RESEARCH ARTICLE



Possible application of quantum computing in the field of ocean engineering: optimization of an offshore wind farm layout with the Ising model

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Abstract

As a possible application of quantum computing in the field of ocean engineering, an attempt is made to identify the optimum layout of an offshore wind farm with the quasi-quantum annealing method while assuming a wake model that conforms to the standard Jensen–Katic wake model but can be implemented in the Ising model. The optimum layouts obtained in the present study are compared with those identified with other conventional methods such as the genetic algorithm. It is confirmed that the quasi-quantum annealing method works as well as other methods in identifying the optimum layouts, while the optimum layout of an offshore wind farm composed of even more than 100 wind turbines can be identified with reasonable CPU time.

Keywords Quantum computing · Simulated annealing · Quasi-annealing method · Wind farm layout · Ising model

1 Introduction

It is expected that quantum computing, if realized, could carry out computations tens of thousands times faster than even the current fastest supercomputers. At present, however, quantum computers have not reached a practically usable level due to several unsolved technical problems such as limited number of usable quantum bits or error correction mechanism.

On the other hand, Kadowaki and Nishimori (1998) introduced quantum fluctuations, instead of thermal fluctuations, into the simulated annealing of optimization problems with the aim toward faster convergence to the global optimum state.

The technique of simulated annealing itself was first proposed by Kirkpatrick et al. (1983) to effectively identify a global minimum in optimization problems. Attempts to apply simulated annealing technique to identify the optimum layout of a wind farm have been done by many researchers [e.g., Elkinton et al. (2008), Yang and Cho (2019)], resulting in the conclusion that simulated annealing works as well as other conventional numerical techniques such as generic algorithm but with less computation time. In contrast to the

conventional simulated annealing that uses thermal fluctuations, Kadowaki and Nishimori (1998) made use of quantum fluctuation processes that could converge more effectively to the global minimum, which they called quantum annealing.

Based on the theoretical proposal of Kadowaki and Nishimori (1998), a Canadian company D-Wave Systems has successfully developed and commercialized a quantum annealing machine in 2011, which is a machine particularly specialized in solving combinatorial optimization problems. Hancock et al. (2023) solved the windfarm layout optimization problem mapped to a quadratic unconstrained binary optimization (QUBO) problem using quantum optimization algorithms, that is quantum annealing, but with a classical simulator instead of quantum annealers.

Since the D-Wave machine necessitates a large refrigerator to use superconducting circuits and the number of quantum bits is not large enough for practical use, quasiquantum annealing methods that search global minimum in a optimization problem in a similar manner to quantum annealing but can operate in the conventional machines have been developed by quite a few companies worldwide, and some of them are providing their quantum software development kit (SDK) for free through Internet for prospective users.

Under these circumstances, the present work aims, as an attempt toward possible applications of quantum computing in the field of ocean engineering, to examine if an optimum



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offshore wind farm layout can be identified with the quasiquantum annealing machine.

In an offshore wind farm, more than a single wind turbine, usually several tens of wind turbines, are deployed. If the wind turbines are installed in close proximity, since the wind speed is attenuated and the wind field is disturbed behind a wind turbine, the power generated by the leeside wind turbine is reduced. To avoid these aerodynamic interactions among wind turbines, the leeside wind turbine needs to be placed a certain distance away from the upwind wind turbine. It is known that such interactions could persist for quite a long distance, 8 ~ 12 times the rotor diameter of the corresponding wind turbines [e.g., Patel (1999)]. On the other hand, however, there usually exists a limitation of the area that the corresponding wind farm can occupy, because, for example, it could hinder nearby fishing, ship routes, and so on. Moreover, if wind turbines are placed with large intervals to avoid aerodynamic interactions among them, it could also incur an extra cost caused by necessary longer power transmission cables. After all, an offshore wind farm cannot be expanded excessively, while wind turbines in the wind farm need to be installed with a certain distance between them to avoid wake effects, and, therefore, it is practically important to identify the optimal arrangement of wind turbines in an offshore wind farm in the sense that the total power generation of the corresponding wind farm is maximized within a certain limited sea area.

