DE LA SALLE UNIVERSITY - MANILA

FLUID
MACHINERY

ADVANCING COMPETENCY IN ENGINEERING SUPERVISION

PROBLEM 310a:

A fan discharges 265 cu.m/min of air through a duct 92 cm in diameter against a static pressure of 0.022 m of water. The gage fluid density is 995 kg/cu.m. The air temperature is 29°C and the barometer pressure is 730 mm Hg. If the power input to the fan is measured as 3.5 HP, what are the total and static fan efficiencies?

PROBLEM 310a:

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Solution:

a) Use Ideal Gas Equation of State to find the weight density of air.

$$\gamma_a = \frac{pg}{RT} = \frac{\left(730\text{ mm Hg} \times \dfrac{101.325\text{ kPa}}{760\text{ mm Hg}}\right)\left(9.80665\text{ m/s}^2\right)}{\left(0.287\text{ kJ/kg - K}\right)\left(29 + 273.15\right)\text{K}}$$

$$\gamma_a = 11.0063\text{ N/m}^3$$

Using Mass Conservation

$$Q = \frac{\pi}{4}\phi^2 V; \quad V = \frac{4Q}{\pi\phi^2}$$

Using Energy Equation

$$H_s + h_f = H_d + h_l$$

$$h_f = \frac{p_d}{\gamma_a} + \frac{V_d^2}{2g}; \text{but} \quad p_d = \gamma_w h_w$$

$$h_f = \frac{\gamma_w h_w}{\gamma_a} + \frac{V_d^2}{2g}$$

Combining with Mass Conservation

$$h_f = \frac{\rho_w g h_w}{\gamma_a} + \frac{8Q^2}{g\pi^2\phi^4}$$

PROBLEM 310a:

Combining with mass conservation and simplifying

$$h_f = \frac{(995\ \mathrm{kg/m^3})(9.80665\ \mathrm{m/s^2})(0.022\ \mathrm{m})}{11.0063\ \mathrm{N/m^3}} + \frac{8\left(265\frac{\mathrm{m^3}}{\mathrm{min}}\times\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)^2}{\pi^2(9.80665\ \mathrm{m/s^2})(0.92\ \mathrm{m})^4}$$

$$h_f = 19.5041\ \mathrm{m} + 2.2506\ \mathrm{m} = 21.7547\ \mathrm{m}$$

Calculating the static efficiency yields

$$\eta_s = \frac{SAP}{BP}\times 100\% = \frac{g\rho_w Q h_s}{3.5\ \mathrm{hp}}\times 100\%$$

$$\eta_s = \frac{(9.80665\ \mathrm{m/s^2}\times 995\ \mathrm{kg/m^3})\left(265\frac{\mathrm{m^3}}{\mathrm{min}}\times\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)(0.022\ \mathrm{m\ H_2O})}{3.5\ \mathrm{hp}\times\frac{746\ \mathrm{W}}{1\ \mathrm{hp}}}\times 100\% = 36.31\%$$

Calculating the total fan efficiency gives

$$\eta_t = \frac{TAP}{BP}\times 100\% = \frac{\gamma_a Q h_f}{3.5\ \mathrm{hp}}\times 100\%$$

$$\eta_t = \frac{(11.0063\ \mathrm{N/m^3})\left(265\frac{\mathrm{m^3}}{\mathrm{min}}\times\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right)(21.7547\ \mathrm{m})}{3.5\ \mathrm{hp}\times\frac{746\ \mathrm{W}}{1\ \mathrm{hp}}}\times 100\% = 40.50\%$$

PROBLEM 310b:

A large forced draft fan is handling air at 1 atm and 43°C under a total head of 266 mm water gage. The power input to the fan is 224 kW and the fan is 75% efficient. Determine the volume of air handled per minute considering local acceleration of 9.80665 m/s^2.

PROBLEM 310b:

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Solution:

a) Using the total fan efficiency

$$\eta_t = \frac{TAP}{BP} \times 100\%$$

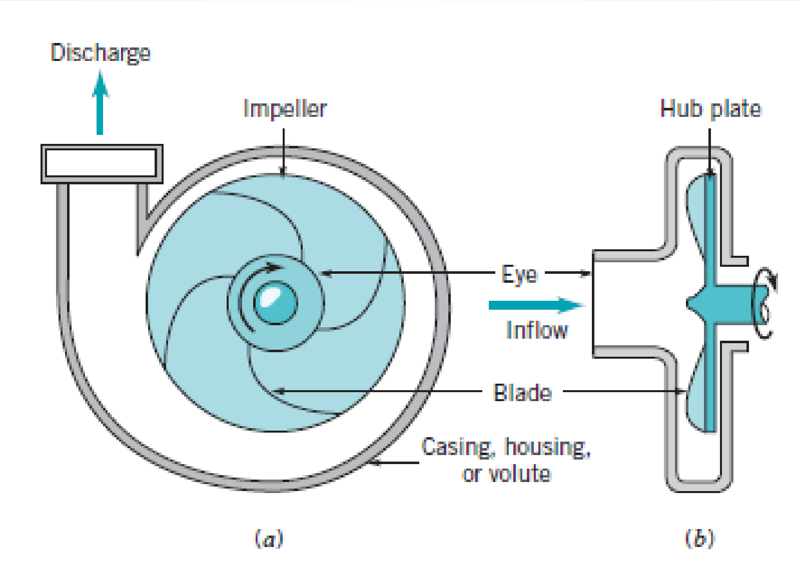
$$\eta_t = \frac{\gamma_a Q h_t}{BP} \times 100\%$$

$$0.75 = \frac{(9.80665\ \text{kN/m}^3)Q(0.266\ \text{m of H}_2\text{O})}{224\ \text{kW}}$$

$$Q = 64.403\ \text{m}^3/\text{s} \times \frac{60\ \text{s}}{1\ \text{min}} = 3{,}864.19\ \text{m}^3/\text{min}$$

