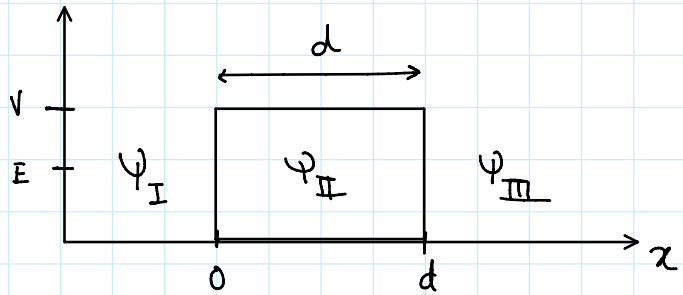
Square Barrier Problem



$$\hat{H}\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\cdot\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_{I/III}}{\partial x^2} = E\psi_{I/III} \Rightarrow \frac{\partial^2\psi}{\partial x^2} = \overbrace{\frac{2mE}{\hbar^2}}^{k^2}\cdot\psi$$

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$$\psi_{III} = Ee^{ikx} + Fe^{-ikx}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_{II}}{\partial x^2} + V\psi = E\psi \qquad k_2 = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$$\psi_{II} = Ce^{+ik_2x} + De^{-ik_2x}$$

BOUNDARY CONDITIONS

$$\boxed{\begin{aligned} &\psi_I(x=0) = \psi_{II}(x=0) \\ &\psi_{II}(x=d) = \psi_{III}(x=d) \\ &\left.\frac{\partial\psi_I}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=0} \\ &\left.\frac{\partial\psi_{II}}{\partial x}\right|_{x=d} = \left.\frac{\partial\psi_{III}}{\partial x}\right|_{x=d} \end{aligned}}$$

$$t = \frac{4k_1k_2 e^{-id(k_1-k_2)}}{(k_1+k_2)^2 - e^{2iak_2}(k_1-k_2)^2}$$

$$\boxed{T = |t|^2 = \frac{1}{1 + \frac{V^2 \sinh^2(k_2 d)}{4E(V-E)}} \quad (E<V)}$$

Transmitted Probability Current

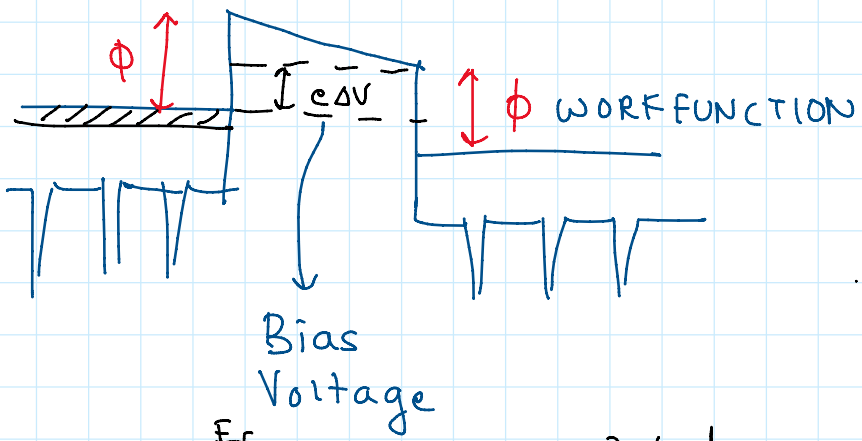
$$j_t = \frac{-i\hbar}{2m_e}\left(\psi_{III}^* \frac{\partial}{\partial x}\psi_{III} - \psi_{III} \cdot \frac{\partial}{\partial x}\psi_{III}\right)$$

$$= \underbrace{\frac{\hbar k}{m_e}}_{j_i = \text{ Impinging Probability Current}} |t|^2$$

with approximations : $\boxed{j_t \propto j_i \cdot e^{-2k_2 d}}$

$$\rho = \psi^* \psi \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 \quad \text{Continuity eq}^n \text{ for QM}$$

Slightly More Realistic Case :=



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Factors : 1.] no. of e^-s closer to E_F

