Substitute value of y in eq (1)

$6x + 3(0) = 0$

$6x = 0$

$x = 0$

critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0, 0)$

$= 24(0) + 6(0) = 0$

$r = 0$

$rt - s^2 = 0(-2) - (0)^2$

$= 0 - 0 = 0$

$r = 0 \quad rt - s^2 = 0$

(Nothing to say)

$\therefore f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} = -1 \quad \therefore x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} = 4$

$y = 4$

$\therefore$ critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = 33$

05/02/2021

critical points are $(0,2)$

$r = f_{xx} = 6$, $t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$\therefore$ $3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$f_y = 0$

$3x^2 - 2y = 0$

multiply eq ① with ③

② with ④

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$17y = 0$

Q.4)

$x^2 - 2y^2 + 3y + zy = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + 2$

$f_x = 2x + 2$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$

Eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}.$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

$3xyz = x + y + z - 4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$f_x = 3(-1)(2) - 1 = -7$

$f_y = 3(1)(2) - 1 = 5$

$f_z = 3(1)(-1) + 1 = -2$

eq of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

Eq of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

$f_y = x^2(-\sin y) + e^y x$

$(x_0, y_0) = (1, 0) \quad x_0 = 0 \quad y_0 = 0$

eq of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x - 1) + 1(y - 0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ It is the require eq of tangent

$= ax + by + c = 0$

$bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$d = 1$

$x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \quad x_0 = 2 \quad y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x - 2) + (-(y + 2)) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eq

$= ax + by + c = 0$

$bx + ay + d = 0$

$-1(x) + 2(y) + d = 0$

$-x + 2y + d = 0 \quad (2, -2)$

$-2 + 2(-2) + d = 0$

$-2 + 4 + d = 0$

$-6 + d = 0$

$d = 6$

$$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$$

$$\left(\frac{1}{2}; \frac{\pi}{\cancel{4}_2}\cancel{(2)}\right)$$

$$\left(\frac{1}{2}, -\frac{\pi}{2}\right)$$

2) $f(x, y, z) = xyz - e^{x+y+z} \quad a = (1, -1, 0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x, y, z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz e^{x+y+z}, xy e^{x+y+z}$

$f(1, -1, 0) = (-1)(0) - e^{(1+(-1)+0)}, (1)(0) e^{(1-1+0)}, (1)(-1) e^{(1-1+0)}$

$= 0 - e^0, 0 - e^0, -1 \cdot e^0)$

$= (-1, -1, -2)$

3) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f_x = \cos y \, 2x + e^{xy} y$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} = 4 \frac{ + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

Here $u = i + 5j$ is not a unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{20}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{26}\right)$

$f(a+hu) = \left(4 + \frac{5h}{\sqrt{20}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} \quad \frac{4h}{26} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$

$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x + 3y \quad a = (1, 2), \; u\,(3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+h) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$L(x,y) = \frac{2-\pi}{2} + (-1)(x - \pi/2) + 1(y)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$L(x,y) = 1 - x + y$

3) $f(x,y) = \log x + \log y$

$f(1,1) = \log 1 + \log 1$

$f(1,1) = 0$

$f_x = \frac{1}{x}$ $f_y = \frac{1}{y}$

$f_x(1,1) = 1$ $f_y(1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + (y-1)$

25/01/2020

$L(x,y) = x + y - 2$

1. $f(x,y) = x + 2y - 3$ $a = (1,-1)$ $u = 3i - j$

Here $u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1, \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$f_{xy} = \frac{\partial}{\partial y} f_x$

$= \frac{\partial}{\partial y}\, y\cos(xy) + e^x \cdot e^y$

$f_{xy} = -xy\sin(xy) + \cos(xy) + e^x \cdot e^y$

$f_{yx} = \frac{d}{dx} f_y$

$= \frac{d}{dx}\, x \cdot \cos(xy) + e^x \cdot e^y$

$f_{yx} = -xy\sin(xy) + \cos(xy) + e^x e^y$

$\therefore f_{xy} = f_{yx}$

c) $(a,b) = (1,1)$

$f(x,y) = \sqrt{x^2+y^2}$ $\qquad f(1,1) = \sqrt{1+1} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$ $\qquad f_y = \frac{2y}{2\sqrt{x^2+y^2}}$

$f_x = \frac{x}{\sqrt{x^2+y^2}}$ $\qquad f_y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}}$ $\qquad f_y(1,1) = \frac{1}{\sqrt{2}}$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{2+x+y-2}{\sqrt{2}}$

$L(x,y) = \frac{x+y}{\sqrt{2}}$

2) $f(x,y) = 1 - x + y\sin x$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 \times \sin \pi/2$

$f(\pi/2, 0) = \frac{2-\pi}{2}$

$f_x = -1 + y\cos x$ $\qquad f_y = \sin x$

$f_x(\pi/2, 0) = -1 + 0 \times \cos \pi/2$ $\qquad f_y(\pi/2, 0) = \sin \pi/2$

$f_x(\pi/2, 0) = -1$ $\qquad f_y(\pi/2, 0) = 1$

$L(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2) + f_y(\pi/2, 0)(y - 0)$

a. $f_{yy} = \frac{\partial}{\partial y} f_y$

$= \frac{\partial}{\partial y} 6x^2y$

$f_{yy} = 6x^2$

$f_{xy} = \frac{\partial}{\partial y} f_x$

$\frac{\partial}{\partial y} \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$f_{xy} = 12xy$