CENTRIFUGAL PUMP

The schematic diagram of basic elements of a centrifugal pump is shown in this figure



The pump head is derived from energy equation in the form of

$$\frac{p_s}{\gamma} + z_s + \frac{V_s^2}{2g} + h_{pump} = \frac{p_d}{\gamma} + z_d + \frac{V_d^2}{2g} + h_{loss}$$

$$h_{pump} = \left[\frac{p_d - p_s}{\gamma}\right] + \left(z_d - z_s\right) + \left[\frac{V_d^2 - V_s^2}{2g}\right] + h_{loss} \qquad \textbf{Eq. 3-32}$$

CENTRIFUGAL PUMP

Since mass is conserved in the pump, then $\dot{m} = \dot{m}_1 = \dot{m}_2$

$$T_{shaft} = \dot{m}_2 \left(r_2 V_{\theta 2} \right) - \dot{m}_1 \left(r_1 V_{\theta 1} \right)$$

$$T_{shaft} = \dot{m} \left(r_2 V_{\theta 2} - r_1 V_{\theta 1} \right)$$

$$T_{shaft} = \rho Q \left(r_2 V_{\theta 2} - r_1 V_{\theta 1} \right)$$ **Eq. 3-33**

For a rotating shaft, the power transferred is given by

$$W_{shaft} = T_{shaft} \omega$$ **Eq. 3-26**

And therefore from Equation 3-33

$$\dot{W}_{shaft} = \rho Q \omega \left(r_2 V_{\theta 2} - r_1 V_{\theta 1} \right)$$

$$\dot{W}_{shaft} = \rho Q \left(U_2 V_{\theta 2} - U_1 V_{\theta 1} \right)$$

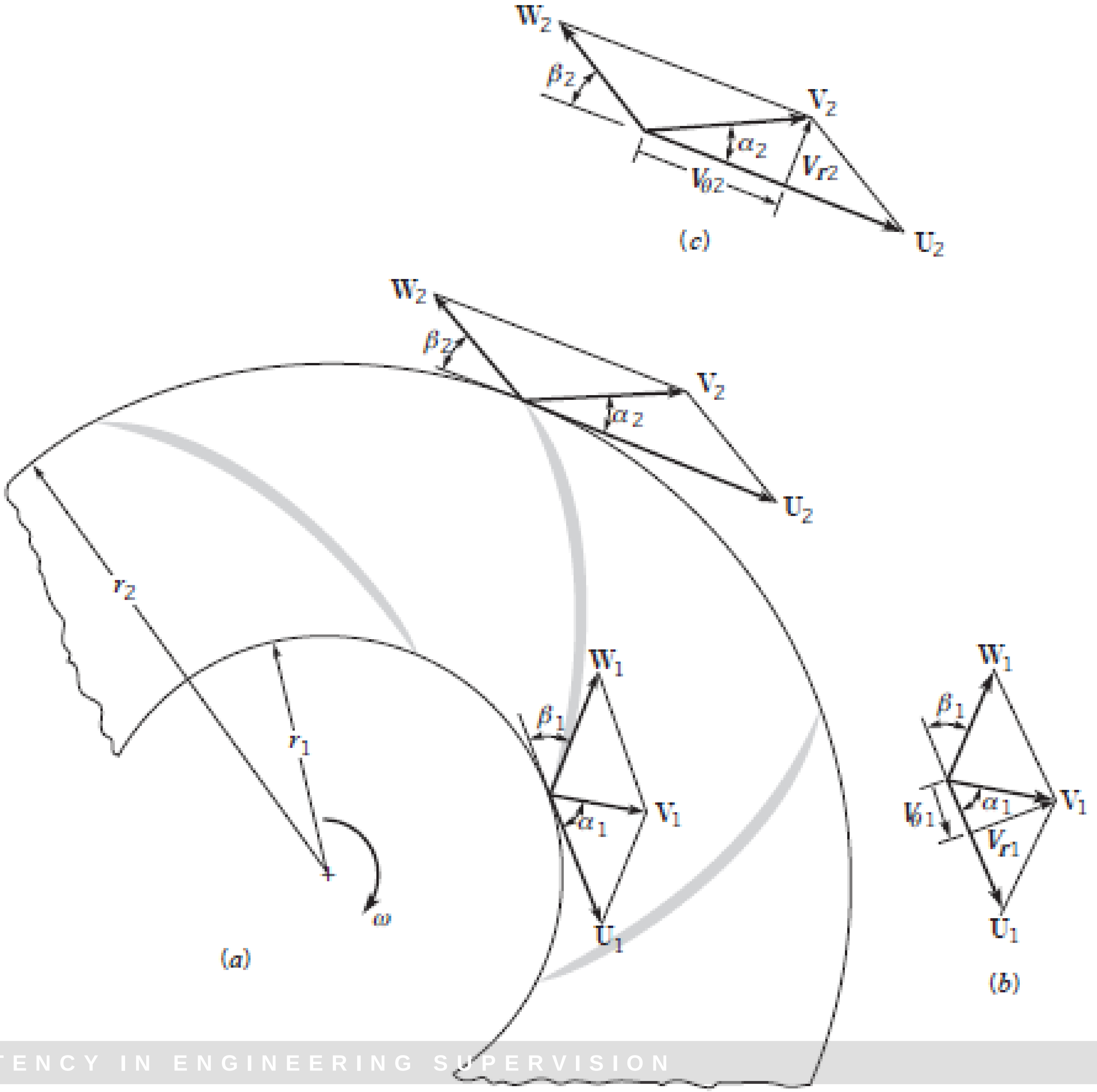
$$w_{shaft} = \frac{\dot{W}_{shaft}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$ **Eq. 3-34**

Also, for an incompressible pump flow, we get

$$w_{shaft} = g h_{pump} = \left[\frac{p_{out} - p_{in}}{\rho} \right] + g \left(z_{out} - z_{in} \right) + \left[\frac{V_{out}^2 - V_{in}^2}{2} \right] + g h_{loss}$$ **Eq. 3-35**

CENTRIFUGAL PUMP

- Velocity diagrams at the inlet and outlet of a centrifugal pump impeller.



