VINIT KUMAR SINGH: 16PH20036

Assignment – 8

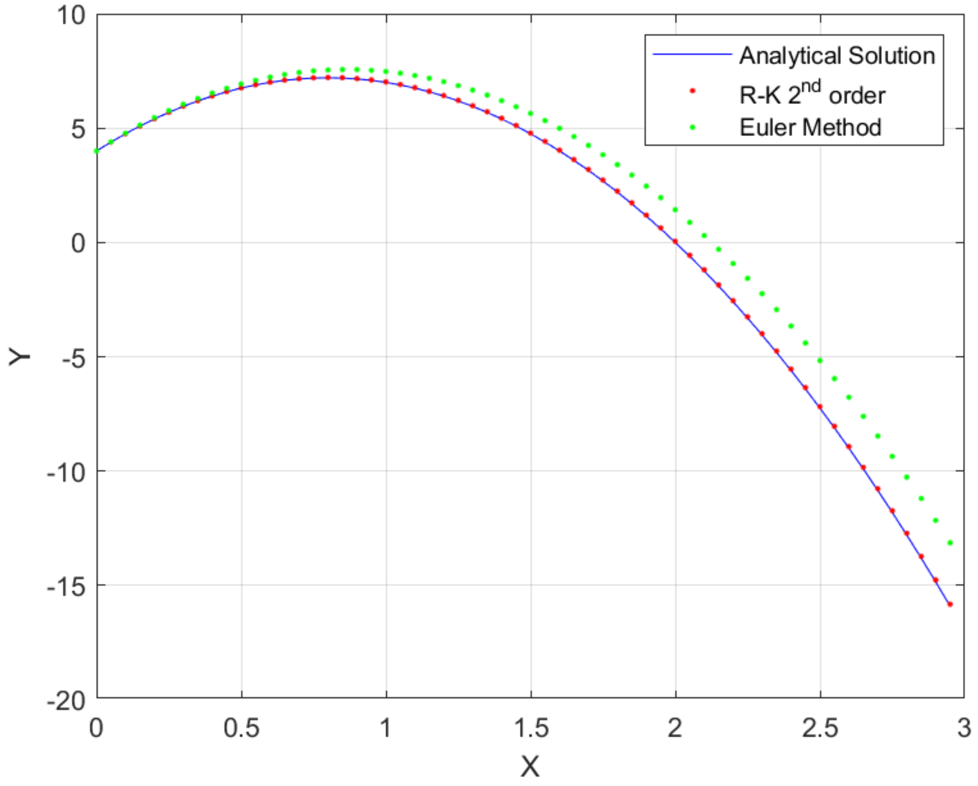
R-K 2nd order: Solve the ODE (1 + x) dy/dx − 2y + 18x = 0 with y(0) = 4 and increment h=0.05 in the interval (0, 3) (i.e. 60 steps) by using 2nd order Runge-Kutta (R-K) method. Plot the analytical solution (y = −5x2 + 8x + 4) and the numerical solution obtained by Euler's method from the last class.

ANSWER:

```
clear
clc
f=@(x,y) (2*y-18*x)/(1+x);

N=60;
a=0;
b=3;
h=(b-a)/N;
x=a:h:b-h;

y1=RK2(f,a,b,4,N); %R-K Method
y2=Euler(f,a,b,4,N); %Euler Method

plot(x,-5*x.^2+8*x+4,'b-',x,y1,'r.',x,y2,'g.')
xlabel('X')
ylabel('Y')
legend('Analytical Solution','R-K 2^{nd} order','Euler
Method')
grid on


Function RK2
function y = RK2(f,a,b,y0,N)
h=(b-a)/N;
x=a:h:b-h;
y=zeros(1,N);
y(1)=y0;


for i=1:N-1
    S1=f(x(i),y(i));
    S2=f(x(i)+h,y(i)+h*S1);
    y(i+1)=y(i)+h*(S1+S2)/2;
end
end
```

Function Euler

```
function y = Euler(f,a,b,y0,n)
h=(b-a)/n;
x=a:h:b-h;
y=zeros(1,n);
y(1)=y0;

for i=1:n-1
    y(i+1)=y(i)+h*f(x(i),y(i));
end
```



Coupled ODE 1: Solve the following coupled ODE by R-K 2nd order method for x=[0.0 0.5] with h=0.05. I.C. are y(0)=0, z(0)=1.

$dy/dx = -x - yz$
$dz/dx = -y - xz$

Plot the solutions y(x) and z(x).

