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Assignment – 9

FMD 1: Solve the 1st order ODE $y' + 2y + 3 = 0$ with both implicit and explicit Euler method with I.C. $y(0)=1$ and compare with the analytical solution:

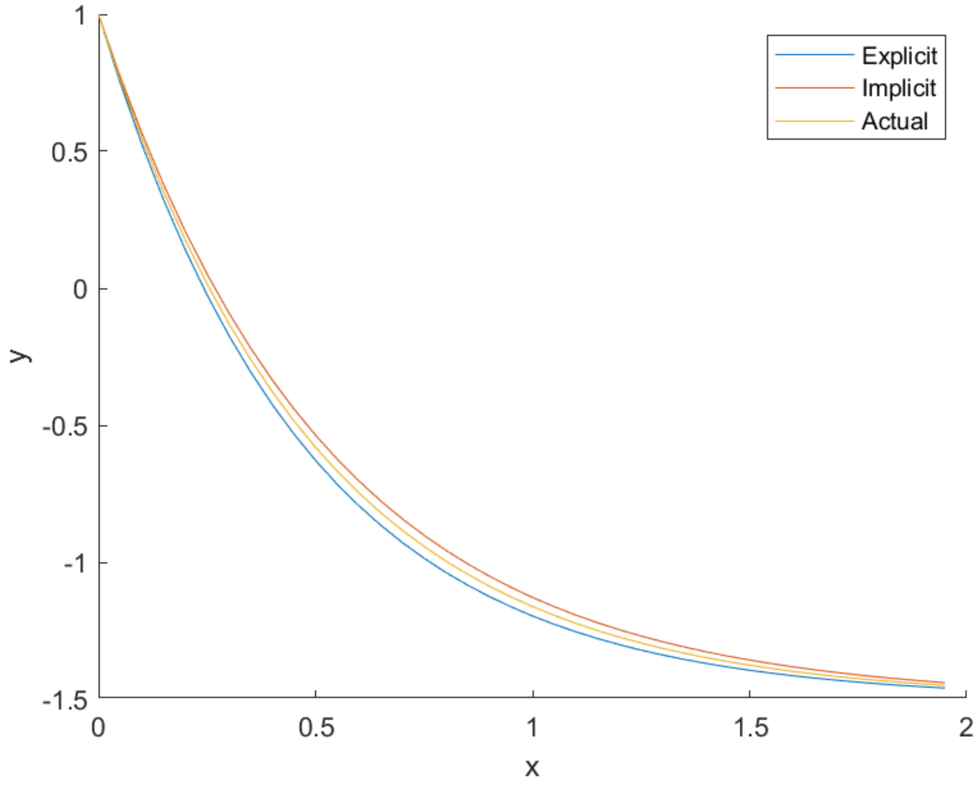
$$y(t) = (5/2)*\exp(-2t) - 3/2.$$

Show that the explicit method becomes unstable for $\Delta t>0.5$ whereas implicit method is unconditionally stable.

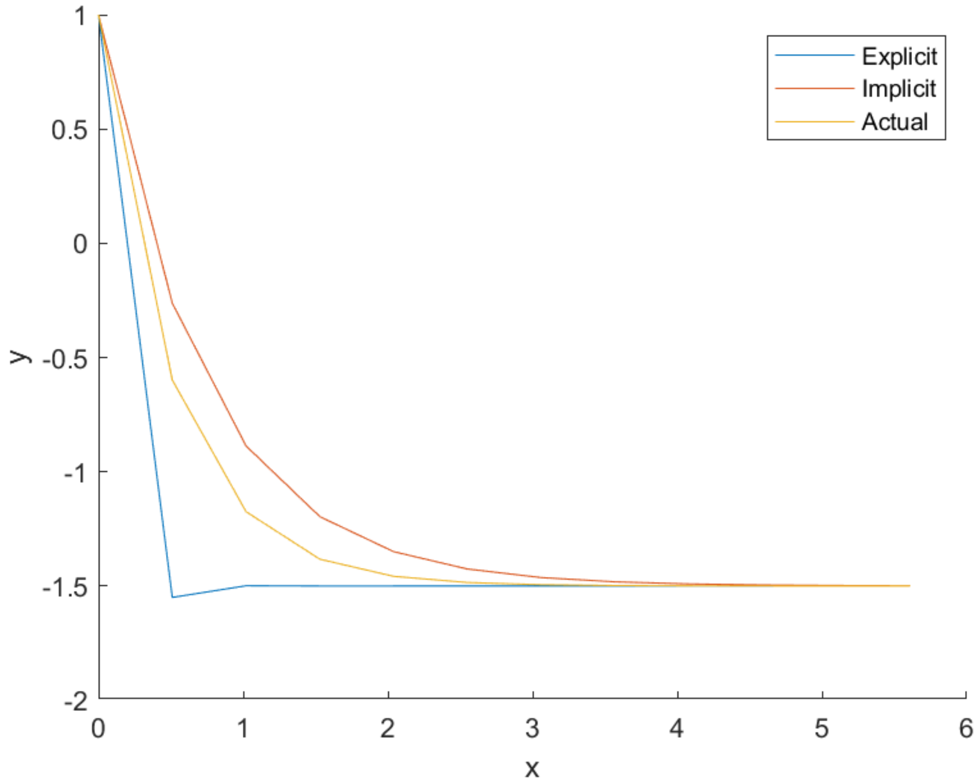
ANSWER:

```
clear
clc
h=0.51;
N=6.12/h;

y1=zeros(N,1);
y2=zeros(N,1);

x=0:h:h*(N-1);
y1(1)=1;
y2(1)=1;
for i=1:N-1
    y1(i+1)=y1(i)*(1-2*h)-3*h;
    y2(i+1)=(y2(i)-3*h)/(1+2*h);
end
hold on
plot(x,y1,x,y2,x,exp(-2*x)*5/2-3/2)
xlabel('x')
ylabel('y')
legend('Implicit','Explicit','Actual')
hold off
```

h=0.05



h=0.51 (Implicit Methods becomes unstable after h=0.5)