CENTRIFUGAL PUMP

Combining Equations 3-34 & 3-35

$$U_2V_{\theta 2} - U_1V_{\theta 1} = \left[\frac{p_{out} - p_{in}}{\rho}\right] + g\left(z_{out} - z_{in}\right) + \left[\frac{V_{out}^2 - V_{in}^2}{2}\right] + gh_{loss}$$ **Eq. 3-36**

Dividing both sides of the equation by acceleration due to gravity

$$\frac{U_2V_{\theta 2} - U_1V_{\theta 1}}{g} = \left[\frac{p_{out} - p_{in}}{g\rho}\right] + \left(z_{out} - z_{in}\right) + \left[\frac{V_{out}^2 - V_{in}^2}{2g}\right] + h_{loss}$$

$$\frac{U_2V_{\theta 2} - U_1V_{\theta 1}}{g} = H_{out} - H_{in} + h_{loss}$$ **Eq. 3-37**

where H is the total head and h_l is head loss.

From this equation we see that the left-hand side equation is the shaft work head added to the fluid by the pump. Neglecting head loss, the ideal head rise possible, h_i, is

$$h_i = \frac{U_2V_{\theta 2} - U_1V_{\theta 1}}{g}$$ **Eq. 3-38**

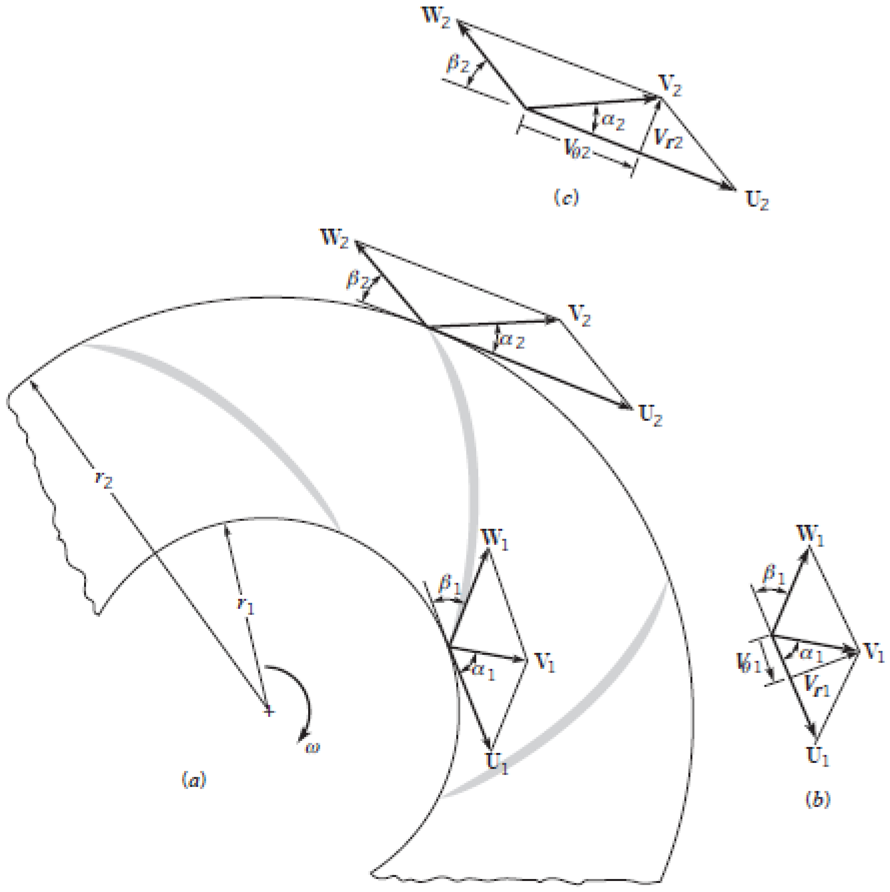
The actual head rise yields $h_a = H_{out} - H_{in}$ or

$h_a = h_i - h_{loss}$ **Eq. 3-39**

PROBLEM 311

Water is pumped at the rate of 1400 gpm through a centrifugal pump operating at a speed of 1750 rpm. The impeller has a uniform blade height, b, of 2 in. with r_1 = 1.9 in. and r_2 = 7.0 in., and the exit blade angle β_2 is 23º. Assume ideal flow conditions and that the tangential velocity component, $V_{\theta 1}$, of the water entering the blade is zero ($\alpha_1 = 90º$).

Determine (a) the tangential velocity component at the exit, (b) the ideal head rise, and (c) the shaft power transmitted to the fluid. Discuss the difference between ideal and actual head rise.