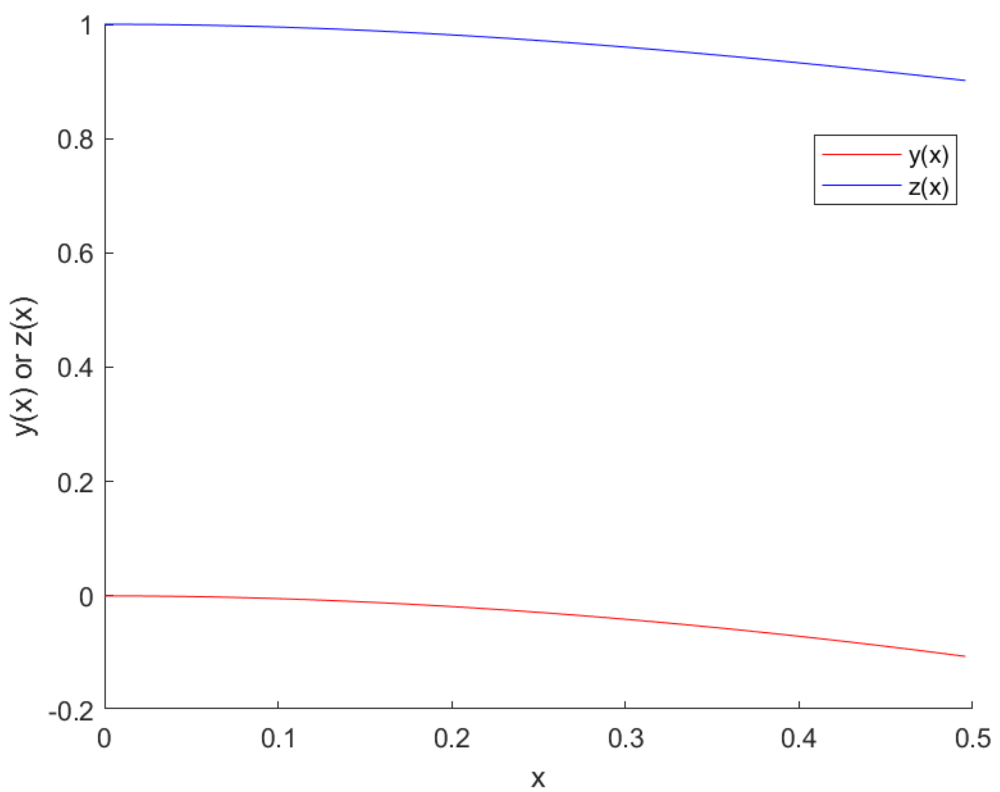
Answer

```
clear
clc
f=@(x,y,z) -x-y.*z;
g=@(x,y,z) -y-x.*z;

N=120;

a=0;
b=0.5;
h=(b-a)/N;
x=a:h:b-h;

[y,z]=COD(f,g,a,b,0,1,N);

hold on
plot(x,y,'r',x,z,'b')
xlabel('x')
ylabel('y(x) or z(x)')
legend('y(x)','z(x)')

hold off
```

Function COD

```
function [y,z] = COD(f,g,a,b,y0,z0,varargin)

if nargin==7
  N=varargin{1};
  w=1;
elseif nargin==8
  N=varargin{1};
  w=varargin{2};
else
  error('Kn accepts up to 2 input arguments!')
end

h=(b-a)/N;

x=a:h:b-h;
z=zeros(N,w);
y=zeros(N,w);
z(1,:)=z0;
y(1,:)=y0;

for i=1:N-1
```

```
    s1=f(x(i),y(i,:),z(i,:));
    p1=g(x(i),y(i,:),z(i,:));
    s2=f(x(i)+h,y(i,:)+h*s1,z(i,:)+h*p1);
    p2=g(x(i)+h,y(i,:)+h*s1,z(i,:)+h*p1);
    y(i+1,:)=y(i,:)+h*(s1+s2)/2;
    z(i+1,:)=z(i,:)+h*(p1+p2)/2;
end
end
```



