ECE355 - Homework II

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1.12

Given the discrete-time signal

$$x[n] = 1 - \sum_{k=3}^{+\infty} \delta[n-1-k]$$

We need to find the constants M and N_0, such that

$$x[n] = u[Mn - n_0] \tag{1}$$

Answer: Just for having some idea of how the behaviour of this signal, we can try some values of n:

$$\begin{aligned}
x[0] &= 1 - \sum_{k=3}^{+\infty} \delta[0-1-k] &= 1 \\
x[1] &= 1 - \sum_{k=3}^{+\infty} \delta[1-1-k] &= 1 \\
x[2] &= 1 - \sum_{k=3}^{+\infty} \delta[2-1-k] &= 1 \\
x[3] &= 1 - \sum_{k=3}^{+\infty} \delta[3-1-k] &= 1 \\
x[4] &= 1 - \sum_{k=3}^{+\infty} \delta[4-1-k] &= 0 \\
x[5] &= 1 - \sum_{k=3}^{+\infty} \delta[5-1-k] &= 0
\end{aligned}$$

Otherwise, it is not hard to realize that the part of the signal is 0 when the second term is 1, i.e., the Dirac-delta function assumes value 1 in some case, and this just happens when $n \geq 4$ because the sum starts in $k = 3$.

We have the definition:

$$u[t] = \begin{cases} 1 & , \quad t \geq 0 \\ 0 & , \quad t < 0 \end{cases}$$

But this signal is 0 starting at 4, it is not 0 until 4. So we must reverse it:

$$u[-t] = \begin{cases} 1 & , \quad t < 0 \\ 0 & , \quad t \geq 0 \end{cases}$$

The signal ends at 3, so we must time-shift it:

$$u[3-t] = \begin{cases} 1 & , \quad t \leq 3 \\ 0 & , \quad t > 3 \end{cases}$$

Turning it equal to 1, we have:

$$u[Mn - n_0] = u[3-t] = u[-1 \cdot t - (-3)]$$

Therefore, $M = -1$ and $n_0 = -3$.

1.13

Given the signal

$$x(t) = \delta(t+2) - \delta(t-2)$$

We need to calculate E_∞ of the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

First of all, let's calculate y(t):

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau = \int_{-\infty}^{t} [\delta(\tau+2) - \delta(\tau-2)]d\tau = \int_{-\infty}^{t} \delta(\tau+2)d\tau + \int_{-\infty}^{t} -\delta(\tau-2)d\tau$$

Let's define $y_1 = \int_{-\infty}^{t} \delta(\tau+2)d\tau$ and $y_2 = \int_{-\infty}^{t} -\delta(\tau-2)d\tau$. Using the definition of the Dirac-delta function, we have the following:

$$y_1 = \begin{cases} 0 & , \quad t < -2 \\ 1 & , \quad t > -2 \end{cases} \quad , \quad y_2 = \begin{cases} 0 & , \quad t < 2 \\ -1 & , \quad t > 2 \end{cases}$$

$$\Rightarrow y(t) = \begin{cases} 0 & , \quad t < -2 \\ 1 & , \quad -2 < t < 2 \\ 0 & , \quad t > 2 \end{cases}$$

Note that y(t) is not defined either for $t = -2$ or $t = 2$, so we will not consider this points when integrating, we will use just the limits over them. Therefore, we have the the integral over this points is zero.

E_∞ of a signal $x(t)$ is defined as $E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt$, for $y(t)$ we have:

$$E_\infty = \int_{-\infty}^{+\infty} |y(t)|^2 dt = \int_{-\infty}^{-2} |y(t)|^2 dt + \int_{-2}^{2} |y(t)|^2 dt + \int_{2}^{+\infty} |y(t)|^2 dt$$

$$\Rightarrow E_\infty = \int_{-\infty}^{-2} |0|^2 dt + \int_{-2}^{2} |1|^2 dt + \int_{2}^{+\infty} |0|^2 dt = \int_{-2}^{2} |1|^2 dt = \int_{-2}^{2} dt = 4$$

Therefore, $E_\infty = 4$.

1.15

Given a system S with input $x[n]$ and output $y[n]$, which consists of a series interconnection of the following systems

$$\begin{aligned} S_1 \ : \ y_1[n] &= 2x_1[n] + 4x_1[n-1] \\ S_2 \ : \ y_2[n] &= x_2[n-2] + \tfrac{1}{2}x_2[n-3] \end{aligned}$$

(a) Determine the input-output relationship for system S:

Answer: x_1 of S_1 is $x[n]$, and x_2 of S_2 is $y_1[n]$, while $y_2[n]$ is $y[n]$, so we have:

$$y[n] = y_1[n-2] + \frac{1}{2}y_1[n-3] = (2x[n]+4x[n-1])[n-2] + \frac{1}{2}(2x[n]+4x[n-1])[n-3]$$

$$\Rightarrow y[n] = (2x[n-2]+4x[n-3]) + \frac{1}{2}(2x[n-3]+4x[n-4]) = 2x[n-2]+5x[n-3]+2x[n-4]$$

Therefore, the input-output relationship for the system S is

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

(b) Is the input-output relationship changed if S_2 comes before S_1?