$f_{yx} = \frac{\partial}{\partial x} f_y$

$\frac{\partial}{\partial x} 6x^2y$

$f_{yy} = 12xy$

$f_{xy} = f_{yx}$

Scanned with CamScanner

(a) $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{\partial}{\partial x} \frac{y^2 - xy}{x^2}$

$= -2y^2 x^{-3} + \frac{y}{x^2}$

$f_x = \frac{y}{x^2} - \frac{2y^2}{x^3}$

$f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x} f_x$

$\frac{\partial}{\partial x} \left(\frac{y}{x^2} - \frac{2y^2}{x^3} \right)$

$f_{xx} = \frac{6y^2}{x^4} - \frac{2y}{x^3}$

$f_{yy} = \frac{\partial}{\partial y} y$

$= \frac{2}{x^2}$

$f_{xy} = \frac{d}{dy} f_x = \frac{d}{\partial y} \frac{y}{x^2} \quad \frac{-2y^2}{x^3}$

$f_{xy} = \frac{1}{x^2} - \frac{4y}{x^3}$

$f_{yx} = \frac{\partial}{\partial x} f_y$

$= \frac{\partial}{\partial x} \frac{2y - x}{x^2}$

$\frac{1}{x^2} - \frac{4y}{x^3}$

$f_{xy} = f_{yx}$

Q.2]

$f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x} x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y} x^3 + 3x^2y^2 - \log(x^2+1)$

$= \cancel{0} + 6yx^2 - 0$

$f_y = 6x^2y$

$f_{xx} = \frac{\partial}{\partial x} f_x$

$= \frac{\partial}{\partial x} \left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$f_{xx} = 6x + 6y^2 - \frac{4x - 2x^2 + 2}{(x^2+1)^2}$

$f_y = xe^{x^2}\frac{\partial}{\partial y}\, y e^{y^2}$

$= xe^{x^2}\left[y\frac{\partial}{\partial y} e^{y^2} + e^{y^2}\frac{\partial}{\partial y} y\right]$

$= xe^{x^2}\left[2y^2 e^{y^2} + e^{y^2}\right]$

$f_y = (2y^2+1)\, x\, e^{x^2+y^2}$

2. $f(x,y) = e^x \cdot \cos y$

$f_x = \frac{d}{\partial x} e^x \cos y$

$f_x = e^x \cdot \cos y$

$f_y = \frac{\partial}{\partial y} e^x \cdot \cos y$

$f_y = -e^x \cdot \sin y$

3. $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\rightarrow f_x = \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 2x^3y - 3x^2 + 3y^2$

Q3) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{1}{1-y^2}\frac{\partial}{\partial x} 2x$

$f_x = \frac{2}{1+y^2}$

$f_x(0,0) = \frac{2}{1+0^2} = \frac{2}{1}$

$f_x(0,0) = 2$

$f_y = 2x \frac{\partial}{\partial y}\frac{1}{1+y^2}$

$2x \times \frac{-1}{(1+y^2)^2} \times 2y$

$= \frac{-4xy}{(1+y^2)^2}$

$f_y(0,0) = \frac{-4\times0\times0}{(1+0^2)^2}$

$f_y(0,0) = 0$

Practical - 9

Q.1

1. $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 + 1}{xy + 5}$

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 + 1}{(-4)(-1) + 5}$

$= \dfrac{-64 + 3 + 1 + 1}{4 + 5}$

$= \dfrac{-59}{9}$

2. $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$= \dfrac{(0+1)[2^2 + 0^2 - 4(2)]}{2 + 3(0)}$

$= \dfrac{1 \times (4 + 0 - 8)}{2}$

$= \dfrac{-4}{2}$

$= -2$

3) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$

$= \dfrac{1^2 - (1)^2 \times (1)^2}{1^3 - (1)^2 (1)(1)}$

$= \dfrac{1 - 1}{1 - 1}$

$= \dfrac{0}{0}$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$ is not defined

Q.2.

1. $f(x,y) = xye^{x^2 - y^2}$

$f(x,y) = x \cdot y \cdot e^{x^2} \cdot e^{y^2}$

$f_x = \dfrac{\partial}{\partial x} x \cdot y \cdot e^{x^2} \cdot e^{y^2}$

$= ye^{y^2} \dfrac{\partial}{\partial x} x \cdot e^{x^2}$

$= ye^{y^2} \left[x \dfrac{\partial}{\partial x} e^{x^2} + e^{x^2} \dfrac{\partial}{\partial x} y \right]$