FDM 2: Solve the following 2nd order ODE

$dy^2/dx^2 + 0.5*dy/dx + 4y = 5$

Initial condition: $y(0) = y'(0) = 0$;
by applying finite difference method for the x-range of $0 \le x \le 10$.
a) Determine a suitable h value from the stability criteria for oscillatory solution, i.e. $h < 2*\mathrm{sqrt}(\omega_0^2 - \alpha^2)/\omega_0^2$

b) Show that the solution becomes unstable, i.e. leads to growing instead of decaying oscillation for $h > 2*\mathrm{sqrt}(\omega_0^2 - \alpha^2)/\omega_0^2$

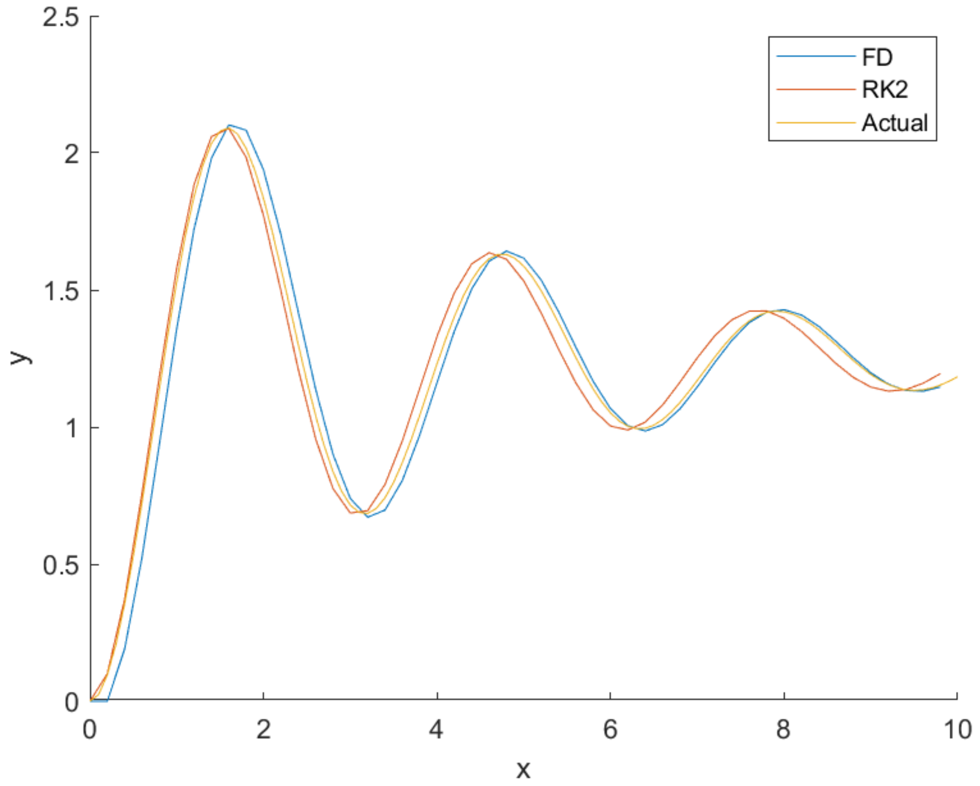
c) Plot y(x) alongside the analytical solution and the one obtained from R-K 2nd order method for different h values for h< 0.5.

Answer

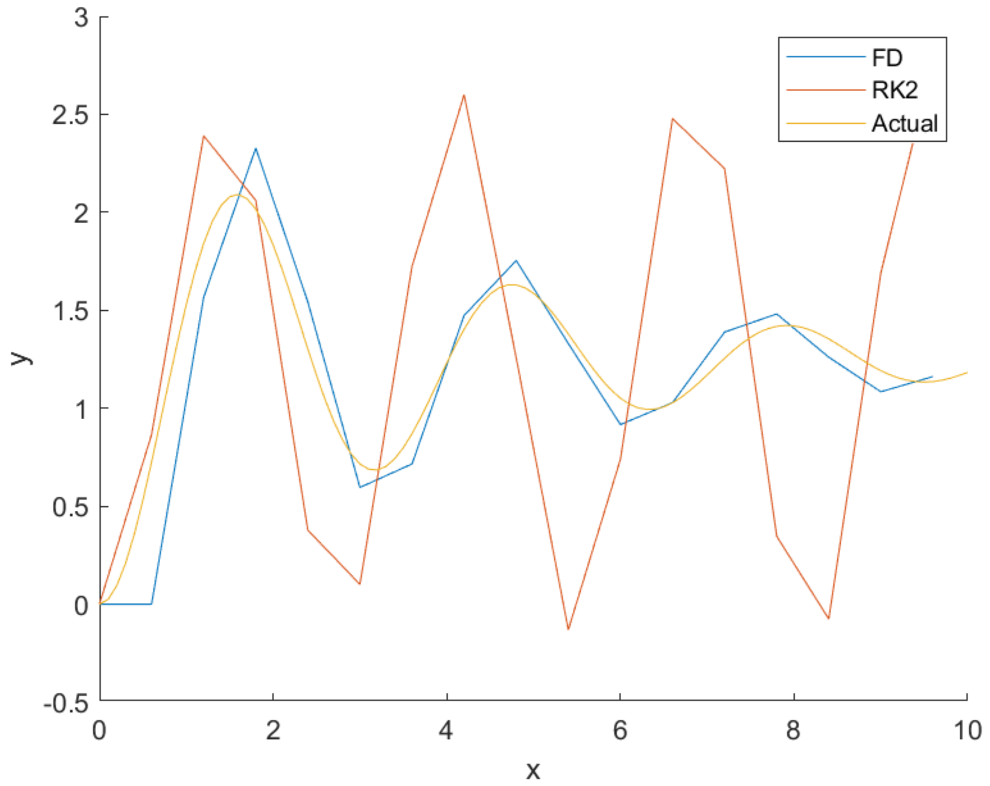
```
clear
clc
w=10;
h=0.65; %0.9682
N=ceil(w/h);
y=zeros(N,1);

x=0:h:h*(N-1);
y(1)=0;
y(2)=0;

b=h^2/(1+0.25*h);
a1=(4*h^2-2)/(1+0.25*h);
a2=(1-0.25*h)/(1+0.25*h);

for i=1:N-2
    y(i+2)=b*5-a1*y(i+1)-a2*y(i);
end

f=@(x,y,z) z;
g=@(x,y,z) 5-4*y-0.5*z;
[y2,z] = COD(f,g,0,w,0,0,N);

x2=0:0.1:10;
hold on
```

```
plot(x,y,x,y2,x2,-0.157485*exp(-x2/4).*sin(1.98431*x2)-
1.25*exp(-x2/4).*cos(1.98431*x2)+1.25)
xlabel('x')
ylabel('y')
legend('FD','RK2','Actual')
hold off
```

