Research

Simple spherical object motion:

- kinematic equations

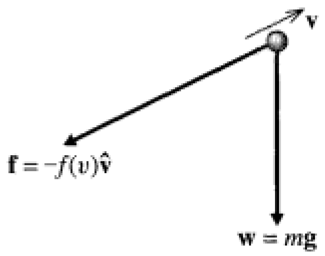


Figure 2.1 A projectile is subject to two forces, the force of gravity, $\mathbf{w} = m\mathbf{g}$, and the drag force of air resistance, $\mathbf{f} = -f(v)\hat{\mathbf{v}}$.

$$
\begin{gathered}
\vec{f} = -f(v)\hat{v} \\
\hat{v} = \vec{v}/|\vec{v}| \\
f(v) = bv + cv^2 = f_{\text{lin}} + f_{\text{quad}} \\
b = \beta D \\
c = \gamma D^2
\end{gathered}
$$

for spherical projectile in air at STP, they have approximate values:

$$\beta = 1.6 \times 10^{-4} N \cdot s/m^2$$

and

$$\gamma = 0.25 \times 10^{-4} N \cdot s^2/m^4$$

$$f_{\text{quad}}/f_{\text{lin}} = \frac{\gamma}{\beta} Dv$$

Reynold's number

$$
\begin{gathered}
Re = \frac{\text{inertial force}}{\text{viscous force}} \\
Re = \frac{\rho v L}{\mu}
\end{gathered}
$$

a high number or ratio indicates laminar flow and a small number indicates laminar flow

Archery arrow aerodynamics

In the experiment, they used two types of arrow heads: one that looks like a bullet, and the other looks like the tip of a sharp pencil

Fig. 1 Archery arrow A/C/E with the bullet and streamlined points

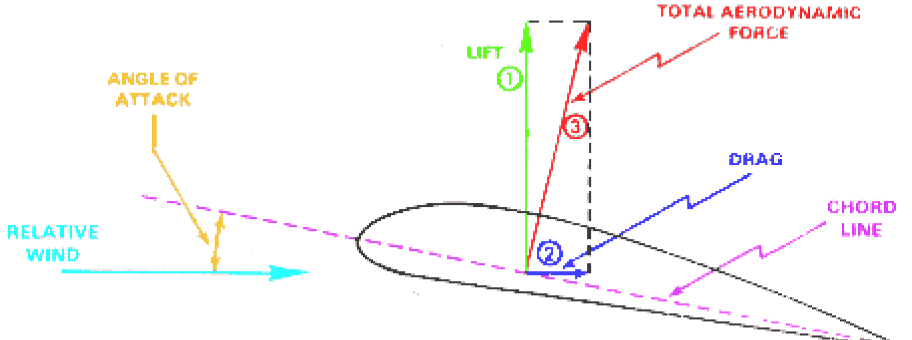
$$Re = \frac{\rho v D}{\mu}$$
$$F_D = \frac{\pi}{8} C_D \rho U^2 D^2$$
$$F_L = \frac{\pi}{8} C_L \rho U^2 D^2$$
$$M = \frac{\pi}{8} C_M \rho U^2 D^2 L$$

D is the diameter of the arrow shaft and U is arrow velocity
C_M is the pitching moment coefficient? What the hell is that?

> In aerodynamics, the pitching moment on an airfoil is the moment (or torque) produced by the aerodynamic force on the airfoil if that aerodynamic force is considered to be applied, not at the center of pressure, but at the aerodynamic center of the airfoil.

aerodynamic force:

- An aerodynamic force is a force exerted on a body by the air in which the body is immersed, and is due to the relative motion between the body and the gas.



- The maximum velocity they did in the experiment (that they could do) was 45 m/s while the typical launching velocity of an arrow is 60 m/s
- measurements are performed in Re ranges from $0.4 \times 10^4 < Re < 1.5 \times 10^4$
- nvm, they did manage 62 m/s with relatively low percent error
- nock yields no substantial difference
- C_D is dependent on Re

what in the heck is a spin parameter?

$$S_p = \frac{\pi D f}{U_2}$$

- the arrows rotate as it flies due to aerodynamic effect of the vanes. What is a vane?
- rotation rate is proportional to arrow velocity
- shape is irrelevant in this case, not necessarily... but between the pencil head and bullet head
- no lift nor moment when angle of attack is zero, but once it is... the coefficients follow:
- nice.

and the open triangles are for the streamlined point. The lift coefficient C_L is independent of the point shape, and the result for the streamlined point is shown. We notice that C_L is proportional to θ with a positive inclination about 0.69 deg^{-1}, indicating that the lift cannot be neglected even if the angle of attack is very small. Nevertheless our two data-analysis methods in the flight experiments, which assume no lift, provide the same values of C_D.

