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Development of collaborative strategies in joint action

by

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My heartfelt gratitude to all my teachers who have been the greatest source of motivation and inspiration in my pursuit of knowledge. This Thesis is dedicated to all of them.

Declaration

I declare that this thesis was composed by me and is the outcome of the original work done in the Neuroengineeing and Neurorobotics Laboratory in the Department of Bioengineering, Robotics, Informatics and Systems Engineering (DIBRIS) of University of Genova, Italy. Due references have been provided on all supporting literatures and resources. The work contained here has not been submitted in support of another degree or professional qualification from this or any other university or institute of learning.

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Abstract

Many tasks in daily life involve coordinating movements between two or more individuals. A couple of dancers, a team of players, two workers carrying a load or a therapist interacting with a patient are just a few examples. Acting in collaboration or joint action is a crucial human ability, and our sensorimotor system is shaped to support this capability efficiently. When two partners have different goals but may benefit from collaborating, they face the challenge of negotiating a joint strategy. To do this, first and foremost both subjects need to know their partner's state and current strategy. It is unclear how the collaboration would be affected if information about the partner is unreliable or incomplete. This work intends to investigate the development of collaborative strategies in joint action. To this purpose, I developed a dedicated experimental apparatus and task. I also developed a general computational framework – based on differential game theory – for the description and implementation of interactive behaviours of two subjects performing a joint motor task. The model allows to simulate any joint sensorimotor action in which the joint dynamics can be represented as a linear dynamical system and each agent's task is formulated in terms of a quadratic cost functional. The model also accounts for imperfect information about dyad dynamics and partner's actions, and can predict the development of joint action through repeated performance. A first experimental study, focused on how the development of joint action is affected by incomplete and unreliable information. We found that information about the partner not only affects the speed at which a collaborative strategy is achieved (less information, slower learning) but also optimality of the collaboration. In particular, when information about the partner is reduced, the learned strategy is characterised by the development of alternating patterns of leader-follower roles, whereas greater information leads to a more synchronous behaviour. Simulations with a computational model based on game theory suggest that synchronous behaviours are close to optimal in a game theoretic sense (Nash equilibrium). The emergence of roles is a compensation strategy which minimises the need to estimate partner's intentions and is, therefore, more robust to incomplete information. A second study addresses how physical interaction develops between adults with Autism spectrum disorder (ASD) and typically developing subjects. ASD remains mostly a mystery and has

therefore generated some theories trying to explain their cognitive disabilities, which involve an impaired ability to interact with other human partners. Although preliminary due to the small number of subjects, our results suggest that ASD subjects display heterogeneity in establishing a collaboration, which can be only partly explained with their ability to perceive haptic force. This work is a first attempt to establish a sensorimotor theory of joint action. It may provide new insights into the development of robots that are capable of establishing optimal collaborations with human partners, for instance in the context of robot-assisted rehabilitation.

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Chapter 1

Introduction

> At every crossway on the road that leads to the future, each progressive spirit is opposed by a thousand men appointed to guard the past.
>
> Maurice Maeterlinck

1.1 Motivation

Many of our everyday behaviours take place in social settings and are coordinated with behaviours of others. Even seemingly simple interactions, like a pair of workers sawing timber with synchronised back and forth movements, a couple executing complex steps in a dance floor, two children playing badminton or a therapist giving physical therapy to a patient – see Figure 1.1, require that two individual minds are connected and their bodies coordinated (Sebanz et al., 2006). Thus joint sensorimotor interaction is the vital aspect of our day to day life. The characteristic feature of such interactions is that the subjects influence each others behaviour through coupled sensorimotor exchange with continuous action spaces over repeated trials or continuously in time.

Many of those interactions require active coordination, which is manifested by physical and cognitive responses related to explicit knowledge of an interacting partner. Let us consider another example: while walking through a corridor, someone is coming towards us; there are two feasible options in this classic game knows as "choosing sides": we could move right while the other person passes on the left or vice versa. Importantly, each of the two scenarios requires that our choices are mutually consistent. In our everyday life, we

Figure 1.1 **Examples of human-human sensorimotor interactions in everyday life**

usually tackle this decision-making situations and we master it, sometimes with awkward readjustments.

The theory of strategic interaction or, game theory, is closely tied to decision theory. In fact, the former can be viewed as the natural extension of the latter. Game theory can offer insights into situations in which two or more interacting players choose actions that jointly affect the payoff of each of them. Despite the importance of joint sensorimotor interactions, there has been limited use of game theoretic models to study it. This limited contact is probably due to the lack of experimental paradigms.

The overall goal of my research is to understand the mechanisms underlying adaptation and negotiation of collaborative strategies in tasks involving sensorimotor joint action. These mechanisms may give insights into the study of patient-therapist interactions in neuro-rehabilitation settings. These insights could be translated into the design and use of better robotic interfaces that could facilitate patient recovery. Secondly the mechanisms underlying sensorimotor collaboration may be altered in persons with Autism Spectrum Disorders (ASD) and may be the basis of their deficits in motor control, social interaction, imitation etc. The mechanisms of sensorimotor interaction between two humans are currently little understood and are the basis for the development of collaborative strategies that are common in everyday life. I plan to investigate these issues from both experimental and modelling points of view, by looking at the optimality criteria predicted by Game Theory.

1.2 Objectives

Objective 1: To investigate sensorimotor joint action in dyads of healthy individuals

I will address how negotiating a collaboration is affected by amount and quality of information about the partner. I will introduce a novel experimental paradigm, based on dual haptic interface that enables investigation of situations in which the subjects have different (and partly conflicting) goals. To establish a collaboration, each subject may need to develop an internal representation of his/her partner's current state and his/her partner's goals (or intention). Alternatively, to establish a collaboration, each subject may not require to develop specific knowledge about his/her partner. In the interaction experiments, I will also study whether the emerging collaborative strategies lead to specialised behaviours ('roles'). I will finally look at their evolution within and across trials.

Objective 2: To develop game-theoretic models to predict interaction strategies in sensorimotor joint action

I will use mathematical modelling to study optimality in interaction. I will develop a modelling framework based on optimal control and differential game theory to understand the extent to which the collaborative strategies are 'optimal'. As a step towards reasoning over the learning pattern of patient in a rehabilitation setting, I will then propose a learning algorithm based on *fictitious play*, a game theory-based learning rule which can be implemented in robot interfaces for rehabilitation.

Objective 3: To investigate sensorimotor joint action in persons with Autism Spectrum Disorders (ASD)

I will study how negotiation of collaboration is evolved in sensorimotor joint action between persons with ASD and typically developing (TD) or neurotypical individuals. In persons with ASD, I will focus in particular on the evolution of internal representations of partner (partner state, partner intentions).

1.3 Thesis outline

In summary, the work presented in this thesis investigates the question of how collaborative strategies are evolving in partly conflicting sensorimotor joint tasks, both in healthy individuals and adolescents with ASD.

The thesis is organised into three parts.

Part I (Background) comprises Chapters 2-4 which contain a review of relevant literature, with emphasis on cognitive and motor control research in sensorimotor joint action (Chapter 2), sensorimotor research on Autism spectrum disorders (Chapter 3), and game theory and learning in games (Chapter 4).

Part II (Methods) includes Chapters 5 and 6 . Chapter 5 describes a simple yet versatile experimental set-up that was used to perform psychophysical experiments and investigate the mechanisms of sensorimotor joint action. It also describes a task which will serve as basis for all subsequent experimental work. Chapter 6 describes a novel modelling framework, based on optimal control and differential game theory, and reports on the model predictions in the context of the task described in the previous Chapter.

Part III (Results) includes Chapters 7 and 8. Both are designed to be self-contained, with an Introduction and problem definition section, and a Discussion/Conclusion section at the end. Chapter 7 investigates how information about the partner affects the development of collaborative strategies in sensorimotor joint action. Chapter 8 addresses how sensorimotor joint action is modified in persons with ASD.

Last, Chapter 9 summarises the contribution of the thesis and discusses about possible future directions.

Part I

Background

Chapter 2

Human-human sensorimotor interaction

> Nature uses only the longest threads to
> weave her patterns, so that each small
> piece of her fabric reveals the
> organization of the entire tapestry.
>
> Richard Feynman

2.1 Introduction

Human-human sensorimotor interaction occurs between two persons (a 'dyad') working towards a common motor goal, such as moving a table together or dancing with a partner. While this is a relatively novel field of research, a number of studies have investigated whether dyads perform better than individuals in motor tasks, and if mutual haptic feedback improves dyadic performance with respect to other forms of feedback. Moreover, these studies have raised the question of whether the interaction forces between agents in the presence of a physical coupling can be considered as a form of implicit communication.

The next few sections review the relevant literature with regard to few aspects of human-human sensorimotor joint action. We begin by reviewing proposed taxonomies of joint motor action. We then examine a few aspects of the literature, analysing the principles of sensorimotor joint action that have been elucidated to date, and the experimental paradigms and metrics used in their identification.

2.2 Taxonomies of joint motor action

Different forms of motor interaction can be identified when looking at the nature of the effort and error terms in the utility function that describes the nature of the task and as well as combination of each agent's behaviour. They were fit into three main categories: collaboration, cooperation and competition (Jarrassé et al., 2012; Sawers and Ting, 2014). During collaborative and cooperative interactions, each agent considers their own effort and error as well as their partner's, that makes them capable to work together to find a mutual beneficial factor to complete a task. In contrast, during competition both agents consider their own effort and error (Jarrassé et al., 2012). Such form of interactions emerges typically during antagonistic tasks, for example in in sports such as wrestling, where the gain of one agents resulted in the loss of the other subject.

Collaborative and competitive interactions can be again divided based on how roles are assigned to each agents and how they impact their contribution to the interaction. In cooperative interactions, roles are assigned a priori to each agent. These assigned roles are maintained throughout the execution of the task. This leads to asymmetric interests between the agents such that while both are working towards the same goal, they are doing it so by executing different parts of the same task. In contrast, during collaborative interactions there is no such prior role assignment. Roles are followed in a spontaneous manner and subject to change. This creates an equal distribution of work between agents (Jarrassé et al., 2012).

2.3 Role assignment in joint motor action

Several studies on joint action address the emergence of asymmetric behaviour due to multiple role switching in joint task (Melendez-Calderon et al., 2011; Reed et al., 2007, 2006; Reed and Peshkin, 2008; Stefanov et al., 2009). Few studies predominantly focused on task where participants have fixed assigned roles (Ikeura and Inooka, 1995; Ikeura et al., 1997). Reed et al. (2006) conducted experiments to study joint action in which two partners are in continous physical interaction. The subject pair (dyad) was connected by a two-handled crank which is mounted on a controlled direct drive motor. This mechanism can measure and interact with a circular movement common to both subjects. Also this can measure if the applied force measured at each handle signifies the interaction strategies. They demonstrated that subjects perform task faster when connected than alone; see Figure 2.1.

In a different study, Reed and Peshkin (2008) suggest that some dyads develop a specialized strategy, in which one of the two partners is master in the first part of the movement

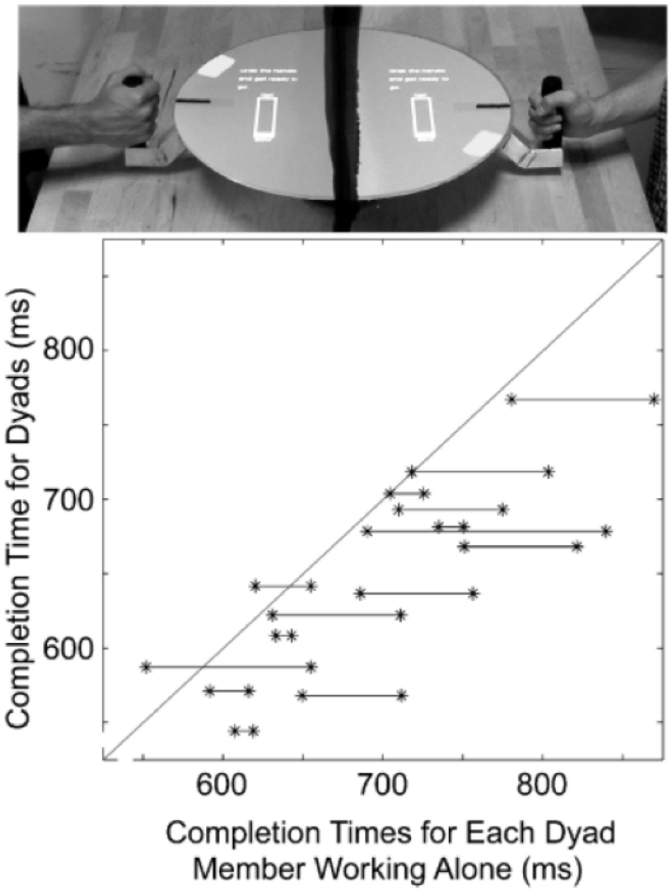


Figure 2.1 **Target reaching with a two-handled crank**. Reaching is consistently faster in dyads than in solo performance. From Reed et al. (2006)

while the other one lets you drive, and then the roles are exchanged in the last one part of the action; see Figure 2.2. In a later study, Reed et al. (2007) implemented a simple model on a robot. They performed a haptic Turing test in which human subjects interacted with a robot. Subjects could not tell whether their partner was a human or a robot, but did not develop the type of role specialisation which was observed in human-human interaction. This suggested that human-human physical interaction is characterised by a very subtle negotiation among partners.

Reed's findings on evolving specialisation strategies in joint action were further studied by Groten et al. (2009), who showed that users prefer a dominance difference among interacting partners in contrast to equally shared control. In this context, here dominance refers to the actual success of influence or control over another and therefore reflects the individual share of the overall share to task success. Stefanov et al. (2009) studied joint interaction during a tracking task and they defined the interaction modality as a tri-state logic composed of two roles and one "no role" condition based on the signs of the interaction velocity, acceleration and force. A role assignment similar to a leader-follower combination involves a conductor-executor strategy. A "conductor" decides what the interacting system should do and express his intention through haptic signal and an "executor" executes the action as determined the conductor. This approach for role determination can be applied to many applications. For example in the analysis of human behaviour in a collaborative task: as the mechanical work distributed among the partners we can calculate contribution of each of them to the total transition work of that certain task.

Melendez-Calderon et al. (2011) studied control of impedance by subjects in a dyad, which is important to maintain stability and robust response to perturbation during interaction in joint action. They estimated joint torque and muscle activation and thereby identified various interaction strategies.

In most of the above-mentioned cases, role specialisation has been noticed, because the participant was not assigned roles a priori. Therefore one has questioned how and why do these specialised roles develop. It is possible that their emergence is due to differences in reaction time, skill level between agents or strength. Masumoto and Inui (2014) studied a leader-follower relationship in a joint action performed by dyad with different skill levels. In this task, participants sat on chairs opposite ends of a table facing the load cell based experiment set up and monitor. They could not see each other's action, and they were instructed not to communicate verbally with each other.In the task, they have to produce discrete isometric forces such that the sum of the forces generated by the right index fingers of the two participants was the target peak force of 10% maximum voluntary contraction.

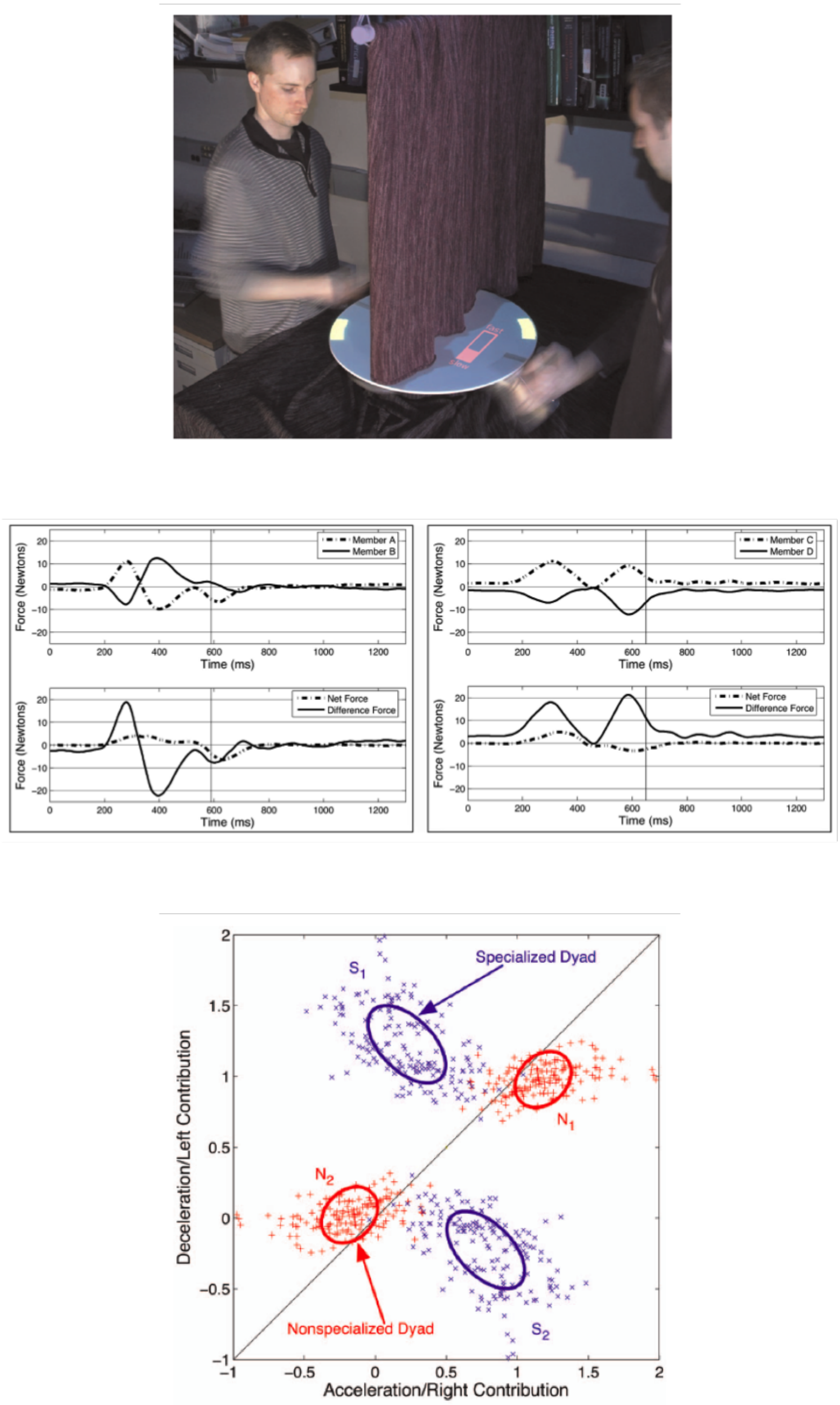


Figure 2.2 **Emergence of roles in dyad reaching with a two-handled crank**. Top: experimental apparatus. Middle and Bottom: In an active/inert dyad (left) one agent is active, the other agent is passive. In a specialized dyad (right), one agent accelerates, the other agent decelerates. From Reed and Peshkin (2008)

The task did not prescribe share force or onset time each participant contributed to the target force. In their novice-novice dyad group, novices with low force variability did to produce an earlier force than those with higher force variability. In the second group, novice-experienced group, experienced participants produced an earlier force than that of the novice. In both group they tend to produce the force complimentary with the training, practice had no effect on the leader-follower relationship. This suggests that leader-follower relations be not always beneficial to the performance in a task (Masumoto and Inui, 2014).

2.4 Joint motor action improves task performance

Prior studies have suggested that haptic interaction forces create a communication channel between partners in a dyad (Ganesh et al., 2014; Melendez-Calderon et al., 2015; Reed et al., 2006; van der Wel et al., 2011). Additional studies have reported that haptically linked dyads can perform collaborative sensorimotor tasks better and faster than either members of the dyad alone, in visuomotor (van der Wel et al., 2011), reaching (Reed et al., 2006), tracking (Ganesh et al., 2014), or in isometric force production tasks (Masumoto and Inui, 2013). Notwithstanding this improvement in performance, communication through haptic interaction is typically perceived as an interference by individual members of dyad compared to their solo performance (Reed et al., 2006).

Bahrami et al. (2010) developed an experimental paradigm for examining joint perceptual decision-making. They specifically addressed the question: Would two people be able to integrate their perceptual information to optimise their decisions? In other words, would two heads be better than one, and in particular, better than the best individual performance in a pair? They found that when two people were given a chance to communicate freely about their level of confidence on a trial-by-trial basis, two heads became better than one.

Few studies have addressed whether interacting with a partner facilitates joint motor action (Ganesh et al., 2014; Masumoto and Inui, 2013).

Masumoto and Inui (2013) studied how two coordination strategies simultaneously facilitate joint action performed by two people. Their study examined the complementary strategies in joint action using an isometric force production task such that the sum of the forces produced by two partners was the target force. They used an experimental apparatus consisting of two load cells and monitors – see Figure 2.3.a. Participants were seated opposite to each other facing the load cell and monitor. The study consisted of a ‘solo’ condition, performed by one participant, and a set of dual conditions, performed jointly by two participants. In the dual conditions, the sum of the forces produced by two partners had to equal a target peak force of 10% MVC or a target valley force of 5% MVC – see Figure 2.3.b – under four conditions (total-force, both-forces, partner-forces and no-feedback) – see Figure 2.3.c. The total-force condition displayed the target forces for the dyad and a sum of the forces produced by the two partners. The both-force condition displayed the force outputs produced by the two partners separately and their personal targets. The partner-force condition displayed only a partner's force output and his target on a monitor and the no-feedback condition removed any visual information from the monitor. Under these four conditions, the participants were instructed to synchronise their force production with their partner's. Both of them were instructed not to verbally communicate to each other.

They found that the two participants produced complementary forces when the total force was visible (indicated by a negative correlation between forces in the total force condition, Figure 2.3.e). This suggested that two participants simultaneously adopted both synchronous and complementary strategies exclusively when the total force was visible. Their results indicated that joint action helps to control force more accurately than the individual action. In a later study Masumoto and Inui (2014), examined effects of a leader-follower strategy on a joint action employing novice and experienced participants, described in the previous section.

Ganesh et al. (2014) developed an experimental paradigm shown in Figure 2.4, consisting of a pair of robots through which the dyad is physically connected through a compliant virtual spring. In this way, they could specifically investigate how the interaction forces of partners can mutually induce adaptation in their motor responses. Each subject of the pair holds one of the two robotic interfaces and receives visual feedback of the position of own hand, represented by a cursor on a screen. They were not allowed to see (the movements of) their arm, which moves on a plane hidden by a panel.

They demonstrated that a physical connection between subjects learning a motor task – visual tracking of a moving target – consistently improves their performance, regardless of their partner's performance – Figure 2.4.b, black trace in Figure 2.5. Moreover, an intermittent connection between the subjects enabled them to learn the task better than subjects who did the task alone for the same duration. In another study van der Wel et al. (2011) demonstrated that subject pairs in a dyad amplify their interaction forces to create a communication channel with their partner. Ganesh et al. (2014) then showed that improvements in interaction task depend not only on their partner's performance but also on the nature of the interacting partner. In their control experiments made it evident that the performance improvement is more prominent when partners are similar, such that interaction with a non-human agent is less beneficial than with a human, and interaction with a peer with similar skill or less skill

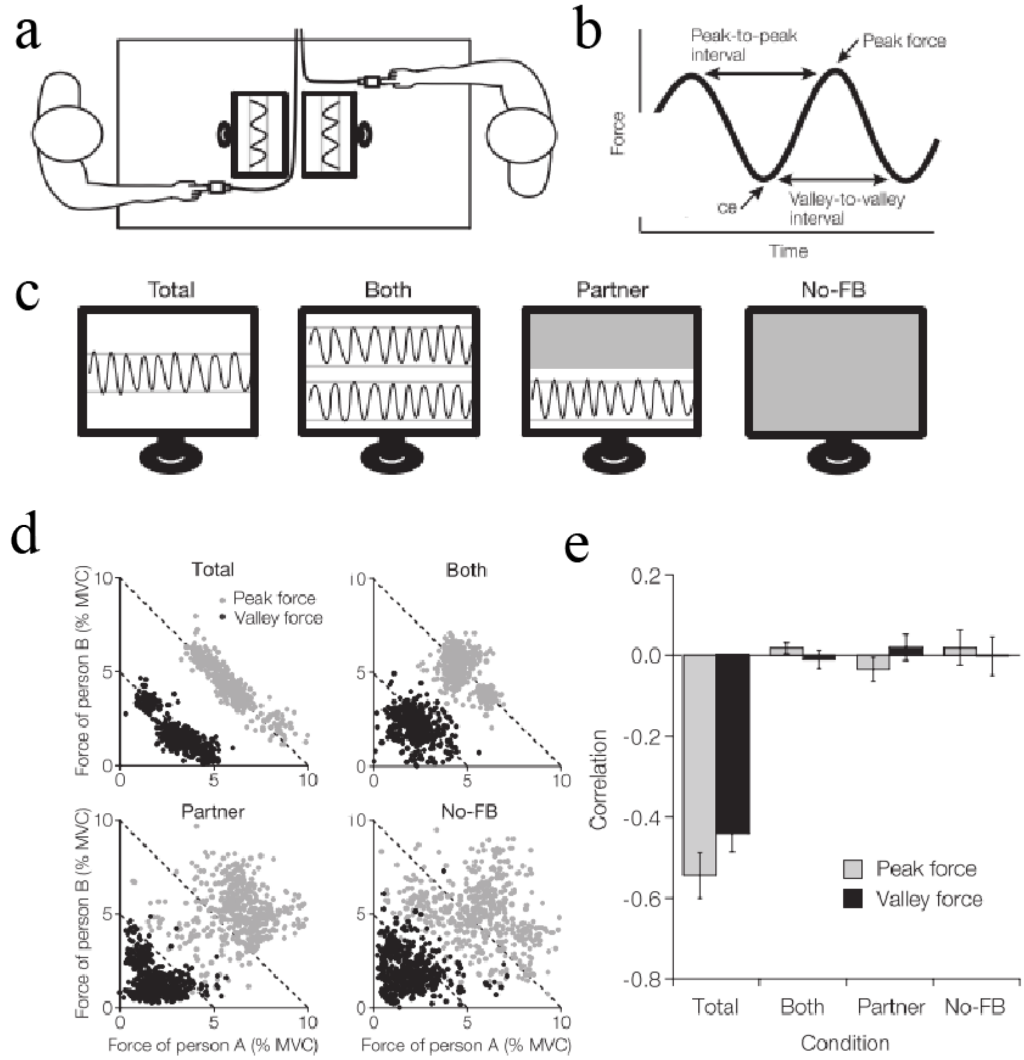


Figure 2.3 **Isometric joint force production task**.(**a**), Experiment setup. (**b**), Definition and measurement of dependent variables. (**c**), Displays under 4 conditions. (**d**), Distribution of forces produced by 10 dyads under four conditons. (**e**), Mean correlation coefficiant between forces produced by 2 participants under four conditons; from Masumoto and Inui (2013)

level is more beneficial than with an expert. This is confirmed by a more recent study by Avila Mireles et al. (2017). It has been suggested that improvements in performance are due to the fact that in a dyad, individuals have fewer actions to deal with, allowing them to each focus on a subset of actions (Knoblich and Jordan, 2003).

How is information being shared between haptically interacting partners? Takagi et al. (2017) proposed a model in which the haptic information provided by touch and proprioception enables individuals to estimate their partner's movements goal and use it to improve their

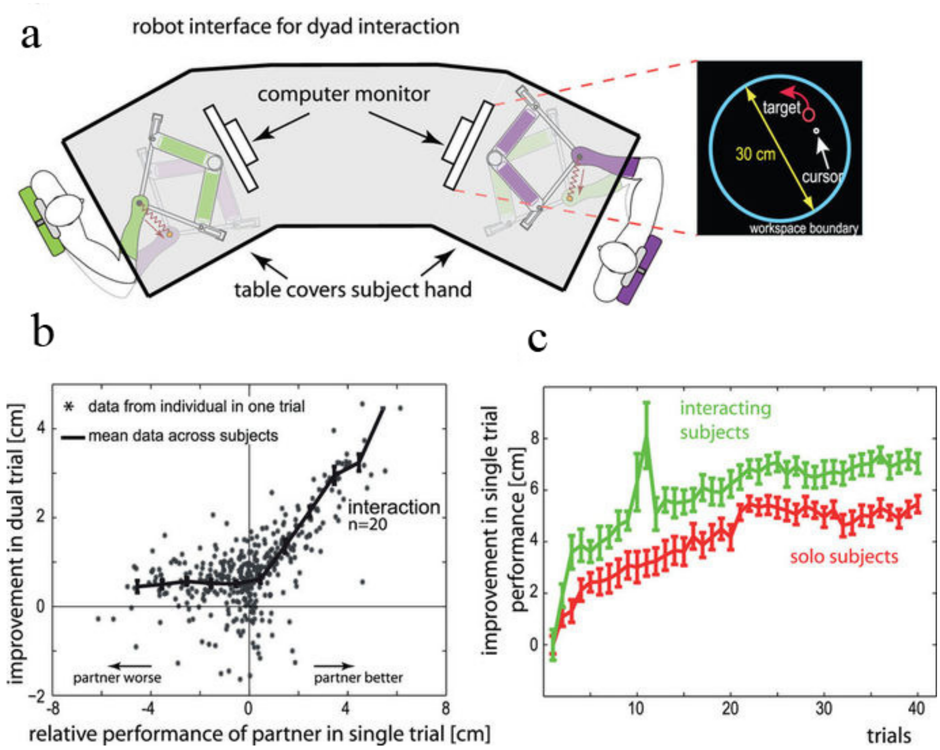


Figure 2.4 **Benefits due to interpersonal interaction in a motor task**. (**a**), Setup: The figure shows a cartoon of the setup used for the experiment. (**b**), Improvement of the performance of both partners during the interaction. (**c**), Learning during the interaction; from Ganesh et al. (2014)

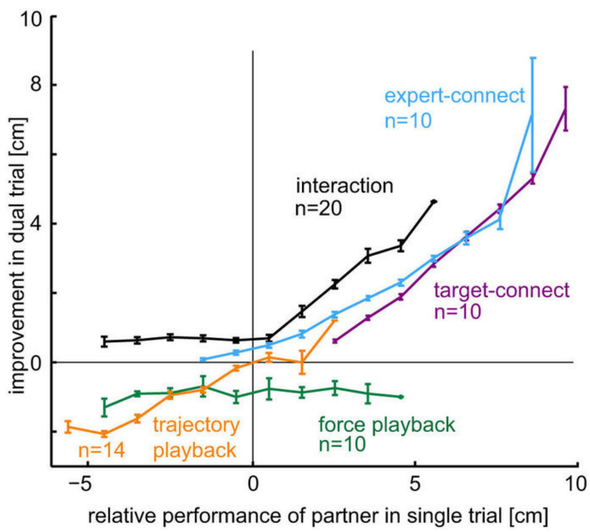


Figure 2.5 **Influence of interaction characteristics**. From Ganesh et al. (2014)

own performance. They focussed on the same task as in (Ganesh et al., 2014), consisting of a sensorimotor estimation problem. Their ‘interpersonal goal integration’ model (Takagi et al., 2017) is computed as an optimally weighted combination of the estimated partner’s target, and their own estimate. This ‘interpersonal goal integration’ model outperformed and explained the experimental results better than other models of interaction proposed in the literature. To experimentally verify the model, Takagi et al. (2017) embodied it into a robot and reported that it induces the same improvements in motor performance in human individuals as those found while interacting with a human partner.

2.5 Sensorimotor interactions as games

Game theory has been used to understand complex decision-making in joint interactions. An overview of game theory is provided in Chapter 4. Here we review experimental studies on joint action which rely on concepts of game theory.

Game theory-based models have been used to understand the decision-making aspects of sensorimotor interactions between self-interested players who aim at optimizing a motor effort (Braun et al., 2009, 2011). Braun et al. (2009) investigated sensorimotor interactions in the realms of the theory of dynamic games where players are continuously interacting over a sequence of time steps. They developed an experimental paradigm in which two subjects sat next to each other and hold handles of separate robot interfaces that were free to move in horizontal plane – see Figure 2.6. The robots were coupled by a simulated virtual spring, so that each player experienced a different force that depends on both players’ movement.

The player-specific payoff matrix of a discrete game can be encoded as a position-dependent force field, so that payoff is associated to motor effort. Using this paradigm, Braun et al. (2009) developed motor versions of two classical games, the *prisoner’s dilemma* and *rope pulling* – see Figure 2.6. In both games, the force field encoded the rewards or costs of the different strategies.

They specifically addressed the question of whether human sensorimotor interactions in such motor games can be interpreted as optimal strategies in a game-theoretic sense.

The prisoner’s dilemma game is described in detail in Chapter 4. In short, there are two prisoners in a jail. Each has the option of cooperating (staying silent) or defeating (betraying their partner). If they both cooperate, they each serve 3 years. If one defeats, he goes free but his partner serves 10 years. Finally, if they both defeat they both serve 7 years. If the two prisoners could agree on a common strategy (cooperative behaviour), their optimal strategy would be to both stay silent. Consider however what happens if they cannot communicate –

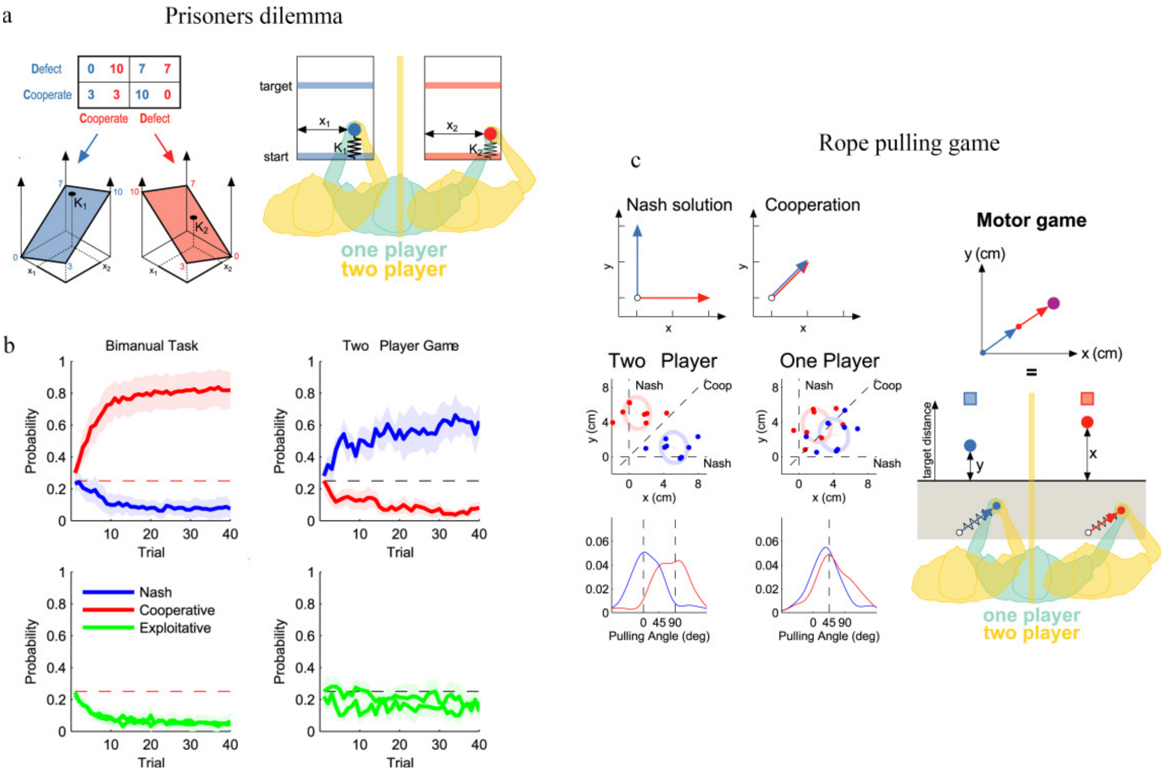


Figure 2.6 **Nash equilibria in motor games**. (**a**), Prisoner's dilemma motor game. (**b**), temporal evolution of game solutions in the prisoner's dilemma.(**c**), Rope-pulling game. A mass is pulled by two players. The arrow shows the direction of force for players for Nash and cooperative solutions. From Braun et al. (2009)

and therefore they don't know what their inmate will do. If they both defeat (betray), they both get 7 years. If one decides unilaterally to remain silent, he may get 10 years. Therefore, he is better off to betray. Same for the other partner. Therefore, the optimal strategy is to both betray. This non-cooperative optimal strategy is called a Nash equilibrium. The experiment involved a two-player and a 'solo' condition, in which one subject grasped the two robots with both hands – see Figure2.6,a. They found that in the bimanual condition subjects converged to the cooperative solution. In the two-subjects condition, they converged to the Nash equilibrium – see Figure2.6, b.

