Peer Effects in Active Learning*

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Abstract

This paper studies peer effects in an active learning environment, where peer interaction is a meaningful learning mechanism. Identification of peer effects relies on the random assignment of students to different disciplines in a higher education institution. We leverage random variation in the peers' ability distribution across groups and in the frequency at which peers meet for group work and show that social proximity among peers is increasing in the number of groups the students share and in how close they are in terms of their predetermined ability. Our main results show that replacing a socially distant peer with a closer one increases the average score of low-ability students in written exams by 2.8%. Also, the probability that low-ability students receive an evaluation corresponding to outstanding performance in group work increases by 30%, which demonstrates that social proximity makes them exert more effort. There are no detectable effects on the high-ability students' grades. This highlights the importance of peer interaction in positive peer effects on the academic performance of low-skilled individuals.

JEL Codes: D62, D85, I21, I23, J24

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1 Introduction

The group of schoolmates is an important input for a student's performance because peer effects matter in education (Sacerdote, 2001; Epple and Romano, 2011). The analysis of different group assignment schemes in the search for a policy to improve performance must consider how unobserved characteristics would shape behavioral responses in different groups (Fruehwirth, 2014) and the consequences for the interaction among peers.

In this paper, we estimate peer effects on students' performance in a higher education institution that adopts an active learning methodology designed to make peer interaction a meaningful learning mechanism. We show that students of low ability levels perform better in written examinations and exert more effort in groups where they have a higher chance of interacting with peers. Our identification strategy exploits the random assignment of students into groups, and we leverage within-student variation of peers to address the empirical challenges in estimating peer effects. We analyzed 132 students in their first academic semester of an undergraduate course in economics. During this period, they completed 6 disciplines and met weekly for classwork in 67 tutorial groups of 13 people, on average. The assignment rule to allocate students into groups ensured random variation of peers in two dimensions: the peers' ability distribution across groups and the number of peers in each group that the student met in at least another group of a different discipline.

The paper contains three main results. First, we show that the number of meetings for group work strongly predicts students' reported social proximity, which also varies according to their predetermined ability level. We define two students as socially proximate if both express the desire to meet each other for future group work. Pairs of students who met in more than one tutorial group were three times more likely to be socially proximate than pairs that met only once. The main takeaway of this analysis is that social proximity increases with the number of groups the students share and the closer they are in terms of their predetermined ability in math. Thus, the assignment rule provides two sources of exogenous variation in social proximity among peers that we leverage to estimate reduced-form peer effects on the students' academic performance.

We show that increasing social proximity among low-ability students and their peers in the tutorial groups increases their average scores in the disciplines' written examinations. Finally, although we cannot rule out that better exam performance results in part from more interaction outside the classroom, we show that social proximity increases the probability that low-ability students receive an evaluation corresponding to outstanding performance, which is a sign of increased effort that students demonstrate during tutorial sessions. There are no significant effects on the performance and effort of high-ability students.

We discuss these results considering a simple peer effects model in which an exogenous variation in peer interaction implies adjustments in the equilibrium effort put into group work. Given the structure of incentives that students face, their achievement production functions, and a reward scheme for participation in group work, low-ability students have room to improve their written exam scores through increased effort put into group work, but this does not imply substantial increase on the performance of high-ability students.

Related literature and contribution In different contexts, there can be positive peer effects on the performance of low-skilled individuals (Carrell et al., 2009; Fafchamps and Gubert, 2007) but researchers have shown that negative effects might also arise (Carrell et al., 2013; Feld and Zölitz, 2017). Peer interaction is an important mechanism to explain these different findings. If peer teaching is a relevant source of peer effects, as it is expected to be in our setting, observing positive spillovers on the low-ability students' achievement requires the existence of interaction with more skilled peers, for instance (Kimbrough et al., 2022).

In the active learning methodology, peer interaction is a meaningful learning mechanism, and in the environment that we analyze, this is reinforced by two characteristics. First, all students have real-world incentives to interact with peers because the evaluation of their effort in group work is part of their final assessment in each discipline they take, and correct incentives are important to foster peer interaction (Li et al., 2014). Besides, we observe students who meet in relatively small task-oriented groups. This proximity should benefit peer interaction (Hong and Lee, 2017; Lu and Anderson, 2015) and provide opportunities for collaboration between peers of different ability levels (Brady et al., 2017). These features make peers an important input of a student's achievement production function in our setting.

Since social proximity matters for peer interaction (Garlick, 2018), it is important to define a student's group of peers as sharply as possible. Observing students in small groups and distinguishing peers within the groups by their likelihood of interaction allows us to circumvent the shortcomings of using a broad definition of peers, and we advance relative to solutions adopted in other studies in two ways. Presler (2022) and Martin et al. (2020) infer peer friendship from records of students who spend time together and leverage the variation of peer interaction captured by this friendship measure to estimate peer effects. However, these types of peers are likely to share unobservables that might confound peer effects estimates. In our approach, it is exogenous variation in the opportunity that peers have to meet that drives the identification of peer effects. Coveney and Oosterveen (2021) adopted a similar strategy to ours by leveraging variation in peer interaction due to the random assignment of students to meetings outside the classroom. However, the strengthening of social ties in this case might occur due to interaction in non-academic activities, while in our approach, this is based on increased interaction in groups where students meet for academic work with the same goals and working under the same incentives. Thus, our reduced-form estimates are less subject to capture effects driven by students' sorting based on unobservable characteristics unrelated to their school performance.

In the rest of the paper, we present the organizational framework, the data, and the research design. Then, we present and discuss our main results. We conclude with some final remarks.

2 Organizational Framework

2.1 Background

The Sao Paulo School of Economics is a private higher education institution established in Brazil in 2003. In 2019, the annual course fee was 65,000 BRL, roughly 3.7 times the estimated Brazilian per capita income for that year. Students in our sample, therefore, rank at the top of the income distribution in Brazil. Between 2003 and 2016, the school used a highly selective admission exam taken by about 1,500 applicants to choose up to 60 students for its undergraduate program in economics, the only one offered by the school. The number of students admitted each year gradually increased to 120 between 2017 and 2021.

Since 2013, all admitted students have completed their coursework within the problem-based learning (PBL) framework. This active learning environment at the Sao Paulo School of Economics is structured in weekly tutorial sessions. There were, on average, 13 students per group, and each group had a tutor – either a professor or a Ph.D. student. Students attended two or three weekly tutorial sessions, contingent on the specific course (e.g., Math, Finance, etc.). In sessions of the same discipline, the group of students remained constant throughout the academic semester, while students faced distinct peer groups across different disciplines. Over the period examined in the paper, the distribution of traditional lectures and tutorial sessions varied depending on the specific course. Nevertheless, tutorial sessions comprised at least 65% of the coursework in any course.

In each tutorial session, the students' main goal is solving a problem by applying concepts and techniques they learn through self or group study outside the class. The approach to a specific problem has three phases. First, students get to know the problem and identify the learning goals. Then, they study outside the class using the bibliographic references and go to the next meeting, where they effectively solve the problem. The tutor's main job is to ensure that all students put effort in the work developed during the sessions. Students are encouraged to participate in the discussions through questions directly posed by the tutor or through individual feedback, publicized at the end of the meeting. The tutor's end-of-session feedback is a mandatory task in which grades between zero and one based on individual performance are assigned to each student. A grade below one denotes that performance fell short of the standard that was expected for the session. Also, it is possible that a student receives a grade slightly above one to denote an outstanding performance. Absence in the session implies a participation grade equal to zero.

2.2 Students Assignment Method

We implemented the allocation of new students from the 2018 and 2019 cohorts in their first semester at the school. During this semester, they took six mandatory courses, and for every discipline, we assigned each student to a tutorial group with 12 students on average.