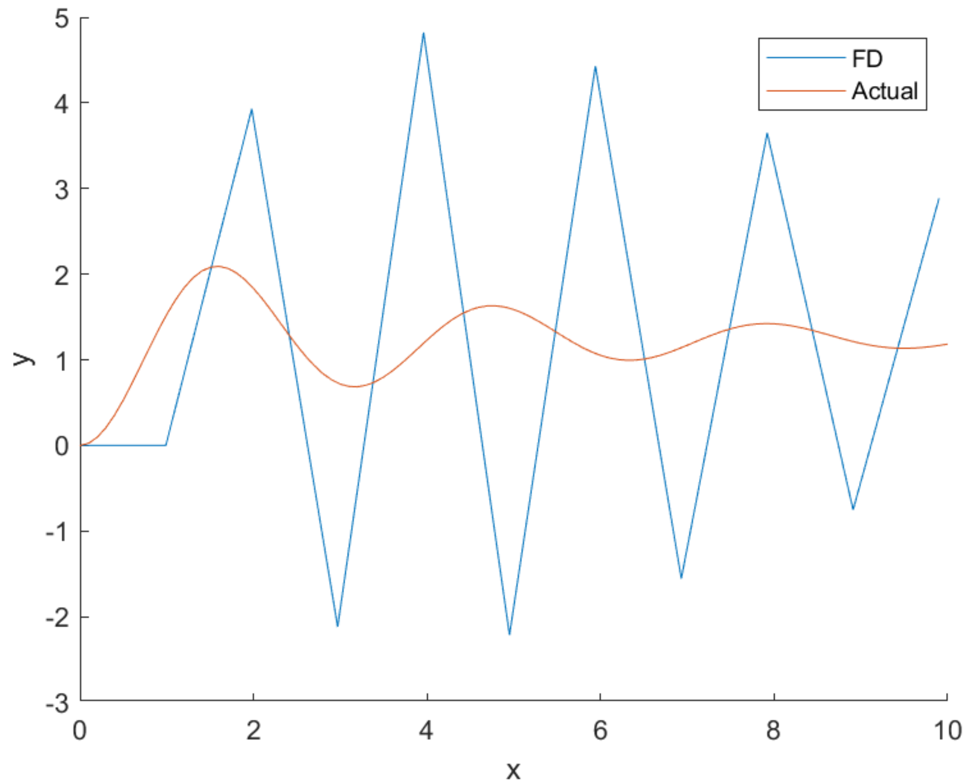
h=0.2



h=0.6 (RK2 starts blowing up after h=0.5)



H=0.99 (FD starts blowing after h=0.9682)



Heat Equation 1: Consider the system in which a thin rod of length L=10cm is placed between two heat reservoirs kept at 100ºC and 50ºC, respectively. The initial temperature of the rod is 0ºC. Write a code to compute heat evolution for 1st 100sec in the rod using **explicit method** of solving heat equation. Plot the result