In the rope pulling game – see Figure2.6 c – two players pull a rope to move a mass on a plane. However, each player has access to visual feedback on either X or Y direction alone. The goal for both is to reach a visual target. In this case, the cooperative solution is that the two players move toward the spatial target in the XY plane. The non-cooperative solution is that each player only moves in the direction for which he has visual feedback, keeping the other to 0. Again, Braun et al. (2009) found that the cooperative solution is preferred in bimanual mode whereas the non-cooperative solution is preferred in the two-player mode.

In a later study Braun et al. (2011), studied motor versions of coordination games, each involving multiple Nash equilibria. They exposed subject pairs to four different classical coordination games: *choosing sides*, *stag hunt*, *chicken* and *battle of sexes* – see Chapter 4 for detailed descriptions of these games. Like the previous study, they used a two-robot apparatus and the for each game the payoff matrix was encoded into a position-dependent force field. They found that players could adapt single movement trials to converge into Nash equilibrium strategies even when the interaction force encoded multiple equilibria.

Overall, these results show that dyads typically converge to optimal non-cooperative strategies (Nash equilibria). However, these tasks put more emphasis on strategic decision-making than sensorimotor control – the movement end position was taken as the decision outcome.

Grau-Moya et al. (2013) studied a decision-making model that forms beliefs about the other player's strategy and its effect of uncertainty on cooperation in a *stag hunt*-like sensorimotor interaction. They adopted a learning model based on *fictitious play* – see Chapter 4. Their key idea is that if a decision maker does not fully trust the model, he will biase his estimates by taking his utility function into account. If he is extremely pessimistic and cautious, he will dismiss the model and assume a worst case scenario. On the other hand if he fully trusts the model, he will end up picking up the strategy with highest utility. But a risk averse decision maker will always compromise between two extremes. Grau-Moya et al. (2013) manipulated the risk sensitivity of the virtual player, and found that humans adapted their behaviours accordingly – by changing the amount of their cooperation. These results suggest that the players develop predictions of their partner's habits. Uncertainty about the partner also plays a role in forming decisions.

In a similar direction, Li et al. (2015) employed game theory to analyse human-robot interactive behaviours. They proposed an adaptation law to achieve a human–robot coordination that automatically adjusts the role of the robot, according to the measured interaction force.

2.6 Cognitive influences during joint motor action

Several studies have addressed the influence of social factors in joint motor action. Shergill et al. (2003) investigated the basis of the force escalation process. Force escalation typically involves an antagonist type of interaction, in which the latter increases intensity over time. Sometimes it arises spontaneously as a result of an interactive pattern. In Shergill et al. (2003)'s example, the interaction is quite minimal and involves subjects applying a downward force with a finger on the other one's hand. This procedure is then repeated, alternating roles. Subjects are instructed to apply the same force as the perceived force applied to them. As the turns alternate, the absolute amount of force escalates. This suggests that subjects tend to underestimate the force they apply in an interaction, self-generated force is perceived as weaker than externally applied force and they compensate by increasing force in the next round, resulting in escalation. Simple as this explanation is, it provides a good model for more general situations. As escalation can originate unintentionally by a reciprocal configuration in which a perceived mismatch between one's own 'moves' and those that we are subject to by external action and somehow this is undesirable in joint motor action experiments.

Recently, Takagi et al. (2016a) studied the influence of social factors involved in facing a partner in the sensorimotor joint action. Similar to Shergill et al. (2003), they asked subjects in a dyad to exchange forces. Their experiment involved two configurations, one where partners were separated by a curtain and another where they seated face to face, and their goal was to reproduce the partner's force accurately. They found the partner's gaze caused significant changes in behaviour so that separated and face-to-face pairs behave differently. However, the social factors influence face-to-face exchange of forces are still unknown. They suggest studies on sensorimotor joint action should take such social influences into account.

2.7 Conclusions

This brief overview of joint sensorimotor action emphasizes a few aspects: (i) interaction implies an exchange of information among partners in a dyad; (ii) interaction involves learning which, at least to some extent, may transfer to 'solo' performance. Like many forms of interaction, sensorimotor joint action may benefit from analytical frameworks, like game theory, which are based on economic concepts – i.e., costs, benefits, and effort. These concepts are increasingly used to interpret human behaviour, in particular decision-making (Glimcher, 2003; Glimcher and Fehr, 2013). In the case of neural control of movement, optimal control theories (Todorov and Jordan, 2002) may be seen as instances of this approach. Some authors (Jarrassé et al., 2012) have suggested to use differential game theory – a natural extension of optimal control – to develop a taxonomy of joint sensorimotor action, but this approach has never been used systematically in conjunction with empirical studies. This is the main goal of this thesis work.

Chapter 3

Sensorimotor symptoms in persons with Autism Spectrum Disorder

Imagination is everything. It is the preview of life's coming attractions

Albert Einstein

3.1 Introduction

Autism Spectrum Disorder (ASD) is a range of conditions characterised by deficits in communication and social skills and repetitive and stereotyped patterns of behaviour and interest (American Psychiatric Association, 2013). Though early intervention has a dramatic impact on reducing symptoms and in increasing the ability of the child to mature and learn new abilities, it is estimated that only 50% of children affected by ASD receive a diagnosis before starting the primary school.

The impaired communication and social skills that so greatly impact the daily lives of those with ASD depend on the motor system, which allows individuals to map the movements, body language and expressions of others onto themselves, and to experience the intentions of others, known as Theory of Mind (Baron-Cohen et al., 1985; Gallese et al., 2004; Iacoboni, 2009; Larson et al., 2008a). Although sensorimotor control is not included in the diagnostic criteria for ASD, many studies have identified a number of motor abnormalities in individuals with autism (Dowell et al., 2009; Ego et al., 2016; Gowen and Hamilton, 2013; Jansiewicz et al., 2006; Mostofsky et al., 2006).

Understanding impaired motor control and motor learning in persons with ASD may offer insight and a potential understanding of the difficulties these children face in developing the higher-order skills that govern social interaction, communication, and cognition.

3.2 Sensorimotor control and motor learning

Due to the broad spectrum of cognitive impairments in persons with ASD, it is not surprising that their observed motor deficits are equally variable. Individuals with ASD repeatedly score low in a generalised set of motor assessments that measures timed movements, balance, gait (Dowell et al., 2009; Dziuk et al., 2007; Jansiewicz et al., 2006; MacNeil and Mostofsky, 2012). Also, they often show an impaired ability to accurately make and imitate gestures, specifically if they are either involving tool use or have a social significance (e.g., waving hand). This is usually referred as dyspraxia (Boria et al., 2009; MacNeil and Mostofsky, 2012; Mostofsky et al., 2006). Also, children with ASD are not only impaired in their ability to perform these skilled gestures, but also in representational or postural awareness of these movements, when performed by others (Dowell et al., 2009). Though overall assessments consistently point at a motor impairment, there is no specific or signature motor deficit for ASD. The above studies report a variety of motor impairments which do not fit into any precise, clinical classification and validation.

Due to the developmental nature of ASD, these deficits may begin with error-based learning, rooted in an unusual mechanism for motor adaptation, and an ineffectiveness to learn how to make movements without error. Proprioception and vision are the two sensory modalities that primarily drive error-based motor learning. As with studies of motor control, individuals with ASD show a broad range of sensorimotor abnormalities. In fact, hypo-reactivity to sensory inputs has been added to DSM-V as part of the diagnostic criteria for autism (American Psychiatric Association, 2013).

Vision-related problems have been demonstrated in several contexts. For instance, individuals with ASD exhibited low performance on a visual-temporal integration tasks (Nakano et al., 2010) and, according to surveys with parents, children with ASD tend to either avoid or seek out visual input (Leekam et al., 2007). With regard to the visual aspects of movement, hypo-reactivity to visual feedback has been manifested as an impairment in recognising biological motion (Cook et al., 2009) and in the recognition and response to visual chains of action (Cattaneo et al., 2007). Taken together, these findings suggest that persons with ASD exhibit an inability to properly process and utilise visual information, especially with regard to movement.

In contrast, studies on the proprioceptive and haptic responses in ASD seem to suggest an overall hypersensitivity. Individuals with ASD are better at haptic-to-visual shape matching (Nakano et al., 2012) and have a lower threshold for high frequency vibro-tactile stimulus detection (Blakemore et al., 2006).

These symptoms are unlikely due to peripheral deficits, as individuals with ASD show normal proprioceptive (Fuentes et al., 2011) and visual acuity (Tavassoli et al., 2011). Rather, they suggest that sensory abnormalities in ASD may arise from differences in the central integration of sensory information.

A number of studies (Haswell et al., 2009; Izawa et al., 2012) – see Figure 3.1 – show evidence of how sensory processing may impact motor learning in children with ASD. Haswell et al. (2009) specifically measured the generalisation patterns in children who learned to control a new tool to quantify the representation of internal models within the brain. In these studies, children experienced velocity-dependent force field perturbations while reaching a target on the left workspace and they were required to learn to compensate for it. They were then asked to make moves to two additional targets on the right workspace. The targets were defined either in extrinsic coordinates – i.e. they were visually matched with the learned targets, but required a different arm configuration to complete the movement – or in intrinsic coordinates – they proprioceptively matched the learned target, i.e. they required the same joint rotations for movement, but were visually different from the initial learned target. The children compensated for the perturbation when reaching to targets in both intrinsic and extrinsic coordinates, or in other words, they generalised their learning, even though they had never experienced a perturbation when reaching to these two additional targets. Interestingly, children with ASD showed normal performance at the learned target, but significantly greater generalisation to the target in the extrinsic coordinate, suggesting a greater reliance on proprioceptive feedback during learning (Haswell et al., 2009). A follow-up study from the same group (Izawa et al., 2012) confirmed this finding with a larger group of subjects, which also included subjects with attention deficit-hyperactivity disorder (ADHD), and confirmed that the observation is specific to ASD.

3.3 Action and intention understanding

Our daily social life is based on our capacity to understand the behaviour of others. How do we understand the goal of our partner by looking at their actions? Humans and monkeys possess a dedicated neuronal network – the mirror neuron system – which maps visual descriptions of actions by others onto their own motor representations of the same actions

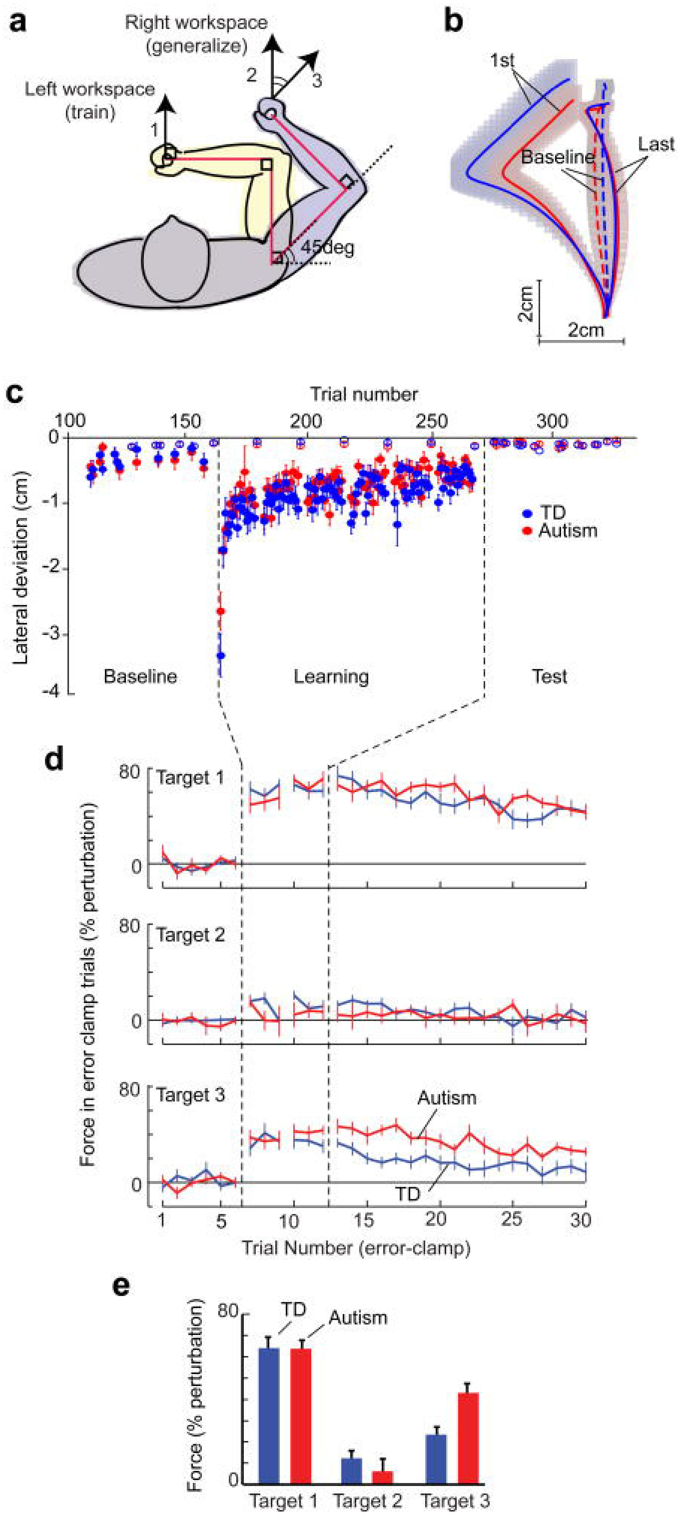


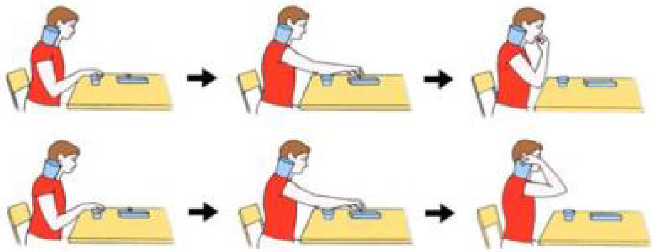
Figure 3.1 **Learning and generalisation of an internal model in typically developing children and children with ASD**. (**a**), Scheme of the motor task. (**b**), Movement trajectories in baseline and last training phase. (**c**), Lateral deviation of movements during the three phases of the task. (**d,e**), Percentage of force perturbation compensated in training and generalisation trials. Typically developing children are shown in blue, ASD children are shown in red. From (Haswell et al., 2009)

(Rizzolatti and Craighero, 2004). Besides this role in action understanding, the mirror neuron system is believed to mediate imitation (Bird et al., 2007; Iacoboni et al., 1999; Rizzolatti and Craighero, 2004), understanding intentions of others (Iacoboni et al., 2005), and emotion recognition (Gallese et al., 2004).

The mirror neuron system encodes the goal of motor acts. Hamilton and Grafton (2006) used a trial-by-trial adaptation technique, where participants observed a couple of video-clips showing goal-directed motion actions in which some of the goals were novel and others were repetitions from previously observed movements. Repeated display of the same goal triggered a response in the left intra-parietal sulcus. Interestingly, this region was not sensitive to the trajectory of the subject's hand.

A number of typical ASD symptoms (impairment in language, communication, and the capacity to understand others) appears to involve functions mediated by the mirror neuron mechanism. The 'broken mirror' hypothesis states that this specific set of deficits might relate to the impairment of the mirror neuron mechanism (Ramachandran and Oberman, 2006; Williams et al., 2001). Evidences from imaging, EEG, and TMS studies supports this hypothesis (Cattaneo et al., 2007; Dapretto et al., 2006). A strong evidence came from (Dapretto et al., 2006). They studied children with high-functioning autism who were MR-scanned while observing and imitating emotional expressions. Children with autism showed weaker activation of mirror neuron areas in the brain than typically developing (TD) children. Interestingly the activation was inversely proportional to symptom severity. However, a few behavioural studies (Bird et al., 2007; Hamilton et al., 2007) do not support the broken mirror hypothesis as they reported that subjects with ASD do not present deficits in understanding goal of motor actions by others.

In a set of neurophysiological studies, Fogassi et al. (2005) investigated the neuronal mechanisms through which a person understands his/her partner's intention. They found that for a chain of motor actions, the mirror neuron system involves a neural population, in which each neuron encodes a specific sequence of motor actions (e.g., reach-grasp-eat or reach-grasp-move away). These sequences of actions also contain 'action-constrained' mirror neurons, which fire only if the motor action they encode is part of a sequence (e.g., grasping for eating, but not grasping for placing, or vice versa). During the observation of others' actions, these 'action-constrained' mirror neurons fire when the observed actions match the actions encoded by the sequence in which those set of neurons are embedded. In a later study, Cattaneo et al. (2007) showed evidence that organisation of sequenced motor actions is impaired in ASD condition. In one of the experiments, ASD children and TD children were asked to perform two action sequences, either grasping an object to eat or a



b

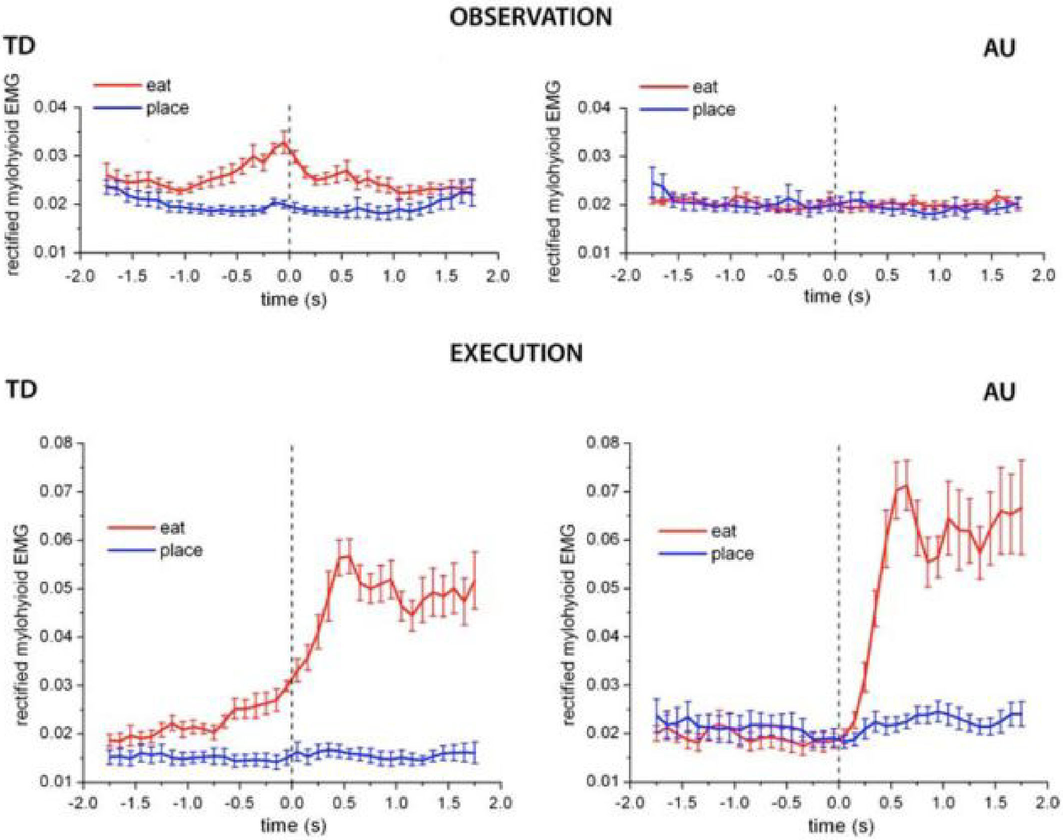


Figure 3.2 **Impairment of the mirror mechanism may explain some deficits in children with autism**. (**a**), Individuals reach food located on a touch sensitive plate, grasp it and bring it to the mouth (top) or put it in a container (bottom). (**b**), Rectified EMG activity of mylohyoid muscle in typically developing (TD) and autism (AU) children during observation (top) and execution (bottom) of the above actions. (Cattaneo et al., 2007)

grasping to place it into a box; see Figure 3.2.a. The EMG activity of the mylohyoid muscle – involved in mouth opening – was recorded.

In TD children, the muscle was active well before they moved the arm to reach a food pellet; in contrast, no muscle activation was observed in ASD children. In these subjects, muscle activation only appeared immediately before they brought food to their mouth. This finding suggests that ASD children are unable to coordinate their motor actions into a sequence characterised by a specific motor intention.

In a separate experiment, subjects were instructed to observe a person while either grasping a food item for eating or grasping a piece of paper for placing it into a box; see Figure 3.2.b. Again, EMG activity of the mylohyoid muscle was recorded. TD children exhibited muscle activation while observing food grasping, but this activation was lacking in ASD children. This finding suggests that in ASD, understanding of other's intention is partially absent. A few follow-up studies claimed this modified mirror neuron hypothesis in both humans (Boria et al., 2009) and in monkeys (Caggiano et al., 2009).

3.4 Prediction and internal models

Predicting the consequences of own motor actions or about future sensory events is a fundamental property of our cognition and is crucial to adapt our actions and behaviours and to interact with the world around us. These abilities have been widely studied in motor control and have highlighted the role of internal models (Wolpert et al., 1998). Empirical evidence supporting the hypothesis that internal models are impaired in ASD is highly controversial – see Gowen and Hamilton (2013) for a review. Many behavioural studies failed to to find a difference in prediction at both motor and perceptual levels in individuals with ASD (Blakemore et al., 2006; Haswell et al., 2009; Larson et al., 2008b; Marko et al., 2015). To further investigate this, Aitkin et al. (2013); Ego et al. (2016) used a framework to test the prediction abilities of persons with ASD in the context of smooth pursuit eye tracking. Ego et al. (2016) recorded eye movements from high-functioning autism children during a tracking with occasional blanking paradigm. They compared them with age-matched controls. Eye movements during the blanking period are controlled on the basis of prediction of the target motion. They defined number and accuracy of predictive saccades and predictive re-acceleration during target blanking as markers of predictive eye movements. They found that these markers are comparable between two populations, suggesting both internal models and predictive abilities are left unaffected in ASD. Aitkin et al. (2013) had found similar results. These studies calls for a more restrictive and careful definition of the impaired prediction hypothesis in ASD.

3.5 Understanding other minds and interpersonal interaction

Humans are great at inferring others' emotional states and on reasoning about how their actions can affect future behaviour of their partner, and also they invent ways to collaborate with them. This has been called a sense of 'theory of mind' (ToM) (Premack and Woodruff, 1978). ToM is central to our social life and is one of the reasons humans are good at collaborating with each other. Hence ToM is essential for physical and social interactions (Häberle et al., 2008; Sebanz et al., 2003).

Between the 1980s and 1990s the mind-blindness theory was proposed to explain the social and communicative difficulties of children with ASD, thus suggesting that they had a delay in developing ToM as the ability to anticipate others' thoughts and feelings (Baron-Cohen et al., 1985; Leslie, 1987).

Leslie (1987) postulated that our ability of mental state representation is based on a dedicated cognitive mechanism. This includes a 'decoupler' and an 'expression raiser' and transform our impressions of the physical world into secondary representations. These are decoupled from reality and mentalising about a situation can be visualized as representing a person's propositional attitude to the states in the world. This is why children are normally not confused when their mother holds something to her face and pretend it as a telephone. According to Leslie (1987), the first marked manifestation of our ability to mentalize is seen in the child's enjoyment of pretence in their early ages, from around 18 months. Here the child acts as if understanding whatever mother is holding as a telephone, which does not make any confusion with the child's learning about the real telephone. A neural system is needed that supports the processing of particular information in relation to agents and is not bound to a particular modality. If such system is dysfunctional from birth, a difficulty will result in mind blindness.

The development of ideas on mentalising and mind blindness as a neurocognitive theory originated from studies published in the late 1970s and early 1980s, on understanding mental states such as beliefs in chimpanzee (Premack and Woodruff, 1978), and in children (Wimmer and Perner, 1983). At around the same time, Wing and Gould (1979) documented that children with ASD lacked spontaneous make-believe play. The theory of mind blindness predicts that normal developmental milestones of mentalizing is absent at the appropriate age in children with ASD. Particularly, they will fail to follow another person's gaze, fail to show or point at objects of interest and failed to understand make belief play. Baron-Cohen et al. (1996) studied these signs in a large population of infants aged 18 months. These early signs were found to predict the ASD diagnosis well at the age of 3, when a firm diagnosis of autism can be made.

The mind-blindness hypothesis was proposed and tested by Baron-Cohen et al. (1985). The conflict on this was that if the social impairment deficit in autism is due to a failure of mentalizing mechanism as abstracted by Leslie (1987), then autism children should be unable to represent mental state such as beliefs. They should be unable to predict and understand behaviour in terms of someone's belief even when having developed the appropriate level of cognitive and verbal development. Wimmer and Perner (1983) first devised a test based on a false belief task. In the Sally-Anne task (Baron-Cohen et al., 1985) – see Figure 3.3

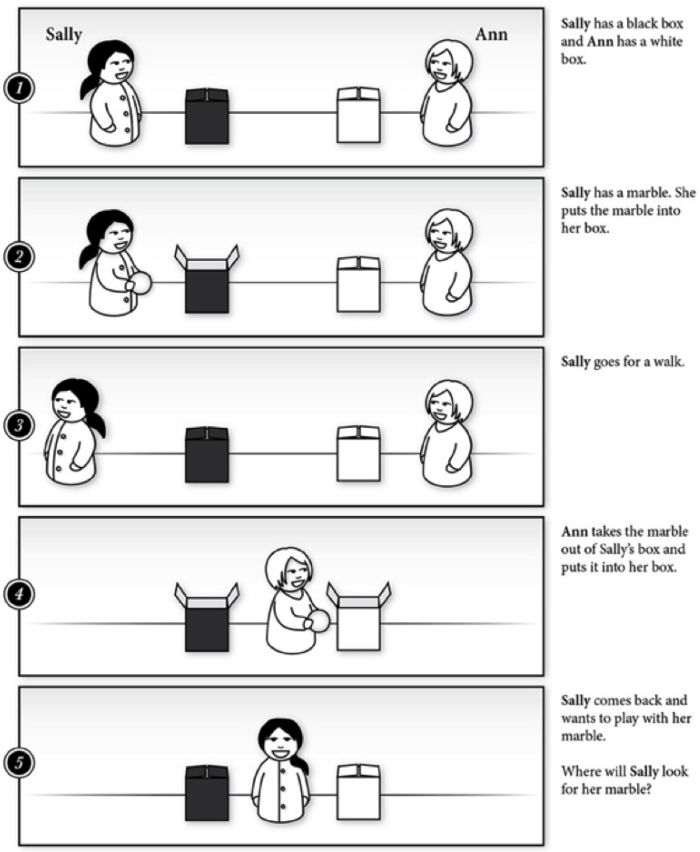


Figure 3.3 **The Sally-Anne test**. From (Baron-Cohen et al., 1985)

– children are told a situation-based story in which there are two people, Sally and Anne. Sally places a marble in her basket, then leaves. Anne later moves the marble to her box while Sally is away. After being told this situation, children are asked where will Sally look for her marble when she returns (action-prediction) or where Sally thinks her marble is (belief). Typically developing children as young as 4 years and children with Down syndrome normally pass such test, whereas younger children and much older children with ASD typically fail. Another reason for studying mechanisms of belief-desire reasoning is to provide an information-processing aspect of successful performance.

The outcome of the false belief test is not always easily understood. The failure on the Sally-Anne test and similar tasks can be predicted by the mentalizing deficit hypothesis, but there are other reasons for failure. For example, the Sally-Anne test requires working memory and ability to suppress reality-oriented responses, i.e., pointing where the object really is. Taken this into consideration, for a good demonstration, it is essential to show success on the test that is in every respect the same but that should not involve thoughts about mental states.

One of the strength of mind blindness hypothesis is that it can explain social communication impairments in autism. It can be also applied to all individuals on the wide range of autistic spectrum, in that age and mental-age appropriate tests are used, which is independent of IQ. Functional imaging studies have reported key areas of the 'social' brain that are activated in particular during false-belief tasks in the typically developing subjects, but are hypo-active in autism conditions.

It is also important to identify shortcomings of the mind blindness hypothesis. It has never claimed to account for the repetitive behaviour and the obsessive interests in autism. It explains neither motor control problems nor perceptual processing impairments. Nonetheless, mind blindness may be able to explain some of the language problems; among other, language delay, muteness, echoing of speech, idiosyncratic use of language. Since today ASD is no longer defined by any firm separation from the "normality", the best way to seeing this "normality" is by referring to the results from Autism Spectrum Quotient (AQ) (Baron-Cohen et al., 2001a,b). This is a questionnaire-based screening test, either done by a parent about his child, or by self-report (if the child is high functioning). The test consists of 50 questions in total, relates to five different aspects, with ten questions each: social skill (S), attention switching (SW), communication (C), imagination (I), attention to details (D). The AQ test needs to be administered to subjects with normal IQ. When administered to a large population its score follows a normal distribution, and separates autism from control groups. Specifically, 93% of the population falling in the mean range of the AQ, and about 99% of the autistic population comes at the high-end of the scale (Baron-Cohen et al., 2001b).

The Empathising-Systemising (ES) theory (Baron-Cohen, 2009) explains the social and communication difficulties in autism condition by reference to delays and deficits in empathy while explaining intact or even superior skill in systemising. Systemising is the drive to analyse or construct systems. This can be any kind of systems, which follows rules. When we systemise, we are trying to identify the rules that regulate the system. Examples of some major kind of systems includes: numerical systems (e.g., bus timetable), collectable systems (e.g., distinguishing between types of plant leaves), mechanical systems (e.g., operating video-recorder), social systems (e.g., a management hierarchy), abstract systems (e.g., the syntax of language). In all these cases, we systemise by understanding regularities and rules. The advantage of ES theory is its usefulness to explain major groups of features in autism spectrum conditions including: repetitive behaviour, local attention to detail and narrow interests. This also explain difficulties in affective reaction to another's mental state. For this reason the ES theory appears better suited to explain the whole set of features characterising in ASD.

As children grow up, they are more able to engage in organized, planned, goal-directed actions. Goal-directed activity depends on some mental processes, including organization, inhibiting impulses, selective attention, planning memory and shifting. The ability to engage in goal-directed activity, along with the mental processes that make this possible, falls under the heading of executive function (EF). The pre-frontal cortex is considered to be mostly responsible for EF skills. Difficulties across the breadth of EFs, including planning, inhibition, cognitive flexibility, generativity and working memory, have all been reported in ASD (see (Hill, 2004) for a review). Jones et al. (2017) studied autistic adolescents and found that social communication difficulties and the presence of restricted and repetitive behaviours related to ToM. In contrast, these behaviours were not associated with EF.

Chapter 4

Game theory and joint action

All stable processes we shall predict.
All unstable processes we shall control.

John von Neumann

4.1 Introduction

Game Theory is an important branch of operational research which aims to study how agents make their decisions restricted to the available information from other or both agents, in order to attain a balanced effect. Following Nash (1951), a 'game' can be defined as *an economic (or other) situation involving two individuals whose interests are neither completely opposed nor completely coincident.*

Simply put, Game Theory is a study of the strategic interaction of self-interested people, in which each of them chooses his actions based on his judgement of how other players will make their decisions. At the beginning of 20th century, John Von Neumann began to systematically study Game Theory (Von Neumann and Morgenstern, 1944). At that time the field was relatively new; the research on game theory was focused on cooperative games, in which agents are assumed they can enforce agreements between them about proper strategies.

In the early 1950s, John Nash initiated a new era of game research, by shifting the focus from cooperative to non-cooperative games. Non-cooperative games are situations in which there are no external means to enforce cooperative behaviours. In other words, each player takes his/her decisions in autonomy. Nash developed a criterion for mutual consistency of players' strategies, known as Nash equilibrium. The notion of 'Nash equilibrium' was first presented in (Nash, 1950), and later in his PhD thesis (Nash, 1951).

The first part of this Chapter will present the basic elements of the game theoretic framework and the notion of Nash equilibrium will be exemplified in various classic games. In the second part we will present a brief overview of differential game theory and its mathematical foundations. We will then introduce the concept of learning in games. The Chapter will be concluded with a review about the application of game theory in behavioural research.

4.2 Basic definitions

Game theory is used to solve the problem of multi-person, multi-object conflict decision-making with constraints under certain rules. The players in a game choose the 'optimal' strategy that maximises their reward based on partial information about each other and the state of the environment to achieve an equilibrium. Unless otherwise mentioned, definitions and notations in the coming subsections are adapted from (Başar and Olsder, 1999; Leyton-Brown, 2008) .

The games studied in game theory are well-defined mathematical objects. To be fully defined, a game must specify the following elements: the players of the game, the information and actions available to each player at each decision point, and the payoffs for each outcome.

1. *Players*. The players are the decision-makers of the game. They must make a decision in order to meet their own demands and interests. Let $N = \{1, 2, \dots, n\}$, denote the set of players. If a player is denoted as i, then his/her partners will be denoted as $-i$ (also termed the 'opponents' of player i). In a two-players game, $-i$ just denotes the opponent of player i.

2. *Actions and Strategies*. Players in a game can take a defined set of actions. If the set is limited, then the game is termed a *finite* game; otherwise it is an *infinite* game. Continuous games, differential games and repeated games are sub-types of infinite games. The notion of differential game will be explained in detail in a dedicated section.

   A strategy is a complete algorithm for playing the game, telling a player what action to take for every possible situation throughout the game.

   A strategy profile (sometimes called a strategy combination) is a set of strategies for all players which fully specifies all actions in a game. A strategy profile must include one and only one strategy for every player. Let S_i be the set of strategies available to player i. If $s_i \in S_i$ the strategy selected by player i, then the strategies selected by each player – the strategy profile – form a vector denoted by $s = \{s_1, s_2, \ldots, s_n\}$.

A strategy can be 'pure' or 'mixed'. In a pure strategy, players that are in exactly the same situation always take the same actions. In a mixed strategy, players select their action randomly. More formally, $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a mixed strategy for all agents if $\sigma_i : S_i \to [0,1]$ with $\sum_{s \in S_i} \sigma_i(s) = 1$ is a probability distribution over the strategy space S_i of player i.

3. *Payoffs*. The win or loss of the players after the decisions are made, which is a function of all game strategies or behaviours. Let $f_i(s_1, s_2, \ldots, s_n)$, $i \in N$, $s_i \in S_i$ be the payoff function value of players obtained under the strategy profile $s = \{s_1, s_2, \ldots, s_n\}$. The income of the player is not only related to the strategy of his choice, but also on the actions taken by the other players. Players' earnings are not only related to their chosen strategies, but also to the actions of other players. In a simple game, a *payoff matrix* is often used to describe the income of each game with different strategy profiles.

4.3 Representation of games

Extensive form A game in extensive form is represented as a tree, in which each vertex (or node) represents a point of choice for a player. The player is specified by a number listed by the vertex. The lines out of the vertex represent a possible action for that player. The payoffs are specified at the bottom of the tree. The extensive form can be viewed as a multi-player generalization of a decision tree. The extensive form can be used to formalize games with a time sequencing of moves. Since choice nodes form is a tree, we can uniquely identify a node with its history, that is, the sequence of choices leads from the root node to it; see Figure 4.1 for an example.

Normal form Also known as strategic or matrix form, it is the common way of representing strategic interaction in a game. A game is represented in this way by specifying every player's utility for every state of the world. The representation of games in normal form is conceptually straightforward. However, that does not always assume players act simultaneously. Also, a normal-form game representation does not incorporate any notion of time or sequence of the actions of players.