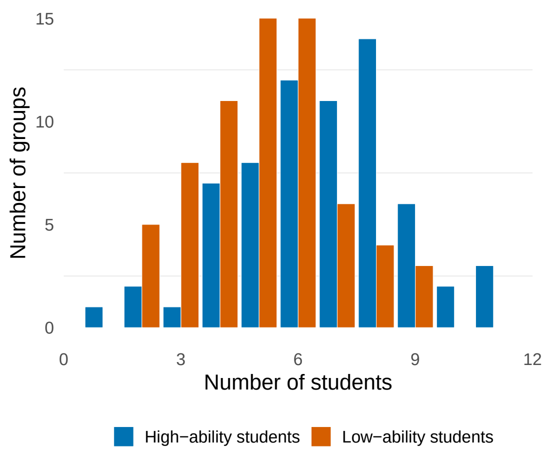
The algorithm to allocate students into tutorial groups ensured random variation in two dimensions. First, considering the number of low- and high-ability students in each group, the algorithm created considerable variation across groups. We classified students by their predetermined ability measured by their ranking in the admission exam, which was the information we had available at the time of the allocation. Besides, since students take six disciplines, some shared more than one group. Thus, some pairs of students met weekly in only one group while others, by chance, met in another group.

Now, we explain the relevant aspects of the assignment mechanism with a simple example. Suppose we have to allocate 18 students in three groups of a given discipline, and the admission ranking determined nine low-ability students and nine high-ability students. Then, we followed these steps:

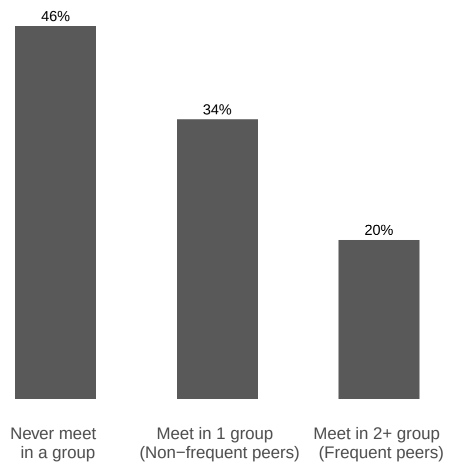
- Step 1) The algorithm randomly chooses how many students of each ability to place in each group. Example: Group A will have 5 low- and 1 high-ability students, group B will have 1 low- and 5 high-ability students and group C will have 3 low- and 3 high-ability students. We run this lottery without replacement (conditional on type) to minimize the chance that two groups have the same composition, and then we increase variation across groups. This step is constrained by group size and total students by ability level.
- Step 2) Given the composition of groups defined in step 1, this step defines, at random, which students will be in each group. That is, if there is a total of 9 low-ability students and the previous step defined that a group must have 5 of them, this step sets the identity of these 5 students. Then, among the 4 remaining low-ability students, the algorithm draws a subset for the next group, and so on.

Random variation in group composition In step 1, the algorithm generated what we call *random variation in group composition* throughout the paper. It means random variation in the number of low- and high-ability students across groups. Besides, step 2 ensured that conditional on the ability level, being in a group with many or a few high-ability peers is a random outcome. The result of such random variation is in figure (1a), where we present the distribution of the number of students by ability level considering the 67 groups created. Although the algorithm used the admission ranking to allocate students, we present group composition with students classified by their predetermined math ability. This is the ability measure we will consider in the paper as it is the strongest predictor of GPA in previous cohorts. Apart from low- and high-ability students, there were few students with no ability classification and students redoing the discipline (see descriptive statistics). These students were randomly allocated after the low- and high-ability ones.

Random variation in the frequency of meetings Consider three students i, j, k randomly selected to be in the same group of some discipline. In a second discipline, a different draw might place the pair ij in the same group while k goes to another group. This implied that ij met twice for group work, while ik jk met only once. This is what we call *random variation in the frequency of meetings*. There were 4,729 possible pairs of students in the cohorts we analyzed. However, in the actual allocation, 45% of those potential pairs never happen to meet for group work, 35% meet in only one group, and 20% meet in more than one group. Thus, among all pairs allocated to some group, more than one-third meet at least once again in a different group.



(a) Group composition



(b) Frequency of meetings

Figure 1: Summary of the assigment mechanism – Figure (1a) displays the distribution of groups by the number of students classified by their math ability level, which is the strongest predictor of GPA in previous cohorts (correlation 0.52). Figure (1b) displays the distribution of pairs of students by number of meetings for group work. The total number of pairs in the sample is 4,729.

After doing the above procedure, we provided the school staff with a list indicating the groups of each student in each discipline. After the school received this information, but before the beginning of the classes, some students withdrew from the enrollment before knowing their allocation (usually to accept offers from other institutions). Students admitted to replace the leaving students were assigned to available slots following the same original assignment rule. Then, considering the pairs student-discipline provided to the school staff, the compliance rate with the random allocation was 98% among those who effectively started the course. Finally, the school staff independently assigned tutors to each group in advance of the students' assignment.

3 Data

The school provided data on students' academic performance, their scores in the admission exam, and the files recording the students' allocation in each discipline (based on the assignment rule). From a survey conducted among the students, provided by the school, we gather data on the peers they indicated as a choice for groups of subsequent courses. In this section, we describe how we construct the variables based these data sets, present descriptive statistics, and perform an evaluation of the random assignment procedure.

Peer reporting During the period we analyzed, the school applied a survey to get information on students' perceptions about the PBL method. The survey was applied some weeks after the end of the first academic quarter. About 2/3 of the students answered the questionnaire, and participation was unrelated to ability.[1] At the end of the survey, students indicated up to seven peers they wished to meet again in future groups, independently of sharing some group at the time.[2] We use this information to construct the variable *Match* at the dyad level. Considering the sample of students who answered the questionnaire, *Match* is equal to 1 when both i reports j and j reports i, and 0 otherwise.

We assume that by revealing the desire to meet a specific peer in future tutorial groups, students recognize that the specific peer contributes to their academic performance. Besides, when this choice is reciprocal, we assume that peer interaction on academic matters is likely to explain part of the social proximity between i and j. Thus, we use *Match* as a proxy to indicate social proximity between the pair ij.

Academic performance During their initial semester at the school, students must complete six courses. Three of these courses extend throughout the entire semester, while two conclude at the end of the first academic quarter. The sixth course starts in the second academic quarter. Within the period of one week between the first and second quarters, students take exams for

[1] See Table A1 in the appendix.
[2] The constraint of a maximum of seven peers was due to space in the questionnaire.

the five courses of the first quarter, and at the end of the second quarter, they take exams for the four courses of this period.

We use three performance measures: *Exam*, *Participation*, and *Participation* > 1. *Exam* is the student's score on each discipline's exam, standardized by discipline and year. For the three courses that have two exams we define *Exam* as the average between the two exams. *Participation* is simply the average of participation grades received by the student in each tutorial session of a discipline. The variable *Participation* contains important qualitative information. A student can only achieve a participation grade above 1 if she has outstanding performance in the group work. Thus, we define *Participation* > 1 as an indicator for the case where the student's participation grade in a given discipline signals that she received this distinction at least once.

Peers variables The variables *low-ability peers* and *high-ability peers* count the number of peers in the group by each level of ability. *Frequent* and *non-frequent peers* count how many peers in a student's group appear in at least some other group of that same student. We also count peers classified according to the possible combinations of the two categories, such as *frequent high-ability peers*, *non-frequent high-ability peers*, and so on. There are three types of students we count separately to include in the regression as a control labeled as *other peers*: those doing a discipline for a second time, a few that did not use the school's admission exam and cannot be classified by the same ability measure, and students from other departments of the school.

3.1 Descriptive Statistics and Randomization Check

Table 1 provides descriptive statistics of the variables used in the analysis. We observe data from 132 students allocated in 67 groups of 6 disciplines per year. The average group has 6.5 high-ability and 5.1 low-ability students, comprising an average group size of 12.5 students. The average numbers of frequent low- and high-ability peers in a group are 2.7 and 3.4. The average performance in the exam is 7.1 (the school's passing grade is 6), and the participation grade is skewed towards 1 with low dispersion. There is much more variation in the indicator of outstanding performance as evidenced by 12% of participation grades above 1.

