ECE355 - Homework III

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Determine which of the following characteristics either hold or do not hold for each system bellow, justify it.

1. Memoryless
2. Time Invariance
3. Linearity
4. Causality
5. Stability

(b) $y(t) = cos(3t)x(t)$

1. Memoryless
   Since the system does not depends on its past, i.e., $\neg y(x(t-a)), \forall a > 0$, it is memoryless.
2. Time Invariance
   If we apply a delay t_0 to $x(t)$, we will have $x(t-t_0)$, which generates the output $y(t) = cos(3t)x(t-t_0)$ but it is different from $y(t-t_0)$.

3. Linearity

   Taking the input signal $ax(t)$, we have the output signal $cos(3t)ax(t) = a \cdot cos(3t)ax(t) = ay(t)$.

   Taking the input signal $x_1(t) + x_2(t)$, we have the output signal $cos(3t)[x_1(t) + x_2(t)] = cos(3t)x_1(t) + cos(3t)x_2(t) = y_1(t) + y_2(t)$.

   Therefore, the system is linear.

4. Causality

   Since the system does not depends on its future, i.e., $\neg y(x(t+a)), \forall a > 0$, it is causal.

5. Stability

   Taking a bounded $x(t)$, i.e., $|x(t)| \leq M_x$, and applying either the absolute value function or the magnitude function to $y(t)$ (it depends on $y(t)$'s domain):

$$|y(t)| \leq |cos(3t)||x(t)| \leq |cos(3t)|M_x \leq M_x \Rightarrow |y(t)| \leq M_x$$

   Taking $M_x = M_y$, we have that the system is stable.

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

1. Memoryless

   Since the integral sum all the values of $x(t)$ until $2t$, the system is not memoryless.

2. Time Invariance

   Taking a signal $x(t - t_0)$ and $\tau_0 = t_0$, we have the output signal $y'(t) = \int_{-\infty}^{2t} x(\tau - \tau_0)d\tau$, taking $\mathrm{T} = \tau - \tau_0$: $y'(t) = \int_{-\infty}^{2t-t_0} x(\mathrm{T})d\mathrm{T}$, which is not equals to $y(t - t_0) = \int_{-\infty}^{2(t-t_0)} x(\tau)d\tau$. Therefore, the system is not time invariant.

3. Linearity

   Taking the input signal ax(t), we have the output:

$$\int_{-\infty}^{2t} ax(\tau)d\tau = a\int_{-\infty}^{2t} x(\tau)d\tau = ay(t)$$

Taking the input signal $x_1(t) + x_2(t)$, we have the output:

$$\int_{-\infty}^{2t} (x_1(\tau) + x_2(\tau))d\tau = \int_{-\infty}^{2t} x_1(\tau)d\tau + \int_{-\infty}^{2t} x_2(\tau)d\tau = y_1(t) + y_2(t)$$

Therefore, the system is linear.

4. Causality

   Since the the integral sum all the values of $x(t)$ until $2t$, i.e., more than t, the system is not causal.

5. Stability

   For instance, taking the signal $x(t) = \frac{1}{t}$, the integral would not converge for all bounded values of x(t). Therefore, using a counterexample, the system is not stable.

(f) $y(t) = x(\frac{t}{3})$

1. Memoryless

   Since $y(t)$ may depend on the value of $x(t)$ in a time before t, the system is not memoryless.

2. Time Invariance

   If we apply a delay t_0 to $x(t)$, we will have $x(t - t_0)$, which generates the output $y(t) = x(\frac{t-t_0}{3})$. However, if the delay is applied after the system, we have the output $y'(t) = x(\frac{t}{3} - t_0)$. Therefore, the system is not time invariant.

3. Linearity

   Taking the input signal $ax(t)$, we have the output signal $ax(\frac{t}{3}) = ay(t)$.

   Taking the input signal $x_1(t) + x_2(t)$, we have the output signal $x_1(\frac{t}{3}) + x_2(\frac{t}{3}) = y_1(t) + y_2(t)$.

   Therefore, the system is linear.

4. Causality

   Since the system depends on its future when $t < 0$ ($t < t/3, \forall t < 0$), it is not causal.