2] Free states available in the tip

$$I \propto \sum_{E_F - e\Delta V}^{E_F} |\psi_n(0)| e^{-2\alpha_n d}$$

{ α_n same as k_z for n^{th} energy level in shaded band }

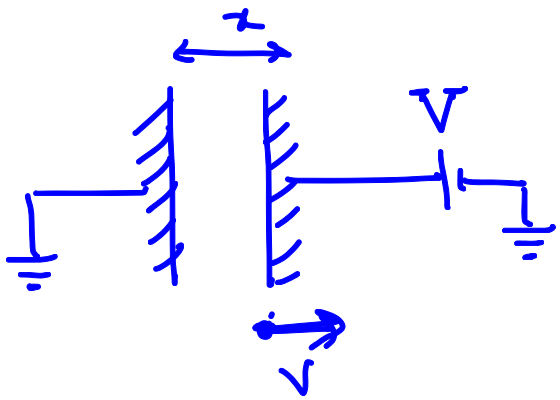
— ✱ Bardeen's Formalism

— Time Dependent

— Perturbative

$$I = \underbrace{I(0)}_{\text{Set Point Current}} e^{-2\alpha d}$$] Decay with distance

Displacement Current



$$q = C \cdot V \qquad C = \frac{A\varepsilon}{d}$$

$$\frac{dq}{dt} = i = V \cdot \frac{dC}{dt}$$

$$= V \cdot A\varepsilon \cdot -\frac{1}{d^2} \cdot \frac{d(d)}{dt}$$

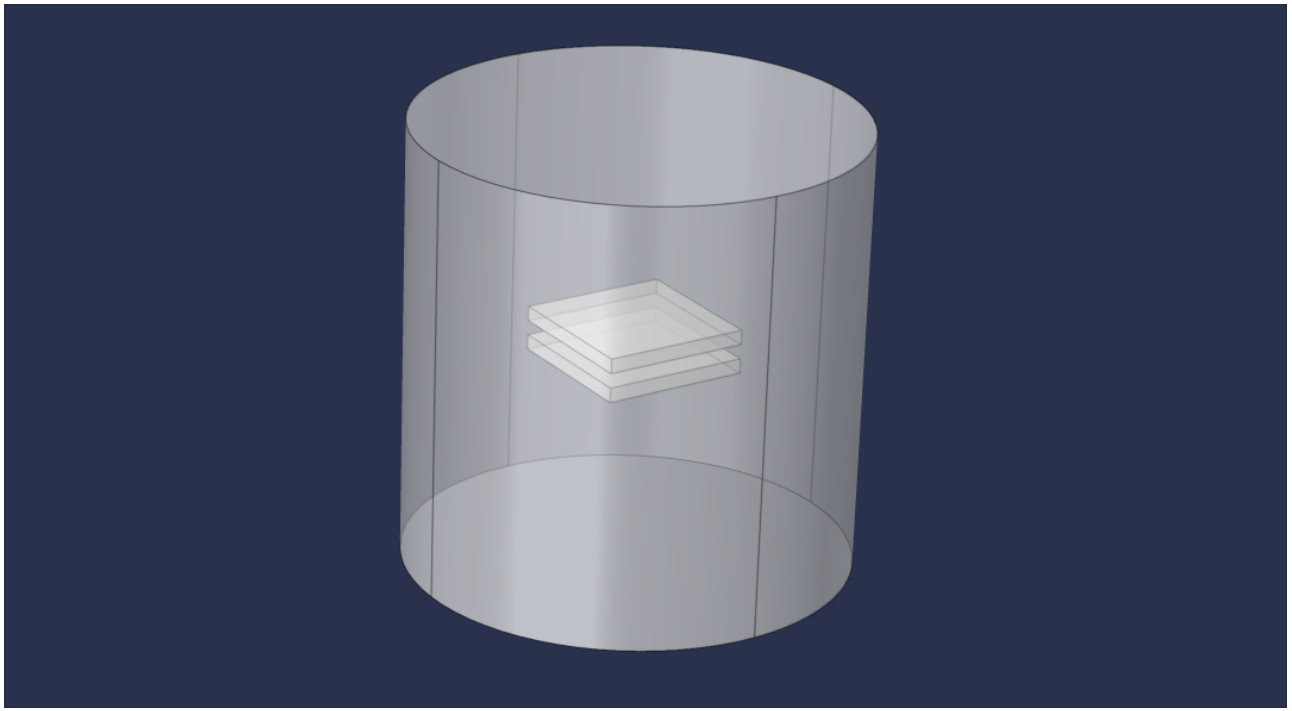
$$\boxed{i_c = -\frac{V A \varepsilon}{(d(t))^2} \cdot \vec{v}(t)}$$