The assignment rule ensured that, in each group, students' average ability was uncorrelated with the number of low- and high-ability students as well as the number of frequent and non-frequent peers. To check this, we show in Table 2 the estimates for regressions of the students' admission score on the peer variables, conditional on randomization controls – year, discipline, and whether the student is classified as either low- or high-ability. Column 1 displays point

Table 1: Descriptive Statistics

	Mean	SD	Min	Max	N
Panel A. Student's ability					
Admission score	5.28	0.55	4.18	6.71	132
Math score	5.56	0.97	3.72	7.90	132
Language score	5.59	0.78	3.21	6.77	132
Panel B. Group variables					
No. of high-ability students	6.54	2.14	1.00	11.00	67
No. of low-ability students	5.13	1.78	2.00	9.00	67
Other types of students	0.72	0.87	0.00	3.00	67
Group size	12.48	2.31	8.00	16.00	67
Panel C. Student x Discipline variables					
No. of frequent high-ability peers	3.41	1.71	0.00	10.00	782
No. of frequent low-ability peers	2.71	1.39	0.00	9.00	782
Final Score in the Discipline	6.53	2.04	0.00	10.00	782
Performance in the 1st Exam	7.12	1.83	0.00	10.00	652
Performance in the 2nd Exam	5.80	2.32	0.00	10.00	518
Participation grade	0.99	0.05	0.34	1.10	782
Participation ¿ 1	0.12	0.33	0.00	1.00	782

Notes: Student's ability measures are their scores in the admission exam and ranges from 0 to 10. High-ability students are those with math score in the admission exam above the median, and low-ability ones those below it. *Frequent peers* are those peers that a student meets in more than one tutorial group. Final score is the participation grade times performance in the exams (an average of the exams when there are more two).

estimates for the coefficients on the number of low- and high-ability peers which are negligible. In column 2, low- and high-ability peers are decomposed into frequent and non-frequent peers, and again, coefficients are very close to zero, which is also true for the coefficient on *Other peers* in both cases.

4 Research Design

A simple approach to estimating peer effects is regressing a student's performance measure on some peer-related variable, such as the number of high-ability peers in the group. The method used to assign students to groups in our setting ensured the random distribution of peer variables in our sample, so we could use this approach to have a consistent estimate of *some* peer effects parameter.

However, changing the group's ability level might also change the opportunity for peer

Table 2: Randomization check – Regression of Admission Score on Group Composition

	Admission score	
	(1)	(2)
High-ability peers	-0.006	
	(0.008)	
Low-ability peers	0.000	
	(0.009)	
Frequent high-ability peers		0.000
		(0.008)
Frequent low-ability peers		-0.004
		(0.009)
Non-frequent low-ability peers		-0.014
		(0.009)
Non-frequent high-ability peers		0.003
		(0.009)
Other peers	0.004	0.003
	(0.011)	(0.011)
Num.Obs.	788	788
Students	133	133
Groups	67	67

Notes: The dependent variable is the raw score used in the admission exam and ranges from 0 to 10. This was the ability measure used to classify students in the group assignment procedure. Peers variables count, for each student, the number of peers in each group categorized by their math ability and by the frequency of meetings with the student. Robust standard errors in parenthesis.

interaction if it depends on peers' ability (Carrell et al., 2013). Thus, estimates obtained from this type of exercise might summarize different mechanisms driving peer effects. For instance, there might be effects arising from peer interaction in the form of peer teaching (Kimbrough et al., 2022), and spillovers that depend on the group's average ability through adjustments in pedagogical practices (Duflo et al., 2011). Besides, the impact in each case might depend not only on the peers' ability level but also on the student's ability level who experiences such peer effects (Carrell et al., 2009).

To investigate the impact of peer interaction entailed by social proximity on performance we distinguish a student's peers in terms of ability level and the frequency with which they meet for group work. This section builds a simple example to discuss how we estimate the effects in our setting.

4.1 Estimating Average Peer Effects

Consider the allocation of students illustrated in table 3. Here, we do not care about the ability of the s students, but we distinguish their peers as types h and l. First, we consider that the pairs ij, formed by s_i and s_j, and mn, formed by s_m and s_n, are more likely to interact because they meet in more than one group: ij in groups A, B and mn in groups A', C'. Notice that no other student appears twice in this allocation. For simplicity, we simply say that ij and mn interact and that everybody else does not. Interacting in a group impacts the student's participation in

Table 3: Simple allocation

Discipline a			Discipline b			Discipline c		
A	A'	A''	B	B'	B''	C	C'	C''
s_i s_j	s_m s_n	h_3 h_4	s_i s_j	s_m h_7	s_n h_{10}	s_i h_{12}	s_m s_n	h_{14} h_{15}
h_1 l_1	h_2 l_2	h_5 l_3	h_6 l_4	h_8 h_9	h_{11} l_5	h_{13} l_6	s_j l_7	h_{16} l_8

that group and the student's exam score of that discipline but does not affect *directly* student's performance in other groups or exams. We also acknowledge that a student's performance in a discipline can impact her own performance in other groups and disciplines – there is a form of own complementarity. To further simplify the analysis, we assume that peers of type l have no effect on the students' performance while peers of type h might positively impact their performance. Finally, define $S = \{i, j, m, n\}$.

We are interested in an outcome y_p^d of student p in discipline/group d. To estimate the effect of interaction using data from discipline a we could compute

$$\Delta^d = \left(\frac{y_i^a + y_j^a + y_m^a + y_n^a}{4}\right) - \frac{1}{5}\sum_{p \notin S} y_p^a$$

The quantity Δ^d compares the average performance of students in the pairs ij and mn, which interact, with the average performance of students who do not interact. But this estimate is likely to be upward biased: interaction of ij in B raises y_i^b and y_j^b, which in turn raises y_i^a and y_j^a. The same holds for the performance of m and n, which interact in both A' and C'. This is just a version of *reflection problem* (Manski, 1993) that arises when estimating peer effects. The same kind of problem would happen if we used data from disciplines b and c.

However, using data from discipline b would entail another problem because then m and n would appear among the students who do not interact in that discipline:

$$\Delta^b = \left(\frac{y_i^b + y_j^b}{2}\right) - \frac{1}{8}\left(y_m^b + y_n^b + \sum_{p \notin S} y_p^b\right)$$

Our simplifying assumptions implied an upward bias in the previous case, but now the sign is ambiguous because the average performance of both *treatment* and *control* subjects might be inflated by the reflection problem. Finally, if we used data from c, a third problem could arise. The estimate would be very similar to what we computed using B data:

$$\Delta^c = \left(\frac{y_m^c + y_n^c}{2}\right) - \frac{1}{8}\left(y_i^c + y_j^c + \sum_{p \notin S} y_s^c\right)$$

As in Δ^b, the reflection problem is still present, but now y_m^c might be further inflated due to its relation with y_m^b. Even though m does not interact in a second group with any of the peers in B', she has three type h peers while n has only two of them in B''. Thus, having more type h peers might have an impact on y_m^b greater than the impact of the h peers on y_n^b. This could derive either from peers of type h having a direct impact on performance or from an increased chance that m interact with some h peer in B' relative to n's chance in B'', since assuming that peers either interact or not is too extreme. Either way, we can to control for group composition in terms of peers' ability to deal with this issue.

To address the first two problems we make the assumption that observed outcome in a discipline d is determined by

$$y_p^d = Y_p^d + \alpha_p$$

in which Y_s^d is the part subject to the peer effects produced in group d. Take student m. Under this assumption we can leverage the within-student variation to estimate the reduced-form effect of interaction for student m as

$$\begin{aligned}\Delta_m = \left(\frac{y_{m,a} + y_{m,c}}{2}\right) - y_{m,b} &= \left(\frac{Y_{m,a} + Y_{m,a}}{2}\right) + \left(\frac{\alpha_m + \alpha_m}{2}\right) - Y_{m,b} + \alpha_m \\ &= \left(\frac{Y_{m,a} + Y_{m,c}}{2}\right) - Y_{m,b}\end{aligned}$$

To get an unbiased estimate, we assume the individual heterogeneity α_p is what mediates student's "own complementarity" of effort in different disciplines. Finally, we can aggregate the estimates across students to get an average effect of peer interaction on performance.

4.2 Empirical Strategy

Here, we present the empirical approach that we adopt to (i) test the hypothesis that two students meeting more frequently are more likely to interact and (ii) estimate peer effects. Our main strategy is to estimate peer effects using the within-student variation of peers following the above discussion. Each regression is weighted by the number of meetings per discipline. At the end of the section, we discuss the inference procedures we adopt.