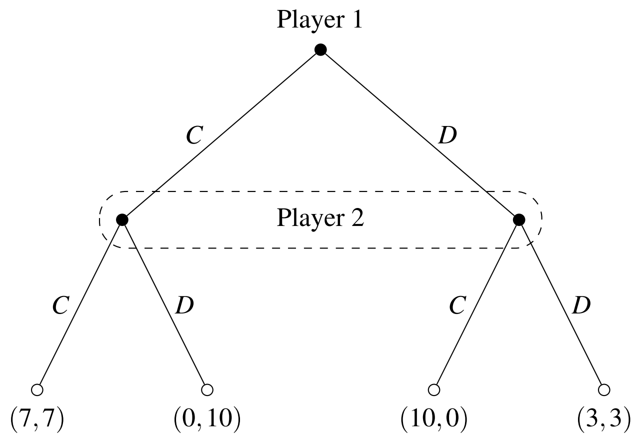


Figure 4.1 **Example of a game (*Prisoner's dilemma*) in extensive form**. C and D are the two actions

4.4 Nash equilibrium

In this section we will look into various classic games in normal form. Consider a game involving two players, player 1 and player 2. Each of them has two available actions, which we call A and B. If both player 1 and player 2 choose A, each of them get 2, whereas if both choose B, they each get 1. If they choose different actions, they each get a payoff of 0. This is a **coordination game** and may be represented as follows, where player 1 chooses column and player 2 chooses a row and the resulting payoffs for each players are listed in parenthesis, where the first element corresponds to player 1 – see Table 4.1. The action outcome (B,B) is

Table 4.1 **Coordination game in normal form**

		Player 1	
		A	B
Player 2	A	$(2,2)$	$(0,0)$
	B	$(0,0)$	$(1,1)$

a 'stable' strategy. In other words we can say it is an equilibrium, since unilateral deviation to A by any of them would result in a lower payoff for the deviating player. Similarly, the action outcome (A,A) is also an equilibrium. This is an example of a pure strategy Nash equilibrium. In general, a Nash equilibrium is defined as follows.

Nash equilibrium in pure strategies A pure strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_n^*)$ is a Nash equilibrium solution if, for all $s_i \in S_i, i \in N$:

$$f_i(s_{-i}^*, s_i^*) \geq f_i(s_{-i}^*, s_i) \tag{4.1}$$

Intuitively, a pure Nash equilibrium is a specification of a strategy for each player such that no player would benefit by changing his strategy unilaterally, i.e. provided the other players don't change their strategies.. The Nash equilibrium provides a very important analysis method for game theory, but does not address how players achieve that equilibrium.

Consider another example:

Matching pennies This game involves two players, each with two actions available. Each of them can choose either heads (H) or tails (T). Player 1 wins a coin from Player 2 if their choice of outcome are the same, and Player 1 loses a coin to Player 2 is they are not; see Table 4.2. This game has no pure-strategy Nash equilibria. If we play this game, we should be

Table 4.2 **Payoff matrix for *Matching pennies* game**

		Player 1	
		H	T
Player 2	H	$(1,-1)$	$(-1,1)$
	T	$(-1,1)$	$(1,-1)$

"unpredictable". That is, we should randomize (or mix) between strategies so that we do not get exploited. Suppose Player 1 plays 0.75 heads and 0.25 tails (i.e. heads with 75% chance and tails with 25%). Then Player 2 by choosing tails (with 100% chance) can get an expected payoff of $(0.75 \times 1 + 0.25 \times (-1) = 0.5)$, but that cannot happen at equilibrium since Player 1 then wants to play tails (with 100% chance) deviating from original mixed strategy. Since this game is completely symmetric, it is easy to see that when players are allowed to use a mixed strategy, at least one Nash equilibrium is guaranteed to exist. Specifically (50%, 50%) mixed strategy, is the only mixed Nash equilibrium for this game.

Nash equilibrium in mixed strategies A mixed strategy $\sigma^*(s) = (\sigma_1^*(s_1), \sigma_2^*(s_2), \ldots, \sigma_n^*(s_n))$ is a mixed-strategy Nash equilibrium if, for $i \in N$, there are

$$f_i(\sigma_{-i}^*(s_{-i}), \sigma_i^*(s_i)) \geq f_i(\sigma_{-i}^*(s_{-i}), \sigma_i(s_i)) \tag{4.2}$$

In other words, all players cannot benefit by deviating unilaterally from a Nash equilibrium.

A few variants of the basic coordination game defined by Table 4.1 are worth mentioning.

Battle of sexes Imagine a couple agreed to meet but they cannot recall if they will be attending the opera or football match. The Player 1 (husband) prefers to go to football whereas Player 2 (wife) would rather prefer opera. Both would prefer to go to together. If they cannot communicate, what would be their decision? The payoff matrix of the scenario is given in Table 4.3. There are two pure Nash equilibria in this case, represented by asterisks.

Table 4.3 **Payoff matrix for *Battle of sexes game* (left) and *Stag hunt game* (right)**

		Husband	
		Opera	*Football*
Wife	*Opera*	$(3^*, 2^*)$	$(0,0)$
	Football	$(0,0)$	$(2^*, 3^*)$

		Player 1	
		Stag	*Hare*
Player 2	*Stag*	$(2^*, 2^*)$	$(1,0)$
	Hare	$(0,1)$	$(1^*, 1^*)$

Stag hunt Suppose that two people are out for hunting. If they work together, they can catch a stag, which is big (would bring highest payoff), but if they work on their own they will catch a hare. The tricky part is that if one hunter alone cannot hunt a stag, he will get nothing. Their strategies are shown in Table 4.3. This is a quite unbalanced coordination game unlike battle of sexes. If they miscoordinate the one who was trying for stag gets penalised more than one who was trying for hare. Here also there are two pure Nash equilibria.

Prisoner's dilemma This is another example of coordination game having one Nash equilibrium – see the payoff matrix Table 4.4. Two subjects (prisoners) have the choice

Table 4.4 **Payoff matrix for *Prisoner's dilemma***

		Subject 1	
		Cooperate	*Defect*
Subject 2	*Defect*	$(0,10)$	$(7^*, 7^*)$
	Cooperate	$(3,3)$	$(10,0)$

between to claim the other subject is innocent (cooperation) and claim other subject is guilty (defection). If both of them cooperate, each will receive a short sentence of 3 years, whereas if both defect they will receive a fairly moderate sentence of 7 years. But if one cooperates and the other defects, the cooperator will receive 10 years of sentence while the defector will be freed. Here it is obvious that the global optimal solution is both subjects to cooperate. However if one decided to defect, the defector will get more advantage at the expense of another subject. In this non-cooperative setting, the pure Nash equilibrium strategy is for both subject to defect (shown by the asterisk). The dilemma arises because here the Nash equilibrium solution is not identical to the globally optimal solution which is cooperative. In extensive form, the *Prisoner's dilemma* game is shown in Figure 4.1.

4.5 Information in games

A complete extensive-form representation specifies:(1) The players of a game. (2) For every player every opportunity they have to move. (3) What each player can do at each of their moves. (4) What each player knows for every move. (5) The payoffs received by every player for every possible combination of moves. Complete information requires that every player know the strategies and payoffs available to the other players but not necessarily the actions taken.

Inversely, in a game with incomplete information, players may not possess full information about their opponents. Some players may possess private information that the others should take into account when forming expectations about how a player would behave. A typical example is an auction: each player knows his own utility function (= evaluation for the item), but does not know the utility function of the other players.

Incomplete information games are studied in the context of Signalling. Signalling is an omnipresent phenomenon in animal and human societies. Examples of signalling in animal kingdom includes odors, pheromones, sounds etc. In humans, the most advanced form of signalling is language undoubtedly. Importantly, we also rely on non-verbal forms of signalling through sensorimotor interactions, for example gestures, facial expressions, interpersonal distance and body orientation (Sebanz et al., 2006; Takagi et al., 2016a). Influence Signalling between self-interested players have become an important issue when one player posses private information about the other partner. Leibfried et al. (2015) investigated how humans behave in a sensory motor version of signalling game with conflicting goals.

4.6 Repeated games

A repeated game is a game in the extensive form that consists of repetitions of the base game (sometimes called a stage game). Repeated games imply that a player will have to take into account the impact of his current action on the future actions of other players; this impact is sometimes called his reputation. Repeated games are broadly divided into two classes, depending on the horizon. The horizon is the player's belief about the number of repetitions of the stage game and may be finite or infinite. In infinite horizon repeated games, players can expect to play indefinitely. In finite horizon repeated games, players expect the game to terminate after playing the stage game some number of times.

4.7 Differential game theory

Differential games are set of problems related to the modelling of conflict in the context of a dynamical system. Especially, a state variable or variables evolve over time according to a differential equation. Differential game theory deals with cooperative or conflict situations in a system is influenced by two or more individuals. Each individual imposes his/her own control to the system in order to gain some utility or sometimes they have to pay costs. A game problem seeks, the controls for all individuals such that each of them achieves his/her own goal, exactly speaking, those controls either minimum or maximum the concerned utility for the players. Since most daily life situations call for making choices of some kind, differential games have a wide range of applications.

Differential games are closely related to optimal control problems. In an optimal control problem, there is single control $u(t)$ and a single criterion to be optimized; differential game theory generalises this to two controls $u_1(t)$, $u_2(t)$ and two criteria, one for each player. Each player tries to control the state of the system so as to achieve the goal.

This section provides background information on differential game theory which will serve as a foundation of the computational model framework described in the Chapter 6. Here I will be discussing various concepts and notions by considering 2-player differential game. A more detailed overview on differential game theory can be found in (Başar and Olsder, 1999).

4.7.1 Problem formulation

A differential game consists of N players, where $N \in \mathbb{Z}^+$. For the case in which $N = 1$ the differential game becomes a problem of standard optimal control. Here we focus our attention on differential games with $N \geq 2$. We will start with the *rope pulling game*, by looking at its cooperative (*nonzero-sum game*) and noncooperative (*zero-sum game*) conditions. We will then define the problem of solving infinite-horizon nonzero-sum differential games.

Rope pulling game A point object can move in a plane which is defined by the standard (x, y) coordinate system (Başar and Olsder, 1999), pp.3-5. Initially, at time $t = 0$, the object is at the origin $(0,0)$. Two forces act on the point object with opposite directions, where one is from Player 1 and the other is Player 2. The directions of forces – measured by angles, counter-clockwise with respect to the positive x-axis – are denoted by u_1 and u_2; see 4.2. Assuming unit mass and unit forces, the system can be described by the following differential equations:

$$\begin{cases} \ddot{x} = cos(u_1) + cos(u_2), \dot{x}(0) = x(0) = 0; \\ \ddot{y} = sin(u_1) + sin(u_2), \dot{y}(0) = y(0) = 0. \end{cases} \tag{4.3}$$

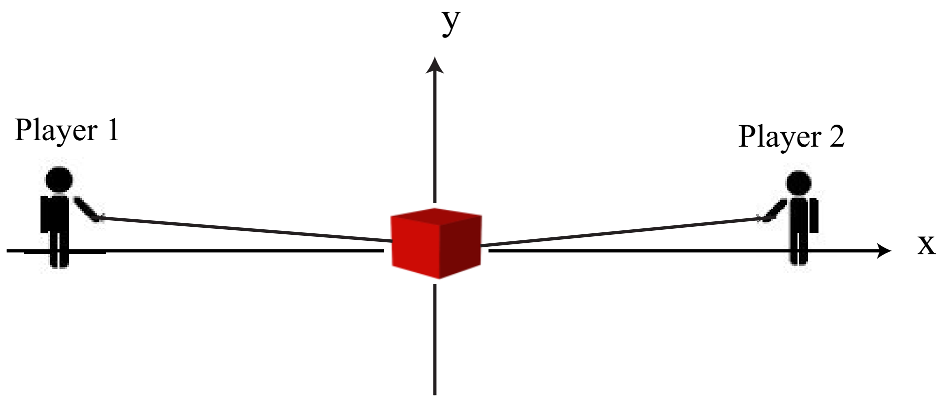


Figure 4.2 ***Rope pulling game***

At time $t = 1$, Player 1 wants to pull the object as far as in the negative x-axis as possible. That is, he wants to minimize $x(1)$. Player 2 would like to pull in the opposite direction of that of Player 1; that is, he wants to maximize $x(1)$. The sum of the utilities of each player is zero. Such a non-cooperative game is called *zero-sum differential game*, and the solution is known as a *saddle point equilibrium*. The solution of this condition follows immediately. Each subjects pulls in his favourite direction. The choice of their action controls are apparently $(u_1, u_2) = (\pi, 0)$. Under these conditions the point object remains at the origin.

Now we change the game slightly. The aim of Player 2 is now to pull the object as far as possible in the negative direction of y-axis; in other words, he aims at minimizing $y(1)$. Instead, Player 1 will maintain his original objective, which is to maximize $x(1)$. This new game is clearly *nonzero-sum*. Now, consider the pair of decisions $(u_1, u_2) = (\pi/2, \pi)$ In this condition, the two players are not in conflict, alternatively, they are of cooperative relationship. The solution here will help both of them to win, is given by , $(u_1, u_2) = (\pi, -\pi/2)$. Their respective payoffs are $L_1 = -1/2$ and $L_2 = -1/2$. If Player 2 sticks to $u_2 = -\pi/2$, the best Player 1 can do is to choose $u_1 = \pi$. Similarly, if Player 1 stays on $u_1 = \pi$, there is no better choice for Player 2 than $u_2 = -\pi/2$. Hence the pair $(u_1, u_2) = (\pi, -\pi/2)$ exhibits an equilibrium behaviour. This kind of solution, where one player cannot improve his utility by changing his choice unilaterally, is called a *Nash equilibrium* solution.

4.7.2 Two-player differential games

Let us consider a dynamical system with state $x(t) \in \mathbb{R}^n$, and the following state-space dynamics:

$$\dot{x} = \bar{f}(x, u_1, u_2) \tag{4.4}$$

where $\bar{f}(x, u_1, u_2)$ is a mapping function and $u_1(t) \in \mathbb{R}^{m_1}$ and $u_2(t) \in \mathbb{R}^{m_2}$,with $m_1 \leq n$ and $m_2 \leq n$ are the control actions of Players 1 and 2. Player 1 aims to select its strategy $u_1(t)$ to minimize the cost functional of this form:

$$J_1[u_1, \ldots, u_N, T] = \int_0^T q_1(x, u_1, u_2)dt + r_1(x(T)) \tag{4.5}$$

where $q_1(x, u_1, u_2)$ is the running cost and $r_1(x(T))$ is the terminal cost respectively for Player 1. Similarly, Player 2 seeks to minimize the cost functional:

$$J_2[u_1, \ldots, u_N, T] = \int_0^T q_2(x, u_1, u_2)dt + r_2(x(T)), \tag{4.6}$$

where $q_2(x, u_1, u_2)$ is the running cost and $r_2(x(T))$ is the terminal cost respectively for Player 2. This problem is known as a *two player finite-horizon differential game*. Here two players must determine their strategies $u_1()$ and $u_2()$ in order to minimize two cost functionals subject to the state dynamics – Equation 4.4. The game theoretic solution of our modelling framework for physical human-human interaction described in Chapter 6 is a *two player finite-horizon differential game*.

In the limit as T tends to infinity the problem becomes a *two player infinite-horizon differential game*. In some applications, for example in an economic situation, the duration of the game may be long or even unknown. In infinite-horizon differential games, it is assumed that by means of their respective control strategies, both Player 1 and 2 aim to minimize cost functionals of the form:

$$\begin{aligned} J_1[u_1, u_2] &= \int_0^\infty \bar{q}_1(x, u_1, u_2)dt \\ J_2[u_1, u_2] &= \int_0^\infty \bar{q}_2(x, u_1, u_2)dt \end{aligned} \tag{4.7}$$

Note that there is no terminal cost term in the cost functionals for the infinite-horizon differential game and it must be taken with care to ensure these cost functionals are bounded.

What if the players in a two player game are in competition, or those cost functionals of the players define a conflicting scenario? Take the non-cooperative condition of above mentioned *rope pulling game*. For this case, when the sum of the cost functionals of two player sis zero, *i.e.* $q_1(x,u_1,u_2) = -q_2(x,u_1,u_2)$, a gain of Player 1 implies a loss for Player 2 and vice versa. Differential games that fall within this categories are called *two player zero-sum differential games*.

In a classical optimal control problem the optimal control is such that the cost functional minimised subjected to the dynamics of the system. For a differential game, on the other hand, optimality concept is not straight forward as intuitive. Consider , an example , a two-player differential game with set of strategies given by $S_1 = \{u_1,u_2\}$ and $S_2 = \{w_1,w_2\}$ assume that the set of strategies S_1 is favourable for player 1 and S_2 is favourable for player 2. Here clearly, in contrast to what we see in a classical optimal control problems, it is not straight forward to determine which set of strategies is better than the other. Thus here different notion of optimality solutions for differential games must be introduced.

The definition of optimality for differential games is not unique and many different solution concepts exist – for example, cooperative conditions and noncooperative condition are treated differently. In the non-cooperative case, the optimal solution is represented by the *Nash equilibrium strategies*.

We specifically focus on closed-loop or feedback Nash equilibrium strategies, which are formalised in the following definitions:

Definition 1. A pair of state feedback control strategies $S = \{u_1,u_2\}$ is said to be fit for the noncooperative differential game – defined by Equations 4.4, 4.7 – if the zero-equilibrium of the system – Equation 4.4 – in closed-loop with S is locally asymptotically stable.

Definition 2. State feedback control strategies given by u_1^* and u_2^* are said to be *Nash equilibrium strategies* of Player 1 and Player 2 , for the non-cooperative differential game of Eq. 4.4- 4.7 if the pairs of strategies $S^* = \{u_1^*,u_1^*\}$ satisfy the following inequalities:

$$\begin{aligned} J_1[u_1^*,u_2^*] &\leq J_1[u_1,u_2^*] \\ J_2[u_1^*,u_2^*] &\leq J_2[u_1^*,u_2] \end{aligned} \tag{4.8}$$

for all admissible feedback strategy pairs $S_1 = \{u_1,u_2^*\}$ and $S_2 = \{u_1^*,u_2\}$, where $u_1 \neq u_1^*$ and $u_2 \neq u_2^*$.

The Nash equilibrium strategies of a differential game are such that if any player deviates from his Nash strategy, while assuming all other players are rational, this results in a loss for the deviating player. This does not imply that the players cannot perform better by changing to different strategies. In fact, strategies that give best outcomes for both players are known as *Pareto-optimal strategies*. Chapter 6 will give a detailed overview of our differential game theory based modelling framework that simulate dyad behaviour in a joint motor action.

4.7.3 Two-player linear-quadratic differential games

This is a category of differential games in which the system satisfies linear dynamics and the cost functionals, which each players seek to minimise are quadratic. Linear quadratic differential games are comparable to its optimal control counter part, linear-quadratic regulator problems. The optimal solution for such a problem relies on the solution of an algebraic Riccati equation.

Consider a two-player differential game in which the system (4.4) is linear, given by he system dynamics:

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2 \tag{4.9}$$

where x, u_1, u_2 state and the players' strategies as defined before. $A \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{n \times m_1}$ and $B_2 \in \mathbb{R}^{n \times m_2}$ are constant matrices. Also, the players aim to minimise the following cost functionals, are quadratic in the state variable and the control strategies:

$$\begin{aligned} J_1 &= \frac{1}{2} \int_0^\infty (x^T Q_1 x + u_1^T R_{11} u_1 + u_2^T R_{12} u_2) dt, \\ J_2 &= \frac{1}{2} \int_0^\infty (x^T Q_2 x + u_1^T R_{21} u_1 + u_2^T R_{22} u_2) dt. \end{aligned} \tag{4.10}$$

where $Q_1 \geq 0, Q_2 \geq 0$ and weighting matrices satisfy $R_{11} = R_{11}^T \geq 0, R_{22} = R_{22}^T \geq 0$. This form of differential games are known as *two-player nonzero-sum differential games.*

4.8 Learning in games

In a game or in a multi-agent system, agents learn their behaviour or adapt based on their experience with the environment and other agents. The amount and rate of learning depends not only on the learning method, but also on how much information is available to the agents.

In game theory the traditional explanation for why and when equilibrium arises is that it results from analysis and self-examination by the players in a situation where the rules of the game, the rationality of the players and the payoffs of players are all common knowledge.

4.8.1 Fictitious play

Classic models in game theory are usually equilibrium models that predict Nash equilibria or its derivatives. In evolutionary game theory, this problem is addressed by developing dynamic learning based models that converge to the equilibria. *Fictitious play* is one of the simplest yet powerful and earliest learning rules, introduced in Brown (1951). It is a "belief-based" learning rule, i.e., players form a belief about their opponent (from the history of past play) and behave rationally with respect to these beliefs.

In a two player game, it works as follows. Assume that players $i = 1,2$, play the game at times $t = 1,2,\dots n$. Define $\eta_i^t : S_{-i} \rightarrow N$ to be number of times Player i has observed the strategies of his partner s_{-i} in the past, and let $\eta_i^0(s_{-i})$ be the starting point (or fictitious past). For example, let $S_2 = \{A,B\}$. If $\eta_1^0(A) = 2$ and $\eta_1^0(B) = 6$, and player 2 plays A,A,B in the first three periods, then $\eta_1^3(A) = 6$ and $\eta_1^3(B) = 4$.

Given the Player i's forecast/belief about his opponents play, he chooses his action at time t to maximise his payoff, that is:

$$s_i^t \in \arg\max_{s_i \in S_{-i}} g_i(s_i, \mu_i^t) \tag{4.11}$$

Shen and Cruz Jr (2007) implemented a form of fictitious play to learn Nash strategies in 2-player linear quadratic discrete-time games with scalar control variables. They assumed that each player uses conventional adaptive control methods to estimate the partner's controller.

4.9 Game theory in behavioural research

The use of innovative experimental methods has allowed researchers to begin to study behavioural functions of the brain as players interact with one another while playing games with real consequences. These games helped to understand the decision making process, in particular the extend to which social motives are important in behavioural and sensorimotor decisions, also the process that may underlie demonstrations of competition or cooperation.

Behavioural game theory is explicitly meant to predict how human subjects behave. Basically it has four components: representation, preferences over outcome, initial conditions and learning. Perception or mental representation of game is the key aspect in strategic interaction. Often players in a game perceive an incomplete representation of the game or some elements of the game may be ignored to avoid computational complexity. This aspect of game is explained by complete or incomplete information game in the previous section.

Social preferences over the outcome imply, when the payoffs in a game are measured, to fully understand the game, a theory is needed to realise preferences over payoff distributions.

When a game is played repeatedly, players can learn from payoffs they get and from the strategies other players choose, also they can learn about what other players likely to do. Many models of this learning process have been proposed *fictitious play*, explained in the previous section is one of them. The general structure of learning models is that strategies have numerical attractions that are updated based on payoffs and actions of the opponent.

Behavioural game theory must be complemented with adequate psychological, neural evidence and motor control based studies. Some examples are described in Section 2.5 of Chapter 2.

The application of game theory in multi-agent motor interaction is becoming more promising and studied over the last few years. A number of studies have recently begun to examine how game theory might be used to analyse the neural architecture of active agents when competitive behaviours are produced. McCabe et al. (2001) initially studied this approach through brain imaging. They examined the brain regions of subjects engaged in a strategic game using functional magnetic resonance imaging (fMRI). In their experiment, the subjects played a two-person game called 'trust and reciprocity'. The game starts with one player who must decide whether to terminate game immediately – in that case, both of them will receive a 45 cents cash reward – otherwise, he/she can decide to turn control of the game over to his partner. If control passes to the partner, then he/she must decide between taking all of a larger share of 405-cents gain for himself or keep only 225 cents and returning 180 to his partner. This conflict if particularly interesting for a game theoretic point of view when subjects face a new opponent on each iteration. In those circumstances, if player two is rational, given the chance, he/she always take all for himself/herself. Here basically cooperating with player one offers no advantage. Player one knows this and should, therefore, always be compelled to end the game at the beginning, which at least guarantees him/her a small but decent outcome.

When opponents encounter each other repeatedly, nevertheless a different optimal strategy emerges. What they found was that a typical player even when he told that he would face a different opponent on text trial, he was very likely to cooperate with a human opponent. Naturally, humans turned out to be more cooperative with their peers than strictly rational, almost as if their brains were learning that assumed their opponent would sooner will encounter again. Notwithstanding, when players were told that they faced a computer opponent, they tend to take a more purely rational approach and they never cooperated. McCabe et al. (2001) found when studying the brains of their subjects over these conditions that when a subject opt to cooperate, a region in the prefrontal cortex was more activated than when they act rationally against a computer player. This suggests on a very rough approximation that frontal areas responsible for game theoretic evaluative choices.

4.9.1 Game theory within theory of minds

Theory of mind (ToM) is a great evolutionary human brain achievement. It is considered as a special type of intelligence that can understand not only one's own beliefs and desires, but also others'. At the core level, this is an ability to assign mental states, belief, desires, knowledge, intent etc. Recent interest in understanding the computational basis of ToM has motivated neuroimaging studies. ToM studies have constantly evidenced the involvement of brain networks comprising the medial pre-frontal cortex (mPFC), posterior superior temporal sulcus (STS) and sometimes also temporal poles (TP) – see for a review, cognition in social setting by (Frith and Singer, 2008).

Although the term 'theory of mind' is rarely seen in the economics literature, the foundations of modern economic theory contain a powerful and elegant mathematical description of ToM. Game theory is basically a theory of social interaction, which can be applied in understanding sociological and psychological behaviours as well. Therefore ToM and its implications can be discussed in the context of game theory. Game theoretic frameworks need a complete , explicit description of people's internal states of mind, including own beliefs and beliefs about the mental states of their opponents.

Harsanyi (1968) initiated most of the effort to understand ToM, by analysing games in which players were lacking information about significant details of the game structure. Harsanyi found – sometimes called *Harsanyi doctrine* – that a complete description of such a game requires explicit representation of an infinite reasoning of reciprocal beliefs. He stated that introduction of a parameter that is unknown to a player necessitates a description of his/her probabilistic beliefs about that unknown parameter.

Aumann (1987) gives a serious account of how the players' internal states of mind seek to provide foundations of equilibrium concepts. An important problem in this regard is: what do players need to know (about game strategy, about opponents, about knowledge of others) for their strategies to constitute an equilibrium? Possibly the best powerful answer to this is correlated equilibrium, a generalisation of Nash equilibrium allowing correlation of in strategies.

4.10 Conclusions

Game theory is useful for developing a precise mathematical model of joint action by linking strategy combinations to payoffs, can be considered as a periodic table of the elements of social life. So far, there has been very limited use of game theoretic models in and experimental tools to study joint action and strategic thinking. This limited contact probably is due to the facts that psychologists and neuroscientists have not used the major tools in game theory, which may be due to the skepticism that the rationality based analysis done in game theory are psychologically inaccurate.

When it comes to study sensorimotor joint action, there is a need of adequate experimental apparatus as well. This chapter gave a brief foundation to Differential Game Theory and learning model, used in the modelling framework explained in Chapter 6.

Behavioural Game theory has progressed rapidly since the term was first proposed 10 years or so ago. It extends the empirical accuracy and cognitive plausibility of game theory, expressing strategic situations in mathematical models that permit rapid progress. One promising point of contact is between ToM and theories of strategic thinking thought to be necessary for understanding desires, beliefs and thoughts of other people.

Game theory could be useful in understanding some psychiatric disorders such as Autism, I study sensorimotor joint action in adults with Autism reported in the Chapter 8.

Part II

Methods

Chapter 5

Experimental apparatus and task[1]

> Somewhere, something incredible is waiting to be known.
>
> Carl Sagan

5.1 Introduction

The main goal of this work is to investigate the mechanisms of joint action during collaborative tasks. Many studies (Braun et al., 2009; Ganesh et al., 2014; Melendez-Calderon et al., 2011; Reed and Peshkin, 2008; Takagi et al., 2016a) use motor tasks based on dual robotic interfaces to study joint action.

For instance, Reed and Peshkin (2008) developed a two-handled crank mechanism, which allows studying point-to-point reaching in a simplified control environment. Their mechanism equipped with a central actuator, which allows them to replace one partner by emulating by simulating a corresponding control strategy using computer simulation. Feth et al. (2009) studied more complex protocols based on lateral arm movements using two linear actuators with dedicated displays placed in front of the partners. They particularly studied how subject pairs in a dyad collaborate in tracking a laterally moving target. Stefanov et al. (2009) similarly used the same system to study the evolution and switching of roles each partner take during a collaborative task. Ganesh et al. (2014) have used a robotic interface to study sensorimotor joint action in a tracking task. In their experimental session there

[1]A part of this chapter has been published as Chackochan, V.T., Tamagnone, I and Sanguineti, V (2015) Development of collaborative strategies in physical human-human interaction. Society for Neuroscience Annual Meeting, Chicago.

are equal number of unconnected solo trials and connected dual trials. In the solo trials each individual performed the tracking alone while in the dual trials the robotic interface implemented a simulated virtual spring that physically connect the hands of individuals in a dyad during the tracking task. Takagi et al. (2016a) points that social factors such gaze can cause altered behaviour in joint motor task. They suggest the use of curtain to mitigate the such effects while investigating motor adaptation in joint motor interaction.

These studies have given us insights on the way humans interact at the level of haptic interaction, but still far from understanding the mechanisms of cooperative strategies in conflict scenarios.

We developed an experimental setup which allows to manipulate all aspects of interaction, by specifying what each agent knows about his/her partner's goal and current actions. This is described in section 5.2.1.

5.2 Experimental set-up

5.2.1 Apparatus

We developed an experimental set-up based on two identical 3DoF haptic interfaces (Novint Falcon), each with a dedicated 19-inch LCD monitor for the presentation of visual information.

The Falcon device (Martin and Hillier, 2009) is a relatively inexpensive three degrees of freedom haptic interface, originally manufactured by Novint for the game industry (`www.novint.com`). The mechanical architecture is based on the Delta parallel robot design, originally introduced by Raymond Clavel (Clavel, 1990, 1991), which consists of three arms connected to universal joints at the base. The key design feature is the use of parallelograms in the arms, which maintain a constant orientation of the end effector and limit its motion to translation in all three dimensions. Due to its low inertia, high stiffness, high power-to-weight ratio, high payload capability, this configuration has proven itself as an efficient platform for operations involving high velocity pick-and-place. Complete technical specifications of the Novint Falcon robot are summarised in the Table 5.1.

The terminal effector can be replaced with different handles, such as a pistol for action games, a pen for writing exercises, or a simple spherical manipulator to generate trajectories in 3D space. This has allowed the application in areas outside the gaming industry, such as robotics and haptics, also thanks to the affordability of this device. It was used as a force-feedback tele-control device (Jin, 2010), as learning aid for visually impaired kids

Table 5.1 **Technical specifications of the Novint Falcon haptic interface**

3D workspace dimensions	10.16 cm $\times$10.16 cm $\times$10.16 cm
Maximum force	$\sim$ 9 N
Resolution	$<$ 0.0635 mm
Communication interface	USB 2.0
Device dimension	22.86 cm $\times$22.86 cm $\times$22.86 cm
Weight	$\sim$ 2.7 kg
Maximum Power	30 W
Power supply	100V-240V AC, 50 Hz-60 Hz, 30 V DC

(Murphy and Darrah, 2015) and as a robot for home-based stroke rehabilitation (Borghese et al., 2013).

The Falcon device is connected with the control computer through a USB interface. The commands sent by the computer through the interface are interpreted by the integrated firmware. Similarly, the data recorded by the optical encoders are transmitted back to the control PC. This interface uses a nominal refresh rate of 1 kHz, with commanded forces maintained by the firmware for about 100 ms. Practically, the 1 kHz sampling rate cannot be sustained over the USB interface, which results in an actual sampling rate of 800 Hz-1kHz.

The experimental apparatus is depicted in Figure 5.1. Each participant sat in front of the computer screen and grasped the handle of the haptic interface. They could not see or hear each other and were not allowed to talk. The subjects were instructed to perform reaching movements in the vertical plane, in which the apparent endpoint inertia is more anisotropic.

5.2.2 Software application

A generic application framework was implemented using CHAI3d (Computer Haptics and Active Interface), is a collection of graphics and haptic libraries based on C++ language intended for computer haptics, visualization and interactive real-time simulation. CHAI3d was originally developed in 2003 at the Robotics and Artificial Intelligence Laboratory of Stanford University (Conti et al., 2003). CHAI3d provides a simple yet powerful development environment that would favour haptics-based psychophysical experiments. With subsequent contributions from EPFL in Switzerland and the University of Siena in Italy, in 2004 the first public version of CHAI3d was released.

The CHAI3d libraries are freely downloadable with examples which could be compiled and executed with Visual Studio 2010 or above. They make use of the underlying low-level graphics library, OpenGL (2.1 or above). They also have separate threads for graphics and haptics loops as explained in next section.

Figure 5.1 **Experimental apparatus for sensorimotor joint action studies**

The software was run on a dual core, 2.2 GHz, Pentium 4 computer running Windows 7.

The software application was programmed to have easy interoperability. The schematic of the software architecture is shown in Figure 5.2. The haptic thread reads encoder information from the haptic interface, performs computation, communicates with the graphics process, and sends motor torques information back to the haptic interface. . Furthermore, as haptic application's refresh rate falls, the system becomes more prone to instability and constraint violation. For this reason, the haptic thread runs at higher priority than graphics thread. To ensure real-time performance, all computations in the haptic thread have to complete within 1 ms (corresponding to a sampling rate of 1 kHz).

The graphics thread is a user-space process that can be run on the same system as the real-time haptics interface which is running at a lower priority than the graphics thread. This thread takes care of user interface that allows visualisation of haptic interaction. Since this thread runs on the user side, updates of the graphics are not deterministic. This thread generally requires low refresh rates (around 60 Hz are sufficient) for graphic rendering and

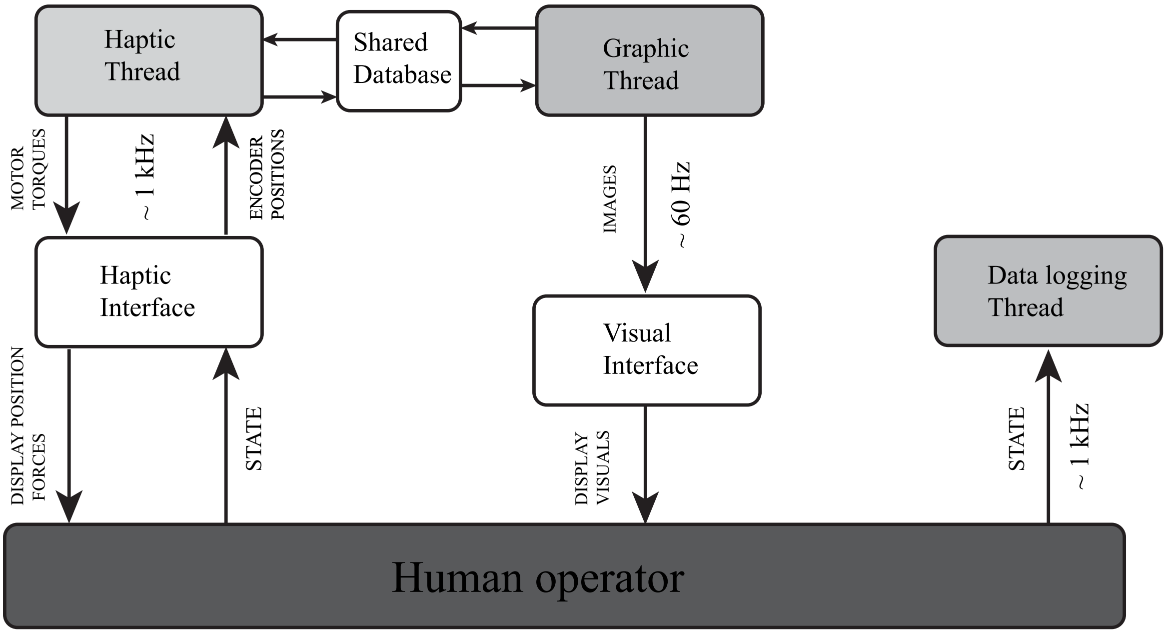


Figure 5.2 **Schematic diagram of the software application**. In the multi-threading structure, both the haptic and the graphic loops share the same database. In this structure, the synchronization of the two loops in accessing the data is important. A separate data logging thread runs in parallel to the haptic thread

have been achieved for complex visual presentation with reasonable hardware requirements. The GUI is implemented with Visual Studio 2010. The visual task is rendered using OpenGL 2.5 using GLUT library.

Data logging is another task that requires high latencies (often over 10 ms), in particular when the bandwidth is high. For my experiment scenarios with a dual-haptic interface we need constantly logging data to disk. It is essential to place a data logging thread that is distinct from the haptic rendering thread with the same priority. We implemented data logging in a separate thread that uses an efficient data structure known as "blocked linked list" (BLL) (Morris, 2006).

5.3 Task

5.3.1 A sensorimotor *Battle of sexes* game

We designed a novel interactive learning paradigm, derived from the classical 'battle of sexes' game, in which subjects have different preferred targets and also a preference to stay together.