2 Ising model

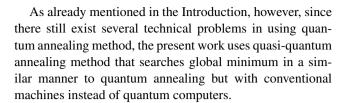
The Ising model was proposed by Wilhelm Lenz in 1920 and named after his Ph.D. student Ernst Ising, who worked on it. The Ising model is rather a simple model in statistical mechanics, but it has proven to be a powerful tool in exploring the ground state of magnetic materials. Subsequently, it has turned out that many of combinatorial optimization problems can be transformed to those of the ground state exploration of magnetic materials [e.g., Lucas (2014)].

In the Ising model, the corresponding combinatorial problem is reduced to a minimization problem of the following polynomial of binary variables:

$$f = \sum_{i < j} Q_{ij} q_i q_j + \sum_i Q_{ii} q_i, \tag{1}$$

where q_i , q_j are either -1 or +1, or either 0 or +1.

In other words, if the corresponding combinatorial problem can be formulated as a minimization problem of a certain polynomial function of binary variables, then the corresponding problem can be solved with the quantum annealing method.



3 Optimization of an offshore wind farm layout with Ising model

3.1 Definition of the problem

As shown in Fig. 1, we define a lattice composed of, for example, 5×5 lattice points in the prospective offshore wind farm area. The 5×5 lattice points are the candidate points where wind turbines are to be placed. Two-dimensional binary variables q(i, j) are assigned at each lattice point as shown in Fig. 1 in such a way that:

- (1) If a wind turbine is placed at the corresponding lattice point, then q(i, j) = 1.
- (2) If a wind turbine is not placed at the corresponding lattice point, then q(i, j) = 0.

The wind direction is, as shown in Fig. 1, assumed to be parallel to the lattice line.

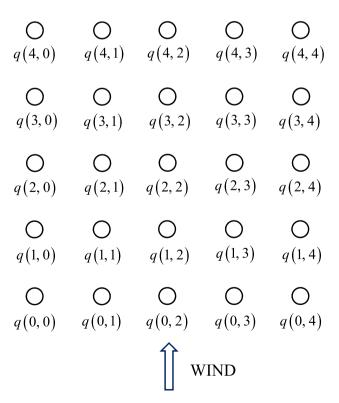


Fig. 1 5×5 lattice



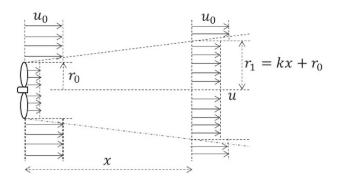


Fig. 2 Jensen-Katic wake model [Reproduced from Yang and Cho (2019)]

3.2 Wake model

When a wind turbine is placed in an air flow, the flow velocity is attenuated behind the wind turbine. In general, the flow velocity reaches its minimum value at just a little distance behind the rotor and then gradually recovers to its inflow value, which could take a significant distance, sometimes even more than ten times the rotor diameter. Other than the velocity deficit in the wake, turbulence of the air flow is enhanced in the wake as well, which makes the proper modeling of the wake flow even more difficult. The fact that the wake pattern is also influenced significantly by the atmospheric stability further complicates the modeling attempt of the wake [e.g., Vermeer et al. (2003)].

As the wake model, the following wake model, Jensen–Katic model (Katic et al. 1986), is recognized as one of the standard models:

$$\left(1 - \frac{u}{u_0}\right) = \frac{2a}{\left(1 + k\frac{x}{r_0}\right)^2},\tag{2}$$

where, as shown in Fig. 2, u_0 : inflow wind velocity, u: wind velocity at the distance x downstream, a: axial induction factor, k: wake decay coefficient, r_0 : turbine radius, x: distance downstream

The parameters a, k in Eq. (2) are not of fixed values but could vary depending on many factors such as the type of the wind turbine, air flow velocity, air flow turbulence, atmospheric stability, etc., and, therefore, it is of little use to attempt to find their values universally adaptable.

Besides, it is difficult to formulate the problem as an Ising model with the wake model given by Eq. (2).