SOLUTION

a) At the exit the velocity diagram is shown in Fig. (c), where ***V*₂** is the absolute velocity of the fluid, ***W*₂** is the relative velocity, and ***U*₂** is the tip velocity of the impeller with

$$U_2 = r_2\omega = \frac{7\text{ in}}{12\text{ in/ft}}\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(1750\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

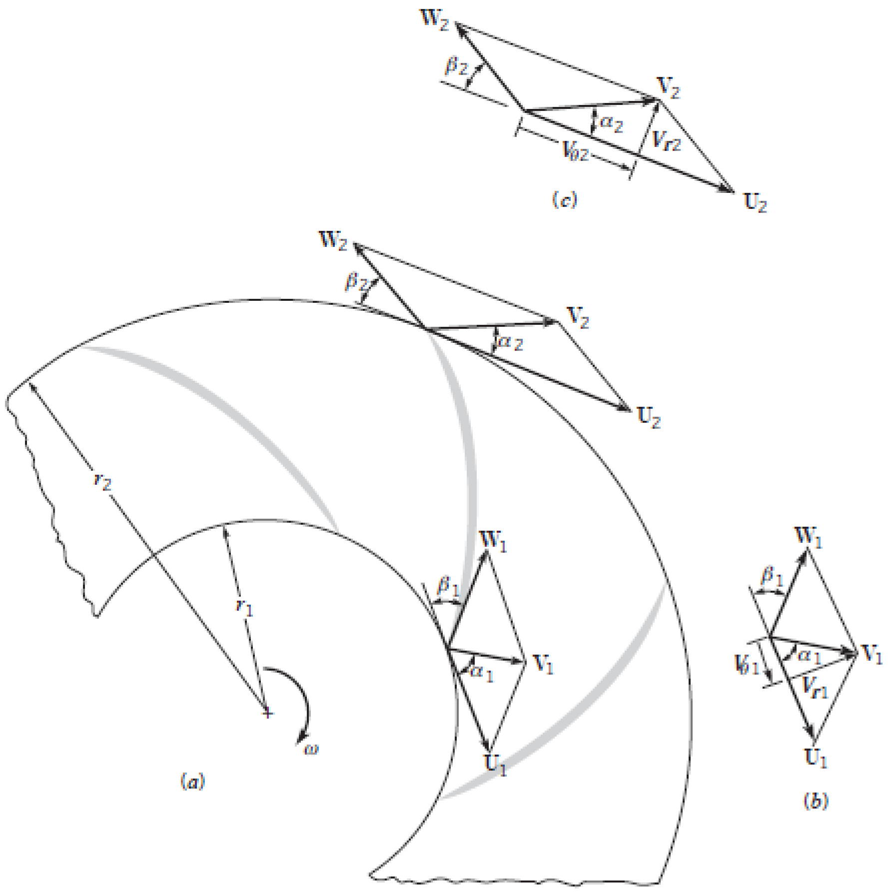
$$U_2 = 106.90\text{ ft/s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi\, r_2 b_2 V_{r2}$$

$$V_{r2} = \frac{Q}{2\pi\, r_2 b_2} = \frac{1400\dfrac{\text{gal}}{\text{min}}\left(\dfrac{1\text{ ft}^3}{7.48\text{gal}}\right)}{\left(\dfrac{60\text{ s}}{\text{min}}\right)(2\pi)\left(\dfrac{7}{12}\text{ft}\right)\left(\dfrac{2}{12}\text{ft}\right)}$$