1) In a 2D contour or surface plot
2) And in animated version

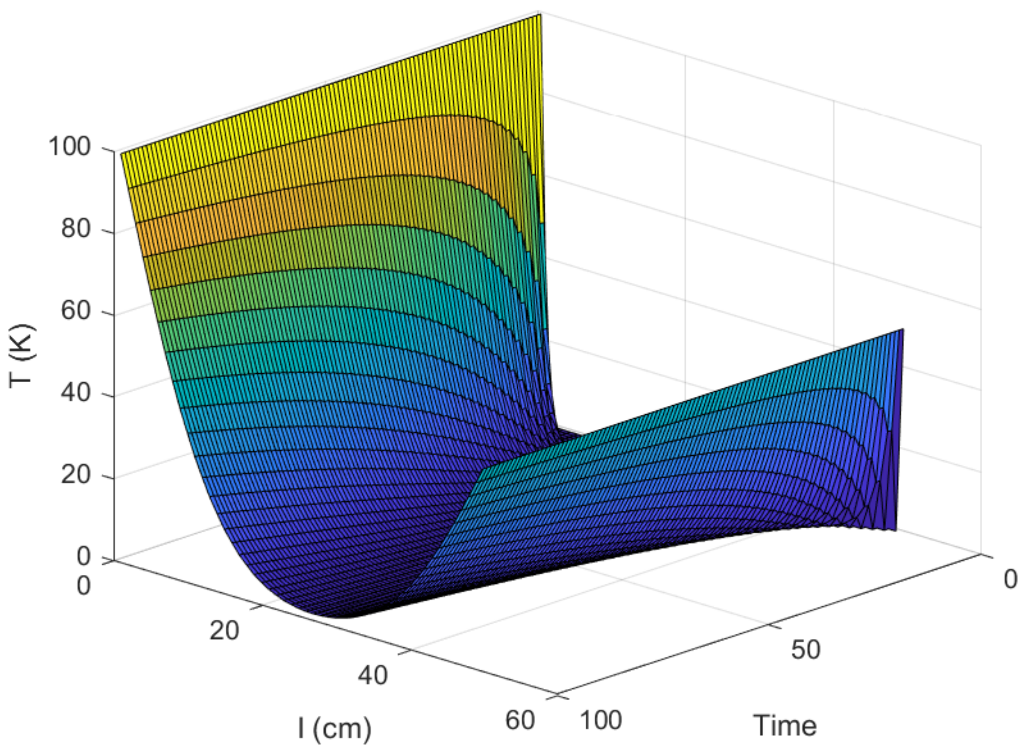
for two values of σ ($=\kappa\Delta t/\Delta x^2$) greater and less than 0.5.
Initial condition: u(0<x<L, t=0) = 0ºC
Boundary condition: u(0,t)= 100ºC and u(50,t) = 50ºC at all t

ANSWER:

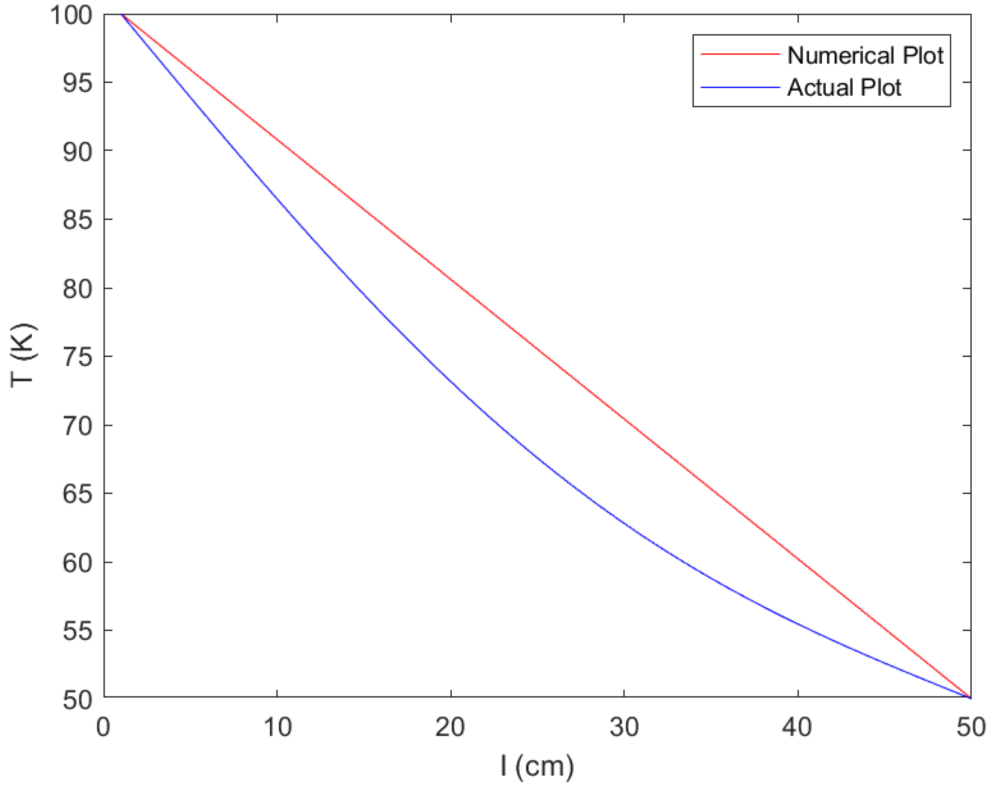
```
clear
clc
N=50;
T=10000;

s=0.499;
a=s;
b=1-2*s;
c=s;

A=sparse(N,N);
A(1,1)=1;
A(N,N)=1;

for i=2:N-1
    A(i,i+1)=a;
    A(i,i)=b;
    A(i,i-1)=c;
end
V=linspace(100,50,50)';

U=zeros(N,1);
U(1)=100;
U(N)=50;

err=zeros(T,1);
M=zeros(N,N);

for i=1:T-1
    if i<=100
        M(:,i)=U;
```

```
    end
    err(i)=mean(V-U);
    U=A*U;

    plot(1:N,100:-1:51,'r',1:N,U,'b')
    xlabel('l (cm)')
    ylabel('T (K)')
    legend('Numerical Plot','Actual Plot')
    getframe;

end

drawnow

figure()
semilogy(err)
xlabel('Time')
ylabel('Mean Error (K)')

figure()
surf(M)
xlabel('Time')
ylabel('l (cm)')
zlabel('T (K)')
```