In the present work, because the main purpose is to examine if the quantum annealing technique, though actually it is the quasi-quantum annealing technique, can be used to identify the optimum layout of an offshore wind farm, we use a rather simple wake model while assuming that the linear

$$\begin{array}{c} q_4 & \bigcirc \\ & \uparrow \\ u_4 = (1-0.3^4 \cdot q_0) - 0.3^3 \cdot q_1 - 0.3^2 \cdot q_2 - 0.3 \cdot q_3 \\ q_3 & \bigcirc \\ & \uparrow \\ u_3 = (1-0.3^3 \cdot q_0) - 0.3^2 \cdot q_1 - 0.3 \cdot q_2 \\ q_2 & \bigcirc \\ & \uparrow \\ u_2 = (1-0.3^2 \cdot q_0) - 0.3 \cdot q_1 \\ q_1 & \bigcirc \\ & \uparrow \\ u_1 = 1-0.3 \cdot q_0 \\ q_0 & \bigcirc \\ & \uparrow \\ \end{array}$$
 Wind velocity: $u_0 = 1$

Fig. 3 The wake model used in the present work

superposition of the wake effects caused by each wind turbine does hold. Figure 3 shows the wake model used in the present study along one column in a 5×5 lattice in an inflow wind velocity $u_0 = 1.0$. It is assumed that there exist no aerodynamic interactions between wind turbines located in neighboring columns.

According to the wake model shown in Fig. 3,

- ① If there is a wind turbine at the very front of the column, that is if $q_0 = 1$, the wind velocity incident to the wind turbine at the second row is reduced by 30%, that is $u_1 = 0.7$. On the other hand, if there is not a wind turbine at the very front of the column, that is if $q_0 = 0$, the wind velocity incident to the wind turbine at the second row remains to be $u_1 = 1.0$.
- ② If there exist no wind turbines at both the 1st, 2nd, rows, that is $q_0 = 0$, $q_1 = 0$, the wind velocity incident to the wind turbine at the 3rd row remains to be $u_2 = 1.0$. If there exist a wind turbine at the 1st row but no wind turbine at the 2nd row, that is $q_0 = 1$, $q_1 = 0$, the wind velocity incident to the wind turbine at the 3rd row is $u_2 = 1.0 0.3^2 = 0.91$. If there exist at both the 1st and 2nd rows, that is $q_0 = 1$, $q_1 = 1$, the wind velocity incident to the 3rd row is $u_2 = (1.0 0.3^2) 0.3 = 0.61$, in which the velocity deficit due to the 1st row and that due to the 2nd row are linearly superposed.
- 3 And so on.

As for the wake decay coefficient k, it is given as [e.g., Herbert-Acero et al. (2014)]

$$k = \frac{0.5}{\ln(z/z_0)},\tag{3}$$



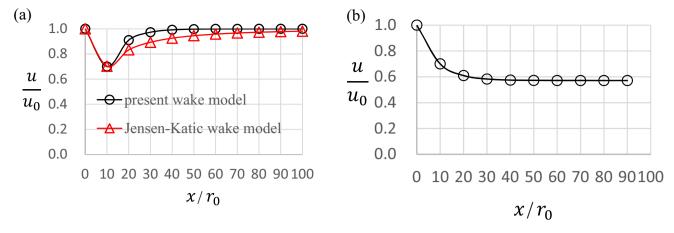


Fig. 4 a Comparisons of wake models behind a single wind turbine. b Wake model among an array of multiple wind turbines

where z is the hub height from the sea surface and z_0 is the surface roughness. The surface roughness of the ocean is given as, for example, $z_0 = 6.1 \times 10^{-3} \text{m}(\text{Golbazi})$ and Archer 2020), with which k results in k ~ 0.05 with the hub height assumed to be z = 100 m, while k in offshore is given as k = 0.04 in Ma et al. (2022). Referring to these values and assuming k = 0.05, Fig. 4a compares the wind velocities behind a single wind turbine given by the present wake model and those given by Jensen-Katic wake model while assuming a = 1/3, k = 0.05 in Eq. (2). (a = 1/3 is chosen, because at which the power coefficient)is to be maximum theoretically according to Betz law.) The distance between adjacent lattice points is assumed to be five times the rotor diameter. In both the wake models, wind velocity is attenuated significantly just behind the wind turbine and, after that, it recovers gradually toward the inflow velocity. Although there exist some discrepancies between the two models, the present wake model is adapted in the following analyses, because, as will be shown in the followings, it enables to construct the objective function in the form of Ising model.