Coupled ODE 2: Solve the following 2nd order ODE

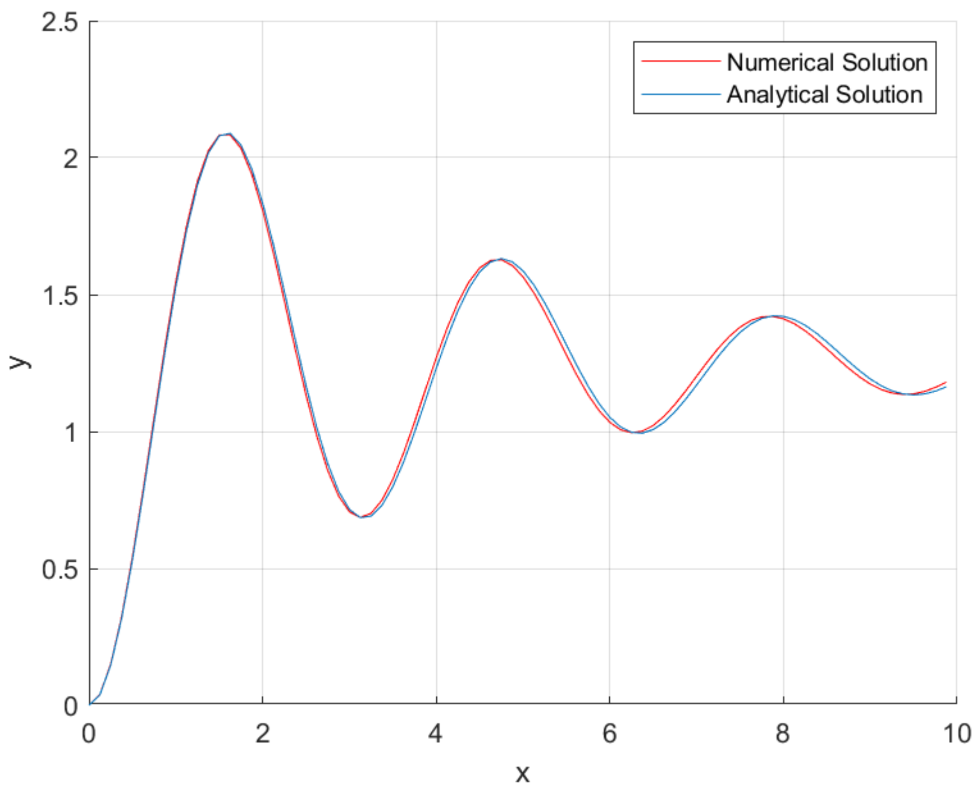
d2y/dx2 + 0.5dy/dx + 4y = 5

Initial condition: y(0) = y'(0) = 0;

by applying of R-K 2nd order method for x-range of $0 \le x \le 5$ and h=0.1. Plot y(x) alongside the analytical solution.

ANSWER:

```
clear
clc
f=@(x,y,z) z;
g=@(x,y,z) 5-4*y-0.5*z;

N=80;

a=0;
b=10;
h=(b-a)/N;
x=a:h:b-h;

[y,z]=COD(f,g,a,b,0,0,N);

plot(x,y,'b',x,-pi/20*exp(-0.25*x).*sin(1.98431*x)-
1.25*exp(-0.25*x).*cos(1.98431*x)+1.25,'r')
xlabel('x')
ylabel('y')
legend('Numerical Solution','Analytical Solution')
grid on
```



Coupled ODE 3: Using the 2nd Order Runge-Kutta Methods find the phase space trajectory of a particle of unit mass in a potential V(x). You need to solve the Hamilton's equations of the system, given by:

$dp/dt = -\,\partial H/\partial x$

$dx/dt = \partial H/\partial p$

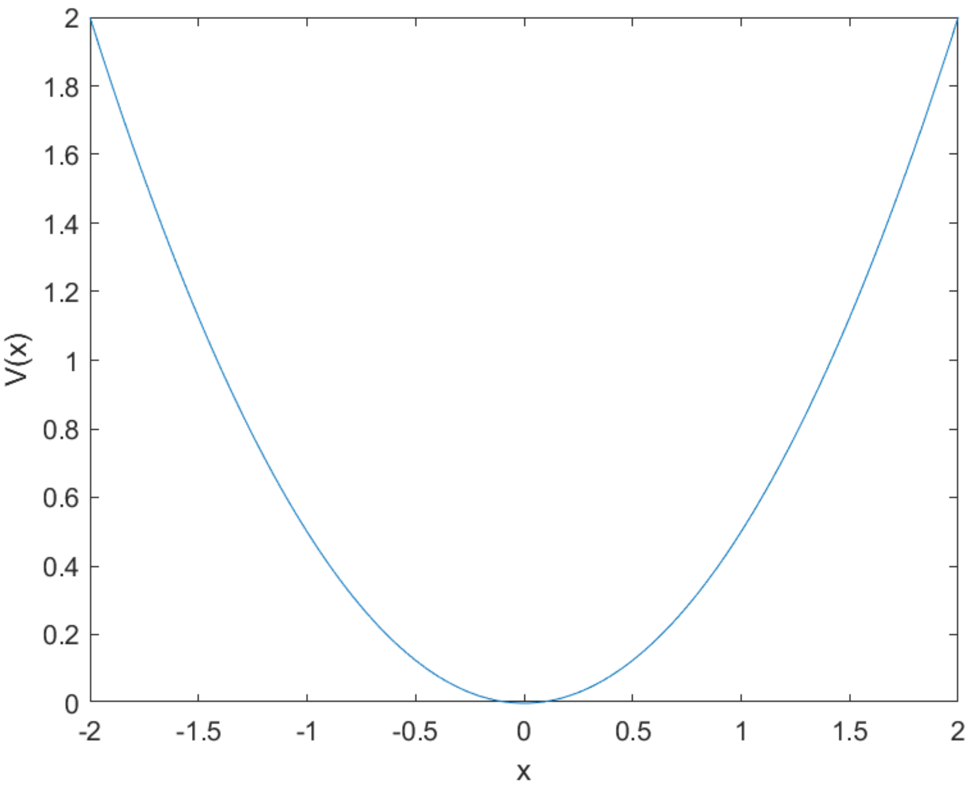
a) Start by taking simple harmonic oscillator potential V(x) = 0.5*kx^2, k=1N/m