$$V_{r2} = 5.1066\text{ ft/s}$$



SOLUTION

From Fig. (c) we see that

$$\tan \beta_2 = \frac{V_{r2}}{U_2 - V_{\theta 2}}$$

$$\cot \beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

$$V_{\theta 2} = U_2 - V_{r2} \cot \beta_2$$

$$V_{\theta 2} = 106.9 - 5.1066 \cot 23^{\circ}$$

$$V_{\theta 2} = 94.87 \text{ ft/s}$$

b) The ideal head rise is given by

$$h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$$

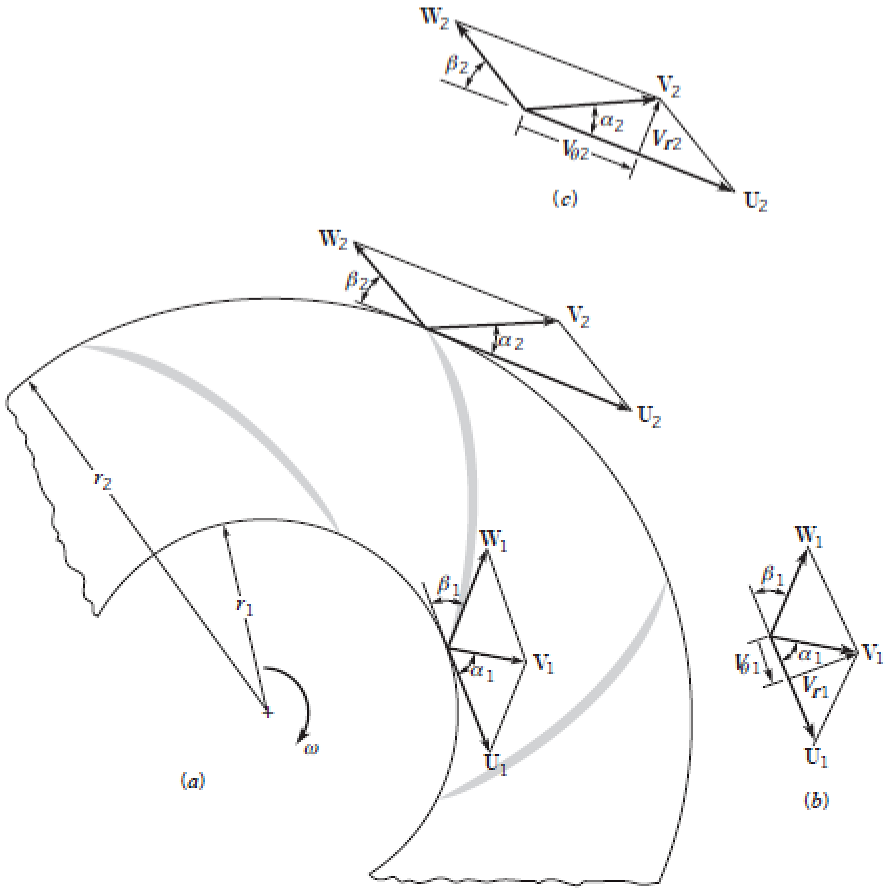
But,

$$V_{\theta 1} = V_1 \cos \alpha_1$$

$$V_{\theta 1} = V_1 \cos 90^{\circ} = 0$$

$$h_i = \frac{U_2 V_{\theta 2}}{g} = \frac{\left(106.9 \frac{\text{ft}}{\text{s}}\right)\left(94.87 \frac{\text{ft}}{\text{s}}\right)}{32.2 \frac{\text{ft}}{\text{s}^2}}$$

$$h_i = 314.96 \text{ ft}$$



SOLUTION

c) Since $V_{\theta 1} = 0$, the power transferred to the fluid is given by

$$\dot{W}_{shaft} = \rho Q\left(U_2 V_{\theta 2} - U_1 V_{\theta 1}\right)$$

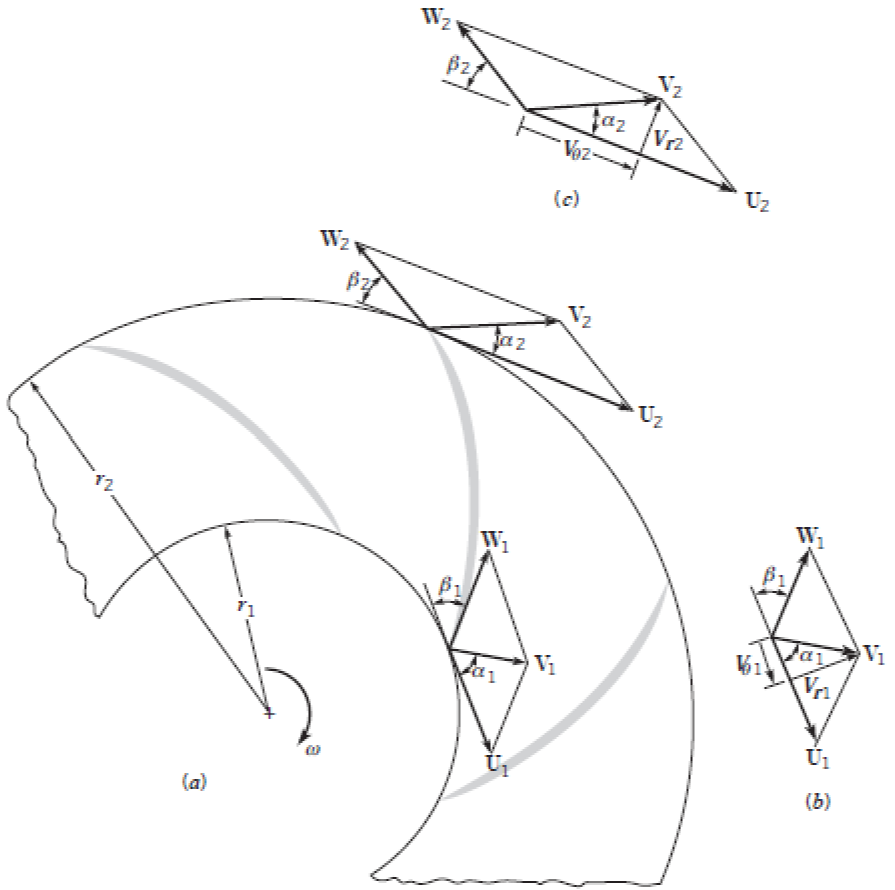
$$\dot{W}_{shaft} = \rho Q U_2 V_{\theta 2}$$

$$\dot{W}_{shaft} = \frac{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1400\,\frac{\text{gal}}{\text{min}}\right)\left(106.9\,\frac{\text{ft}}{\text{s}}\right)\left(94.87\,\frac{\text{ft}}{\text{s}}\right)}{\left(\frac{1\,\text{slug - ft/s}^2}{1\,\text{lb}}\right)\left(7.48\,\frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{60\,\text{s}}{\text{min}}\right)}$$

$$\dot{W}_{shaft} = 61{,}373.87\,\frac{\text{ft - lb}}{\text{s}} \times \frac{1\,\text{hp}}{550\,\frac{\text{ft - lb}}{\text{s}}} = 111.6\,\text{hp}$$

Note that the ideal head rise and the power transferred to the fluid are related through the relationship

