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Assignment – 7

Numerical integration 1: Numerical integration 4: We consider the 1-D motion of a particle of mass m in a time, independent potential V(x). The fact that the energy E will be conserved allows us to integrate the equation of motion and obtain a solution in closed form. We consider a case of SHM where the particle of mass m is in bound motion between two points a and b where V(a)=E and V(b)=E and V(x)<E for a<x<b. The time period of the oscillation T is given by:

$$T = \int_a^b \frac{\sqrt{2m}}{\sqrt{E - V(x)}} dx$$

Consider a particle with m = 1Kg in the potential V(x) = α(x^2)/2 with α = 4 Kg/sec2.

a) Numerically calculate the time period of oscillation by integrating the equation with Trapezoidal method and check this against the expected value. Note that the integrand will diverge at the limits. So, the limits must be redefined i.e. (b-ε) in place of b. Numerically obtained value of T, on the other hand, will diverge for very low values of ε. Make a log-log plot of ε vs. T. Then choose a suitable value of ε that will provide a reasonably accurate value of T.

b) Verify that T does not depend on the amplitude of oscillation.

ANSWER:

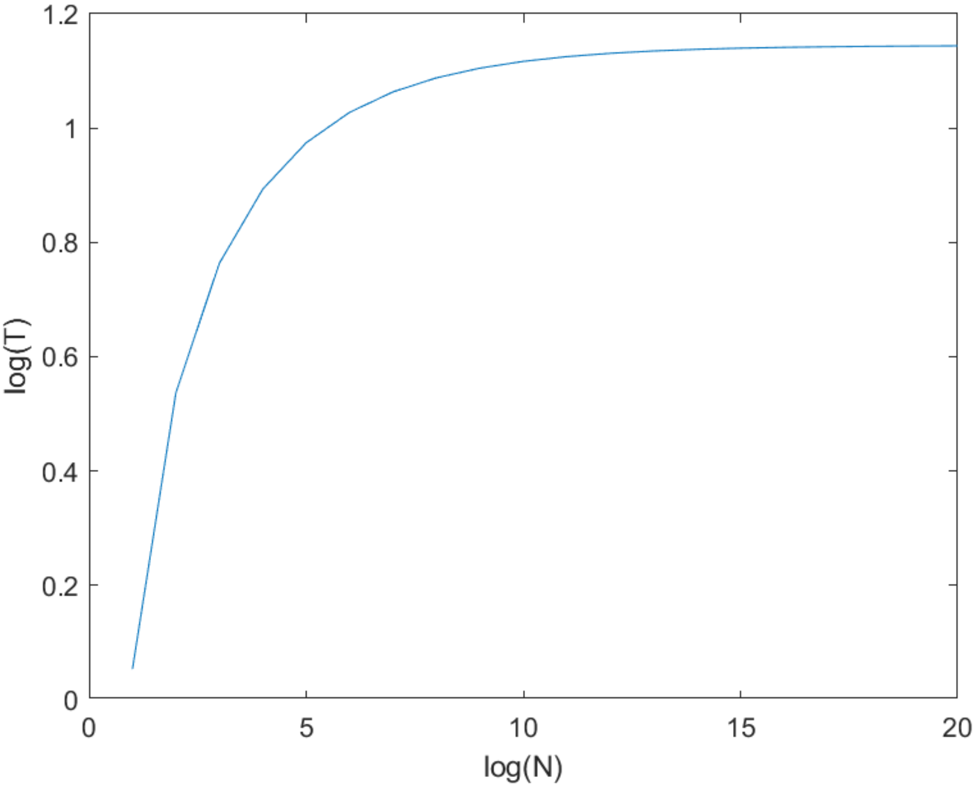
Expected Value: pi ~ 3.141592

Numerical Value: 3.138915 (N = 2^20)