$= ye^{y^2} \left[2x^2 e^{x^2} + e^{x^2} \right]$

$f_x = (2x^2 + 1) ye^{x^2 + y^2}$

$$\frac{dv}{dx} = 1-\sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + C$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$

$$2 + \frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{v-1}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= 3\left(\frac{v+1}{v+2}\right)$$

$$\int\left(\frac{v+2}{v+1}\right) dv = 3\int dx$$

$$\int \frac{v+1}{v+1} dv + \int \frac{1}{v+1} dv = 3x + c$$

$$v + \log(v+1) = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + C$$

$$3y = x - \log|2x+3y+1| + C$$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = 2xe^{-2x}$

$I.F = e^{\int P(x)dx} = e^{\int 2dx} = e^{2x}$

$y(I.F) = \int Q(x)(I.F)dx + C$

$y e^{2x} = \int 2x * e^{-2x} \times e^{2x} dx$

$\therefore ye^{2x} = \int 2x\, dx + C$

$ye^{2x} = x^2 + C$

vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$\frac{\sec^2 x\, dx}{\tan x} = \frac{\sec^2 y\, dy}{\tan y}$

$\int \frac{\sec^2 x\, dx}{\tan x} = \int \frac{\sec^2 y\, dy}{\tan y}$

$\therefore \log(\tan x) = -\log(\tan y) + c$

$\log(\tan x \cdot \tan y) = c$

$\tan x \cdot \tan y = e^c$

vi) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1$

Differentiate both sides

$x-y+1 = v$

$1-\frac{dy}{dx} = \frac{dv}{dx}$

$1-\frac{dv}{dx} = \frac{dy}{dx}$

$1-\frac{dv}{dx} = \sin^2 v$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$

$P(x) = 2/x \qquad Q(x) = \cos x/x^2$

$I.F = \int P(x)dx$

$= e^{\int 2/x\, dx}$

$= e^{2\ln x}$

$= e^{\ln x^2}$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + C$

$x^2y = \int \frac{\cos x}{x^2} \times x^2\, dx + C$

$= \cos + C$

$x^2y = \sin x + C$

$x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3}{x}y = \frac{\sin x}{x^3}$

$P(x) = 3/x \qquad Q(x) = \sin x/x^3$

$I.F = e^{\int P(x)dx}$

$= e^{\int 3/x\, dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)\,dx + C$

$x^3y = \int \frac{\sin x}{x^3} \times x^3\, dx + C$

$= \int \sin x\, dx + C$

$x^3y = -\cos x + C$

Practical-7

i) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = 1/x \qquad g(x) = e^x/x$

$I.F = e^{\int P(x)dx}$

$= e^{\int \frac{1}{x}dx}$

$= e^{\log x}$

$I.F = x$

$y(I.F) = \int (I.F)dx + c$

$y x = \int \frac{e^x}{x} \times x \, dx + c$

$= \int e^x dx + c$

$xy = e^x + c$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{x}$

$I.F = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)dx + c$

$y e^{2x} = \int e^{-x} \times e^{2x} dx + c$

$= \int e^{-x+2x} dx + c$

$= \int e^x dx + c$

$e^{2x} = e^x + c$

a.i) $\int_0^2 e^{x^2}\,dx$ with $n=4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982

$\int e^{x^2}\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+(y_2)\right]$

$= \frac{0.5}{3}\left[(1+54.5982)+4(1.284+9.4877)+2(2.7183)\right]$

$= \frac{0.5}{3}\left[55.5982+43.0868+5.436\right]$

$\int_0^2 e^{x^2}\,dx \; 17.3535$

ii) $\int_0^4 x^2\,dx \quad n=4$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$\int x^2\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$

$= \frac{1}{3}\left[0+16+4(1+9)+2\times 4\right]$

$= \frac{1}{3}\left[16+4(10)+8\right] = \frac{64}{3}$

$\int_0^4 x^2\,dx = 21.3333$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad n=6$

$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$
y	0	0.4167	0.5848	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^{\pi/3} \sqrt{\sin x}\,dx = \frac{h}{3}\left[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$

$= \frac{\pi/18}{3}\left[0.4167+0.9306+4(0.4167+0.7071+0.8752)+2(0.5848+0.8017)\right]$

$= \frac{\pi}{54}\left[1.3473+4(1.999)+2(1.3865)\right]$

$= \frac{\pi}{54}\left[1.3473+7.996+2.773\right]$

$= \frac{\pi}{54} \times 12.1163$

$\int_0^{\pi/3} \sqrt{\sin x}\,dx = 0.7049.$

1.2

$x = 3\sin t \qquad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\pi\, dt$

$= 3\int_0^{2\pi} dt$

$= 3[x]_0^{2\pi}$

$3(2\pi - 0)$

$L = 6\pi$ units

5. $x = \frac{1}{6}y^3 + \frac{1}{2y}$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{1 + \left(\frac{(y^4-1)^2}{4y^4}\right)}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4-1)^2 + 4y^4}{4y^4}}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left\{\frac{17}{6}\right\} = \frac{17}{12}$ units.

