Exploring Quantum Comprehension through the Elitzur-Vaidman Bomb Testing Problem

Vinícius Pereira Pinto[1*], Dr Bruno Pereira de Oliveira[1], Dr Fátima Maria Mitsue Yasuoka[1, 2], Prof Philippe Wilhelm Courteille[1], Prof Jarbas Caiado de Castro Neto[1]

[1]São Carlos Institute of Physics, University of São Paulo, São Carlos, 13560-970, São Paulo, Brazil.

[2]BR Labs Tecnologia Óptica e Fotônica Ltda, São Carlos, 13567-020, São Paulo, Brazil.

*Corresponding author(s). E-mail(s): viniciusppinto@usp.br;

Abstract

The Elitzur-Vaidman bomb testing problem is a thought experiment in quantum mechanics that explores the concepts of quantum superposition and entanglement. The problem creates a scenario where a bomb is activated by absorbing a single photon and suggests testing that it works without exploding it, so that classically it is impossible to know if the bomb works without disturbing the system. Therefore, an interferometer with two arms and two outputs is proposed, mounted in such a way that only one of them has positive interference. However, when placing a pump that works in one of the arms, the interference pattern breaks down, and it can be inferred that the pump works without interacting with it. In this work, a methodology is adopted that combines computational simulations and a simple experimental proposal to investigate the concept of Interaction-Free Measurements. The IBM quantum cloud programming language (Qiskit Runtime) is utilized to simulate the experiments, and an experimental approach is proposed that uses simple laser sources to qualitatively test the theory, offering a practical perspective for developing an understanding of quantum concepts.

Keywords: quantum superposition, interference, quantum programming, interaction-free measurement, quantum Zeno effect

1 Introduction

Quantum mechanics, as a fundamental theory, provides a comprehensive framework for understanding the behavior of particles at the microscopic level. Although numerous concepts contribute to the richness of this theory, two crucial aspects that challenge classical intuitions are superposition and entanglement. Delving into these ideas leads us to the fascinating concept of non-locality in quantum mechanics.

Superposition refers to the remarkable phenomenon in which a quantum system can exist partially in multiple states simultaneously before being measured. Unlike classical systems, which are generally confined to well-defined properties such as position or momentum, quantum particles can be in a state that spans a range of possibilities. This is exemplified by the famous thought experiment of Schrödinger's cat [1], in which the state of a cat is conceived as a superposition of the states of both dead and alive until its state is observed.

On the other hand, entanglement reveals a profound correlation between particles that defies classical explanations. When two or more particles become entangled, their states become intertwined, regardless of the distance between them. Measurements made on one particle instantaneously affect the state of the other, regardless of spatial separation. This concept, famously described as "spooky action at a distance" by Einstein, Podolsky, and Rosen [2], challenges our classical intuition that information cannot propagate faster than the speed of light.

To explore the implications of superposition and entanglement, Elitzur and Vaidman developed the Bomb Testing Problem as a thought experiment [3]. The problem is set up as follows: a bomb is placed inside a chamber with two entrances, each equipped with a photon detector. The bomb is designed to either explode or not, depending on the polarization of the photon entering the chamber. However, the experiment is configured in such a way that it is impossible to know whether the bomb has exploded or not without disturbing the system. This is because the photon entering the chamber is in a superposition of two polarizations, and the system becomes entangled with the bomb. This experiment raises the question of how to determine whether the bomb has exploded or not without actually disturbing the system.

2 The Bomb Testing Problem

The Elitzur-Vaidman experiment consists of a Mach-Zehnder interferometer setup, comprising beam splitters with reflectivity R, to explore the wave-particle duality of light and the concept of quantum superposition. In this experiment, a single-photon light source is employed to ensure the emission of only one photon at a time. The experiment commences with the photon arriving at the first beam splitter, which evenly splits the incoming light into two paths: Path A (upper) and Path B (lower), as depicted in Figure 1. Each path is equipped with mirrors that redirect the photon towards the final beam splitter.

