### Flood Frequency Distribution (FFD 2.1)

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#### Purpose of the software

Flood Frequency Distribution (FFD 2.1) is free software to analyze flood and estimate Quantile for Different return Periods and Flood frequency relations.

#### What news in FFD 2.1:

1- In FFD 2.1, we have introduced ten (10) Probability Distributions, (8 in FFD 1.0).

Probability Distributions are:

- 1-Normal Distribution
- 2-Log Normal Distribution (2p)
- 3-Gumbel Distribution
- 4-Racine-Normal Distribution
- 5-General Extreme Values Distribution (GEV)
- 6-Gamma Distribution (2p)
- 7-Log Pearson 3 Distribution
- 8-Goodrich Distribution
- 9-LogNormal (3p) Distribution
- 10-Weibull Distribution (2p)
- 2-In FFD 2.1, Estimation of parameters method of some of distributions can be with:
  - a- Method of Moment or b- L-Moment
- 3-The results of quantiles are estimated with lower and upper bounds 95% (Except Goodrich distribution).
- 4-Quantile-Quantile plot (QQ-plot) is displayed with Correlation coefficient R.
- 5-Displays Root mean squared Error (RMSE) between quantile and observed data.
- 6-Return periods are: 2, 5 10, 20, 50 100, 500, 1000 and 10000 years
- 7-Diplays the calculated quantiles and lower-upper Bounds in excel File.

#### Help & Appendix:

#### 1-Normal Distribution:

The PDF for Normal Distribution is:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The parameters of Normal PDF are:  $\mu$  and  $\sigma$ :  $\mu$  is the mean and  $\sigma$  is the standard deviation, and are estimated by Method of Moment (6).

The quantile will be estimated as follow:

$$Xp(\%)=ug\sigma+\mu$$

#### 2-Log Normal Distribution:

The PDF for log-Normal Distribution is the same to Normal except X=log(x), thus the parameters are  $\mu_1$  is the mean of (log x) and  $\sigma_1$  is the standard deviation of (log x).

#### 3-Gumbel Distribution:

Gumbel is Extreme value (EV) distributions case, thus Extreme value type I corresponds to Gumbel distribution, the PDF is:

$$f(x) = \frac{1}{\alpha} \exp\left[-\frac{x-u}{\alpha} - \exp\left(-\frac{x-u}{\alpha}\right)\right]$$
$$\alpha = \frac{\sqrt{6}s_x}{\pi} \qquad u = \overline{x} - 0.5772\alpha$$

 $\alpha$  and  $\mu$  are the parameters of Gumbel Distribution, estimated by Method of Moments (see Chow, [1]) or L-moment.

<u>Warning</u>: to estimate Parameters of Gumbel Distribution in <u>Matlab (with MLE method)</u>, you must write: Par=Evfit(-P),

Thus the parameters of Gumbel distribution are: -Par(1), Par(2).

#### **4-Racine Normal Distribution:**

Racine Normal Distribution is "unknown" Distribution, it allows to normalize data observation when they present high dimensional variance, the purpose of this distribution is to avoid dissymmetric and undefined domain of data observed in LogNormal case, so this distribution can be used for any data (0<X<+∞)

The PDF for Normal Distribution is:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The parameters of Normal PDF are:  $\mu$  and  $\sigma$ .

In the Racine-Normal case, u variable is calculated as follow:

$$u = \frac{\sqrt{x} - \sqrt{\bar{x}}}{\delta_{\sqrt{x}}}$$

 $\sqrt{\bar{x}}$  is mean of observed data x=X<sup>0.5</sup> and  $\delta_{\sqrt{x}}$  is the standard deviation of observed data (x=X<sup>0.5</sup>).

