ECE355 - Homework VI

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3.28

Determine the Fourier series coefficients of the following signals. In addition, plot the magnitude and phase of each set of coefficients a_k.

(a.a)

$$x[n] := \begin{cases} \mathcal{U}[n] & , -2 \le n \le 5 \\ x[n+7] \end{cases}$$

We have that:

$$a_k = \frac{1}{N}\sum_{0}^{N-1} x[n]e^{-jk\omega_0 n} = \frac{1}{7}\sum_{0}^{6} x[n]e^{-jk\omega_0 n} = \frac{1}{7}\sum_{0}^{5}\mathcal{U}[n]e^{-jk\omega_0 n}$$
$$\Rightarrow a_k = \frac{1}{7}\sum_{0}^{4} e^{-jk\omega_0 n}$$

For $k = 0$, we have $a_0 = \frac{5}{7}$.

Otherwise, let us take a Geometric Series with first term a_0, a ratio q, and n' elements, its sum S may be calculated by:

$$S = \frac{a_0 - a_0 q^{n'}}{1-q} \tag{1}$$

Using (1), we have:

$$a_k = \frac{1}{7}\frac{1-e^{-5jk\omega_0}}{1-e^{-jk\omega_0}} = \frac{1}{7}\frac{e^{-\frac{5}{2}jk\omega_0}(e^{\frac{5}{2}jk\omega_0} - e^{-\frac{5}{2}jk\omega_0})}{e^{-\frac{1}{2}jk\omega_0}(e^{\frac{1}{2}jk\omega_0} - e^{-\frac{1}{2}jk\omega_0})}$$

However, from the Euler's formula:

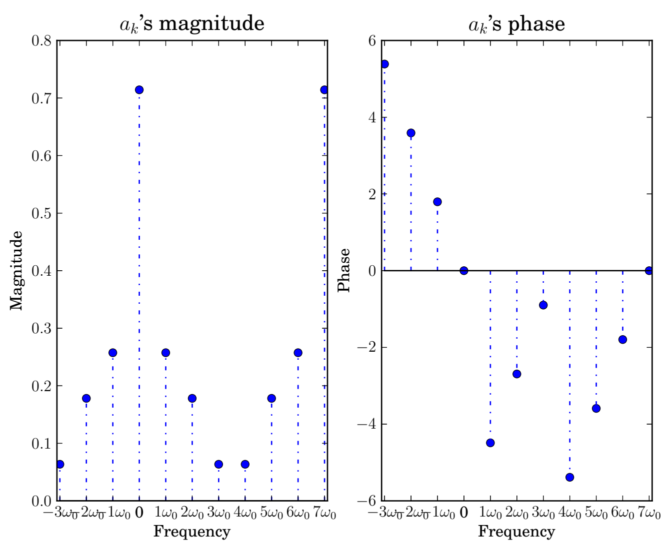
$$\begin{array}{c} e^{jz} = cos(z) + jsin(z) \\ e^{-jz} = cos(z) - jsin(z) \\ \hline e^{jz} - e^{-jz} = 2jsin(z) \end{array} \tag{2}$$

Using (2), we have:

$$a_k = \frac{1}{7}\frac{e^{-\frac{5}{2}jk\omega_0}[2jsin(\frac{5}{2}k\omega_0)]}{e^{-\frac{1}{2}jk\omega_0}[2jsin(\frac{1}{2}k\omega_0)]} = \frac{e}{7}^{-2jk\omega_0}\frac{sin(\frac{5}{2}k\omega_0)}{sin(\frac{1}{2}k\omega_0)}$$

But we have that $\omega_0 = \frac{2\pi}{N}$, so:

$$a_k = \begin{cases} \frac{5}{7} & \text{, for } k = 0 \\ \frac{e}{7}^{-\frac{5\pi}{7}jk}\frac{sin(\frac{5\pi}{7}k)}{sin(\frac{\pi}{7}k)} & \text{, for } k \neq 0 \end{cases},$$



(a.b)

$$x[n] := \begin{cases} \mathcal{U}[n] & , -2 \leq n \leq 4 \\ x[n+6] \end{cases}$$

We have that:

$$a_k = \frac{1}{N}\sum_{0}^{N-1} x[n]e^{-jk\omega_0 n} = \frac{1}{6}\sum_{0}^{5} x[n]e^{-jk\omega_0 n} = \frac{1}{6}\sum_{0}^{3}\mathcal{U}[n]e^{-jk\omega_0 n}$$
$$\Rightarrow a_k = \frac{1}{6}\sum_{0}^{3} e^{-jk\omega_0 n}$$

For $k = 0$, we have $a_0 = \frac{2}{3}$.