Although wake patterns behind a single wind turbine are discussed in many works, there exist few works on wake patterns among an array of multiple wind turbines placed along the wind direction. In Fig. 4b, as an example, the wind velocities among 10 wind turbines arrayed at every lattice point, that is with the same distance equal to 5 times the rotor diameter, along a straight line parallel to the wind direction given by the present wake model is shown.

3.3 Objective function and constraints

Assuming that we seek an optimum layout of a wind farm in which the total sum of the wind velocities incident to all the wind turbines in the wind farm is maximized, the objective

function f under the wake model shown in Fig. 3 in the example case of 5×5 lattice is written as in Eq. (4).

The first term '- 1.0' of the right-hand side of Eq. (4) is added to convert the maximization problem to the minimization problem, because the quasi-quantum annealing method the present work uses is supposed to seek a global minimum.

As for the constraint(s) in the present problem, it is that the number of wind turbines should be equal to the given number. For example, if N wind turbines are to be deployed in the 5×5 lattice, minimization of the objective function given by Eq. (4) with the constraint can be converted to the minimization of the following objective function given by Eq. (5) without constraints

$$f = -1.0 \times [q(0, 0) + q(0, 1) + q(0, 2) + q(0, 3) + q(0, 4) + q(1, 0) \times (1.0 - 0.3 \times q(0, 0)) + q(1, 1) \times (1.0 - 0.3 \times q(0, 1)) + q(1, 2) \times (1.0 - 0.3 \times q(0, 2)) + q(1, 3) \times (1.0 - 0.3 \times q(0, 3)) + q(1, 4) \times (1.0 - 0.3 \times q(0, 4)) + q(2, 0) \times \{(1.0 - 0.3^2 \times q(0, 0)) - 0.3 \times q(1, 0)\} + q(2, 1) \times \{(1.0 - 0.3^2 \times q(0, 1)) - 0.3 \times q(1, 1)\} + q(2, 2) \times \{(1.0 - 0.3^2 \times q(0, 2)) - 0.3 \times q(1, 2)\} + q(2, 3) \times \{(1.0 - 0.3^2 \times q(0, 3)) - 0.3 \times q(1, 3)\} + q(2, 4) \times \{(1.0 - 0.3^2 \times q(0, 4)) - 0.3 \times q(1, 4)\} + \cdots$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$+ \cdots$$

$$\vdots$$

$$(4)$$

$$\begin{split} f &= -1.0 \times [q(0,\ 0) + q(0,\ 1) + q(0,\ 2) + q(0,\ 3) + q(0,\ 4) \\ &+ q(1,\ 0) \times (1.0 - 0.3 \times q(0,\ 0)) + q(1,\ 1) \\ &\times (1.0 - 0.3 \times q(0,\ 1)) + q(1,\ 2) \times (1.0 - 0.3 \times q(0,\ 2)) \end{split}$$



$$+ q(1, 3) \times (1.0 - 0.3 \times q(0, 3)) + q(1, 4)$$

$$\times (1.0 - 0.3 \times q(0, 4)) + q(2, 0)$$

$$\times \left\{ \left(1.0 - 0.3^2 \times q(0, 0) \right) - 0.3 \times q(1, 0) \right\}$$

$$+ q(2, 1) \times \left\{ \left(1.0 - 0.3^2 \times q(0, 1) \right) - 0.3 \times q(1, 1) \right\}$$

$$+ q(2, 2) \times \left\{ \left(1.0 - 0.3^2 \times q(0, 2) \right) - 0.3 \times q(1, 2) \right\}$$

$$+ q(2, 3) \times \left\{ \left(1.0 - 0.3^2 \times q(0, 3) \right) - 0.3 \times q(1, 3) \right\}$$