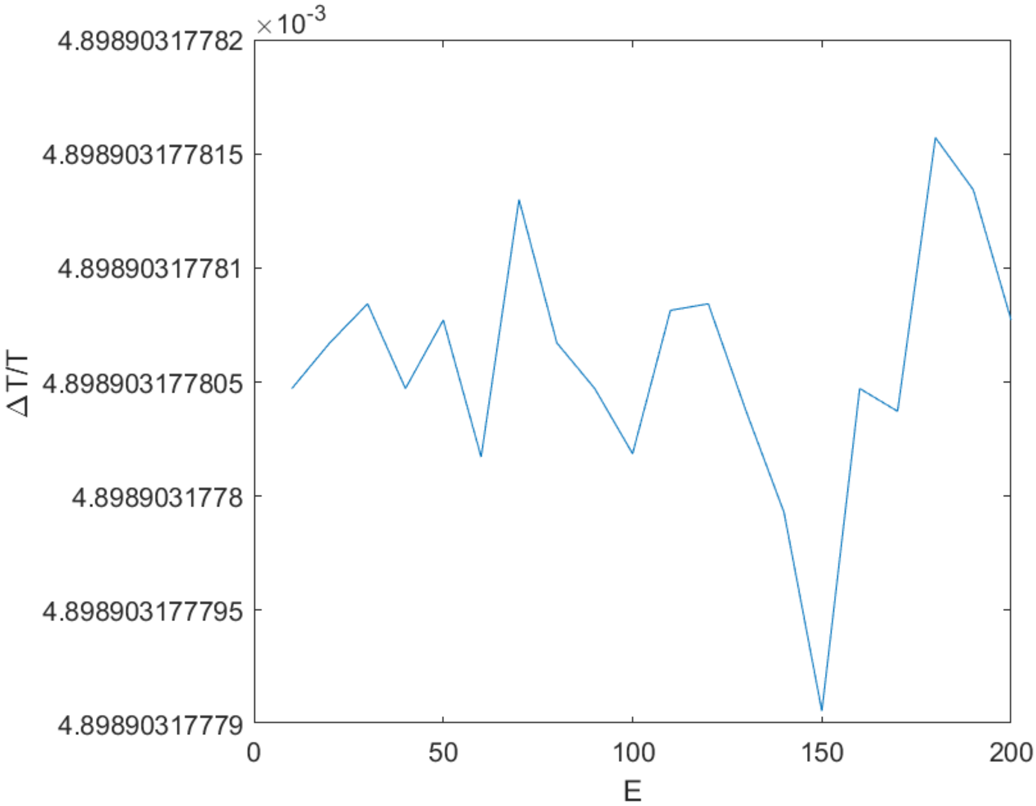
b) Value of T does not depend on the amplitude of oscillation.

Plot of ΔT/T v/s E shows negligible variation.

```
clear
clc
m=1;
alpha=4;
E=100;
a=sqrt(2*E/alpha); %Turning Points
V=@(x) alpha*x.^2/2; %Potential Function

T=zeros(20,1);
for i=1:20
    N=2^i;
    eps=a/N; %Epsilon is equal to the step size.
    T(i)=2*sqrt(2*m)*Trapezoidal(@(x) (1./sqrt(E-
V(x))),0,a-eps,N);
end
plot(log(T))
xlabel('log(N)')
ylabel('log(T)')


%Study of Variation of T with varying amplitude
E=10:10:200;
T=zeros(20,1);
for i=1:20
    a=sqrt(2*E(i)/alpha); %Turning Points
    N=2^20;
    eps=a/N;
    T(i)=2*sqrt(2*m)*Trapezoidal(@(x) (1./sqrt(E(i)-
V(x))),0,a-eps,N);
end
figure()
plot(E,T)
xlabel('E')
ylabel('T')
```



1.2
1
0.8
0.6
0.4
0.2
0
log(T)
0
5
10
15
20
log(N)



×10-3
4.89890317782
4.898903177815
4.89890317781
4.898903177805
4.8989031778
4.898903177795
4.89890317779
ΔT/T
0
50
100
150
200
E

2D integral 1: In order to compute ∫∫ f(x, y)dxdy with h=k=0.25,

1) Formulate 2 dimensional Trapezoidal and Simpson coefficient matrices. You need to use the MATLAB function 'meshgrid' for this.
2) Evaluate the integral for:
a) f(x, y) = 5*(x^3)*y + 2x. Simpson method must match exactly with the analytical value.
b) f(x, y) = 5*x*y – y^4