The interferometer setup is such that, under normal conditions, the beams interfere constructively at one of the detectors, referred to as D1, while interfering destructively at the other detector, D2. This implies that when both paths are unobstructed, we

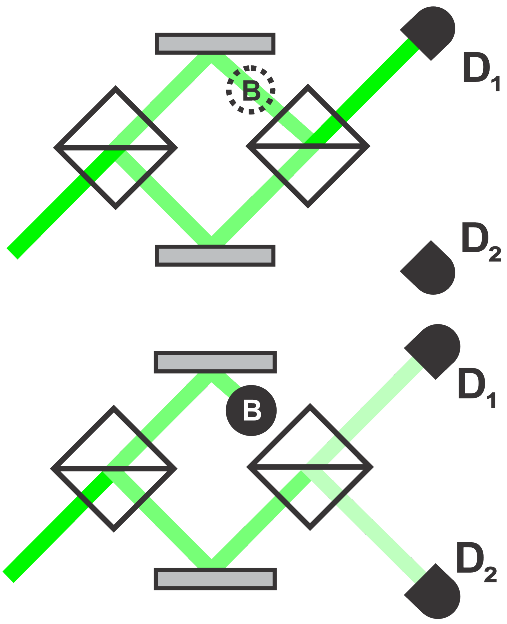

Fig. 1 Elitzur-Vaidman experiment without the bomb (top) and with the bomb (bottom). Adapted from [4].

expect to detect the photon exclusively at D1. This behavior is intriguing because it demonstrates that even when using single photons, when they are in a superposition of states, interference between the states of the single photon is observed.

However, this is where the experiment becomes intriguing. Elitzur and Vaidman introduce a clever twist: they propose a specific set of conditions in which one of the paths can be intentionally blocked. Surprisingly, even when one path is obstructed, the experiment reveals a counterintuitive result. Instead of detecting the photon solely at D1, the experiment demonstrates that an equal amount of light is detected at both detectors, D1 and D2. Adding to the intrigue, they propose that the path be blocked by a bomb that is triggered by a photon detector when it absorbs a single photon.

In the case of single-photon emissions, the results of the interferometer can be categorized as follows:

- **No detections at the detectors:** This result occurs when the emitted photon interacts with an object placed in the path of the beam, preventing the photon from reaching either of the detectors. In this scenario, the presence of the object obstructs the photon's path, leading to the absence of detection.
- **Detector D1 clicks:** This result can occur in two situations. Firstly, when the object is present in the photon's path, causing it to interact and be redirected towards D1. Secondly, even in the absence of the object, D1 can still register a detection. Therefore, when D1 clicks, it indicates that the measurement was unsuccessful and requires a new attempt to obtain conclusive results.

- **Detector D2 clicks:** This is the desired result as it indicates the measurement of the presence of an object without directly interacting with it, unlike the first case. The occurrence of a detection at D2 suggests the absence of any obstruction in the photon's path, indicating the likely presence of the object being measured.

It is evident that the behavior of the photon is dependent on the experimental setup. In this case, by blocking one of the paths with an object that absorbs the photon (equivalent to performing an intermediate measurement), the system exhibits distinct behavior. In other words, obtaining information about the photon's path destroys the interference pattern.

It is important to emphasize that the presence of an object can only be reliably inferred when a detection occurs at D2. This is due to the experimental setup, which is adjusted to produce destructive interference at D2. Destructive interference ensures that a detection at D2 can only be attributed to the presence of the object, as any obstruction in the path of the photon would destroy the interference. The entanglement of the system is established because the state of the photon depends on the state of the bomb, and vice versa. This quantum connection between the photon and the bomb is fundamental to the phenomenon of measurement without interaction. Therefore, the entanglement of states between the photon and the bomb plays a crucial role in the Elitzur-Vaidman Problem, allowing information about the presence or absence of the object to be inferred without direct interaction between them.

Using the formalism of quantum mechanics, the state of a photon passing through the lower path can be described as $|1\rangle$, while passing through the upper path is denoted as $|2\rangle$. Let it consider a 50/50 beam splitter (BS). The operation for the beam splitter is as follows:

$$\begin{aligned} |1\rangle &\xrightarrow{BS} \tfrac{1}{\sqrt{2}}(|1\rangle + i\,|2\rangle) \\ |2\rangle &\xrightarrow{BS} \tfrac{1}{\sqrt{2}}(|2\rangle + i\,|1\rangle) \end{aligned} \tag{1}$$

and the operation for a mirror (E) is:

$$\begin{aligned} |1\rangle &\xrightarrow{E} i\,|2\rangle \\ |2\rangle &\xrightarrow{E} i\,|1\rangle \end{aligned} \tag{2}$$

When the object is absent in the system, as illustrated in the first example in Figure 1, the evolution is as follows:

$$\begin{aligned} |1\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}(|1\rangle + i\,|2\rangle) \xrightarrow{E} \frac{1}{\sqrt{2}}(i\,|2\rangle - |1\rangle) \\ &\xrightarrow{BS} \frac{1}{2}(i\,|2\rangle - |1\rangle) - \frac{1}{2}(|1\rangle + i\,|2\rangle) \\ &= -\,|1\rangle \end{aligned}$$

In this case, only detector D1 is triggered, with a probability of 1.

