G ̈odel’s incompleteness theorem  
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1 Introduction

Kurt G ̈odel’s incompleteness theorems are two theory of mathematical logic.  
They concern the limits of probability in formal axiomatic theories. For any  
consistent formula system, there will always be statements about natural num-  
bers that are true but unprovable within the system. The first in comply states  
that no consistence of axioms can be listed by an effective procedure capable of  
proving all truths about the arithmetic of natural numbers.

2 Formal systems: completeness, consistency,  
and effective axiomatization

The incompleteness theorems apply to formal systems that are of sufficient com-  
plexity to express the basic arithmetic of the natural numbers. They show that  
the systems which contain a sufficient amount od arithmetic cannot possess all  
three of these properties. Theoretical systems can only be consistent if they are  
consistent, and effectively axiomatized. A formula system is said to be effec-  
tively axiomatized if its sets of theorems is a recursively enumerable set. This  
means that there is a computer program that could enumerate all the theorem of  
the system without any listing any statements that are not true. Incompleteness  
theorems apply only to formula systems which can prove a sufficient collection  
of facts about the natural numbers. Some systems, such as peano arithmetic,  
can directly express statements about natural numbers into their language. In  
the standard systems of first-order logic, an inconsistent set of axioms will prove  
every statement in its language. It is not even possible for an infinite list of ax-  
ioms to be complete, consistent, and effectively axiomatized. It has two type of  
incompleteness theorem that is A) First incompleteness theorem First incom-  
pleteness theorem is “any consistent formula system F within which a certain  
amount of elementary arithmetic can be carried out is incomplete; i.e., there are  
statements of the language of F which can neither be proved nor disproved in F.”  
B) Second incompleteness theorem Second incompleteness theorem is “assume  
F is a consistent formalized system which contains elementary arithmetic. This  
theorem is stronger than the first incompleteness theorem because the state-  
ment constructed in the first incompleteness theorem does not directly express  
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the consistency of the system. The proof of the second incompleteness theorem  
is obtained by formalizing the proof of the first incompleteness theorem within  
the system F itself.

3 Relationship with computability

Smorynski (1997) shows how the existence of recursively inseparable sets can be  
used to prove first incompleteness theorem. This proof is often extended to show  
that the systems such as peano arithmetic are essentially undecidable, Kleene  
(1967) says. Matiyasevich proved that there is no algorithm that determines  
whether there is an integer solution to the equation p= 0 . the incompleteness  
theorem is closely related to several results about undecidable sets in recursion  
theory. It can be used to obtain a proof to G ̈odel’s first incompletely.

4 conclusion

Incompleteness is a result of the fact that the all states in a system of formalism  
can be proved to be true. The results affect the philosophy od mathematics,  
particularly versions of formalistic formalism, which use a single system of logic  
to define their principles.