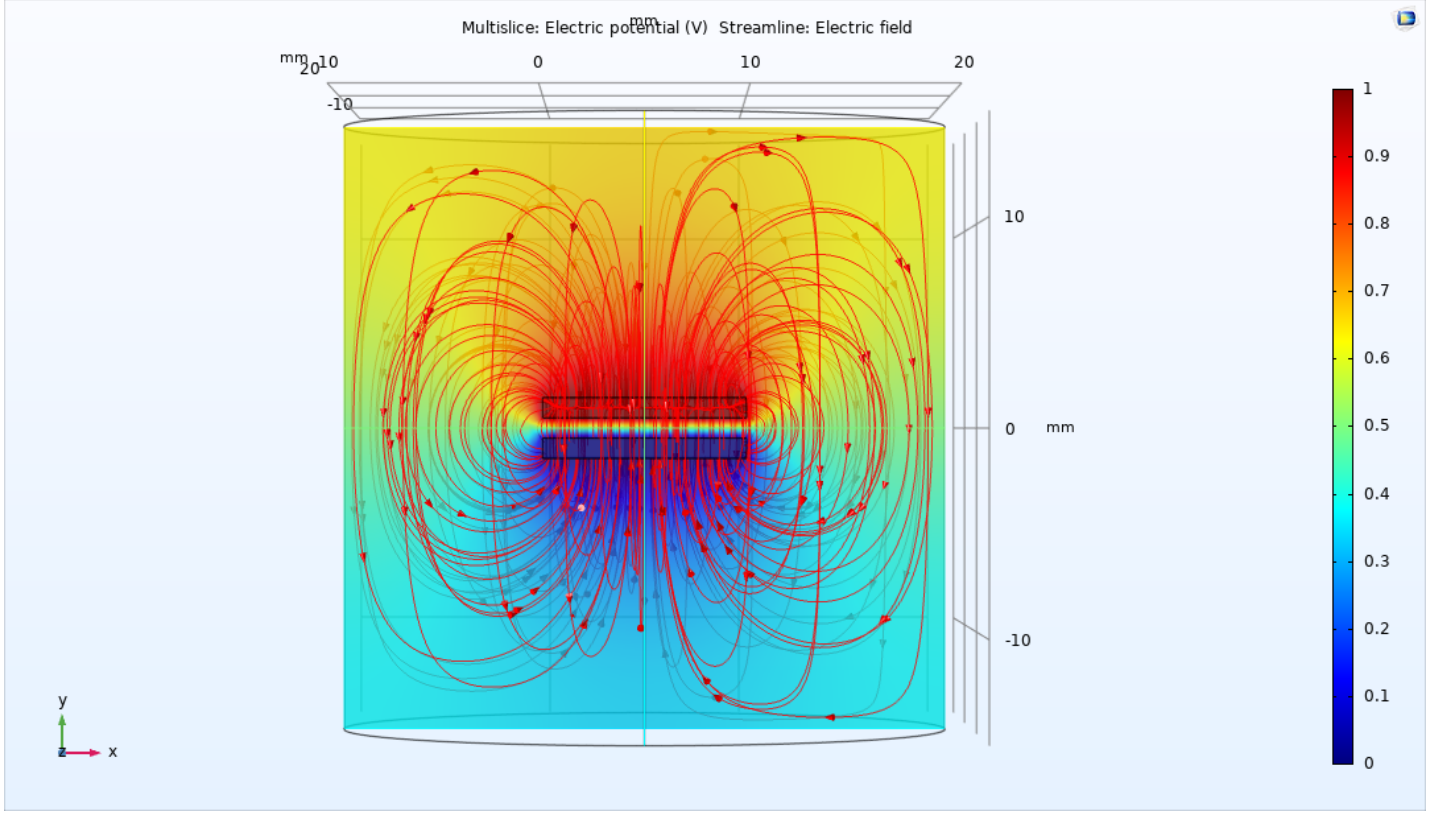
$$\vec{E} : \frac{q}{A\varepsilon}$$