5. Stability

   Taking a bounded $x(t)$, i.e., $|x(t)| \leq M_x$, we will have $|y(t)| \leq M_x = M_y$ as well. Therefore, the system is stable.

(g) $y(t) = \frac{d}{dt}x(t)$

1. Memoryless

   Since $y(t)$ cannot be determined using only $x(t)$, it is not memoryless.

2. Time Invariance

   If we apply a delay t_0 to $x(t)$, we will have $x(t-t_0)$, which generates the output $y(t) = \frac{d}{dt}x(t-t_0)$. In addition, if we apply the same delay in an output signal $y'(t)$, we will have the final output $y(t) = \frac{d}{dt}x(t-t_0)$. Therefore, the system is time invariant.

3. Linearity

   Taking the input signal $ax(t)$, we have the output signal $a\frac{d}{dt}x(t) = ay(t)$.

   Taking the input signal $x_1(t) + x_2(t)$, we have the output signal $\frac{d}{dt}[x_1(t) + x_2(t)] = \frac{d}{dt}x_1(t) + \frac{d}{dt}x_2(t) = y_1(t) + y_2(t)$.

   Therefore, the system is linear.

4. Causality

   Since the system may anticipate its future, it is not causal. Take as example the following signal $x(t)$:

$$x(t) := \begin{cases} 1, & \text{if } x \leq 2 \\ 2, & \text{if } x > 2 \end{cases}$$

   At $t = 2$, $y(t)$ will anticipate the discontinuity.

5. Stability

   For instance, taking the signal $x(t) = log(1-x)$, the integral would not converge for all bounded values of x(t). Therefore, using a counterexample, the system is not stable.

1.28

Determine which of the following characteristics either hold or do not hold for each system bellow, justify it.

1. Memoryless
2. Time Invariance
3. Linearity
4. Causality
5. Stability

(c) $y[n] = nx[n]$

1. Memoryless

   Since the system depends only on the present, it is memoryless.

2. Time Invariance

   If we apply a delay n_0 to $x[n]$, we will have $x[n-n_0]$, which generates the output $y[n] = nx[n-n_0]$. However, if we apply the delay after the system, we will have the final output $y'[n] = (n-n_0)x[n-n_0]$. Therefore, the system is not time invariant.

3. Linearity

   Taking the input signal $ax[n]$, we have the output signal $anx[n] = ay[n]$.

   Taking the input signal $x_1[n] + x_2[n]$, we have the output signal $n[x_1[n] + x_2[n]] = nx_1[n] + nx_2[n] = y_1[n] + y_2[n]$.

   Therefore, the system is linear.

4. Causality

   Since the system cannot anticipate its future, it is causal.

5. Stability

   Taking a bounded $x[n]$, i.e., $|x[n]| \leq M_x$, and applying either the absolute value function or the magnitude function to $y[n]$ (it depends on $y[n]$'s domain):

$$|y[n]| \leq |n||x[n]| \leq |n|M_x$$

Taking $M_y = n \cdot M_x$, we have that $y[n]$ is bounded as well. Therefore, the system is stable.

(d) $y[n] = Ev\{x[n-1]\} = \frac{1}{2}\{x[n-1] + x[1-n]\}$

1. Memoryless

   Since the system depends on its past, it is not memoryless.

2. Time Invariance

   If we apply a delay n_0 to $x[n]$, we will have $x[n-n_0]$, which generates the output $y[n] = \frac{1}{2}\{x[(n-n_0)-1] + x[1-(n-n_0)]\}$. In addition, if we apply the delay after the system, we will have the final output $y'[n] = \frac{1}{2}\{x[(n-n_0)-1] + x[1-(n-n_0)]\} = y[n]$. Therefore, the system is time invariant.

3. Linearity

   Taking the input signal $ax[n]$, we have the output signal $\frac{1}{2}\{ax[n-1] + ax[1-n]\} = \frac{a}{2}\{x[n-1] + x[1-n]\} = ay[n]$.
   Taking the input signal $x_1[n] + x_2[n]$, we have the output signal $\frac{1}{2}\{x_1[n-1] + x_2[n-1] + x_1[1-n] + x_2[1-n]\} = \frac{1}{2}\{x_1[n-1] + x_1[1-n]\} + \frac{1}{2}\{x_2[n-1] + x_2[1-n]\} = y_1 + y_2$.
   Therefore, the system is linear.