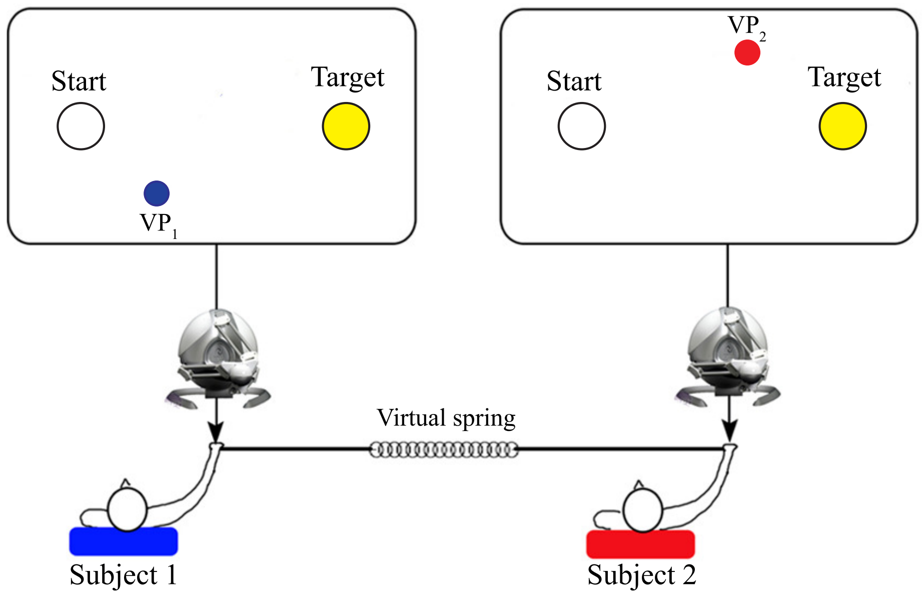


Figure 5.3 **Experimental protocol of the sensorimotor *Battle of sexes* game**

In our task, two subjects are mechanically connected but cannot see each other. Both are instructed to perform reaching movements with the same start and end positions, but through different via-points (VP). Both are also instructed to keep the interaction force as low as possible during movement, see Figure 5.3. Subjects had the option of establishing a collaboration - negotiating a path through both VPs, which would lead to a minimisation of the interaction forces – or to ignore each other, by only focusing on their own goal.

In Chapter 7, we manipulated the information available on partner's actions by providing it either haptically, through the interaction force (haptic group, H); by additionally displaying the interaction force vector on the screen (visuo-haptic group, VH); or by displaying the partner's trajectory (partner-visible group, PV).

The experimental protocol is described in more detail in the next Chapters.

5.3.2 Performance indicators

During each trial we measured the movement trajectories of both partners, $p_1(t)$ and $p_2(t)$, and the interaction forces, $F_1(t)$ and $F_2(t) = -F_1(t)$. From the trajectories we also calculated the partners' velocities, $\dot{p}_1(t)$ and $\dot{p}_2(t)$. From these data we then calculated a number of performance indicators, at either dyad or individual subject level – see Figure 5.4.

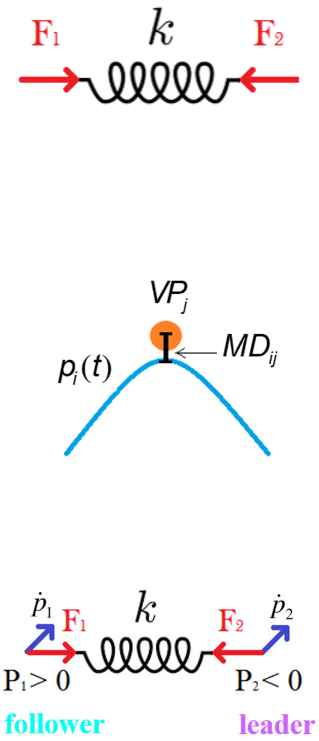


Figure 5.4 **Summary of performance indicators**. Top: interaction force. Middle: Minimum distance to partner's via-point. Bottom,: Leadership index

Dyad-level

Interaction force The average interaction force (IF) is an overall measure of the extent to which the two agents share the same trajectory when they are mechanically connected. IF is defined as:

$$IF = \frac{1}{T}\sum_{t=1}^{T} ||F_i(t)|| \tag{5.1}$$

where $F_i(t)$ is the interaction force at time t; see Figure 5.4, top.

Subject-level

Minimum distance from partner's VP The minimum distance of subject i from the j-th VP is calculated as:

$$MD_{ij} = \min_t \sqrt{(p_i^x(t) - p_{VPj}^x)^2 + (p_i^y(t) - p_{VPj}^y)^2} \tag{5.2}$$

When $i \neq j$ this indicator represents, for each subject, his/her ability to cross his/her partner's VP; see Figure 5.4, middle.

Leadership index Looking at the power developed by each subject would provide information on whether the subjects move actively, or are passively pulled by their partner through the mechanical coupling.Individual agents at a given time may behave as 'leaders' or as 'followers'. Similar to (Stefanov et al., 2009), we defined a subject's index of leadership in terms of the power associated to the interaction force, defined as: $P_i(t) = F_i(t)^T \cdot \dot{p}_i(t)$; see Figure 5.4, bottom.

$$\mathrm{LI}_{ij} = \frac{1}{\tau} \sum_{t=T_{vp}-\tau}^{T} P_i(t) \tag{5.3}$$

We reasoned that, at a given time, a negative power would mean that the subject is controlling his/her motion (i.e. he/she is behaving as a 'leader'). Conversely, a positive power would indicate that the subject is being pulled toward the other (i.e., he/she is behaving as a 'follower').

We specifically focused on the average power calculated in the 300-ms interval taken just before and just after the crossing of each via-point. We denote as LI_{ij} this value for the i-th subject and the j-th via-point.

5.4 Conclusions

This chapter presents a dual-haptic interface to investigate sensorimotor joint action between two humans. A generic, modular software is developed using CHAI3d library, which allows us to implement and save experimental behavioural tasks having moderate graphical requirements. We introduced a sensorimotor version of a classic game *Battle of sexes* in this Chapter. Various performance indicators have been proposed to validate the joint action experiments. We present the experiments in detail in Chapter 7 and Chapter 8. In particular, Chapter 7 addresses joint motor action experiments with healthy individuals. Chapter 8 focuses on individuals with Autism Spectrum Disorder.

Chapter 6

Computational model[1]

We do not describe the world we see, we see the world we can describe.

Rene Descartes

6.1 Introduction

Different from most previous studies – see Chapter 2 – we are specifically interested in situations in which the two interacting agents have slightly different goals. This chapter describes a computational framework to model joint actions that involve situations in which – different from most previous studies – the two interacting agents have slightly different goals. This situation can be modelled using a particular form of game theory – differential game theory – in which the agents' goals are described by two separate cost functionals and their strategies are described by two feedback controllers (Başar and Olsder, 1999). Differential game theory represents a generalisation of optimal feedback control, a well established theory of sensorimotor control (Todorov and Jordan, 2002) which describes actions in terms of the trade-off of performance and effort. Here this notion is extended to the joint optimisation of the individual agents' goals.

In this Chapter we define a general modelling framework, which we then use to run computer simulations of the interaction experiments. The purpose of simulations is to characterise scenarios in which each partner autonomously determines his/her actions, based

[1]An earlier version of this chapter has been published as Chackochan, V.T. and Sanguineti, V (2017) Modelling collaborative strategies in physical human-human interaction. Biosystems and Biorobotics 15:253-258.

on a variety of assumptions about his/her knowledge about the dyad, the task and the partner. Specifically, we focus on two separate objectives.

First, we use the general game-theoretic framework to provide predictions for the optimal dyad behaviours. We compare two different scenarios: (i) each partner has a perfect knowledge of their partner's control policy – this corresponds to the *Nash* strategies; and (ii) each player completely ignores their partner when determining his/her control policy – these will be referred as *optimal no-partner* strategies.

Second, we develop a model, based on fictitious play, of how the agents develop a joint coordination as a result of repeated task performance. We use this model to assess how lack of information about the partner affects the learned strategy.

6.2 Modelling framework

6.2.1 Dyad dynamics

We assume there is one single plant, reflecting dyad dynamics – i.e. both agents' body dynamics and their mechanical interaction. The dyad state trajectory, $x(t)$, is determined by both partner's control commands, $u_1(t)$ and $u_2(t)$. We approximated plant dynamics as a discrete-time linear dynamical system with two inputs:

$$x(t+1) = A \cdot x(t) + B_1 \cdot [u_1(t) + \eta_1(t)] + B_2 \cdot [u_2(t) + \eta_2(t)] \tag{6.1}$$

The state variable $x(t)$ accounts for position, velocity and muscle activation dynamics of both partners. Both inputs are affected by process or 'motor' noise, assumed to be Gaussian and zero-mean: $\eta_i(t) \sim N(0, \Sigma_i^\eta)$, with $i = 1,2$. Model parameters Σ_i^η, $i = 1,2$ reflect the non-deterministic component of plant dynamics which is due to motor command uncertainty.

6.2.2 Control model

Consistent with the optimal control model of sensorimotor control (Todorov and Jordan, 2002), we completely specified the agents' goals by a pair of cost functionals:

$$\begin{cases} J_1[u_1, u_2] = \sum_{t=1}^{T} \left[x(t)^T \cdot Q_1(t) \cdot x(t) + u_1(t)^T \cdot R_1(t) \cdot u_1(t)\right] \\ J_2[u_1, u_2] = \sum_{1=1}^{T} \left[x(t)^T \cdot Q_2(t) \cdot x(t) + u_2(t)^T \cdot R_2(t) \cdot u_2(t)\right] \end{cases} \tag{6.2}$$

Assuming that the two agents have a perfect knowledge of the plant state, the dyad's control system can be modelled as a pair of feedback controllers, one per each agent:

$$u_i(t) = -L_i(t) \cdot x(t) \tag{6.3}$$

with $i = 1,2$. This is called a linear-quadratic discrete-time differential game (Başar and Olsder, 1999).

We assume that each agent autonomously determines his/her own control policy, $L_i(t)$ with no explicit agreement with his/her own partner - which in game theory is called a non-cooperative scenario.

The controller gains can be calculated on the basis of the assumptions that each agent makes on his/her partner. We specifically focused on two situations.

Nash controllers

The optimal non-cooperative solution is known as Nash equilibrium (Nash, 1951). A pair of control policies i $[u_1^*, u_2^*]$ is a Nash equilibrium if none of the agents can achieve a lower cost magnitude by unilaterally changing his/her own control policy:

$$J_1\,[u_1^*, u_2^*] \leq J_1\,[u_1, u_2^*] \;\; \forall u_1 \neq u_1^* \tag{6.4}$$

and similarly

$$J_2\,[u_1^*, u_2^*] \leq J_2\,[u_1^*, u_2] \;\; \forall u_2 \neq u_2^* \tag{6.5}$$

The definition of a Nash equilibrium is depicted in Figure 6.1.

The optimal Nash feedback controllers can be determined through the following iterative algorithm (Başar and Olsder, 1999):

$Z_i(T) \leftarrow Q_i(T)$
for $t \leftarrow T, 0$ **do**
  solve for $i = 1,2$**:**
  $\left[R_i(t) + B_i^T \cdot Z_i(t+1) \cdot B_i\right] \cdot L_i(t) + \left[B_i^T \cdot Z_i(t+1) \cdot B_{-i}\right] \cdot L_{-i} = B_i^T \cdot Z_i(t+1) \cdot A$
  $F(t) \leftarrow -B_1 \cdot L_1(t) - B_2 \cdot L_2(t)$
  $Z_i(t) \leftarrow Q_i(t) + F(t)^T \cdot Z_i(t+1) \cdot F(t) + L_i(t)^T \cdot R_i(t) \cdot L_i(t)$
end for

In the above equation and in all the following, i denotes an agent and $-i$ denotes his/her partner.

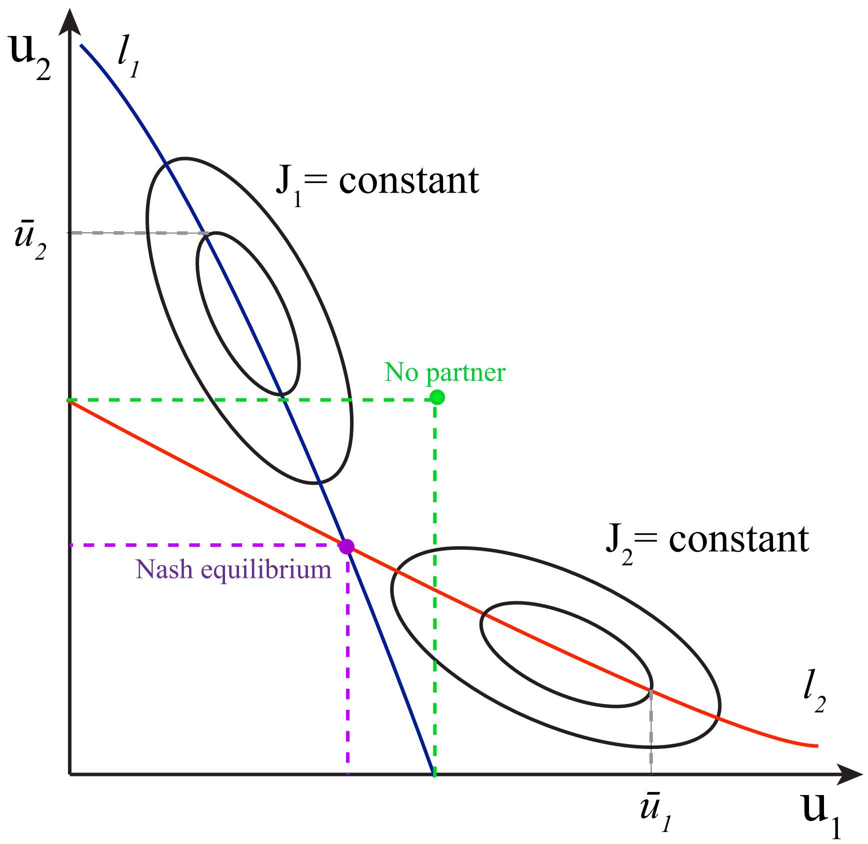


Figure 6.1 **Nash equilibria and 'no partner' solution in a two-agents non-cooperative game**. The two agents have different goals and hence separate cost functionals, J_1 and J_2, depicted as iso-cost curves in the $[u_1,\, u_2]$ (action) plane. For a given u_1, say $\bar{u}_1$, the best agent 2 can do is to choose $u_2 = l_2(\bar{u}_1)$, where l_2 is the agent 2's 'reaction curve' (in red), i.e. the curve defined by $u_2 = \arg\min_u J_2[\bar{u}_1, u]$. Similarly, agent 1 has his/her own reaction curve l_1 (depicted in blue). The set of Nash equilibria is determined by the intersections of l_1 and l_2 – here we assume that the Nash equilibrium is unique. A second (sub-optimal) scenario is represented by the situation in which each agent assumes that his/her partner is inactive. This 'no partner' solution, is determined by the intersections of the two reaction curves with the coordinate axes, adapted from (Başar and Olsder, 1999).

Optimal no-partner controllers

A second (sub-optimal) scenario is represented a the situation in which each agent assumes that his/her partner is inactive, i.e. $u_{-i}(t) = 0$.

In this case, the optimal controllers are calculated independently, as two separate LQG optimal control problems:

$Z_i(T) \leftarrow Q_i(T)$
for $t \leftarrow T, 0$ **do**
$L_i(t) = \left[R_i(t) + B_i^T \cdot Z_i(t+1) \cdot B_i\right]^{-1} B_i^T \cdot Z_i(t+1) \cdot A$
$Z_i(t) \leftarrow Q_i(t) + L_i(t)^T \cdot R_i(t) \cdot L_i(t) + [A - B_i \cdot L_i(t)]^T \cdot Z_i(t+1) \cdot [A - B_i \cdot L_i(t)]$
end for

The above algorithm is separately applied to both agents, i.e. for $i = 1, 2$.

6.2.3 State observer and partner model

The above calculations of the optimal control policy assume that each agent has a perfect information on the plant state vector, $x(t)$, but this is not the case. However, each agent has its own sensory system, described by:

$$y_1(t) = H_1 \cdot x(t) + v_1(t) \tag{6.6}$$

$$y_2(t) = H_2 \cdot x(t) + v_2(t) \tag{6.7}$$

where v_1 and v_2 are zero-mean, Gaussian sensory noise processes: $v_i(t) \sim N(0, \Sigma_i^v)$, with $i = 1, 2$. Eq. 6.7 implies that each agent's sensory system has a (partial and noisy) knowledge of the whole plant state. Model parameters H_i and Σ_i^v, $i = 1, 2$ reflect such knowledge.

In a single-agent situation, agent i may predict the dyad state at time t by combining prior knowledge of dyad dynamics (forward model), including a copy of his/her own motor command, u_i (efferent copy) with his/her own sensory information, $y_i(t)$. The combination of using own sensory feedback and forward model to estimate the current state is known as state observer.

In the case of linear systems with Gaussian noise, the Kalman algorithm is an optimal (Bayesian) solution to this state estimation problem. The posterior estimate of the next state, $\hat{x}^+(t+1)$, has the general structure:

$$\hat{x}^+(t+1) = \hat{x}^-(t+1) + K_i(t+1) \cdot [y_i(t+1) - H_i \hat{x}^-(t+1)] \tag{6.8}$$

The two components of the observer are, respectively, the ‘prediction’ and the ‘correction’ (or ‘innovation’) terms.

In particular, the optimal ‘prior’ prediction of the next state, $\hat{x}^-(t+1)$ – i.e. the estimation obtained before the sensory feedback $y_i(t+1)$ is measured – is given by:

$$\hat{x}^-(t+1) = A\hat{x}^+(t) + B_i u_i(t) \tag{6.9}$$

The Kalman gain $K(t)$, $t = 1, \cdots, T$ of the ‘innovation’ term is determined by the Kalman iterative algorithm and reflects the trade-off of the reliability of the prediction and correction terms. If the prediction term is highly reliable – i.e., if we have a good knowledge about plant dynamics – the innovation term will add little to the state estimation and the Kalman gain will be small. In contrast, if the sensory input is highly reliable the Kalman gain will be large and the state estimation will be largely determined by the innovation term.

However, when there are two agents acting on the same plant, unbiased estimation of the plant state also requires the partner’s input, $u_{-i}(t)$ which is generally unknown. Several authors – among others, Gillijns and Moor (2007) – have proposed general solutions to the problem of joint estimation of input and state when no prior information about the input, thus resulting in a generalisation of the Kalman algorithm.

However, under the reasonable assumption that the partner’s input is smooth, the problem of estimating $u_{-i}(t)$ can be formulated as a simple extension of the Kalman algorithm. The assumption that the partner’s input is smooth can be formalised in the following expression:

$$u_{-i}(t+1) = A_u \cdot u_{-i}(t) + \varepsilon_{-i}(t) \tag{6.10}$$

where $0 < A_u < 1$ and $\varepsilon_{-i}(t) \sim N(0, \Sigma^{\varepsilon}_{-i})$. Eq. 6.10 corresponds to the prior belief that the partner’s input is a low-pass filtered Gaussian noise.

We can now define, for each agent, an augmented state: $X_i(t) = [x(t)\,,\, u_{-i}(t)]^T$. Equations 6.1 and 6.10 can be grouped together as:

$$X_i(t+1) = \begin{bmatrix} A & B_{-i} \\ 0 & A_u \end{bmatrix} \cdot X_i(t) + \begin{bmatrix} B_i \\ 0 \end{bmatrix} \cdot u_i(t) + w_i(t) \tag{6.11}$$

where $w_i(t) = N(0, \Sigma^w_i)$ and $\Sigma^w_i = \mathrm{diag}(B_i \Sigma^{\eta}_i B_i^T, \Sigma^{\varepsilon}_{-i})$.

We then assume that each agent has a state observer for the augmented dynamics of Eq. 6.11. In this way, each agent’s state observer combines information on plant dynamics and on own sensory information to predict both dyad dynamics and partner’s input.

This formulation implies that in joint action each agent has his/her own sensory system, control policy and state observer. The latter also includes an internal representation of the partner's input. We will refer to this as the agent's 'partner model'. In other words, the model assumes that joint action requires that each agent infers what the other partner intends to do.

By varying the parameters A_u and $\Sigma^{\varepsilon}_{-i}$, the above model can be used to capture a variety of situations, ranging from perfect information to no information at all about partner's actions. Therefore, the model provides a general modelling framework but makes no strong assumptions on the actual ability to predict the partner's actions. The model only constitutes a reference to understand the consequences of different assumptions.

The extent to which an agent actually uses information about his/her partner during interaction will be investigated empirically in the next chapters.

The overall control model is summarised in Figure 6.2.

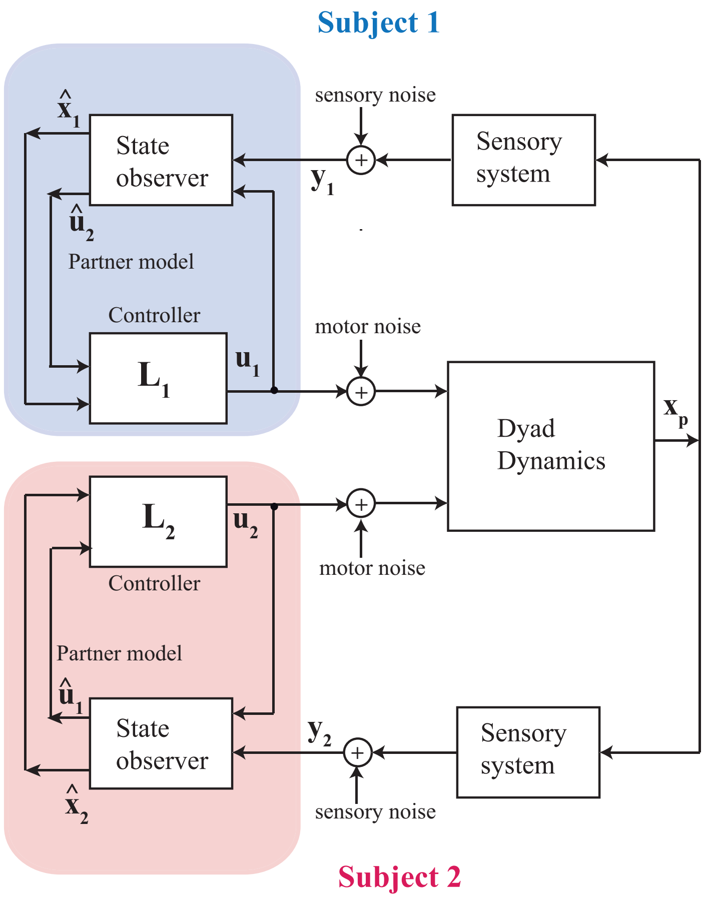


Figure 6.2 **Optimal joint control of a dyad**. Each subject has his/her own sensory system, optimal controller, state observer and partner model

6.2.4 Learning through 'fictitious play'

In the previous sections we introduced a general optimality framework to account for dyad behaviours. In the case of perfect information about the plant, the task and the partner, Nash equilibrium is the predicted optimal behaviour if the two agents act independently in determining their respective control policies (non-cooperative play).

However, Nash equilibria describe the optimal collaborative behaviour but do not tell us how do agents achieve it. Collaborative behaviour is the result of repeated task performance, during which the agents gradually gain knowledge about dyad dynamics, the task requirements, and the partner's actions. This suggests that collaboration, if any, is a result of learning and adaptation.

One possible solution of the problem of iteratively calculating a Nash equilibrium is represented by the classical learning process known as fictitious play or as the Brown-Robinson learning process, originally introduced by Brown (1951) as an algorithm for finding the value of a zero-sum game, and first studied by Robinson (1951). In fictitious play, two agents play the game repeatedly. After arbitrary initial moves in the first round, in every round each agent determines its best response against the empirical strategy distribution of his/her partner.

In fictitious play, strict Nash equilibria are absorbing states (Fudenberg and Levine, 1998). In other words, if at any time period all the players play a Nash equilibrium, then they will do so for all subsequent rounds. Further, if fictitious play converges to any distribution, those probabilities correspond to a Nash equilibrium of the underlying game. Convergence does not occur in general, but many authors have identified classes of games for which such convergence holds; see Berger (2007) for review.

Fictitious play has two basic properties: (i) It is only adequate if the partner uses a stationary strategy; (ii) It does not require that each agent knows the partner's task as it only requires a model of the strategy distribution. In other words, players don't have to know anything at all about their opponent's payoffs. All they do is to form beliefs about how their opponents will play (Fudenberg and Levine, 1998).

Alternatively, players need to incorporate beliefs about opponent's strategies or require players to have a 'model' of the game. While many studies agree that humans can form 'models' of their opponents and/or they 'understand' their intentions, the exact nature of these models remains elusive. Here we use fictitious play as it represents the simplest form of 'partner model'.

Within our modelling framework, we implemented a simplified version of fictitious play by assuming that each agent 'sees' a plant that incorporates partner's input estimated at the previous trial. For agent i, the augmented dyad dynamics is defined as:

$$\begin{bmatrix} x(t+1) \\ x_F(t+1) \end{bmatrix} = \begin{bmatrix} A & B_{-i}\hat{u}_{-i}(t) \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ x_F(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} \cdot [u_i(t) + \eta_i(t)] \tag{6.12}$$

where x_F is a 'dummy' state variable, initialised as $x_F(1) = 1$ so that it remains constant for the whole duration of the movement. If we define $\tilde{x}_i = [x, x_F]^T$ the above can be rewritten as:

$$\tilde{x}_i(t+1) = \tilde{A}(t) \cdot \tilde{x}_i(t) + \tilde{B}_i \cdot [u_i(t) + \eta_i(t)] \tag{6.13}$$

This augmented dynamics can be used to calculate the optimal control policy by using the LQG algorithm described in section 6.2.2.

Our implementation of fictitious play only uses the most recent estimate of partner's input. This is less robust than estimating the distribution of partner inputs over multiple repetitions, but may be adequate for practical purposes.

6.3 Model implementation

To study how joint coordination is influenced by uncertainty about the goals and actions of their partner, we focused on a novel experimental task, a sensorimotor version of classic game *Battle of sexes*, explained in Chapter 4. Partners were mechanically connected through a compliant virtual spring and they have partly conflicting goals – reaching the same target through different via-points.

6.3.1 Dyad dynamics

In our simulated dyad movements, we approximated each agent's upper limb and robot dynamics as a point mass m_i, $i = 1,2$:

$$m_i \ddot{p}_i = f_i + k \cdot (p_{-i} - p_i) - b \cdot \dot{p}_i + m_i g \tag{6.14}$$

where $p_i(t)$ and p_{-i} are the hand position vectors of, respectively, agent i and his/her partner $-i$; m_i is the agent's mass, $f_i(t)$ is the muscle-generated force vector. We also assumed that each agent is subjected to gravity and to a small viscous force accounting for the damping caused by muscles and soft tissue. In all simulations, consistent with the actual experiments – see the next Chapters – we took $m_1 = m_2 = 2$ kg, $b = 10$ Ns/m and $k = 150$ N/m.

As in (Todorov and Jordan, 2002), we modelled the dynamics of muscle force generation as a second order system:

$$\tau^2 \ddot{f}_i + 2\tau \dot{f}_i + f_i = u_i \tag{6.15}$$

where $u_i(t)$ is the activation vector, which is taken as system's input, and τ is the activation time constant. We took $\tau = 40$ ms.

By defining the overall state vector as $x = [p_1^T, \dot{p}_1^T, f_1^T, \dot{f}_1^T p_2^T, \dot{p}_2^T, f_2^T, \dot{f}_2^T]^T$, the dyad dynamics can be rewritten in state-space form:

$$\dot{x} = A \cdot x + B_1 \cdot (u_1 + \eta_1) + B_2 \cdot (u_2 + \eta_2) + G \tag{6.16}$$

where $G = [0,0,0,-m_1 g,0,0,0,0,0,0,0,-m_2 g,0,0,0,0]^T$ is a constant vector which accounts for gravity and η_1 and η_2 are process noise sources (one per subject), assumed to be Gaussian with covariance Σ_i^η.

Eq. 6.16 can be rewritten as:

$$\dot{x} = A \cdot x + B_1 \cdot u_1 + w_1 + B_2 \cdot u_2 + w_2 + G \tag{6.17}$$

where $w_i = B_i \eta_i$, with variance $\Sigma_i^w = B_i \Sigma_i^\eta B_i^T$.

In all simulations we took $\Sigma_i^u = \mathrm{diag}(0.25, 0.25)\,\mathrm{N}^2$, identical for both subjects.

For simulation purposes, the above model equations were discretised by using a first-order hold method, with a sampling rate $\Delta t = 1$ ms over a movement duration of $T = 2$ s.

After model discretisation, we added three extra state variables to store information about the position of the target x_T and of the two via-points, x_{VP_1} and x_{VP_2} so that the new state is: $X = [x, x_T, x_{VP_1}, x_{VP_2}]^T$ – a 22-dimensional vector.

6.3.2 Task and cost functionals

The task (reaching a target through a via-point) was specified in terms of the following cost functionals ($i = 1,2$):

$$\begin{aligned} J_i[u_1,u_2] = & w_p \cdot \|x_T - x_i(T)\|^2 + \\ & w_v \cdot \|\dot{x}_i(T)\|^2 + \\ & w_{vp} \cdot \|x_{VP_i} - x_i(T_{VPi})\|^2 + \\ & w_f \cdot \frac{1}{T} \sum_{t=1}^{T} \|x_{-i}(t) - x_i(t)\|^2 + \\ & r \cdot w_u \cdot \frac{1}{T} \sum_{t=1}^{T} u_i(t)^2 \end{aligned}$$

The cost functional has five terms. The first two terms enforce stopping on target at the end of the movement. The third term reflects the requirement to pass through the via-point. The fourth term accounts for minimising the distance between agents throughout the movement. The last term penalises the effort incurred during the movement.

The weight coefficients determine the relative importance of the corresponding constraint. We set these weights by assuming (Bryson's rule) a maximum acceptable displacement (in the via-point and in the final target) equal to, respectively, the radius of the via-point (2.5 mm) and that of the target (5 mm). The value of the 'velocity' weight was calculated by assuming a maximum acceptable speed at the target of 5 mm/s. We made a similar normalisation in the maximum inter-agent distance (15 mm) and maximum activation (10 N). In all simulations we used the following weights: $w_{vp} = 160000\ 1/\mathrm{m}^2$, $w_p = 40000\ 1/\mathrm{m}^2$ and $w_v = 40000\ \mathrm{s}^2/\mathrm{m}^2$, $w_f = 40001/\mathrm{m}^2$ and $w_u = 0.011/\mathrm{N}^2$.

The scalar coefficient r – the only free parameter in the model – specifies the trade-off between task-related accuracy and effort. With $r \gg 1$, the optimal strategy is not moving at all. With $r \ll 1$, the optimal strategy pays little attention to effort requirements. In all simulations we used r=1.

6.3.3 Sensory systems

Each agent has his/her own sensory system $y_i(t)$, which provides information about the dyad state. Reliability of the sensory information is determined by the magnitude of the sensory noise, $v_i(t)$, assumed to be Gaussian. The sensory system of each agent is described by

Eq.6.7:

$$y_i(t) = H_i \cdot x(t) + v_i(t) \tag{6.18}$$

The structure of the H_i matrix depends on the available sensory information. In the H and VH groups (see previous Chapter) the sensory information is defined as $y_i = [p_i, \dot{p}_i, k(p_{-i} - p_i), x_T, x_{VP_i}]^T$. For subject 1 we have:

$$H_1 = \begin{bmatrix} I_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & I_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ -k \cdot I_2 & 0_2 & 0_2 & 0_2 & k \cdot I_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & I_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & I_2 & 0_2 \end{bmatrix} \tag{6.19}$$

A similar expression is found for H_2.

The subjects in the PV group are assumed to also see their partner's position so that the sensory information is defined as $y_i = [p_i, \dot{p}_i, p_{-i}, \dot{p}_{-i}, k(p_{-i} - p_i), x_T, x_{VP_i}]^T$ and H_1 and H_2 are modified accordingly.

The measurement noise is assumed to be Gaussian with variance:

$$\Sigma_i^v = \mathrm{diag}(\sigma_x^2, \sigma_x^2, \sigma_{xd}^2, \sigma_{xd}^2, \sigma_f^2, \sigma_f^2, \sigma_x^2, \sigma_x^2, \sigma_x^2, \sigma_x^2) \tag{6.20}$$

We set $\sigma_x^2 = 1.7^2\,\mathrm{mm}^2$, $\sigma_{xd}^2 = 35^2\,\mathrm{mm}^2/\mathrm{s}^2$. For the H and VH group, we respectively set $\sigma_f^2 = 0.1^2\,\mathrm{N}^2$ and $\sigma_f^2 = 0.01^2\,\mathrm{N}^2$.

6.3.4 Partner model

Partner's control input was estimated as part of the state observer. We made the prior assumption that partner input is described by a low-pass filtered white noise:

$$u_{-i}(t+1) = A_u \cdot u_{-i}(t) + \varepsilon_{-i}(t) \tag{6.21}$$

In all simulations, we set $A_u = 1$ and $\Sigma_{-i}^{\varepsilon} = 0.5\mathrm{N}^2$.

6.4 Simulation results

Nash equilibrium for this task consists of the two subjects following the same trajectory by crossing both via-points at the same time. Therefore there are two Nash equilibria, which differ in the order of crossing (VP_1 first or VP_2 first). The second Nash equilibrium is characterised by greater path lengths and therefore greater costs, and is therefore never chosen. In the following we only refer to the first Nash equilibrium, and will refer to it as 'the' Nash equilibrium.

6.4.1 Optimal collaboration

We first simulated joint movements under three situations: (i) unconnected – the mechanical link among the two agents has been removed; (ii) no-partner – the two agents are mechanically connected but, when calculating the optimal control policy, each agent ignores his/her partner i.e. he/she considers him/her as noise; (iii) Nash – the two agents establish an optimal collaboration (Nash equilibrium).

The simulation results, based on the H condition, are summarised in Figure 6.3.

The trajectories look similar in the two models, but a closer look suggests that in the no-partner case – see Figure 6.3.a, left – each subject crosses his/her own via-point before their partner.

This is also reflected in the interaction power. The interaction power (dot product of interaction power and subject velocity) allows to identify when a subject behaves as a leader (moving against the interaction force – negative interaction power) or as a follower (pulled by the other partner – positive interaction power). In the non-collaborative strategy, each subject actively moves toward his/her own via-point, but gets closer to the other only because he/she is pulled by the partner. This is why he/she approaches the partner's via-point with a delay.

In the collaborative scenario (Nash equilibrium) – see Figure 6.3, right – the two subjects approximately follow the same trajectory, by crossing each via-point at approximately the same time. Both the interaction force and the interaction power remain low over the whole movement, and there are no clear leader-follower roles.