Otherwise, using (1), we have:

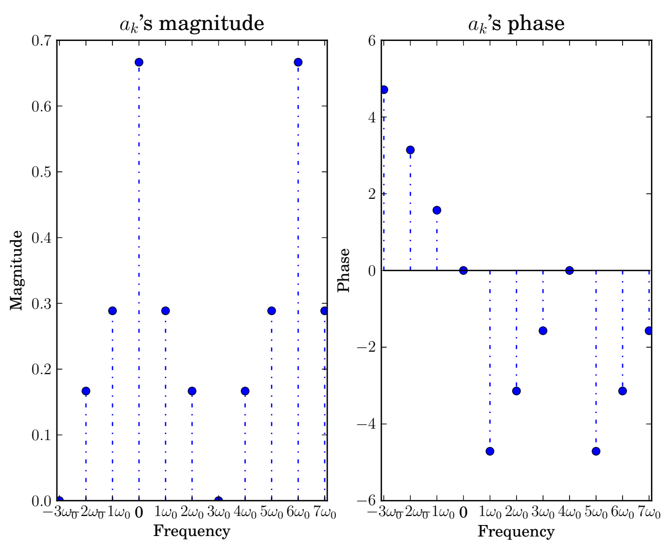
$$a_k = \frac{1}{6}\frac{1-e^{-4jk\omega_0}}{1-e^{-jk\omega_0}} = \frac{1}{6}\frac{e^{-2jk\omega_0}(e^{2jk\omega_0}-e^{-2jk\omega_0})}{e^{-\frac{1}{2}jk\omega_0}(e^{\frac{1}{2}jk\omega_0}-e^{-\frac{1}{2}jk\omega_0})}$$

Using (2), we have:

$$a_k = \frac{1}{6}\frac{e^{-2jk\omega_0}[2jsin(2k\omega_0)]}{e^{-\frac{1}{2}jk\omega_0}[2jsin(\frac{1}{2}k\omega_0)]} = \frac{e}{6}^{-\frac{3}{2}jk\omega_0}\frac{sin(2k\omega_0)}{sin(\frac{1}{2}k\omega_0)}$$

But we have that $\omega_0 = \frac{2\pi}{N}$, so:

$$a_k = \begin{cases} \frac{2}{3} & \text{, for } k = 0 \\ \frac{e}{6}^{-\frac{\pi}{2}jk}\frac{sin(\frac{2\pi}{3}k)}{sin(\frac{\pi}{6}k)} & \text{, for } k \neq 0 \end{cases},$$



(a.c)

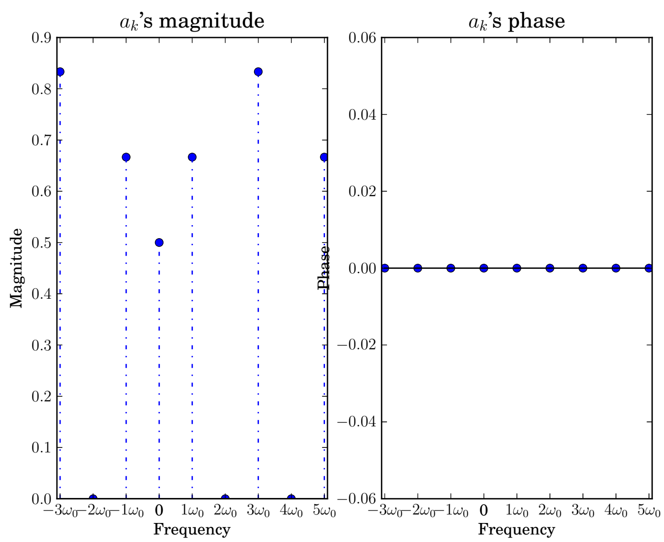
$$x[n] := \begin{cases} 1 & , n = 0 \\ 2 & , n = 1 \text{ or } n = 5 \\ -1 & , n = 2 \text{ or } n = 4 \\ 0 & , n = 3 \\ x[-n] & \\ x[n+6] & \end{cases}$$

We have that:

$$a_k = \frac{1}{N}\sum_{0}^{N-1} x[n]e^{-jk\omega_0 n} = \frac{1}{6}\sum_{0}^{5} x[n]e^{-jk\omega_0 n} = \frac{1}{6}\sum_{0}^{5} x[n]\left[cos(k\omega_0 n) - jsin(k\omega_0 n)\right]$$

Since $x[n]$ is even, it will not have any sine component, so:

$$a_k = \frac{1}{6}\sum_{0}^{5} x[n]cos(k\omega_0 n)$$
$$\Rightarrow a_k = \frac{1}{6}\left\{1 + 2\left[cos(\tfrac{k\pi}{3}) + cos(\tfrac{5k\pi}{3})\right] - 1\left[cos(\tfrac{2k\pi}{3}) + cos(\tfrac{4k\pi}{3})\right]\right\}$$
$$\Rightarrow a_k = \frac{1}{6}\left\{1 + 2\left[cos(\tfrac{k\pi}{3}) + cos(\tfrac{k\pi}{3})\right] - 1\left[cos(\tfrac{2k\pi}{3}) + cos(\tfrac{2k\pi}{3})\right]\right\}$$
$$\Rightarrow a_k = \frac{1}{6} + \frac{2}{3}cos(\tfrac{k\pi}{3}) - \frac{1}{3}cos(\tfrac{2k\pi}{3})$$



3.29

Given the Fourier series coefficients of the following discrete-time signals, which are periodic with period 4, determine the signal x[n].

(a)

$$x[n]\begin{cases} 4(\delta[n-1] + \delta[n-7]) + 4j(\delta[n-3] - \delta[n-5]) & , 0 < n < 7 \\ x[n+8] & \end{cases}$$

(c)

$$x[n]\begin{cases} 1 + (-1)^n + 2cos\left(\frac{\pi n}{4}\right) + 2cos\left(\frac{3\pi n}{4}\right) & , 0 < n < 7 \\ x[n+8] & \end{cases}$$

(d)

$$x[n] \begin{cases} 2 + 2cos\left(\frac{\pi n}{4}\right) + cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}cos\left(\frac{3\pi n}{4}\right) & , 0 < n < 7 \\ x[n+8] \end{cases}$$