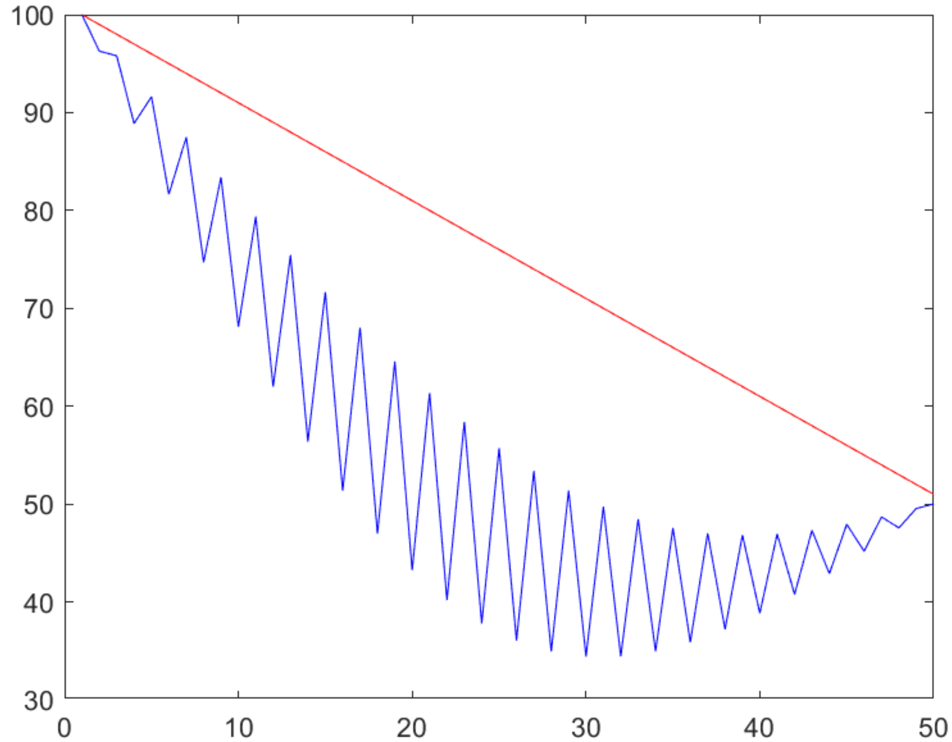
Surface Plot:



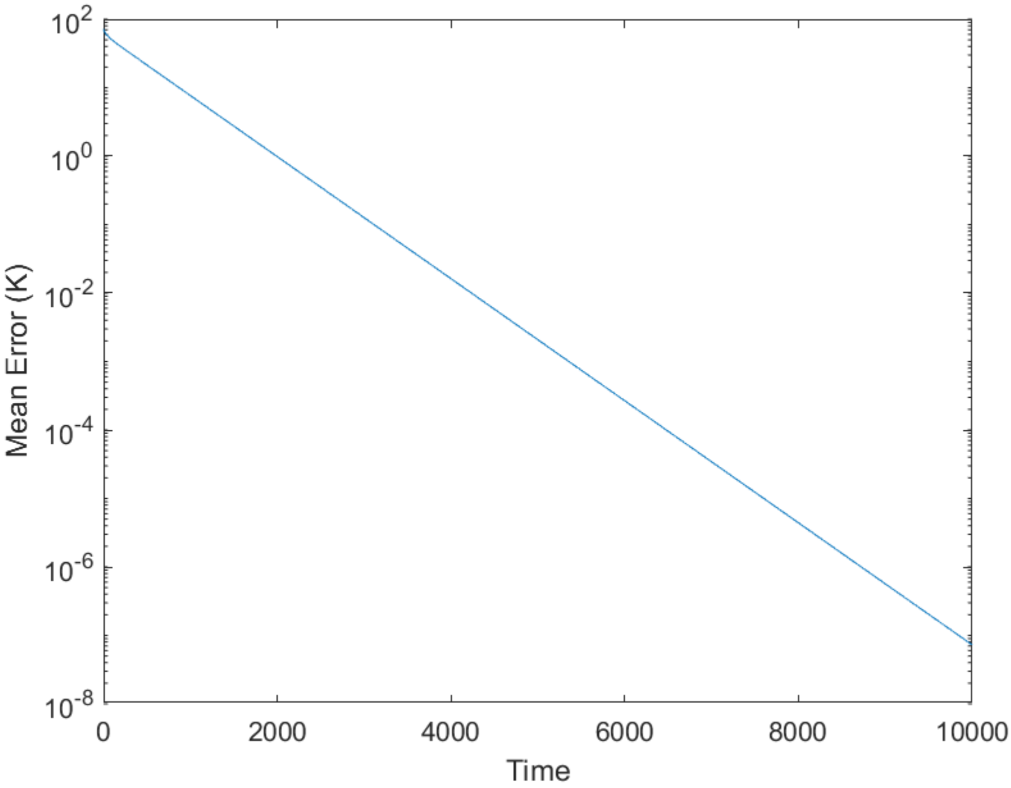
Animated Version: σ<0.5



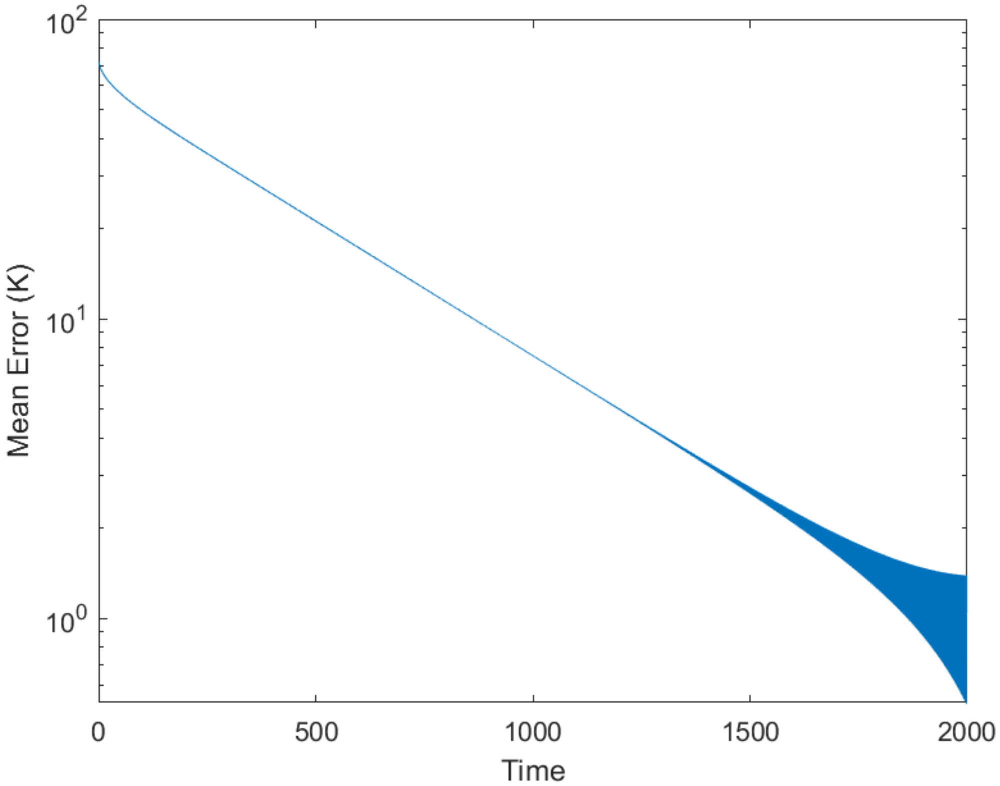
Animated Version: σ>0.5



Error: σ<0.5



Error: σ>0.5



Heat equation 2: Consider the system in which an l=10 cm heating coil is placed at the centre of a long thin rod of length L=100cm. The initial temperature of the rod is 300K at that of the hater is 1200K. The heater is always on so that the temperature at the central part stays constant. The temperature of the end point does not change. Write a code to compute heat evolution for 1st 100sec in the rod using **explicit method** of solving heat equation. Plot the result in a:

1) In a real time animation

2) In a 2D contour or surface plot

for two values of $\sigma>0.5$ and $\sigma<0.5$.

I.C.: $u(-l/2<x<l/2, t=0) = 1200K$, $u(x, t=0)=300K$ for other x

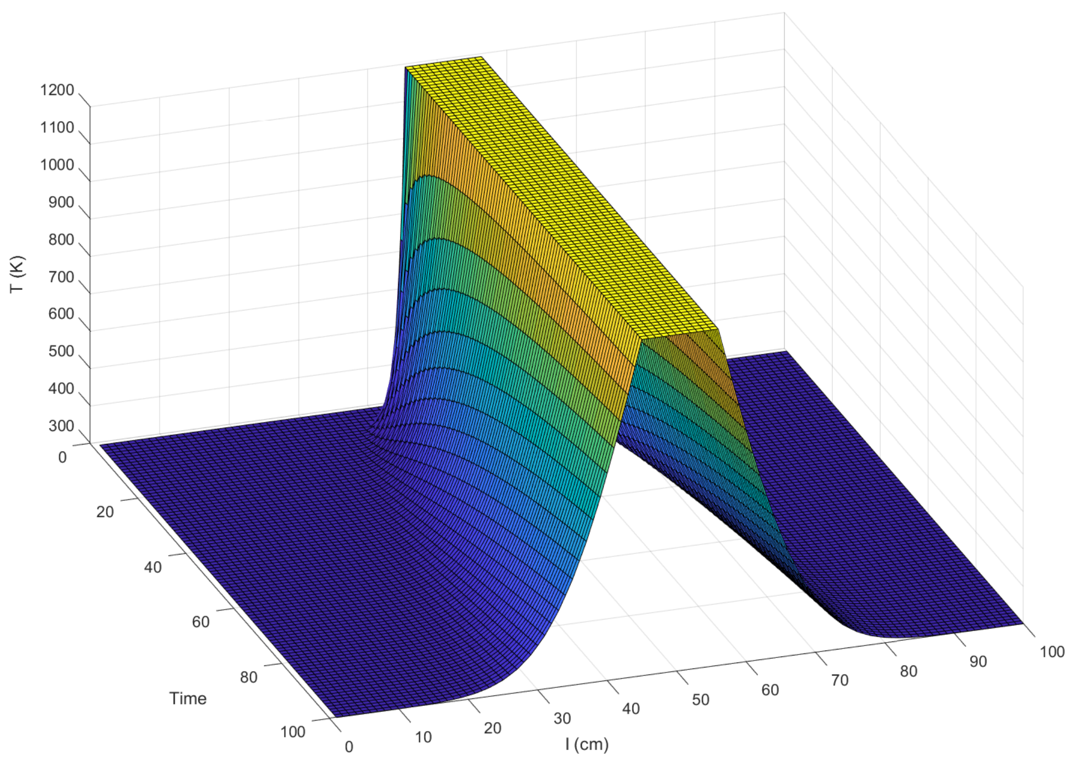
B.C.: $u(-L/2,t)= u(L/2,t) = 300K$, $u(-l/2<x<l/2\ ) = 1200K$ at all t

ANSWER:

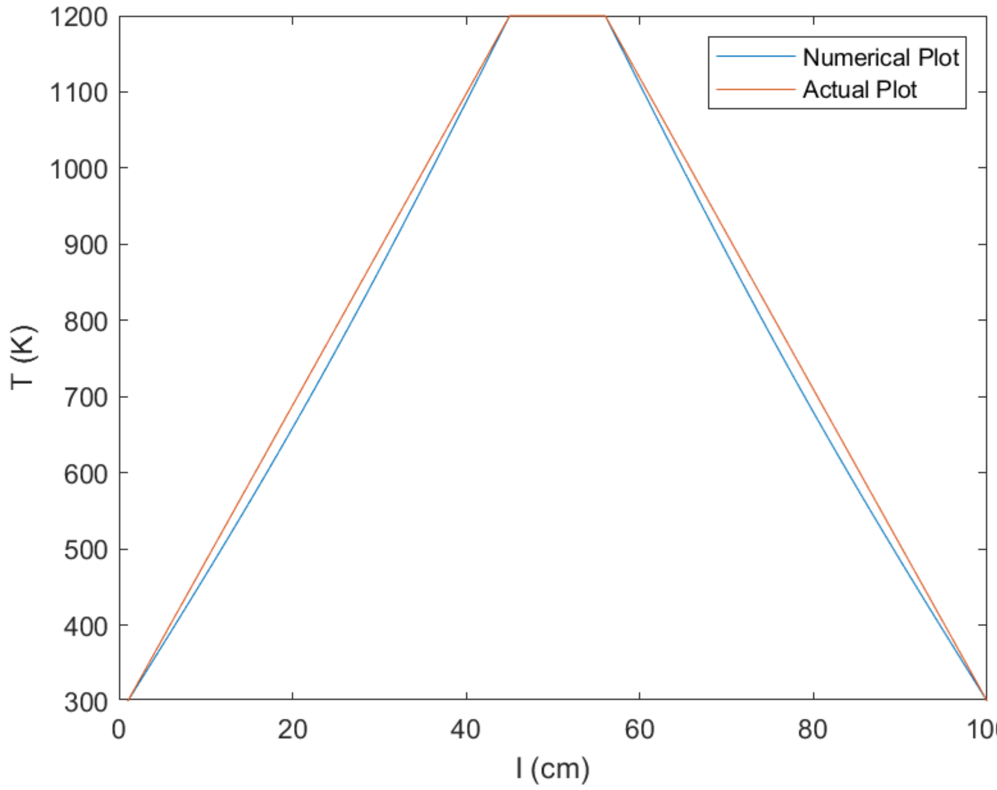
```
clear
clc
L=100;
l=10;

T=10000;

s=0.499;
a=s;
b=1-2*s;
c=s;

A=sparse(L/2-l/2,L/2-l/2);
A(1,1)=1;
A(L/2-l/2,L/2-l/2)=1;

for i=2:L/2-l/2-1
    A(i,i+1)=a;
    A(i,i)=b;
    A(i,i-1)=c;
end
```

```
V=[linspace(300,1200,45) linspace(1200,1200,10)
linspace(1200,300,45)]';

U=ones(L,1)*300;
U(L/2-l/2:L/2+l/2+1)=1200;
U(1:L/2-l/2-1)=300;
U(L/2+l/2+2:L)=300;
err=zeros(T,1);
M=zeros(L,L);
for i=1:T
    if i<=100
        M(:,i)=U;
    end
    err(i)=mean(V-U);
    U(1:L/2-l/2)=A*U(1:L/2-l/2);
    U(L/2+l/2+1:L)=A*U(L/2+l/2+1:L);
    %{
    plot(1:L,U(:,i+1),1:L,V)
    xlabel('l (cm)')
    ylabel('T (K)')
    legend('Numerical Plot','Actual Plot')
    getframe;
    %}
end
%drawnow;

semilogy(err)
xlabel('Time')
ylabel('Mean Error (K)')

surf(M)
xlabel('Time')
ylabel('l (cm)')
zlabel('T (K)')
```

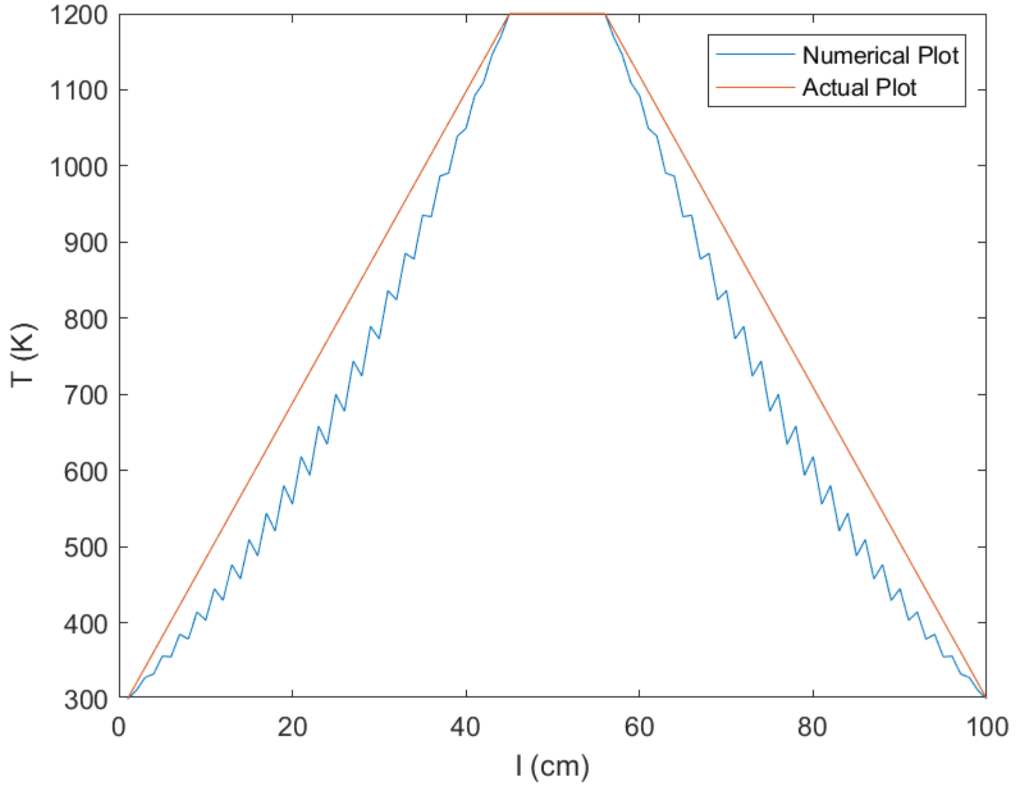
Surface Plot:



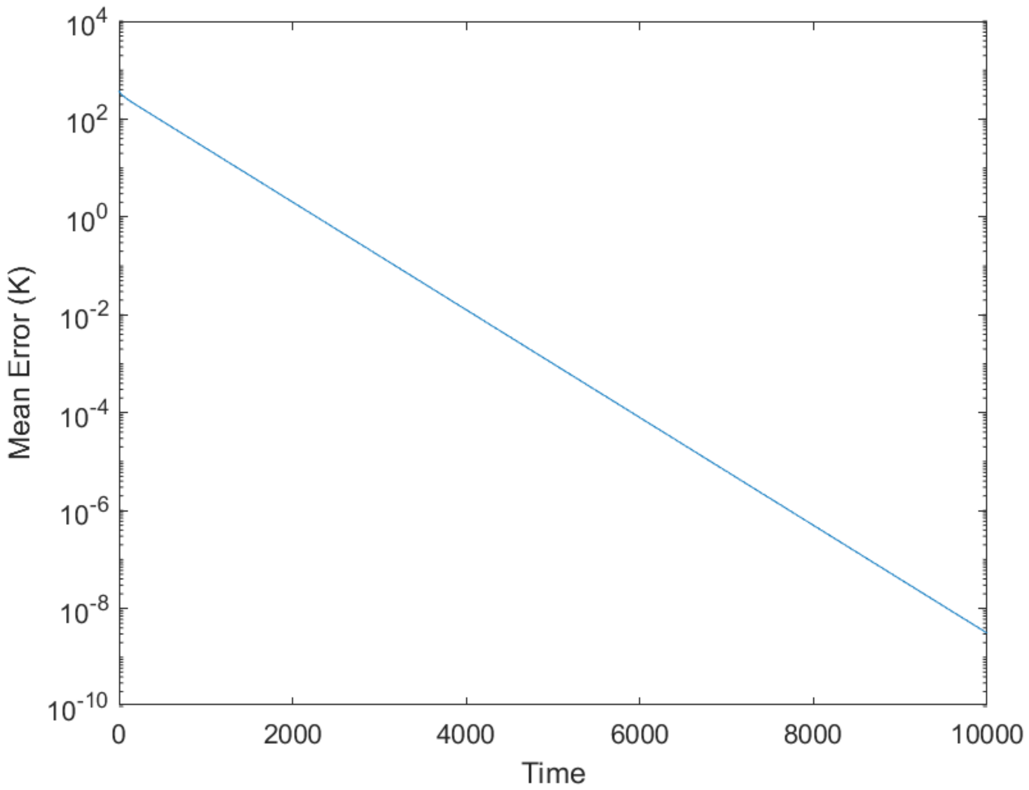
Animation: σ<0.5



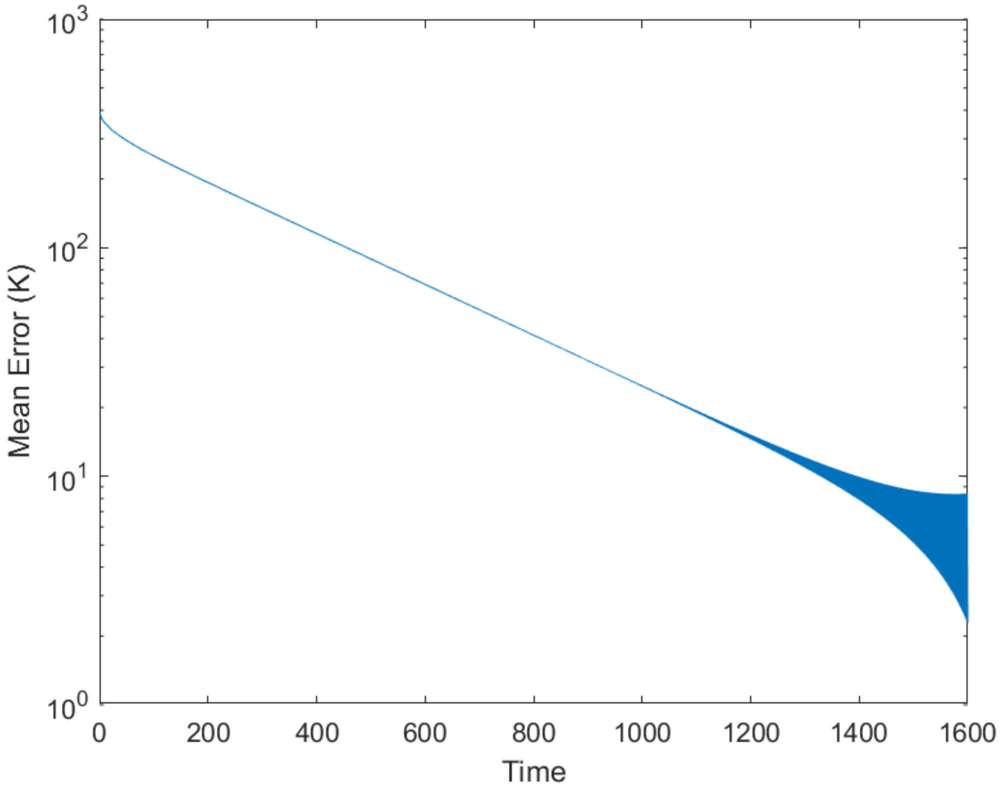
Animated: σ>0.5



Error: σ<0.5



Error: σ>0.5



10^3
10^2
10^1
10^0
Mean Error (K)
0
200
400
600
800
1000
1200
1400
1600
Time