Statistical analysis (t-test) confirmed these qualitative observations. Specifically, we found significant group differences in the interaction force ($t_{38} = 83.7$, $P < 10^{-6}$) in the interaction power ($t_{38} = 84.6$, $P = 2.2 \cdot 10^{-6}$), and in the via-point crossing times (via-point 1: $t_{38} = 38$, $P < 10^{-6}$; via-point 2: $t_{38} = 6.21$, $P < 10^{-6}$). At individual subjects level, we observed significant differences in the minimum distance from partner's via point (subject 1: $t_{38} = 24$, $P < 10^{-6}$; subject 2: $t_{38} = 63.4$, $P < 10^{-6}$) and in the leadership index, at via-point

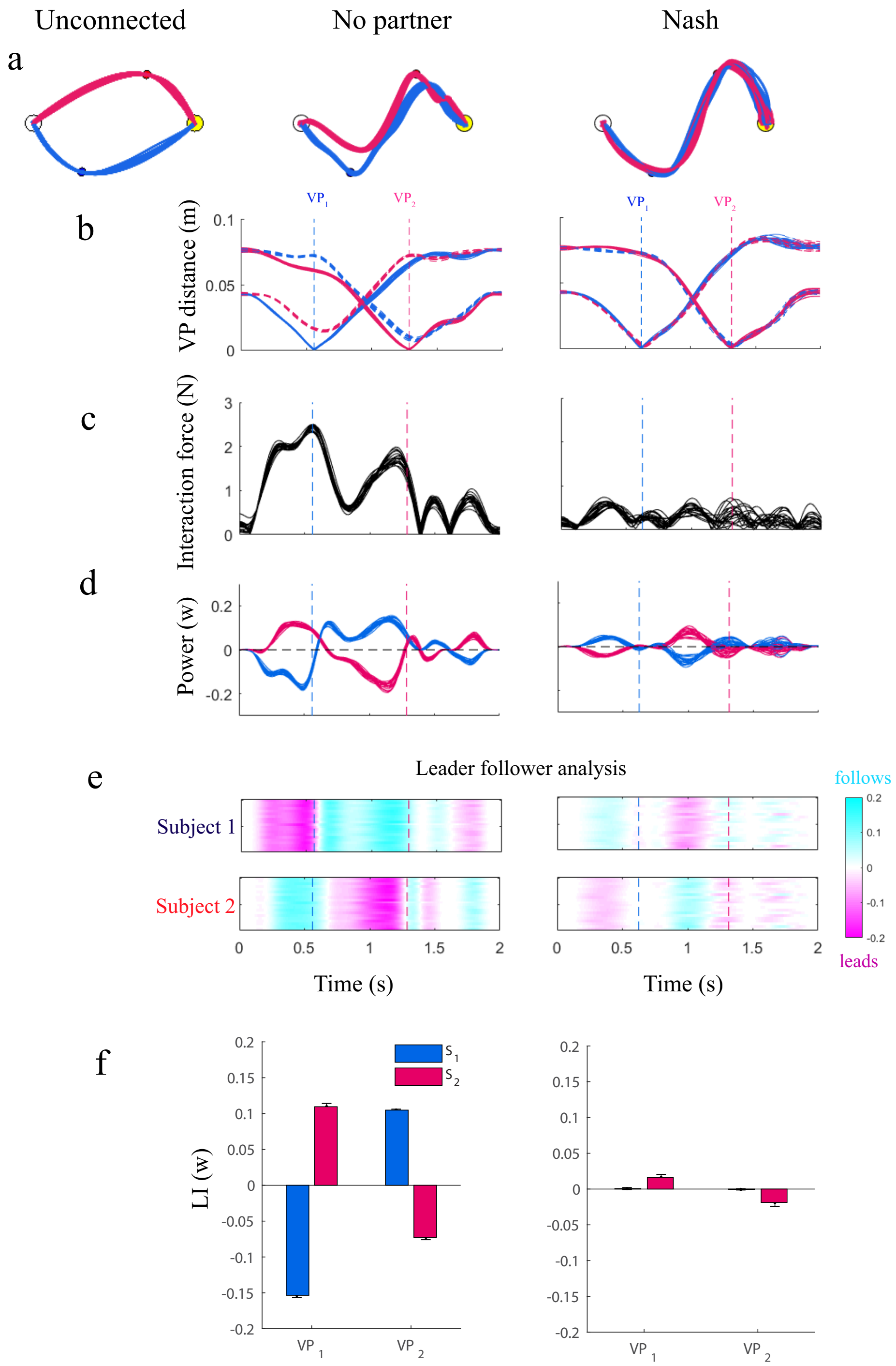


Figure 6.3 **Optimal collaboration**. Simulation results for unconnected (left), no-partner (middle), Nash (right) agents. (**a**), Simulated trajectories for agent 1 and agent 2 (blue and red). (**b**), Distances from via-points as a function of time. (**c**), Interaction force, as a function of time. (**d**), Interaction power. (**e**), Leader-follower patterns for the two subjects. The different colours denote the different signs of the interaction power: negative (magenta) denotes leading, positive (cyan) denotes following. (**f**), Leadership indexes for the two subjects, calculated as the average interaction power in the 300-ms interval before via-point crossing

1 (subject 1: $t_{38} = -92$, $P < 10^{-6}$; subject 2: $t_{38} = 62$, $P < 10^{-6}$) and at via-point 2 (subject 1: $t_{38} = 18$, $P < 10^{-6}$; subject 2: $t_{38} = -11$, $P < 10^{-6}$).

In conclusion, with respect to optimal collaboration, in addition to a greater cost (greater interaction force), a distinctive feature of the no-partner scenario is the alternation of leader and follower roles - each subject acts as a leader when crossing his/her own via-point, and as a follower when crossing that of the partner. This is also reflected in different crossing times (with respect to the leader, the follower lags behind).

6.4.2 Estimation of partner input

Figure 6.4 depicts, for both agents, the results of the prediction of the partner's motor command, in comparison to the corresponding true command (agent 1: blue; agent 2: red). We repeated the simulation for the different scenarios used in the experiments described in Chapter 7, i.e. H (haptic feedback alone), VH (visual and haptic feedback) and PV (partner visible).

6.4.3 Learning to collaborate

We then simulated the trial-by-trial learning process of a collaborative strategy. When two naive subjects are mechanically connected, at least three learning and/or adaptation processes take place:

- **Model adaptation** Each agent adapts the forward model of dynamics which is part of their state observer in order to incorporate the interaction force and the dynamics of the connected partner.
- **Task learning** Each agent modifies his/her control policy in order to incorporate the additional constraint of keeping the interaction force low. This form of learning affects the feedback controller and aims at maximising the performance score.
- **Partner learning** Each agent may establish an internal representation of the partner's motor command, to be incorporated into the computed control policy. Estimating the partner's motor command can be achieved as a relatively straightforward extension of the state observer – see section 6.2.3 – but it is unclear to what extent human subjects are actually capable of incorporating it into their control policy.

In our simulations, we assumed that model adaptation is instantaneous, i.e. changes in dyad dynamics are immediately incorporated in each agent's internal models. As regards task

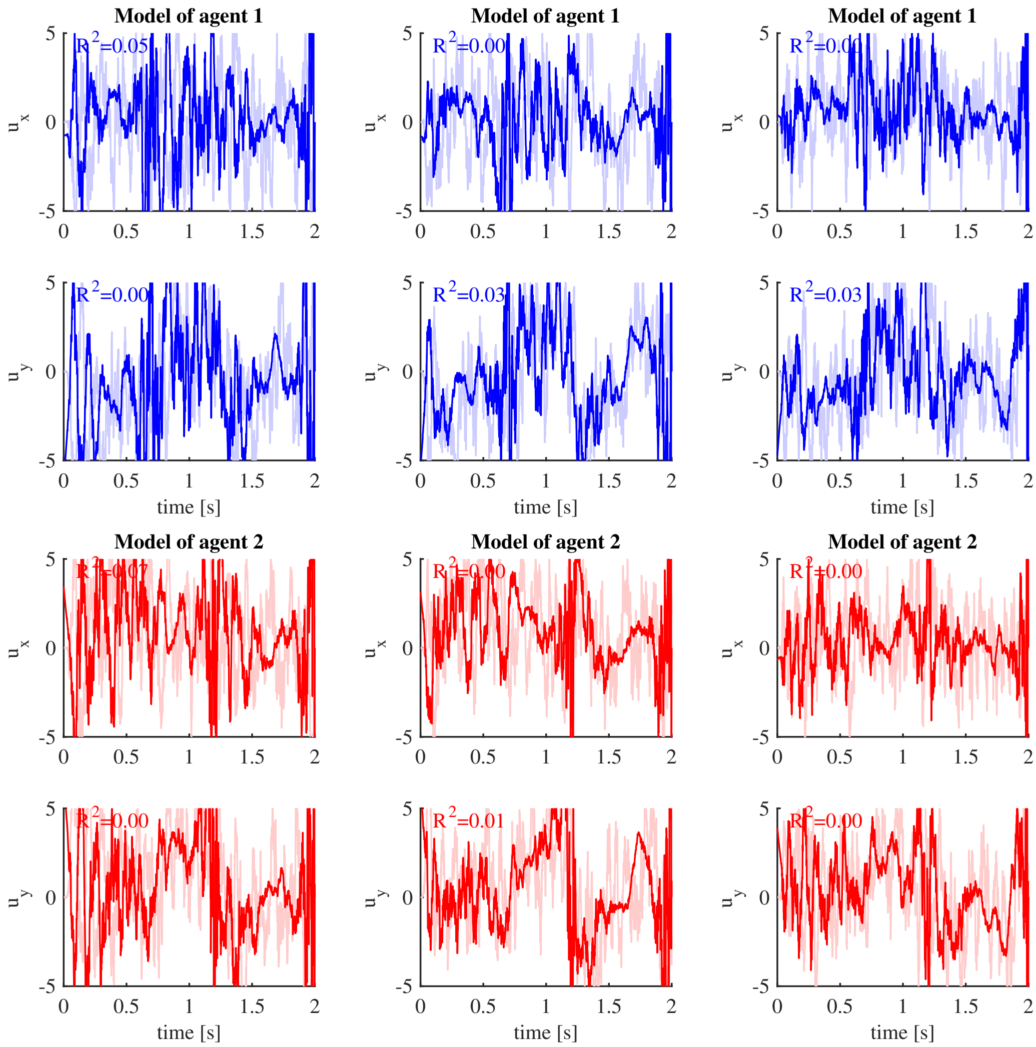


Figure 6.4 **Prediction of partner's input from sensory information and state prediction**. On-line prediction of partner's input from sensory information and state prediction, for agent 1 (left) and agent 2 (right). Dark colours denote the true inputs, light colours their estimations. Simulations are based on the H scenario (the only information about partner is provided by haptic interaction)

learning, we assumed that when the mechanical connection is established, the controller incorporates the additional constraint on minimising the interaction force – see Eq. 6.18 – gradually, within 12 trials (one epoch). As regards incorporation of the partner model into the control policy (partner learning), we assumed that it occurs more gradually, within 24 trials (two epochs). This prevents abrupt changes of the partner model when the mechanical connection is established or turned out.

The learning outcome is summarised in Figure 6.5 for all three scenarios (H, VH PV). The results of the simulation are quantified in Figure 6.6 (average within epoch 11 or 'late'

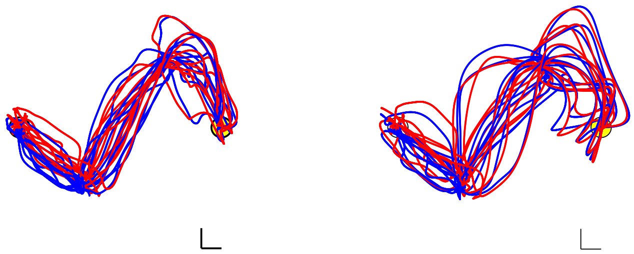


Figure 6.5 **Learning outcome**. Trajectories in the last epoch for all three H (left), VH (middle) and PV scenarios (right)

learning). We found that increasing the amount of information about the partner makes the final performance closer to the ideal situation (Nash equilibrium).

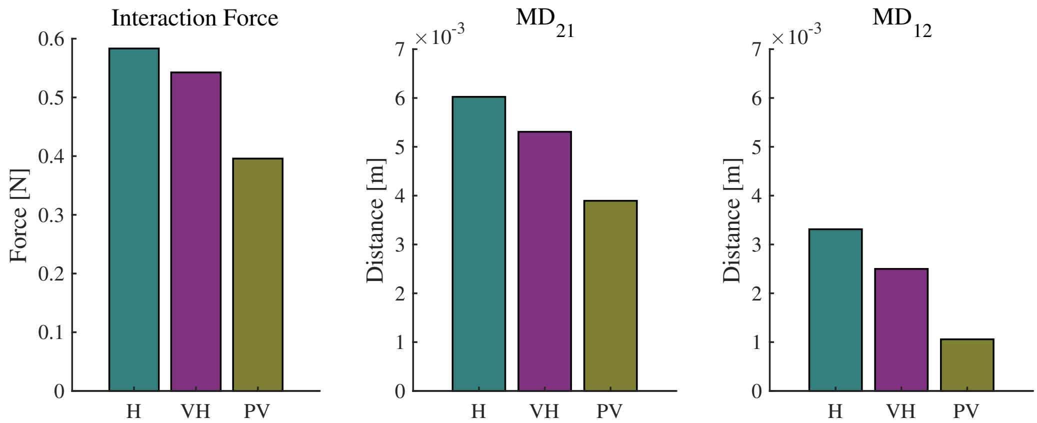


Figure 6.6 **Learning performance**. From left to right: Interaction force and minimum distance from VP_1 (MD_{21}) and VP_2 (MD_{12}), for all three H, VH and PV scenarios

This is also confirmed when looking at the leadership index; see Figure 6.7. The switching of roles (each subject leads when aiming at his/her own VP and follows when aiming at the partner's VP) – which is an indication of lack of consideration of partner intentions when developing their own control policy – decreases as the amount of information about the partner increases.

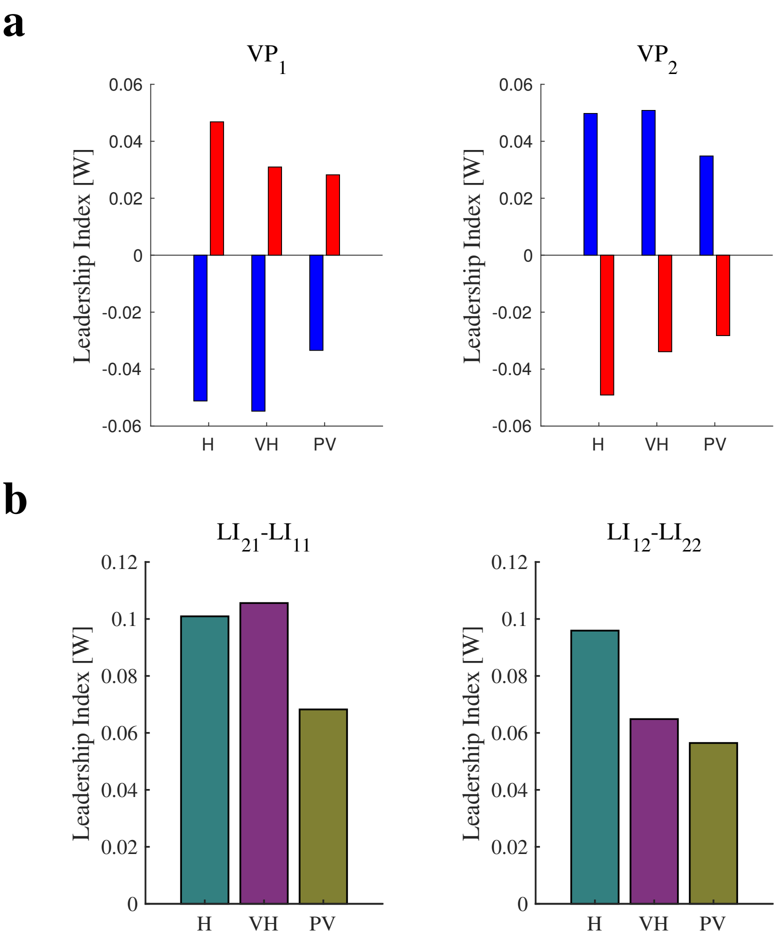


Figure 6.7 **Leadership index for model**. (**a**), Leadership index for subject 1 (blue) and subject 2 (red) at VP_1 (left) and VP_2 (right).(**b**), For all three H, VH and PV scenarios

6.5 Discussion

6.5.1 A general computational model for joint action

In this Chapter we defined a general modelling framework to analyse human-human sensory motor collaborative strategies. Our modelling framework uses a differential game-theoretic approach, which is a natural extension of optimal feedback control models which are widely used to capture sensorimotor control of individuals (Todorov and Jordan, 2002). Game theory methods yields shared decision making, allowing subjects to have different utility functions, which help to characterize the existence and uniqueness of an 'optimal collaboration', defined by a Nash equilibrium point (Nash, 1951).

In our model, dyad dynamics is described by a linear state-space model with two control inputs, each with Gaussian process noise. The task is described by a pair of quadratic cost functionals (one per partner). The model accounts for incomplete information about the partner and includes, for each agent, a separate sensory system with Gaussian measurement noise. Each partner is equipped with a feedback controller and a state observer – which also estimates the partner's motor command. We modelled the time course of learning a joint action through the biologically-plausible notion of 'fictitious play' (Brown, 1951), which requires minimal assumptions on the partner's goals and ongoing actions.

To our knowledge, this is the first and most complete computational model which accounts for learning joint action with partial or incomplete information. Previous attempts of applying game theory to physical joint action are less general (Braun et al., 2009) and/or address simple situations or toy problems (Jarrassé et al., 2012). More recently, Li et al. (2015) applied differential game theory and Nash strategies to a linear model of human sensorimotor control in the context of human-robot interaction. Takagi et al. (2017) modelled the interaction of two mechanically connected partners while tracking a moving target. They show that the experimental observations are better explained by assuming that each partner maintains a model of their partner's predicted target movement. However, their model is not based on game theory. No existing model addresses learning of collaboration and the effect of incomplete information.

The model could be useful to interpret pathological interactive motor behaviours (e.g. in case of Autism spectrum disorder) – a preliminary exploration of this issue is done in Chapter 8 – or to interpret pattern of interaction that imply the establishment of a collaboration which is driven by mutual utility. In principle, this same framework could be used to investigate the patient-therapist or patient-robot interaction in neuromotor rehabilitation. In particular, it may explain 'slacking' – a distinctive observation in human-robot or robot-assisted rehabilitation in which voluntary control diminishes over trials when assistance keeps the errors low (Emken et al., 2007; Franklin et al., 2008, 2003; Thoroughman and Shadmehr, 1999).

6.5.2 Nash equilibria in a dynamic 'battle of sexes' game

We used our model to simulate the 'battle of sexes' game described in Chapter 4, to predict the optimal non-cooperative strategy (Nash equilibrium) and to simulate the learning process under a variety of conditions, characterised by different amount of information about the partner. The game is characterised by two Nash equilibria, in which both subjects cross both via-points. The two Nash equilibria only differ in the order the two via-points are crossed, and in terms of overall path length and therefore energetic cost. The Nash equilibria are characterised by overlapping paths, approximately same crossing times at both via points and near-zero interaction forces.

Nash equilibria require that each agent knows what his/her partner is doing (partner model). This is not a realistic assumption in interpersonal joint action. For this reason, we also simulated an opposite situation, in which each agent has no knowledge at all about his/her partner's actions. In this no-partner model, the overall trajectories are very similar – they too come close to both via-points – but the overlap between paths is incomplete and each partner crosses their partner's via-point with a significant delay.

In the next Chapter these simulations will be compared to experimental results in which we studied how nature and quality of the collaboration is affected by the incomplete or unreliable information about the partner.

6.5.3 Roles are an emergent strategy to compensate the lack of information about the partner

The simulations with the battle of sexes game make a distinctive prediction about how the two agents assume different 'roles' (leader, or follower). We specifically found that 'roles' are an optimal compensation strategy, which results from lack or incomplete information about the partner. Therefore, 'roles' are a signature of this lack of information. We will use this prediction in Chapter 7 and 8 to understand the extent to which subjects establish a model of their partner's actions.

6.5.4 Model limitations

Optimal feedback control models of individual sensorimotor control (Todorov and Jordan, 2002) typically assume that process noise increases its magnitude with that of the motor command – signal-dependent or multiplicative noise. Multiplicative noise is ubiquitous in the motor system. In its current implementation, our model only uses additive, Gaussian noise. However, the model can be easily extended by using results in differential game theory in which multiplicative noise is used in differential games with both finite (Huiying and Liuyang, 2011; Sun et al., 2013) and infinite horizon (Sun et al., 2012).

6.6 Conclusions

We developed a general computational framework – based on differential game theory – for the description and implementation of interactive behaviours of two subjects performing a joint motor task. The model allows to simulate any joint sensorimotor action in which the joint dynamics can be represented as a linear dynamical system and each agent's task is formulated in terms of a quadratic cost functional. The model also accounts for imperfect information about dyad dynamics and partner's actions, and can predict the development of joint action through repeated performance. In Chapter 7, I will use this computational framework to study the role of sensory information about partner in the development of collaborative strategy in joint action.

Part III

Experiments

Chapter 7

Information about the partner affects the development of collaborative strategies in joint motor action

> All our knowledge has its origin in our perceptions.
>
> Leonardo da Vinci

7.1 Introduction

Many tasks in daily life involve coordinating movements between two or more individuals. A couple of dancers, a team of players, two workers carrying a load or a therapist interacting with a patient are just a few examples. Acting in collaboration or joint action (Sebanz et al., 2006) is a crucial human ability, and our sensorimotor system is shaped to efficiently support this capability.

Two partners (a 'dyad') may have a common goal – e.g. reaching a target (Reed et al., 2006), spatio-temporal control of isometric force (Masumoto and Inui, 2013, 2014), or tracking a moving target (Ganesh et al., 2014). In this case, dyad behaviour can be compared with individual performance. Although the partner is often perceived as an impediment, dyads generally perform better than individual subjects. For instance, dyads tend to move faster than individuals (Reed et al., 2006). In visual tracking, dyad performance is always better than that of the partner who was most skilled in 'solo' performance (Ganesh et al., 2014). Further, the performance achieved by training as a dyad transfers to the subsequent 'solo' performance. In a related study, Takagi et al. (2016b) suggested that rigidly coupled subject pairs accomplish joint reaching movements by relying on distinct motion plans that are independent of the partner's behaviour throughout the trial. However, Takagi et al. (2017) give evidence that human individuals establish models of their partner in a shared tracking task.

In many situations, partners in a dyad have different and partly conflicting goals (Braun et al., 2009, 2011; Grau-Moya et al., 2013; Leibfried et al., 2015). Game theory is a natural framework to address interpersonal interaction. Braun et al. (2009) used a pair of haptic robots to implement 'motor' versions of classical non-cooperative games, like the prisoner's dilemma and rope pulling, in which the player-specific payoff was substituted by a position-dependent force field, encoding costs or rewards of the interaction. These authors compared bimanual and dyad behaviours. In the bimanual version of the task there is only one controller, whereas in the dyad version there are two controllers which may (or may not) negotiate a common strategy. They found that the bimanual version of the task ended up in a cooperative solution, whereas the dyad converged to the optimal non-cooperative solution, which can be interpreted as a Nash equilibrium (Nash, 1951).

Asymmetric behaviours (i.e., 'roles') have been consistently observed within a dyad. Reed and Peshkin (2008) reported the emergence of a variety of roles (e.g., acceleration, deceleration) in individual partners within a dyad. Based on the signs of velocity, acceleration and applied force, Stefanov et al. (2009) proposed an analytic framework in which roles are described in terms of 'execution' (whether a partner contributes or resists to the overall movement) and 'conductorship' indicators (to what extent a partner is responsible for initiating or stopping a movement). Masumoto and Inui (2014) distinguished leader and follower roles, where leaders tend to initiate the action and contribute most effort. Roles can be fixed, or change with time. In a visual tracking task subject to force perturbations, Melendez-Calderon et al. (2015) identified a variety of interaction strategies, differing in terms of patterns of joint torques and muscle activations. They also observed that perturbations often induced a switch among strategies. Role assignment in human-human interaction has often been a source of inspiration for applications involving the interaction between humans and robots - see Jarrassé et al. (2014) for a review.

Further, if two partners have different goals, they need to negotiate a joint strategy. However, if information about the partner is incomplete or unreliable collaboration may be less effective or even unfeasible (Oguz et al., 2012). Optimal collaboration (Nash equilibrium) is an ideal situation, which can only be achieved if both subjects know everything about their own and partner's goals. If knowledge about the partner is partial or incomplete, optimal collaboration may be difficult to achieve. In a human-computer negotiation game, Oguz et al. (2012) found that the combined effect of visual and haptic sensory feedback leads to better performance than that attained by subjects who played with visual feedback alone. However, it is unclear how partial information affects establishing a collaboration in two interacting humans – see also Grau-Moya et al. (2013).

Here we address how negotiating a collaboration is affected by amount and quality of information about the partner. We designed a novel interactive learning paradigm, in which two subjects are mechanically connected but cannot see each other. Both subjects were instructed to perform reaching movements with the same start and end positions, but through different via-points(VP); see Figure 7.1. Both subjects were instructed to keep the interaction force as low as possible during movement. Subjects had the option of establishing a collaboration - negotiating a path through both VPs, which would lead to a minimisation of the interaction forces - or to ignore each other, by only focusing on their own goal. The task can be seen as a sensorimotor version of the classical 'battle of sexes' game. We manipulated the information available on partner's actions by providing it either haptically, through the interaction force (haptic group, H); by additionally displaying the interaction force vector on the screen (visuo-haptic group, VH); or by continuously showing the partner movements (partner visible, PV).

7.2 Materials and Methods

7.2.1 Experimental apparatus and task

Each experiment involved a pair of subjects (a dyad). Each participant sat in front of a computer screen and grasped the handle of a three-dimensional haptic interface (Novint Falcon). They could not see or hear each other, and were not allowed to talk. The experimental apparatus is depicted in Figure 7.1. The subjects were instructed to perform reaching movements in the vertical plane, between the same start point (displayed as a white circle, $\oslash$ 1 cm) and the same target point (yellow circle, $\oslash$ 1 cm), but through different via-points. In a reference frame centred on the robot workspace (one for each subject), with the X axis aligned with the left-right direction and the Y axis aligned with the vertical direction, the start point was placed in the (-5, 0, 0) cm position and the target point was placed in the (5, 0, 0) cm position. Hence the start and the target point had a horizontal distance of 10 cm. The subjects were also instructed to keep their movements as planar as possible, i.e. by keeping the depth, Z coordinate within the range (18-26 cm) from the origin of the workspace.

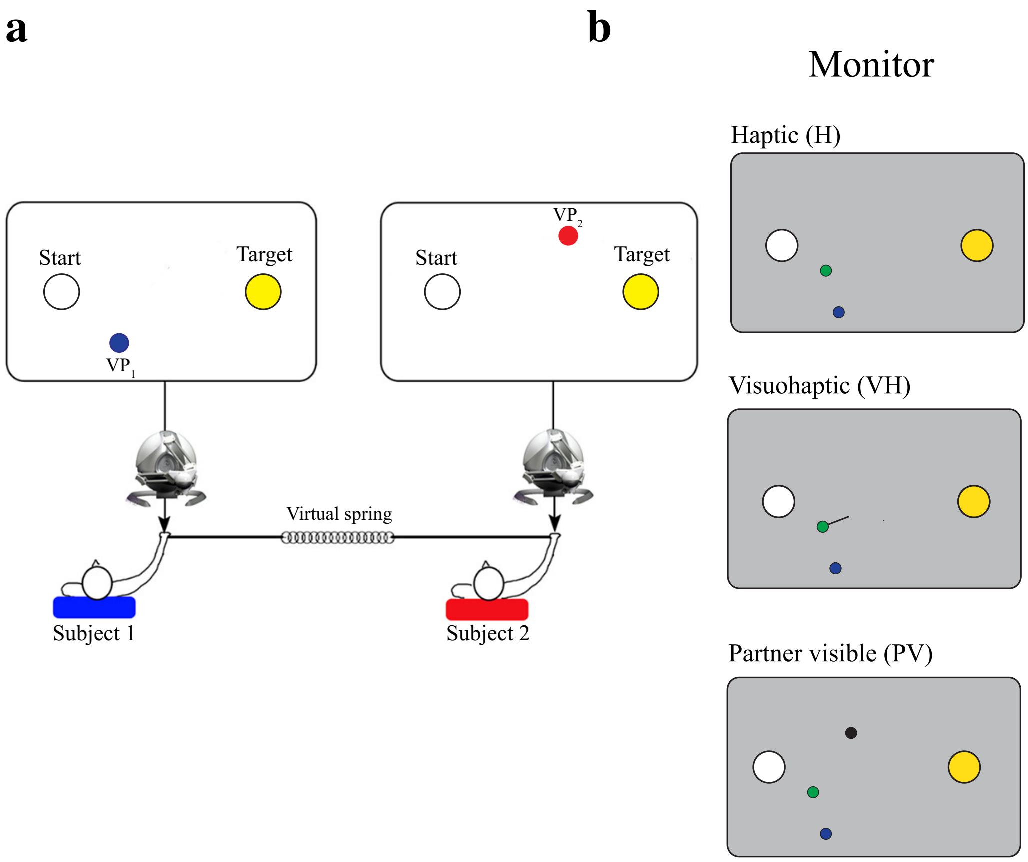


Figure 7.1 **Experimental apparatus and protocol**. (**a**), Partners in a dyad were connected through a virtual spring. Both subjects were instructed to perform reaching movements in the vertical plane, between the same start point (displayed as a white circle, ⊘ 1 cm) and the same target point (yellow circle, ⊘ 1 cm), but through via-points (VP). Each subject could only see his/her own VP, but not their partner's. Both were instructed to keep the interaction force as low as possible during movement. A trial started when both subjects placed their cursor inside the start region. Then the target and a VP (⊘ 0.5 cm circle) appeared. The VPs were different for the two subjects and were placed, respectively, at locations $VP_1 =$ (-3,-2,0) cm and $VP_2 =$(3,2,0) cm. At the end of each movement each subject received a 0-100 reward, calculated as a function of the minimum distance of their movement path from his/her own VP and of the average interaction force. The subjects were instructed to aim at maximising this score. The experiment was organised into epochs of 12 movements each. The experimental protocol consisted of three phases: (i) baseline (one epoch), (ii) training (ten epochs) and (iii) after-effect (two epochs) for a total of $13 \times 12 = 156$ movements. During the baseline phase the interaction forces were turned off, and each subject performed on their own ('solo' performance). During the training phase the subjects were mechanically connected, but in randomly selected trials (catch trials) within each epoch (1/6 of the total, i.e. 2 trials per epoch) the connection was removed. The connection was permanently removed during the after-effect phase. (**b**), We manipulated the information available on partner's actions by providing it either haptically, through the interaction force (Haptic group, H) or by additionally displaying the interaction force vector on the screen (Visuohaptic group, VH) or displaying partner's cursor itself (Partner visible group, PV). The yellow and white circles denote, respectively, the start and target position. The green circle is the cursor location. In the VH group, direction and magnitude of the interaction force is depicted by a line originating from the cursor. In the PV group, partner's cursor is shown by black circle.

The current positions of the end effectors, x_1 and x_2, were continuously displayed to each partner, as $\oslash$ 0.5 cm circular cursors on their respective screens, coloured in green if the depth was correct and in red otherwise. Audio queues were provided at the start and end of the movements.

A trial started when both subjects placed their cursor inside the start region. Then the target and a via-point ($\oslash$ 0.5 cm circle) appeared. The via-points were different for the two subjects and were placed, respectively, at locations VP_1 = (-3,-2,0) cm and VP_2 =(3,2,0) cm. The haptic interfaces generated a force proportional to the difference of the two hand positions:

$$F_1 = K \cdot (x_2 - x_1) \tag{7.1}$$

$$F_2 = K \cdot (x_1 - x_2) \tag{7.2}$$

with $k = 150$ N/m. Hence, the two subjects were mechanically connected. At the end of each movement each subject received a 0-100 reward, calculated as a function of the minimum distance of their movement path from his/her own via-point and of the average interaction force, according to the following formula:

$$\text{score}_i = \frac{100}{1 + \exp^{-k \cdot (d_0 - d_i)}} \tag{7.3}$$

where $d_i = d_{VP_i} + c \cdot d_{12}$ and $i = 1, 2$. The quantities d_{VP_i} and d_{12} are, respectively, the minimum distance between the movement trajectory and the subject's own 'via-point' (VP_i) and the average distance between the two subjects' hand positions. In the disconnected trials we took $c = 0$, i.e. the score only depended on how close the subjects got to their own via-point. Parameters k and d_0 were calculated so that the score was maximum (100) for $d_i \leq$0.005 m (i.e., the VP radius), and minimum (0) for $d_i \geq 0.02$ m. To encourage subjects to establish a collaboration, in trials in which the two subjects were mechanically connected we took $c = 0.5$, so that in order to get a maximum score subjects also had to keep their relative distance as low as possible. The subjects were instructed to aim at maximising this score. Specifically, they were told that performance depends on how close they would get to the via-point. They were also told that they might experience a force while performing the task. They were warned that force magnitude also affects the score, and were instructed to also keep this force to a minimum. To encourage subjects to maintain an approximately constant movement duration, after each movement a text message and changes in the color of the target (either green or red) warned the subjects if the movement was either too fast (duration < 1.85 s) or too slow (duration > 2.15 s). However, if their movement duration did not remain within the recommended range the participants received no further penalisation.

The subjects pairs were randomly assigned to three groups which differed for the feedback provided about the interaction force. In the haptic (H) group, interaction could only be sensed haptically. In the visuo-haptic group (VH), interaction force (magnitude, direction) was also displayed as an arrow attached to the cursor (scale factor: 10 N/cm). A third group, namely partner-visible (PV) was recruited as control group. In the PV group subjects see their partner's cursor at all times. Therefore, subjects in the PV group receive highly reliable information about partner's movement. The experiment was organised into epochs of 12 movements each. The experimental protocol consisted of three phases: (i) baseline (one epoch), (ii) training (ten epochs) and (iii) after-effect (two epochs) for a total of $13 \times 12 = 156$ movements. During the baseline phase the interaction forces were turned off, and each subject performed on their own ('solo' performance). During the training phase the subjects were mechanically connected. During this phase, in randomly selected trials (catch trials) within each epoch (2 trials per epoch, i.e. 1/6 of the total) the connection was removed. The connection was permanently removed during the after-effect phase. During the training phase the subjects had the option to establish a collaboration - negotiating a path through both via-points, which would lead to a minimisation of the interaction forces and a maximum score for both - or to ignore each other - each partner would only focus on their own via-point and on maximising his/her own score. The application has been developed using CHAI3D, an open-source software environment for the control of haptic devices (Conti et al., 2003) (see Chapter 5 for more details about experiment setup).

7.2.2 Subjects

A total of 30 subjects participated in this study. All subjects were right-handed, as assessed using the Edinburgh Handedness Inventory (Oldfield, 1971), naive to the task and with no known neurological or motor impairment at the upper limb. From the list of participants, we randomly formed three groups of ten subjects, which were randomly assigned to the H and VH and the control group, PV. The subjects' demographic data are summarized in Table 7.1. The research conforms to the ethical standards laid down in the 1964 Declaration of Helsinki that protects research subjects. Each subject signed a consent form conforming to these guidelines.

Table 7.1 **Participants demographics for study 1**. For the three groups, we indicate the range and mean (± SE) age, the total number of subjects, subject label, dyad label, age and sex.

Group	# Subjects	Label	Dyad	Age	Sex
Haptic (H)	10	S11-S20	D1-D5	25 ± 5	9 M+ 1 F
Visuohaptic (VH)	10	S1-S10	D6-D10	24 ± 3	8 M+ 2 F
Partner visible (PV)	10	S21-S30	D11-D15	24 ± 3	6 M+ 4 F

7.2.3 Data Analysis

Hand trajectories and robot-generated forces were sampled at 100 Hz and stored for subsequent analysis. The data samples were smoothed by means of a 4th order Savitzky-Golay filter with a 370 ms time window (cut-off frequency: 7.5 Hz). We used the same filter to estimate velocity and acceleration. We identified the start and end times of each trajectory as the time instants at which the speed crossed a threshold of 2 cm/s. In the analysis, we specifically focused on the temporal evolution of the trajectories and on signs of collaboration between partners within the same dyad. Collaboration can be characterised in terms of both movement kinematics and movement kinetics. We developed a number of performance indicators to characterise collaboration. All data were analysed using MATLAB.

The performance indicators can be broadly divided into Dyad-level and Subject-level indicators – see Chapter 5 for details.

Dyad-level indicators

Interaction force (IF) is calculated as $IF = \frac{1}{T}\sum_{t=1}^{T} \|F_i(t)\|$, where $F_i(t)$ is the interaction force (equal and opposite for the two partners in the dyad) – see Eq. 7.2. Less interaction force would point at a greater collaboration.

Subject-level indicators

Another sign of collaboration is that each subject, while passing through his/her own via-point, also gets very close to his/her partner's. This can be quantified in terms of the Minimum Distance to the Via-Point (MD_{ij}), defined as the minimum value of the distance of subject i to the j-th via-point ($MD_{ij} = \min_t ||x_i(t) - x_{VPj}||$ with $i, j = 1, 2$. If $i \neq j$, this quantity reflects how close each subject gets from his/her partner's via-point.

Looking at the power developed by each subject would provide information on whether the subjects move actively, or are passively pulled by their partner through the mechanical coupling. To quantify this, we calculated the power (P_i), defined as the scalar product of the interaction force $F_i(t)$ and the velocity vector $v_i(t)$ of each of the subjects. At a given time, a positive power would mean that the subject is controlling his/her motion (i.e. he/she is behaving as a 'leader'). Conversely, a negative power would indicate that the subject is being pulled toward the other (i.e., he/she is behaving as a 'follower'). We specifically focused on the average power calculated in the 300ms interval taken just before the crossing of each via-point. We denote as LI_{ij} this value for the i-th subject and the j-th via-point.

