. This means that the distance between both location is calculated in line with the travel direction of the aisle. The equation is made up of three elements; the difference in distance between the sections, the width of the cross-aisle and the width of the pick aisle. The second and third part of the equation describe a situation in which both i and j are in the same pick aisle, but the picker is forced to change aisles due to the fact that the pick aisles are unidirectional and j is located at a lower location number than i. The terms are similar to the first element of the equation, however, now both turning corners as well as the distance for the depth of the racks are taken into account. As previously stated, a second scenario arises when i and j are not located in the same pick aisle. This forces the picker to change aisles at least one time. Furthermore, the travel distance depends on starting or ending in an odd or even aisle. The total distance covered between i and j is therefore made up of a horizontal and a vertical component. To ease notation, the variable r_i is used to represent the combination of the pick aisle and the side of the aisle, as shown in Equation A.2. Using this equation, the horizontal distance can be formulated Equation A.3. When considering the options of starting and/or ending in an odd or even aisle, four options arise for calculating the vertical distance. These options are displayed in Equation A.4.

Products i and j in the different aisles:

$$r_i = z_i + 2(x_i - 1) \tag{A.2}$$

$$h = (|r_i - r_j| - 1)\gamma + |x_i - x_j|2\alpha \tag{A.3}$$

$$d_{i,j} = \begin{cases} (2S - y_i - y_j)\beta - max(0, 3 - b_i - b_j)\beta S + |b_i - b_j|\delta + 2\Omega + h & \text{if } x_i \text{ is odd, } x_j \text{ is even} \\ (S - y_i + y_j)\beta + |b_i - b_j|(\beta S + \delta)S + 4\Omega + h & \text{if } x_i \text{ is odd, } x_j \text{ is odd} \\ (y_i + y_j)\beta - max(0, b_i + b_j - 3)\beta S + |b_i - b_j|\delta + 2\Omega + h & \text{if } x_i \text{ is even, } x_j \text{ is odd} \\ (S + y_i - y_j)\beta + |b_i - b_j|(\beta S + \delta)S + 4\Omega + h & \text{if } x_i \text{ is even, } x_j \text{ is even} \end{cases} \tag{A.4}$$

Table A.1: Distance Model Variables

Variable	**Value**
S	72
x_i	$x_i \in \{1, ..., 44\}$
y_i	$y_i \in \{1, ..., 72\}$
z_i	$z_i \in \{1, 2\}$
b_i	$b_i \in \{1, 2\}$
m	$m \in \{0, ..., 6335\}$
α	1.025 m
β	1.35 m
δ	4.25 m
γ	3.2 m
Ω	1.0 m