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MATH.APP.730-2024-2025-1: Inverse Problems, Tampere University, Spring 2025, Sampsa Pursiainen. Project Work.

As an alternative to the exam (and exercises), students may choose to submit a project work. It is also possible to complete both the exam and the project work, in which case the higher of the two grades will be used as the final course grade.

Project Work Instructions

Choose one of the two topics listed below. Write a code to solve the presented problem and prepare a short 3–4 page document of your work, including the following sections:

1. Introduction,
2. Materials and Methods,
3. Results,
4. Discussion.

Include your computation code as an attachment. Collaboration with other students is allowed for computing the results; however, the document should be written independently. The final file should be submitted via the Moodle course page no later than Monday, March 10th, 2025, by 23:59.

Topics

Topic 1: EEG Dipole Source Recovery

Utilize the hierarchical Bayesian approach to recover an analytic EEG dipole source within a unit disc with constant conductivity $\sigma = 1$. Assume that data is measured using a set of electrodes (points) positioned on the boundary, with the number of electrodes ranging from 10 to 30.

Place a lattice of analytic dipoles within the disc and compute the lead field matrix $\mathbf{L}$ with respect to that lattice and the electrode points. The potential field u corresponding to a dipole with position $\vec{r}$ and dipole moment $\vec{d}$ is given by:

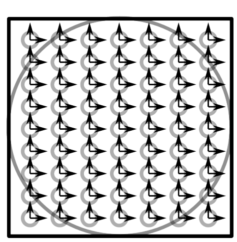

Figure 1: A regular lattice of dipoles placed within a unit disk. Each lattice point includes a dipole for both x- and y-axis orientation.

Table 1: Diameter and center coordinates of Ω and foam cylinders **A**–**C**.

Target	x (cm)	y (cm)	Diameter (cm)
A	-11.0	12.0	15.0
B	12.0	10.0	10.0
C	-2.5	-13.0	10.0
Ω	0.0	0.0	29.0

$$u = \frac{\vec{d} \cdot (\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3}$$

Assume noisy measurements and employ either the IAS algorithm (simpler) or the Gibbs sampler (more complex) to recover the dipole using a hierarchical Gaussian prior. Each lattice point should include dipoles oriented along both x- and y-axes, as shown in Figure 1.

To prevent *inverse crime*, avoid using lattice points for the exact dipole location. Instead, recover the exact dipole as a linear sum of lead field dipoles:

$$\vec{d} = \sum_{i=1}^{n} \alpha_i \vec{d}_i, \quad \vec{r} = \sum_{i=1}^{n} \gamma_i \vec{r}_i.$$

Investigate localization accuracy concerning dipole position and orientation separately. Analyze the impact of dipole depth on localization accuracy. Compare the results to an alternative prior, such as a white noise prior. Try different hyperparameter values and two noise levels, for example, 5–10% Gaussian noise relative to the signal amplitude.

Topic 2: Acoustic Setup for Object Localization

An acoustic setup with one speaker and two microphones was used to gather the attached waveform data. The computational domain Ω is a 59 cm diameter disk on a 0.5 cm thick soft foam cover (Figure 2). Three foam cylinders, labeled **A**, **B**, and **C**, with diameters 15 cm, 10 cm, and 10 cm respectively, were placed upright within Ω. The perimeter of Ω consists of 64 control points (labeled **1**–**64**) for localizing the transmitter and receiver. A top-down view is shown in Figure 2. Object diameters and positions are provided in Table 1.

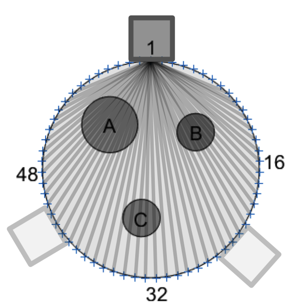

Figure 2: Top-down view of the setup. Control points **1**–**64** are marked as blue crosses along the perimeter. Four points are numbered in a clockwise sequence. Transmitter positions include **1**, **24**, and **43**.

Localize the foam cylinders using signal travel-time data. Approximate the signal paths with linear line segments. To determine travel-time, employ either thresholding, integration, or the Akaike Information Criterion (a special case of the Bayesian information criterion).

Forward simulations can be performed using the ground layer imaging example and script from Lecture 2. Choose an appropriate inversion strategy from those introduced during the course.