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## Fast

Abstract...We use an unconditionally stable numerical scheme Astract—We use an unconditionally stable numerical scheme in implement a fast version of the goodesic active contour model. The proposed scheme is useful for object segmentation in images, ide tracking moving objects in a sequence of images. The method is based on the Weickert-Romeney-Viergever (additive operator splitting) AOS scheme. It is applied at small regions, modivated by distable and the proposed of th Sethian's fast marching method for re-initialization. Experimental results demonstrate the power of the new method for tracking in

Index Terms—Additive operator splitting, color, geodesic active contours, level sets, numerical scheme, partial differential equations, segmentation, tracking.

## I. INTRODUCTION

N important problem in image analysis is object segmen-A tation. It involves the isolation of a single object from the rest of the image that may include other objects and a background. Here, we focus on boundary detection of one or several objects by a dynamic model known as the "geodesic active contour" introduced in [4]-[7] (see also [19] and [30]).

Geodesic active contours were introduced as a geometric alternative for "snakes" [18], [32]. Snakes are deformable models that are based on minimizing an energy along a curve. The curve, or snake, deforms its shape so as to minimize an "internal" and "external" energies along its boundary. The internal part causes the boundary curve to become smooth, while the external part leads the curve toward the edges of the object in the where  $C_p \equiv \{\partial_p x(p), \partial_p y(p)\}$  and  $\alpha$  and  $\beta$  are positive con-

In [2] and [23], a geometric alternative for the snake model was introduced, in which an evolving curve was formulated by the Osher-Sethian level set method [24]. The method works on a fixed grid, usually the image pixels grid, and automatically handles changes in the topology of the evolving contour.

The geodesic active contour model was born latter. It is both a geometric model as well as energy functional minimization. In [4] and [5], it was shown that the geodesic active contour model is related to the classical snake model. Actually, a simplified snake model yields the same result as that of a geodesic active contour model, up to an arbitrary constant that depends on the initial parameterization. Unknown constants are an undesirable property in most automated models.

Although the geodesic active contour model has many advantages over the snake, its main drawback is its nonlinearity

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that results in inefficient implementations. For example, explicit Euler schemes for the geodesic active contour limit the numerical step for stability. In order to overcome these speed limitations, a multi-resolution approach was used in [34] and additional heuristic steps were applied in [25], like computationally preferring areas of high energy.

In this paper, we introduce a new method that maintains the numerical consistency and makes the geodesic active contour model computationally efficient. The efficiency is achieved by cancelling the limitation on the time step in the numerical scheme, by limiting the computations to a narrow band around the the active contour and by applying an efficient re-initialization technique.

## II. FROM SNAKES TO GEODESIC ACTIVE CONTOURS

Snakes were introduced in [18] and [32] as an active contour model for boundary segmentation. The model is derived by a variational principle from a nongeometric measure. The model starts from an energy functional that includes "internal" and "external" terms that are integrated along a curve.

Let the curve  $C(p) = \{x(p), y(p)\}$ , where  $p \in [0, 1]$  is an arbitrary parameterization. The snake model is defined by the energy functional

$$S[C] = \int_{0}^{1} (|C_p|^2 + \alpha |C_{pp}|^2 + 2\beta g(C)) dxdy$$

The last term represents an external energy, where g() is a positive edge indicator function that depends on the image I(x, y), it gets small values along the edges and higher values elsewhere. For example  $g(x,y) = 1/(|\nabla I|^2 + 1)$ . Taking the variational derivative with respect to the curve,  $\delta S[C]/\delta C$ , we obtain the Euler-Lagrange equations

$$-C_{pp} + \alpha C_{pppp} + \beta \nabla g = 0.$$

One may start with a curve that is close to a significant local minimum of S[C] and use the Euler-Lagrange equations as a gradient descent process that leads the curve to its proper position. Formally, we add a time variable t and write the gradient descent process as  $\partial_t C = -\delta S[C]/\delta C$ , or explicitly

$$\frac{dC}{dt} = C_{pp} - \alpha C_{pppp} - \beta \nabla g.$$

The snake model is a linear model and thus an efficient and powerful tool for object segmentation and edge integration, especially when there is a rough approximation of the boundary location. There is however an undesirable property that characterizes this model. It depends on the parameterization. The model is not geometric.

- read the formulas
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- x check the results
- reproduce the results
- see images in details
- x see graphics in details