Assignment 2. USC ID: 2867581111. Name: Vinit Jain.

Theory Problem 1: Color Theory

Problem 1)

Given equation is,

$$C(X,Y,Z) = \alpha_1 P_1(X_1,Y_1,Z_1) + \alpha_2 P_2(X_2,Y_2,Z_2) + \alpha_3 P_3(X_3,Y_3,Z_3)$$

i) Normalized chromacity coordinates -

for $P_1(X_1,Y_1,Z_1) \Rightarrow x_{cp_1} = \frac{X_1}{X_1+Y_1+Z_1}$, $y_{cp_1} = \frac{Y_1}{X_1+Y_1+Z_1}$, $z_{cp_1} = \frac{Z_1}{X_1+Y_1+Z_1}$

for $P_2(X_2,Y_2,Z_2) \Rightarrow x_{cp_2} = \frac{X_2}{X_2+Y_2+Z_2}$, $y_{cp_2} = \frac{Y_2}{X_2+Y_2+Z_2}$, $z_{cp_3} = \frac{Z_2}{X_2+Y_2+Z_2}$

for $P_3(X_3,Y_3,Z_3) \Rightarrow x_{cp_3} = \frac{X_3}{X_3+Y_3+Z_3}$, $y_{cp_3} = \frac{Y_3}{X_3+Y_3+Z_3}$, $z_{cp_3} = \frac{Z_3}{X_3+Y_3+Z_3}$

ii) For $C(X,Y,Z)$, normalized chromacity coordinates are

$$x_c = \frac{X}{X+Y+Z}, \quad y_c = \frac{Y}{X+Y+Z}, \quad z_c = \frac{Z}{X+Y+Z}$$

we have, $X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$

$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$

$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$

$$\therefore x_c = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3 + \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}$$

$$\Rightarrow x_c = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$||^{ly}$

$$y_c = \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$z_c = \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

from i) we know

$$x_{cp1} = \frac{X_1}{X_1+Y_1+Z_1}, \quad x_{cp2} = \frac{X_2}{X_2+Y_2+Z_2}, \quad x_{cp3} = \frac{X_3}{X_3+Y_3+Z_3} \quad \text{— (1)}$$

Multiply & divide RHS by α_1, α_2 & α_3 for x_{cp1}, x_{cp2} & x_{cp3} respectively

$$x_{cp1} = \frac{\alpha_1 X_1}{\alpha_1 (X_1+Y_1+Z_1)}, \quad x_{cp2} = \frac{\alpha_2 X_2}{\alpha_2 (X_2+Y_2+Z_2)}, \quad x_{cp3} = \frac{\alpha_3 X_3}{\alpha_3 (X_3+Y_3+Z_3)}$$

$$\therefore \quad \alpha_1 X_1 = \alpha_1 (X_1+Y_1+Z_1) \cdot x_{cp1} \quad \text{— (2)}$$

$$\alpha_2 X_2 = \alpha_2 (X_2+Y_2+Z_2) \cdot x_{cp2}$$

$$\alpha_3 X_3 = \alpha_3 (X_3+Y_3+Z_3) \cdot x_{cp3}$$

from ii) & (2)

$$x_c = \frac{\alpha_1 (X_1+Y_1+Z_1) \cdot x_{cp1} + \alpha_2 (X_2+Y_2+Z_2) x_{cp2} + \alpha_3 (X_3+Y_3+Z_3) \cdot x_{cp3}}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$\therefore \boxed{x_c = A \, x_{cp1} + B x_{cp2} + C x_{cp3}}$$

where $A = \frac{\alpha_1 (X_1+Y_1+Z_1)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$

$$B = \frac{\alpha_2 (X_2+Y_2+Z_2)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$C = \frac{\alpha_3 (X_3+Y_3+Z_3)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

Similarly we can prove for y_c, z_c

Hence Chromaticity coordinates of C is a linear combination of chromaticity coordinates of respective primaries

. **Theory Problem 2: Generic Compression Problem**

Problem 2:

i) 22, 24, 24, 28, 28, 28, 25, 26, 26, 26, 21, 19, 20, 20, 22, 24, 24, 24, 23, 24, 20, 16, 10, 10, 8, 11, 6, 9, 9, 12, 15, 19

ii) $32 = 2^5 \Rightarrow 5$ bits per Signal

$32 * 5 = 160$ bits

iii) Differences Maximum = 1 (After excluding first number)