#### **5-GEV Distribution:**

The GEV distribution is a family of continuous probability distributions that combines the Gumbel (EV1), Frechet and Weibull distributions. GEV distribution makes use of 3 parameters: location, scale and shape [2]

The PDF is:

$$f_{\xi,\mu,\sigma}(x) = \frac{1}{\sigma} \left( 1 + \xi \frac{(x-\mu)}{\sigma} \right)^{-1-1/\xi} \exp \left( -\left( 1 + \xi \frac{(x-\mu)}{\sigma} \right)^{-1/\xi} \right) \qquad \xi \neq 0$$

And:

$$f_{0,\mu,\sigma}(x) = \frac{1}{\sigma} e^{-(x-\mu)/\sigma} \exp(-e^{-(x-\mu)/\sigma})$$
  $\xi = 0$ 

To estimate parameters of GEV, the Method is Weighted Moments [2], [3], [4]:

$$\kappa = 7.8590c + 2.9554c^{2}$$
in which  $c = \frac{2}{3+\tau_{3}} - \frac{ln2}{ln3}$ 

$$\alpha = \frac{\lambda_{2}k}{(1-2^{-k})\Gamma(1+k)}$$

$$\xi = \lambda_1 - \alpha \{1 - \Gamma(1+k)\}/k$$

<u>Important</u>: Note that for 3-parameters Probability distribution, (Gamma, GEV, Log-Pearson, Goodrich) the confidence interval is sometimes very complex.

Thus for GEV, the interval confidence contains sometimes undefined domain for some quantile and complex Numbers (ex: Log (<0)), and returns a message like:

In Run\_Flood at 34

Warning: Imaginary parts of complex X and/or Y arguments ignored

> In D:\Exchange\Flood\_Tarik\GEV.p>GEV at 55

In D:\Exchange\Flood\_Tarik\Flood\_Tarik.p>Flood\_Tarik at 52

In Run\_Flood at 34

Please see References of interval confidence of GEV Distribution.

We hope more Research to avoid this problem ...

#### 6-Gamma Distribution (Gamma2):

In probability theory and statistics, the gamma distribution is a two parameter family of continuous probability distributions.

The PDF is:

$$f(x) = \frac{(x - \gamma)^{\alpha - 1}}{\beta^{\alpha} \Gamma(\alpha)} \exp(-(x - \gamma)/\beta)$$

In our case we have used Gamma-2P with PDF:

$$\frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}\exp\left(-x/\beta\right)$$

 $\alpha$ ,  $\beta$  (scale and Shape) parameters estimated with Weighted Moments.

#### 7-Log Pearson 3 Distribution

The pdf is:

$$f(x) = \frac{\lambda^{\beta} (y - \varepsilon)^{\beta - 1} e^{-\lambda(y - \varepsilon)}}{\Gamma(\beta)} \qquad y = \log x \ge \varepsilon$$

The parameters were estimated according to Water Resources Council Method.

-Note that the quantile for Gamma\_2 and Log Pearson3 are calculated as follow:

For Gamma X=(m+K\*s) and for Log Pearson3  $X=10.^{(m+K*s)}$ 

The quantile depends on K factor, that last depends on skew coefficients (Cs) of data and the frequency.

The k factor can be interpolated from K-Table or calculated as follow [8]:

$$k_T = \frac{2}{C_s} \left\{ \left[ \frac{C_s}{6} \left( z - \frac{C_s}{6} \right) + 1 \right]^3 - 1 \right\}$$

The confidence limits of logPearson3 are calculated according Bulletin 17B:

$$U_{P,c}(X) = \overline{X} + s. K_p^U$$

$$L_{P,c}(X) = \overline{X} + s.\,K_p^L$$

In which  $\overline{X}$  and s are the logarithmic mean and standard deviation of the observed data,  $K_p^U$  and  $K_p^L$  are the upper and lower confidence coefficients. (See Bulletin 17B)

#### **8-Goodrich Distribution:**

The PDF is (Lamas 1985);

$$|F(x) = 1 - Exp \left[ -a * (x - b)^{1/n} \right]$$
  
$$b \le x \le +\infty$$

A, b, n are the parameters of Goodrich Distribution: See Lamas (1985).