$$+ q(2, 4) \times \left\{ \left(1.0 - 0.3^2 \times q(0, 4) \right) - 0.3 \times q(1, 4) \right\}$$

$$+ \cdots$$

$$\vdots$$

$$+ \cdots$$

$$\vdots$$

$$+ q(0, 0) + q(0, 1) + q(0, 2) + q(0, 3) + q(0, 4) + q(1, 0)$$

$$+ q(1, 1) + q(1, 2) + q(1, 3) + q(1, 4) + q(2, 0) + q(2, 1)$$

$$+ q(2, 2) + q(2, 3) + q(2, 4) + q(3, 0) + q(3, 1) + q(3, 2)$$

$$+ q(3, 3) + q(3, 4) + q(4, 0) + q(4, 1) + q(4, 2) + q(4, 3)$$

$$+ q(4, 4) - N \right\}^2 \times 1000.$$

$$(5)$$

The last five lines in Eq. (5), which are appended to Eq. (4), take care of the constraint, because, as the total sum of q(i, j) should be equal to the number of wind turbines N, if the total sum of q(i, j) is either smaller or larger than N, the resultant value due to the last five lines should be of large quantity and thus the total sum of q(i, j) has no choice but be equal to N in the annealing process that explore the minimum value of Eq. (5). (The last term '1000' of Eq. (5) can be somewhat of an arbitrary large positive number, because the role of the number is to make the resultant value due to the last 5 lines very large in case the total sum of q(i, j) differs from the given number N.)

After all, since the function f is now written as a polynomial of binary variables q(i, j), that is as an Ising model, the quasi-quantum annealing method can be applied in searching q(i, j) that minimize the value of f.

3.4 Implementation of the Ising model

In developing the computer code for the calculations conducted in the present work, open software development kit Amplify SDK (Software Development Kit) provided by the Japanese software developing company Fixstars was used. The Amplify SDK is a Python library that runs the solver of combinatorial optimization problems and Amplify AE (Fixstars Amplify Annealing Engine) is used as a solver. In implementing the present Ising model in a Python code in a Windows machine, a Linux virtual environment needs to be built in the machine. Figure 5 shows the part of the Python code written by the author, in which, as can be seen, the exploration of the global optimum state of the given objective function is performed by the 'solve()' function.

```
model=BinaryQuadraticModel(f)
client=FixstarsClient()
client.token="AE/Gf9RLPwN1PJMU0o4ZJ5x
client.parameters.timeout=1000
solver=Solver(client)
result=solver.solve(model)
q_values=q.decode(result[0].values)
print(q_values)
```

Fig. 5 The part of the Python code used in the present analyses

3.5 Results and discussion

Table 1 shows the example results of the optimal wind farm layout identified with the quasi-quantum annealing method. Here, 'the optimal wind farm layout' is defined as the optimal layout in the sense that the total sum of the wind velocities incident to all the wind turbines in the wind farm is maximized.

The specific definitions of each column in the tables are: Lattice: For example, 3×10 indicates that the assumed lattice points are composed of 3 rows and 10 columns.

N: The assumed number of wind turbines to be deployed in the wind farm.

Results: The identified optimal layout of the wind farm. $\sum_{i=1}^{N} u_i$: Total sum of the wind velocities incident to all the wind turbines.

The 'Results' shown in Table 1 are the screen shots of the results that appeared in the Linux screen where the computations were carried out. Figure 6 explains how to interpret the screen shots shown in the 'Results' in the tables. In the screen shots in the tables, the identified value of q(i, j) at each lattice point in the optimum layout is shown, in which q(i, j) = 1 indicates that a wind turbine should be placed at the corresponding lattice point, while q(i, j) = 0 indicates that no wind turbines are to be placed at the corresponding lattice point.

From the results shown in Table 1, we can find out the followings.

From the result obtained with 'Lattice 3×10 ', 'N = 20': Intuitively, we can know that the obtained result is the correct one, in which all the front row is occupied with wind turbines and the remaining 10 wind turbines are placed as far away from the front row as possible. The total sum of the wind velocities is 19.1000, while it should be 20.0 if there are no aerodynamic interferences among the 20 wind turbines.