Answer

f(x)	5*(x^3)*y+2*x	*5*x*y-y^4*
Actual Result	1.6250	1.0500
Trapezoidal Rule	1.6231	1.0480
Simpson's Rule	1.6250	1.0500

```
%Simpson's Method
function [I,f] = Simpson2D(F,a,b,c,d,N)
h=(b-a)/N;
k=(d-c)/N;

x=linspace(a,b,N);
y=linspace(c,d,N);

[xx, yy]=meshgrid(x,y);

f=F(xx,yy);

sc= 2*ones(N,1);
sc(2:2:N-1)=4;
sc(1)=1;
sc(N)=1;
w=sc*sc';

I=h*k*sum(sum(w.*f))/9;
end
```

```
%Trapezoidal Method
function [I,f] = Trapezoidal2D(F,a,b,c,d,N)
h=(b-a)/N;
k=(d-c)/N;

x=linspace(a,b,N);
y=linspace(c,d,N);

[xx, yy]=meshgrid(x,y);

f=F(xx,yy);

sc= 2*ones(N,1);
sc(1)=1;
sc(N)=1;
w=sc*sc';

I=h*k*sum(sum(w.*f))/4;
end


%Main Function
clear
clc
F=@(x,y) 5*x.*y-y.^4;

e=zeros(2,10);
a=0;