Identifying social proximity To test the hypothesis that peers meeting in more than one group are more likely to interact we perform a regression at the dyad level. Consider the pair n composed by student i and peer j. Using the sample restricted to students that answered the school's questionnaire, we estimate the equation

$$\begin{aligned}\text{Match}_n = {} & \alpha\text{never\_meet}_n \\ & + \beta\text{nonfrequent\_peer}_n + \gamma\text{frequent\_peer}_n + \delta\text{math\_distance}_n + \varepsilon_n\end{aligned} \tag{1}$$

where Match_n is a variable indicating a match in peer reporting: i reported the desire of having j in her groups of subsequent courses *and* j reported i for the same reason. This is our proxy variable for interaction between i and j. The pair n not necessarily met in some group – the question was unconditional on meeting – thus, never_meet_n indicates that the pair n never met for group work, $\text{nonfrequent\_peer}_n$ indicates they met in only one group, and frequent_peer_n indicates they met in more than one group. The variable math_distance_n is the absolute distance between the math ability measures of i and j.

In equation (1), α, β, and γ give estimates for the probabilities of a match among pairs who never met in some tutorial group, those who met in one tutorial group, and those who met in more than one tutorial group, respecively. Actually, there is no need to control for math_distance_n in principle. The exercise controlling for this difference intends to demonstrate that the effect of frequent meetings is not driven by the fact that students seeing each other several times ended up learning about the ability of his colleague and then make the peer choice in the survey based on this information. We report results with and without this variable

Ability peer effects A basic regression to estimate peer effects on performance using within-student variation of a peer-related variables is

$$y_{i,d} = \beta\text{low\_peers}_{i,g(d)} + \mathbf{z}'_{i,g(d)}\pi + \eta_i + \theta_{c(i),d} + \varepsilon_{i,g(d)} \tag{2}$$

where $y_{i,d}$ is the outcome of student i in discipline d, $\text{low\_peers}_{i,g(d)}$ is the number of low-ability peers the student has in the group of that discipline, and $\mathbf{z}_{i,g(d)}$ controls for the quantity of peers either not classified by ability or redoing the course, and group size. The individual fixed-effect η_i captures the randomization controls (student's cohort and student ability level), plus the characteristics that could bias our estimates as discussed in section 4.1. Also, we use a cohort-discipline fixed-effect $\theta_{c(i),d}$ to account for occasional discipline-specific changes regarding contents and methods. Finally, $\varepsilon_{i,g(d)}$ is an unobserved random shock of i in group

$g(d)$. The ordinary least squares estimate of β then gives the average effect of moving i to a group with one more low-ability peer in replacement of a high-ability one, since group size and other types of students are constant. This is produced by comparing a given student in different groups.

Peer interaction The potential problem in estimating an equation like (2) is that changing the group ability distribution will likely affect peer interaction if peers' ability is important in determining who interacts with whom. However, in our setting we can distinguish subsets of peers that are more likely to interact with one another and estimate the equation

$$y_{i,d} = \beta \text{frequent\_peers}_{i,g(d)} + \mathbf{z}'_{i,g(d)}\pi + \eta_i + \theta_{c(i),d} + \varepsilon_{i,g(d)} \tag{3}$$

where $\text{frequent\_peers}_{i,g(d)}$ is the number of peers that i meets in group $g(d)$ and also in some other group $g(d')$. The estimate for β gives the average effect of moving i to a group with one more frequent peer in replacement of a non-frequent peer, which increases the probability of interaction for i in the group.

Social proximity The variable *frequent peers* does not fully capture the heterogeneity in the potential interaction within groups because social proximity depends on both the frequency of meetings and on the peers' ability level, as the results based on equation (1) will demonstrate. Thus, based on these results, we define the variable *close peers* by counting the peers with the highest chance of being socially proximate to i. For the low-ability students, *close peers* are everybody except the non-frequent high-ability peers. For the high-ability students, the excluded category is the non-frequent low-ability peers. Then, we estimate the equation

$$y_{i,d} = \beta \text{close\_peers}_{i,g(d)} + \text{peers\_ability}_{g(d)} + \mathbf{z}'_{i,g(d)}\pi + \eta_i + \theta_{c(i),d} + \varepsilon_{i,g(d)} \tag{4}$$

While in (3), replacing a non-frequent peer by a frequent one leaves the group average ability unchanged, this is no longer true when replacing a distant peer by a closer one. Thus, we control by $\text{peers\_ability}_{g(d)}$ so that the estimate for β is not biased by ability peer effects and gives the average effect of moving i to a group with one more socially close peer in replacement of a distant one, which increases i's average social proximity in the group.

Finally, for all equations, from (2) to (4), we present results for versions of these equations that display the effects separately for low- and high-ability students. We do so by interacting the peer variables with dummies of the student's own ability level.

4.2.1 Inference

For the dyadic regression (1) we compute standard errors following the estimator proposed by Aronow et al. (2015). For any equation derived from (2) or (4) we report standard errors clustered at both student and group levels (Cameron et al., 2011). However, since we know the data-generating process of our regressors of interest, we can report p-values calculated through a randomization inference approach. This is helpful since we do not have a large number of clusters and the correlation across error terms can be quite complex in a setting like ours.

The procedure consisted in replicating the assignment rule described in section 2.2 10,000 times to generate a set of allocations that could have been implemented instead of the actual one. Then, we use students' actual performance to run each regression in each of these placebo allocations. Thus, assuming that potential outcomes are unchanged, we compare the estimate obtained in the actual data with the mean zero distribution of estimates computed out of the placebo estimates. For each coefficient, we report the p-value associated with the test $H_0 : \beta = 0$ and $H_1 : \beta \neq 0$. The reported number is the frequency at which the placebo estimate is larger in absolute value than the actual estimate.

5 Results

In this section, we present three sets of results. Firstly, we show that pairs of students meeting more frequently are more likely to be socially proximate. Then, we show that social proximity is an important source of peer effects on the performance of low-ability students on the exams, and affects the effort they put into group work.

5.1 The Effect of Frequent Meetings on Social Proximity

The main takeaway from this section is that meeting a second time for group work increases social proximity among peers relative to meeting only once. Besides, social proximity also increases the closer peers are in terms of their math ability.

Column 1 of Table 4 shows that among pairs of students who never meet in a group, there is a 2.3% chance of them reporting each other as a choice to share some group in the future. Pairs that meet in only one group would have a 1.4% probability of reporting each other, but the estimate is not statistically significant at the usual levels, and we also do not reject the equality with the "never meet" coefficient. Pairs that interact in more than one group – the frequent peers – are three times more likely to report each other than pairs that never meet. The

Table 4: The Effect of Frequent Meetings on Social Proximity

Dependent variable:	(1) Match	(2) Match	(3) Match	(4) Match	(5) Match
Never meet in a group	0.023*	0.039*	-0.002	0.014	0.072*
	(0.006)	(0.011)	(0.010)	(0.012)	(0.029)
Non-frequent peers (meet in 1 group)	0.014	0.014	0.056*	0.011	-0.011
	(0.010)	(0.010)	(0.028)	(0.010)	(0.016)
Frequent peers (meet in 2+ groups)	0.071*	0.072*	0.095*	0.055*	0.079*
	(0.014)	(0.014)	(0.033)	(0.013)	(0.029)
Distance in math ability		-0.013*	0.004	-0.001	-0.019
		(0.005)	(0.024)	(0.006)	(0.021)
Sample	Any pair	Any pair	Low-Low	Low-High	High-High
Dyads	1921	1921	336	960	625

Notes: The dependent variable is a binary variable indicating whether in a pair of students that answered the survey reported each other as a choice to be in some tutorial group in the future. Dyadic cluster-robust standard errors in parentheses calculated according to Aronow et al. (2015). $^{+}$ $p < 0.1$, * $p < 0.05$

estimated coefficient for the probability of a match between frequent peers is 0.071, significant at 5%, and we reject the equality with the other two coefficients (p-values < 0.01).