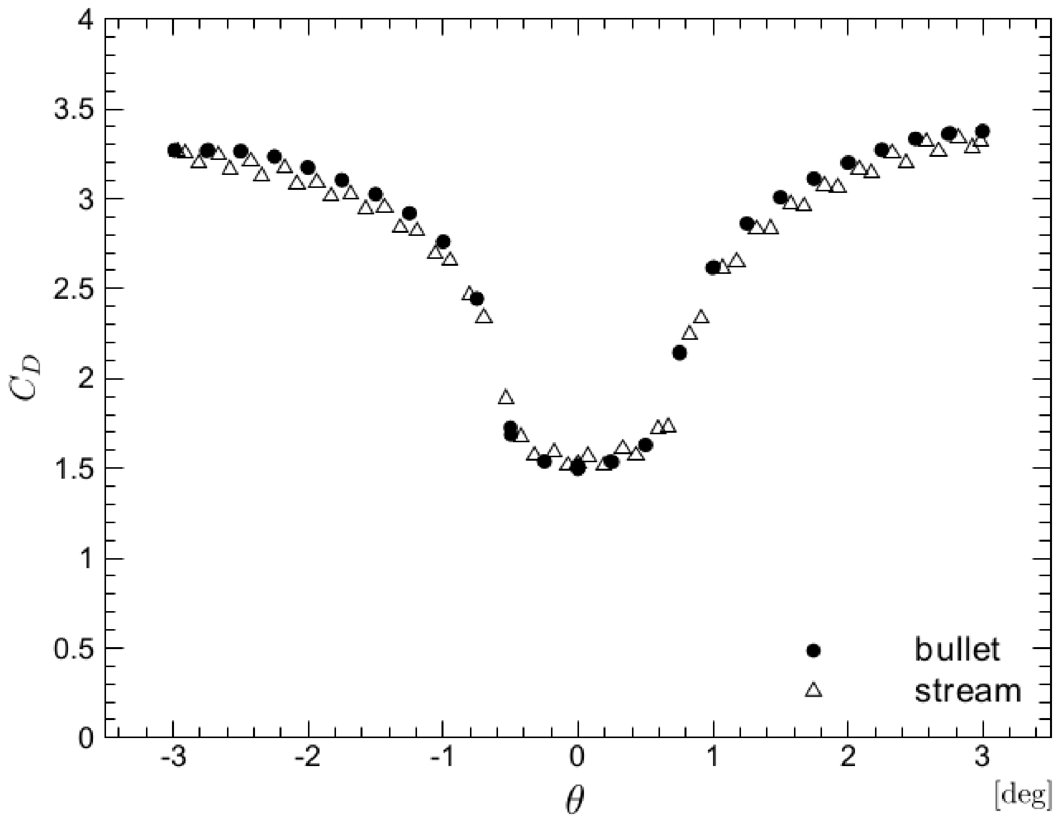


Fig. 13 C_D as a function of the angle of attack θ at $Re = 1.0 \times 10^4$

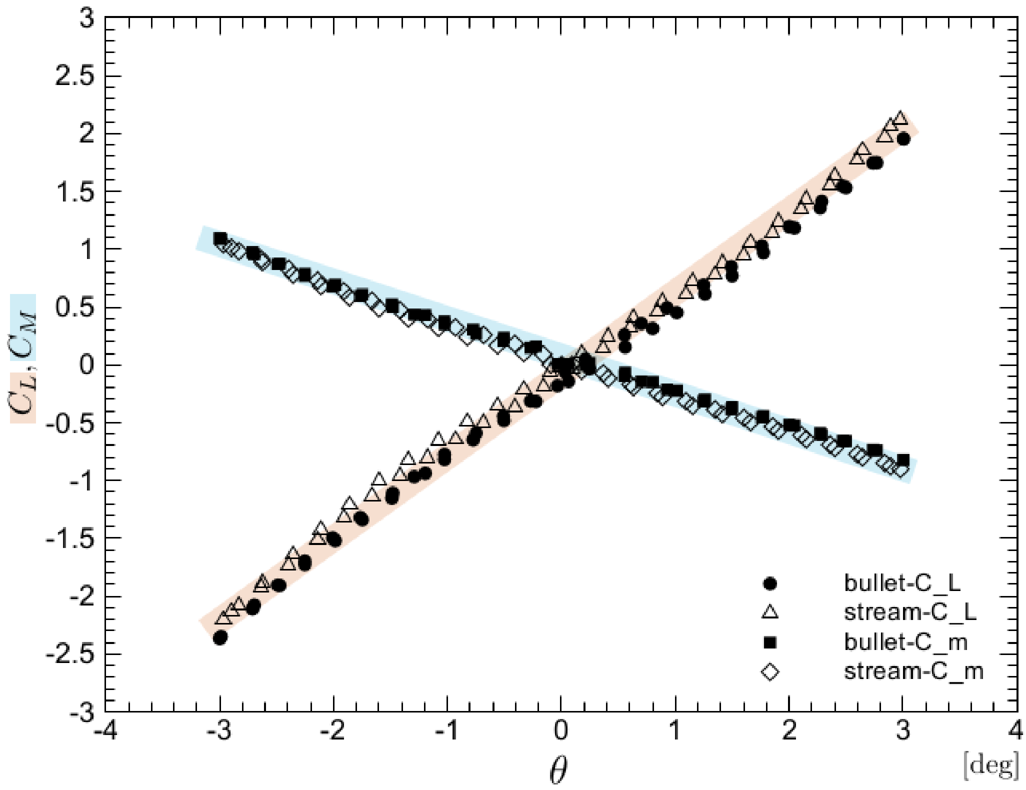


Fig. 14 C_L as a function of the angle of attack θ at $Re = 1.1 \times 10^4$

tionally, two values of C_D are found at $Re = 1.8 \times 10^4$. When the streamlined point is attached to the arrow nose, two values are found for $Re = 1.3 \times 10^4$ and the lower value is about 1.6 (laminar boundary layer) and the larger value is about 2.6 (turbulent boundary layer). The drag coefficient of the arrow launched by an expert archer is about 2.65 at $Re = 2.0 \times 10^4$ (58 m/sec). The reason why two values are observed in the flight experiments can be attributed to the insufficient control of the attitude of flying arrows. We show by MSBS wind tunnel tests that a very small angle of attack θ can induce the boundary layer transition even at $Re = 1.0 \times 10^4$. Similarly, the oscillation of the arrow from a bow makes the boundary layer turbulent.