$$\dot{W}_{shaft} = \rho g Q h_i$$

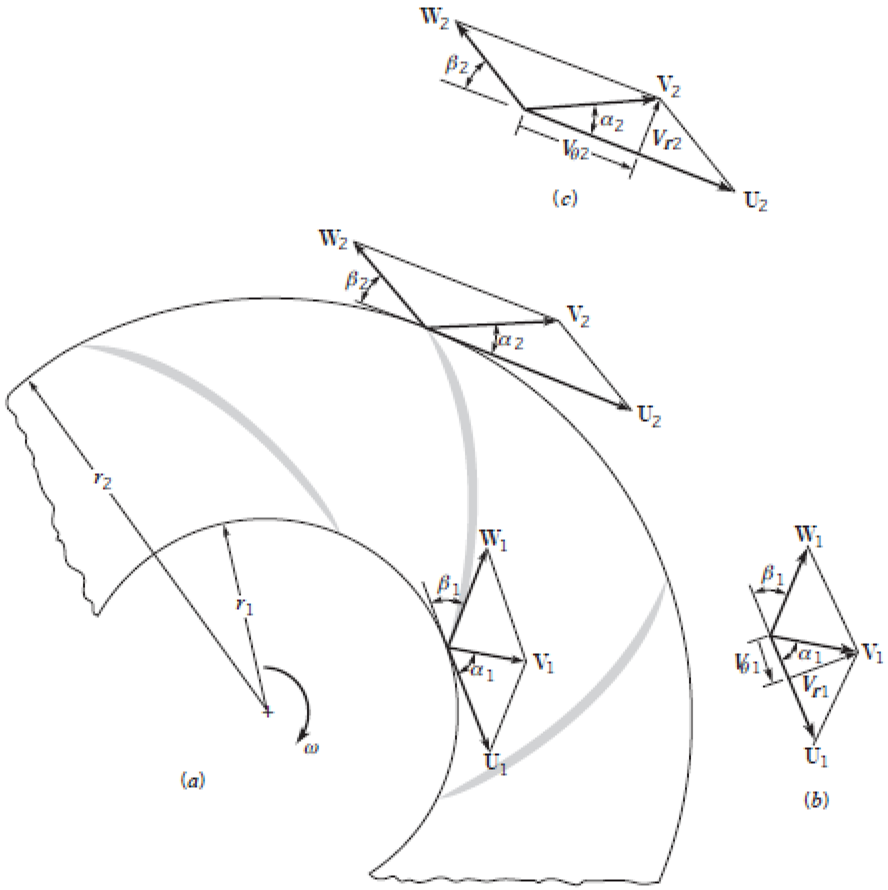


COMMENTS

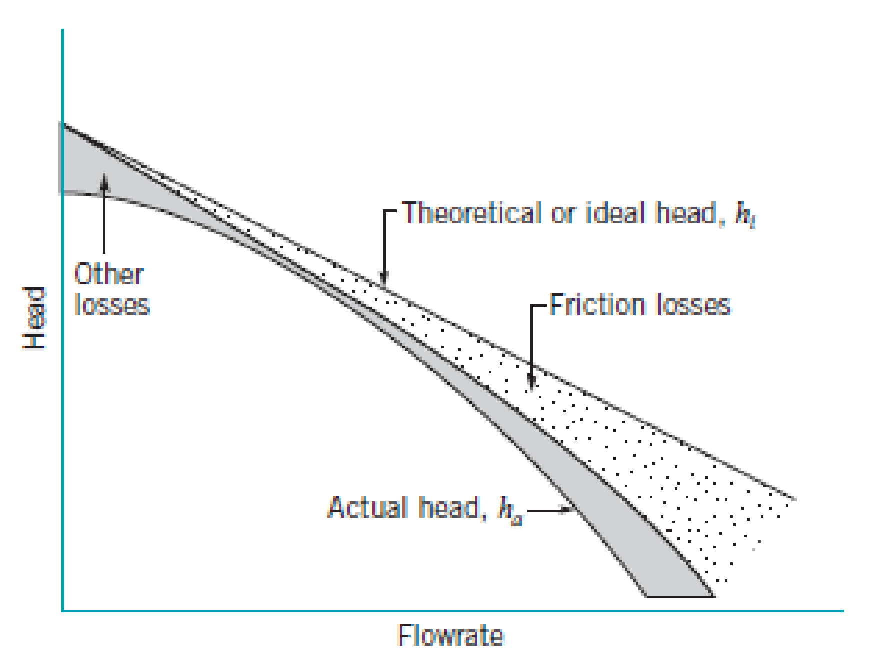
It should be emphasized that results given in the previous equation involve the ideal head rise.

The actual head rise performance characteristics of a pump are usually determined by experimental measurements obtained in a testing laboratory.

The actual head rise is always less than the ideal head rise for a specific flowrate because of the loss of available energy associated with actual flows.



EFFECT OF LOSSES ON THE PUMP HEAD – FLOWRATE CURVE



This figure shows the ideal head versus flowrate curve for a centrifugal pump with backward curve vanes ($\beta 2 < 90°$).

Since there are simplifying assumptions (i.e., zero losses) associated with the equation for h_i, we would expect that the actual rise in head of fluid, h_a would be less than the ideal head rise.

As shown in this figure, the h_a versus Q curve lies below the ideal head rise curve and shows a nonlinear variation with Q.

The differences between the two curves (as represented by the shaded areas between the curves) arise from several sources. These differences include losses due to fluid skin friction in the blade passages, which vary as Q^2, and other losses due to such factors such as flow separation, impeller blade-casing clearance flows and other three-dimensional flow effects.