If there is an object in the path, the photon may be absorbed, described by the state $|s\rangle$. The evolution is then given by:

$$\begin{aligned}|1\rangle &\xrightarrow{BS} \frac{1}{\sqrt{2}}(|1\rangle + i\,|2\rangle) \xrightarrow{E} \frac{1}{\sqrt{2}}(i\,|2\rangle + i\,|s\rangle)\\ &\xrightarrow{BS} \frac{1}{2}(i\,|2\rangle - |1\rangle) + \frac{1}{\sqrt{2}}\,|s\rangle\end{aligned}$$

Thus, the detectors collapse this quantum state into:

$$|1\rangle \xrightarrow{\text{Bomb test}} \begin{cases} |1\rangle\,, & \text{D1 is triggered, } P = 1/4 \\ |2\rangle\,, & \text{D2 is triggered, } P = 1/4 \\ |s\rangle\,, & \text{no trigger, } P = 1/2 \end{cases}$$

Therefore, it can be predicted that detector D2 is only triggered when there is an object in the path. In this case, the photon performs a measurement without directly interacting with the object in 25% of the measurements.

The success rate of interaction-free measurements can be quantified by the ratio of the object detection probability to the sum of the object detection probability and the photon absorption probability by the object:

$$\eta_{\text{EV}} = \frac{P(\text{det})}{P(\text{abs}) + P(\text{det})} \tag{3}$$

If 50/50 beam splitters are used in the experiment, we have $P(\text{det}) = 1/4$ and $P(\text{abs}) = 1/2$, thus $\eta_{\text{EV}} = 1/3$. In the general case, for beam splitters with reflectivity R, we have $\eta_{\text{EV}} = (1-R)/(2-R)$, which approaches the limit $\eta_{\text{EV}} \leq 0.5$.

3 Interaction-Free Measurements

Since the proposal by Elitzur and Vaidman (EV), further advancements have been made in the field of interaction-free measurements with the aim of increasing the 25% chance of detecting the bomb without effectively interacting with it. A notable line of research involves the use of the Quantum Zeno Effect to increase the probability of performing interaction-free measurements, approaching probabilities close to 100%.

The Quantum Zeno Effect, named after the ancient Greek philosopher Zeno of Elea, refers to the phenomenon where frequent observations or measurements can significantly delay or even halt the evolution of a quantum system. This effect can be observed when horizontally polarized light passes through polarization rotators. In one scenario, a single measurement is made after the light passes through a horizontal polarizer at the end, resulting in a measurement intensity of 0%. In another scenario, measurements are performed at each rotation, resulting in non-zero measurements, as illustrated in Figure 2.

In the context of the EV problem, the Zeno effect can be harnessed to increase the probability of successfully detecting an object without interacting with it. A couple

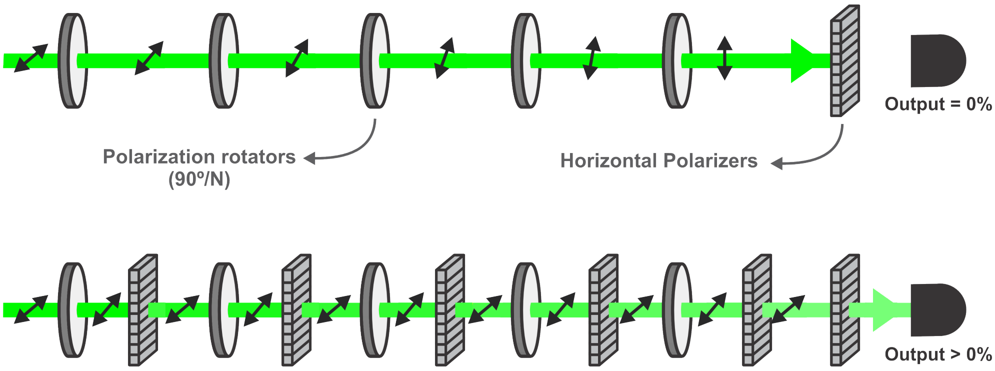

Fig. 2 Horizontal polarization of a photon passing through N polarization rotators (each rotating by $90/N$) converts it to vertical polarization, leading to its blockage by the horizontal polarizer (top). Inserting a horizontal polarizer after each rotator inhibits the polarization change, allowing detection of light beyond the horizontal polarizer (bottom).