From the result obtained with 'Lattice 5×5 ', 'N = 10': Again, intuitively, we can know the obtained result is the correct one, in which all the front row is occupied with wind turbines and the remaining 5 wind turbines are placed as far away from the front row as possible. The total sum of the



wind velocities is 9.9595, while it should be 10.0 if there are no aerodynamic interferences among the 10 wind turbines.

From the two results obtained with 'Lattice 5×5 ', 'N = 12':

Intuitively, the two results seem to be reasonable, as out of 12 wind turbines, 5 wind turbines are placed along the front row and 5 wind turbines are placed as far away from the front row as possible, and the remaining 2 wind turbines are placed at the middle row in order to minimize the aerodynamic interferences with both the wind turbines placed along the front row and those placed along the end row. It is also understandable that the places where the two wind turbines are placed along the middle row are different between the two results although the assumed 'Lattice' and 'N' are the same, because, it is assumed that there are no aerodynamic interferences among the wind turbines placed in the same raw.

From the two results obtained with 'Lattice 10×10 ', 'N = 20':

The two results are different, but they were obtained with the same assumptions 'Lattice 10×10 ', 'N = 20', that is, the results can be different even if exactly the same computations are repeated. Moreover, it can be said that the two results are not the right solutions, because we know that the

correct solution should be a layout in which 10 wind turbines are placed at the very front row of the lattice while the remaining 10 wind turbines are placed at the very end row of the lattice. It is also that, abiding by the wake model shown in Fig. 3, the total sum of the wind velocities incident to the right layout mentioned above should be 19.9998. As we can observe in the two results, although the total sum of the wind velocities incident to the 20 wind turbines in the two results are smaller than the right solution, they are quite close to that of the right solution, that is, 19.9934/19.9998 = 0.9997 and 19.9928/19.9998 = 0.9996. This fact, in turn, suggests that there should exist a large number of local minima, the values of which are quite close to that of the global minimum, and this must be the reason why it is difficult to identify the global minimum even with the quasi-quantum annealing method. Anyway, we understand that even the quantum annealing method does not guarantee that the global minimum can always be identified.

Since the power extracted by a wind turbine is proportional to u^3 instead of u, using the quasi-quantum annealing method, another attempt was made to identify the optimal layout in which the total sum of u^3 is maximized. (When the wind velocity exceeds the rated wind velocity, the power generated by the wind turbine is usually kept to be the rated

Table 1 Example results of the identified optimal wind farm layout $(\sum_{i=1}^{N} u_i \text{ is maximized})$

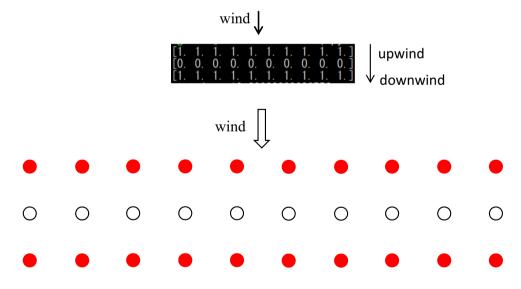
Lattice	N	Results (optimal wind farm layout)	$\sum_{i=1}^{N} u_{i}$
3×10	20	[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [1. 1. 1. 1. 1. 1. 1. 1. 1.]	19.1000
5×5	10	[1. 1. 1. 1. 1.] [0. 0. 0. 0. 0.] [0. 0. 0. 0. 0.] [0. 0. 0. 0. 0.] [1. 1. 1. 1. 1.]	9.9595
5×5	12	[Ĭ. 1. 1. 1.] [0. 0. 0. 0. 0.] [1. 1. 0. 0. 0.] [0. 0. 0. 0. 0.] [1. 1. 1. 1.]	11.5995
5×5	12	[1. 1. 1. 1. 1.] [0. 0. 0. 0. 0.] [0. 0. 1. 1. 0.] [0. 0. 0. 0. 0.] [1. 1. 1. 1.]	11.5995



Table 1 (continued)