PUMP PERFORMANCE CHARACTERISTICS

The actual head rise, h_a, gained by fluid flowing through a pump can be determined with an experimental arrangement of the type shown below. Using the energy equation with $h_a = h_s - h_l$ where h_s is the shaft work head and is identical to h_i, and h_l is the pump head loss.

$$h_a = \left[\frac{p_2 - p_1}{\gamma}\right] + (z_2 - z_1) + \left[\frac{V_2^2 - V_1^2}{2g}\right] \qquad \textbf{Eq. 3-40}$$

with sections (1) and (2) at the pump inlet and exit, respectively. Typically, the differences in elevations and velocities are small so that

$$h_a \approx \left[\frac{p_2 - p_1}{\gamma}\right] \qquad \textbf{Eq. 3-41}$$

The power gained by the fluid is given by the equation

$$P_f = \gamma Q h_a \qquad \textbf{Eq. 3-42}$$

PUMP PERFORMANCE CHARACTERISTICS

and this quantity, expressed in terms of horsepower is traditionally called the *water horsepower*. Thus

$$P_f = \text{water horsepower} = \frac{\gamma Q h_a}{550} \qquad \textbf{Eq. 3-43}$$

In addition to the head or power added to the fluid, the *overall efficiency, η*, is of interest, where

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{P_f}{\dot{W}_{shaft}} \qquad \textbf{Eq. 3-44}$$

The denominator of this relationship represents the total power applied to the shaft of the pump and is often referred to as *brake power* (bhp). Thus,

$$\eta = \frac{\gamma Q h_a / 550}{\text{bhp}} \qquad \textbf{Eq. 3-45}$$

PUMP PERFORMANCE CHARACTERISTICS

The volumetric efficiency is equal to actual flowrate denoted as Q divided by the sum of actual flowrate and fluid leakages, Q_l given by

$$\eta_v = \frac{Q}{Q + Q_l} \qquad \textbf{Eq. 3-46}$$

The hydraulic efficiency is the ratio of net head delivered to the fluid by the head transferred from the rotor to the fluid. The hydraulic head loss is denoted as h_f.

$$\eta_h = \frac{h}{h_i}$$

$$\eta_h = \frac{h_i - h_f}{h_i} \qquad \textbf{Eq. 3-47}$$

The mechanical efficiency is the ratio of the difference of brake power and power loss to mechanical friction in the bearings and stuffing boxes, fp divided by the brake power.

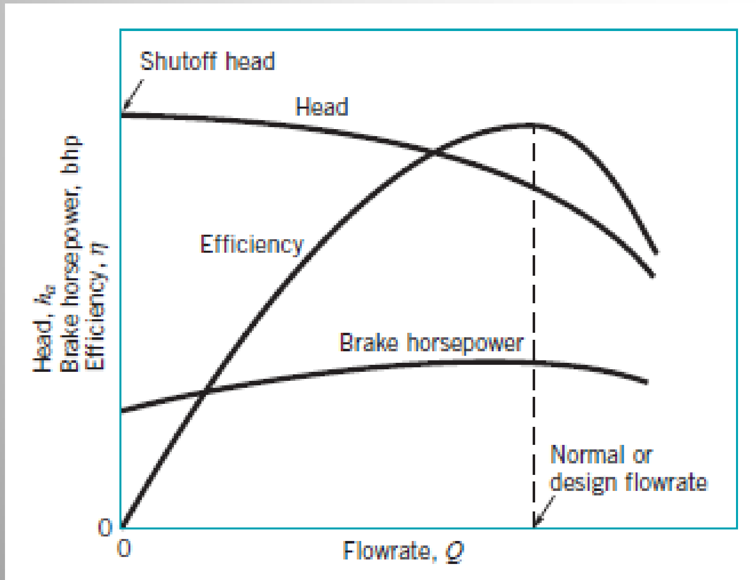
$$\eta_m = \frac{bp - fp}{bp} \qquad \textbf{Eq. 3-48}$$

PUMP PERFORMANCE CHARACTERISTICS

The overall efficiency is the product of hydraulic, volumetric, and mechanical efficiencies given as

$$\eta_o = \eta_h \eta_v \eta_m \qquad \textbf{Eq. 3-49}$$

Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed is shown in the figure below.



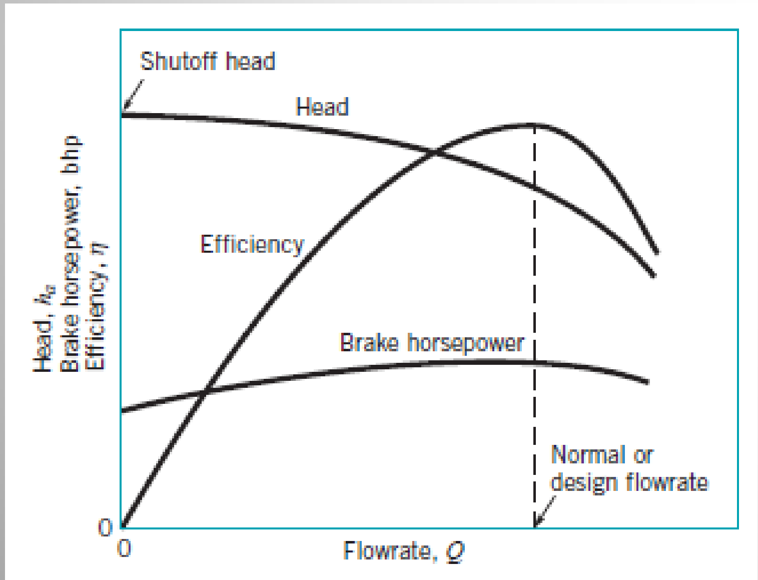
As the discharge is increased from zero the brake horsepower increases, with a subsequent fall as the maximum discharge is approached.

As previously noted, with h_a and bhp known, the efficiency can be calculated. The efficiency is a function of the flowrate and reaches a maximum value at some particular value of the flowrate, commonly referred to as the ***normal*** or ***design*** flowrate.

