ECE355 - Homework IV

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3.22

Determine the Fourier series representations for the following signals.

(a)

$$\begin{cases} x(t) = t & , -1 < t < 1 \\ x(t) = & x(t+2) \end{cases}$$

We have that:

$$a_k = \frac{1}{T}\int_{-1}^{1} te^{-jkw_0 t}dt \tag{1}$$

For $k = 0$:

$$a_0 = \frac{1}{T}\int_{-1}^{1} tdt = 0$$

For $k \neq 0$:

$a_k = \frac{1}{2}\int_{-1}^{1} te^{-jk\pi t}dt$

$\Rightarrow 2a_k = \frac{tj}{k\pi}e^{-jk\pi t}\Big|_{t=-1}^{1} - \frac{j}{k\pi}\int_{-1}^{1} e^{-jk\pi t}dt$

$\Rightarrow 2\pi kja_k = -e^{-jk\pi} - e^{jk\pi} + \int_{-1}^{1} e^{-jk\pi t}dt$

$\Rightarrow 2\pi kja_k = -e^{-jk\pi} - e^{jk\pi} + \frac{j}{k\pi}e^{-jk\pi t}\Big|_{t=-1}^{1}$

$\Rightarrow 2\pi kja_k = -e^{-jk\pi} - e^{jk\pi} + \frac{j}{k\pi}e^{-jk\pi t} - \frac{j}{k\pi}e^{jk\pi t}$

$\Rightarrow 2\pi kja_k = -e^{-jk\pi}(1 - \frac{j}{k\pi}) - e^{jk\pi}(1 + \frac{j}{k\pi})$

$\Rightarrow 2\pi kja_k = [-cos(k\pi) + jsin(k\pi)](1 - \frac{j}{k\pi}) + [-cos(k\pi) - jsin(k\pi)](1 + \frac{j}{k\pi})$

$$\Rightarrow 2\pi kja_k = 2cos(k\pi) - jsin(k\pi)(\frac{2j}{k\pi})$$
$$\Rightarrow \pi ka_k = -jcos(k\pi) - jsin(k\pi)(\frac{1}{k\pi})$$
$$\Rightarrow a_k = \frac{1}{\pi kj}[cos(k\pi) + \frac{1}{k\pi}sin(k\pi)], \text{ for } k \neq 0$$

(b)

$$\begin{cases} x(t) = t+2 & , -2 < t < 1 \\ x(t) = 1 & , -1 < t < 1 \\ x(t) = t-2 & , 1 < t < 2 \\ x(t) = & x(t+6) \end{cases}$$

Using similar reasoning and symmetry, we have that $a_0 = 0$, and $a_k = \frac{3j}{2\pi^2 k^2}[e^{jk\frac{2\pi}{3}} sin(k\frac{2\pi}{3}) + 2e^{jk\frac{\pi}{3}} sin(k\frac{\pi}{3})]$ otherwise.

3.23

Given the Fourier series coefficients of the following continuous-time signals, which are periodic with period 4, determine the signal x(t).

(a)

$$\begin{cases} x(t) = -\frac{1}{4} & , -0.5 < t < 2.5 \\ x(t) = \frac{3}{4} & , 2.5 < t < 3.5 \\ x(t) = & x(t+4) \end{cases}$$

(b)

$$\begin{cases} x(t) = 0 & , 0 < t < \frac{7}{4} \\ x(t) = \frac{1}{2} & , \frac{7}{4} < t < \frac{11}{4} \\ x(t) = & x(t+4) \end{cases}$$