6.1 Method 1: using the horizontal velocity components

Under the previous assumption, the equations of motion are written as,

$$\frac{du}{dt} = -\frac{1}{2}\hat{D}u\sqrt{u^2 + w^2}, \tag{3}$$

$$\frac{dw}{dt} = -g - \frac{1}{2}\hat{D}w\sqrt{u^2 + w^2}, \tag{4}$$

$$\frac{ds}{dt} = \sqrt{u^2 + w^2}. \tag{5}$$

Here, u and w denote the horizontal and vertical velocity components, respectively. The horizontal and vertical coordinates are x and z, respectively, and s means the length along the trajectory. The velocity decay rate $\hat{D}$ (m^{-1}) is linked with the drag coefficient C_D as

$$C_D = \frac{4m\hat{D}}{\rho\pi D^2}. \tag{6}$$

Here, m denotes the mass of the arrow.

We can eliminate the time variable t from (3) and (5):

$$\frac{du}{ds} = -\frac{1}{2}\hat{D}u. \tag{7}$$

It is solved to give a simple relation:

$$u = u_1 \exp\left(-\frac{1}{2}\hat{D}s\right). \tag{8}$$

Miyazaki, T., Mukaiyama, K., Komori, Y., Okawa, K., Taguchi, S., & Sugiura, H. (2013). Aerodynamic properties of an archery arrow. Sports Engineering, 16(1), 43-54.