Adapted from [9]

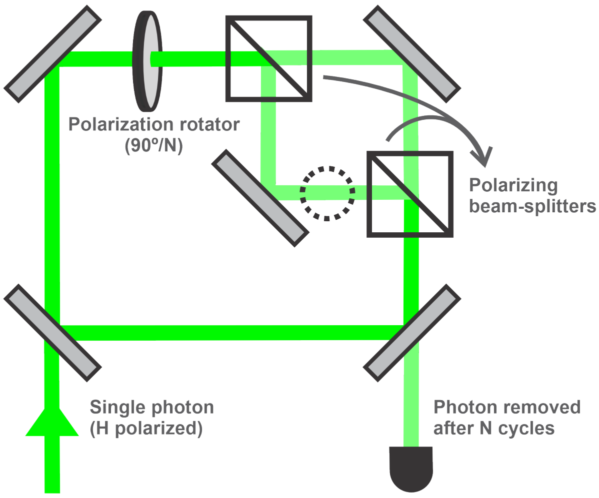

Fig. 3 Variation of the Elitzur-Vaidman bomb testing problem using a cavity for n repetitions of the injected single photon.

Adapted from [6].

years after the EV proposal [3], Kwiat *et al.* proposed a different method that enables an increase of η_{EV} close to unity [5] and explored further versions a few years later [6].

The proposed approach involves merging the EV experiment with the previously presented polarized object detection technique by injecting a horizontally polarized single photon into a cavity passing through a polarization rotator of $90°/N$ for N cycles, such that the same photon exits with vertical polarization, as illustrated in Figure 3. By utilizing polarized beam splitters, the photon's path depends on its polarization. If the path is unobstructed, the photon's state splits and reconstructs at the end of the interferometer, rotating the photon's polarization to vertical after N cycles. However, if there is an object in the path, the wavefunction collapses to the upper path with probability of $\cos^2 \pi/2N$, once again presenting entirely horizontal polarization. Clearly, the probability of the photon surviving without being absorbed after N cycles is given by:

$$P = \left[\cos^2\left(\frac{\pi}{2N}\right)\right]^N . \tag{4}$$

Thus, the experiment can result in three possible scenarios: (1) the photon is not detected, indicating it was absorbed; (2) the photon exhibits vertical polarization, indicating an object-free path; and (3) the photon exhibits horizontal polarization, indicating it was not absorbed in a system with an object. As the value of N increases ($N \geq 4$), the probability of achieving interaction-free measurements surpasses 50% of the maximum probability in the original EV configuration. Furthermore, as N becomes larger, proximity to the efficiency parameter $\eta_{EV} = 1$ is achieved, enabling the inference of the object's presence without interacting with it in any of the cases, thus performing an Interaction-Free Measurement.

This variation was implemented using single-photon sources, produced using a $LiIO_3$ crystal, in resonant cavities with Michelson interferometers, obtaining experimental results of η_{EV} that agree with the theoretical prediction of Equation 4, with $\eta_{EV} \approx 0.5$ being the highest value obtained in the experiment, and pointing towards developments that may experimentally achieve $\eta_{EV} = 2/3$ [5].

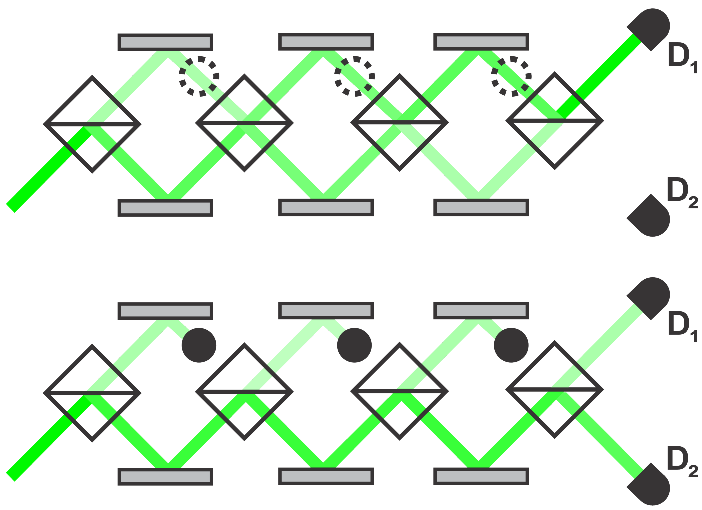

Fig. 4 Variation of the Elitzur-Vaidman bomb testing problem with repeated tests known as Interaction-Free Measurement.