Lattice	N	Result (optimum wind farm layout)	$\sum_{i=1}^{N} u_i$
10×10	20	[1. 0. 1. 1. 1. 0. 0. 1. 1. 0.] [0. 1. 0. 0. 0. 0. 1. 0. 0. 1.] [0. 0. 0. 0. 0. 0. 1. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [1. 0. 0. 0. 0. 0. 0. 1. 1.] [1. 0. 0. 0. 0. 0. 1. 0. 0. 0.] [0. 1. 0. 0. 0. 1. 0. 1. 0. 0.] [0. 0. 1. 0. 0. 0. 0. 0. 0. 0. 0.]	19.9934
10×10	20	[0. 1. 1. 1. 0. 0. 0. 0. 0. 1.] [1. 0. 0. 0. 1. 1. 1. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 1. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 1. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.] [1. 1. 0. 1. 0. 0. 1. 0. 1. 0.] [0. 0. 0. 0. 0. 1. 0. 1. 0.] [0. 0. 0. 0. 0. 1. 0. 1. 0.]	19.9928

Fig. 6 Explanation of the result (red circle:: wind turbine, white circle:: no wind turbine)



power and thus does not increase proportionately to u^3 . However, since, as can be found in, e.g., Morgan et al. (2011), the offshore wind velocity is mostly, 80–90%, less than the rated wind velocity, which is typically around 12 m/s, seeking the wind farm layout in which the total sum of u^3 is maximized may be justified.)

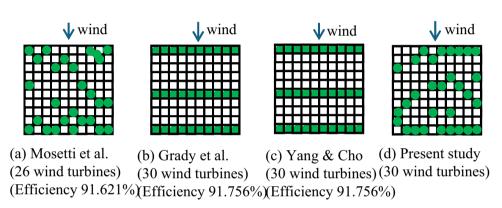
The results are shown in Table 2. In the example results shown in the table, we can confirm right solutions are obtained. From Table 2, we can also know such things that the total power generated in the corresponding wind farm is reduced by 12.3% in the first example (3×10 Lattice) and by 1.2% in the second example (5×5 Lattice) due to the aerodynamic interactions among the wind turbines.



Table 2 Example results of the identified optimum wind farm layout $(\sum_{i=1}^{N} u_i^3)$ is maximized)

Lattice	N	Results (optimum wind farm layout)	$\sum_{i=1}^{N} u_i^3$
3×10	20	[1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	17.5357
5×5	10	[1. 1. 1. 1.] [0. 0. 0. 0. 0.] [0. 0. 0. 0. 0.] [0. 0. 0. 0. 0.] [1. 1. 1. 1.]	9.8795

Fig. 7 The optimum layouts identified in other works and in the present study [Reproduced by the author based on the figure shown in Yang and Cho (2019)]

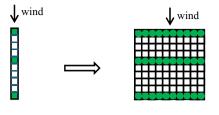


3.6 Comparisons with other works

Optimum layouts of offshore wind farms identified by Mosetti et al. (1994) or by Grady et al. (2005) with a genetic algorithm are often used as benchmark ones that are to be compared with those identified with other models and/or methods. For example, Yang and Cho (2019) explored optimum layouts of offshore wind farms with the simulated annealing and compared their results with those of Mosetti et al. (1994) and those of Grady et al. (2005). Figure 7 shows the comparisons reproduced by the author based on the figure shown in Yang and Cho (2019). They are the optimum layouts of wind turbines placed in unidirectional wind of its speed 12 m/s. The possible wind farm of area 2000 m \times 2000 m is divided into 10×10 cells, on each of which a wind turbine can be placed. The definition of 'efficiency' specified in Fig. 7 is the ratio of the 'wind farm power conversion' to the 'ideal wind farm conversion that should take place if no wake and turbulence effects are taken into account'. In Fig. 7, the optimum layout identified in the present study is also shown. Since the layout obtained as an optimum one in the present study is not necessarily the same but varies a little as we repeat the present calculations, the optimum layout identified in the present study shown in the figure is that of the highest efficiency obtained while repeating the calculations 20 times. The efficiency of the layout obtained in the present study shown in the figure is 94.5%. The efficiencies of the layouts obtained in the present study while repeating the calculations 20 times were 90.0% ~ 94.5%. (The definition of the efficiency in the present study is the ratio of the total sum of u^3 in the identified optimum wind farm to that of the wind farm in which no aerodynamic interactions are accounted for.) The direct comparisons of the efficiencies shown in Fig. 7 and that of the present study are not fair, because premises such as the objective functions, wake model are not the same. For example, the objective function assumed in Yang and Cho (2019) is the ratio of cost to power conversion, whereas the objective function in the present study is the total sum of u^3 .

