MA202 Project

Unsteady heat conduction equation in Cartesian coordinate using the Crank-Nicolson scheme.

Project 2

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1 Problem Statement

The temperature distribution in a long solid bar of length L is governed by the following equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

where t is time, T(x, t) is temperature and α is called the thermal diffusivity of the material. The above equation is known as the unsteady heat diffusion equation, which governs transient heat transfer only along one direction. The above equation is subject to the following initial condition and boundary conditions:

$$T(x\ ,\ 0) = 300\ K \tag{2}$$

$$T(0\ ,\ t) = 350\ K \tag{3}$$

,

$$\left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \tag{4}$$

Given that

$$\alpha = 1.2 \times 10^{-4}\ m^2/s \tag{5}$$

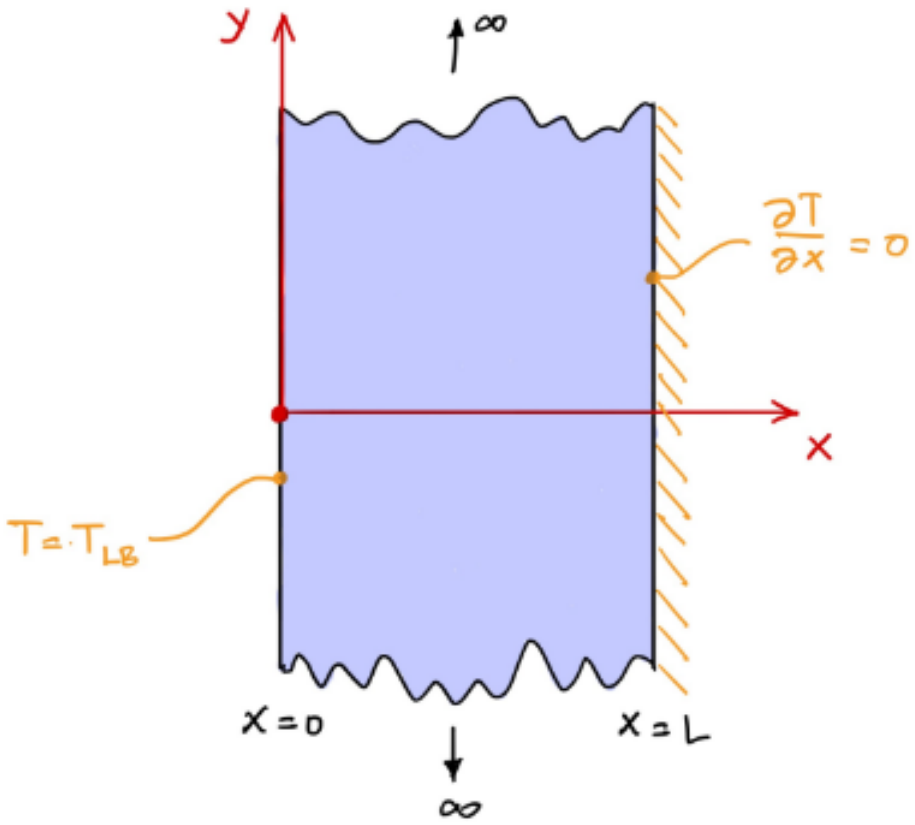

Figure 1: Schematic for the problem

2 Background

The Crank-Nicolson method is a widely used numerical method for solving partial differential equations, particularly those that describe time-dependent phenomena. It is a finite difference method that approximates the solution to a partial differential equation at discrete points in both space and time. The method is known for its accuracy and stability and is particularly useful for problems that require long time integration. The Crank-Nicolson method is a central difference scheme that uses the average of the values of the dependent variable at the current time level and the next time level to approximate the time derivative. This gives rise to a system of equations that can be solved iteratively using numerical techniques such as matrix inversion. The method provides a stable solution, even for stiff problems, and is unconditionally stable for linear problems. The Crank-Nicolson method has found wide applications in various fields of science and engineering. It is commonly used to solve problems in fluid dynamics, heat transfer, electromagnetics, and quantum mechanics. In fluid dynamics, the method is used to simulate the flow of fluids in complex geometries and to study the behaviour of turbulence. In heat transfer, the method is used to simulate the temperature distribution in materials and to design efficient cooling systems. In electromagnetics, the method is used to study the propagation of electromagnetic waves in complex structures, such as antennas and waveguides. In quantum mechanics, the method is used to study the behaviour of quantum systems and to solve the Schrödinger equation. The Crank-Nicolson method has several advantages over other numerical methods for solving partial differential equations. The method is accurate, providing solutions that are close to the exact solution of the problem. Additionally, the method is easy to implement and can be used to solve problems with complex geometries. In summary, the Crank-Nicolson method is a powerful numerical method for solving partial differential equations that has found wide applications in various fields of science and engineering. Its accuracy, stability, and ease of implementation make it a popular choice for researchers and practitioners in these fields.

