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Assignment – 6

Numerical integration 1: Integrate the tabulated data given below by a suitable method. Take the end points of the table as integration limits.

x	1.34	1.46	1.52	1.6	1.87	2.03	2.18	2.8	3.2	3.8	4.15
f(x)	1.5	2.3	2.3	2.4	2.5	3.2	4.9	4.7	3.4	7.8	17.1

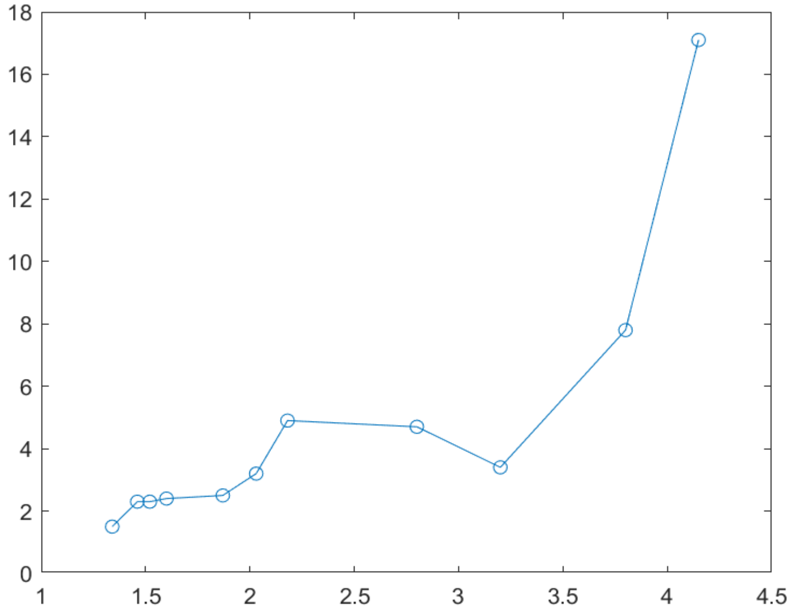
ANSWER:

I = 14.5925

```
A=[1.34 1.5; 1.46 2.3; 1.52 2.3; 1.6 2.4; 1.87 2.5; 2.03
3.2; 2.18 4.9; 2.8 4.7; 3.2 3.4; 3.8 7.8; 4.15 17.1];
X=A(:,1);
Y=A(:,2);

n=length(X);
I=0;

for i=1:n-1
    I=I+(Y(i+1)+Y(i))*(X(i+1)-X(i))/2;
end
disp(I);
plot(X,Y,'o-')
```



Numerical integration 2: Evaluate the following integrals in limits *a* to *b* for [*a*,*b*] interval divided into *N* (=2^n, *n*=1, 2, 3, ...10) **intervals** with:

1) trapezoidal rules
2) Simpson's rule

f(x)	x^4	*sin(x)*	*x*sin(x)*
a	0	0	0
b	1	π	π

Let y_N be the numerical result for *N* number of intervals used. Let y_{AN} be the analytic result. We define the relative error as

$$e(N) = (yN - yAN/yAN)$$

where the error depends on *N*. Show plot of *e(N)* with varying *N* for all the integrals. Are these results keeping with how you expect the error to scale with N?

Answer

f(x)	x^4	*sin(x)*	*x*sin(x)*
a	0	0	0
b	1	π	π
Trapezoidal Rule	0.2	2	3.1416
Simpson's Rule	0.2	2	3.1416

```
%F=@(x) x.^4 ;
F=@(x) sin(x);
%F=@(x) x.*sin(x);

e=zeros(3,10);
a=0;
b=pi;

for i=1:10
    n=2^i;
    e(1,i)=Mid_point(F,a,b,n);
    e(2,i)=Trapezoidal(F,a,b,n);
    e(3,i)=Simpson(F,a,b,n);
end
e_an=2;
e=abs(e-e_an)/e_an;
hold on
grid on
```