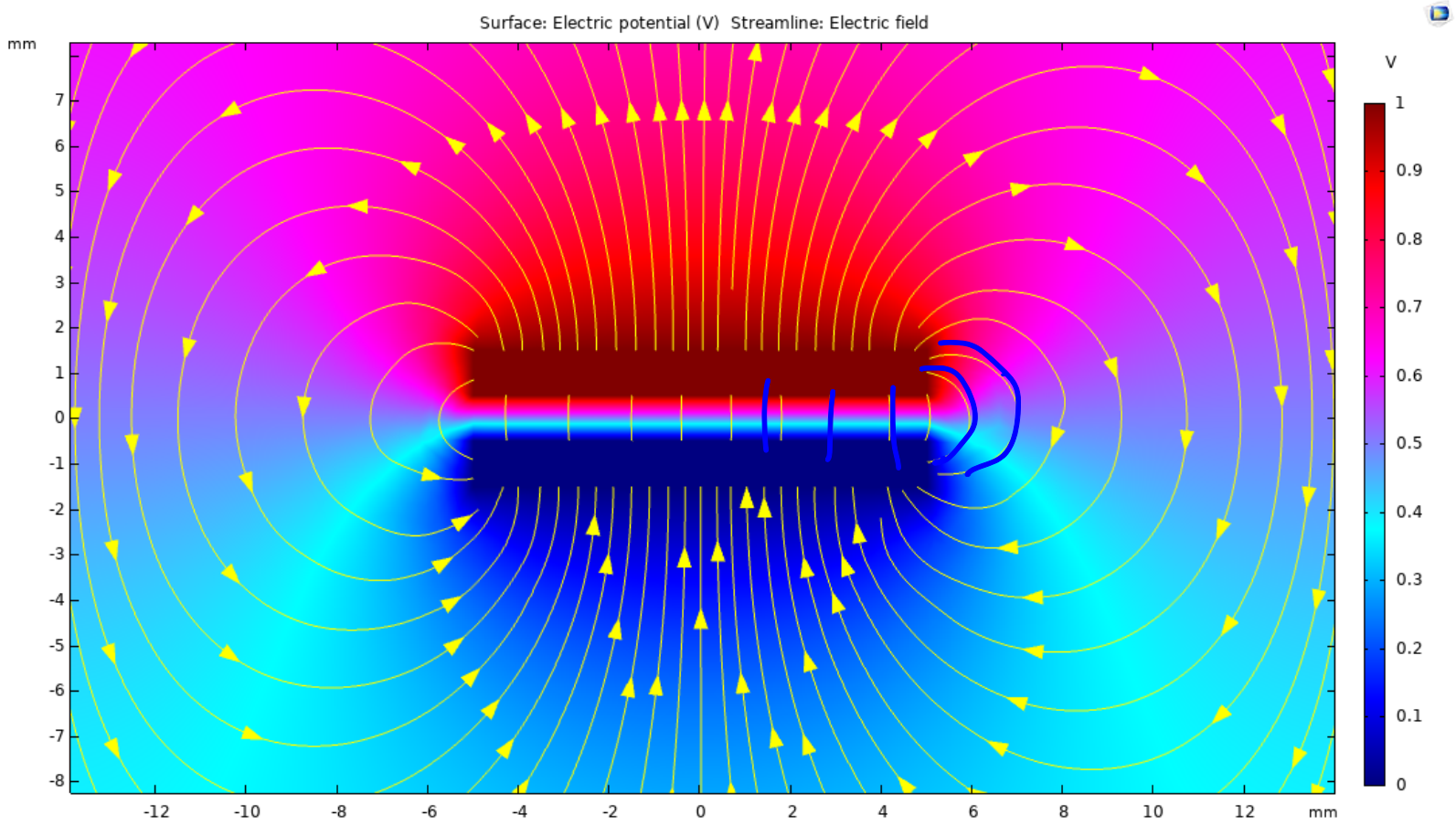
$$\boxed{\frac{d\vec{E}}{dt} = \frac{1}{A\varepsilon} \cdot i_c}$$