Adapted from [5].

Another proposed approach involves passing a single photon through the Mach-Zehnder interferometer multiple times [5], where the photon gradually transitions from the lower left half to the upper right half of the system, as illustrated in Figure

4. By incorporating detectors in each cycle, the photon acquires a probability, $P = \cos^2(\pi/2n)$, where n represents the number of cycles, to persist along the lower path. In this case, the system is analogous to the previous proposal, such that for large n, the results will be: (1) the photon is not detected, indicating it was absorbed by one of the objects; (2) D1 is triggered, indicating an object-free path; and (3) D2 is triggered, indicating it was not absorbed in a system with an object. The interpretation is also analogous.

These experimental configurations also serve to explore quantum erasure phenomena using interaction-free measurements [7].

4 Experiments

To qualitatively observe the experiment proposed by Elitzur and Vaidman, a Mach-Zehnder interferometer was constructed using two 50/50 beam splitters and a HeNe laser operating at a wavelength of 632.8 nm, as illustrated in Figure 5.

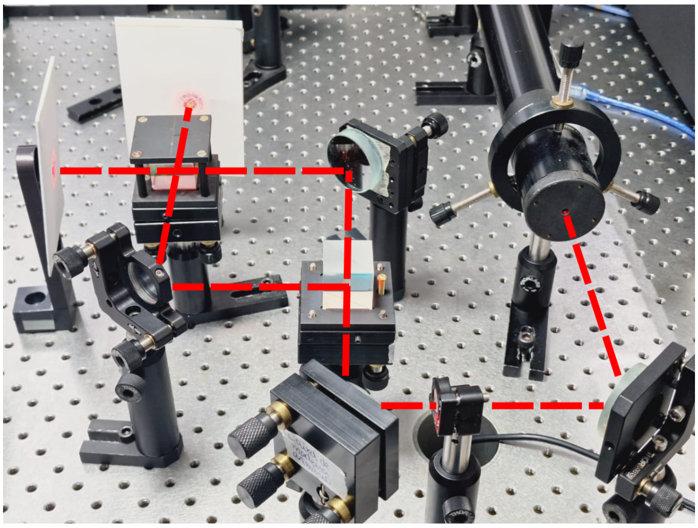

Fig. 5 Reproduction of the original experimental setup of Elitzur-Vaidman Bomb Testing Problem using a Mach-Zehnder interferometer with two 50/50 beam splitters.

During the experiment setup, it becomes evident that when both paths are unobstructed, an interference pattern emerges at one output, while the complementary interference pattern emerges at the other output. However, when one of the paths is blocked by an object, the interference pattern disappears, resulting in the previously dark region becoming illuminated, as illustrated in Figure 6.

It is important to note that this experimental setup utilizes continuous light sources rather than single-photon sources. Therefore, while it is possible to assess the quantum behavior of light based on the experiment setup (with or without intermediate measurements), a rigorous analysis of the problem proposed by Elitzur and Vaidman cannot be performed since single photons are not utilized.