(i) Show the trajectory for the initial conditions (x, px) = (1.0, 0.1), (-1.0, 0.1), (1.0, 10.0) at zero time. It should be an ellipse for all initial conditions.

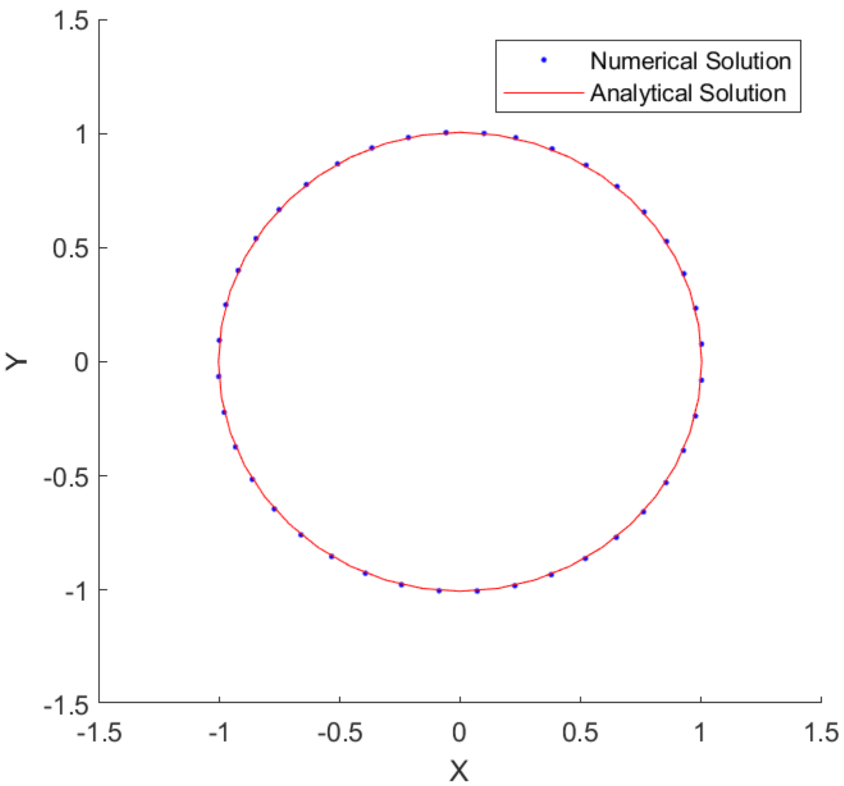
(ii) Plot the analytical solution obtained from H = p^2/2m + 0.5*kx^2 alongside numerical results for each set of initial conditions.