7.2.4 Statistical analysis

We expect that task performance at subjects and dyad level evolves with time (learning) and is affected by the amount of information each subject has available about his/her own partner. To test this, for all the above indicators we ran a repeated-measures ANOVA with group (VH, H, PV) and Time (early - epoch 1, middle - epoch 6 and late - epoch 11) as factors. In the case of the leadership index, we only focused on the final epoch. Hence in this case we looked at a 1-way ANOVA with group as the only factor. If a main effect was found, Tukey's HSD (honest significant difference) post hoc test was used to examine the differences. Statistical tests were performed using R. Statistical significance was considered at $P < 0.05$ level for all tests.

7.3 Results

7.3.1 Collaboration in dyads and the role of information

In all three haptic (H), visuo-haptic (VH) and partner-visible (PV) groups, all dyads converged to stable and consistent behaviours; see Figure 7.2. At a first glance, the learned movement paths at the end of the training phase look quite similar in all groups. When the connection was removed, both agents quickly returned to the baseline situation.

These observations are confirmed when looking at the score, the interaction force and the minimum distance from the partner's VP. All are expected to decrease if subjects establish a collaboration. The temporal evolution of score for subject pairs is summarised in Figure 7.3 (a,b).

Subjects in the VH group achieved a greater score than those in the H group at the end of training, which is confirmed by statistical analysis. Overall the subject pairs improved their score with training ($F_{2,24} = 47$; $P < 10^{-4}$) and exhibited significant group differences ($F_{2,12} = 56.07$; $P < 10^{-4}$).

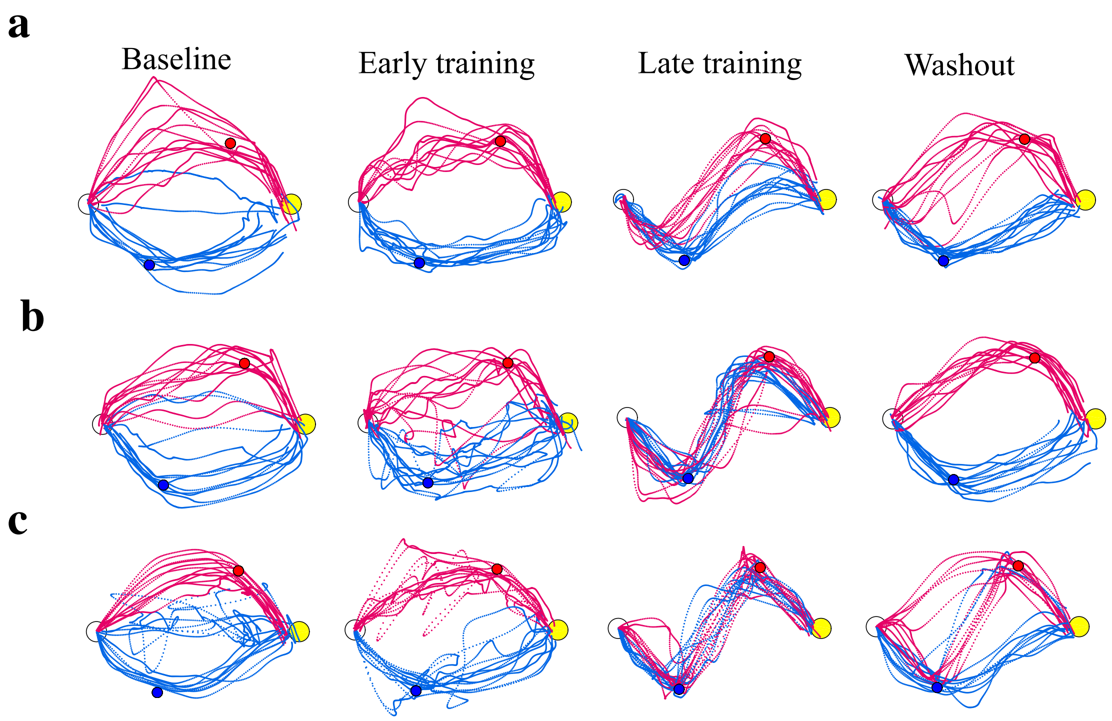


Figure 7.2 **Movement trajectories**. Movement trajectories in baseline (unconnected), early-training, late-training and washout phases of the experiment from Haptic group (**a**), Visuo-haptic group (**b**) and Partner-visible group (**c**).

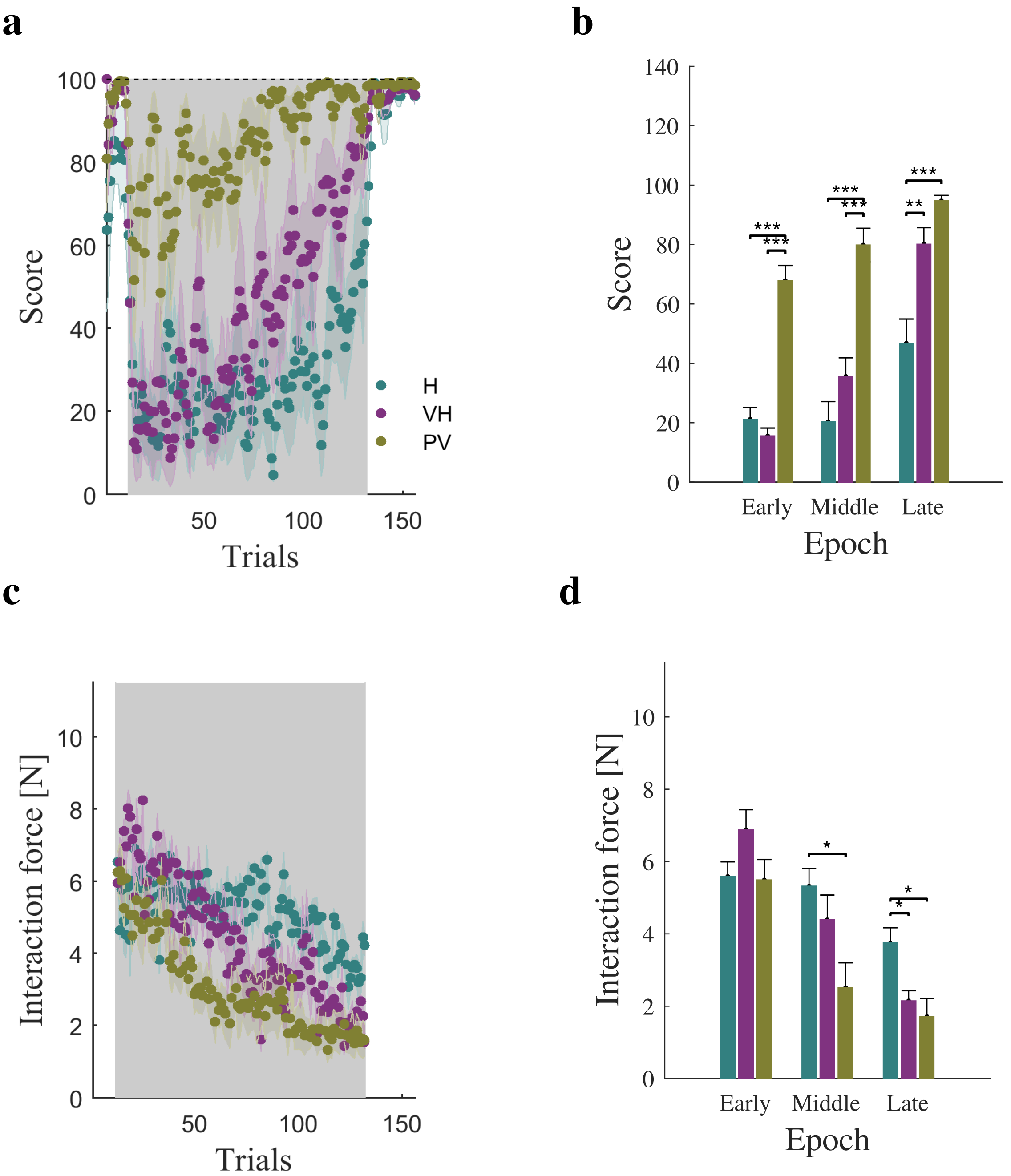


Figure 7.3 **Score and Interaction force changes with training**. Magnitude of score (**a**) over trials, for the visuo-haptic (VH) , haptic (H) and Partner-visible(PV) group respectively). (**b**), Score at the beginning, middle and at the end of training. **c**, Magnitude of the interaction force (rms) over trials. (**d**), Interaction force at the beginning, middle and at the end of training. The areas in grey denote the training phase. Error bars denote the standard error (SE). Asterisks indicate statistically significant differences ($*: P < 0.05$, $**: P < 0.005$, $***: P < 0.0005$)

We also found a significant group×time interaction ($F_{4,24} = 5.2$; $P = 0.0034$). Specifically, post-hoc analysis confirmed that subject pairs in the VH group achieved a significantly greater score at the end of training phase than the H group ($P = 0.004$). Also the PV group differs significantly from H ($P < 0.0002$). At the middle time, PV again differs from H ($P < 10^{-3}$) and from VH ($P < 10^{-3}$), see Figure 7.3.(b).

The interaction force is the main determinant of the score, and its temporal evolution exhibits a similar behaviour in the three groups – see Figure 7.3 c, d. Overall, we found a significant training (time) effect ($F_{2,24} = 37.4$; $P < 10^{-4}$), a significant group effect ($F_{2,12} = 6.2$; $P = 0.014$), and a significant group × time interaction effect ($F_{2,12} = 6.2$; $P = 0.014$) – see Figure 7.3.(d). Post-hoc analysis showed that groups PV-H (p=0.02) but not VH-H and PV-VH differ significantly at middle time. In addition, groups VH-H ($P = 0.04$) and PV-H ($P = 0.01$) but not PV-VH differ significantly at late time.

In summary, the temporal evolution of both Score and Interaction Force are faster in the PV group and slower in the H group.

A similar behaviour can be observed in the temporal evolution of the minimum distance from the partner's via-point is depicted in Figure 7.4.(a,b). In both groups and in both subjects in the dyad, the minimum distance (MD) decreases over trials and quickly washes out when the connection is permanently removed (after-effect phase). The magnitude of the decrease is very similar in all groups.

Statistical analysis confirmed this observation. We found a significant time effect ($F_{2,24} = 40$; $P < 10^{-4}$) for subject 1 and ($F_{2,24} = 36.4$; $P = 0.0067$) for subject 2. We also found significant group effects ($F_{2,12} = 40$; $P < 10^{-4}$) for subject 1 and ($F_{2,12} = 8.9$; $P = 0.004$) for subject 2. However, we only found a significant group×time interaction for subject 2 ($F_{2,16} = 5.64$; $P = 0.014$), but not for subject 1. Post-hoc analysis showed that for subject 1, in the groups (PV-H) the MD value is significantly different (lower in the PV group) in the late time ($P = 0.0296$) and also group combinations (PV-H and PV-VH) differ significantly at the middle time ($P = 0.0002$, $P = 0.01$ respectively). For subject 2, post-hoc analysis showed that group pairs (VH-H and PV-H , but not PV-VH) differ significantly at the late time ($P = 0.0009$, $P = 0.0003$ respectively) and groups (PV-H , but not VH-H and PV-VH) significantly differ at the middle time ($P = 0.04$). In other words the three groups – specially H and PV – differed in both magnitude and rate of decrease of their via-point distance. Figure 7.5 summarises the effect of learning in terms of MD in three groups. The Figure also suggests that in the H group learning is less complete for subject 2 than subject 1.

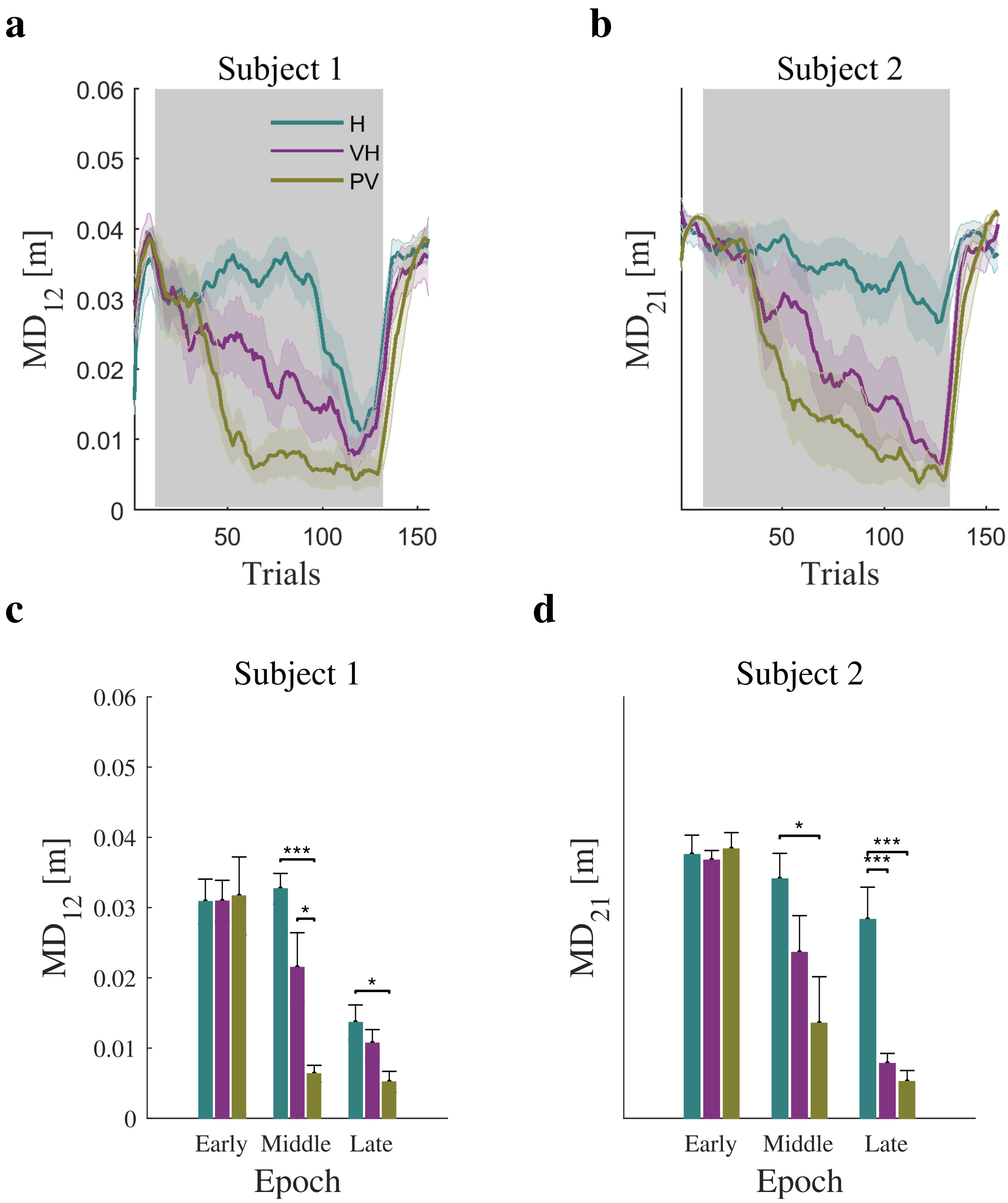


Figure 7.4 **Minimum distance from the partner's VP decreases with training**. Magnitude of the distance from partner's VP for subject 1 (**a**) , subject 2 (**b**), over trials, for the visuo-haptic (VH) , haptic (H) and Partner-visible(PV) group respectively). (**c,d**), Mean of minimum distances of subject 1 and subject 2 at the beginning, middle and at the end of training. The areas in grey denote the training phase. Error bars denote the standard error (SE). Asterisks indicate statistically significant differences ($* : P < 0.05$, $** : P < 0.005$, $*** : P < 0.0005$)

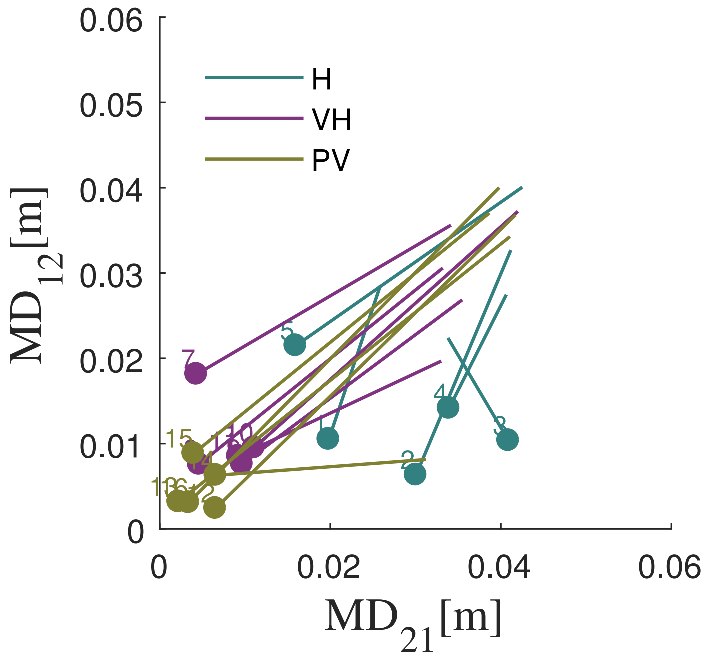


Figure 7.5 **Learning in dyads**. Minimum distance from partner's VP for subject 1 (MD_{12}) vs subject 2 (MD_{21}) from baseline to end training (filled circles) for all three groups. The PV group ended up with better learning by minimising distance at the end of training which is followed by VH and H groups

Subjects also consistently adapted with catch trials throughout the training phase. We found significant catch trial error ($F_{2,36} = 15.11$; $P < 10^{-4}$), which shows the adaptation within the training phase. However, we found neither group nor group $\times$ time effects.

Overall, these results suggest that in the PV group learning is faster and results in a better performance, followed by VH (greater score, lower interaction force).

7.3.2 Optimal interaction: model predictions

The above results still say little on how the collaboration is developed and about the underlying mechanisms.

We developed a computational model, based on differential game theory Başar and Olsder (1999), to predict the 'optimal' interaction behaviours. We modelled the dyad dynamics and the subjects' sensory systems as a pair of point masses connected by a spring. We assumed that each subject operates his/her own point mass by applying a force to it. As in the H group in this experiment, we also assumed that each partner's sensory system provides visual and proprioceptive information about his/her own position, plus haptic information about the interaction force ; the latter indirectly provides information about partner's position. The task is specified by a pair of quadratic cost functionals (one per partner). Consistent with the score provided to subjects at the end of each trial, each cost functional is a combination of distance from own via-point and interaction force with the partner, plus an effort term – see Chapter 6 for details. The interaction strategy is completely specified by a pair of feedback controllers (one per partner); see Figure 6.2. Consistent with computational models of sensorimotor control of individual movements (Shadmehr and Krakauer, 2008), we posited that each partner must use a state observer to predict the dyad state from sensory and motor information. The state observer is easily extended to also account for the partner's control action; see Figure 6.2.

Using the model we simulated an optimal collaboration, which occurs when no partner can improve his/her strategy unilaterally (Nash equilibrium)– see Nash (1951). The two controllers are calculated from differential game theory (Başar and Olsder, 1999), by iteratively solving a system of two Riccati equations. We also considered an opposite scenario, in which the subjects determine their control actions by assuming that they are alone in controlling the dyad dynamics. As a consequence, they just focus on their own via-point and on minimising the interaction. The two controllers are calculated separately, by solving two separate linear quadratic Gaussian (LQG) optimal control problems. In this case, each subject does not need to know what the other partner is doing. Therefore, this scenario defines the maximum compliance with the task achievable with the minimum amount of collaboration between partners. We refer to this scenario as the 'no-partner' strategy – see Figure 6.3. Movement trajectories look similar in the two models, but a closer look suggests that in the no-partner case each subject actively moves toward his/her own via-point – thus behaving as a 'leader', but gets closer to the other only because he/she is pulled by the partner – thus switching to a 'follower' role. This effect is clearly visible when looking at the average interaction power calculated just before crossing the via-point – se Figure 6.3.(f). As a consequence, the no-partner scenario exhibits temporal delays between the via-points crossing times and a greater magnitude of interaction force and interaction power. In contrast, in the optimal (Nash) scenario the two subjects approximately follow the same trajectory, by crossing each via-point at approximately the same time. Both the interaction force and the interaction power remain low over the whole movement, and there are no clear leader-follower roles. Therefore, a distinctive feature of the 'no-partner' scenario is the alternation of leader and follower roles - each subject acts as a leader when crossing his/her own via-point, and as a follower when crossing that of the partner. This is also reflected in the different crossing times (with respect to the leader, the follower lags behind). In conclusion, establishing roles can be seen as a form of compensation for poor integration of the partner's intentions into the subject's own control strategy.

The model was also used to simulate the process of establishing a collaboration through repeated task performance. The model uses a form of 'fictitious play', in which each subject is assumed to estimate the most likely partner's action and to incorporate it into his/her own control policy. The model is attractive as it requires minimum information about the partner – it does not need to establish a model of the partner's task or intentions. We simulated all three scenarios (H, VH, and PV) and found – see Figure 6.5 and 6.6 – that greater information leads to more stable and more 'Nash-like' collaboration, characterised by greater synchronisation and less distinct roles; see Figure 6.7.

7.3.3 Emergence of roles

Motivated by the model predictions, in our experimental data we looked into the emergence of distinctive roles. Figure 7.6 shows the leader-follower strategies from three sample dyads – one per group – as they appear from the sign and magnitude of the interaction power. Figure 7.7 summarises the leadership indices (LI) – average interaction power in the 300 ms interval before via-point crossing – calculated in the late epochs. As regards LI_{11} – leadership index for subject 1 at VP_1 – we found significant differences between H and VH ($t_{6.65} = -2.98, P = 0.022$), H and PV ($t_{5.72} = -4.82, P = 0.003$) and VH and PV ($t_{7.52} = -2.3, P = 0.047$). Similarly, for LI_{12}, we found significant differences for the following pairs: H-VH ($t_{6.58} = 2.42, P = 0.04$), H-PV ($t_{7.32} = 2.87, P = 0.023$) but not VH-PV. Also, we found no significant effects for LI_{21} and LI_{22}.

These results indicate that when there is limited information about the partner (group H), Subject 1 exhibits a transition from a leader role near VP_1 and a follower role near VP_2. The effect decreases and tend to vanish when the amount of available information about the partner increases (from H – minimum information – to PV – maximum information). Although not statistically significant, Subject 2 exhibits a similar trend – leader near VP_2 and follower near VP_1. When comparing these results with the simulations, at the end of the training phase the dyads in the PV group are more similar to the optimal (Nash) strategy. To compare model predictions and experimental results, . For a quick comparison with the model, we calculated the difference in the interaction power for both partners at VP_1 and VP_2 and for the corresponding Nash (green) and 'No partner' (yellow) scenarios. The experimental results are summarised in Figure 7.8. These results suggest that dyads with more available information (PV group) about the partner are closest to the optimum (Nash)

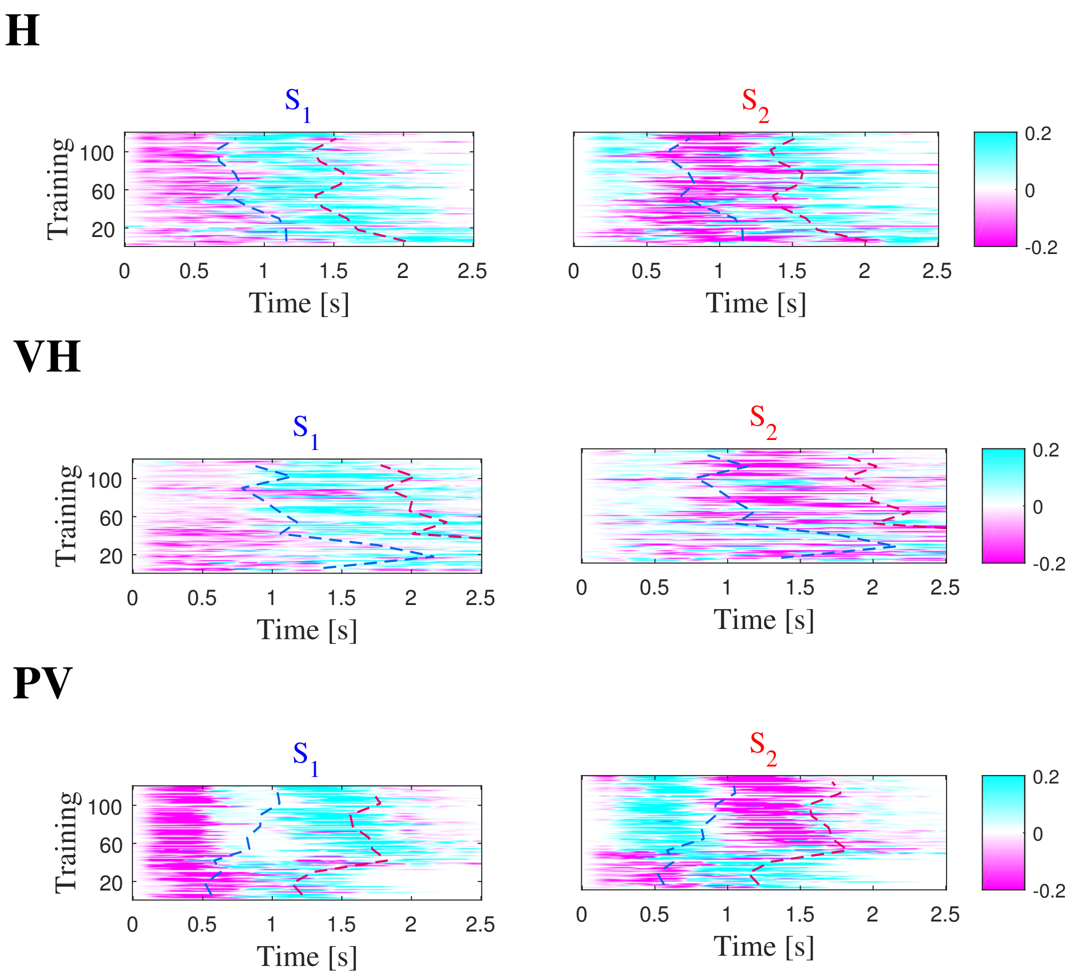


Figure 7.6 **Leader-follower strategies from typical dyads**. From top to bottom: H, VH and PV groups. The crossing times for VP_1 and VP_2 are represented, respectively, by a blue and a red dotted line

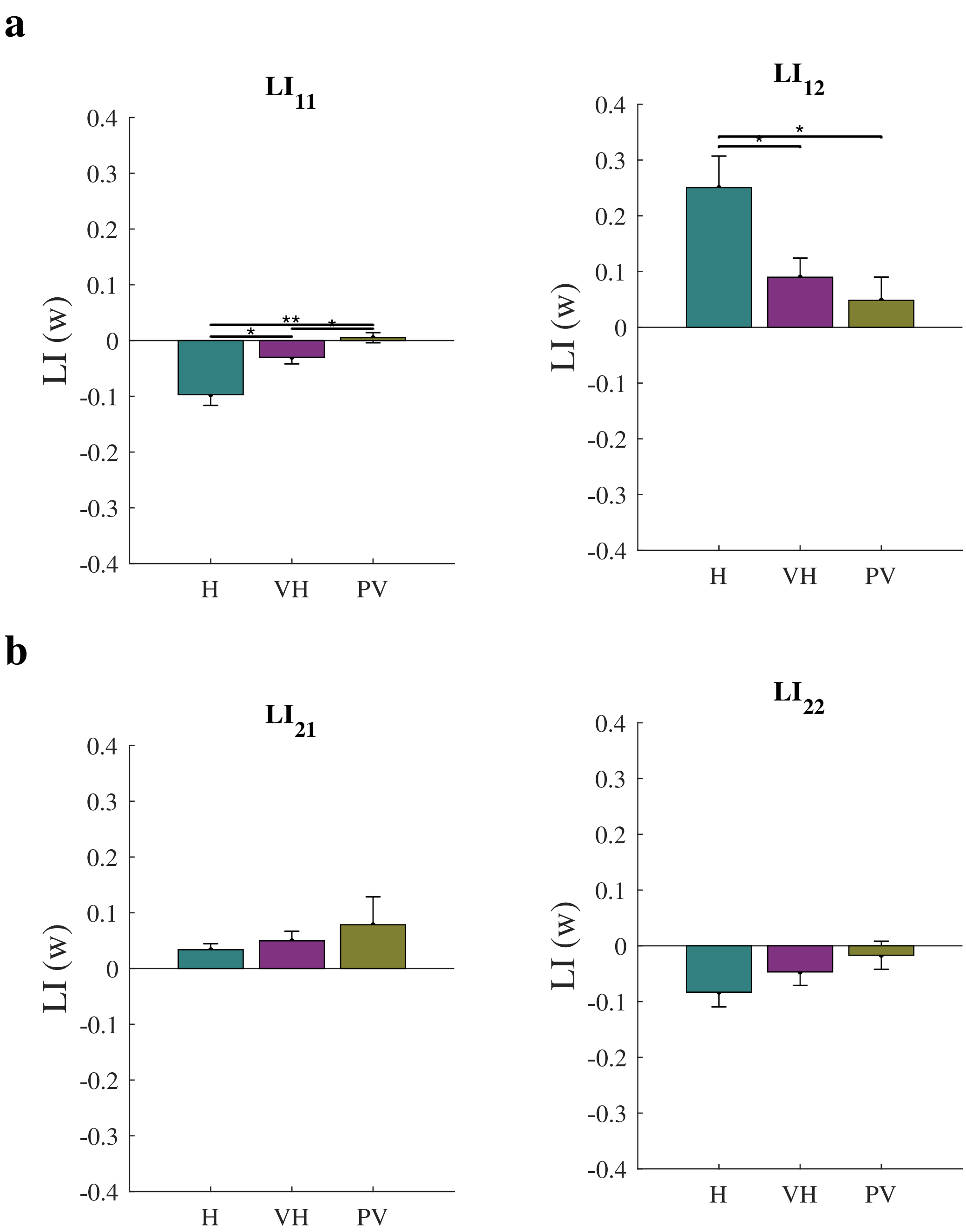


Figure 7.7 **Leadership Index (LI) in late epochs**. LI is calculated as the average power calculated in the 300 ms interval just before each via-points. (**a**), LI_{11} (left) is the leadership index for subject 1 at VP_1 at the late epoch, similarly LI_{12} is for subject 1 at VP_2. Negative value of power shows subject 1 is the leader till his via-point and follower at his partner's via-point. Similarly (**b**), leadership index for subject 2 at VP_1 (left) and VP_2 (right).

scenario, whereas dyads with less reliable information (H group) are closest to the 'no partner' scenario.

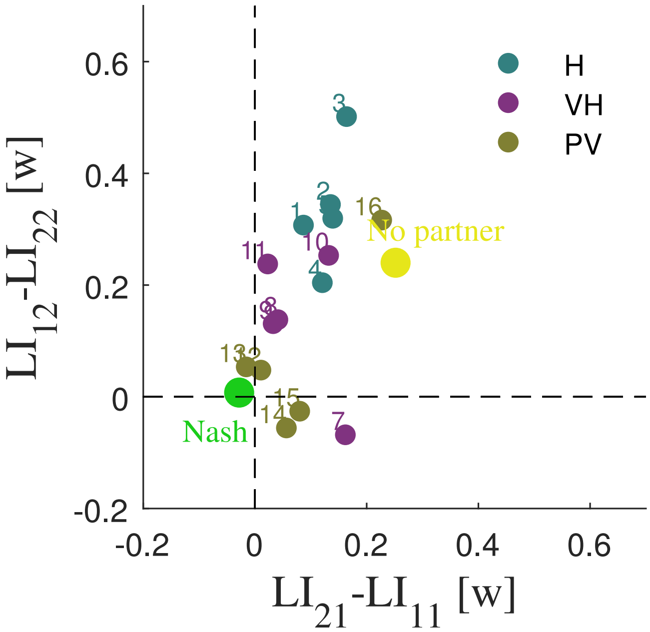


Figure 7.8 **Difference in power for subject pairs at each VPs**. additionally the same parameter for Nash and No partner models are shown. Nash model have more balanced leader-follower strategy at VP, meaning at each VP neither of them leads nor follows. Out of three dyad groups, PV have similar characteristics to Nash model

7.4 Discussion

We investigated how subject pairs in a dyad establish a collaboration in a partially conflicting reward-based point-to-point reaching task – a sensorimotor form of *Battle of sexes* game. We compared the learning performance and the final modality of interaction in dyads performing the same task, but with different types of information about partner's state and movements: haptic information alone (H group); haptic and visual display of the interaction force (VH group); and visual display of partner movements (PV group). We also compared the experimental results with the predictions of a computational model based on differential game theory and the notion of 'fictitious play'. The experiments confirmed the prediction of the learning model, that the type of interaction strategy achieved is determined by the amount and reliability of the information about the partner.

7.4.1 Dyads gradually develop stable strategies

Dyads in all the three groups gradually converged to a stable interaction strategy. The final practice epoch shows less variability than the early stage of training in terms of score, interaction force, minimum distance from partners via point. Convergence required repeated training ('play'). Therefore, stable interaction seems the effect of a learning process.

In a joint reaching task involving two rigidly coupled subjects, Takagi et al. (2016b) did not observe systematic trial-to-trial changes. Rather, subjects relied on a pre-programmed motion plan that was independent of the partner. On the other hand, in subjects with a weaker coupling a learning effect was clearly observed (Ganesh et al., 2014). Likewise, Reed and Peshkin (2008) reported that emergence of stable strategies in the joint control of crank rotations did not occur instantaneously, but required trial-by-trial adaptation. These observations suggest that dyads need time to learn and accustom to an interaction strategy. This is particularly true if task difficulty increases or – as in the present study – if the subjects have different and partly conflicting goals.

7.4.2 The learned interaction is influenced by the amount of available information

In our study, we manipulated the amount of information available about the partner. We found that when more reliable information about the partner actions is available, the interaction strategies come closer to optimal collaboration (Nash equilibrium).

Oguz et al. (2012) studied human-robot interaction in the context of conflicting and collaborative settings and their negotiation behaviours. They found that haptic cues provided a statistically significant increase in the human-recognition accuracy of machine-displayed behaviours.

In a stag-hunt game between a human and a virtual player, Grau-Moya et al. (2013) manipulated the risk sensitivity of the virtual player. They found that humans adapted their behaviours accordingly – by changing the amount of their cooperation. In contrast, in our experiment we did not directly manipulate opponent behaviour. Rather, we altered the amount of available information about him/her.

We used a computational model not only to determine the optimal behaviours predicted by game theory, but also to understand how these behaviours are learned. Our model predicts that dyads converge to a Nash equilibrium if players have reliable information about their opponent. In contrast, if there is more uncertainty on the partner action, the dyad converge to a pragmatic form of interaction, which requires minimal or no negotiation with the partner.

Model predictions are confirmed by our experimental results. Dyads in which more information about the partner is available (PV group) get closer to a Nash strategy. Dyads with less reliable information about the partner (H group) develop strategies that are less sensitive to their partner's actions. These results also suggest that the ability to predict interaction is indicative of a key aspect of 'acting together'.

7.4.3 Taking roles compensates for uncertainty about the partner

The computational model allowed to identify one specific signature of the extent to which subjects use information about their partner's actions when planning their movements. In simulations in which each subject computes his/her control action by ignoring the partner, subjects alternate leader and follower roles within the same movement. Subject 1 leads in via-point 1 and then follows his partner in via-point 2. Conversely, Subject 2 follows his partner in via point 1 and leads at via point 2.

This behaviour can be interpreted in terms of the 'minimum intervention principle' of optimal feedback control (Todorov and Jordan, 2002). For subject 1, via-point 2 is task-irrelevant and therefore getting close to this point is not controlled explicitly; vice versa for subject 2.

We found that dyads characterised by less reliable partner information (group H) converge to this strategy. In contrast, clear roles disappear when information is more reliable – similar to the Nash equilibrium situation. Similarly, in a tracking task Stefanov et al. (2009) identified the same roles – leader, follower, or neither of them.

Does training have an effect on roles? Masumoto and Inui (2014) studied joint action in a task where participants produced discrete isometric forces such that sum of the forces determine the target force. They observed a leader-follower relation in the novice-experienced pairs, but found that practice had no effect in leader-follower relationship. In contrast, in both simulations and experiments we found that roles evolve with the knowledge gained about the partner. As a consequence, in all groups early trials exhibit distinct roles, which gradually disappear as high-information dyads come close to Nash equilibrium.