b=1;
c=0;
d=1;

for i=1:10
    n=2^i+1;
    e(1,i)=Trapezoidal2D(F,a,b,c,d,n);
    e(2,i)=Simpson2D(F,a,b,c,d,n);
end
e_an=21/20;
e=abs(e-e_an)/e_an;
hold on
grid on
plot(log(e(1,:)))
```

```
plot(log(e(2,:)))
legend('Trapezoidal','Simpsons');
xlabel('log(N)')
ylabel('log(error)')
hold off
```

ODE1:

a) Solve the ODE (1 + x) dy/dx − 2y + 18x = 0 with y(0) = 4 and interval h=0.05 in the interval (0, 3) (i.e. 60 steps) by using Euler's method. Plot the analytical solution (y = −5x2 + 8x + 4) and the numerical solution in the same figure to visually inspect the outcome.

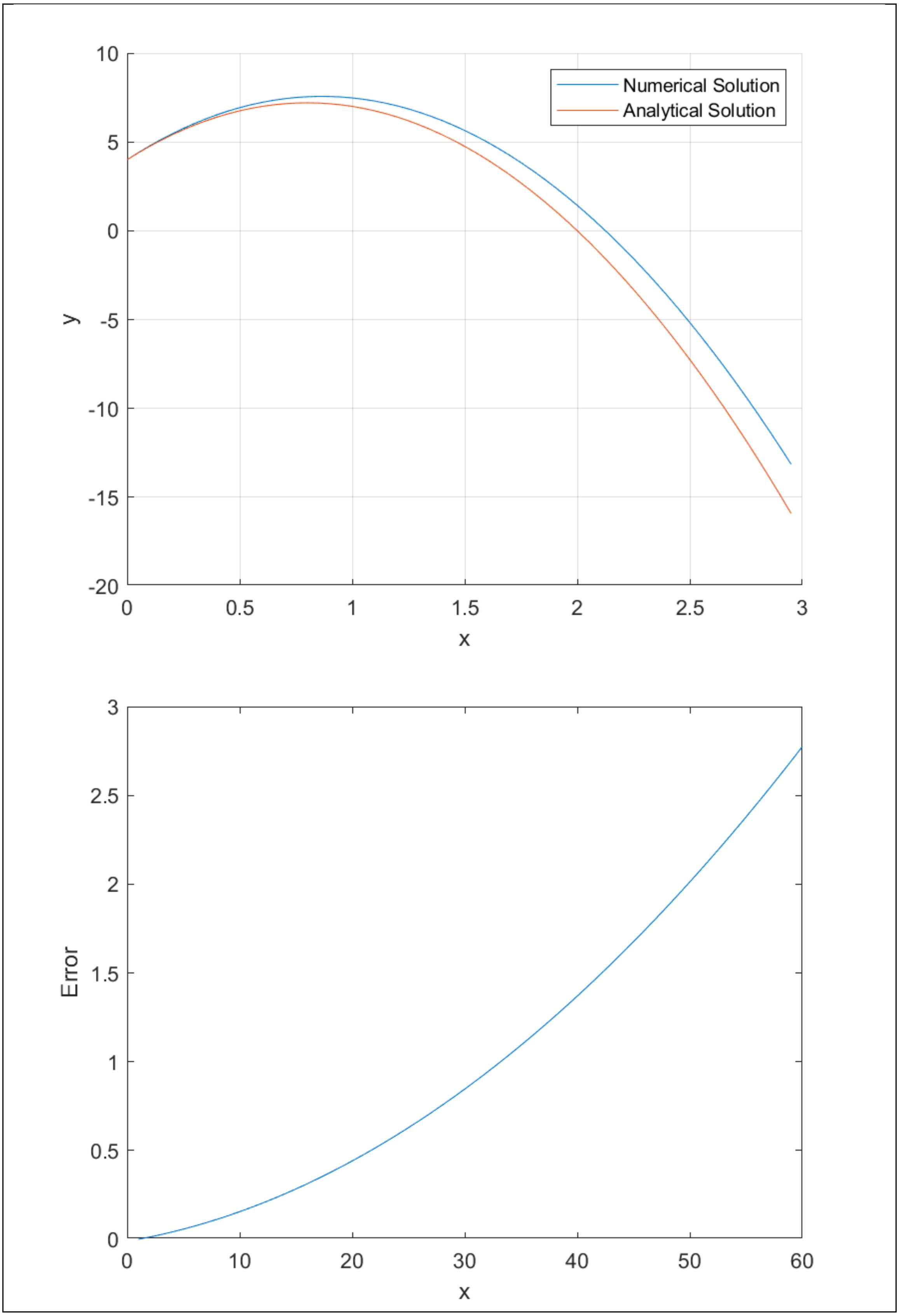
b) Plot the absolute error at each point for Euler's method as a function of x.