In the present study, since it is assumed that there do not exist the aerodynamic interactions between the wind turbines located along the line perpendicular to wind direction, as shown in Fig. 8, the optimum layout of 30 wind turbines in 10×10 lattice should be the one similar to that of Grady et al. (2005) or that of Yang and Cho (2019). Anyway, as the number of possible locations of wind turbines and the number





1 column of possible locations of 3 wind turbines (Identified as the optimum layout in the present study) 10 columns of possible locations of 30 wind turbines (Efficiency 98.95%)

Fig. 8 The optimum layout of 30 wind turbines in a 10×10 lattice that should be identified in the present study

of wind turbines itself increases, the number of local minima of the corresponding objective functions should increase explosively, and therefore, it must become quite difficult to identify the global minimum even with the quantum annealing method, although it has been confirmed that the present method can somehow come out with the optimum layout of an offshore wind farm composed of even more than 100 wind turbines with reasonable CPU time.

Other than the work of Yang and Cho (2019) described above, do Couto et al. (2013), using an MATLAB code PCA (Production Calculation Algorithm) and the commercial software WindFarmer, compare the effects of wake models on the total conversion power, the efficiency and the capacity factor while subjecting the optimum layout identified by Mosetti et al. (1994) shown in Fig. 7 to their calculations, and they conclude that their PCA optimization tool by genetic algorithm gives as good results as those given by the well-established WindFarmer [e.g., Schlez et al. (2006)].

3.7 Partial shadowing in wakes

In the present study, it is assumed that the downwind wind turbine is fully wrapped in the wake caused by the upwind wind turbine, but, in reality, in the cases of oblique wind incident to the corresponding wind farm or in the case of a staggered array of wind turbines, the downwind turbine may not be fully wrapped but be partially shadowed by the upwind turbines and thus is not fully involved in the wake or it may be exposed to the wakes due to multiple wind turbines. Optimum layouts in these cases are dealt with in, e.g., do Couto et al. (2013) or Yang and Cho (2019), in which the wake effects are assumed to be certain functions of the ratio of the partially shadowed area of downwind wind turbines to their rotor areas. Anyway, it is the subject of future work to implement the partially shadowed wake effects in the Ising model.

4 Conclusions

As a possible application of quantum computing in the field of ocean engineering, an identification problem of an optimum offshore wind farm layout has been dealt with. In the present work, it has been shown in what way the corresponding problem can be mathematically modeled as a minimization of a polynomial function of binary variables, that is, as an Ising model, and thus can be solved with a quasi-quantum annealing method.

Through example calculations, it has been confirmed that the quasi-quantum annealing method works as well as other methods in identifying the optimum layouts, while the optimum layout of an offshore wind farm composed of even more than 100 wind turbines can be identified with reasonable CPU time.

Actually, the optimization of offshore wind farm layout can be solved and has been solved by many other numerical methods such as simulated annealing and genetic algorithm [e.g., Mosetti et al. (1994), Grady et al. (2005), do Couto et al. (2013), Yang and Cho (2019)], but, since the distinguished advantage of quantum computing is that the computing could be done much faster even than conventional supercomputers, the results of the present work, which were obtained with quasi-quantum annealers, encourage the development of quantum annealers.

The present work is a preliminary one as there exist several shortcomings from the practical point of view, such as (1) the wake model behind a wind turbine assumed in the present work is rather a simple one, (2) oblique wind incident to a wind farm is not dealt with, and so on. However, the author wishes that the present work will initiate the application of quantum computing in the field of ocean engineering.

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Declarations

Conflict of interest The authors declare no competing interests.

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