7.4.4 Do partners understand each other intentions?

One crucial question is whether whether the two partners within the dyad develop an understanding of their partner's 'intentions'. Many studies have suggested that humans actually have this capability. For instance, Grau-Moya et al. (2013) show that subjects adapt their behaviours to changes of their partner's risk variability.

Factors like gaze are known to alter behaviour in joint motor tasks. This effect is easily eliminated by using curtains (Takagi et al., 2016a). In our task, the partners in a dyad are separated by a curtain. Also, they have complete information about their own task, but no information at all about their partner's – in particular, partner's via-point (and whether they have one).

In our experiment, the gradual decrease in the leadership indices suggests that subjects incorporate information about their partner into their motor plans. However, model simulations suggest that minimal partner information about the partner is sufficient to converge to quasi-optimal behaviours (Nash equilibrium). In particular, learning through fictitious play does not require to establish a model of the partner's task – this is what should properly referred as 'intention' – but simply require to account for the partner's most likely control actions – inferred from previous trials. Future experiments, possibly involving generalisation to other tasks or interacting with a virtual partner will be necessary to clarify this important point.

7.5 Conclusions

Several attempts have been made before to use game theory to explain joint sensorimotor behaviours, but until now the emphasis has been on tasks which do not involve different goals for each participant (Takagi et al., 2016a) or in tasks that, although sensorimotor in nature, ultimately focus on making discrete decisions (Braun et al., 2009, 2011; Grau-Moya et al., 2013). Here for the first time we use game theory to understand a purely motor interaction task, in which the goal of collaboration is to develop coordinated dyad movements.

We have shown that information about the partner deeply affects speed and outcome of learning a joint collaboration. In future studies, the same experimental and computational framework could be used to transfer capabilities to establish a collaboration to virtual or artificial agents. This may open the way to novel approaches to e.g. neuromotor rehabilitation, in which an artificial agent – e.g. a robot – automatically adapts its action as a consequence of incorporating a model of the patient's action.

Chapter 8

Sensorimotor joint action in adults with Autism spectrum disorders

> Love, for instance. Everybody experiences it, craves it, requires it for his or her very existence, knows it's there. But no one can explain it, break it down into physics and chemistry.
>
> Rupert Isaacson, *The Horse Boy*

8.1 Introduction

Autism spectrum disorder (ASD) is a developmental disorder characterised by deficits in communication and social skills, and stereotyped and repetitive patterns of behaviour and imagination. Even though motor impairments are not considered a diagnostic feature of ASD, motor deficits have reported in many studies (Dowell et al., 2009; Dziuk et al., 2007; Gowen and Hamilton, 2013; Jansiewicz et al., 2006). Similarly, abnormalities in sensory function have been repeatedly demonstrated in few studies (Blakemore et al., 2006; Leekam et al., 2007; Nakano et al., 2012; Paton et al., 2012), but ASD lacks any clear signature impairment. Rather, sensory and motor impairments manifested in ASD conditions lack any clinical classification and are broad and generalised – see Chapter 3 for a more comprehensive review. On our activities of daily living, deficits in motor abilities can have far-reaching effects, as motor control is believed essential for language, communication, decision making and even understanding of others thoughts, intention and to act jointly (Gallese et al., 2004; Iacoboni, 2009).

It is likely that motor impairments shown by adolescents with ASD may be present right from their infancy (Teitelbaum et al., 1998) and are rooted in the ability to acquire motor learning or new motor skills. In sensorimotor tasks, interestingly the effect of these motor impairments sometimes leads to improved performance in some aspects of motor control relative to healthy individuals. For instance in a set of studies Haswell et al. (2009) and Izawa et al. (2012) assessed generalisation of motor learning during adaptation. Children with ASD show increased generalisation in the proprioceptive coordinates. Similarly, in a haptic-to-visual shape matching task in which subjects explored an object by touching and tracing with their fingers, Nakano et al. (2012) showed that individuals with ASD are better at recognising the object. Further, individuals with ASD have less susceptibility to proprioceptive drift during a rubber band illusion task (Paton et al., 2012) and show lower thresholds in the tactile perception of vibrotactile stimulus (Blakemore et al., 2006).

Predicting the consequences of own motor action or about future sensory events is a fundamental property of our cognition to enable us to adapt our actions and behaviours and to interact with the world around us. Empirical evidence supporting the hypothesis that internal models are impaired in ASD is highly controversial – see (Gowen and Hamilton, 2013) for a review. Many behavioural studies failed to find a difference in prediction at both motor and perceptual levels in individuals with ASD (Blakemore et al., 2006; Ego et al., 2016; Haswell et al., 2009; Larson et al., 2008b; Marko et al., 2015). Our daily social life is based on our capacity to understand the behaviour of others. How does one understand the goal of his partner by looking at the motor action? Rizzolatti and Craighero (2004) reported that humans and monkeys possess a system of neurons called mirror neuron system, which maps visual description of actions by others onto their partner's motor representation of the same action. Subsequent studies demonstrated various functional aspects of the mirror neuron system in mediating imitation (Bird et al., 2007; Iacoboni et al., 1999), understanding intention of others (Iacoboni et al., 2005), and emotion recognition (Gallese et al., 2004). Does this aspect of cognition have any implication on sensorimotor joint action in individual with ASD?

Here we address how negotiation of a joint action evolves in persons with ASD and in typically developing (TD) individuals. We specifically looked at differences between dyads of TD individuals and mixed dyads, involving one ASD and one TD individual.

8.2 Materials and Methods

8.2.1 Participants

We recruited 26 subjects, aged 22-27 years. Twenty were typically developing (TD) subjects, and six were diagnosed with high-functioning ASD with no intellectual impairments – see Table 8.1. ASD participants were recruited at the local psychiatric outpatient facility (Centro Interzonale Autismo, ASL3 Genovese). The protocol and procedures were conducted in accordance with the Declaration of Helsinki. All participants provided written informed consent. Experiments took place in a dedicated room with the presence of a professional educator and a psychologist.

Table 8.1 **Participants demographics for study 2**. For the two groups, we indicate the range and mean ($\pm$ SE) age, the total number and the number of males, weight, height and the Autism-Spectrum Quotient (AQ) score

Group	# Subjects	Label	Age	Sex	Weight (kg)	Height (m)	AQ score
TD	20	S1-S20	25 ± 3	10 M+ 10 F	70 ± 20	1.73 ± 0.2	12 ± 9
ASD	6	S21-26	25 ± 3	6 M+ 0 F	76 ± 11	1.78 ± 0.03	22 ± 8

Autism diagnosis was established using the Autism Diagnostic Interview – Revised (ADI-R) and were confirmed by a child psychiatrist with experience of autism spectrum diagnosis. Participants were excluded if they had other neuromotor disorders, obsessive compulsive disorder, attention deficit hyperactivity disorder (ADHD), with exception of anxiety disorder. Before the experiment session, each individual were evaluated with Autism-Spectrum Quotient (AQ) questionnaire (Baron-Cohen et al., 2001b), Italian version (Ruta et al., 2012). The AQ questionnaire comprises 50 questions, assessing 5 different areas (10 questions each): social skills, attention switching, attention to detail, communication, and imagination.

Participants were paired by matching their body mass index (BMI) and their Edinburgh Handedness score Oldfield (1971) and were grouped into 2 categories (either TD + TD, control dyad or ASD + TD, mixed dyad). The demographic data of dyad groups are shown in Table 8.2.

Table 8.2 **Dyad description**. Subjects were grouped into 2 categories (control or mixed) by matching with their BMI.

Dyad #	Type	$Subject_1$	BMI_1	$Subject_2$	BMI_2
D1	control	S1	22.6	S11	20.2
D2	control	S2	18.9	S12	19.5
D3	control	S3	23.4	S13	21.5
D4	control	S4	17.9	S14	18.8
D5	control	S5	23.5	S15	20.7
D6	control	S6	19.1	S16	19.6
D7	control	S7	23.6	S17	21.8
D8	mixed	S21	24.1	S8	26.5
D9	mixed	S22	20.9	S9	21.2
D10	mixed	S23	20.7	S10	21.6
D11	mixed	S24	24.9	S18	26.1
D12	mixed	S25	24.5	S19	26.2
D13	mixed	S26	22.6	S20	23.3

8.2.2 Experimental setup and protocol

The joint motor task for the this study was a modified version of the task used for healthy subjects in the previous chapter. Briefly, subjects sat in front of a computer screen and grasped the handle of a three-dimensional haptic interface (Novint Falcon). They could not see or hear each other, and were not allowed to talk. The experimental apparatus is depicted in Figure 5.1.a. The subjects were instructed to perform reaching movements in the vertical plane, between the same start point (displayed as a white circle, $\oslash$ 1 cm) and the same target point (yellow circle, $\oslash$ 1 cm), but through different via-points. In a reference frame centred on the robot workspace (one for each subject), with the X axis aligned with the left-right direction and the Y axis aligned with the vertical direction, the start point was placed in the (-5, 0, 0) cm position and the target point was placed in the (5, 0, 0) cm position. Hence the start and the target point had a horizontal distance of 10 cm. The subjects were also instructed to keep their movements as planar as possible, i.e. by keeping the depth, Z coordinate within the range (18-26 cm) from the origin of the workspace. The current positions of the end effectors, x_1 and x_2, were continuously displayed to each partner, as $\oslash$ 0.5 cm circular cursors on their respective screens, coloured in green if the depth was correct and in red otherwise.

In a separate experiment – an haptic tracking task – we assessed the participants' ability to perceive an haptic force during movements guided by an expert – see Figure 8.1.b.

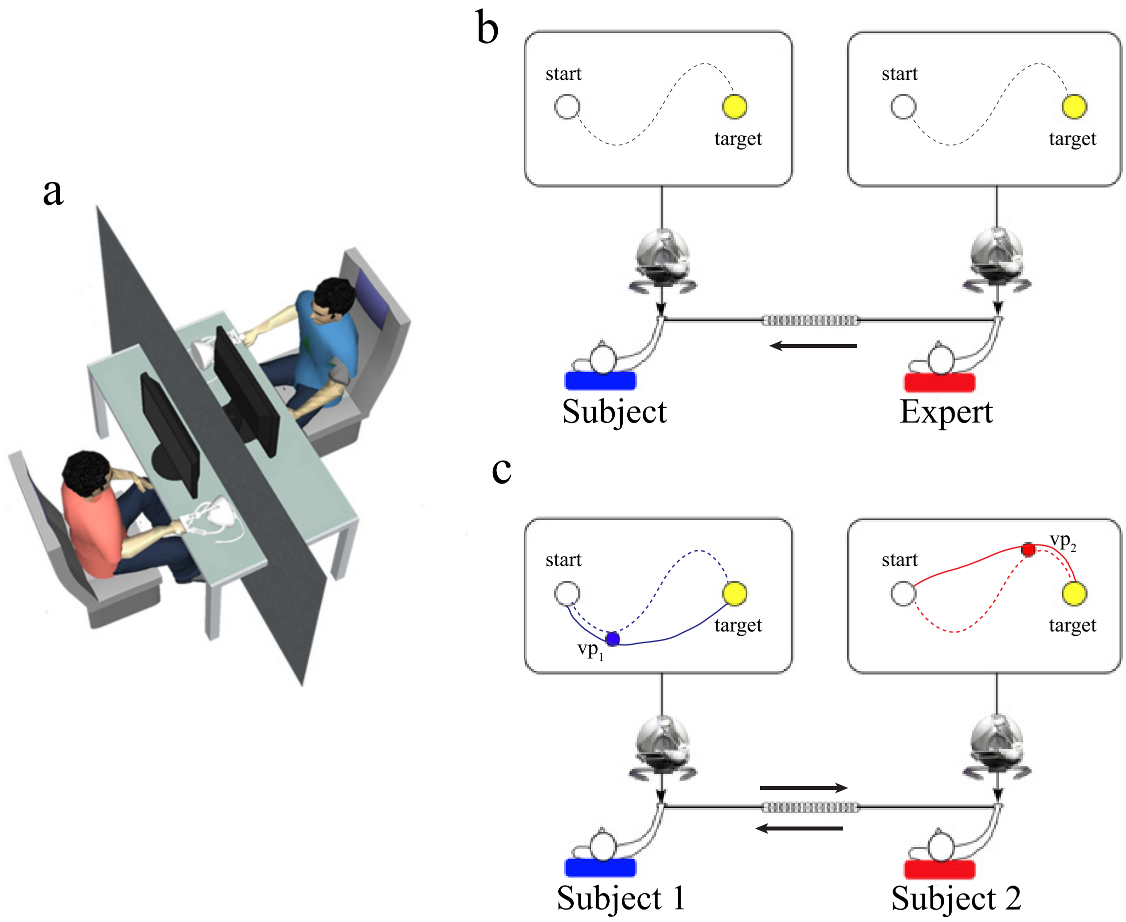


Figure 8.1 **Experimental apparatus and protocol**. (**a**), Partners in a dyad were connected through a virtual spring. (**b**), Force perception task. Subjects must track the movements of a 'master', by feeling the interaction force. (**c**), Joint action task

The experiment was organised into epochs of 12 movements each. The experimental protocol consisted of three phases: (i) baseline (3 epochs), (ii) training (12 epochs) and (iii) after-effect (2 epochs) for a total of 18 $\times$ 12 = 216 movements. During the baseline phase the interaction forces were turned off, and each subject performed on their own ('solo' performance). During the training phase the subjects were mechanically connected, but in randomly selected trials (catch trials) within each epoch (1/6 of the total, i.e. 2 trials per epoch) the connection was removed. The connection was permanently removed during the after-effect phase. The experiment duration for joint motor action task was about 45 minutes – see Figure 8.1.(c).

The experimental protocol for the haptic tracking task consisted of two phases: (i) baseline (one epoch), and (ii) training (five epochs) for a total of 6 $\times$ 12 = 72 movements. Each experiment lasted for about 20 minutes. The two tasks were administered in random order.

To modify the paradigm to make it more suitable for ASD subjects, we created three game-like scenarios –see Figure 8.2. We replaced start, target and via-points with images and made it more colourful and engaging. Additionally, we provided 2 baseline sessions for familiarisation with the setup and task. After each movement, a score was displayed – computed as explained in the previous chapter – accompanied by different sounds of appreciation.

8.2.3 Data analysis

Hand trajectories and robot-generated forces were sampled at 100 Hz and stored for subsequent analysis. The data samples were smoothed by means of a 4th order Savitzky-Golay filter with a 370 ms time window (cut-off frequency: 7.5 Hz). We used the same filter to estimate velocity and acceleration. We identified the start and end times of each trajectory as, the time instants at which the speed crossed a threshold of 2 cm/s.

In the analysis, we specifically focused on the temporal evolution of the trajectories and on signs of collaboration between partners within the same dyad. Collaboration can be characterised in terms of both movement kinematics and movement kinetics. We developed a number of performance indicators to characterise collaboration. Data were analysed using MATLAB.

Most of the performance indicators are defined as same as in the previous chapter. In short, we have interaction force (IF) as dyad level indicator, is calculated as $IF = \frac{1}{T}\sum_t \|F(t)\|$,

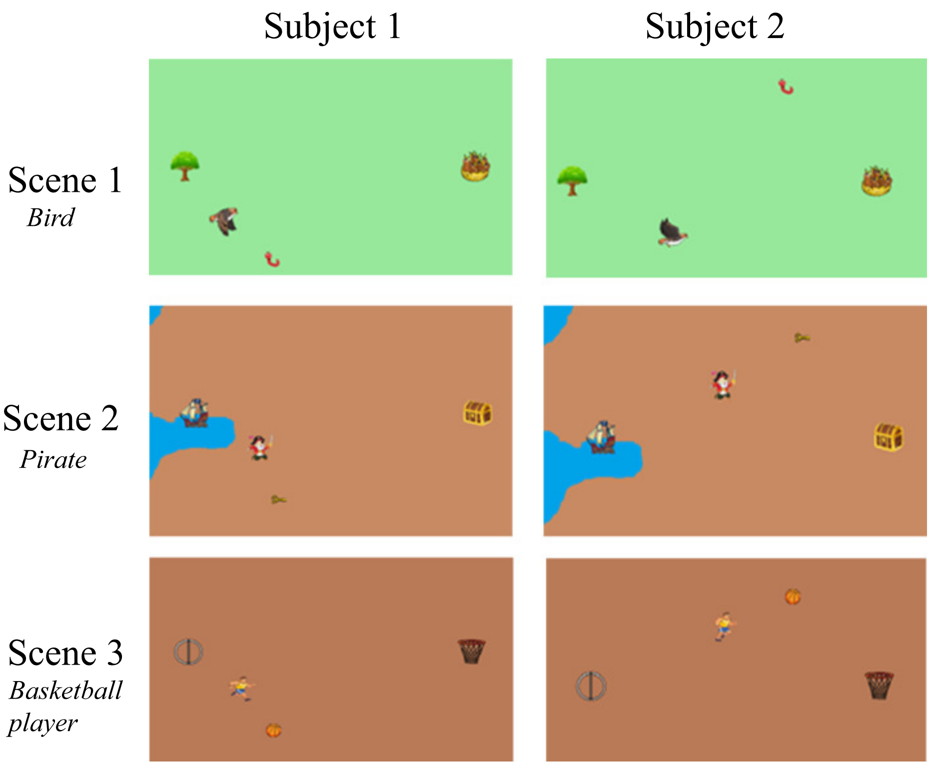


Figure 8.2 **Graphic environment**. The graphic environment includes three different scenarios, switched after each epoch of training to keep the level of motivation of the participants. The same environments have been used both experiments. Each scene consists of a background and four small images, called sprites, each corresponding to one of the characteristic elements of each our basic motor task, the start and end points and the two via points. The cursor is one animated sprite, obtained by alternating the display of two still images, which gives more reality to each of these scenes. For example, in scene 1, a tree (start) and nest (target), a bird (cursor) have to fly from tree to nest by picking a worm (via point). Similarly, for scene 2, a pirate has to move from ship to a treasure box by picking a key, in scene 3, a player has to run from start point to basket by collecting a ball. Audio queues were provided at the start and end of the movements.

where $F(t)$ is the interaction force (equal and opposite for the two partners in the dyad). less interaction force would point at a greater collaboration.

A sign of collaboration is that each subject, while passing through his/her own via-point, also gets very close to his/her partner's. This can be quantified in terms of the Minimum via-point distance (MD_{ij}), defined as the minimum value of the distance of subject i to the j-th via-point($MD_{ij} = \min_t ||x_i(t) - x_{VPj}||$ with $i, j = 1, 2$. If $i \neq j$ this quantity reflects how close each subject gets from his/her partner's via-point.

Looking at the power developed by each subject would provide information on whether the subjects move actively, or are passively pulled by their partner through the mechanical coupling. To quantify this, we calculated the power (P_i), defined as the scalar product of the interaction force $F_i(t)$ and the velocity vector $v_i(t)$ of each of the subjects. We specifically focused on the average power calculated in the 300ms interval taken just before the crossing of each via-point. We defined this as Leadership Index, denoted as LI_{ij} this value for the i-th subject and the j-th via-point.

As for the haptic tracking task, as indicator of the performance of each subject we took the mean value of Tracking Error (TE_i) , defined as the average difference between the $x_i(t)$ positions of the subject's hand with that of an expert, $x_M(t)$ for each movement ($TE_i = \frac{1}{T} \sum_t \|x_i(t) - x_M(t)\|$).

We expect that task performance at subjects and dyad level evolves with time (learning) and is affected by the amount of information that each subject has available about his/her own partner. To test this, for all the above indicators we ran a repeated-measures ANOVA with group – control dyad (TD+TD) vs mixed dyad (ASD+TD) – and Time (early - epoch 1 and late - epoch 11) as factors. If a main effect was found, Tukey's HSD (honest significant difference) post-hoc test was used to examine the differences. Data were analysed using MATLAB and statistical tests were performed using R and Microsoft Excel. Statistical significance was considered at $P < 0.05$ level for all tests.

8.3 Results

8.3.1 Questionnaire data

We first looked at the AQ score of the two subjects groups. The score of subjects in the TD group was 12 ± 9 (mean ± SE) whereas that of ASD subjects was 22 ± 8 – see also Table 8.1. The difference between the two groups (TD vs ASD) is statistically significant (t-test 2 tailed for unequal variances, $t_6 = -3.6, P = 0.01$). Significant differences were also

Table 8.3 **Summary of AQ score for all participants**

No	Subject label	Type	Total AQ	Social skill	Attention switching	Local details	Imagination	Communication
1	S1	TD	13	0	2	5	4	2
2	S2	TD	12	1	3	4	1	3
3	S3	TD	17	5	3	4	3	2
4	S4	TD	20	0	4	9	6	1
5	S5	TD	9	1	3	3	1	1
6	S6	TD	13	2	2	6	2	1
7	S7	TD	4	0	1	1	2	0
8	S8	TD	8	1	3	3	0	1
9	S9	TD	15	1	4	7	1	2
10	S10	TD	15	2	4	4	3	2
11	S11	TD	15	1	6	3	3	2
12	S12	TD	16	1	5	4	4	2
13	S13	TD	11	0	5	2	1	3
14	S14	TD	11	1	1	6	2	1
15	S15	TD	17	0	7	5	2	3
16	S16	TD	8	1	1	2	1	3
17	S17	TD	18	1	5	6	3	3
18	S18	TD	13	3	3	3	0	4
19	S19	TD	11	1	1	2	4	3
20	S20	TD	3	0	1	2	0	0
21	S21	ASD	14	1	1	5	2	5
22	S22	ASD	28	4	5	4	7	8
23	S23	ASD	26	4	6	4	6	6
24	S24	ASD	20	4	6	5	2	3
25	S25	ASD	30	8	9	4	7	2
26	S26	ASD	11	2	1	4	4	0

found when looking at individual subsections of the AQ questionnaire, namely social skill ($t_6 = -3.9, P = 0.008$), and imagination ($t_7 = -3.1, P = 0.01$), but not attention switching ($t_7 = -1.9, P = 0.08$), attention to local detail ($t_{14} = -0.86, P = 0.41$), or communication ($t_5 = -1.7, P = 0.14$). The AQ score for individual subjects are displayed in Table 8.3

8.3.2 Movement trajectories

We are specifically interested in comparing the interaction strategies in a mixed dyad with participants with ASD and age-matched, BMI matched control dyads. We studied a total of thirteen dyads, out of which seven control (TD+TD) and six mixed (ASD+TD). The

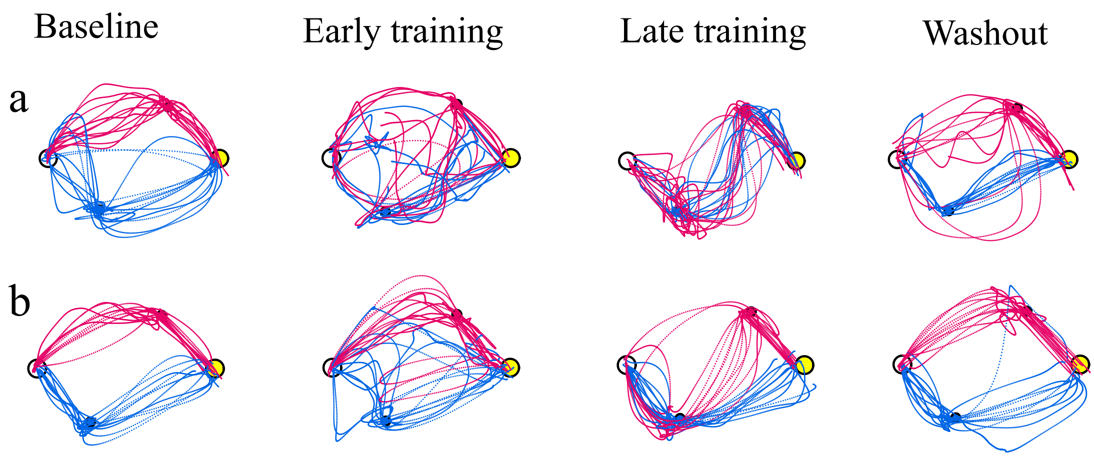


Figure 8.3 **Movement trajectories**. (**a**), Movement trajectory of control dyads (TD+TD). Subject 1 (blue), subject 2 (red). (**b**), Mixed dyad (ASD+TD)

movement trajectories from two typical control and mixed dyads are displayed in Figure 8.3. Movements were categorised as early training, late training and after-effects. In the control (TD+TD) group, dyads generally converged to stable and consistent behaviours. Only one subject in the control group exhibited poor convergence. In the mixed (ASD+TD) group, half of the ASD subjects (three out of six) exhibited few signs of convergence to a joint behaviour. The other ASD subjects looked more similar to TD subjects. When the connection was removed, both TD and ASD subjects quickly returned to the baseline situation. Only few subjects showed retention in the after-effect phase.

8.3.3 Dyad behaviour: Interaction force and score

The overall temporal evolution of interaction force and average dyad score for both groups are shown in Figure 8.4. Overall, subject pairs improved their score with training ($F_{1,10} = 11.05$; $P = 0.0005$) but we did not find any difference between the groups ($P = 0.45$). Interaction force also decreases with training ($F_{1,10} = 15.35$; $P = 0.003$) but, again, we found no differences between groups ($P = 0.69$).

8.3.4 Minimum distance to partner's via-point

A similar behaviour can be observed in the temporal evolution of the minimum distance from the partner's via-point – see Figure 8.5. In both groups and in both subjects in the dyad, the minimum distance (MD) decreases over trials and quickly washes out when the connection is permanently removed (after-effect phase). The overall time effect was significant for

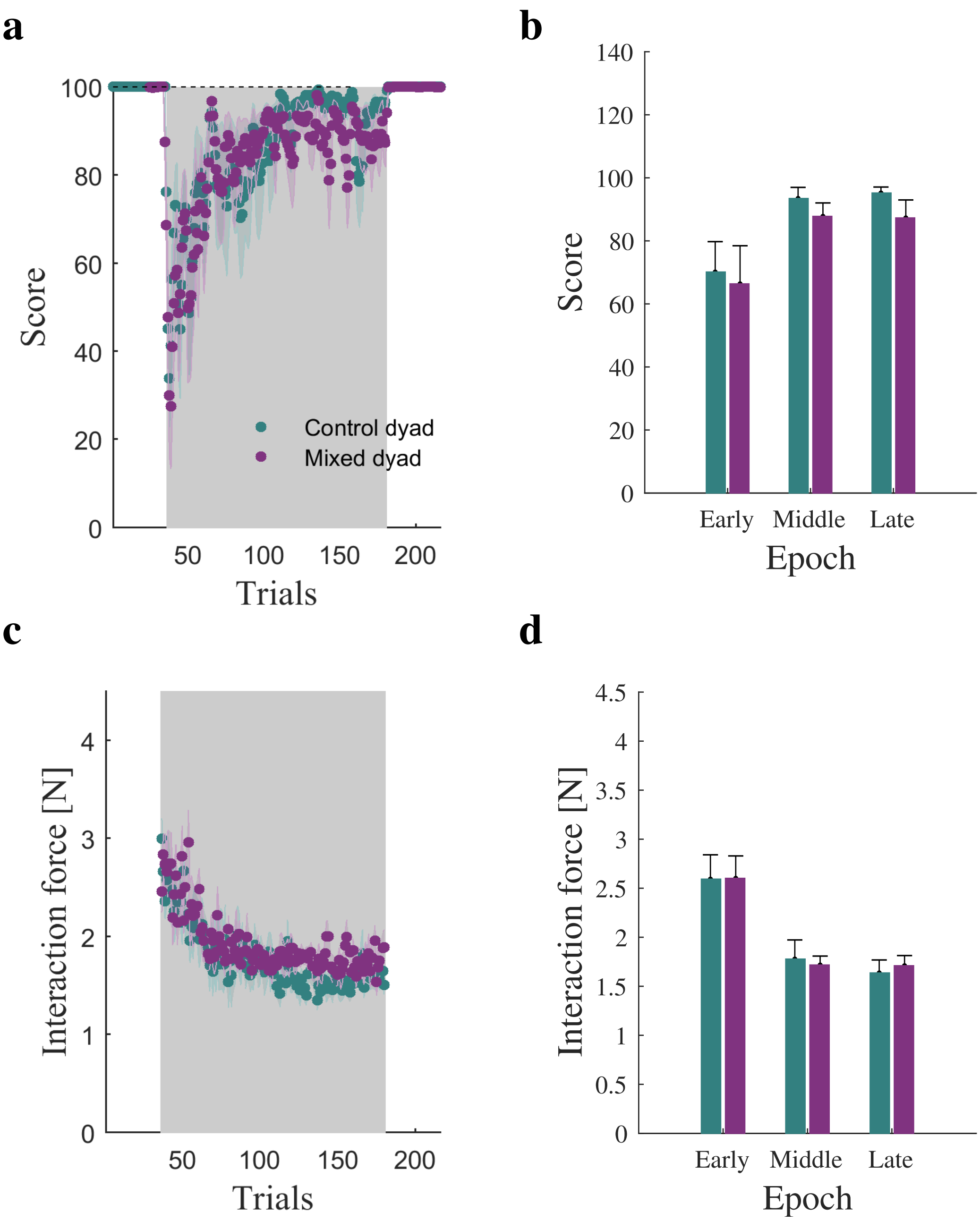


Figure 8.4 **Score and Interaction force change with training**. Magnitude of score (**a**) , Magnitude of the interaction force (rms), (**c**), over trials, for the control dyad , mixed dyad groups respectively). (**b**), Score at the beginning, middle and at the end of training. (**d**), Interaction force at the beginning, middle and at the end of training. The areas in grey denote the training phase. Error bars denote the standard error (SE)

both Subject 1 and Subject 2 (MD_{12}: $F_{1,10} = 18.19$; $P = 0.0016$; MD_{21}: $F_{1,10} = 22.58$; $P = 0.0008$). However, we found no significant group effects and no significant group-time interactions. Although the ASD subject (subject 1) in the mixed group exhibited on average a greater minimum distance to VP_2 (MD_{12}) at the end of training, post-hoc analysis revealed no significant group differences in the final (late) minimum distances. Figure 8.6 summarises the effect of learning in terms of MD in both groups.

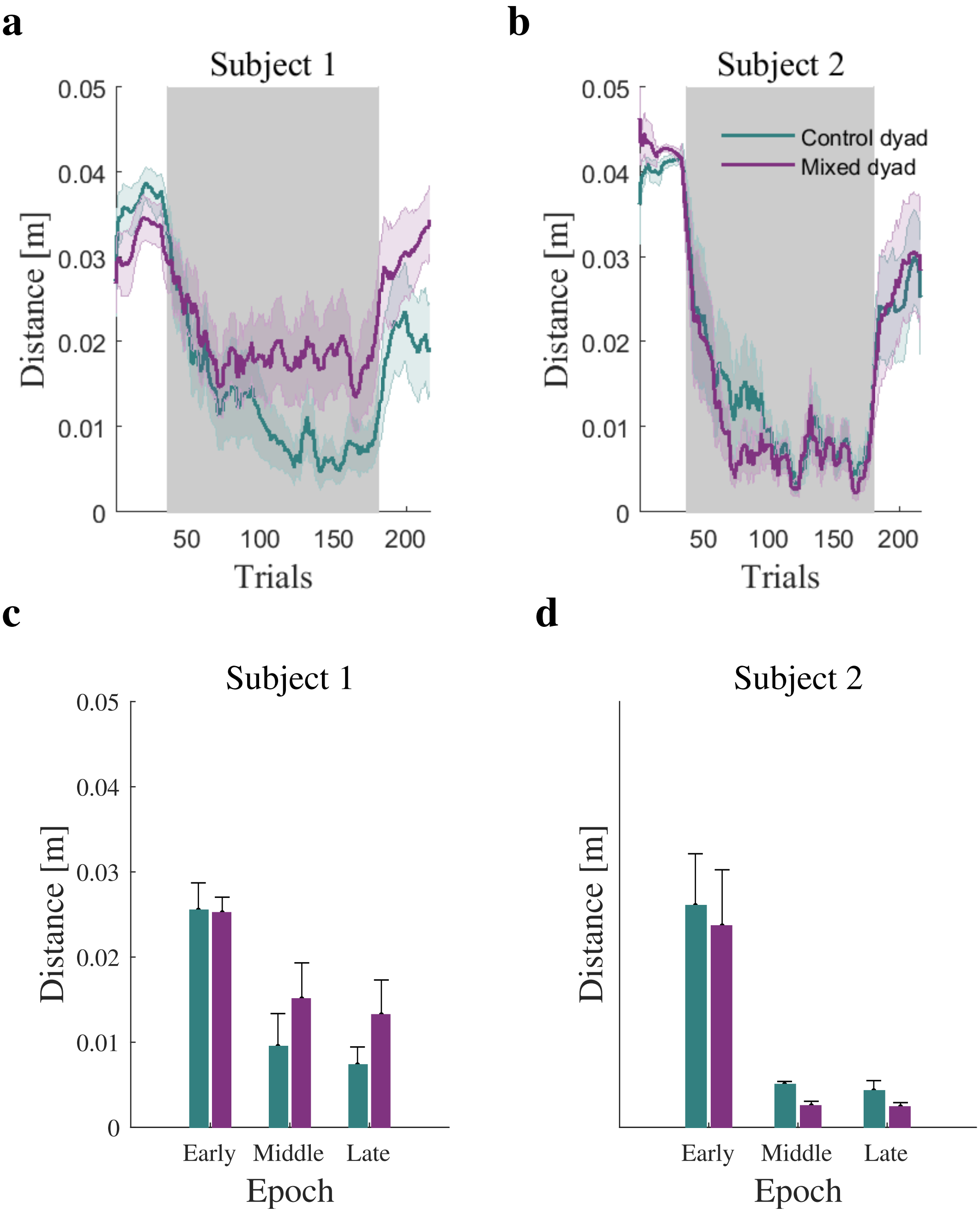


Figure 8.5 **Minimum distance in dyad groups**. (**a,b**). Minimum distance from partner's via-point for subject 1 (MD_{12}) vs subject 2 (MD_{21}) for control dyads and mixed dyads (**b**). Mean of minimum distance for each dyad groups in the early, middle and late training phase for subject 1 and subject 2 (**c,d**). In mixed dyads, Subject 1 is always ASD

Figure 8.6 summarises the findings at individual subjects level. The figure confirms the initial observations, that only one subject in the control group exhibits poor convergence of MD_{12}. In the mixed group (ASD+TD), half of the ASD subjects (three out of six) did not reduce their MD_{12} after training, whereas the other ASD subjects looked more similar to TD subjects.

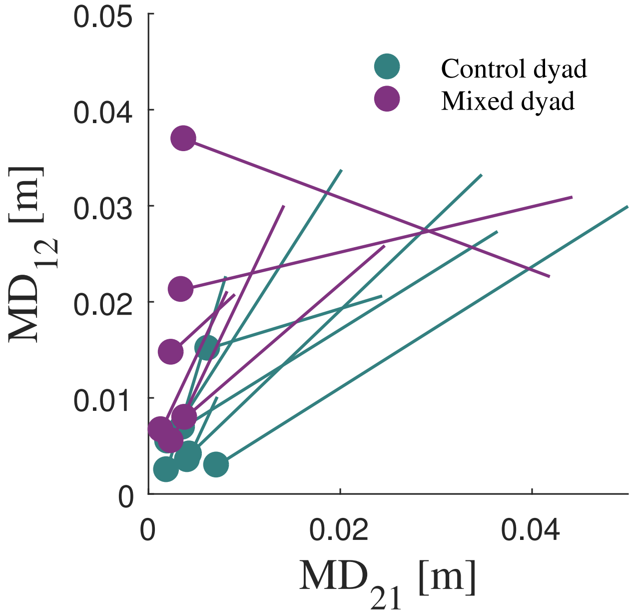


Figure 8.6 **Learning in dyads**. Minimum distance from partner's VP for subject 1 (MD_{12}) vs subject 2 (MD_{21}) from baseline to end training (filled circles) for both groups.

8.3.5 Leadership index and emergence of roles

We then examined if there is a specialised leader-follower strategy in mixed dyads. Figures 8.7 and 8.8 show the leader-follower strategies – expressed in terms of interaction power – from the control and mixed dyads, for both subject 1 and subject 2.

