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**Assignment – 9**

**FMD 1:** Solve the 1st order ODE y′ + 2y + 3 = 0 with both implicit and

explicit Euler method with I.C. y(0)=1 and compare with the analytical solution:

y(t) = (5/2)\*exp(−2t) – 3/2.

Show that the explicit method becomes unstable for ∆t>0.5 whereas implicit

method is unconditionally stable.

**ANSWER:**

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| clear  clc  h=0.51;  N=6.12/h;    y1=zeros(N,1);  y2=zeros(N,1);    x=0:h:h\*(N-1);  y1(1)=1;  y2(1)=1;  for i=1:N-1  y1(i+1)=y1(i)\*(1-2\*h)-3\*h;  y2(i+1)=(y2(i)-3\*h)/(1+2\*h);  end  hold on  plot(x,y1,x,y2,x,exp(-2\*x)\*5/2-3/2)  xlabel('x')  ylabel('y')  legend('Implicit','Explicit','Actual')  hold off  **h=0.05**  A close up of a mans face  Description automatically generated  **h=0.51** (Implicit Methods becomes unstable after h=0.5)  A close up of a map  Description automatically generated |

**FDM 2:** Solve the following 2nd order ODE

dy2/dx2 + 0.5\*dy/dx + 4y = 5

Initial condition: y(0) = y’(0) = 0;

by applying finite difference method for the x-range of 0 ≤ x ≤ 10.

a) Determine a suitable h value from the stability criteria for oscillatory

solution, i.e. h < 2\*sqrt(ω02−α2)/ω02

b) Show that the solution becomes unstable, i.e. leads to growing instead of

decaying oscillation for h > 2\*sqrt(ω02−α2)/ω02

c) Plot y(x) alongside the analytical solution and the one obtained from R-K

2nd order method for different h values for h< 0.5.

**Answer**

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| clear  clc  w=10;  h=0.65; %0.9682  N=ceil(w/h);  y=zeros(N,1);    x=0:h:h\*(N-1);  y(1)=0;  y(2)=0;    b=h^2/(1+0.25\*h);  a1=(4\*h^2-2)/(1+0.25\*h);  a2=(1-0.25\*h)/(1+0.25\*h);    for i=1:N-2  y(i+2)=b\*5-a1\*y(i+1)-a2\*y(i);  end    f=@(x,y,z) z;  g=@(x,y,z) 5-4\*y-0.5\*z;  [y2,z] = COD(f,g,0,w,0,0,N);    x2=0:0.1:10;  hold on  plot(x,y,x,y2,x2,-0.157485\*exp(-x2/4).\*sin(1.98431\*x2)-1.25\*exp(-x2/4).\*cos(1.98431\*x2)+1.25)  xlabel('x')  ylabel('y')  legend('FD','RK2','Actual')  hold off  **h=0.2**  A close up of a map  Description automatically generated  **h=0.6** (RK2 starts blowing up after h=0.5)  A close up of a map  Description automatically generated  **H=0.99** (FD starts blowing after h=0.9682)  A close up of a map  Description automatically generated |

**Heat Equation 1:** Consider the system in which a thin rod of length L=10cm is

placed between two heat reservoirs kept at 100oC and 50oC, respectively. Theinitial temperature of the rod is 0oC. Write a code to compute heat evolution for

1st 100sec in the rod using **explicit method** of solving heat equation. Plot the

result

1) In a 2D contour or surface plot

2) And in animated version

for two values of σ (=κ∆t/∆x2) greater and less than 0.5.

Initial condition: u(0<x<L, t=0) = 0oC

Boundary condition: u(0,t)= 100oC and u(50,t) = 50oC at all t

**ANSWER:**

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| --- |
| clear  clc  N=50;  T=10000;    s=0.499;  a=s;  b=1-2\*s;  c=s;    A=sparse(N,N);  A(1,1)=1;  A(N,N)=1;    for i=2:N-1  A(i,i+1)=a;  A(i,i)=b;  A(i,i-1)=c;  end  V=linspace(100,50,50)';    U=zeros(N,1);  U(1)=100;  U(N)=50;    err=zeros(T,1);  M=zeros(N,N);    for i=1:T-1  if i<=100  M(:,i)=U;  end  err(i)=mean(V-U);  U=A\*U;    plot(1:N,100:-1:51,'r',1:N,U,'b')  xlabel('l (cm)')  ylabel('T (K)')  legend('Numerical Plot','Actual Plot')  getframe;    end    drawnow    figure()  semilogy(err)  xlabel('Time')  ylabel('Mean Error (K)')    figure()  surf(M)  xlabel('Time')  ylabel('l (cm)')  zlabel('T (K)')  **Surface Plot:**  **A close up of a piece of paper  Description automatically generated**  **Animated Version: σ<0.5**  **A close up of a map  Description automatically generated**  **Animated Version: σ>0.5**  **A close up of a map  Description automatically generated**  **Error: σ<0.5**  **A screenshot of a cell phone  Description automatically generated**  **Error: σ>0.5**  **A close up of a map  Description automatically generated** |

**Heat equation 2:** Consider the system in which an l=10 cm heating coil is

placed at the centre of a long thin rod of length L=100cm. The initial

temperature of the rod is 300K at that of the hater is 1200K. The heater is

always on so that the temperature at the central part stays constant. The

temperature of the end point does not change. Write a code to compute heat

evolution for 1st 100sec in the rod using **explicit method** of solving heat

equation. Plot the result in a:

1) In a real time animation

2) In a 2D contour or surface plot

for two values of σ>0.5 and σ<0.5.

I.C.: u(-l/2<x<l/2, t=0) =1200K, u(x, t=0)=300K for other x

B.C.: u(-L/2,t)= u(L/2,t) = 300K, u(-l/2<x<l/2 ) = 1200K at all t

**ANSWER:**

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| --- |
| clear  clc  L=100;  l=10;    T=10000;    s=0.499;  a=s;  b=1-2\*s;  c=s;    A=sparse(L/2-l/2,L/2-l/2);  A(1,1)=1;  A(L/2-l/2,L/2-l/2)=1;    for i=2:L/2-l/2-1  A(i,i+1)=a;  A(i,i)=b;  A(i,i-1)=c;  end    V=[linspace(300,1200,45) linspace(1200,1200,10) linspace(1200,300,45)]';    U=ones(L,1)\*300;  U(L/2-l/2:L/2+l/2+1)=1200;  U(1:L/2-l/2-1)=300;  U(L/2+l/2+2:L)=300;  err=zeros(T,1);  M=zeros(L,L);  for i=1:T  if i<=100  M(:,i)=U;  end  err(i)=mean(V-U);  U(1:L/2-l/2)=A\*U(1:L/2-l/2);  U(L/2+l/2+1:L)=A\*U(L/2+l/2+1:L);  %{  plot(1:L,U(:,i+1),1:L,V)  xlabel('l (cm)')  ylabel('T (K)')  legend('Numerical Plot','Actual Plot')  getframe;  %}  end  %drawnow;    semilogy(err)  xlabel('Time')  ylabel('Mean Error (K)')    surf(M)  xlabel('Time')  ylabel('l (cm)')  zlabel('T (K)')  **Surface Plot:**  **A picture containing stationary  Description automatically generated**  **Animation: σ<0.5**  **A screenshot of a social media post  Description automatically generated**  **Animated: σ>0.5**  **A close up of a map  Description automatically generated**  **Error: σ<0.5**  **A screenshot of a cell phone  Description automatically generated**  **Error: σ>0.5**  **A close up of a map  Description automatically generated** |