#### 9-Log Normal Distribution 3p

The probability density function of the three-parameter lognormal distribution is:

$$\frac{1}{(X-x_0)\sigma\sqrt{2\pi}}exp\left\{-\frac{[ln(X-x_0)-\mu^2]}{2\sigma^2}\right\}$$

where  $0 \le x_0 < x$  ,  $-\infty < \mu < \infty$  ,  $\sigma > 0$  .  $\mu$ ,  $\sigma$  and  $x_0$  are the parameters of the distribution.

The distribution becomes the two-parameter lognormal distribution when  $x_0 = 0$ . In our case:

 $\mu$  =m1 and  $\sigma$ = s1 are the respectively mean and standard deviation of log of observed data, the third parameter x0 depend on minimum and length of data.

#### 10-Weibull Distribution

The 2-parameter Weibull distribution has a scale and shape parameter. The 3-parameter Weibull includes a location parameter.

When  $t \ge 0$  then the probability density function formula is:

$$f(t) = rac{eta t^{eta-1}}{\eta^eta} e^{-\left(rac{t}{\eta}
ight)^eta}$$

In our case the parameter  $\beta$  is always equal to 1 the distribution has a constant failure rate, then Weibull reduces to an Exponential distribution with  $\beta=1$ .

#### Important: How to use Flood Frequency Distribution (FFD 2.1)

In matlab :Open Main File: "Run\_flood.m" thus run it, and will displays:

After displays Statistics of Sample,

```
*** ---- Statistics of Sample ----- Statistiques de la Serie *** -----
Mean of Sample----Moyenne
                                                551.5
Variance of Sample --- Variance
                                                 5323.2
Stand_Deviation---Ecart type
                                                 73.0
                                                 710.8
Maximum of Values
Minimum of Values
                                                 429.3
Median of Values
                                                 544.5
Coefficient of variation
                                                 0.13
Coefficient of Skewness--- C.Asymetrie
                                                0.58
Coefficient of Kurtosis--- C.Kurtois
                                                -0.05
```

The user must choose a probability distribution:

Disp ('Flood Frequency Distribution -- Ajustement a une loi statistique. Choose : ')

```
')
disp (' 1 - Normal Distribution
                                           ---- Loi Normale
                                          ---- Loi Log Normale
                                                                    ')
disp (' 2 - LogNormal-2p Distribution
                                                                   ')
                                           ---- Loi de Gumbel
disp (' 3 - Gumbel Distribution
disp (' 4 - Racine-Normal Distribution
                                          ---- Loi Racine_Normale
disp (' 5 - Gen Ext-Va(GEV)Distribution
                                           ---- Loi Gene-Extre-Va(GEV) ')
disp (' 6 - Gamma Distribution (Pearson3) ---- Loi GAMMA
                                                                    ')
                                                                     ')
disp (' 7 - Log Pearson3 Distribution
                                          ---- Loi Log Pearson3
disp (' 8 - Goodrich Distribution
                                           ---- Loi de Goodrich
                                                                    ')
                                                                      ')
disp (' 9 - LogNormal-3p Distribution
                                           ---- Loi LogNormale3P
                                                                   ')
disp (' 10- Weibull-2p Distribution
                                           ---- Loi Weibull-2p
```

Choose a Distribution-- Choix de loi de Distribution:

Choose a Distribution, by example: 1 for Normal Distribution

#### Then choose Probability Plot position:

- 1- Hazen Method
- 2- Weibull Method
- 3- Cunane Method
- 4- Chegodayev Method
- 5- Gringorten Method

Choose 2: for Weibull plotting Position

#### Thus the results are displayed:

---- Normal Distribution --- Method of Moments- Methode des moments ----

1-KS test with H and p value:

\*\* [Kolmogorov\_Smirnov Test : H p-value] 0 0.6887

We Accept H0 at Sign.Level :5%

2- Khi squared with H hypothesis:

\*\* [Chi2.goodness Test: H] 0

Chi2.goodness of fit: We Accept H0 at Sign.Level: 5%

3-and we have added Anderson-Darling with H and p value:

\*\* [Anderson-Darling Test: H p-value] 0 0.4011

Anderson-Darling Test: We Accept H<sub>0</sub> at Sign.Level :5%

4-The parameters of Plot distribution:

\*\*Parameters of Normal Distribution [m s] 551.49 73.0

5- and correlation coefficient Observed/Quantiles:

\*\* C. Correlation (Observed/Quantiles): 0.958

\*\* Root Mean Squared Error (RMSE) : 54.65

Then FFD displays the principal quantiles for different Return periods:

# 1- The Quantiles for different Return Periods and with confidence limits (95%:)

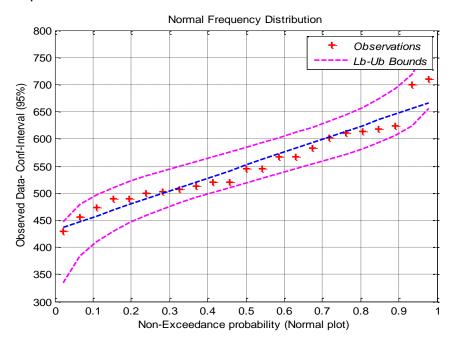
#### Note: FR = Non-Exceedance Probability:

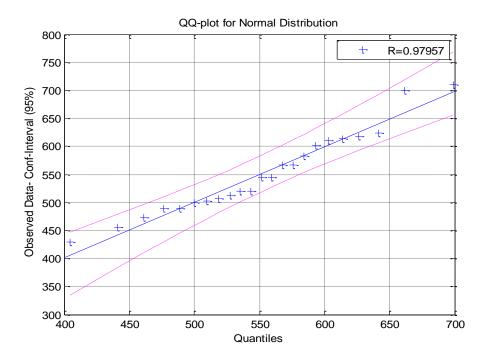
**Estimation of Quantiles Calcul des Quantiles				
FR = Non-Exceedance	Probability	Quantile(%)	Low_Bo	Upp_Bo
XP2 year	FR_0.5	526.5	491.7	561.3
XP5 year	FR_0.8	655.0	574.2	735.7
XP10 year	FR_0.9	752.1	636.6	867.7
XP20 year	FR_0.95	849.3	699.0	999.6
XP50 year	FR_0.98	977.8	781.5	1174.1
XP100 year	FR_0.99	1075.0	843.9	1306.0
XP500 year	FR_0.998	1300.6	988.8	1612.4
XP1000year	FR_0.999	1397.8	1051.2	1744.4
XP10000year	FR_0.9999	1720.7	1258.5	2182.8

#### 2-Plot 2 graphs:

In FFD 2.1 we display the two principal graphs:

- 1-Frequency distribution (in X axis FR = Non-Exceedance Probability and Y axis the calculated quantiles)
- 2-QQplot Quantiles versus observed data.





**Note:** Results saved in Results\_Flood.txt and Quantile & LB-UB in Data\_Sim excel. <u>Important:</u> If you receive message:

Error using xlswrite (line 219)

The file D:\ Flood\_T 2.0\_Matlab File EXchange\Data\_Sim.xls is not writable. It may be locked by another process.

Then you must close Data\_Sim.xls and run Run\_Flood.m again.

The user must refer to many References and documents in the Web:
1-Chow
2-Bobee & Ashkar
3- Laborde
4-Water Resources Research Report: the comparison of GEV- Log Pearson
5- Meylan et al
6-Roche
7- Lamas
8- Ashkar & Ouarda

9- Bulletin-B17 Bulletin-C17, USGS...

Many references in the web ...

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