$= -4\left(\cos \frac{2\pi}{2} - \cos 0\right)$

$= -4(-1 - 1)$

$= -4 \times -2$

$L = 8 \text{ units}$

2) $y = \sqrt{4 - x^2}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$

$= \frac{-x}{\sqrt{4-x^2}}$

$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$

$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$

2) $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

$x^3 - 3x^2 + 1 = t$

$\therefore 3x^2 - 6x = \frac{dt}{dx}$

$\therefore (x^2 - 2x)\,dx = \frac{dt}{3}$

$I = \frac{1}{3} \int \frac{1}{t}\, dt$

$= \frac{1}{3} \log(t) + c$

$I = \frac{1}{3} \log(x^3 - 3x^2 + 1) + c$

Practical - 6

1). $x = A \sin t \qquad y = 1 - \cos t$

$\frac{dx}{dt} = \sin t$

$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(1 - \cos t)^2 + \sin^2 t}\; dt$

$\int_0^{2\pi} 1 - 2\cos t + \cos^2 t + \sin^2 t$

$\int \sqrt{2 - 2\cos t}\; dt$

$\int_0^{2\pi} \sqrt{2} \sqrt{1 - \cos t}\; dt$

$\int_0^{2\pi} \sqrt{2} \times \sqrt{2} \sqrt{\sin^2 t/2}\; dt$

$= 2 \int_0^{2\pi} \sin t/2\; dt$

$= -2 \left[\frac{\cos t/2}{1/2} \right]_0^{2\pi}$

vii) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$\frac{1}{x^2} = t$

$\therefore \frac{-2}{x^3} = \frac{dt}{dx}$

$\therefore \frac{dx}{dt} = \frac{-1}{2} dt$

$I = \frac{-1}{2} \int \sin t \, dt$

$= \frac{-1}{2} = \cdot \cos t + C$

$= \frac{\cos t}{2} + C$

$I = \frac{\cos(1/x^2) + C}{2}$

viii) $I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$ $2x$

Put $\sin x = t$

$\therefore \cos dx = dt$

$I = \int \frac{1}{t^{2/3}} dt$

$\int t^{-2/3} dt$

$= \frac{t^{-2/3+1}}{-2/3+1} + c$

$= 3t^{1/3} + c$

$= 3\sqrt[3]{\sin x} + C$

ix) $I = \int e^{\cos^2 x} \cdot \sin 2x \, dx$.

Put $\cos^2 x = t$

$-2\cos x \sin x = \frac{dt}{dx}$

$\sin 2x \, dx = -dt$

$I = -\int e^t dt$

$= e^t + C$

$I = -e^{\cos^2 x} + C$

$$I = \frac{1}{4}\left(x\int \sin 2x\,dx - \int \frac{d}{dx}x \int \sin 2x\,dx\right)dx$$

$$\frac{1}{4}\left[-\frac{x\cdot\cos 2x}{2} + \frac{1}{2}\right)\cos 2x\,dx\Big]$$

$$= \frac{1}{16}\sin 2x - \frac{x\cdot\cos 2x}{8} + c$$

$$I = \frac{1}{16}\sin 2t^4 - \frac{t^4\cdot\cos 2t^4}{8} + C$$

vi) $I = \int \sqrt{x}\,(x^2-1)\,dx$

$$I = \int x^2\sqrt{x}\,2x - \int \sqrt{x}\,dx$$

$$= \int x^{5/2}dx - \int x^{1/2}dx$$

$$\frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + C$$

$$= \frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x\sqrt{x} + C$$

v) $I = \int t^7 \sin(2t^4)\,dt$

$\int t^4 \sin(2t^4)\; t^3$

Put $t^4 = x$

$4t^3 = \frac{dx}{dt}$

$t^3 dt = \frac{1}{4}dx$

$$I = \frac{1}{4}\int x\sin(2x)\,dx$$

$$I = \frac{1}{4}\left[x\int \sin 2x\,dx - \int\left(\frac{d}{dx}x \int \sin 2x\,dx\right)dx\right]$$

$$= \frac{1}{4}\left[-\frac{x\cdot\cos 2x}{2} + \frac{1}{2}\int \cos 2x\,dx\right]$$

$$= \frac{1}{16}\sin 2x - \frac{x\cdot\cos 2x}{8} + c$$

$$I = \frac{1}{16}\sin 2t^4 - \frac{t^4\cdot\cos 2t^4}{8} + C$$

vi) $I = \int \sqrt{x}\,(x^2-1)\,dx$

$$I = \int x^2\sqrt{x}\,dx - \int\sqrt{x}\,dx$$

$$= \int x^{5/2}dx - \int x^{1/2}dx$$

$$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + C$$

$$= \frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x\sqrt{x} + C$$

Practical 5

i) $I = \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$

$= \int \frac{1}{\sqrt{(x+1)^2-2^2}}\,dx$

$I = \ln\left|x+1+\sqrt{x^2+2x-3}\right| + C$

ii) $I = \int (4e^{3x}+1)\,dx$

$4\int e^{3x}dx + \int 1\,dx$

$I = \frac{4e^{3x}}{3} + x + C$

iii) $I = \int (2x^2 - 3\sin x + 4\sqrt{x})\,dx$

$= 2\int x^2 dx - 3\int \sin x\,dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{5\times 2}{3} x^{3/2} + C$

$= \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x\sqrt{x} + C$

iv) $I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

put $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx} \quad \therefore \frac{dx}{\sqrt{x}} = 2dt$

