Exercise 01.

1. Given:

$A = 3\,mm \quad \widehat{=}\ C_2$

$f = 30$ cycle/s

Find:

a) max. velocity $= \dot{y}_{max}$

b) max. acceleration $= \ddot{y}_{max}$

Solution:

a) Displacement response of a SHM yields

$$y(t) = C_1 \cos \omega_n t + C_2 \sin \omega_n t \quad \ldots\ (1)$$

$$f = \omega / 2\pi \quad \ldots\ (2)$$

$$\omega = 2\pi f$$

$$\therefore \omega = 2\pi\,(30\ \text{cycle/s}) = 60\pi\ \text{rad/s} \quad \ldots\ (3)$$

Combine Equations (1) & (3)

$$y(t) = C_1 \cos 60\pi t + C_2 \sin 60\pi t$$

when $t = 0$; $y(0) = 0$

$$0 = C_1 \underbrace{\cos 0}_{1.0} + C_2 \overbrace{\sin 0}^{0}$$

$$\therefore C_1 = 0$$

$$y(t) = C_2 \sin 60\pi t \quad \text{.... (4)}$$

Differentiating

$$\dot{y}(t) = C_2(60\pi) \cos[60\pi t] \quad \text{... (5)}$$

NB: MAX. VELOCITY OCCURRED WHEN THE DISPLACEMENT IS 0;

I.E. WHEN $t=0$. USING EQN (5)

$$\dot{y}_{max} = C_2(60\pi) \cos[60\pi(0)] \quad ; \quad C_2 = 3\ mm$$

$$\dot{y}_{max} = 3\ \cancel{mm}(60\pi)(1)[1m/1000\ \cancel{mm}]$$

$$\therefore \dot{y}_{max} = 0.56549\ m/s$$

b) max. acceleration $\Rightarrow \ddot{y}_{max}$

MAX. ACCELERATION OCCURRED WHEN YOU HAVE THE MAX. DISPLACEMENT. USING EQN (4)

$$y(t) = C_2 \sin 60\pi t \quad \text{where } y(t) = C_2$$

$$\cancel{C_2} = \cancel{C_2} \sin 60\pi t$$

$$t = \frac{1}{60\pi} \sin^{-1}[1]$$

$$\therefore t = 0.477465\ s.\ °/rad$$

DIFFERENTIATE EQN (5)

$$\ddot{y}(t) = -C_2(\omega_n)^2 \sin[\omega_n t]$$

$$\ddot{y}_{max} = -3\cancel{mm}[60\pi]^2 \sin[60\pi(0.477465\ s\text{-}°/rad)]$$

$$\times\ 1m/1000\ \cancel{mm}$$

$$\therefore \ddot{y}_{max} = -106.59\ m/s^2$$

2. Given:

$\dot{y}_{max} = 3.2\ m/s$

$T = 0.15\ s$

Find:

a) Amplitude of Motion

b) Max. acceleration

Solution:

a) $T = \dfrac{1}{f}$ .... (1)

$\therefore f = 1/T = 1/0.15\ s = 6.667\ cycle/s$

$\omega = 2\pi f$ .... (2)

$\therefore \omega = 2\pi(6.667\ cycle/s) = 41.8879\ rad/s$

THE DISPLACEMENT RESPONSE GIVES

$y(t) = C_1 \cos w_n t + C_2 \sin w_n t$ WHEN $t=0$; $y=0$

$0 = C_1 \cancel{\cos} 0 + C_2 \cancel{\sin} 0$

$1 \qquad 0$

$\therefore C_1 = 0$

$y(t) = C_2 \sin w_n t$ ... (3)

DIFFERENTIATE EQN (3)

$\dot{y}(t) = C_2 w_n \cos[w_n t]$ ... (4)

$3.2 m/s = C_2 [41.8879 \text{ rad/s}] \cos[41.8879 (0)]$

$\therefore C_2 = \dfrac{3.2 m/s}{41.8879 \frac{rad}{s} \cos 0} = 0.07639 m$

amplitude.

b) DIFFERENTIATE EQN (4)

$\ddot{y}(t) = -C_2 w_n^2 \sin[w_n t]$ ... (5)

USING THE DISPLACEMENT EQN (3)

$y(t) = C_2 \sin w_n t$ where: $y(t) = C_2$

$\cancel{C_2} = \cancel{C_2} \sin w_n t$

$1 = \sin w_n t$

$t = \dfrac{1}{w_n} \sin^{-1}[1]$

$$t = \frac{1}{41.8879\ rad/s} \sin^{-1}[1]$$

$$\therefore t = 2.1486\ s \cdot {}^{\circ}/rad$$

USE EQN (5)

$$\ddot{y}(t) = -C_2 W_n^2 \sin[W_n t]$$

$$\ddot{y}(t) = -0.07639\ m\left[41.8879\ \frac{rad}{s}\right]^2 \times \sin\left[41.8879\ \frac{\cancel{rad}}{\cancel{s}} \times 2.1486\ \cancel{s}\ {}^{\circ}/ra\cancel{d}\right]$$

$$\therefore \ddot{y}_{max} = -134.04\ \frac{m}{s^2}$$