ANSWER:

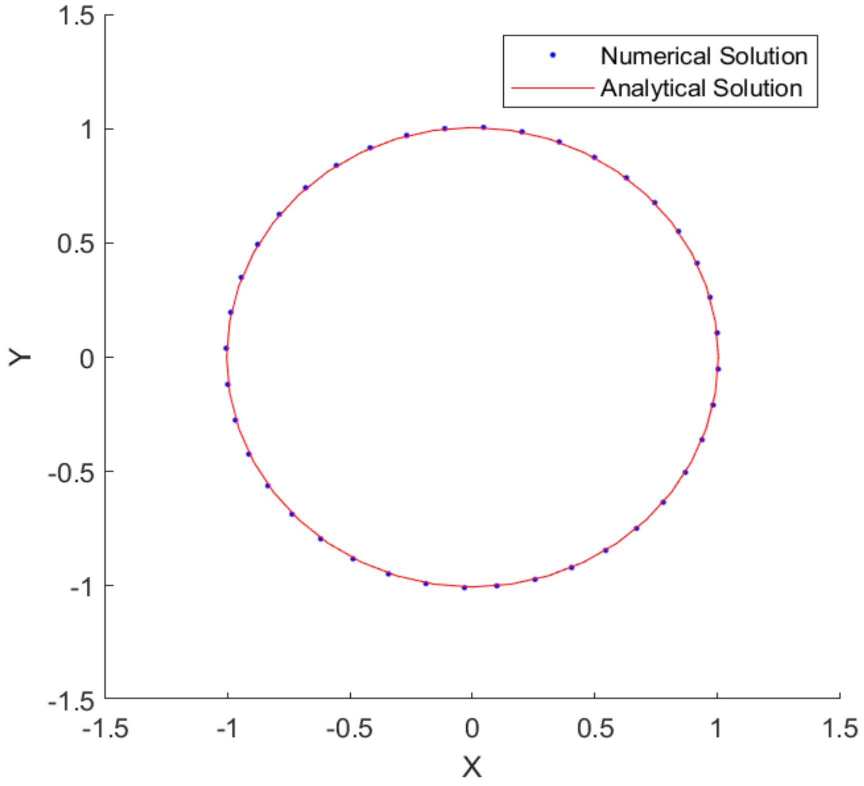
```
clear
clc
k=1;
m=1;
V=@(x) k*x.^2/2; %(x.^2-1).^2;
H=@(x,p) p.^2/(2*m) + V(x);

f=@(t,x,p) p/m;
g=@(t,x,p) -k*x;%4*x-4*x.^3;

N=40;

a=0;
b=2*pi;
h=(b-a)/N;
t=a:h:b-h;

hold on
[x,p]=COD(f,g,a,b,1,10,N);
r=sqrt(1+10^2);
t=a:h:b;
plot(p,x,'b.',r*cos(t),r*sin(t),'r-')
xlabel('X')
ylabel('Y')
legend('Numerical Solution','Analytical Solution');
hold off
```


V(x)
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

`(x, p) = (1,0.1)`

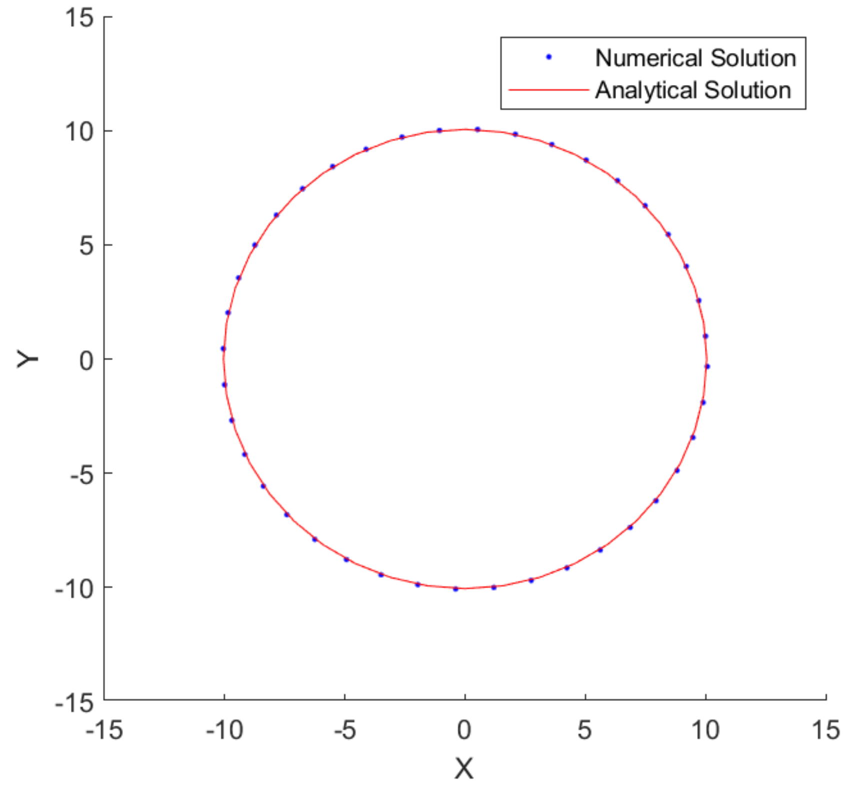

Numerical Solution
Analytical Solution
Y
X
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

(x, p) = (-1,0.1)



Numerical Solution
Analytical Solution
1.5
1
0.5
0
-0.5
-1
-1.5
Y
-1.5
-1
-0.5
0
0.5
1
1.5
X

(x, p) = (1,10)



Numerical Solution
Analytical Solution
15
10
5
0
-5
-10
-15
Y
-15
-10
-5
0
5
10
15
X

b) Now consider a double well potential V(x) = (x2 − 1)^2. Start by plotting this potential in x = [-2 2].