Although the assignment rule ensures that the frequency of meetings and peers' average ability are uncorrelated, students could learn about their peers' skills by meeting more frequently and indicate in the survey not the socially proximate peers, but only those more skilled. In column 2, we control for the peers' distance in math ability and it does not change the average effect of meetings on *Match*, but the estimate for the probability of a match among peers that never met for group work increases, which reveals a homophily pattern: conditional on the frequency of meetings, peers of similar math ability levels are more likely to report each other.

We consider this pattern in columns 3 to 5, where we analyze pairs of students according to the ability levels that we will use in our peer effects analysis. If pairs of low-ability students never meet in a tutorial session, the probability that they match in the survey (column 3) is statistically zero. Among pairs of non-frequent peers, there is a 5.6% significant chance of reporting a match, and among pairs of frequent peers, this chance increases by 70%, as indicated by the significant 0.095 coefficient, although we do not reject the equality between these coefficients. Among pairs formed by low- and high-ability students (column 4), the probabilities of a match among pairs that never meet and pairs that meet once are zero. However, among pairs with at least two meetings, there is a significant 5.5% chance of reporting a match, and we reject the equality with each of the other coefficients at usual levels.

Finally, among high-ability pairs (column 5), there is a 7.2% chance of a match among pairs that never meet for group work. The high rate of match among high-ability peers that never meet for group work could be explained in our setting because since the beginning of the course, students usually engage in organizations such as junior enterprises, and these students are likely high-ability ones. The probability of a match among non-frequent peers is statistically zero, and if the high-ability pair meets at least twice, the chance of a match is similar to that among peers never meeting, as indicated by the significant 0.079 coefficients (we do not reject the equality).

5.2 Peer Effects on Performance

Peer effects on the exam scores Columns 1 to 3 from Table 5 present estimates of peer effects on students' performance in the exams of each discipline (standardized by discipline and year). We first analyze the results based on equation (2), displayed in Panel A.

Column 1 of Panel A shows that the average effect of replacing a high-ability peer with a low-ability one on Exam is indistinguishable from zero. However, this zero hides a heterogeneity displayed in column 2: Adding a low-ability peer in replacement of a high-ability one causes a 5% standard deviation average decrease in Exam scores of high-ability students and an average 4.1% standard deviation increase in the Exam scores of low-ability students. Raising the number of low-ability students in a group of fixed size implies a decrease in the group's average ability, which could explain these estimated effects. Adding peers' ability measured by their average math score in the admission exam as a control in column 3 causes the estimate to lose statistical significance, but the pattern and magnitude of the point estimate remain.

While the negative estimates for high-ability students suggest the existence of ability peer effects in the sense that more skilled peers would benefit their performance, the positive estimates for low-ability students deserve more attention. Exchanging peers of different ability levels impacts not only the group's average ability but also the supply of high- and low-skilled peers with whom students can interact. Thus, a positive effect of low-ability peers on low-ability students could stem from the 70% higher chance of interaction among low-ability students (compared to pairs of low- and high-ability students). Since the distribution of ability across groups is independent of the frequency with which peers meet for group work, we investigate this in Panel B by estimating the average effect of having an additional frequent peer in the group in replacement of a non-frequent one.

Column 1 of Panel B shows that there is a positive estimated effect on Exam, and Column 2 shows that this is entirely driven by an increase in the average performance of low-ability students. Having one more frequent peer in the group produces a 5% standard deviation increase in the low-ability students' average Exam scores. There are no detectable effects on the performance of high-ability students. The independence between group ability and the number of meetings implies that adding the peers' average ability as a control in column 3 leaves these results unchanged.

Although replacing a non-frequent peer with a frequent peer certainly increases the chance of i's interaction with colleagues in the group, it does not necessarily produce the maximum increase in social proximity and peer interaction. According to Table 4, a non-frequent high-ability peer is the type of peer with the lowest probability of reporting social proximity for low-ability students, for instance. This means that, for low-ability students, adding a non-frequent *low*-ability peer to the group would still increase social proximity if it replaces a non-frequent *high*-ability peer. We take this into account in Panel C, which shows estimates based on equation (4).

Column 2 shows that replacing a socially distant peer with a closer one for low-ability students implies an average 6% standard deviation increase in the Exam scores. However, this change decreases the average ability group because distant peers are high-ability colleagues. Thus, in Column 3, we control by peers' average ability and estimate an 8.1% standard deviation increase in the Exam scores for them. For high-ability students, we do not reject the hypothesis of no effect on performance, although estimates in Columns 2 and 3 are approximately two times larger than the corresponding estimates in Panel B.

Peer effects on participation in group work Social proximity among students could raise the average performance of low-ability students in the written by encouraging them to study harder outside the classroom. A different possibility is that by having socially proximate peers in the group, students increase the effort they put into their work during the tutorial sessions and improve their understanding of the course contents. Although we do not have data to analyze what happens outside the classroom environment to test the first hypothesis, we can investigate the effect on the participation grades to see whether frequent peers impact the student's effort in group work.

Columns 4 to 6 of Table 5 present estimates of peer effects on the students' average participation grade. Panel A shows no detectable effects of group composition on the average participation grades, and the same columns in Panel B show only a barely significant 0.2 percentage point average increase for high-ability students, whose average participation grade is 0.99, and Panel C, again, shows no detectable effects of close peers on the average participation grades.

One practical limitation imposed by this measure of participation in group work is that it is highly concentrated around 1 and has little variation. Nevertheless, 12% of observations have an average participation grade slightly above 1 (see Table 1), which is relevant qualitative information since it means that a student performed outstandingly in at least one tutorial session. In columns 7 to 9, we analyze peer effects on this indicator of outstanding performance.

Table 5: Peer Effects on Performance

Dependent variable:	Exam			Participation Grade			Participation > 1		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A. Ability peer effects									
Low-ability peers	-0.007			-0.001			0.018*		
	(0.019)			(0.001)			(0.014)		
	[0.675]			[0.592]			[0.043]		
Low-ability student × Low-ability peers		0.039	0.044		0.000	-0.001		0.037*	0.013
		(0.023)	(0.052)		(0.001)	(0.002)		(0.016)	(0.027)
		[0.136]	[0.270]		[0.875]	[0.592]		[0.002]	[0.492]
High-ability student × Low-ability peers		-0.050*	-0.045		-0.001	-0.002		0.000	-0.023
		(0.027)	(0.053)		(0.000)	(0.001)		(0.017)	(0.034)
		[0.032]	[0.238]		[0.470]	[0.379]		[0.970]	[0.226]
Peers' average math ability			0.031			-0.006			-0.144
			(0.264)			(0.007)			(0.134)
			[0.865]			[0.564]			[0.128]
Observations	782	782	782	782	782	782	782	782	782
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132
Panel B. Peer interaction									
Frequent peers	0.029+			0.001			0.009		
	(0.018)			(0.001)			(0.009)		
	[0.094]			[0.291]			[0.291]		
Low-ability student × Frequent peers		0.050*	0.050*		0.000	0.000		0.020+	0.020+
		(0.024)	(0.024)		(0.001)	(0.001)		(0.015)	(0.015)
		[0.049]	[0.049]		[0.982]	[0.980]		[0.098]	[0.097]
High-ability student × Frequent peers		0.015	0.015		0.002	0.002		0.002	0.002
		(0.023)	(0.023)		(0.001)	(0.001)		(0.011)	(0.011)
		[0.500]	[0.502]		[0.103]	[0.104]		[0.873]	[0.865]
Peers' average math ability			0.039			0.001			-0.118*
			(0.104)			(0.004)			(0.062)
			[0.679]			[0.895]			[0.017]
Observations	782	782	782	782	782	782	782	782	782
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132
Panel C. Social Proximity									
Close peers	0.043*			0.000			0.019+		
	(0.020)			(0.002)			(0.014)		
	[0.045]			[0.940]			[0.066]		
Low-ability student × Close peers		0.060*	0.081*		-0.002	-0.001		0.020	0.031*
		(0.028)	(0.026)		(0.003)	(0.002)		(0.019)	(0.014)
		[0.027]	[0.004]		[0.320]	[0.752]		[0.110]	[0.023]
High-ability student × Close peers		0.027	0.031		0.001	0.001		0.018	0.017
		(0.027)	(0.026)		(0.002)	(0.001)		(0.017)	(0.015)
		[0.305]	[0.243]		[0.274]	[0.396]		[0.193]	[0.215]
Peers' average math ability			0.092			-0.001			-0.105+
			(0.118)			(0.005)			(0.065)
			[0.385]			[0.825]			[0.056]
Observations	782	782	782	782	782	782	782	782	782
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132

