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**Assignment – 9**

**Heat Equation 2:** Consider the system in which a thin rod of length L=10cm is

placed between two heat reservoirs kept at 100 and 50oC, respectively. The initial temperature of the rod is 0oC. Write a code to compute heat evolution for

1st 100sec in the rod using implicit method of solving heat equation. Plot the

result

1) In a 2D contour or surface plot

2) And in animated version

for two values of σ (=κ∆t/∆x^2) greater and less than 0.5.

Initial condition: u(0<x<L, t=0) = 0oC

Boundary condition: u(0,t)= 100oC and u(50,t) = 50oC at all t.

**ANSWER:**

|  |
| --- |
| clear  clc  N=50;  T=1000;    s=5;  a=-s;  b=1+2\*s;  c=-s;    A=sparse(N,N);  A(1,1)=1;  A(N,N)=1;    for i=2:N-1  A(i,i+1)=a;  A(i,i)=b;  A(i,i-1)=c;  end  V=linspace(100,50,50)';    U=zeros(N,1);  U(1)=100;  U(N)=50;    err=zeros(T,1);  M=zeros(N,N);    for i=1:T-1  if i<=100  M(:,i)=U;  end  err(i)=mean(V-U);  U=A\U;  %{  plot(1:N,V,'r',1:N,U,'b')  xlabel('l (cm)')  ylabel('T (K)')  legend('Numerical Plot','Actual Plot')  getframe;  %}  end  %drawnow    figure()  semilogy(err)  xlabel('Time')  ylabel('Mean Error (K)')    figure()  surf(M)  xlabel('Time')  ylabel('l (cm)')  zlabel('T (K)')  **Surface Plot:**    **Animation:**  **A close up of a map  Description automatically generated**  **Error Propagation:**  A screenshot of a cell phone  Description automatically generated |

**Heat equation 3:** Consider the system in which an l=10 cm heating coil is

placed at the centre of a long thin rod of length L=100cm. The initial

temperature of the rod is 300K at that of the hater is 1200K. The heater is

always on so that the temperature at the central part stays constant. The

temperature of the end point does not change. Write a code to compute heat

evolution for 1st 100sec in the rod using implicit method of solving heat

equation. Plot the result in a real time animation for two values of σ>0.5 and

σ<0.5.

I.C.: u(-l/2<x<l/2, t=0) =1200K, u(x, t=0)=300K for other x

B.C.: u(-L/2,t)= u(L/2,t) = 300K, u(-l/2<x<l/2 ) = 1200K at all t

**Answer**

|  |
| --- |
| clear  clc  L=100;  l=10;    T=200;    s=5;  a=-s;  b=1+2\*s;  c=-s;    A=sparse(L/2-l/2,L/2-l/2);  A(1,1)=1;  A(L/2-l/2,L/2-l/2)=1;    for i=2:L/2-l/2-1  A(i,i+1)=a;  A(i,i)=b;  A(i,i-1)=c;  end    V=[linspace(300,1200,45) linspace(1200,1200,10) linspace(1200,300,45)]';    U=ones(L,1)\*300;  U(L/2-l/2:L/2+l/2+1)=1200;  U(1:L/2-l/2-1)=300;  U(L/2+l/2+2:L)=300;  err=zeros(T,1);  M=zeros(L,L);  for i=1:T  if i<=100  M(:,i)=U;  end  err(i)=mean(V-U);  U(1:L/2-l/2)=A\U(1:L/2-l/2);  U(L/2+l/2+1:L)=A\U(L/2+l/2+1:L);  plot(1:L,U,1:L,V)  xlabel('l (cm)')  ylabel('T (K)')  legend('Numerical Plot','Actual Plot')  getframe;  end  drawnow;  figure()  semilogy(err)  xlabel('Time')  ylabel('Mean Error (K)')    figure()  surf(M)  xlabel('Time')  ylabel('l (cm)')  zlabel('T (K)')  **Surface Plot:**    **Animation:**  **A screenshot of a map  Description automatically generated**  **Error Propagation:**  A screenshot of a cell phone  Description automatically generated |

**Laplace equation:** Write a program to solve the two dimensional Laplace

equation Txx + Tyy = 0 describing the steady state temperature distribution on a

square plate of sides L=100 cm. Use same length for x- and y- increment.

Show the temperature distribution T(x, y) using a surface plot for an x-y grid of

minimum 20×20 segments. Vary number of greed points to fill the table below.

The boundary conditions are:

T(x = 0) = T(x = L) = 0° C, T(y = 0) = -100°, T(y = L) = 100° at all time, corner

points are assumed to have T equals to the average of adjoining sites.

**ANSWER:**

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| --- |
| clear  clc    N=100;  T=2000;  U=zeros(N,N);  %Boundary Conditions  U(1,:)=0;  U(N,:)=0;  U(:,1)=-100;  U(:,N)=100;  %Corner Points  U(1,1)=-50;  U(1,N)=50;  U(N,1)=-50;  U(N,N)=50;    beta=1;    err=zeros(T,1);  U\_prev=zeros(N,N);    for t=1:T  for i=2:N-1  for j=2:N-1  U(i,j)=(U(i-1,j)+beta\*(U(i+1,j)+U(i,j+1)+U(i,j-1)))/(2\*(1+beta^2));  end  end  err(t)=norm(U-U\_prev);  U\_prev=U;  imagesc(U)  getframe;  end  semilogy(err(2:T))  xlabel('iteration')  ylabel('Error')  **Temperature Profile:**  **A screenshot of a computer  Description automatically generated**  **Error Propagation:**  **A close up of a map  Description automatically generated** |

|  |  |  |  |
| --- | --- | --- | --- |
| **Sr. No.** | **No. of grid segments (N)** | **No. of iterations to converge (e=1e-2)** | **Total no. of calculations** |
| 1 | 20 | 202 | 80,800 |
| 2 | 30 | 401 | 3,60,900 |
| 3 | 40 | 635 | 10,16,000 |
| 4 | 50 | 896 | 22,40,000 |
| 5 | 60 | 1181 | 42,51,600 |

**Poisson equation:** In the previous problem, incorporate a point heater of 500oC

at the centre of the square plate and determine the equilibrium temperature distribution for a given error limit. Use an x-y grid of minimum 20×20 segments

The boundary conditions are:

T(x = 0) = T(x = L) = 0° C, T(y = 0) = -100°, T(y = L) = 100° at all time, corner

points are assumed to have T equals to the average of adjoining sites.

**ANSWER:**

|  |
| --- |
| clear  clc    N=100;  T=2000;  U=zeros(N,N);  %Boundary Conditions  U(1,:)=0;  U(N,:)=0;  U(:,1)=-100;  U(:,N)=100;  %Corner Points  U(1,1)=-50;  U(1,N)=50;  U(N,1)=-50;  U(N,N)=50;    beta=1;    err=zeros(T,1);  U\_prev=zeros(N,N);    for t=1:T  for i=2:N-1  for j=2:N-1  U(i,j)=(U(i-1,j)+beta\*(U(i+1,j)+U(i,j+1)+U(i,j-1)))/(2\*(1+beta^2));  end  end  U(N/2,N/2)=U(N/2,N/2)+500/4;  err(t)=norm(U-U\_prev);  U\_prev=U;  contour(U,100)  colorbar  getframe;  end  semilogy(err(2:T))  xlabel('iteration')  ylabel('Error')  **Temperature Profile:**  **A screenshot of a cell phone  Description automatically generated**  **Error Propagation:**  **A screenshot of a cell phone  Description automatically generated** |