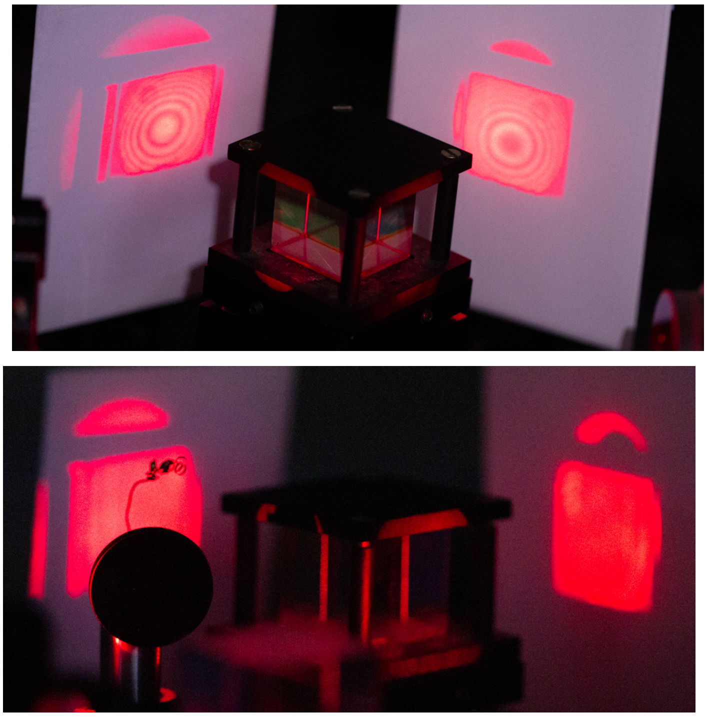

Fig. 6 Comparison between the original EV experiment without an object in the path (top) and with an object in the path (bottom).

5 Simulations with Quantum Circuits

The EV problem can also be investigated through experiments using quantum circuits implemented via high-level programming languages on commercially available superconducting quantum processors, such as those provided by IBM [8]. The quantum computing formalism used in this work is sophisticated and timely, but not necessary, and mathematically not even significantly advantageous for the system under consideration. After all, it is possible to obtain an analytical expression for the probabilities. However, it serves as a starting point to understand the translation of this problem into quantum circuits, which can be valuable for developing new quantum programs and technologies.

By employing a quantum circuit with two qubits, representing the photon $|q_0\rangle$ and the bomb $|q_1\rangle$, we can use a Hadamard gate (H) to split the initial state $|q_0\rangle = |0\rangle$ into $|0\rangle$ and $|1\rangle$, encoding the information of which path. Then, a CNOT gate entangles $|q_0\rangle$ with the bomb qubit $|q_1\rangle$, simulating the role of the bomb. Treating $|q_0\rangle$ as the signal, a second Hadamard gate is applied to probe the interference patterns. The desired outcome of this EV experiment relies on the $|00\rangle$ state, indicating a successful interaction-free measurement. As illustrated in Figure 7, it is observe that $|00\rangle$ is obtained in 25% of the measurements.

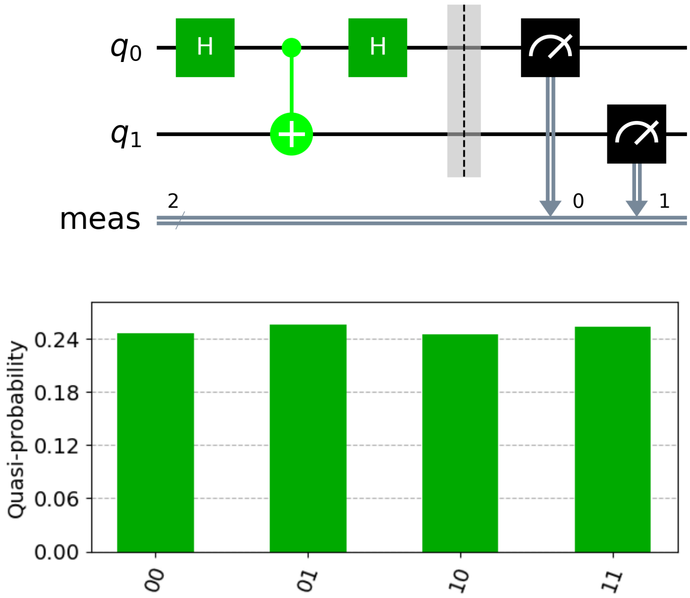

Fig. 7 Quantum circuit for the original EV experiment, using the H gate to represent a 50/50 beam splitter and the C-NOT gate to represent the bomb; and quasi-probabilities of the quantum circuit using the IBM cloud-based quantum computing service (Qiskit Runtime).

To increase the success rate of interaction-free measurements, as in the proposal in Fig. 4, it can build a quantum circuit using n rotation operators, specifically R_x gates, which individually rotate the qubit around the x-axis by an angle of θ/n (in radians) instead of using the Hadamard gate. In this configuration, as the number of cycles increases, the probability of achieving interaction-free measurements improves. Consequently, for sufficiently large values of n, it can approach an efficiency parameter of $\eta_{\mathrm{EV}} = 1$, as illustrated in Figure 8.

6 Discussion

This intriguing result arises from the wave-particle duality inherent in quantum mechanics. Due to the principle of superposition, the single photon exists simultaneously in both paths A and B until a measurement is made. By blocking one of the paths, a "either-or" scenario emerges: if the blocked path is chosen, the photon must be in the unblocked path, and vice versa. Consequently, the photon appears to exhibit an intriguing property called "quantum nonlocality," where its presence in one path seems to affect the measurement outcome at the other detector.