ANSWER:

Error increases with x.

```
clc
f=@(x,y) (2*y-18*x)./(1+x);

n=60;
h=3/n;

y=zeros(1,n);
x=zeros(1,n);
x(1)=0;
y(1)=4;

for i=1:n-1
    x(i+1)=x(1)+i*h;
    t=f(x(i),y(i));
    y(i+1)=y(i)+h*t;
end
grid on
hold on
plot(x,y)
plot(x,-5*x.^2+8*x+4)
legend('Numerical Solution','Analytical Solution')
xlabel('x')
ylabel('y')
hold off
```

```
figure()
plot(abs(y+5*x.^2-8*x-4))
xlabel('x')
ylabel('Error')
```



ODE2: a) Solve the one-dimensional trajectory of a projectile fired from cannon located at the origin using the Euler method. Assume the initial

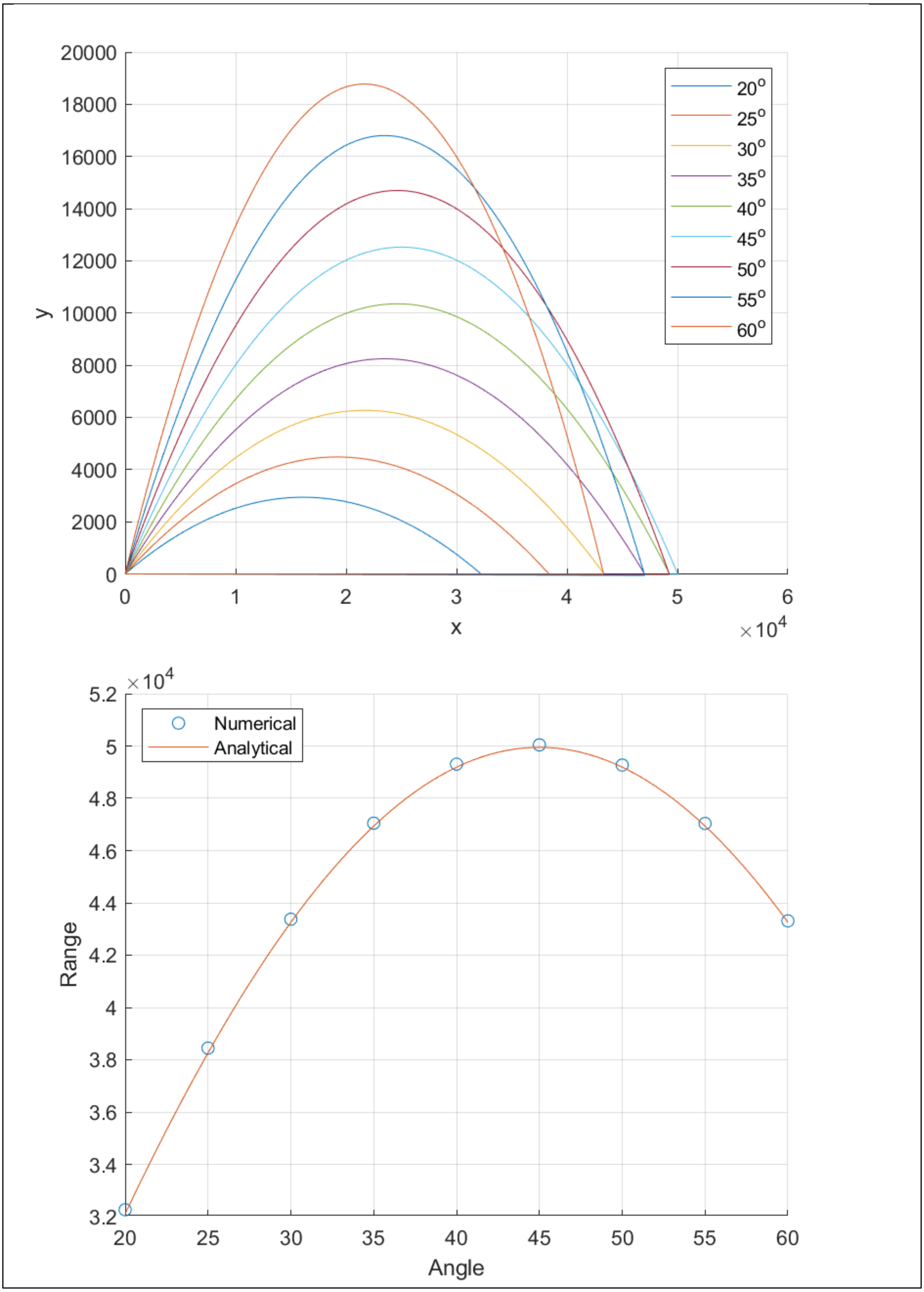
projected speed is 700 m/s, and using different firing angles starting from 20 degrees to 60 degrees with an interval of 5 degrees. Neglect the effects of the air resistance. Plot the trajectories of the projectile for different firing angles. Also plot the range of the projectile against the firing angles and show numerically that the maximum range of the projectile corresponds to a firing angle of 45 degrees.