$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}}\,dx$

$= 2\int t^6 + 3t^2 + 4\,dt$

$= 2\left[\frac{t^7}{7} + \frac{3t^3}{3} + 4t\right] + C$

$= 2\left[\frac{x^{7/2}}{7} + x^{3/2} + 4x^{1/2}\right] + C$

c) $I = \int t^3 \sin(2t^4)\,dt$

$\int t^4 \sin(2t^4) \times t^3 dt$

put $t^4 = x$

$4t^3 = \frac{dx}{dt}$

$t^3 dt = \frac{1}{4}dx$

$I = \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$f(x_3) = -0.0005$

$f'(x_3) = 17.9757$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7065 - \frac{0.0009}{17.9757}$$

$x_4 = 2.7065$

iv) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

-2.2 is close to 0 so the number is

$x_0 = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{-2.2}{-5.2}$$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5769) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5769 - \frac{0.6762}{-8.217}$$

$x_2 = 1.6592$

$f(x_2) = 0.0204$

$f'(x_2) = -7.7143$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 - \frac{0.0204}{-7.7144}$$

$x_3 = 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$f(x_3) = 0$

1.6618 is the root of the function

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_1 = 0$

$f(x_0) = 9.5$

$f'(x_0) = -55$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0 - \frac{9.5}{-55}$

$x_1 = 0.1727$

$f(x_1) = -0.0828$

$f'(x_1) = -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0828}{-55.9467}$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$f'(x_2) = -55.9393$

$x_3 = x_2 - \frac{f(x_1)}{f(x_2)} = 0.1712 - \frac{0.0011}{-55.9345}$

$\therefore x_3 = 0.1712$

$x = 0.1712$ is the root of the equation

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = -9$

$f(3) = 6$

6 is close so 3 and the number is

$x_0 = 3$

$f(x_0) = 6$

$f'(x_0) = 23$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5942$

$f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f(x_2)}$

$= 0.5942 / 18.508$

$x_2 = 2.701$

$f(x_2) = 0.0085$

$f'(x_2) = 17.9835$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.707 - \frac{0.0065}{17.9835}$

$x_3 = 2.7065$

Scanned with CamScanner

iii)

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

For maxima / minima

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$f(x)$ is maximum at $x = 0$ and minimum $x = 2$

$f(0) = 1$

$f(2) = -3$

10) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

maxima / minima

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f''(2) = 24 - 6 \quad 18 > 0$

$f(x)$ is maximu at $x = 1$ and minimum at $x = 2$

$f(-1) = 8$

$f(2) = -19$

Practical 4

a) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

For maxima / minima

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = f''(-2) = 2 + \frac{96}{(\pm 2)^4} = \frac{2 \times 96}{16} = 8 > 0$

$f(x)$ is minimum at $x = \pm 2$

$f(x) = 8$ is the minimum value

ii) $f(x) = 3 - 5x^3 + 3x^3$

$f'(x) = 15x^4 - 15x^2$

For maxima / Minima

$f'(x) = 15x^4 - 15x^2 = 0$

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

$x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$f(x)$ is maximum at -1 and minima at 1

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

iii) $y = x^3 - 27x + 5$

$y' = 3x^2 - 27$

$y'' = 6x$

For concave upwards

$y'' > 0$

$6x > 0$

$x > 0$

f(x) is concave upwards for $x \in (0, \infty)$

To concave downwards

$y'' < 0$

$6x < 0$

$\therefore \cancel{x}\, x < 0$

f(x) is concave downwards for $x \in (-\infty, 0)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

$y' = 6x^2 - 18x - 24$

$y'' = 12x - 18$

To concave upward

$y'' > 0$

$12x > 18$

$x > 3/2$

f(x) is concave upward for $x \in (3/2, \infty)$

For concave downwards.

$y'' < 0$

$12x - 18 < 0$

$\therefore x < 3/2$

$\therefore$ f(x) is concave downwards for $x \in (-\infty, 3/2)$

(2) $f(x) = 2x^3 + x^2 - 20x + 4$

$y' = 6x^2 + 2x - 20$

$\therefore y'' = 12x + 2$

For concave upwards.

$y'' > 0$

$12x + 2 > 0$

$x > -2/12$

$x > -1/6$

f(x) is concave upwards for $x \in (-1/6, \infty)$

To concave downwards

$y'' < 0$

$12x + 2 < 0$

$x < -1/6$

$\therefore$ f(x) is concave downwards for $x \in (-\infty, -1/6)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$y' = 4x^3 - 18x^2 + 24x + 5$

$y'' = 12x^2 - 36x + 24$

For concave upwards

$y'' > 0$

$12x^2 - 36x + 24 > 0$

$x^2 - 3x + 2 > 0$

$x^2 - 2x - x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$

Case 1:

$x - 2 > 0$ and $x - 1 > 0$

$x > 2$ $\quad$ $x > 1$

$x > 2$

OR

Case 2:

$x - 2 < 0$ and $x - 1 < 0$

$x < 2$ and $x < 1$

$x < 1$

y is concave upward for

$x \in (-\infty, 1) \cup (2, \infty)$

For concave downwards

$y'' < 0$

$12x^2 - 36x + 24 < 0$

$x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

Case 1

$x - 2 < 0$ and $x - 1 > 0$

$x < 2$ and $x > 1$

$1 < x < 2$

$x \in (1, 2)$

Case 2:

$x - 2 > 0$ and $x - 1 < 0$

$\therefore x > 2$ and $x < 1$

This is not possible

$\therefore$ case 2 is invalid

$\therefore$ y is concave downwards for

$x \in (1, 2)$

To decreasing

$f'(x) < 0$

$6x^2 - 18x - 24 < 0$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

Case 1

$x-4 < 0$ and $x+1 > 0$

$x < 4$ and $x > -1$

$x \in (-1, 4)$

or case 2

$x-4 > 0$ and $x+1 < 0$

$x > 4$ and $x < -1$

This is not possible

$\therefore$ case 2 is invalid

$f(x)$ is decreasing in the interval $x \in (-1, 4)$

2) i) $y = 3x - 2x^3$

$y' = 6x - 6x^2$

$y'' = 6 - 12x$

For concave upwards

$y'' > 0$

$6 - 12x > 0$

$6 > 12x$

$12x < 6$

$x < 1/2$

y is concave upwards for $x \in (-\infty, 1/2)$

For concave downwards

$y'' < 0$

$6 - 12x < 0$

$12x > 6$

$x > 1/2$

y is concave downwards for $x \in (1/2, \infty)$

$x > 2/3$ and $x < -2$

∴ This is not possible
case 2 is invalid

f(x) is decreasing in the interval
$x \in (-2, 2/3)$

iv) $f(x) = x^3 - 27x + 5$

∴ $f'(x) = 3x^2 - 27$

for increasing

$f'(x) > 0$

$3x^2 - 27 > 0$

$x^2 - 9 > 0$

$x^2 > 9$

$x < -3$ OR $x > 3$

f(x) is increasing in the interval $x \in (-\infty, -3) \cup (3, \infty)$

For decreasing

$f'(x) < 0$

$3x^2 - 27 < 0$

$x^2 < 9$

$x > -3$ OR $x < 3$

$-3 < x < 3$

∴ f(x) is decreasing in the interval $x \in (-3, 3)$

c) $f(x) = 64 - 24x - 9x^2 + 2x^3$

$f'(x) = 6x^2 - 18x - 24$

for increasing

$f'(x) > 0$

∴ $6x^2 - 18x - 24 > 0$

∴ $x^2 - 3x - 4 > 0$

∴ $x^2 - 4x + x - 4 > 0$

∴ $x(x-4) + 1(x-4) > 0$

∴ $(x-4)(x+1) > 0$

case 1

$x - 4 > 0$ and $x + 1 > 0$

$x > 4$ $\quad$ $x > -1$

$x > 4$

OR

cas 2

$x - 4 < 0$ and $x + 1 < 0$

$x < 4$ and $x < -1$

$x < -1$

f(x) is increasing in the interval
$x \in (-\infty, -1)$ $(4, \infty)$

ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

For increasing

$f'(x) > 0$

$\therefore 2x - 4 > 0$

$2x > 4$

$x > 2$

$f(x)$ is increasing in the interval $x \in (2, \infty)$

For decreasing

$f'(x) < 0$

$2x - 4 < 0$

$2x < 4$

$x < 2$

$\therefore f(x)$ is decreasing in the interval $x \in (-\infty, 2)$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

For increasing

$f'(x) > 0$

$\therefore 6x^2 + 2x - 10 > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 2(x+2) > 0$

$\therefore (3x-2)(x+2) > 0$

case 1

$3x - 2 > 0$ and $x + 2 > 0$

$3x > 2$ and $x > -2$

$x > 2/3$ and $x > -2$

$\therefore x > 2/3$

or case 2:

$3x - 2 < 0$ $\quad x + 2 < 0$

$x < 3/2$ $\quad x < -2$

$x < -2$

$f(x)$ is increasing in the interval $x \in (2/3, \infty) \cup (-\infty, -2)$

To decreasing

$f'(x) < 0$

$6x^2 + 2x - 10 < 0$

$3x^2 + x - 10 < 0$

$(3x-2)(x+2) < 0$

case 1

$3x - 2 < 0$ and $x + 2 > 0$

$x < 2/3$ and $x > -2$

$x \in (-2, 2/3)$

case 2

$3x - 2 > 0$ and $x + 2 < 0$

(3)

$L.H.D = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$

$\lim_{x \to 3} \frac{4x+7-4(3)-7}{x-3}$

$= 4 \lim_{x \to 3} \frac{x-3}{x-3}$

$L.H.D = 4$

$R.H.D = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3} \frac{x^2+3x+1-3^2-3(3)-1}{x-3}$

$= \lim_{x \to 3} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3} \frac{(x+6)(x-3)}{x-3}$

$= \lim_{x \to 3} x+6$

$= 9$

$L.H.D \neq R.H.D$

$\therefore$ The function is not differentiable at $x = 3$

2) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

For increasing:-

$f'(x) = 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$ $\therefore x < -\sqrt{\frac{5}{3}}$ OR $x > \sqrt{\frac{5}{3}}$