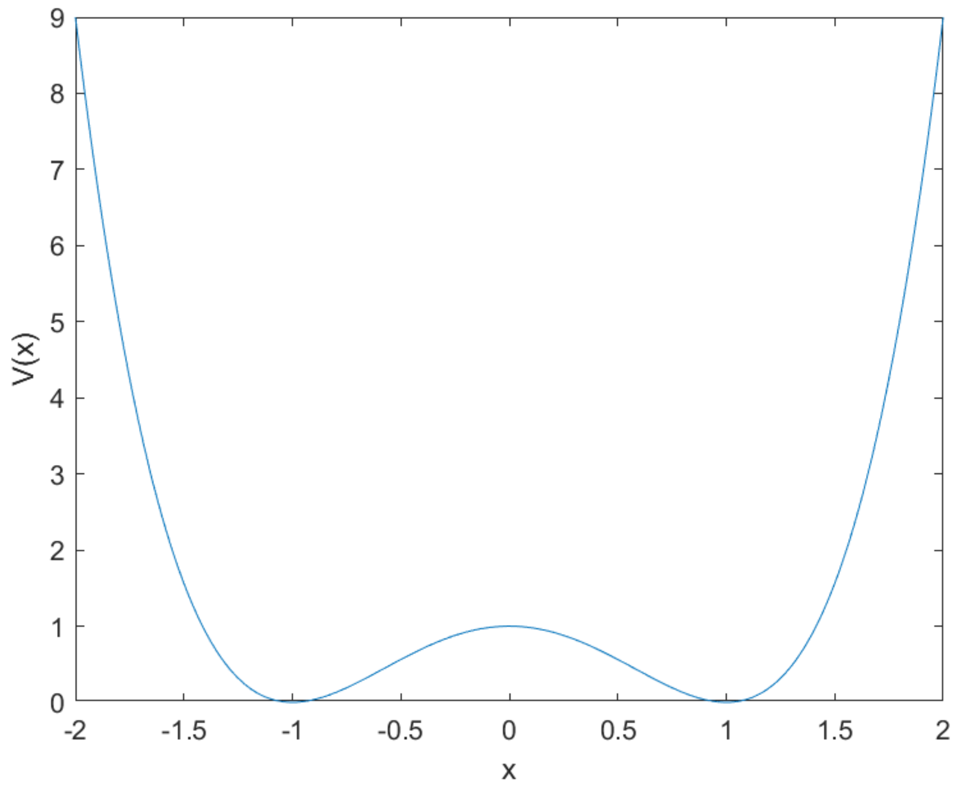
(i) Show the trajectory for the initial conditions (x, px) = (1.0, 0.1), (-1.0, 0.1), (1.0, 10.0) at zero time.

(ii) Check for UNBOUND MOTION of the particle moving in a double well by enhancing the simulation time. You should see the motion in phase space goes out of bound for any initial condition if the simulation time is sufficiently long.

