Student's name: **Vinicius Lopes**

Question 1:

As we know, the mean is the sum of all samples divided by the number of samples, in order to know the new mean, we must sum the new value to the old sum of all samples and divide it by the new number of samples, which is n+1. Consider:

$\bar{x}_{n+1}$ as the new mean

X as the new value added to samples

n is the old number of samples

To know the new mean, we must follow the sample mean formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} xi$$

Now N to multiply the mean value

$$n * \bar{x} = \sum_{i=1}^{n} xi$$

$$sm = (n * \overline{x)}$$

We know that to know the new mean we have to add the new value to the old sum of samples and divide it by n+1, then:

$$\frac{\bar{x}_{n+1} = X + (n * \overline{x)}}{(n+1)}$$

According to question 1 we know that X is equal to 20, n is equal to 9, and $\bar{x}$ is equal to 10, then:

$$\frac{\bar{x}_{n+1}=20+(9*\overline{10)}}{(9+1)} \qquad \frac{\bar{x}_{n+1}= 20+90}{10} \qquad \bar{x}_{n+1} = 11$$

Then we have function f(.) equal to 11.

μ' = 11

Now regarding to the new variance v' consider:

s_n^2 as the new variance

s_{n-1}^2 as the old variance

$\bar{x}_n$ as the new mean

$\bar{x}_{n-1}$ as the old mean

n is the new number of samples for the new variance

and X as the new value added to samples

Following the sample variance formula:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(xi - \bar{x})^2$$

To solve this problem, we can use the **Welford's method**, where it can be derived by the difference between the sums of squared differences for N and N-1 samples, then:

$$s_n^2 - s_{n-1}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(xi - \bar{x}_n)^2 - \frac{1}{n-2}\sum_{i=1}^{n-1}(xi - \bar{x}_{n-1})^2$$

Then:

$$(n-1)s_n^2 - (\mathrm{n}-2)s_{n-1}^2 = \sum_{i=1}^{n}(xi - \bar{x}_n)^2 - \sum_{i=1}^{n-1}(xi - \bar{x}_{n-1})^2$$

Deriving and using the X as the new added value:

$$= (X - \bar{x}_n)^2 + \sum_{i=1}^{n-1}((xi - \bar{x}_n\ )^2 - (xi - \bar{x}_{n-1}\ )^2)$$

$$= (X - \bar{x}_n)^2 + \sum_{i=1}^{n-1}(xi\ \ - \bar{x}_n + xi - \bar{x}_{n-1})(\bar{x}_{n-1} - \bar{x}_n\ )$$

$$= \ (X - \bar{x}_n)^2 + (\bar{x}_n - X)(\bar{x}_{n-1} - \ \bar{x}_n)$$

$$= \ (X - \bar{x}_n) \ \ (X - \ \bar{x}_n - \ \bar{x}_{n-1} + \bar{x}_n)$$

$$(n-1)s_n^2 - (\mathrm{n}-2)s_{n-1}^2 = \ (X - \bar{x}_n) \ \ (X - \ \bar{x}_{n-1})$$

Finally, we get to:

$$s_n^2 = \frac{(\mathrm{n}-2)\ s_{n-1}^2 \ + \ (X - \bar{x}_n) \ \ (X - \ \bar{x}_{n-1})}{n-1}$$

According to question 1 we know that the old variance, s_{n-1}^2 as 18. X as the new value added 20. $\bar{x}_n$ as the new mean, which was previously calculated as 11, $\bar{x}_{n-1}$ as the old mean 10 and finally the new amount of samples n as 10.

$$s_n^2 = \frac{(10-2)18 + \ (20-11) \ \ (20 - \ 10)}{10-1}$$

$$s_n^2 = \frac{144 + \ 9 \quad * 10}{9}$$

$$s_n^2 = \frac{234}{9}$$

Then we have the new variance as **26.**

v’ = 26