Notes: The dependent variables are students' standardized score in the disciplines' exams, their participation grades, and an indicator of average participation grade greater than 1. Standard errors clustered at both student and group levels displayed in parentheses. We show in brackets p-values calculated through the randomization inference procedure based on the 10,000 placebo allocations of students.
$+\ p < 0.1, *\ p < 0.05$

Columns 7 and 8 of Panel A show that replacing a high-ability peer with a low-ability peer increases the probability of receiving a participation grade above 1. This is driven by a 3.7 percentage point increase in this probability for low-ability students, which disappears once the control for the peers' average ability is added in column 9. If the tutor evaluates the student's relative performance in the group, this result can reflect that it would be easier for a low-ability student to outperform peers in a group of lower average ability level.

However, column 8 of Panel B shows that there is a 2 percentage point increase in the probability of participation grade above 1 for low-ability students when a frequent peer replaces a non-frequent one. Following our previous discussion, in Panel C we look at what happens when social proximity is maximized. Column 9 displays a 3.1 percentage point increase, which means a 31% increase relative to the 10% rate of participation grades above 1 among low-ability students. There is no effect on the probability of some outstanding performance of high-ability students, although estimates are again larger when compared to the corresponding figure in Panel B.

Summary Increasing the social proximity of students in a group by replacing socially distant peers with closer ones improves average exam performance for low-ability students by 8.1% of a standard deviation, which translates to a 2.8% increase in their average raw score in the exams. There are no detectable effects on high-ability students' scores. Although there aren't substantial effects on the intensive margin of average participation grade, increasing the potential for peer interaction raises the probability of outstanding performance in group work among low-ability students.

5.2.1 Additional Results and Robustness Exercises

Decomposing Social Proximity In Table 6, we present an exercise in which we consider each type of close peer separately. That is, we can now look at how frequent peers of low and high ability levels affect one's performance differently, for instance. We conduct separate regressions in the samples of low- and high-ability students. In each case, we exclude the socially distant type of peer from the estimating equation: for low-ability students, the baseline type is the non-frequent high-ability peers; for high-ability students, it is the non-frequent low-ability peers.

For low-ability students, Panel A shows the estimated effects of replacing a non-frequent high-ability peer with peers with a higher chance of social proximity on the exam scores and on the indicators for outstanding and below-standard performance. Overall, there does not seem to be a differential effect depending on which type of peer replaces the socially distant type. At least, Panel B shows that we cannot reject that frequent peers of low and high ability levels have the same average effect that a non-frequent low-ability peer has when replacing a non-frequent high-ability peer. This is in line with our estimates in Table (4), which shows that low-ability students have a positive probability of being socially close to any peer included in the equation.

Although we also do not reject the equality of estimates for the effects on the exam scores and probability of outstanding performance among high-ability students, we see that socially proximate peers are likely to have a modest impact on participation grades, as already evidenced in Panel B of Table 5.[3]

Table 6: Peer Effects on Performance - Social Proximity

	Low-ability students			High-ability students		
	Exam	Part.	Part. > 1	Exam	Part.	Part. > 1
Panel A. Close peers: Regression estimates						
Frequent low-ability peers (β_1)	0.081	-0.002	0.057*	-0.011	0.003+	-0.004
	[0.168]	[0.606]	[0.044]	[0.775]	[0.075]	[0.838]
Frequent high-ability peers (β_2)	0.078*	-0.002	0.033+	0.035	0.003	0.032
	[0.034]	[0.519]	[0.061]	[0.505]	[0.140]	[0.255]
Non-frequent low-ability peers (γ_1)	0.058	-0.003	0.040			
	[0.312]	[0.455]	[0.139]			
Non-frequent high-ability peers (γ_2)				0.007	0.002	0.032
				[0.889]	[0.317]	[0.250]
Peers' average math ability	-0.076	-0.007	-0.039	0.173	-0.006	-0.198
	[0.784]	[0.727]	[0.774]	[0.480]	[0.566]	[0.143]
Panel B. Tests for the equality of coefficients						
$\beta_1 - \beta_2$	0.003	0.000	0.024	-0.045	0.000	-0.036
	[0.963]	[0.913]	[0.400]	[0.404]	[0.873]	[0.231]
$\beta_1 - \gamma_1$	0.023	0.001	0.016			
	[0.598]	[0.743]	[0.427]			
$\beta_1 - \gamma_2$				-0.018	0.001	-0.036
				[0.736]	[0.739]	[0.206]
$\beta_2 - \gamma_1$	0.020	0.001	-0.008			
	[0.734]	[0.720]	[0.778]			
$\beta_2 - \gamma_2$				0.028	0.001	0.000
				[0.397]	[0.448]	[0.999]
Observations	344	344	344	438	438	438

Notes: The dependent variables are: the students' standardized final score; standardized performance in the exams; and the indicator of average participation grade greater or equal to 1. We show in brackets p-values calculated through the randomization inference procedure based on the 10,000 placebo allocations of students. $+\, p < 0.1, *\, p < 0.05$

[3] If we look at the impact of frequent peers without distinguishing their ability but holding constant the non-frequent high-ability ones, there is a 0.3 percentage point increase in the participation grades (p-value $= 0.043$, not reported), which is still a very modest.

Share of peers If we use the share of different types of peers instead of counting them, the results are basically the same. The 0.711 coefficient presented in Column 1 of Panel B of Table 7, for example, means that the average effect of replacing a non-frequent peer with a frequent one on the performance of low-ability students would be of a 5.7% standard deviation, since one peer represents 8% of a group, on average. This is close to the corresponding 5% standard deviation increase presented in Table 5.

Only first quarter disciplines Our main analysis used students' achievement in all the 6 disciplines they take in their first semester at the school. However, one might be concerned about whether the definitions of frequent and non-frequent peers we used are appropriate for the discipline *Probability* that begins in the second quarter. This is because a peer is defined as "frequent" if she or he shares more than one group with a given student. But once the first quarter ends, this definition could break in some cases, where students were defined as frequent peers because they met in a discipline that also ends. In this situation, they only share a single group in the second quarter. To check whether this is a relevant problem, we replicate our main results from tables 5 removing the observations of *Probability*. The outcome *Exam* is now only the score in the exams taken at the end of the first quarter. Unfortunately, we do not have participation grades for this period only and we are using the same participation grade as before in the restricted sample. Overall, the estimates are qualitatively similar and even larger. Thus, it does not seem to be a major problem for our main analysis.

Peers classified by their writing ability Peers' ability level throughout the paper was based on their performance in the math exam of admission score since this is the score with the highest correlation with performance for previous cohorts in the school. However, considering the dynamics of group work, peers' language skills can be an important input for one's performance. Table 7 presents the results for the ability peer effects specification defining low- and high-ability peers based on their ability in the writing admission exam. Now, the results based on the ability peer effects specification in Panel A show the same signs as those in Table 5, but for high-ability students, replacing a peer with high writing skills with a lower-skilled one implies a statistically significant decrease in the average participation grade. Although the 0.4 average percentage point decrease is a modest effect, it seems to be enough to imply a lower probability of outstanding performance by high-ability students, as evidenced by the 4 percentage point decrease, which is a 28% decrease relative to the 14% rate of outstanding performance among high-ability students. Using the peers' writing ability in the definition of close peers (Panel C) produces qualitatively similar estimates, but the effects are weaker than those that consider the peers' math ability.