3 Solution

In the code, we first define the geometry and the x grid parameters, such as the length of the bar and the number of spatial divisions based on the spatial step size. We also define the material properties, such as the thermal diffusivity.

Next, we define the CrankNicolson function, which defines the time grid parameters and the initial conditions for the temperature distribution. It then performs the simulation by iterating over time and updating the temperature distribution using the Crank-Nicolson scheme. The boundary conditions are also defined within this function.

```
def CrankNicolson(t):
    for times in t:
        # time grid
        t_end = times  # in s
        t_step = 0.05  # in s
        nt = int(t_end/t_step)
        # initial value
        T_i = 300  # in K

        # define results parameter
        T = np.ones(x_divisions)*T_i

        # simulation
        for n in range(1, nt):
            Tn = T.copy()

            T[1:-1] = Tn[1:-1] + t_step * alpha * \
                (Tn[2:] - 2*Tn[1:-1] + Tn[0:-2])/(x_step)**2

            # defining boundary conditions
            T[0] = 350  # in K

            # gradient is zero at x=L so Last T value gets copied to last but one T value
            T[-1] = T[-2]

        #plotting the results
        plt.plot(x, T)
```

Figure 2: Code Snippet of our implementation

Taking Endtime from the user as an input and dividing it into 20 equal divisions, the program plots the T graph at each interval. The output is 20 graphs following the initial and boundary conditions.

Equation for finite size method in Crank-Nicolson Scheme is given by

$$T_i^{n+1} = T_i^n + \frac{\alpha \Delta t}{2\Delta x^2} \left(T_{i-1}^n - 2T_i^n + T_{i+1}^n \right) \tag{6}$$

Overall, this code is a useful tool for simulating and visualizing the temperature distribution within a solid bar as heat is transferred through it.

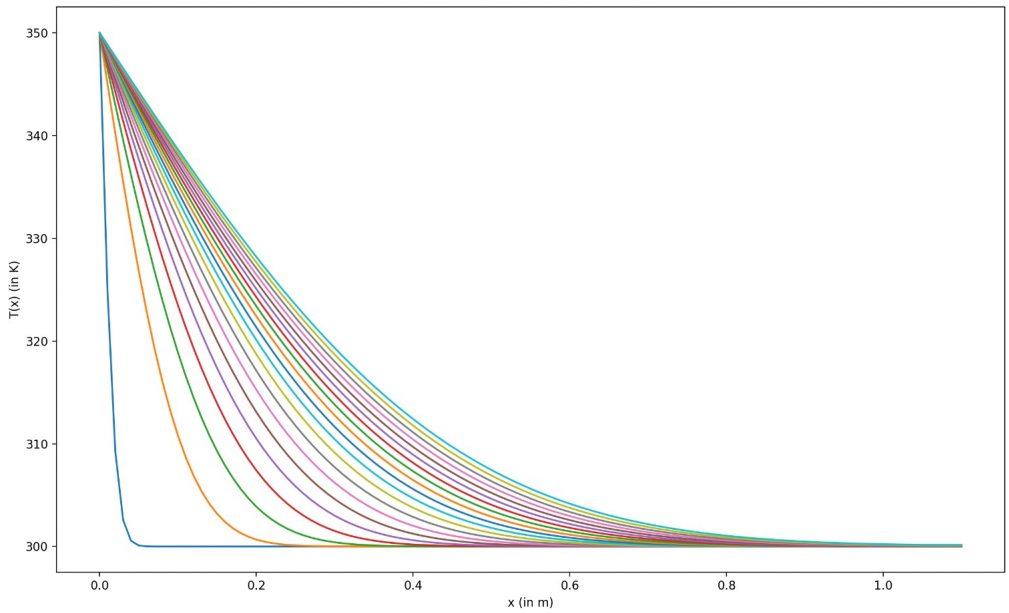

Figure 3: T vs x when plot τ_{end} =500s