4. Causality

   Since the system may anticipate its future if $n < 0$, it is not causal.

5. Stability

   foo

(e) $y[n] = \begin{cases} x[n] & , n \geq 1 \\ 0 & , n = 0 \\ x[n+1] & , n \leq -1 \end{cases}$

1. Memoryless

   Since the system may depend on its future for $n \leq -1$, it is not memoryless.

2. Time Invariance

   The system is not time invarant, it is easy to see taking $n = 0$.

3. Linearity

   The system is linear, because each of its pieces is linear.

4. Causality

   Since the system may anticipate its future, it is not causal.

5. Stability

   foo

(g) $y[n] = x[4n + 1]$

1. Memoryless

   Since the system depends on its future, it is not memoryless.

2. Time Invariance

   Applying a delay either before or after the system results in the same output signal. Therefore, the system is time invariant.

3. Linearity

   Taking the input signal $ax[n]$, we have the output signal $ax[4n+1] = ay[n]$.

   Taking the input signal $x_1[n] + x_2[n]$, we have the output signal $x_1[4n + 1] + x_2[4n + 1] = y_1 + y_2$.

   Therefore, the system is linear.

4. Causality

   Since the system may anticipate its future, it is not causal.

5. Stability

   foo

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2.2

The signal h[k] is non-zero only in the interval $-3 \leq k \leq 9$, so the signal h[-k] is non-zero in the interval $-9 \leq k \leq 3$, if we shift it n units, we will have the interval $n-9 \leq k \leq n+3$. Therefore, $A = n-9$ and $B = n+3$.

2.9

Given the signal:

$$h(t) = e^{2t}u(-t+4) + e^{-2t}u(t-5)$$

Find A and B such that

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & , \ if \tau < A \\ 0 & , \ if A < \tau < B \\ e^{2(t-\tau)} & , \ if \tau > B \end{cases}$$

The first part of the sum will be 0 if $\tau < t-4$, while the second part of the sum will be 0 if $\tau > t-5$. Therefore, we have that $A = t-5$ and $B = t-4$.

2.21

Given the signals:

$$\left.\begin{aligned} x[n] = \alpha^n u[n], \\ h[n] = \beta^n u[n], \end{aligned}\right\} \alpha \neq \beta$$

Calculate $y[n] = x[n] * h[n]$. By the definition of convolution, we have:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k, \text{ for } n \geq 0$$

$$\Rightarrow y[n] = \left(\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}\right) u[n], \text{ for } \alpha \neq \beta$$

2.28

(a) Given the signal:

$$h[n] = \left(\frac{1}{5}\right)^n u[n]$$

As the impulse response of a discrete-time LTI system, determine if this system is causal or stable.

The system is causal because $h[n] = 0$ for $n < 0$. It is stable as well because $\sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n = \frac{5}{4} < +\infty$.

(c) Given the signal:

$$h[n] = \left(\frac{1}{2}\right)^n u[-n]$$

As the impulse response of a discrete-time LTI system, determine if this system is causal or stable.

The system is not causal because $h[n] = 0$ for $n > 0$.Also, it is not stable because $\sum_{n=-\infty}^{0} = +\infty$.

(e) Given the signal:

$$h[n] = \left(-\frac{1}{2}\right)^n u[n] + (1.01)^n u[n-1]$$

As the impulse response of a discrete-time LTI system, determine if this system is causal or stable.

The system is causal because $h[n] = 0$ for $n < 0$. Also, it is not stable because the second term tends to infinite as $n \to +\infty$.

(g) Given the signal:

$$h[n] = n \left(\frac{1}{3}\right)^n u[n-1]$$

As the impulse response of a discrete-time LTI system, determine if this system is causal or stable.

The system is causal because $h[n] = 0$ for $n < 0$. Also, it is stable because $\sum_{n=-\infty}^{+\infty} |h[n]| = 1 < +\infty$.