FLUID
MACHINERY

Hello!

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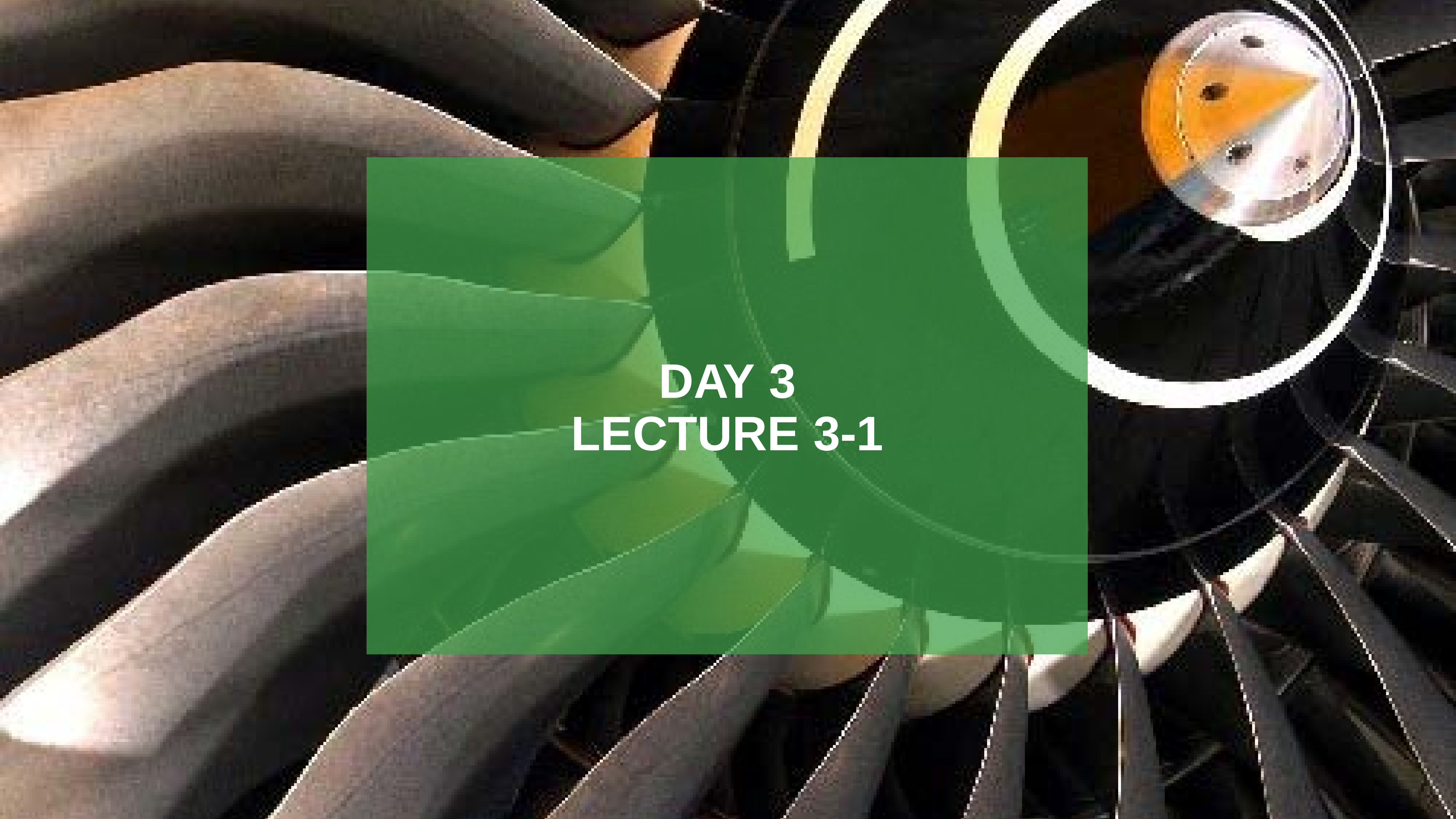


DAY 3
LECTURE 3-1

TOPICS

- Contents of Fluid Machinery
- Dimensionless Coefficients
- Specific Speed
- Similarity Rules
- Centrifugal Pumps
- Pump Head
- Cavitation
- Net Positive Suction Head

LEARNING OBJECTIVES

After completing this study, you should be able to:

- Determine the dimensionless characteristics of centrifugal pump.
- Derive the Affinity Laws using Buckingham Π Theorem
- Identify the pump specific speed.
- Apply similarity laws for pumps.
- Select an appropriate class of pump for a particular application.

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

- Centrifugal pump characteristics (i.e., specific energy gH, power P, and efficiency η curves plotted vs. flow capacity Q) at different pump operating speeds are important to pumps best efficiency points during part load and full load conditions.
- The pump characteristics are treated in a dimensionless form. To complete the dimensional analysis successfully, the most important condition is to find all variables influencing the system studied.

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

- Based on the list of influencing variables, it is possible to construct the so-called dimensional matrix. This matrix is divided into two parts: quadratic core matrix and residual matrix.
- By rearranging the core matrix to the unity matrix and using the same arithmetic operations for the residual matrix, one gets dimensionless numbers corresponding to the system of interest.
- In the case of centrifugal pump characteristics, the chosen variables are specific energy of pump (gH) and efficiency η.

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

- Then, all independent variables influencing the target variable should be determined and the so-called relevance list of variables must be formed.
- For the chosen centrifugal pump the following independent variables are pump rotational speed n, volumetric flow rate Q, density ρ, and dynamic viscosity of pumped liquid μ.
- Besides this, the diameter of pump impeller is also chosen as a characteristic length. Mathematical formulation of the functional dependence between target variable and independent variables for the centrifugal pump is given by the following equations

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

$$F(gH, Q, \mu, \rho, D, n) = 0 \qquad \textbf{Eq. 3-1a}$$
$$G(\eta, Q, \mu, \rho, D, n) = 0 \qquad \textbf{Eq. 3-1b}$$

Using Eq. 3-1a, the dimensional matrix consists of a core and residual matrices which can be expressed as

$$\begin{array}{c|ccc|ccc} & \rho & D & n & gH & Q & \mu \\ M/kg & 1 & 0 & 0 & 0 & 0 & 1 \\ L/m & -3 & 1 & 0 & 2 & 3 & -1 \\ T/s & 0 & 0 & -1 & -2 & -1 & -1 \end{array}$$

core matrix residual matrix

Multiplying the 1st row by 3 then, add the 2nd row then the new equation yields

$$\begin{array}{c|ccc|ccc} & \rho & D & n & gH & Q & \mu \\ M/kg & 1 & 0 & 0 & 0 & 0 & 1 \\ L/m & 0 & 1 & 0 & 2 & 3 & 2 \\ T/s & 0 & 0 & -1 & -2 & -1 & -1 \end{array}$$

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

The dimensional matrix reveals that for 6 dimensional variables only three (3) dimensions (M,L,T) exist. Then, three (3) independent dimensionless variables may be obtained. However, it is necessary to recalculate the above-mentioned matrix by linear operations, the result of which is to transform the core matrix to a unity matrix.

$$\begin{array}{c|ccc|ccc} & \rho & D & n & gH & Q & \mu \\ M/kg & 1 & 0 & 0 & 0 & 0 & 1 \\ L/m & 0 & 1 & 0 & 2 & 3 & 2 \\ T/s & 0 & 0 & 1 & 2 & 1 & 1 \end{array} \qquad \textbf{Eq. 3-2}$$

core matrix residual matrix

Extracting the dimensionless numbers resulting from the dimensional analysis may now be defined as

$$\pi_1 = \frac{gH}{n^2 D^2}; \quad \pi_2 = \frac{Q}{nD^3}; \quad \pi_3 = \frac{\mu}{\rho n D^2} \qquad \textbf{Eq. 3-3}$$

$$\frac{gH}{n^2 D^2} = F_1\left(\frac{Q}{nD^3}, \frac{\mu}{\rho n D^2}\right) \qquad \textbf{Eq. 3-4}$$

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

The three (3) equations presented in Eq. 3-3 are the so-called head coefficient, flow coefficient and inverse of Reynolds number, respectively. This simply indicates that the head coefficient is a function of flow coefficient and Reynolds number.

Evaluating Eq. 3-1b, the dimensional matrix consisting of a core and residual matrices can be rewritten as

$$
\begin{array}{c|ccc|ccc}
 & \rho & D & n & \eta & Q & \mu \\
M/kg & 1 & 0 & 0 & 0 & 0 & 1 \\
L/m & -3 & 1 & 0 & 0 & 3 & -1 \\
T/s & 0 & 0 & -1 & 0 & -1 & -1
\end{array}
$$

core matrix residual matrix

Simplifying the above matrix yields

$$
\begin{array}{c|ccc|ccc}
 & \rho & D & n & \eta & Q & \mu \\
M/kg & 1 & 0 & 0 & 0 & 0 & 1 \\
L/m & 0 & 1 & 0 & 0 & 3 & 2 \\
T/s & 0 & 0 & 1 & 0 & 1 & 1
\end{array}
$$

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

The dimensionless numbers derived from the dimensional analysis can be expressed as

$$\pi_1 = \eta; \quad \pi_2 = \frac{Q}{nD^3}; \quad \pi_3 = \frac{\mu}{\rho n D^2} \qquad \textbf{Eq. 3-5}$$

$$\eta = G_1\left(\frac{Q}{nD^3}, \frac{\mu}{\rho n D^2}\right) \qquad \textbf{Eq. 3-6}$$

Buckingham Π Theorem:

Another method of identifying the dimensionless characteristics of centrifugal pump is by using the Buckingham Π Theorem. One of the applications is deriving the pump performance curve that would represent a set of similar pumps (Augusto et al. 2013).

Dimensional analysis with a generalized approach commonly known as Buckingham Π Theorem was adopted to derive pertinent parameters and to predict the performance of pump model under selected conditions of operation.

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

When the length scale was neglected the following variables were used.

$$f\left(h, Q, n, D, \varepsilon, \rho, \mu, T, \eta, g\right) = 0 \qquad \textbf{Eq. 3-7}$$

The dimensionless parameters can be expressed as

$$\theta\left(\frac{h}{D}, \frac{T}{\rho n^2 D^5}, \frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\varepsilon}{D}, \eta, \frac{g}{n^2 D}\right) = 0 \qquad \textbf{Eq. 3-8}$$

Simplifying the above parameters, the new functional relations yield the head coefficient, torque coefficient and efficiency as shown in Equations 3-9a, 3-9b and 3-9c, respectively.

$$C_H = \frac{gh}{n^2 D^2} = \theta_1\left(\frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\varepsilon}{D}\right) \qquad \textbf{Eq. 3-9a}$$

$$C_T = \frac{T}{\rho n^2 D^5} = \theta_2\left(\frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\varepsilon}{D}\right) \qquad \textbf{Eq. 3-9b}$$

$$\eta = \theta_3\left(\frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\varepsilon}{D}\right) \qquad \textbf{Eq. 3-9c}$$

DIMENSIONLESS CHARACTERISTICS OF CENTRIFUGAL PUMP

Subsequently, based on the above-mentioned equations, the Affinity Laws for geometrically similar pumps such as head, torque and volume flow rate were derived as shown in Equations 3-10a, 3-10b, and 3-10c.

$$\left[\frac{gh}{n^2D^2}\right]_1 = \left[\frac{gh}{n^2D^2}\right]_2$$ **Eq. 3-10a**

$$\left[\frac{T}{\rho n^2D^5}\right]_1 = \left[\frac{T}{\rho n^2D^5}\right]_2$$ **Eq. 3-10b**

$$\left[\frac{Q}{nD^3}\right]_1 = \left[\frac{Q}{nD^3}\right]_2$$ **Eq. 3-10c**

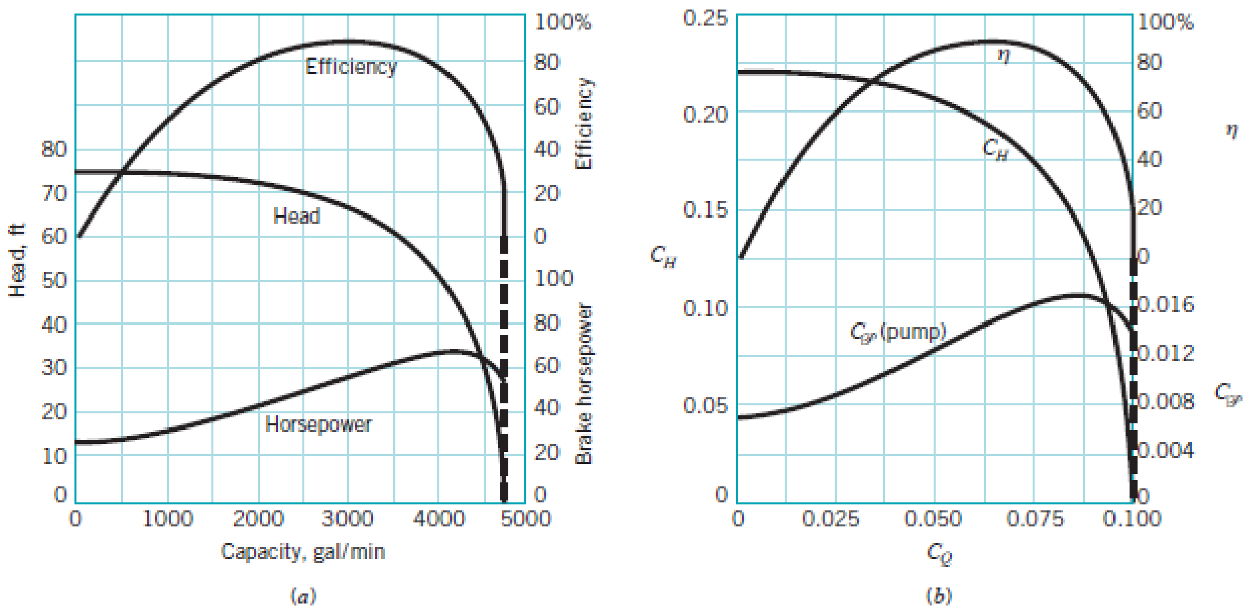
Simplifying Eq. 3-10b into shaft power then,

$$\left[\frac{2\pi T}{\rho n^2D^5}\left(\frac{n}{n}\right)\right]_1 = \left[\frac{2\pi T}{\rho n^2D^5}\left(\frac{n}{n}\right)\right]_2$$

$$\left[\frac{\dot{W}_1}{\rho n^3D^5}\right]_1 = \left[\frac{\dot{W}_2}{\rho n^3D^5}\right]_2$$ **Eq. 3-10d**

PROBLEM 301

An 8-in diameter centrifugal pump operating at 1200 rpm is geometrically similar to the 12-in diameter pump having the performance characteristics as shown below while operating at 1000 rpm. The working fluid is water at 60°F. For peak efficiency, predict the discharge, actual head rise and shaft horsepower for this smaller pump.



Typical performance data for a centrifugal pump: (a) characteristic curves for a 12-in centrifugal pump operating at 1000 rpm, (b) dimensionless characteristic curves.

SOLUTION

As indicated in Equation 3-9c, for a given efficiency the flow coefficient has the same value for a given family of pumps. Form Fig. b we see that at peak efficiency C_Q*=0.0625*. Thus, for the 8-in pump

$$Q = C_Q n D^3$$

$$Q = 0.0625\left(1200\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{8\text{ in}}{12\text{ in/ft}}\right)^3$$

$$Q = 2.327\frac{\text{ft}^3}{s}$$

The actual head rise and the shaft horsepower can be determined in a similar manner since at peak efficiency C_H*=0.19* and C_P*=0.014*, so that

$$h = C_H n^2 D^2 / g$$

$$h = 0.19\left[\left(1200\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2\left(\frac{8\text{ in}}{12\text{ in/ft}}\right)^2 \Bigg/ 32.2\frac{\text{ft}}{\text{s}^2}$$

$$h = 41.413\text{ ft}$$

SOLUTION

and using Equation 3-9b, deriving the power coefficient gives

$$C_T = \frac{T}{\rho n^2 D^5}$$

$$(2\pi)C_T = \frac{T}{\rho n^2 D^5}\left(\frac{n}{n}\right)(2\pi)$$

$$(2\pi)C_T = \frac{2\pi T n}{\rho n^3 D^5}$$

$$C_P = \frac{2\pi T n}{\rho n^3 D^5} \qquad \textbf{Eq. 3-11}$$

$$C_P = \frac{2\pi T n}{\rho n^3 D^5} = \frac{\dot{W}}{\rho n^3 D^5}$$

$$\dot{W} = C_P \rho n^3 D^5$$

$$\dot{W} = 0.014\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left[\left(1200\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\right]^3\left(\frac{8\text{ in}}{12\text{ in/ft}}\right)^5$$

$$\dot{W} = 7{,}097.46\text{ ft - lb/s} \times \frac{1\text{ hp}}{550\text{ ft - lb/s}} = 12.9045\text{ hp}$$

The last result gives the shaft horsepower, which is the power supplied to the pump shaft. The power actually gained by the fluid is equal to $\gamma Q h$ which is

$$P = \gamma Q h = 62.4\frac{\text{lb}}{\text{ft}^3}\left(2.327\frac{\text{ft}^3}{s}\right)(41.413\text{ ft}) = 6{,}013.37\text{ ft - lb/s}$$

SOLUTION

Thus, the efficiency, η is

$$\eta = \frac{P}{\dot{W}} = \frac{6{,}013.37}{7{,}097.46} = 84.73\%$$

SPECIAL PUMP SCALING LAWS

Two special cases related to pump similitude commonly arise. In the first case we are interested in how a change in the operating speed, ω, for a given pump, affects pump characteristics. It follows from Eq. 3-10c that for the same flow and therefore the same efficiency with $D_1 = D_2$ (the same pump)

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2}$$

Eq. 3-12a

The subscripts 1 and 2 now refer to the same pump operating at two different speeds at the same flow coefficient. Also, from Equations 3-10a and 3-10d it follows that

$$\frac{h_1}{h_2} = \left[\frac{\omega_1}{\omega_2}\right]^2$$

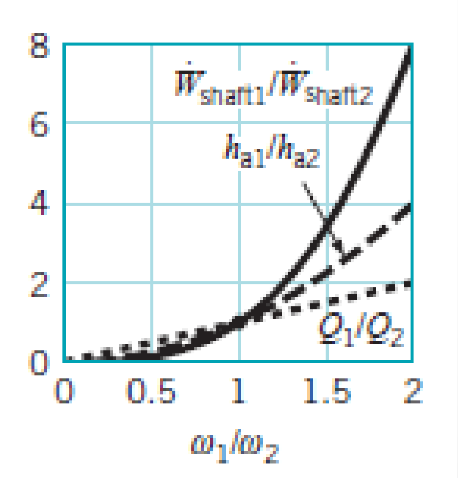
Eq. 3-12b

SPECIAL PUMP SCALING LAWS

and

$$\frac{\dot{W}_1}{\dot{W}_2} = \left[\frac{\omega_1}{\omega_2}\right]^3 \qquad \textbf{Eq. 3-12c}$$

Thus, for a given pump operating at a given flow coefficient, the flow varies directly with speed, the head varies as the speed squared, and the power varies as the speed cubed. These effects of angular velocity variation are illustrated in the sketch.



The scaling laws are useful in estimating the effects of changing pump speed when some data are available from a pump test obtained by operating the pump at a particular speed.

SPECIAL PUMP SCALING LAWS

In the second special case we are interested in how a change in the impeller diameter *D*, of a geometrically similar family of pumps, operating at a given speed, affects pump characteristics. It follows that for the same flow coefficient with $\omega_1 = \omega_2$ the equation yields

$$\frac{Q_1}{Q_2} = \left[\frac{D_1}{D_2}\right]^3$$

Eq. 3-13a

Similarly,

$$\frac{h_1}{h_2} = \left[\frac{D_1}{D_2}\right]^2$$

Eq. 3-13b

$$\frac{\dot{W}_1}{\dot{W}_2} = \left[\frac{D_1}{D_2}\right]^5$$

Eq. 3-13c

The pump similarity laws expressed by Equations 3-12a through 3-13c are sometimes referred to as the ***pump affinity laws***.

MOODY STEP-UP FORMULA

The effects of viscosity and surface roughness have been neglected in the foregoing similarity relationships. However, it has been found that as the pump size decreases these effects more significantly influence efficiency because of smaller clearances and blade size.

An approximate empirical relationship to estimate the influence of diminishing size on efficiency is

$$\frac{1-\eta_2}{1-\eta_1} \approx \left(\frac{D_1}{D_2}\right)^{1/5} \qquad \textbf{Eq. 3-14}$$

In general, it is to be expected that the similarity laws will not be very accurate if tests on a model pump with water are used to predict the performance of a prototype pump with a highly viscous fluid, such as oil, because at the much smaller Reynolds number associated with the oil flow. The fluid physics involved is different from the higher Reynolds number flow associated with water.

SPECIFIC SPEED

A useful pi term can be obtained by eliminating diameter D between the flow coefficient and the head rise coefficient. This is accomplished by raising the flow coefficient to an appropriate exponent (1/2) and dividing this result by the head coefficient raised to another appropriate exponent (3/4) so that

$$\frac{\left(Q/\omega D^3\right)^{1/2}}{\left(gh/\omega^2 D^2\right)^{3/4}} = \frac{\omega(\mathrm{rad/s})\sqrt{Q(\mathrm{m^3/s})}}{\left[g(\mathrm{m/s^2})\,h(\mathrm{m})\right]^{3/4}} = N_s \qquad \textbf{Eq. 3-15}$$

The dimensionless parameter N_s is called the ***specific speed***. Specific speed varies with flow coefficient just as the other coefficients and efficiency. However, for any pump it is customary to specify a value of specific speed at the flow coefficient corresponding to peak efficiency only.

Specific speed as defined by Eq. 3-15 is dimensionless, and therefore independent of the system units used in its evaluation as long as a consistent unit system is used.

SPECIFIC SPEED

However, in the United States a modified, dimensional form of specific speed, N_{sd}, is commonly used, where

$$N_{sd} = \frac{n(\mathrm{rpm})\sqrt{Q(\mathrm{gpm})}}{\left[h(\mathrm{ft})\right]^{3/4}} \qquad \textbf{Eq. 3-16}$$

In this case N_{sd} is said to be expressed in ***US customary units***. Typical values of N_{sd} are in the range of $500 < N_{sd} < 4000$ for centrifugal pumps. Both N_s and N_{sd} have the same physical meaning, but their magnitudes will differ by a constant conversion factor ($N_{sd} = 2733\ N_s$) when ω is expressed in rad/s.

For SI unit, the specific speed is ranging from $0.183 < N_s < 1.464$.

Each family or class of pumps has a particular range of values of specific speed associated with it. Thus, pumps that have low-capacity, high-head characteristics will have a specific speeds that are smaller than pumps that have high-capacity and low-head characteristics.

SPECIFIC SPEED

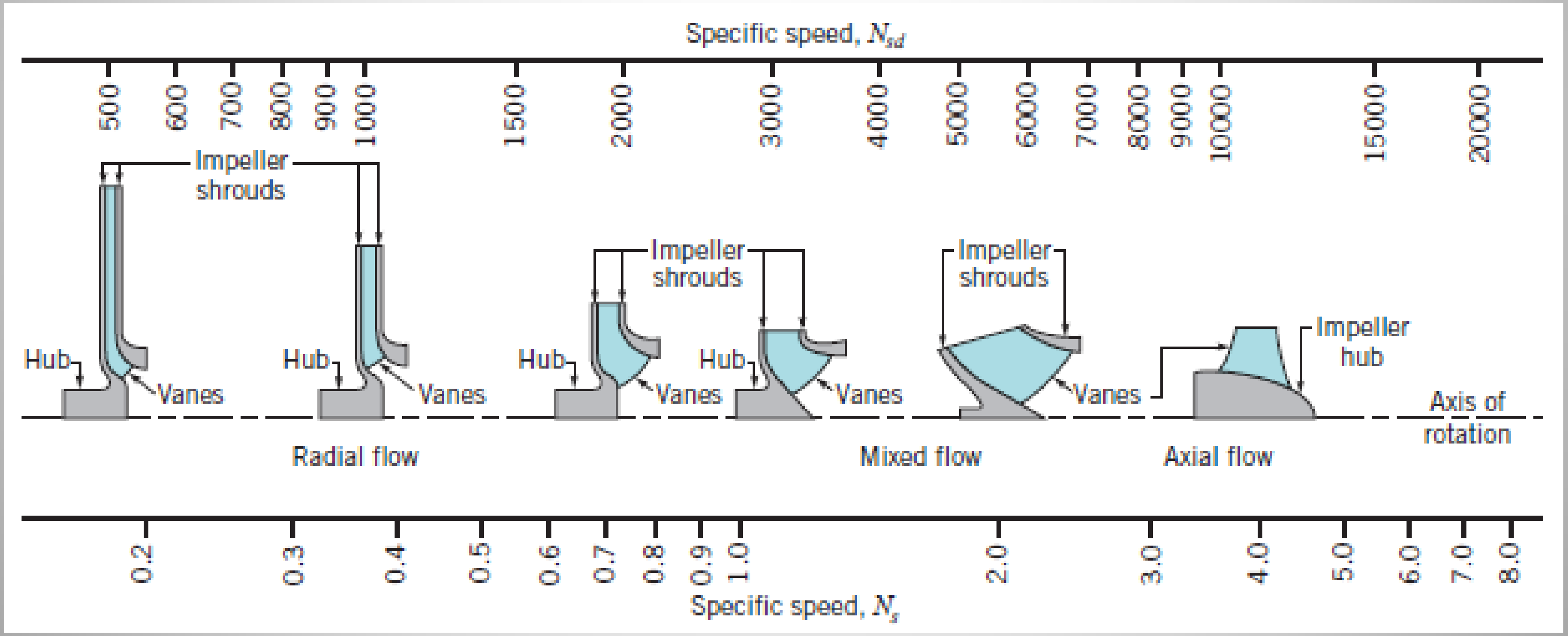
The concept of specific speed is very useful to engineers and designers, since if the required head, flowrate, and speed are specified, it is possible to select an appropriate and most efficient type of pump for a particular application.

For example as shown in the figure, as the specific speed, N_{sd} increases beyond about 2000 the peak efficiency, η of the purely radial flow centrifugal pump starts to fall off, and other types of more efficient pump design are preferred.

In addition to the centrifugal pump, the axial flow pump is widely used. In an axial flow pump the direction of flow is primarily parallel to the rotating shaft rather than radial as in the centrifugal pump. Axial flow pumps are essentially high capacity, low-head pumps, and therefore have large specific speeds ($N_{sd} > 9000$) compared to centrifugal pumps.

SPECIFIC SPEED

The figure below shows the variation in specific speed at maximum efficiency with different types of pump.



SPECIFIC SPEED

Another equation of specific speed is the one proposed by Daugherty, Franzini and Finnemore which can be found from the book entitled "Fluid Mechanics with Engineering Applications". The equation can be expressed as

$$N_S^* = \frac{n(\mathrm{rpm})\sqrt{Q(\mathrm{m}^3/s)}}{[h(\mathrm{m})]^{3/4}} \qquad \textbf{Eq. 3-17}$$

Also,

$$N_{sd} = 51.65771\, N_S^*$$