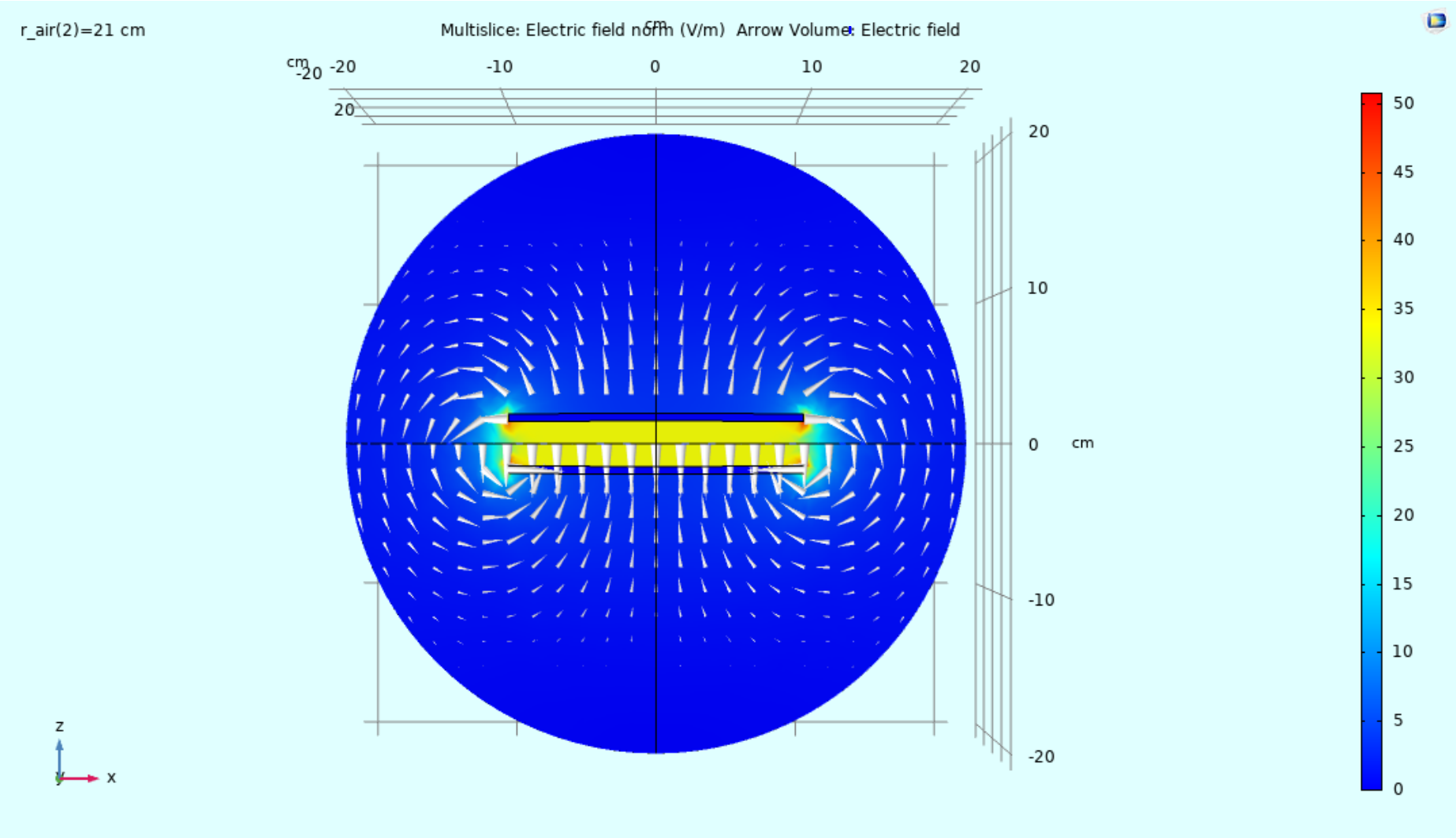
Displacement Current

$$\underbrace{I_D = \epsilon \frac{\partial \Phi_E}{\partial t}}_{\Phi_E = \vec{E} \cdot \vec{A}} = A\varepsilon \frac{\partial \vec{E}}{\partial t} = A\varepsilon \cdot \frac{i_c}{A\varepsilon} = i_c \Rightarrow \boxed{I_D = i_c}$$




Multislice: Electric potential (V) Streamline: Electric field
mm
mm
-10
0
10
20
20
-10
10
0
mm
-10
y
z
x
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


Surface: Electric potential (V) Streamline: Electric field
mm
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
mm
V
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


r_air(2)=21 cm
Multislice: Electric field norm (V/m) Arrow Volume: Electric field
cm
-20
-10
0
10
20
cm
20
10
0
-10
-20
cm
50
45
40
35
30
25
20
15
10
5
0
z
x