$\therefore$ f is increasing in $x \in (-\infty, \sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

To decreasing

$3x^2 - 5 < 0$

$x^2 < \frac{5}{3}$

$x < \sqrt{\frac{5}{3}}$ OR $x > -\sqrt{\frac{5}{3}}$

$-\sqrt{\frac{5}{3}} < x < \sqrt{\frac{5}{3}}$

$\therefore f(x)$ is decreasing ind introde $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

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iii) Ans. $\sec x$

if $f'(x)$ exists

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$

$\lim_{x \to a} \frac{\cos a - \cos x}{x - a} \times \sec x \cdot \sec a$

$= 2 \lim_{x \to a} \frac{\sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)}{x - a} \sec x \cdot \sec a$

$x - a = h, \quad h \to 0 \quad x \to a$

$f'(a) = \lim_{h \to 0} \sin\left(\frac{2a+h}{2}\right) \sin(h/2) \cdot \sec x \cdot \sec a$

$= 2 \times \frac{1}{2} \times \sin a \times \frac{1}{\cos a} \times \sec a$

$f'(a) = \tan a \cdot \sec a$

Q.2

Ans. $f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2} \frac{4x + 7 - 4(2) - 7}{x - 2}$

$= 4 \lim_{x \to 2} \frac{x - 2}{x - 2}$

LHD = 4

RHD = $\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2} \frac{x^2 + 5 - 2^2 - 5}{x - 2}$

$= \lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2} \quad x^2 - 2^2$

$= 2' \times 2'$

$= 4$

RHD = 4

LHD = R.H.D

The function is differentiable at $x = 2$

Practial-2

Q.1

i) $f(x) = \cot x$

if $f'(a)$ exists

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f'(a) = \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{\frac{\cos x}{\sin x} - \frac{\cos a}{\sin a}}{x - a}$

$f'(a) = \lim_{x \to a} \frac{\sin a \cos x - \sin x \cos a}{\sin x \sin a (x - a)}$

$\lim_{x \to a} \frac{\sin (a - x)}{(x - a) \sin x \sin a}$

$\lim_{x \to a} \frac{\sin (x - a)}{x - a} \operatorname{cosec} x \operatorname{cosec} a$

$x - a = h \quad x \to a \quad h \to 0$

$= -\lim_{h \to 0} \frac{\sin h}{h} \operatorname{cosec}(a+h) \operatorname{cosec} a$

$= -1 \times \operatorname{cosec}(a+0) \times \operatorname{cosec}(a)$

$f'(a) = -\operatorname{cosec}^2 a$

ii) $f(x) = \operatorname{cosec} x$

if $f'(x)$ exists.

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$= \lim_{x \to a} \frac{\sin a - \sin x}{x - a} \times \operatorname{cosec} x \operatorname{cosec} a$

$= 2 \lim_{x \to a} \sin \left(\frac{a - x}{2}\right) \cos \left(\frac{x + a}{2}\right) \times \operatorname{cosec} x \cdot \operatorname{cosec} x$

$= x - a = h \quad x \to a \quad h \Rightarrow 0$

$\therefore f'(a) = 2 \lim_{h \to 0} \sin \left(\frac{-h}{2}\right) \cos \left(\frac{2a + h}{2}\right) \times \operatorname{cosec}(a+h) \operatorname{cosec} a$

$= 2 \times -\frac{1}{2} \times \cos (a + 2a/2) \times \operatorname{cosec}(a+0) + \operatorname{cosec} a$

$= -1 \frac{\times \cos a}{\sin a} + \operatorname{cosec} a$

$f'(a) = -\cot a \operatorname{cosec} a.$

RH·L ≠ LHS

The function can be made continous by redefing f(0)

or f(0) = 9/2

ii) sol

$RHS = \frac{\pi}{60}$

$LHS = \lim_{x\to 0} \frac{(e^{3x}-1)\sin x^\circ}{x^2}$

$= 3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \times \lim_{x\to 0} \frac{\sin\left(\frac{3x}{180}\right)}{x}$

$= 3 \times 1 \times \frac{\pi}{180}$

$= \frac{\pi}{60}$

LHL = RHL

f(x) is continous at x = 0

g) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $\quad x \neq 0$

is continous at $x = 0$

$\therefore f(0) = \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$

$= \lim_{x\to 0} \frac{e^{x^2} - 1 + (-\cos x + 1)}{x^2}$

$= \lim_{x\to 0} \frac{e^{x^2} + 1}{x^2} + 2\lim_{x\to 0} \frac{\sin^2 x/2}{x^2}$

$= 1 + 2 \times \left(\frac{1}{2}\right)^2$

$= 1 + 1/2$

$= 3/2$

$LHS = \lim_{x \to 6^-}$

$= \lim_{x \to 6} x + 3$

$= 6 + 3$

$LHS = 9$

$RHS = \lim_{x \to 6^+} f(x)$

$\lim_{x \to 6} \frac{x^2 + 9}{x + 3}$

$= \frac{36 + 9}{6 + 9}$

$= \frac{45}{15}$

$= 5/3$

$RHL \neq LHS$

$\therefore f(x)$ is not continous at $x = 6$

6)