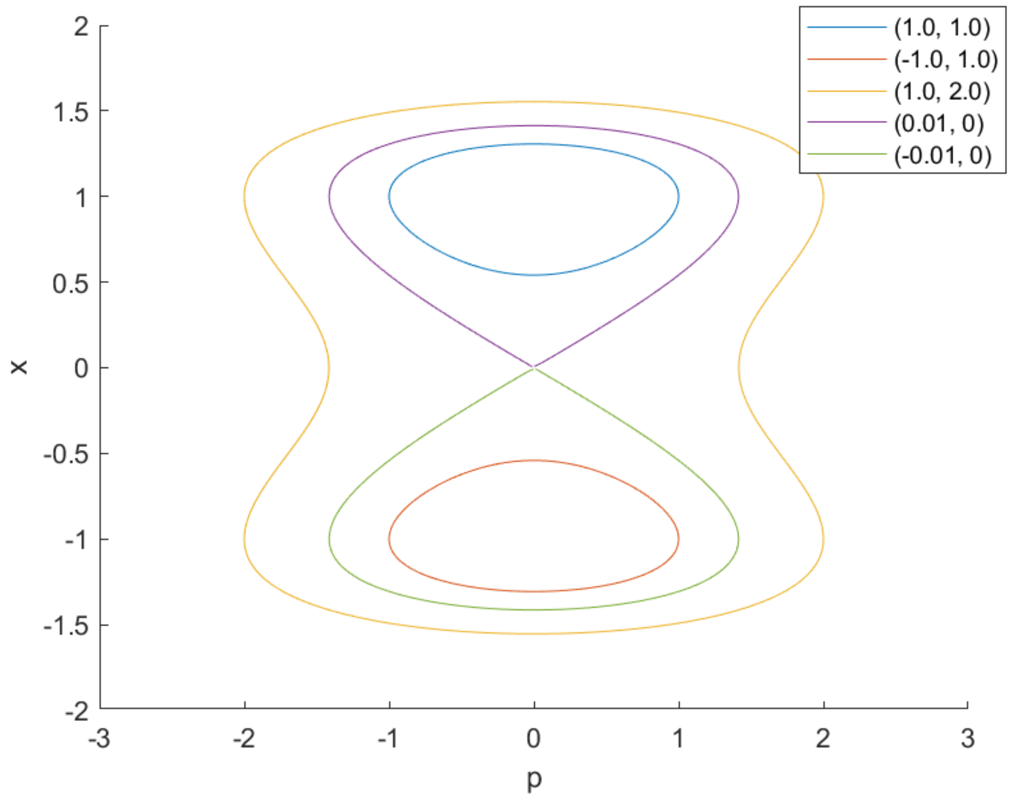
ANSWER:

```
clear
clc
k=1;
m=1;
V=@(x) (x.^2-1).^2; %k*x.^2/2;
H=@(x,p) p.^2/(2*m) + V(x);

f=@(t,x,p) p/m;
g=@(t,x,p) 4*x-4*x.^3; %-k*x;

N=400;

a=0;
b=2*pi;
h=(b-a)/N;
t=a:h:b-h;

hold on
[x,p]=COD(f,g,a,b,1,1,N);
plot(p,x)
[x,p]=COD(f,g,a,b,-1,1,N);
plot(p,x)
[x,p]=COD(f,g,a,b,1,2,N);
plot(p,x)
[x,p]=COD(f,g,a,b,0.01,0,N);
plot(p,x)
[x,p]=COD(f,g,a,b,-0.01,0,N);
plot(p,x)
xlabel('p')
ylabel('x')
legend('(1.0, 1.0)','(-1.0, 1.0)','(1.0, 2.0)','(0.01,
0)','(-0.01, 0)');
hold off
```

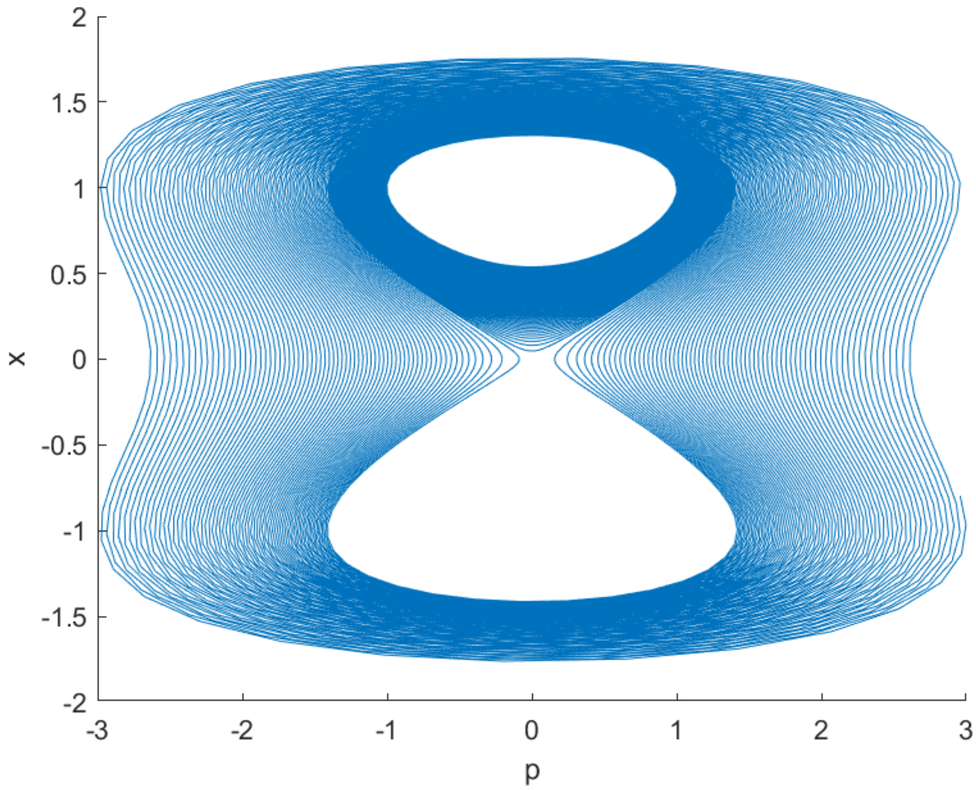
Potential:



