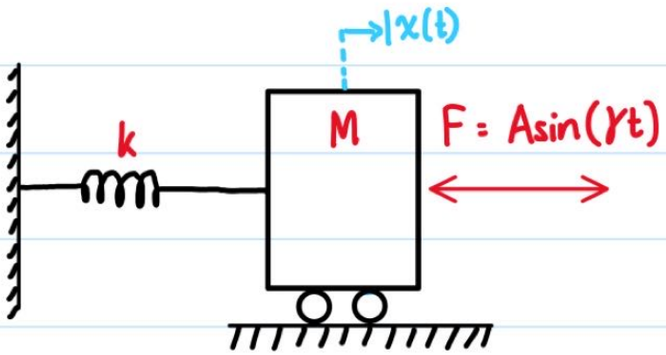
VIBRATION ENGINEERING

LONG EXAM 3 NOTES

Non-conservative Systems

Saturday, March 23, 2024 9:16 AM



* let:

		initial:
$M = 1$	$A = 15$	$x(0) = 0$
$k = 16$	$\gamma = 1$	$\dot{x}(0) = \sqrt{3}$

Find displacement & velocity.

~ identify T, V, & F_i:

$$T = \frac{1}{2} M \dot{x}^2$$

$$V = \frac{1}{2} k (x - \cancel{x_0})^2 = \frac{1}{2} k x^2$$

$$F_i = A \sin(\gamma t)$$

~ use LEM: ($q_i = x$)

$$\frac{\partial}{\partial t}\left[\frac{\partial T}{\partial \dot{x}}\right] - \cancel{\frac{\partial T}{\partial x}} + \frac{\partial V}{\partial x} = Q_i = F_I - \cancel{\frac{\partial \mathcal{F}}{\partial \dot{x}}}$$

$$\frac{\partial}{\partial t}\left[\frac{\partial T}{\partial \dot{x}}\right] = \frac{\partial}{\partial t}\left[\frac{1}{\cancel{2}}(\cancel{2}) M \dot{x}\right] = \underline{M \ddot{x}}$$

$$\frac{\partial V}{\partial x} = \frac{1}{2}(2) k x = \underline{k x}$$

$$F_i = A \sin(\gamma t)$$

$$\left[M \ddot{x} + k x = A \sin(\gamma t)\right] \frac{1}{M}$$

$$\ddot{x} + \frac{k}{M} x = \frac{A}{M} \sin(\gamma t)$$

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$$\ddot{x} + \frac{k}{M}x = \frac{A}{M}\sin(\gamma t)$$

$$\boxed{\ddot{x} + \omega_n^2 x = \frac{A}{M}\sin(\gamma t)} \; ; \; \omega_n^2 = \frac{16}{1} \; , \; \underline{\omega_n = 4}$$

~ let $F(x(t)) = 0$

$$M^2 + \omega_n^2 = 0$$

$$M = \pm \omega_n i \dashrightarrow \text{complex root}$$

$$\therefore \; x_c = c_1\cos(\omega_n t) + c_2\sin(\omega_n t) \; (4)$$

$$x_p = c_3\cos(\gamma t) + c_4\sin(\gamma t) \; (5)$$