

## Calculo da integral eliptica

$$T = \sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = \sqrt{\frac{2\ell}{g}} \int_{-\theta_0+\varepsilon}^{\theta_0+\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} \quad \text{numérica/avaliado}$$

$$\underbrace{+\sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{-\theta_0+\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}}_A + \underbrace{\sqrt{\frac{2\ell}{g}} \int_{\theta_0-\varepsilon}^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}}_B$$

A analítico                      B analítico

$$= N + 2A \quad \text{pois } \theta \rightarrow -\theta \Rightarrow A=B$$

$$A = \sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{-\theta_0+\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} \quad \leftarrow \theta \text{ próximo de } -\theta_0$$

$$\theta = -\theta_0 + \varphi \quad \varphi \text{ pequeno}$$

$$\cos\theta - \cos\theta_0 = \cos(-\theta_0 + \varphi) = \cos\theta_0 \cos\varphi + \sin\theta_0 \sin\varphi - \cos\theta_0$$

$$\approx \sin\theta_0 \cdot \varphi,$$

$$A = \sqrt{\frac{2\ell}{g}} \int_0^\varepsilon \frac{1}{\sqrt{\sin\theta_0}} \frac{1}{\sqrt{\varphi}} d\varphi = 2 \underbrace{\sqrt{\frac{2\ell}{g}} \frac{1}{\sqrt{\sin\theta_0}} \sqrt{\varepsilon}}_{\text{adicionar ao número}}$$