

Total energy of the system is

$$F[\vec{m}(\vec{r})] = \int \mathcal{F}(\vec{m}(\vec{r})) d^3r \quad (1)$$

Where

$$\mathcal{F}(\vec{m}(\vec{r})) = \frac{J_2}{4!}(\nabla \vec{m})^4 - \frac{J_1}{2!}(\nabla \vec{m})^2 + D\vec{m} \cdot ((\hat{z} \times \nabla) \times \vec{m}) + Km_z^2 - hm_z \quad (2)$$

So the total energy can be expressed as $F[\vec{m}(\vec{r})] = F_{J_2} + F_{J_1} + F_D + F_K + F_h$. Each terms represents the energy coming from the individual terms in equation (2). The magnetization vector can be expressed as

$$\vec{m}(\vec{r}) = \hat{e}_1 m_1 \cos(\vec{q} \cdot \vec{r}) + \hat{e}_2 m_2 \sin(\vec{q} \cdot \vec{r}) + \hat{e}_3 m_3$$

where \hat{e}_1 , \hat{e}_2 and \hat{e}_3 are the unit vector in the new coordinate basis and they are mutually orthonormal to each other. From the normalization condition, we can write $m_3 = \sqrt{1 - (m_1 \cos(\vec{q} \cdot \vec{r}))^2 - (m_2 \sin(\vec{q} \cdot \vec{r}))^2}$. The new basis is related the old basis (\hat{x}_1 , \hat{x}_2 , \hat{x}_3) via relation

$$\hat{e}_i = \mathcal{R}_z(\theta_3) \mathcal{R}_y(\theta_2) \mathcal{R}_z(\theta_1) \hat{x}_i$$

where $\hat{x}_1 = \hat{x}$, $\hat{x}_2 = \hat{y}$ and $\hat{x}_3 = \hat{z}$. θ_1 , θ_2 , and θ_3 are the angle of rotation with x , y and z are the axis of rotation respectively. So there are eight variational parameters: θ_1 , θ_2 , θ_3 , m_1 , m_2 and \vec{q} . Without loss of the generality we can choose \vec{q} to lie in the $x - z$ plane leaving 7 parameters. So we expressed the total energy in terms of the 7 parameters. Those are

$$F_{J_2} = \frac{J_2}{4!} \frac{(q_x^2 + q_z^2)^2}{4} \left(4 - 2(2 + m_1^2 + m_2^2) \sqrt{(1 - m_1^2)(1 - m_2^2)} \right) \quad (3)$$

$$F_{J_1} = -\frac{J_1}{2!} \frac{q_x^2 + q_z^2}{4} \left(1 - \sqrt{(1 - m_1^2)(1 - m_2^2)} \right) \quad (4)$$

$$F_D = -Dq_x m_1 m_2 \sin \theta_1 \sin \theta_2 \quad (5)$$

$$F_K = \frac{K}{2} \cos^2 \theta_2 (2 - m_1^2 - m_2^2) + \frac{K}{2} \sin^2 \theta_2 ((m_1 \cos \theta_3)^2 + (m_2 \sin \theta_3)^2) \quad (6)$$

$$\begin{aligned} F_h &= -\frac{h}{2\pi} \cos \theta_2 \int_0^{2\pi} \sqrt{1 - (m_1 \cos u)^2 - (m_2 \sin u)^2} \\ &= -\frac{h}{\pi/2} \cos \theta_2 \sqrt{1 - m_1^2} E\left(-\frac{m_1^2 - m_2^2}{1 - m_1^2}\right) \end{aligned} \quad (7)$$

Where $E(k) = \int_0^{\pi/2} \sqrt{1 - k \sin^2 x} dx$ is the elliptic integral