$\because f(x)$ is continous at $x = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0} \frac{1 - \cos 6x}{x^2} = k$

$\therefore k = \frac{2 \lim \sin^2 3x}{x^2}$

$= 2 \times 4 \lim_{x \to 0} \frac{\sin^2 2x}{(2x)^2}$

$= 8 \times 1$

$K = 8$

$\because f(x)$ is continous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} f(x)$

$K = \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$= \lim_{x \to 0} \left(1 + \frac{1}{\cot^2 x}\right)^{\cot^2 x}$

$= \lim_{x \to 0} (1 + \tan^2 x)^{1/\tan^2 x}$

w.k.T, $\lim_{x \to 0} (1 + px)^{1/x} = e$

$K = e$

7) $RHL = 9$

$LHL \lim_{x \to 0} \frac{1 - \cos 3x}{x \tan x}$

$= 2 \lim_{x \to 0} \frac{\sin^2 3x/2}{x \tan x}$

$= 2 \lim_{x \to 0} \frac{\sin^2 3x/2 \, / x^2}{\tan x / x}$

$= 2 \times \left(\frac{3}{2}\right)^2 \times 1$

$= 9/2$

4. $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$

$= \lim_{x \to \infty} \left[\frac{x^2+5 - x^2+3}{x^2+3 - x^2-1} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$

$= \frac{8}{2} \lim_{x \to \infty} \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+5} + \sqrt{x^2-3}}$

$= 4 \lim_{x \to \infty} \frac{\sqrt{1+3/x^2} + \sqrt{1+1/x^2}}{\sqrt{1+5/x^2} + \sqrt{1+3/x^2}}$

$= 4 \left(\frac{\sqrt{1+0} + \sqrt{1+0}}{\sqrt{1+0} + \sqrt{1+0}} \right)$

$= 4$

5) i) LHS $\lim_{x \to \pi/2^-} f(x)$

$= \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x \to \pi/2} \frac{2 \sin x \cos x}{\sqrt{2} \sin x}$

$= \sqrt{2} \times 0$

$= 0$

RHL $= \lim_{x \to \pi/2^+} f(x)$

$= \lim_{x \to \pi/2} \frac{\cos x}{\pi - 2x}$

$x - \frac{\pi}{2} = h$

$x \to \frac{\pi}{2}, h \to 0$

RHL $= \lim_{h \to 0} \frac{\cos(\pi/2 + h)}{-2h}$

$= \frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{1}{2}$

LHS $\neq$ RHL

$f(x)$ is discontinuous at $x = \pi/2$

ii). LHS $\lim_{x \to 3^-} f(x)$

$= \lim_{x \to 3} \frac{x^2-9}{x-3}$

$= \lim_{x \to 3} \frac{x^2-3^2}{x-3}$

$= 2(3) = 6$

RHL $\lim_{x \to 3^+} x+3$

$= 3+3$

$= 6$

LHS = RHS

No.

$$= \lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}}$$

$$\lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$$

$$= \lim_{y \to 0} \frac{y}{y\left(a+y+\sqrt{a^2+ay}\right)}$$

$$= \lim_{y \to 0} \frac{1}{a+y+\sqrt{a^2+ay}}$$

$$= \frac{1}{a+0+\sqrt{a^2+0}}$$

$$= \frac{1}{a+a}$$

$$= \frac{1}{2a}$$

7) (w.r.t.) write the function have removable discontinuity and redefine them to remove the discontinuity

i) $f(x) = \dfrac{1-\cos 3x}{x \tan x}$ $x \neq 0$
$= 9$ $x = 0$ $\}$ $x = 0$

ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$ $x \neq 0$
$= \dfrac{\pi}{6}$ $x = 0$ $\}$ at $x = 0$

If $f(x) = e^{x^2} - \cos x$ for $x \neq 0$ continuous at $x = 0$, find $f(0)$

1. $\lim_{x\to a} \left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$= \lim_{x\to a} \left[\dfrac{a+2x-3x\ (\sqrt{3a+x}+2\sqrt{x})}{3a+x-4x\ (\sqrt{a+2x}+\sqrt{3x})}\right]$

$= \dfrac{1}{3} \lim_{x\to a} \left[\dfrac{\cancel{a-x}}{\cancel{a-x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$

$= \dfrac{1}{3}\left[\dfrac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}\right]$

$= \dfrac{1}{3}\left[\dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3}\sqrt{a}+\sqrt{3}\sqrt{a}}\right]$

$= \dfrac{1}{3} \times \dfrac{2}{\sqrt{3}}$

$= \dfrac{2}{3\sqrt{3}}$

Practical 1

$f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x < \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\pi/2 \le x < \pi$

} at $x = \pi/2$

$f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x < 6$

$= \frac{x^2-3}{x+3}$ $6 \le x < 9$

} at $x = 3$ and $x = 6$

Find the value of k the individual point to the funtion f(x) in continous at

$f(x) = \frac{1-\cos 4x}{x^2}$ $x < 0$

$= k$ $x \geq 0$

} at $x = 0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

$= k$

$x < 0$

} at ...

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