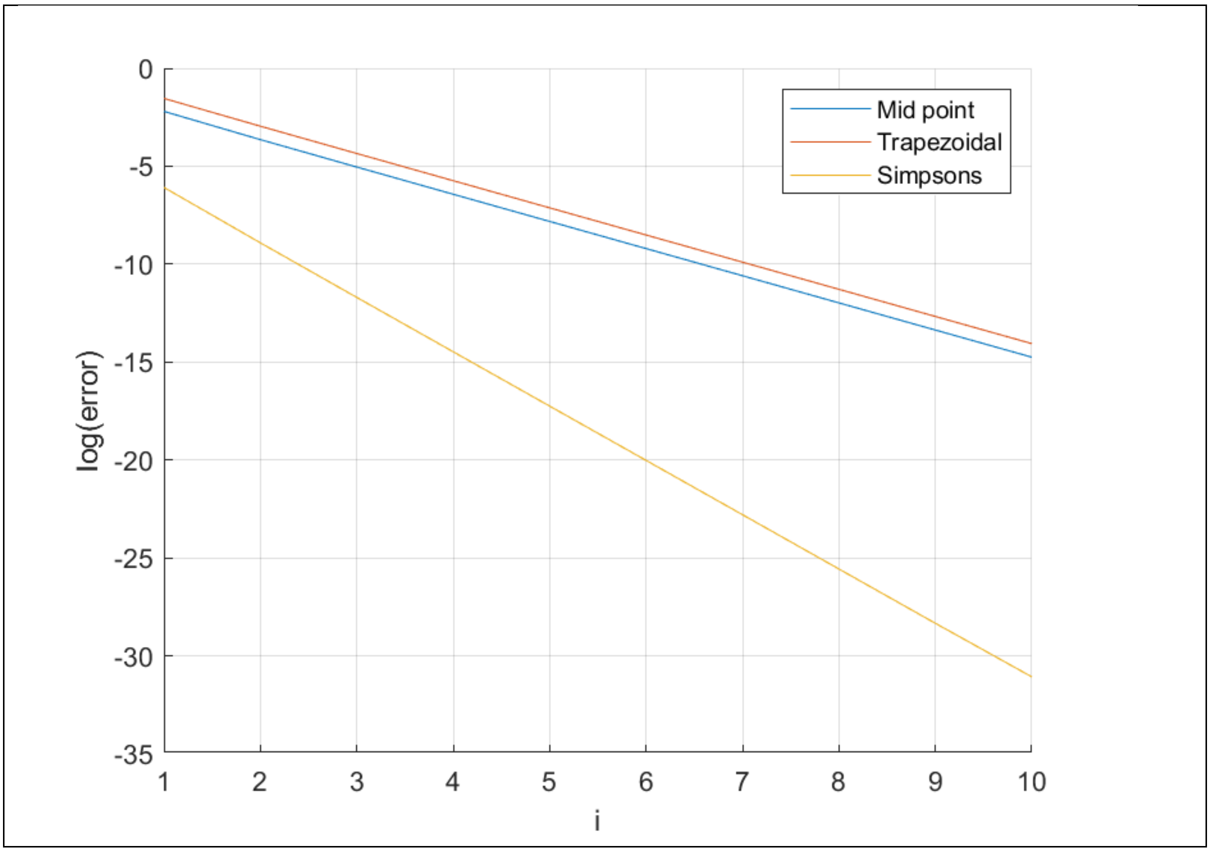
```
plot(log(e(1,:)))
plot(log(e(2,:)))
plot(log(e(3,:)))
legend('Mid point','Trapezoidal','Simpsons');
ylabel('log(error)')
hold off
```

Functions Used

```
function I = Mid_point(F,a,b,N)
h=(b-a)/N;
I=0;
for i=0:N-1
    I=I+F(a+(i+1/2)*h)*h;
end
end


function I = Trapezoidal(F,a,b,N)
h=(b-a)/N;
I=0;
for i=0:N-1
    I=I+(F(a+i*h)+F(a+(i+1)*h))*h/2;
end
end


function I = Simpson(F,a,b,N)
h=(b-a)/N;
I=0;
for i=0:N-1
    I=I+(F(a+i*h)+F(a+(i+1)*h)+4*F(a+(i+1/2)*h))*h/6;
end
end
```



Numerical integration 3:

Evaluate the following integrals

f(x)	$x^6 - 7x^3 + 5$	$x^2 e^{x-1}$	$\sin x/\sqrt{x}$
A	-1	0	0
B	1	5	1

by using

1) Trapezoidal rule with 100 points

2) Simpson's rule for with 51 point

3) Gauss quadrature method for 6 points (Please see the table below)

And compare the values in a table

ANSWER:

f(x)	$x^6 - 7x^3 + 5$	$x^2 e^{x-1}$	$\sin x/\sqrt{x}$
A	-1	0	0
B	1	5	1
Trapezoidal: n=100	10.2861	927.8309	0.6203
Simpson's: n=51	10.2857	927.4329	0.6205
Gauss Quadrature	10.2857	927.4326	0.6209

```
clear
clc

F=@(x) x.^2.*exp(x-1);
%F=@(x) x.^6 -7*x.^3 +5;
%F=@(x) sin(x)./sqrt(x);

a=1e-18;
b=1;
n=51;

Trapezoidal(F,a,b,n)
Simpson(F,a,b,n)
Gauss_Quad(F,a,b,6)
```

Functions Used

```
function I = Trapezoidal(F,a,b,N)
h=(b-a)/N;
I=0;
for i=0:N-1
    I=I+(F(a+i*h)+F(a+(i+1)*h))*h/2;
end
end


function I = Simpson(F,a,b,N)
h=(b-a)/N;
I=0;
for i=0:N-1

I=I+(F(a+i*h)+F(a+(i+1)*h)+4*F(a+(i+1/2)*h))*h/6;
end
end
```

```
function I = Gauss_Quad(F,a,b,n)

s(2).w=[1.0000000000000000 1.0000000000000000];
s(2).x=[-0.5773502691896257 0.5773502691896257];

s(3).w=[0.8888888888888888 0.5555555555555556
0.5555555555555556];
s(3).x=[0.0000000000000000 -0.7745966692414834
0.7745966692414834];

s(4).w=[0.6521451548625461 0.6521451548625461
0.3478548451374538 0.3478548451374538];
s(4).x=[-0.3399810435848563 0.3399810435848563 -
0.8611363115940526 0.8611363115940526];

s(5).w=[0.5688888888888889 0.4786286704993665
0.4786286704993665 0.2369268850561891
0.2369268850561891];
s(5).x=[0.0000000000000000 -0.5384693101056831
0.5384693101056831 -0.9061798459386640
0.9061798459386640];

s(6).w=[0.3607615730481386 0.3607615730481386
0.4679139345726910 0.4679139345726910 0.1713244923791704
0.1713244923791704];
s(6).x=[ 0.6612093864662645 -0.6612093864662645 -
0.2386191860831969 0.2386191860831969  -
0.9324695142031521  0.9324695142031521];

p=(a+b)/2;
q=(b-a)/2;

I=q*sum(s(n).w.*F(p+q*s(n).x));
end
```