For a more quantitative analysis at population level, we focused on the leadership indices (LI) for both subjects at both via-points, at the end of training ('late' epoch). These results are summarised in Figure 8.9. As expected, the subjects in the control (TD+TD) group alternated leader roles when crossing their own via-point (negative LI_{11} and LI_{22}) and follower roles when crossing their partner's (positive LI_{12} and LI_{21}).

The mixed dyads exhibited a qualitatively different behaviour. When crossing VP_1 – where Subject 1 (ASD) is aiming – both subjects exhibit small interaction power, suggesting

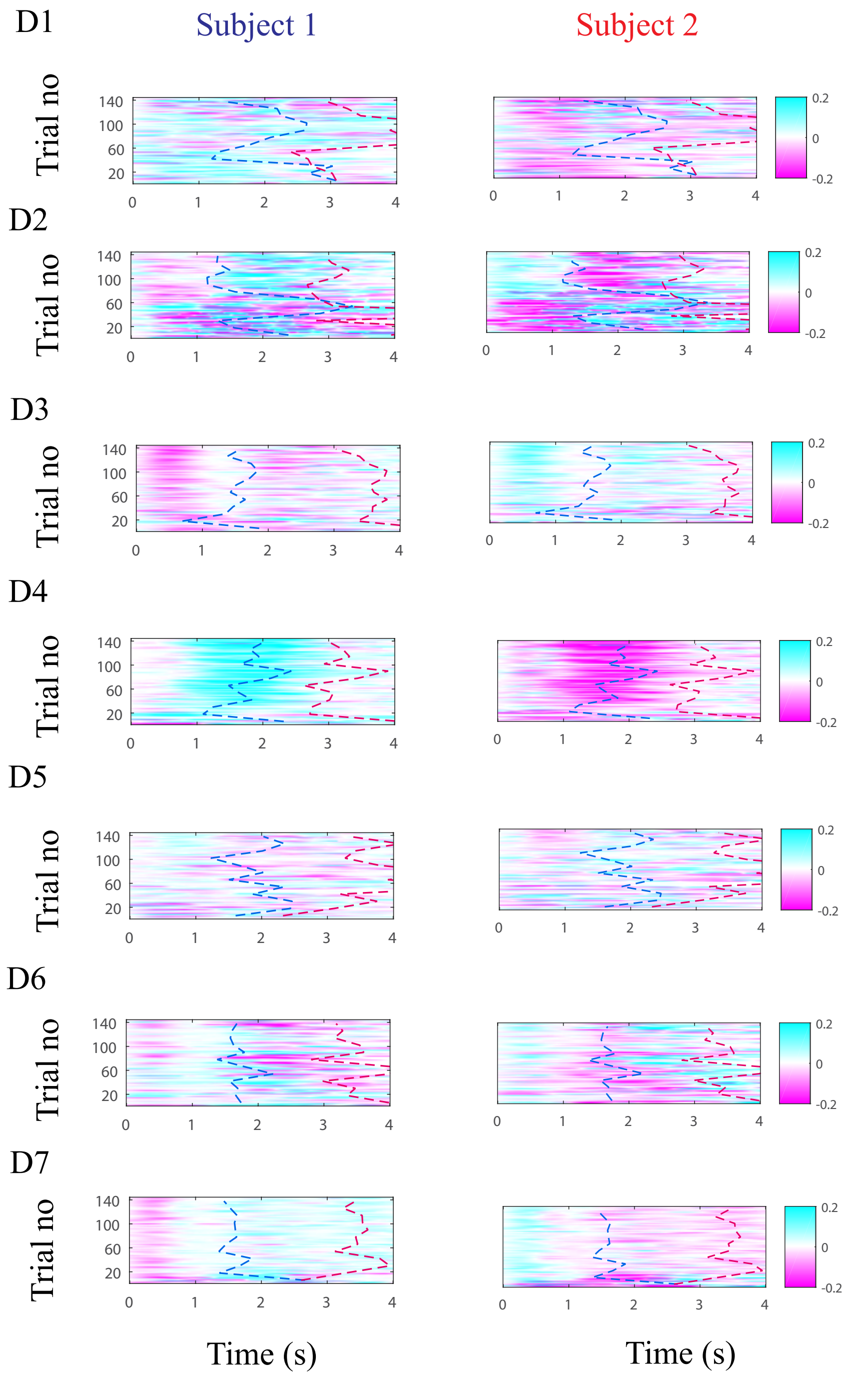


Figure 8.7 **Leader-follower strategy in control dyads**. The colours indicate the magnitude of the interaction power, per time instant and per trial. Magenta: negative power, i.e. leader; Cyan: positive power, i.e. follower. Blue dotted lines indicate the time of crossing of via-point 1. Red dotted lines represent the average time of crossing of via-point 2

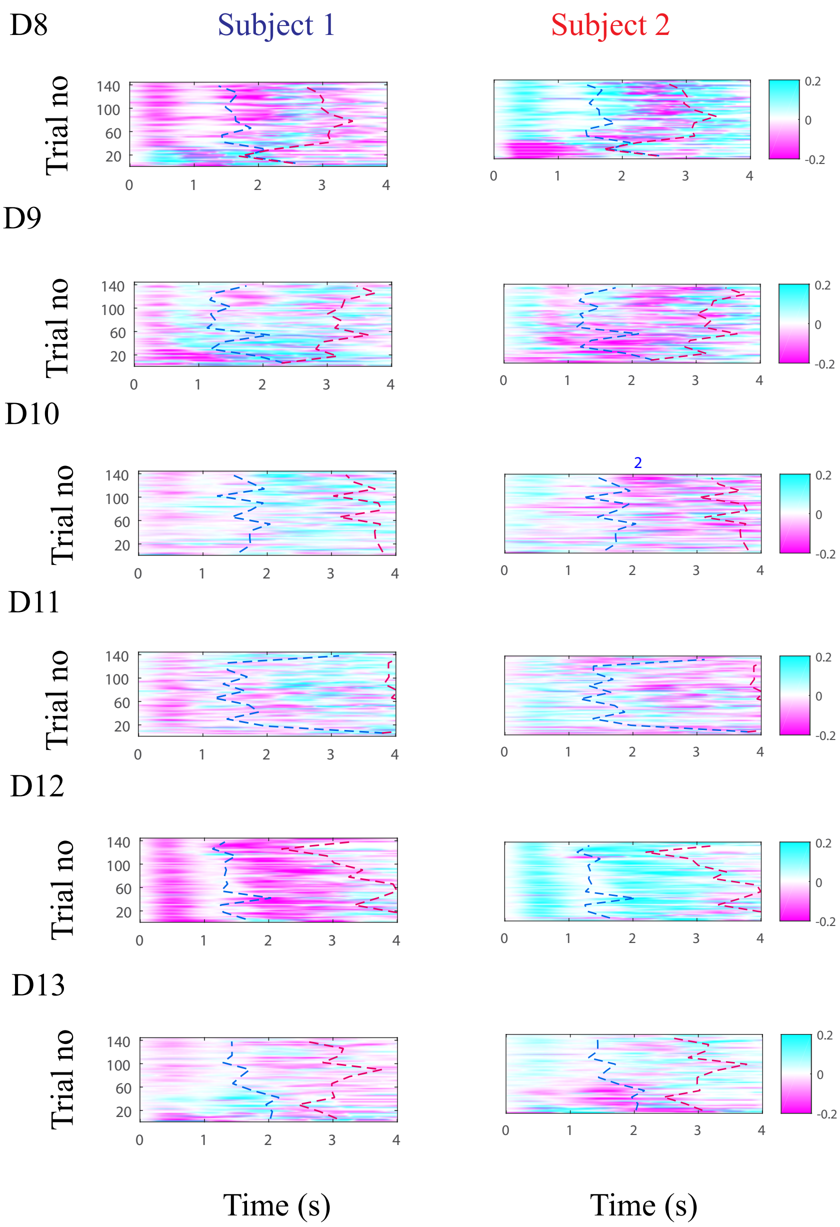


Figure 8.8 **Leader-follower strategies in mixed dyads**. The colours indicate the magnitude of the interaction power, per time instant and per trial. Magenta: negative power, i.e. leader; Cyan: positive power, i.e. follower. Blue dotted lines indicate the time of crossing of via-point 1. Red dotted lines represent the average time of crossing of via-point 2

close collaboration. However, when crossing VP_2, not only Subject 2 (TD) but also Subject 1 (ASD) behave as leaders. In other words, both subjects move in such a way that their interaction forces increase. Subject 2 moves toward his via-point, but Subject 1 moves away from it. This is a sign that Subject 1 moves against collaboration. However this behaviour is not consistent among all ASD subjects, so that we found no group difference for any of LI_{11}, LI_{12}, LI_{21}, LI_{22}.

8.3.6 Tracking error

We then turned to the results of the haptic perception experiment. The tracking error (TE) is shown in Figure 8.10.(a). Although some ASD subject exhibited a greater error, we found no significant group difference in haptic force perception capability.

Can the poor performance of some ASD subjects in the joint task (Experiment 1) be explained by poor haptic perception? To test this, we looked at the correlation between minimum distance from the partner's via-point (MD) in both dyads with the tracking error measured in Experiment 2; see Figure 8.10.(b). Indeed we found a significant ($N = 26$, $R = 0.206$, $P = 0.01$), but only one ASD subject exhibiting large MD_{12} also exhibited a large TE. For this subject the inability to collaborate may be explained by a poor haptic perception, but this does not seem to be the case in general.

8.3.7 Correlation with the AQ score

To further analyse the variability of performance observed in ASD subjects, we also looked at the correlation between MD, TE and the total AQ score – see Figure 8.11.(a,b). Both correlations were not significant for either TD or ASD subjects. Somehow surprisingly, the ASD subjects exhibiting lack of collaboration were the ones with a closer to normal AQ score (Figure 8.11).

8.4 Discussion

The main objective of the study was to understand if people with ASD are capable of establishing collaboration with a partner in a partially conflicting joint motor task. I adopted the same paradigm used in Chapter 7. I compare the joint motor action strategy of a typically developing subject pair (control dyad) with that performed by a mixed dyad formed by an ASD and a typically developing subject.

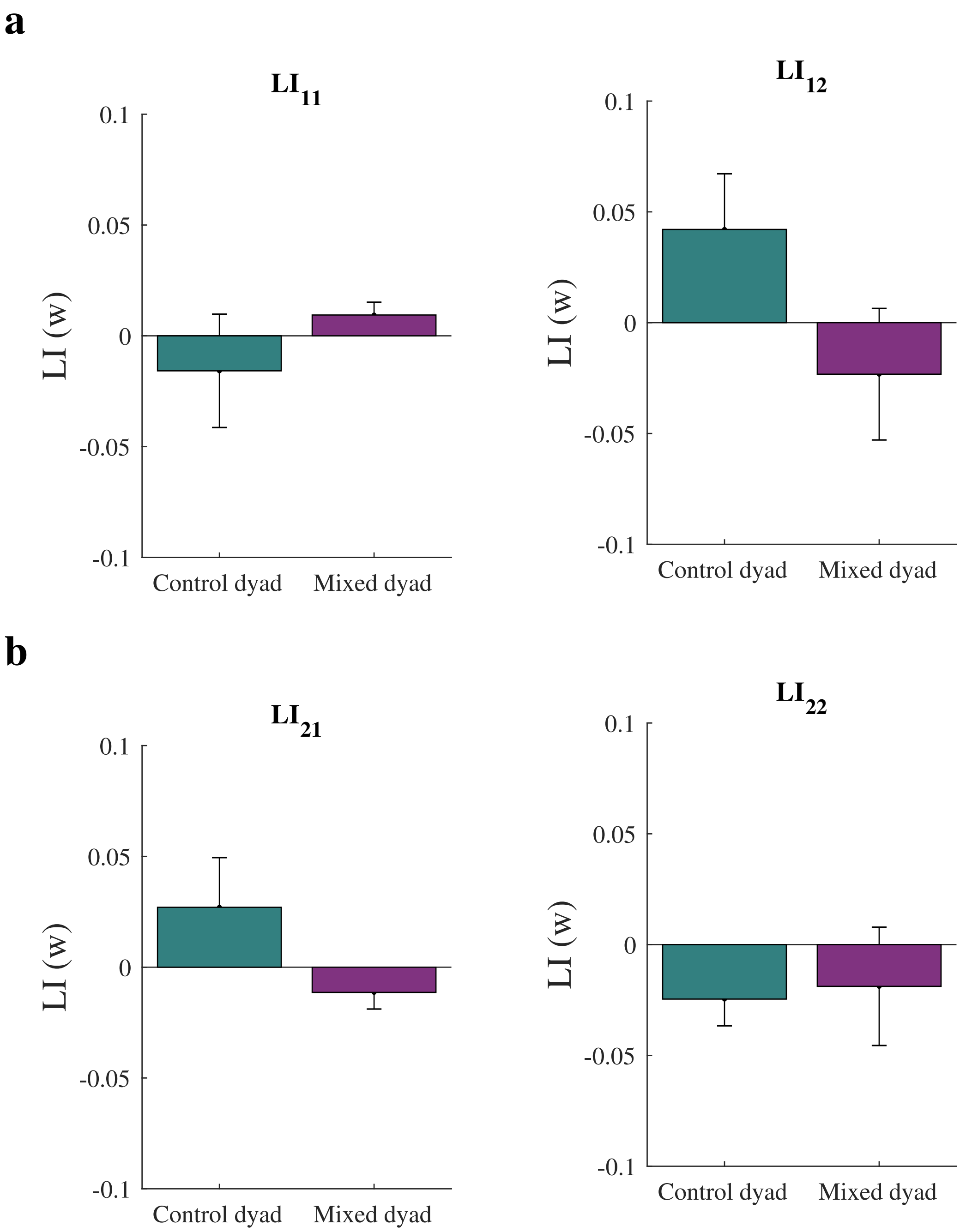


Figure 8.9 **Leadership index (LI) in the late epochs for control and mixed dyad subjects**. Leadership index (LI) is the average power calculated in the 300ms interval just before each via-points. (**a**), LI_{11} (left) is the leadership index for subject 1 at VP_1, similarly LI_{12} is for subject 1 at VP_2. Negative value of power shows subject 1 is the leader till his via-point and follower at his partner's via-point. Similarly, (**b**), leadership index for subject 2 at VP_1 (left) and VP_2 (right).

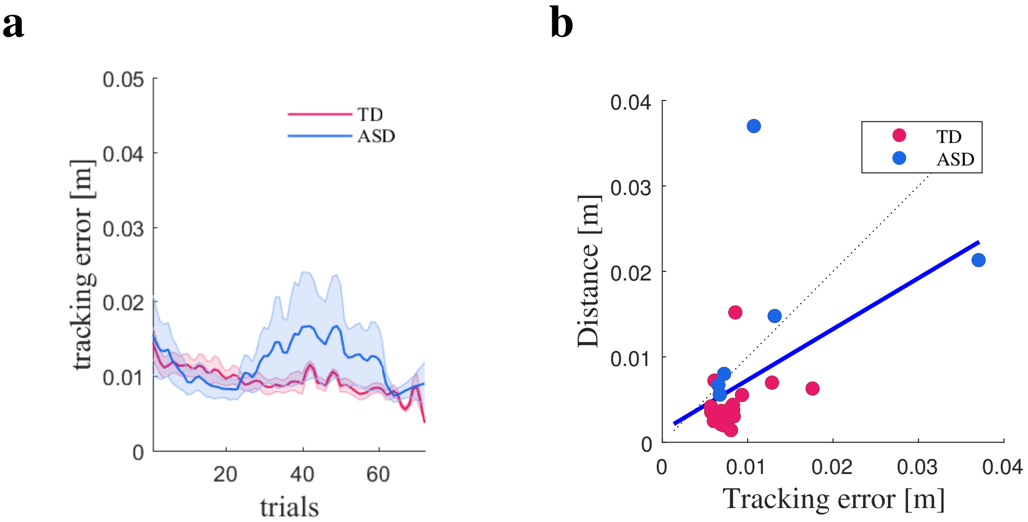


Figure 8.10 **Haptic force perception and correlations**. (**a**), Mean tracking error in each subject group (TD or ASD) over trial is shown. (**b**), Correlation between minimum distance from partner's via-point and tracking error

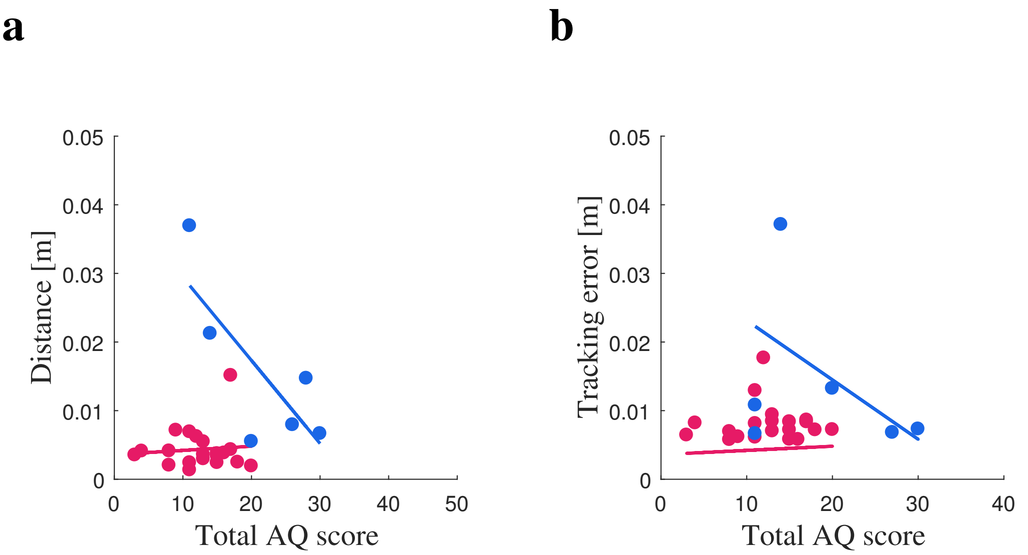


Figure 8.11 **Correlation with the AQ score**. (**a**), Correlation between minimum distance from partner's VP and total total AQ score. (**b**), Correlation between tracking error and total AQ score

Since their infancy, persons with an autism spectrum condition demonstrate motor impairments and an inability to appropriately adapt their movements (Teitelbaum et al., 1998). In addition, individuals with autism commonly lack the ability to extrapolate their partner's goal by observing their actions alone (Cattaneo et al., 2007). Based on these findings, we hypothesised that ASD subjects may be unable to estimate a partner's desired movement from observing haptics forces alone. If internal models are crucial for understanding partner's intentions (Mostofsky et al., 2009), then individuals who exhibit an ability to perform joint motor actions may also have better success learning more complex social interactions.

Our experimental results suggest that some ASD subjects exhibit a difficulty in establishing a collaboration, which can only be partly explained in terms of defective haptic perception. However, other ASD subjects are not distinguishable from TD subjects. Because of the small number of subjects these results should be taken cautiously and need to be confirmed in a larger study.

However, the literature on impaired coordination in persons with ASD is somehow inconsistent. Several studies reported impairments (i) in the perceptual level (Leekam et al., 2007; Paton et al., 2012), (ii) in adaptive capabilities (Teitelbaum et al., 1998), (iii) in the ability to predict the outcomes of own actions or of external stimuli, and (iv) in the ability to understand their partner's intentions (Cattaneo et al., 2007). However, other studies reported no differences with respect to normal subjects or an even better performance (Blakemore et al., 2006; Nakano et al., 2012). For instance persons with ASD exhibit normal anticipatory eye movements (Aitkin et al., 2013; Ego et al., 2016).

Taken together, these results suggest that ASD subjects are characterised by a wide spectrum of symptoms which may involve different aspects of the sensorimotor system (sensory performance, ability to establish and adapt and internal model of dynamics and/or the partner, etc) and therefore may result in different capabilities of establishing a collaboration with a partner.

8.5 Conclusions

Although the results are not conclusive, we still believe that our proposed experimental framework may be an useful starting point to achieve a better understanding of the neurophysiological and computational substrate of individuals' ability to coordinate actions with a partner. In addition to probing the motor aspects of Theory of Mind, motor paradigms involving physical interpersonal interaction may be used to develop forms of exercise to train persons with ASD in order to improve their interaction capabilities.

Chapter 9

Conclusions and future directions

> In three words I can sum up everything
> I've learned about life: It goes on.
>
> Robert Frost

9.1 Conclusions

We human beings possess a remarkable ability to coordinate our actions with others to reach common goals. Several mechanisms can be identified that are involved in joint motor action. Studies on psychology address theories on non-verbal and non-haptic joint motor actions. However, joint motor action has not been investigated much until the last ten years or so. In this regard, there is still a great number of questions that need to be addressed on joint motor actions. From a mathematical point of view, joint motor actions can be studied using game theory, using a set of cost functions to organise, understand and reproduce human motor behaviours of interactions with partners. Once the nature of the underlying motor task has been characterised, existence and uniqueness of a Nash equilibrium can be established. This thesis's main focus is on the sensorimotor interactions between two individuals that perform joint motor tasks. It is the first study to use psychophysics and a differential game theory-based computational model that accounts for learning in joint motor action with partial or incomplete information.

Chapter 5 presented a dual-haptic interface to investigate joint motor interaction between two humans. A generic, modular software environment has been developed. The modularity of the software allows to implement various functionality for the experimental task in an easy and coordinated manner. In Chapter 6, I introduced a modelling framework to study sensory motor collaborative strategies between to humans, where the task and interacting subjects' constraints are described by a pair of cost functionals. In the specific example of a sensorimotor version of the 'battle of sexes' game, simulation results demonstrate that the model predicts that optimal collaboration between partners (Nash equilibrium) is characterised by overlapping paths, approximately same crossing times at both via-points and near-zero interaction forces. In contrast, in the no-partner model, the overlap between paths is incomplete and each partner crosses their partner's via-point with a significant delay. Also, the interaction powers suggest that partners switch between leader-follower roles during movements. I modelled the time course of learning a joint action under the theoretical framework of 'fictitious play'. To our knowledge, this is the first and most complete computational model which accounts for learning joint action with partial or incomplete information.

The simulations suggest that different 'roles' during an interaction can be predicted by the interaction modality. The modelling framework uses differential game theoretic based approach, using set of quadratic cost functionals to detect, understand and reproduce motor behaviours of joint interaction with a partner. Game theory methods yield shared decision making, allowing subjects to have different utility functions, which help to characterize the existence and uniqueness of a Nash equilibrium point. For the simplicity our framework omitted multiplicative noise in defining sensory and motor system, although those models are more practical than additive noise model, is much more complex when it comes to game theoretic models.

In Chapter 7, I studied how subject pairs in a dyad establish collaboration in partially conflicting reward based point-to-point reaching task–a sensorimotor form of 'battle of sexes' game. In the task, I manipulated the amount of information available to the partner. I have found that information about the partner deeply affects speed and outcome of learning a collaboration also when more information about the partner action is available, the interaction strategies come closer to Nash equilibrium.

In Chapter 8, I investigated how negotiation of a joint action evolves in persons with ASD and in typically developing (TD) individuals. We specifically looked at differences between dyads of TD individuals and mixed dyads, involving one ASD and one TD individual. Our experiment results suggest that some ASD subjects exhibit difficulty in establishing a collaboration, which can only be partly explained with their ability to perceive haptic force. Since autism is characterised by large spectrum symptoms, this need to tested with large population study.

9.2 Future directions

Any interesting body of research inevitably leads to more questions beyond those that stimulated the initial studies. Our results suggest that sensorimotor joint action can be understood by game theoretic framework. Moreover, the general design of our experiments provides a tool to translate classical games into continuous motor games and might provide a new avenue for studying human motor interactions, many questions remain open and will have to be addressed in future work.

While the studies presented in this thesis were not directly linked to rehabilitation, one of the major motivational factors that drove me to initiate this study was to gain a better understanding on the processes behind interactions that are commonly seen during a patient-therapist interaction. Physiotherapist assisting stroke or spinal cord injury patient is a common example of sensorimotor joint action. I strongly believe that better understanding of these processes will translate into simpler and more efficient rehabilitation protocols without compromising recovery in future.

Finally, this thesis raises new questions that, when answered, may further expand current definitions and a small step towards an ultimate understanding of autism spectrum disorder. It is not clear how the brain implement the internal model of a partner in sensorimotor interactions. So far, there has been only limited use of game theory and experimental tools to link strategic thinking Theory of Mind (ToM). Game theoretic models could also be useful in understanding autism. Future studies based on psychophysics and neuroimaging offers a useful method for future exploration of whether key subcomponents of formal ToM models predict brain activity in ToM regions.

⋆⋆⋆

Appendix A

Autism-Spectrum Quotient (AQ) Questionnaire (Italian and English versions)

Il Quoziente di Spettro Autistico (AQ)

Eta' 16+

SPECIMEN, SOLO PER USO DI RICERCA.

Per informazioni dettagliate si prega di vedere:

S. Baron-Cohen, S. Wheelwright, R. Skinner, J. Martin and E. Clubley, (2001)
The Autism Spectrum Quotient (AQ) : Evidence from Asperger Syndrome/High Functioning Autism, Males and Females, Scientists and Mathematicians
Journal of Autism and Developmental Disorders 31:5-17

Nome e Cognome:.. Sesso:……….……

Data di nascita:.. Data odierna:..

Come compilare il questionario

Sotto sono riportate una lista di affermazioni. Leggi ciascuna affermazione molto attentamente e indica quanto fortemente sei in accordo o in disaccordo con esse, cerchiando la tua risposta.

NON SALTARE ALCUNA AFFERMAZIONE.

Esempio:

E1. Sono disposto a correre dei rischi.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
E2. Mi piace giocare ai giochi da tavolo.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
E3. Trovo facile imparare a suonare strumenti musicali.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
E4. Sono affascinato dalle altre culture.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo

1. Preferisco svolgere le attivita' con gli altri piuttosto che da solo.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
2. Preferisco fare le cose sempre nello stesso modo.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
3. Se cerco di immaginare qualcosa, trovo molto semplice creare un'immagine nella mia mente.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
4. Frequentemente vengo cosi' fortemente assorbito da una cosa che perdo di vista le altre cose.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
5. Spesso noto piccoli suoni che gli altri non notano.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
6. Di solito noto i numeri di targa delle macchine o simili sequenze di informazioni.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
7. Frequentemente le altre persone mi dicono che quanto ho detto e' scortese, anche quando io penso sia cortese.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
8. Quando leggo una storia, riesco facilmente a immaginare come i personaggi potrebbero apparire.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
9. Sono affascinato dalle date.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
10. In un gruppo sociale, riesco facilmente a seguire le conversazioni di parecchie persone.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
11. Trovo le situazioni sociali semplici.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
12. Ho la tendenza a notare dettagli che gli altri non notano.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
13. Preferisco andare in biblioteca piuttosto che ad una festa.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
14. Trovo semplice inventare racconti.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
15. Mi trovo attratto piu' fortemente dalle persone che dalle cose.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
16. Tendo ad avere interessi molto forti e mi innervosisco se non posso perseguirli.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
17. Mi piace chiacchierare.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo

18. Quando parlo, non e' sempre facile per gli altri inserirsi nella conversazione.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
19. Sono affascinato dai numeri.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
20. Quando leggo una storia, trovo difficile capire le intenzioni dei personaggi.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
21. Non amo particolarmente leggere romanzi.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
22. Trovo difficile fare nuove amicizie.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
23. Noto costantemente degli schemi nelle cose.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
24. Preferisco andare al teatro piuttosto che al museo.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
25. Non mi infastidisco se le mie routine quotidiane vengono disturbate.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
26. Mi capita frequentemente di non sapere come continuare una conversazione.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
27. Trovo semplice "leggere tra le righe" quando qualcuno mi parla.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
28. Di solito mi concentro di piu' sull'intera figura che su piccoli dettagli.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
29. Non sono molto bravo a ricordare i numeri telefonici.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
30. Di solito non noto piccoli cambiamenti in una situazione, o nell'aspetto di una persona.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
31. So distinguere se chi mi ascolta si sta annoiando.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
32. Trovo semplice fare piu' di una cosa contemporaneamente.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
33. Quando parlo al telefono, non sono sicuro quando e' il mio turno di parlare.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
34. Amo fare le cose spontaneamente.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
35. Sono spesso l'ultimo a capire il punto di una battuta.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo

36. Trovo semplice capire cosa una persona sta pensando o provando, semplicemente guardandola in faccia.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
37. Se c'e' un'interruzione, io posso ritornare a cio' che stavo facendo molto velocemente.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
38. Sono bravo nella conversazione sociale.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
39. Le persone spesso mi dicono che persevero sempre sulla stessa cosa.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
40. Quando ero piccolo, mi piaceva fare giochi di finzione con altri bambini.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
41. Mi piace raccogliere informazioni su categorie di cose (es. tipi di macchine, tipi di uccelli, tipi di treni, tipi di piante, etc.).	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
42. Trovo difficile immaginarmi nei panni di qualcun'altro.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
43. Mi piace pianificare attentamente ogni attivita' a cui partecipo.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
44. Mi piacciono gli eventi sociali.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
45. Trovo difficile capire le intenzioni delle persone.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
46. Le situazioni nuove mi rendono ansioso.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
47. Mi piace incontrare persone nuove.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
48. Sono un buon diplomatico.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
49. Non sono molto bravo a ricordare la data di nascita delle persone	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo
50. Trovo semplice fare giochi di finzione con i bambini.	Assolutamente daccordo	Parzialmente daccordo	Parzialmente in disaccordo	Assolutamente in disaccordo

Grazie per aver compilato il Questionario!*

* **Sviluppato dall' *Autism Research Centre*, Cambridge University.**

Traduzione validata eseguita dalla Dott.ssa Liliana Ruta, specialista in Neuropsichiatria Infantile, Universita' di Catania – ruta@policlinico.unict.it

The Adult Autism Spectrum Quotient (AQ)

Ages 16+

SPECIMEN, FOR RESEARCH USE ONLY.

For full details, please see:

S. Baron-Cohen, S. Wheelwright, R. Skinner, J. Martin and E. Clubley, (2001)
The Autism Spectrum Quotient (AQ) : Evidence from Asperger Syndrome/High Functioning Autism, Males and Females, Scientists and Mathematicians
Journal of Autism and Developmental Disorders 31:5-17

Name:.. Sex:..

Date of birth:.................................. Today's Date................................

How to fill out the questionnaire

Below are a list of statements. Please read each statement very carefully and rate how strongly you agree or disagree with it by circling your answer.

DO NOT MISS ANY STATEMENT OUT.

Examples

E1. I am willing to take risks.	definitely agree	slightly agree	slightly disagree	definitely disagree
E2. I like playing board games.	definitely agree	slightly agree	slightly disagree	definitely disagree
E3. I find learning to play musical instruments easy.	definitely agree	slightly agree	slightly disagree	definitely disagree
E4. I am fascinated by other cultures.	definitely agree	slightly agree	slightly disagree	definitely disagree

1. I prefer to do things with others rather than on my own.	definitely agree	slightly agree	slightly disagree	definitely disagree
2. I prefer to do things the same way over and over again.	definitely agree	slightly agree	slightly disagree	definitely disagree
3. If I try to imagine something, I find it very easy to create a picture in my mind.	definitely agree	slightly agree	slightly disagree	definitely disagree
4. I frequently get so strongly absorbed in one thing that I lose sight of other things.	definitely agree	slightly agree	slightly disagree	definitely disagree
5. I often notice small sounds when others do not.	definitely agree	slightly agree	slightly disagree	definitely disagree
6. I usually notice car number plates or similar strings of information.	definitely agree	slightly agree	slightly disagree	definitely disagree
7. Other people frequently tell me that what I've said is impolite, even though I think it is polite.	definitely agree	slightly agree	slightly disagree	definitely disagree
8. When I'm reading a story, I can easily imagine what the characters might look like.	definitely agree	slightly agree	slightly disagree	definitely disagree
9. I am fascinated by dates.	definitely agree	slightly agree	slightly disagree	definitely disagree
10. In a social group, I can easily keep track of several different people's conversations.	definitely agree	slightly agree	slightly disagree	definitely disagree
11. I find social situations easy.	definitely agree	slightly agree	slightly disagree	definitely disagree
12. I tend to notice details that others do not.	definitely agree	slightly agree	slightly disagree	definitely disagree
13. I would rather go to a library than a party.	definitely agree	slightly agree	slightly disagree	definitely disagree
14. I find making up stories easy.	definitely agree	slightly agree	slightly disagree	definitely disagree
15. I find myself drawn more strongly to people than to things.	definitely agree	slightly agree	slightly disagree	definitely disagree
16. I tend to have very strong interests which I get upset about if I can't pursue.	definitely agree	slightly agree	slightly disagree	definitely disagree
17. I enjoy social chit-chat.	definitely agree	slightly agree	slightly disagree	definitely disagree
18. When I talk, it isn't always easy for others to get a word in edgeways.	definitely agree	slightly agree	slightly disagree	definitely disagree

19. I am fascinated by numbers.	definitely agree	slightly agree	slightly disagree	definitely disagree
20. When I'm reading a story, I find it difficult to work out the characters' intentions.	definitely agree	slightly agree	slightly disagree	definitely disagree
21. I don't particularly enjoy reading fiction.	definitely agree	slightly agree	slightly disagree	definitely disagree
22. I find it hard to make new friends.	definitely agree	slightly agree	slightly disagree	definitely disagree
23. I notice patterns in things all the time.	definitely agree	slightly agree	slightly disagree	definitely disagree
24. I would rather go to the theatre than a museum.	definitely agree	slightly agree	slightly disagree	definitely disagree
25. It does not upset me if my daily routine is disturbed.	definitely agree	slightly agree	slightly disagree	definitely disagree
26. I frequently find that I don't know how to keep a conversation going.	definitely agree	slightly agree	slightly disagree	definitely disagree
27. I find it easy to "read between the lines" when someone is talking to me.	definitely agree	slightly agree	slightly disagree	definitely disagree
28. I usually concentrate more on the whole picture, rather than the small details.	definitely agree	slightly agree	slightly disagree	definitely disagree
29. I am not very good at remembering phone numbers.	definitely agree	slightly agree	slightly disagree	definitely disagree
30. I don't usually notice small changes in a situation, or a person's appearance.	definitely agree	slightly agree	slightly disagree	definitely disagree
31. I know how to tell if someone listening to me is getting bored.	definitely agree	slightly agree	slightly disagree	definitely disagree
32. I find it easy to do more than one thing at once.	definitely agree	slightly agree	slightly disagree	definitely disagree
33. When I talk on the phone, I'm not sure when it's my turn to speak.	definitely agree	slightly agree	slightly disagree	definitely disagree
34. I enjoy doing things spontaneously.	definitely agree	slightly agree	slightly disagree	definitely disagree
35. I am often the last to understand the point of a joke.	definitely agree	slightly agree	slightly disagree	definitely disagree
36. I find it easy to work out what someone is thinking or feeling just by looking at their face.	definitely agree	slightly agree	slightly disagree	definitely disagree
37. If there is an interruption, I can switch back to what I was doing very quickly.	definitely agree	slightly agree	slightly disagree	definitely disagree

38. I am good at social chit-chat.	definitely agree	slightly agree	slightly disagree	definitely disagree
39. People often tell me that I keep going on and on about the same thing.	definitely agree	slightly agree	slightly disagree	definitely disagree
40. When I was young, I used to enjoy playing games involving pretending with other children.	definitely agree	slightly agree	slightly disagree	definitely disagree
41. I like to collect information about categories of things (e.g. types of car, types of bird, types of train, types of plant, etc.).	definitely agree	slightly agree	slightly disagree	definitely disagree
42. I find it difficult to imagine what it would be like to be someone else.	definitely agree	slightly agree	slightly disagree	definitely disagree
43. I like to plan any activities I participate in carefully.	definitely agree	slightly agree	slightly disagree	definitely disagree
44. I enjoy social occasions.	definitely agree	slightly agree	slightly disagree	definitely disagree
45. I find it difficult to work out people's intentions.	definitely agree	slightly agree	slightly disagree	definitely disagree
46. New situations make me anxious.	definitely agree	slightly agree	slightly disagree	definitely disagree
47. I enjoy meeting new people.	definitely agree	slightly agree	slightly disagree	definitely disagree
48. I am a good diplomat.	definitely agree	slightly agree	slightly disagree	definitely disagree
49. I am not very good at remembering people's date of birth.	definitely agree	slightly agree	slightly disagree	definitely disagree
50. I find it very easy to play games with children that involve pretending.	definitely agree	slightly agree	slightly disagree	definitely disagree

Developed by:
The Autism Research Centre
University of Cambridge

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