Differences Minimum = -1.5

Range is $[-1.5, 1]$. There are 11 levels so

$11 = 2^{3.51}$, therefore 4 bits per Signal

$32 * 4 = 128$ bits

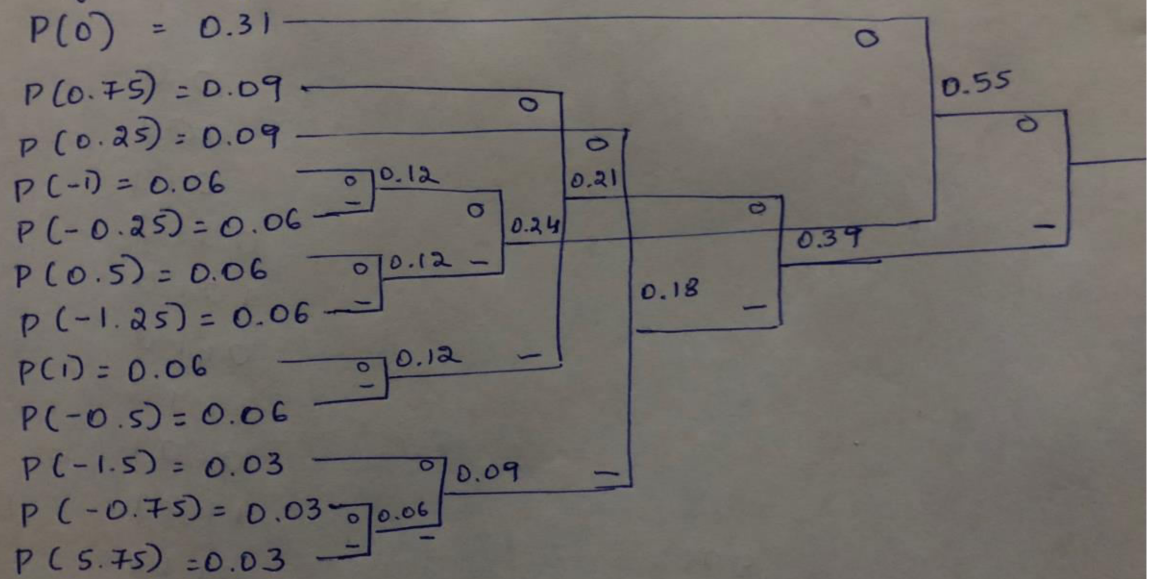
iv) Compression Ratio =

$$\frac{\text{Number of bits required to encode without DPCM}}{\text{Number of bits required to encode with DPCM}}$$

$$= \frac{160}{128} = \frac{5}{4}$$

128 4

v) Ignoring I^{st} number, Huffman coding is



Symbol	Huffman code
-0.75	11110
-1.5	1110
-0.5	1011
1	1010
-1.25	0111
0.5	0110
-0.25	0101
-1	0100
0.25	110
0.75	100
0	00

vi) compression Ratio

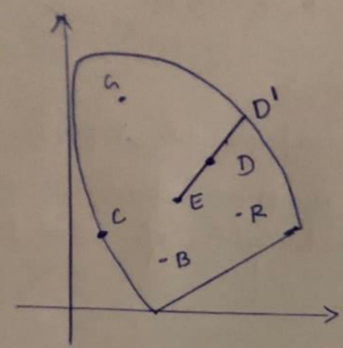
$$= \frac{160}{103}$$

v) Number of bits = 103

Theory Problem 3: Color Theory

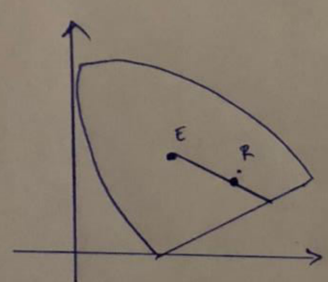
Problem 3)

i)



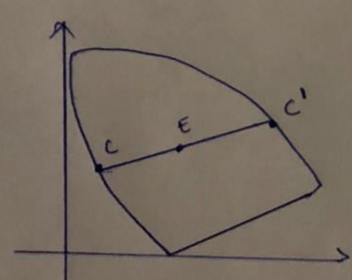
The wavelength represented by D' is the dominant wavelength of Color D.

ii)



NO. All colors do not have a dominant wavelength. for eg. R is non-spectral since the intersection of the rays hit the boundary in the flat part.

iii)



The wavelength represented by C' is the complimentary color of C