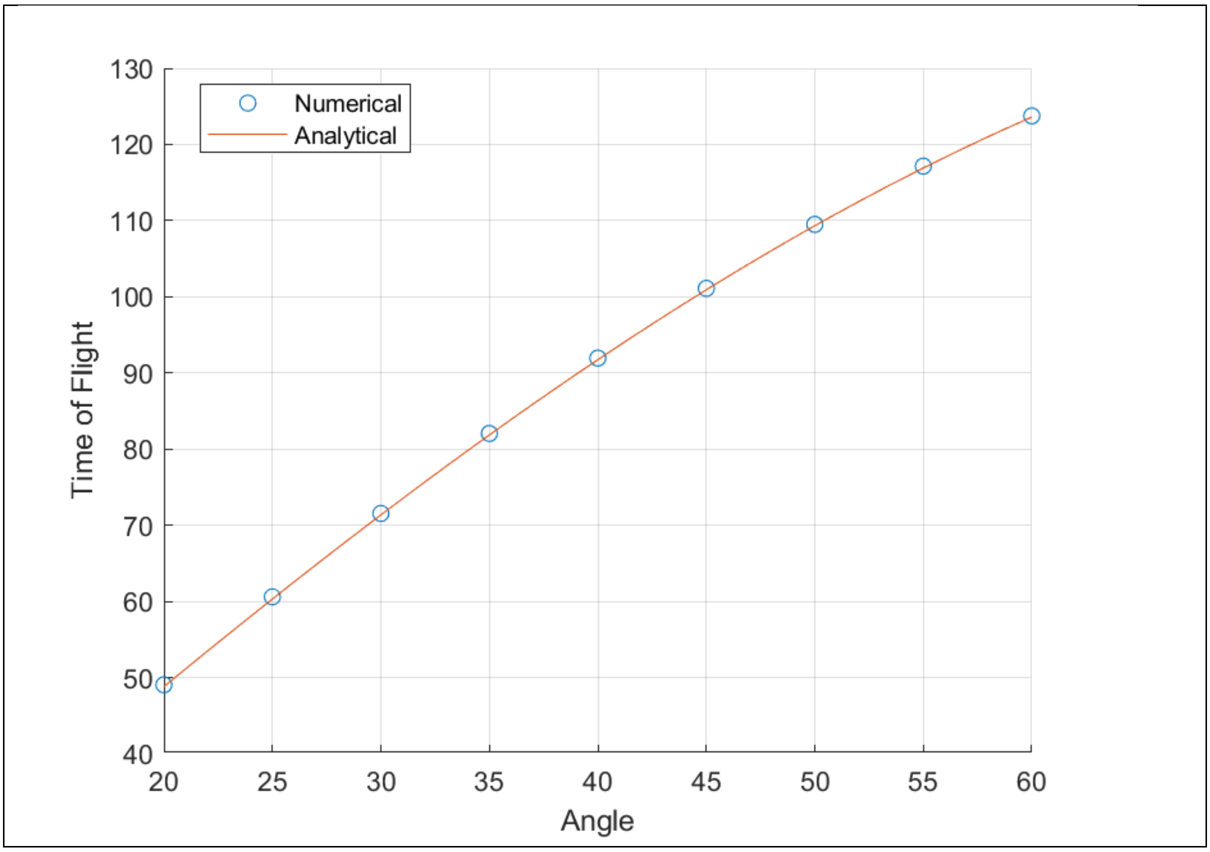
b) Plot the numerical results with the analytical solutions for the range and the duration of the projectile.

ANSWER:

```
clc
clear

v=700;
g=9.81;
n=1000;
h=150/n; %Maximum Time of Flight ~ 150s

R=zeros(9,1); %Stores Range
T=zeros(9,1); %Stores Time of Flight
theta=pi/9:pi/36:pi/3;

hold on
grid on
for j=1:9
t=zeros(1,n);
y=zeros(1,n);
x=zeros(1,n);
x(1)=0;
y(1)=0;
t(1)=0;

vx=v*cos(theta(j));
vy=v*sin(theta(j));
for i=1:n-1
    t(i+1)=t(i)+h;
    x(i+1)=x(i)+h*vx;
    y(i+1)=y(i)+h*(vy-g*t(i));
    if y(i+1)<0
        R(j)=x(i+1);
```

```
        T(j)=t(i+1);
        break
    end
end
plot(x,y)
end
legend('20^o','25^o','30^o','35^o','40^o','45^o','50^o',
'55^o','60^o')
xlabel('x')
ylabel('y')
hold off

%Range Analysis
figure()
grid on
hold on
scatter(theta*180/pi,R);
x=pi/9:pi/360:pi/3;
y=v^2*sin(2*x)/g;
plot(x*180/pi,y)
xlabel('Angle')
ylabel('Range')
legend('Numerical','Analytical');
hold off

%Time of Flight Analysis
figure()
grid on
hold on
scatter(theta*180/pi,T);
x=pi/9:pi/360:pi/3;
y=2*v*sin(x)/g;
plot(x*180/pi,y)
xlabel('Angle')
ylabel('Time of Flight')
legend('Numerical','Analytical');
hold off
```


20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
y
0
1
2
3
4
5
6
x
×10^4
20°
25°
30°
35°
40°
45°
50°
55°
60°
×10^4
5.2
5
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
Range
20
25
30
35
40
45
50
55
60
Angle
Numerical
Analytical



Numerical
Analytical
Time of Flight
Angle
130
120
110
100
90
80
70
60
50
40
20
25
30
35
40
45
50
55
60