By carefully controlling the experimental conditions and analyzing the detection probabilities at D1 and D2, Elitzur and Vaidman effectively demonstrate a quantum "interaction-free measurement." This peculiar phenomenon provides a unique perspective on the counterintuitive behaviors exhibited by quantum systems, and

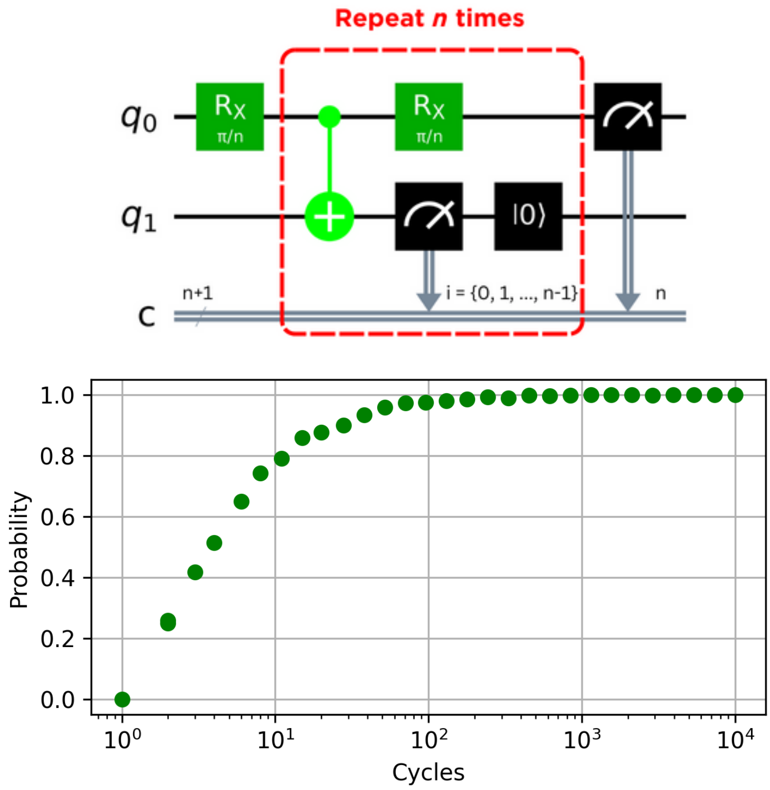

Fig. 8 Quantum circuit for the interaction-free measurement experiment proposed by [5] for n cycles, where $R_x(\pi/n)$ replaces the H gate to control the reflectivity, in which intermediate measurements of qubits representing the state of the bomb q_1 are performed and a final measurement of the photon qubit q_0; and experimental data of quantum circuits from 1 to 10000 cycles using the IBM cloud-based quantum computing service (Qiskit Runtime).

various perspectives have been proposed to explain the experimental results, each offering unique insights into the fundamental nature of quantum phenomena.

The *Many-Worlds Interpretation* introduces the concept of parallel universes: according to this interpretation, when the Elitzur-Vaidman experiment is conducted, the universe splits into different branches corresponding to each possible measurement outcome [12, 13]. Each branch represents a different reality where the photon either interacts with the bomb or avoids it, and the experimental results can be understood as the observer's experience in one of the many coexisting parallel worlds. Therefore, this interpretation claims that when the observer successfully predicts that there is a bomb without interacting with it, the photon was absorbed by another universe, and the bomb indeed exploded.

The interpretation known as Bohmian Mechanics offers an alternative perspective. It proposes the existence of *local hidden variables* that determine the particle's trajectory, even in the presence of superposition and entanglement. In the Elitzur-Vaidman experiment, this interpretation posits that the photon's path is guided by its interaction with the bomb, providing an explanation for the observed measurement results [11].

One interpretation focuses on the notion of a *"single real result"*, which is one of the most widely accepted interpretations currently. It suggests that the measurement outcome corresponds to the actual interaction between the photon and the bomb. The experiment demonstrates that a conclusive result can be obtained without direct interaction, challenging classical intuitions about measurement processes [3]. Another aspect explored in the context of the experiment is the violation of Bell inequalities. These mathematical inequalities provide a criterion for assessing the presence of nonlocal correlations in entangled systems [10]. The observed violation in the experiment suggests the existence of nonlocal influences, highlighting the non-classical nature of quantum entanglement and contradicting the idea of local hidden variables.