6 Discussion

In this section, we discuss how adjustments in the effort put into the work during the tutorial sessions in response to a denser network of peers could have produced different impacts on the performance of students of different ability levels. The arguments follow a simple peer effects model, which we present in the sequence, that we tailored to clarify the potential mechanisms at work under the structure of incentives in the environment we analyze

A student's final score in a discipline is the product of their performance in the written examinations and their average participation grade in that discipline. It means that there are two ways of raising the final score: achieving better performance on the exams or receiving higher participation grades during the semester. Importantly, students want to have participation grades as close to 1 as possible to maximize the contribution of their exam scores to the final score.

The effects on the exam scores Achievement on the exam depends on the effort students put into group work because tutorial sessions are opportunities for them to strengthen their understanding of the material they studied prior to the meeting. This occurs by raising and discussing different questions related to the learning goals. Importantly, when a student makes the effort to start a discussion, the quality of that discussion will likely depend on the effort that peers employ in the discussion. Besides being an example of how effort in group work can produce better performance in the exam, it highlights that peer interaction introduces a complementarity that makes the return to achievement of the student's own effort increase with peers' effort. However, this is not the only way that social proximity can lead to better performance. Making effort is costly. In an environment of high-income peers, students might be less likely to ask a clarification question if they believe it will give peers a bad signal of their capabilities (Bursztyn et al., 2019). Thus, having closer friends in the group can reduce the cost due to this type of peer pressure.

But why would this mechanism entail that social proximity improves performance in the exam for low-ability students but not for high-ability ones? Students' scores in the exam are bounded above by the maximum possible grade. If the production function that transforms effort into achievement is heterogeneous in the student's ability in a way that high-ability students get close to the upper bound of the exam with lower effort levels[4], then a marginal increase in effort could lead to a sizable increase in performance for low-ability students but only a negligible change for the high-ability ones. From a practical point of view, this means, for example, that highly skilled students could arrive at the tutorial sessions with a very good level of under-

[4]the function for high-ability students is "more concave".

standing about the subject and make an effort only to ensure a participation grade large enough to internalize achievement. On the other hand, low-ability students would still exert some effort to grasp the core of the learning goals to achieve an appropriate exam score.

The effects on participation The small variance in participation grades does not mean that all students exert the same level of effort. Different from the written examination, which is the same and uniformly applied to all students, the student's evaluation about participation in the tutorial sessions might depend on ability if the tutor wants to maximize students' effort subject to the constraint that it is not too costly that students opt to quit the course. If the cost of effort is decreasing in ability, a low-ability student would get close to participation grade equal to 1 with less effort than a high-ability one. Finally, if social proximity affects effort, why don't we identify any effects on participation grades? The simple answer is that if students are close enough to a participation grade equal to 1 and if the tutor rewards effort through a function with diminishing returns, then participation grades might respond little to increases in effort. However, a significant increase in effort would still produce a small variation in the participation grade but would be enough to put the participation grade slightly above 1 as a signal of outstanding performance. This explains why we do not find effects of social proximity on participation grades but do find effects on the probability of outstanding performance of low-ability students.

To conclude, it is worth noting that besides the effect of increased effort on performance in the exam, the public recognition that a student had outstanding performance might amplify this effect by making the rewarded student to exert more effort and do even better on the exams (Moreira, 2016).

6.1 A Simple Model of Peer Effects

The basics Any active action within the group, such as raising a question, answering someone else's questions (including the tutor's), or volunteering to explain some concept, entails some level of effort. This is the type of effort the tutor rewards in each tutorial session. Of course, a student who is only sitting in the class and paying attention to the discussion is making some cognitive effort and can learn from peers. But we are interested in the "active" effort, which we simply call "effort" from now on. For simplicity, we refer to "friends" as a pair of students who have some level of interaction during group work.

Here, we discuss four elements needed to set up and solve the student's problem of optimal effort choice in group work: the production function for student's achievement in the exam, the cost of effort, a social interaction component, and the tutor's participation reward scheme. In the model, a student i is characterized by the ability level θ_i. The vector containing the ability of i's peers is $\boldsymbol{\theta}_{-i}$. The effort in group work is e_i and $\mathbf{e}_{-i}$ for the student and peers, respectively, and there is an interaction matrix $\mathbf{G}$ describing the group's network. Students will maximize performance – achievement times participation – net of their effort cost plus a social component that affects the incentive to make effort.

Student's achievement in the exam We assume the student's production function has two components:

$$A(e_i, \theta_i; \mathbf{e}_{-i}, \boldsymbol{\theta}_{-i}) = z(\mathbf{e}_{-i}, \boldsymbol{\theta}_{-i}) + f(e_i, \theta_i)$$

In our environment, the most common form of participation usually involves some form of peer teaching in which a student develops ideas on the blackboard. This is why both peers' effort and ability impact i's achievement through a continuous and concave function z that we assume to be increasing in both arguments. This makes feasible that students present in the session can, in principle, learn from peers independent of their own effort.

Besides, we assume that a student internalizes the output of their own participation through f, a continuous, strictly increasing, and concave function of effort ($f_e > 0$, $f_{ee} < 0$). We assume that more skilled students perform better for a given level of effort ($f_\theta > 0$). Analogous to the effect of peer teaching in z, the function f is able to capture effects apart from peer interaction, such as developing an exercise on the blackboard without any feedback, which could serve as some practice that improves achievement. The separability in A allows for these types of independent effects. But notice that A can still account for the output from student interactions, where questions and answers increase f or z depending on the role played by the student. We will introduce a premium for interaction below.

The cost of effort The cost of exerting effort is given by $c(e_i, \theta_i)$, which we assume to be an increasing and convex function of effort ($c_e > 0, c_{ee} > 0$), decreasing in θ_i ($c_\theta < 0$), and with the marginal cost of effort also decreasing in θ_i ($c_{e\theta} < 0$). The decreasing effects of ability characterize students' heterogeneity in terms of the cognitive requirements for each student to exert a certain level of effort and, potentially, a peer pressure channel that makes a low-ability student less likely to raise questions (Bursztyn et al., 2019).

The social interaction component A social interaction component

$$s(e_i; \mathbf{e}_{-i}, \mathbf{G}) = \left(\sum_{j \neq i} g_{ij} e_j \right) e_i$$

can be interpreted as either a reduction of cost or an increase in performance due to an increased incentive to exert effort once the student has some level of interaction in the group as indicated by $g_{ij} \in (0, 1]$ for at least some j. Social proximity is important in our context for two reasons. The ability heterogeneity in cost due to peer pressure can decrease with friends in the group. Besides, even a minimal level of interaction produces a complementarity in effort that raises performance. Importantly, it would be reasonable that g_{ij} depends on effort in the sense that making an effort to help someone increases the likelihood of either establishing or strengthening a friendship. However, we make a simplifying assumption that g_{ij} represents only the exogenous portion of such a friendship (the frequent meetings in our environment, for example), which turns $\mathbf{G}$ into an exogenous interaction matrix.

The participation reward scheme We assume the tutor evaluates participation through an increasing and strictly concave function $p(e_i - \theta_i)$ bounded above by 1 (the maximum participation grade). Remember that to internalize their achievement in the exam, students must be as close to having participation in grade 1 as possible. Thus, the function p implies that getting close to participation grade 1 requires more effort the greater the θ_i. [5]

Student's optimal effort Each student must solve

$$\max_{e_i} \quad p(e_i - \theta_i)A(e_i, \theta_i; \mathbf{e}_{-i}, \boldsymbol{\theta}_{-i}) - c(e_i, \theta_i) + s(e_i; \mathbf{e}_{-i}, \mathbf{G}) \tag{5}$$

Given that $\mathbf{G}$ is exogenous, the concavity and continuity of the objective function plus the assumption that students have a finite set of choices for e_i (e.g. seconds talking in the class) ensure the existence of a Nash equilibrium $\mathbf{e}^*$. Thus, each student takes $\mathbf{e}^*_{-i}$ as given, and the problem's first-order condition for each student i

$$\underbrace{p'(e_i^* - \theta_i)}_{(i)} \left[z(\mathbf{e}^*_{-i}, \boldsymbol{\theta}_{-i}) + f(e_i^*, \theta_i)\right] + p(e_i^* - \theta_i) \underbrace{f_e(e_i^*, \theta_i)}_{(ii)} = c_e(e_i^*, \theta_i) - \sum_{j \neq i} g_{ij} e_j^* \tag{6}$$

is necessary and sufficient for optimization.