Answer: Using the same ideas used in (a), but with a reverse order of the systems. Let's call the final output of $y'[n]$:

$y'[n] = 2y_2[n]+4y_2[n-1] = 2(x_2[n-2]+\frac{1}{2}x_2[n-3])[n]+4(x[n-2]+\frac{1}{2}x[n-3])[n-1]$

$\Rightarrow y'[n] = 2(x_2[n-2]+\frac{1}{2}x_2[n-3])+4(x[n-3]+\frac{1}{2}x[n-4]) = y'[n] = 2x[n-2]+5x[n-3]+2x[n-4]$

Therefore, $y[n] = y'[n]$.

1.25

Determine if the following signals era periodic, if they are, give their fundamental period.

(a) $x(t) = 3cos(4t + \frac{\pi}{3})$

$x(t)$ is a signal dependent of a cosine, which is a periodic signal, so $x(t)$ is periodic as well. Its fundamental period is given by $T = \frac{2\pi}{4} = \frac{\pi}{2}$, according to the equation (1.24) of the book.

(c) $x(t) = [cos(2t - \frac{\pi}{3})]^2$

$x(t)$ can be expressed as a function of $cos(2t - \frac{\pi}{3})$, which is a periodic signal, so $x(t)$ is periodic as well. Using a trigonometric identity $cos(x)^2 = \frac{1+cos(2x)}{2}$:

$$x(t) = [cos(2t - \frac{\pi}{3})]^2 = \frac{1}{2}[1 + cos(4t - \frac{2\pi}{3})]$$

Using again the equation (1.24), x(t)'s fundamental period is given by $T = \frac{2\pi}{4} = \frac{\pi}{2}$.

(e) $x(t) = Ev\{sin(4\pi t)u(t)\}$

$x(t) = Ev\{sin(4\pi t)u(t)\} = \frac{1}{2}[sin(4\pi t)u(t)+sin(-4\pi t)u(-t)] = \frac{1}{2}[sin(4\pi t)u(t)-sin(4\pi t)u(-t)]$

Because of the unit-step function, we have the signal defined by parts:

$$x(t) = \begin{cases} \frac{1}{2}[sin(4\pi t) & , \quad t > 0 \\ -\frac{1}{2}[sin(4\pi t) & , \quad t < 0 \end{cases}$$

This signal is not periodic because it is formed by to different signals across $t = 0$.

(f) $x(t) = \sum\limits_{n=-\infty}^{+\infty} e^{-(2t-n)}u(2t-n)$

Defining $k = 2t - n$, we have the signal as a sum of infinite functions in the form $e^{-k}u(k)$, so it is not periodic.

1.26

Determine if the following signals era periodic, if they are, give their fundamental period.

Note that a discrete time signal is periodic if

$$\Omega = 2\pi\tfrac{p}{q} \quad ; \quad p, q \in \mathbb{Z}$$

Where Ω is the signal's angular frequency, and q its fundamental period.

(a) $x[n] = sin(\frac{6\pi}{7}n + 1)$

$$\Omega = \frac{6\pi}{7} = 2\pi \cdot \frac{3}{7}$$

$x[n]$ has a fundamental period, which is 7.

(c) $x[n] = cos(\frac{\pi}{8}n^2)$

If $x[n]$ is periodic with a period N, then $x[n+N] = x[n]\ \forall n$.

$$x[n+N] = cos(\frac{\pi}{8}(n+N)^2) = cos(\frac{\pi}{8}[n^2 + 2nN + N^2])$$

Using the initial hypothesis:

$$x[n] = x[n+N] \Rightarrow cos(\frac{\pi}{8}n^2) = cos(\frac{\pi}{8}[n^2 + 2nN + N^2])$$

Knowing that cosine is a periodic function with period 2π, the previous identity gives us that:

$$\frac{\pi}{8}(2nN + N^2) = 2\pi\alpha \Rightarrow N(2n + N) = 16\alpha$$

N cannot be a odd number, or we would have a *reductio ad absurdum*, since 16α is a even number. A similar situation happens if we take either $N = 2$ or $N = 4$ or $N = 6$. The smallest possible number is $N = 8$, so it is the fundamental period.

(d) $x[n] = cos(\frac{\pi}{2}n)cos(\frac{\pi}{4}n)$

$x[n]$ can be rewritten as

$$x[n] = \frac{1}{2}[cos(\frac{\pi}{4}n) + cos(\frac{3\pi}{4}n)]$$

Defining $x_1[n] = \frac{1}{2}cos(\frac{\pi}{4}n)$, and $x_2[n] = \frac{1}{2}cos(\frac{3\pi}{4}n)$, we have:

$$\Omega_1 = \tfrac{\pi}{4} = 2\pi \cdot \tfrac{1}{8} \quad , \quad \Omega_2 = \tfrac{3\pi}{4} = 2\pi \cdot \tfrac{3}{8}$$

Both $x_1[n]$ and $x_2[n]$ have a fundamental period of 8 (because $q = 8$, according to the definition). Therefore, $x[n]$'s fundamental period is 8.