These interpretations offer diverse and thought-provoking explanations for the results of the Elitzur-Vaidman bomb testing. They deepen our understanding of the intricate nature of quantum mechanics and its implications for the nature of reality, measurement, and the behavior of quantum systems.

7 Conclusion

The Bomb Testing Problem illustrates the nonlocal nature of entanglement in quantum mechanics. The ability to obtain information about the state of a particle at a remote location through entanglement with another particle implies a connection that transcends traditional notions of spatial distance and challenges our understanding of causality. While the exact mechanisms behind this nonlocality are still the subject of ongoing research and debate, experimental observations have consistently confirmed the validity of entanglement and its influence on distant particles.

These remarkable perspectives on the nonlocal nature of entanglement have profound implications for the advancement of quantum technology. The exploration of entanglement has the potential to revolutionize fields such as quantum communication, quantum cryptography, and quantum computing. It was even a topic that earned researcher Anton Zeilinger, cited here, the Nobel Prize in Physics in 2022 for applying these concepts in experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science. For example, protocols based on entanglement enable secure and efficient distribution of quantum keys, facilitating secure communication over long distances. Furthermore, the ability to manipulate and control entangled particles is a fundamental requirement for quantum computing, where quantum bits (qubits) exhibit superior computational capabilities compared to classical bits [14]. Consequently, understanding and harnessing the nonlocality of entanglement not only enriches worldwide understanding of quantum mechanics but also drives innovations in quantum technology with transformative implications in various scientific and technological domains.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon request.

Funding. The authors acknowledge the support provided by FAPESP (São Paulo Research Foundation) Grant 2013/07276-1 (CEPOF-CEPID Program).

Disclosures. The authors declare no conflicts of interest.

References

[1] Schrödinger, Erwin. "Die gegenwärtige Situation in der Quantenmechanik". *Naturwissenschaften*, **23** (48): p. 807-812, (1935).

[2] A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?". *Phys. Rev.*, **47**, p. 777, (1935)

[3] Elitzur, Avshalom C.; Lev Vaidman. "Quantum mechanical interaction-free measurements". *Foundations of Physics*, **23** (7): p. 987-997 (1993).

[4] Koppell, Stewart et al. "Transmission electron microscopy at the quantum limit". *Appl. Phys. Lett.*, **120** (19), (2022)

[5] Kwiat, P. G. ; Weinfurter, H.; Herzog, T.; Zeilinger, A.; Kasevich, M. A. "Interaction-free Measurement". *Phys. Rev. Lett.*, **74** (24): p. 4763-4766 (1995).

[6] Kwiat, P. G.; White, A. G.; Mitchell, J. R.; Nairz, O.; Weihs, G.; Weinfurter, H.; Zeilinger, A. "High-Efficiency Quantum Interrogation Measurements via the Quantum Zeno Effect". *Phys. Rev. Lett.*, **83** (23): p. 4725-4728, (1999).

[7] Karlsson, A.; Björk, G.; Tsegaye, T. "Theory of interaction-free measurements in cavity resonators". *J. Opt. Soc. Am. B* **15** (12), 2958-2966 (1998)

[8] Tran, D. M.; Nguyen, D. V.; Le, B. H.; Nguye, H. Q. "Experimenting quantum phenomena on NISQ computers using high level quantum programming". *EPJ Quantum Technology* **9**: 6 (2022)

[9] Venugopalan, Anu. "The quantum Zeno effect-watched pots in the quantum world". *Resonance*, **12** (4): p. 52-68, (2007)

[10] Leifer, M. S., "Is the Quantum State Real? An Extended Review of ψ-ontology Theorems". *Quanta*, **3** (1): 67, (2014).

[11] Bricmont, J., "The de Broglie-Bohm Theory". *Making Sense of Quantum Mechanics*, p. 129-197, (2016).

[12] Vaidman, L., "On the Paradoxical Aspects of New Quantum Experiments". *Philosophy of Science Association 1994*, p. 211-217, (1994).

[13] McQueen, K. J.; Vaidman, L., "In Defence of the Self-Location Uncertainty Account of Probability in the Many-Worlds Interpretation". *Studies in History and Philosophy of Science*, **66**: p. 14-23, (2019).

[14] Liu, C.; Liu, J.; Zhang, J. et al. "The Experimental Demonstration of High Efficiency Interaction-free Measurement for Quantum Counterfactual-like Communication". *Sci Rep* **7**, 10875 (2017).