[5] This assumption can be justified by a setting in which the tutor knows each θ_i and acts to maximize aggregate effort in the group, subject to the constraint that the cost of effort is below a personal threshold for each student so that they do not quit:

$$\begin{aligned} \max_{\mathbf{e}} \quad & \sum_i e_i \\ \text{s.t.} \quad & c(e_i, \theta_i) \leq \bar{c}_i \text{ for all } i \end{aligned}$$

If possible, the tutor would like to induce $\mathbf{e}^*$ so that $c(e_i^*, \theta_i) = \bar{c}_i$ for all i. If the student's cost threshold is not "too much" decreasing in ability, the tutor would like to have high-ability students exerting more effort than low-ability ones to get the same participation grade. However, the tutor does not know c exactly and the best he can do is designing a reward scheme with this property.

Equation 6 makes clear that a student is likely to adjust effort in response to changes in social proximity with peers ($\mathbf{g}_i$) and that this adjustment impact (i) the student's participation grade and (ii) the achievement in the exam. It is worth noting that the effects we can analyze through 6 are still reduced-form: changing g_{ij} implies an adjustment through 6 for i but also for j, which would induce further adjustment for i, and so on. This is just an example of the *reflection problem* at work in the group (Manski, 1993). This means that the student's own achievement changes through f, but also through z.

Explaining our findings The assumption on the particiaption reward scheme implies that low-ability students get close to 1 at lower effort levels than high-ability students. It means that the marginal contribution of effort to the participation grade $p'(e_i^* - \theta_i)$ approaches zero earlier for the low-ability students (low θ_i). However, for these students, the equilibrium effort level should not be too low so that it jeopardizes achievement through f or if it is insufficient to internalize the achievement through p. Thus, the equilibrium choice e_L^* of a low-ability student θ_L can be such that $p'(e_L^* - \theta_L)$ is very small and then the adjustment in effort from a change in $\mathbf{g}_L$ would have, at most, a negligible effect on the participation grade while still having substantial effect on achievement through $A(e_L^*, \theta_L; \mathbf{e}_{-L}^*, \boldsymbol{\theta}_{-L})$.

To explain the results for high-ability students, we need to add assumptions on the behavior of f that do not change the above interpretation for low-ability students. Since f is intended to describe how the student's own effort translates into achievement, we assume that f has an upper limit common to all students (e.g., the maximum score in the exam). Also, we assume that at low effort levels, f increases much faster for high-ability students than for low-ability ones.[6] In a limiting case, a highly skilled student could reach the achievement's upper limit even without attending the tutorial sessions (f would be constant at the upper limit). On the other hand, low-ability students would need to exert more effort to grasp the core of the learning goals to achieve an appropriate exam score.[7] Since the tutor requires more effort from a high-ability student θ_H, our estimates are consistent with the equilibrium effort level being large enough so that both $f_e(e_H^*, \theta_H)$ and $p'(e_H^* - \theta_H)$ are very small.

[6]Formally, $f_e(e; \theta_H) > f_e(e; \theta_L)$ up to some $\bar{e}$.

[7]The function $f(e, \theta) = 1 - \exp\big(-\theta e\big)$ satisfies the assumptions, for instance.

7 Conclusion

Peer effects estimates might be a valuable resource from a policy perspective since they can guide reallocation schemes that maximize aggregate performance. In line with recent literature on peer effects, our paper highlights the importance of peer interaction in explaining these effects. It provides a subsidy to further understand how peers can generate positive spillovers on one's performance.

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Table 7: Peer Effects on Performance – Robustness Exercises

	Share of peers			Only 1st quarter disciplines			Peers by writing ability		
	Exam	Part.	Part. > 1	Exam	Part.	Part. > 1	Exam	Part	Part. > 1
Panel A. Ability peer effects									
Low-ability student × Low-ability peers	0.578	-0.002	0.355	0.056	-0.001	0.018	0.016	-0.003	0.018
	(0.571)	(0.022)	(0.333)	(0.050)	(0.002)	(0.027)	(0.036)	(0.003)	(0.024)
	[0.257]	[0.944]	[0.158]	[0.194]	[0.795]	[0.391]	[0.607]	[0.207]	[0.205]
High-ability student × Low-ability peers	-0.387	-0.007	-0.161	-0.045	-0.001	-0.017	-0.032	-0.004*	-0.040*
	(0.627)	(0.021)	(0.396)	(0.052)	(0.002)	(0.033)	(0.035)	(0.002)	(0.024)
	[0.425]	[0.751]	[0.516]	[0.280]	[0.527]	[0.414]	[0.242]	[0.002]	[0.008]
Peers' average math ability	0.078	-0.001	-0.080	0.103	-0.005	-0.119	0.031	-0.003	-0.130*
	(0.262)	(0.008)	(0.130)	(0.264)	(0.009)	(0.130)	(0.110)	(0.004)	(0.060)
	[0.683]	[0.923]	[0.408]	[0.618]	[0.598]	[0.252]	[0.761]	[0.633]	[0.012]
Observations	782	782	782	652	652	652	782	782	782
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132
Panel B. Peer interaction									
Low-ability student × Frequent peers	0.711*	0.002	0.301+	0.058*	-0.001	0.024+			
	(0.275)	(0.016)	(0.198)	(0.027)	(0.002)	(0.015)			
	[0.034]	[0.918]	[0.076]	[0.043]	[0.732]	[0.084]			
High-ability student × Frequent peers	0.146	0.023+	0.012	0.005	0.001	-0.001			
	(0.289)	(0.011)	(0.133)	(0.025)	(0.001)	(0.012)			
	[0.609]	[0.071]	[0.944]	[0.836]	[0.167]	[0.924]			
Peers' average math ability	0.040	0.000	-0.117*	0.082	-0.001	-0.118*			
	(0.105)	(0.004)	(0.062)	(0.105)	(0.004)	(0.065)			
	[0.673]	[0.941]	[0.019]	[0.428]	[0.799]	[0.029]			
Observations	782	782	782	652	652	652			
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132
Panel C. Social Proximity									
Low-ability student × Close peers	0.845*	-0.019	0.284	0.069*	-0.003	0.023	0.060*	-0.002	0.019
	(0.300)	(0.032)	(0.227)	(0.030)	(0.003)	(0.017)	(0.029)	(0.003)	(0.018)
	[0.024]	[0.441]	[0.114]	[0.022]	[0.174]	[0.102]	[0.028]	[0.324]	[0.152]
High-ability student × Close peers	0.342	0.024	0.232	0.019	0.001	0.020	0.026	0.001	0.020
	(0.352)	(0.015)	(0.235)	(0.033)	(0.002)	(0.020)	(0.027)	(0.002)	(0.017)
	[0.347]	[0.147]	[0.250]	[0.510]	[0.254]	[0.208]	[0.331]	[0.282]	[0.157]
Peers' average math ability	0.048	0.000	-0.119*	0.090	-0.002	-0.118*	0.045	0.000	-0.119*
	(0.106)	(0.004)	(0.063)	(0.105)	(0.004)	(0.065)	(0.106)	(0.004)	(0.062)
	[0.629]	[0.921]	[0.022]	[0.391]	[0.717]	[0.034]	[0.642]	[0.987]	[0.017]
Observations	782	782	782	652	652	652	782	782	782
Groups	67	67	67	67	67	67	67	67	67
Students	132	132	132	132	132	132	132	132	132

Notes: The dependent variables are students' standardized score in the disciplines' exams, their participation grades, and an indicator of average participation grade greater than 1. Columns under "Share of peers" correspond to equations that uses each indicated variable as a proportion of the tutorial group. Columns under "Only 1st quarter disciplines" correspond to regressions that restricts the sample to disciplines taken during the 1st quarter, both those that end after the quarter and those that continue. Columns under "Peers by writing ability" correspond to equations that consider in each indicated variable the peers classified by their writing score measured in the admission exam. Standard errors clustered at both student and group levels displayed in parentheses. We show in brackets p-values calculated through the randomization inference procedure based on the 10,000 placebo allocations of students.
$+\, p < 0.1, *\, p < 0.05$