Various Phase Space Diagram:



`Unbounded Motion: Initial condition (p, m) = (1, 1).`



Coupled ODE 4: Write a program to follow the motion of an electron (e) in an electric field E(x, t) and a magnetic field B(x, t). Numerically determine the trajectory of an electron for 1 micro second with 1 nano second of time resolution by solving Lorentz force equation:

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\times\vec{B}).$$

Assume that the particle starts at the origin with velocity v=(1.0, 1.0, 1.0) m/sec for the following field configurations:

(i) Uniform magnetic field 10-4 Tesla along the z-axis.

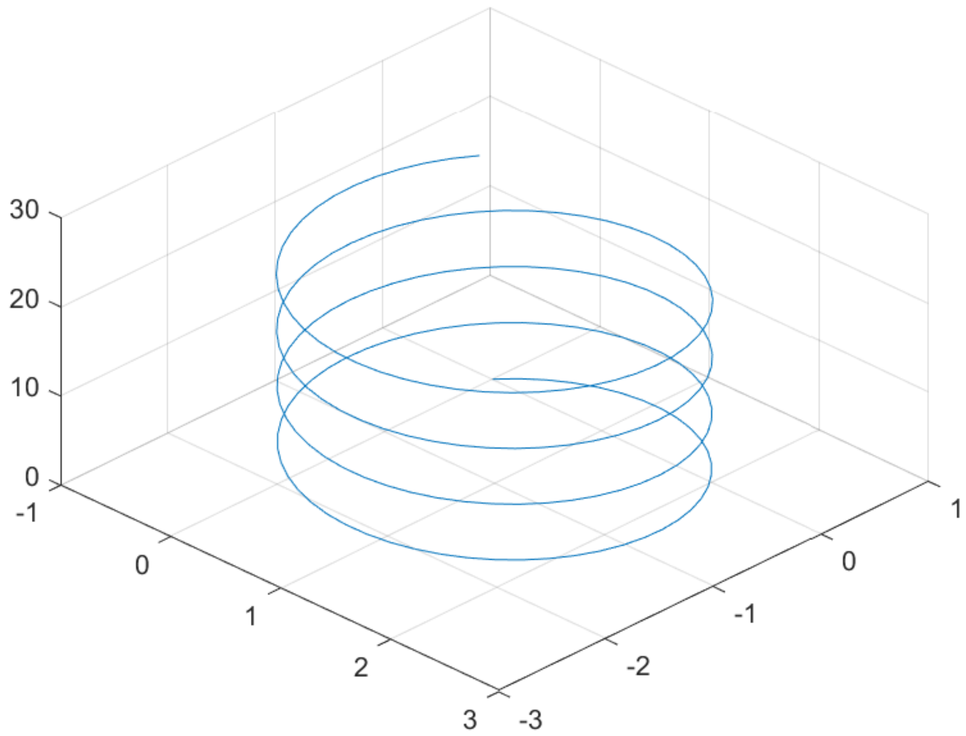
(ii) Uniform magnetic field 10-4 Tesla along the z-axis and a uniform electric field 1 V/m along the y-axis.

You need to use 3D plot function plot3 in MATLAB for this exercise.

[Use parameters q=-1.6×10-19C, me=9.11×10-31 Kg]

```
q=1;
m=1;
E=[0 1 0];
B=[0 0 1];

f=@(t,x,v) v;
g=@(t,x,v) q*(E+cross(v,B))/m;

N=200;
a=0;
b=8*pi;
h=(b-a)/N;
t=a:h:b-h;

[x,v]=COD(f,g,a,b,[0,0,0],[1 1 1],N,3);

grid on
hold on
plot3(x(:,1),x(:,2),x(:,3))
hold off
```

Magnetic Field along z-axis:



Magnetic Field along z-axis and Electric Field along y-axis: