PROBLEM 308:

(a) Determine the specific speed of a pump that is to deliver 125 L/s against a head of 45 m with a rotative speed of 600 rpm. (b) If the rotative speed were doubled, what would be the flowrate and the head developed by this pump? Assume no change in efficiency. (c) Check the specific speed for the conditions given in (b). (d) Find the required operating speed of a two-stage pump to satisfy the requirements in (a).

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Solution:

a) Use Pump Specific Speed by Daugherty, Franzini & Finnemore

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$N_s^* = \frac{600\text{ rpm}\sqrt{0.125\text{ m}^3/s}}{(45\text{ m})^{3/4}}$$

$$N_s^* = 12.21$$

b) Calculating the flowrate and head if the rotating speed were doubled using Affinity Laws

$$\frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3}$$

$$Q_2 = Q_1\left[\frac{n_2}{n_1}\right]$$

$$Q_2 = 0.125\frac{\text{m}^3}{s}\left[\frac{1200}{600}\right] = 0.25\frac{\text{m}^3}{s}$$

$$\frac{gh_1}{n_1^2 D_1^2} = \frac{gh_2}{n_2^2 D_2^2}$$

$$h_2 = h_1\left[\frac{n_2}{n_1}\right]^2$$

$$h_2 = 45\text{ m}\left[\frac{1200}{600}\right]^2 = 180\text{ m}$$

PROBLEM 308:

c) Calculating the Pump Specific Speed for the conditions given in (b).

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$N_s^* = \frac{1200\text{ rpm}\sqrt{0.25\text{ m}^3/s}}{(180\text{ m})^{3/4}}$$

$$N_s^* = 12.21$$

d) Identifying the operating speed of a two-stage pump but satisfying the requirements in (a).

$$h_{new} = \frac{45\text{ m}}{2} = 22.5\text{ m}$$

Also,

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$n = \frac{N_s^* h^{3/4}}{\sqrt{Q}}$$

$$n = \frac{(12.21)(22.5\text{ m})^{3/4}}{\sqrt{0.125\text{ m}^3/s}} = 356.78\text{ rpm}$$

RESTRICTION ON USE OF SIMILARITY LAWS

Similarity laws are of great practical value, but care must be exercised when applying them. Thus in comparing two machines of different sizes, the two must be homologous and the variation in the values of *h*, *D* and *n* should not be too large.

For example, a machine which operates satisfactorily at low speeds may cavitate at high speeds. The values of coefficients change somewhat as *h*, *D* and *n* are varied because the efficiencies of homologous machines are not identical.

Large machines are usually more efficient than smaller ones because their flow passages are larger. Also, efficiency usually increases with speed of rotation because power output varies with the cube of the speed while mechanical losses increase only as the square of the speed.

PERIPHERAL VELOCITY FACTOR

For a pump impeller, the ratio of its peripheral velocity to $\sqrt{2gh}$ is referred to as the *peripheral-velocity factor*, denoted by ϕ. Thus, for a centrifugal pump,

$$u_2 = \phi\sqrt{2gh} \qquad \textbf{Eq. 3-20}$$

But for practical engineering use,

$$n = \frac{60u}{\pi D} = \frac{60\phi\sqrt{2gh}}{\pi D} \qquad \textbf{Eq. 3-21}$$

which may be reduced to the convenient form

$$nD = 84.6\phi\sqrt{h} \qquad \textbf{Eq. 3-22}$$

For any machine its peripheral velocity might be any value from zero up to some maximum under a given head, and ϕ would consequently vary through as wide a range.

But the speed which is of most practical significance is that at which the efficiency is a maximum. The value of this dimensionless factor for this particular speed may be designated as ϕ_e.

EXERCISE 302

1. All dimensions of pump A are one-third as large as the corresponding dimensions of pump B. When operating at 300 rpm, B delivers 6 L/s of water against a head of 15 m. Assuming the same efficiency: (a) What will be the speed and capacity of A when it develops a head of 15 m: (b) what will be the speed and head of A when it delivers 6 L/s; (c) what will be the head and capacity of A when it operates at 300 rpm?

2. A 45 cm-diameter centrifugal-pump runner discharges 0.7 cu.m/s at a head of 30 m when running at 1,200 rpm. (a) If its efficiency is 85%, what is the brake power? (b) If the same pump were run at 1,800 rpm, what would be *h*, *Q*, and brake power for homologous conditions?

3. What head will the pump of Problem 2 develop if it is operating on MARS at 1200 rpm and delivering 0.7 cu.m/s?

4. For the pump in Problem 2, what are the values of flow, head and power coefficients? What is the specific speed?

THANK YOU!

DAY 3
LECTURE 3-2

TOPICS

- Contents of Fluid Machinery
- Dimensionless Coefficients
- Specific Speed
- Similarity Rules
- Centrifugal Pumps
- Pump Head
- Cavitation
- Net Positive Suction Head

LEARNING OBJECTIVES

The objectives of this study are the following:

- Explain how and why a turbomachine works.
- Know the basic fundamentals of pump.
- Recognize the importance of minimizing loss in a turbomachine.
- Appreciate the basic fundamentals of sensibly scaling turbomachines that are larger or smaller than a prototype.
- Move on to more advanced engineering work involving the fluid mechanics of turbomachinery (e.g., design, development, research)

TURBOMACHINES

Pumps and turbines (often turbomachines) occur in a wide variety of configurations. In general, pumps add energy to the fluid – they do work on the fluid to move and / or increase the pressure of the fluid. However, turbines extract energy from the fluid – the fluid does work on them.

The term "pump" will be used to generically refer to all pumping machines, including *pumps*, *fans*, *blowers*, and *compressors*.

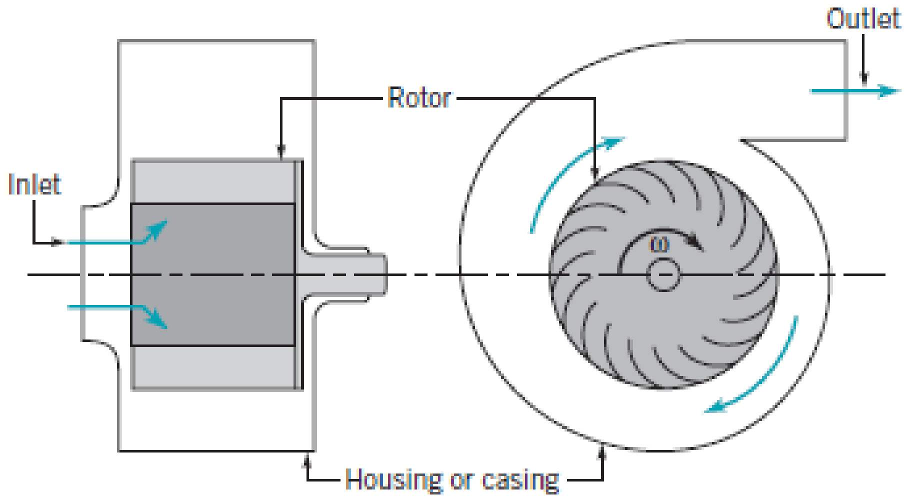
Turbomachines are mechanical devices that either extract energy from a fluid (turbine) or add energy to a fluid (pump) as a result of dynamic interactions between the device and the fluid.

A fluid that is moving can force rotation and produce shaft power. In this case we have a turbine.

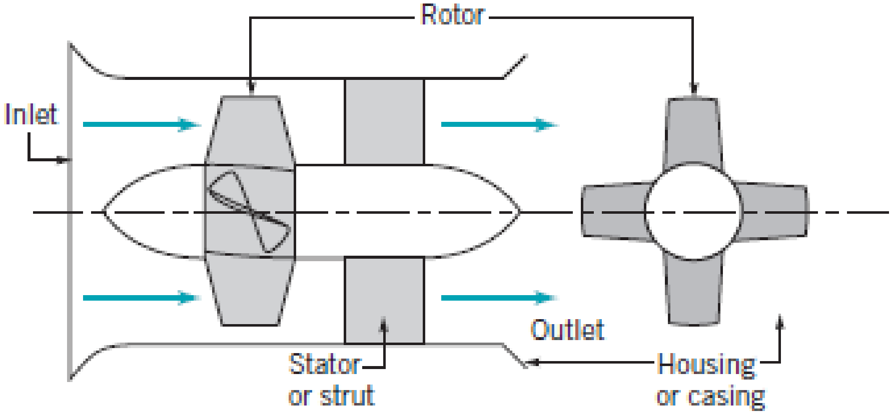
On the other hand, we can exert a shaft torque, typically with a motor, and by using blades, flow channels, or passages force the fluid to move. In this case we have a pump.

TYPICAL EXAMPLES OF TURBOMACHINES

- Typical examples of (a) a radial flow fan, and (b) an axial flow turbomachine.



(a) Radial-flow fan



(b) Axial-flow fan

TYPICAL EXAMPLES OF TURBOMACHINES

Some turbomachines include stationary blades or vanes in addition to rotor blades. These stationary vanes can be arranged to accelerate the flow and thus serve as nozzles. Or, these vanes can be set to diffuse the flow and act as diffusers.

Turbomachines are classified as *axial-flow, mixed-flow,* or *radial-flow* machines depending on the predominant direction of the fluid motion relative to the rotor’s axis as the fluid passes the blades.

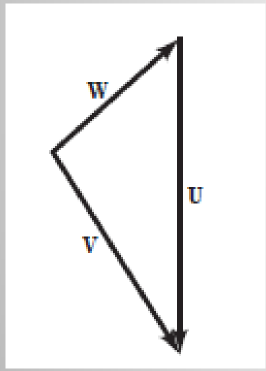
For an *axial-flow* machine the fluid maintains a significant axial-flow direction component from the inlet to outlet of the rotor.

For a *radial-flow* machine the flow across the blades involves a substantial radial-flow component at the rotor inlet, exit or both.

In other machines, designated as mixed-flow machines, there may be significant radial-and axial-flow velocity components for the flow through the rotor row.

BASIC ENERGY CONSIDERATIONS

An understanding of the work transfer in turbomachines can be obtained by considering the basic operation of a household fan (pump) and a windmill (turbine). Although the actual flows in such devices are very complex (i.e., three-dimensional and unsteady), the essential phenomena can be illustrated by use of simplified flow considerations and velocity triangles.

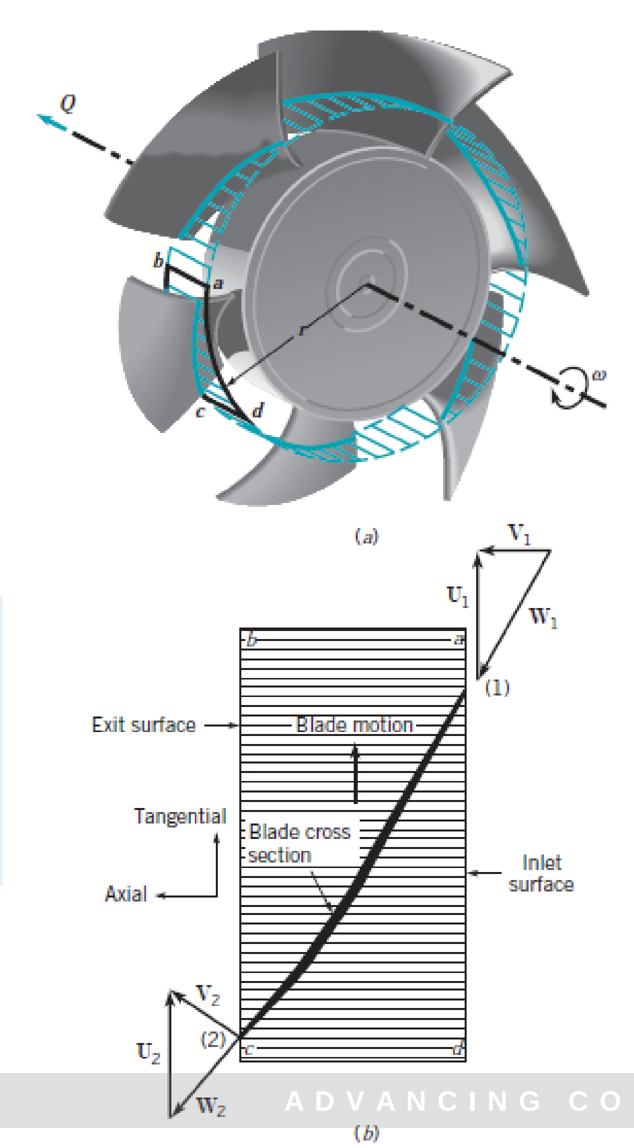


Consider a fan blade driven at constant angular velocity, ω, by a motor as shown in Fig. a. We denote the blade speed as $\boldsymbol{U} = \omega r$, where r is the radial distance from the axis of the fan. The absolute fluid velocity (that seen by a person sitting stationary at the table on which the fan rests) is denoted **V**, and the relative velocity (that seen by a person riding on the fan blade) is denoted **W**.

As shown by the figure in the margin, the actual absolute fluid velocity is the vector sum of the relative velocity and the blade velocity.

$$V = W + U \qquad \textbf{Eq. 3-23}$$

IDEALIZED FLOW THROUGH FAN BLADE GEOMETRY



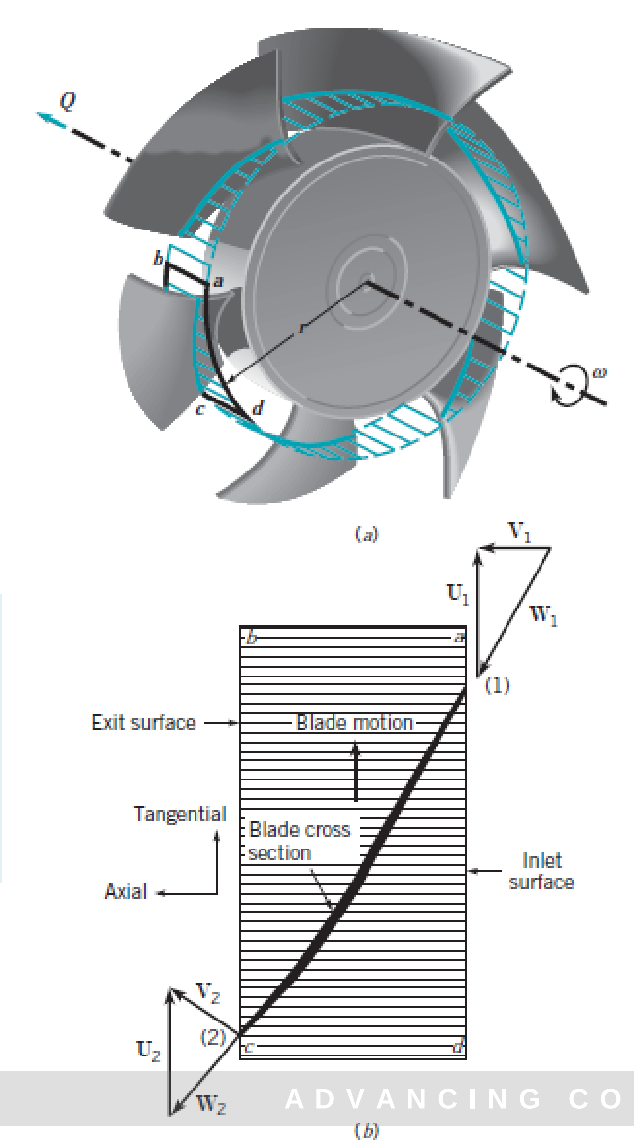
A simplified sketch of the fluid velocity as it "enters" and "exits" the fan at radius r is shown in Fig. b. The shaded surface labeled *a-b-c-d* is a portion of the cylindrical surface (including a "slice" through the blade) shown in Fig. a.

We assume for simplicity that the flow moves smoothly along the blade so that relative to the moving blade the velocity is parallel to the leading and trailing edges (points 1 and 2) of the blade.

For now we assume that the fluid enters and leaves the fan at the same distance from the axis of rotation; thus, $U_1 = U_2 = \omega r$. With this information we can construct the *velocity triangles* shown in Fig. b.

Note that this view is from the top of the fan, looking radially down toward the axis of rotation. The motion of the blade is up; the motion of the incoming air is assumed to be directed along the axis of rotation.

IDEALIZED FLOW THROUGH FAN BLADE GEOMETRY

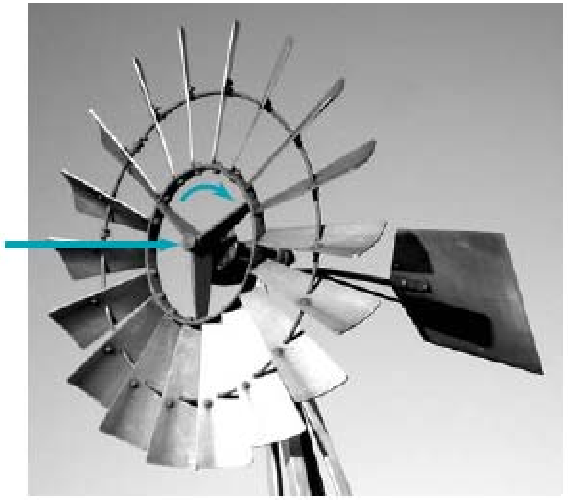


The important concept to grasp from this sketch is that the fan blade (because of its shape and motion) "pushes" the fluid, causing it to change direction. The absolute velocity vector $\boldsymbol{V}$, is turned during its flow across the blade from section (1) to section (2).

Initially the fluid had no component of absolute velocity in the direction of the motion of the blade, the θ (or tangential) direction. When the fluid leaves the blade, this tangential component of absolute velocity is nonzero. For this to occur, the blade must push on the fluid in the tangential direction. That is, the blade exerts a tangential force component on the fluid in the direction of the motion of the blade.

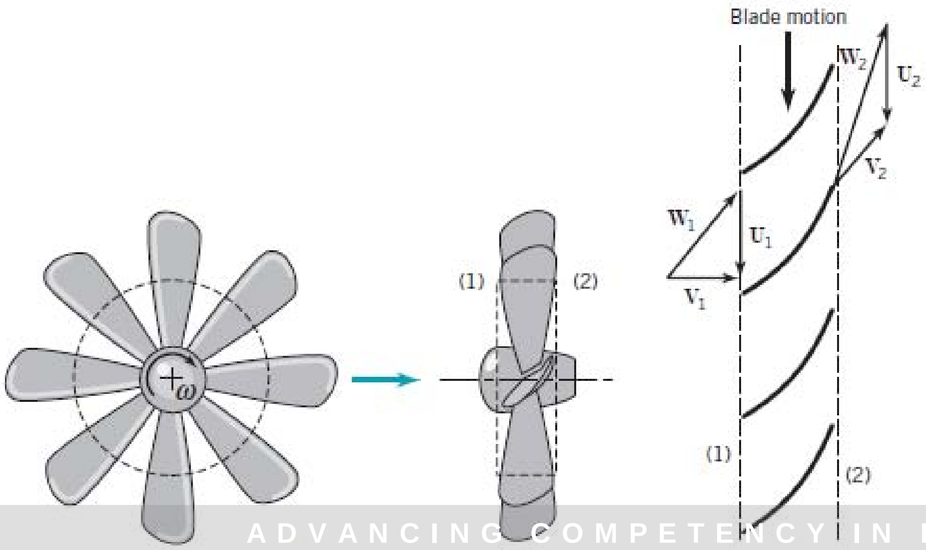
This tangential force component and the blade motion are in the same direction – the blade does work on the fluid. This device is a pump.

IDEALIZED FLOW THROUGH WINDMILL BLADE


(a)

On the other hand, consider the windmill, the blades move in the direction of the lift force exerted on each blade by the wind blowing through the rotor.

Because of the blade shape and motion, the absolute velocity vectors at sections (1) and (2), $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$, have different directions. For this to happen, the blades must have pushed up on the fluid – opposite to the direction of blade motion.



(b) (c)

Because of equal and opposite forces (action/reaction) the fluid must have pushed on the blades in the direction of their motion – the fluid does work on the blades. This extraction of energy from the fluid is the purpose of a turbine.

BASIC ANGULAR MOMENTUM CONSIDERATIONS

Since all turbomachines involve the rotation of an impeller or a rotor about a central axis, then it is appropriate to discuss their performance in terms of torque and angular momentum.

In a turbomachine, a series of particles (a continuum) passes through the rotor. Thus, the moment of momentum equation applied to a control volume is valid. For steady flow condition, the equation yields

$$\sum(r \times F) = \int_{cs} (r \times V) \rho V . \hat{n} dA \qquad \textbf{Eq. 3-24}$$

The left-hand side equation represents the sum of the external torques (moments) acting on the contents of the control volume.

The right-hand side equation is the net rate of flow of moment of momentum (angular momentum) through the control surface.

BASIC ANGULAR MOMENTUM CONSIDERATIONS

The one-dimensional simplification of flow through a turbomachine rotor with section (1) as the inlet and section (2) as the outlet results in

$$T_{shaft} = \dot{m}_2 \left(r_2 V_{\theta 2} \right) - \dot{m}_1 \left(r_1 V_{\theta 1} \right) \qquad \textbf{Eq. 3-25}$$

where T_{shaft} is the *shaft torque* applied to the contents of the control volume. The "-" is associated with mass flowrate into the control volume and the "+" is used with the outflow. The above equation is commonly known as the Euler turbomachine equation.

The sign of V_θ component depends on the direction of V_θ and the blade motion U. If V_θ and U are in the same direction, then V_θ is positive.

The shaft power W_{shaft} is related to the shaft torque and angular velocity by

$$W_{shaft} = T_{shaft}\omega \qquad \textbf{Eq. 3-26}$$

Combining the above equations and using the fact that $U = \omega r$, we obtain

$$W_{shaft} = \dot{m}_2 \left(U_2 V_{\theta 2} \right) - \dot{m}_1 \left(U_1 V_{\theta 1} \right)$$

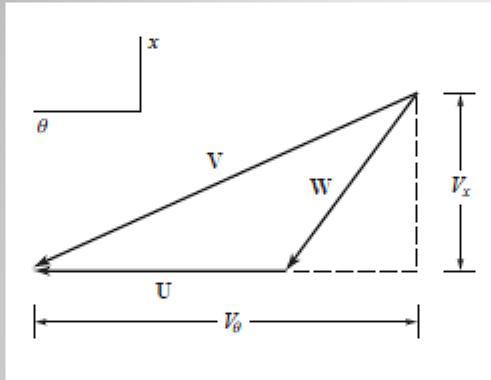
$$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \qquad \textbf{Eq. 3-27}$$

BASIC ANGULAR MOMENTUM CONSIDERATIONS

Another useful equation is

$$V^2 = V_\theta^2 + V_x^2 \qquad \textbf{Eq. 3-28}$$

$$V_x^2 = V^2 - V_\theta^2$$



The absolute velocity, *V* is equal to the vector sum of relative velocity, *W* and blade velocity, *U*.

From the small right triangle, we note that

$$V_x^2 + (V_\theta - U)^2 = W^2 \qquad \textbf{Eq. 3-29}$$

Combining the above equations, eliminating V_x

$$V^2 - V_\theta^2 + (V_\theta - U)^2 = W^2$$

$$V^2 - V_\theta^2 + V_\theta^2 - 2UV_\theta + U^2 = W^2$$

$$UV_\theta = \frac{1}{2}\left[V^2 + U^2 - W^2\right] \qquad \textbf{Eq. 3-30}$$

Combining the Equations 3-27 and 3-30, we obtain

$$w_{shaft} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - \left(W_2^2 - W_1^2\right)}{2} \qquad \textbf{Eq. 3-31}$$