Tasks for 21/01/2019:

Computer arithmetic 3: Consider the logistic map: $x_{n+1} = Ax_n(1 - x_n)$, where, x_n is the nth iteration of x for a starting value of $0 \leq x \leq 1$. Here A is a constant.

(a) Write a code to generate the logistic map. Start by varying the value of *A* to observe the following:

- With *A* between 0 and 1, the value of x_n will eventually go to zero, independent of the initial value of *x*.
- With *A* between 1 and 2, the population will quickly approach the value $\frac{A-1}{A}$, independent of the initial *x* value.
- With *A* between 2 and 3, x_n will also eventually approach the value $\frac{A-1}{A}$, but first will fluctuate around that value for some time.
- With *A* between 3 and $1+\sqrt{6}=3.449$, from almost all initial conditions x_n will approach permanent oscillations between two values.

Plot x_n *vs. n* for all this condition for a value of $n>50$.

(b) For an initial value $x=0.3$, vary the value of A from 0.5 to 3.99 in total 250 steps. For each value of A, note the values of x_n for $n=150$, then make a plot of A *vs.* x_n and see the bifurcation and chaos. Now change the initial value of *x*. Do you see any change in the plot?

(c) For $A = 3.0$, choose two points x and x' close to 1 where, $x' = x + 0.01$ and iterate. Plot $log(|x_n - x'_n| / 0.01)$ as a function of *n* in log-log representation. See if it is approaching a straight line for high *n*. Check for other values around 3, and you will see error dropping to 0 much sooner. This happens because at A=3, near the bifurcation memory, the systems approaches equilibrium in a dramatically slow manner.

Root finding 1: How many real roots does the polynomial $f(x) = 2x^3 - 5$ has? Find the roots using the method of bisection.

Root finding 2: Solve the equation $f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$ using Newton-Raphson method.

1) With initial guess of $x(0) = 0.05$.
2) With initial guess of $x(0)=0.11$,

Why does the 2^{nd} case do not offer any solution? You may start of by plotting the function.

3) Can you find another initial guess which will lead to no solution? Explain why.

Root finding 3: Now find the roots for the polynomial of last problem by method of bisection. See if choosing $x(0)=0.11$ as one of the initial bound work. Compare the number of iterations it takes to converge to a root. Also check x= 1.8 as the starting point