PUMP PERFORMANCE CHARACTERISTICS

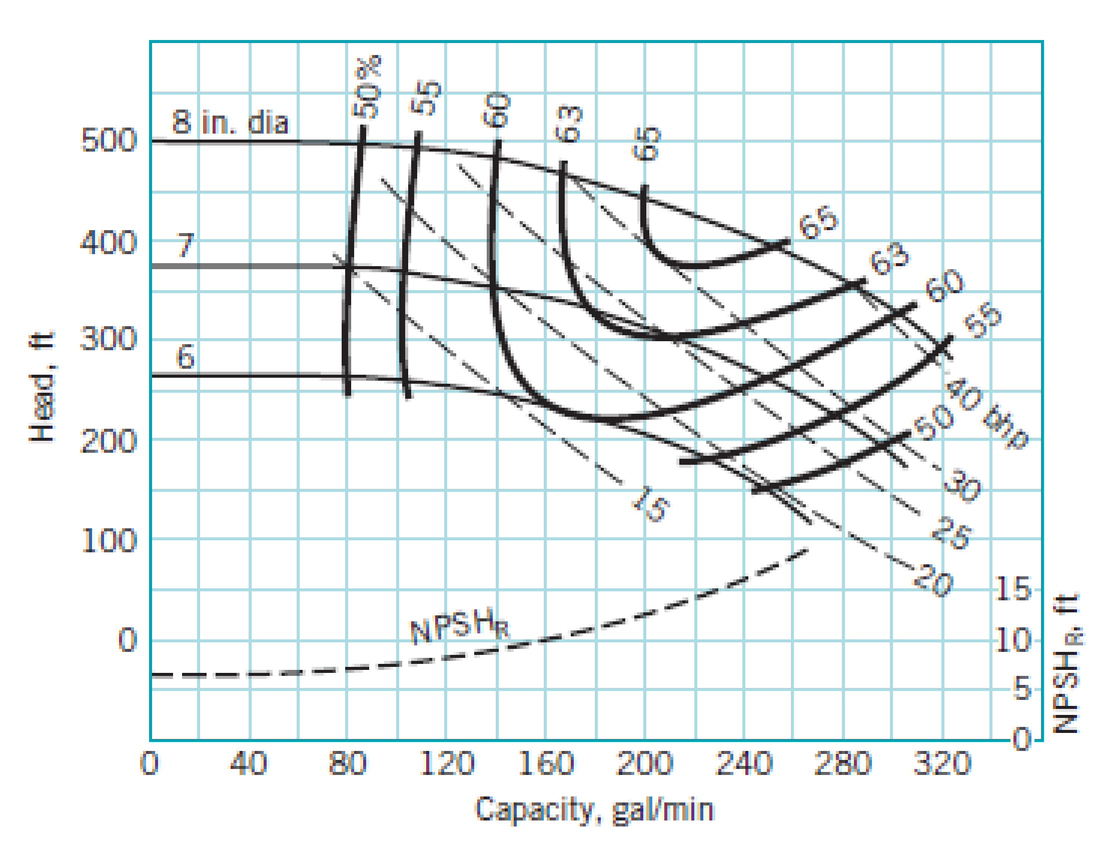
The points on the various curves corresponding to the maximum efficiency are denoted as the ***best efficiency points*** (BEP).

It is apparent that when selecting a pump for a particular application, it is usually desirable to have the pump operate near its maximum efficiency.



Thus, performance curves of the type shown in this figure are very important to the engineer responsible for the selection of pumps for a particular flow system.

PUMP PERFORMANCE CHARACTERISTICS



Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameters.

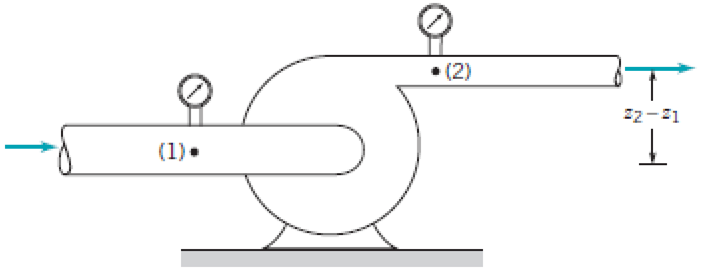
Pump performance characteristics are also presented in charts of the type shown below.

Since impellers with different diameters may be used in a given casing, performance characteristics for several impeller diameters can be provided with corresponding lines of constant efficiency and brake horsepower.

An additional curve is also given in this figure, labeled *$NPSH_R$*, which stands for ***required net positive suction head***.

PROBLEM 312

The performance characteristics of a certain centrifugal pump are determined from an experimental set-up similar to that shown below. When the flowrate of a liquid (SG = 0.9) through the pump is 120 gpm, the pressure gage at (1) indicates a vacuum of 95 mm of Hg and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. If $z_2 - z_1 = 0.5$ m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.



Typical experimental arrangement for determining the head rise gained by a fluid flowing through a pump.

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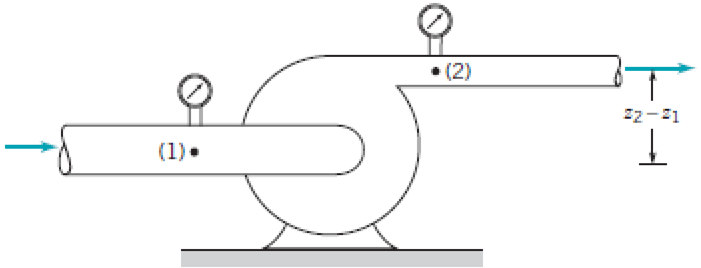
Solution:

a) The actual head rise can be determined using the energy equation which can be expressed as

$$h_a = \left[\frac{p_2 - p_1}{\gamma_f}\right] + (z_2 - z_1) + \left[\frac{V_2^2 - V_1^2}{2g}\right] \qquad \textbf{Eq. 1}$$



Typical experimental arrangement for determining the head rise gained by a fluid flowing through a pump.

SOLUTION

Using Continuity Equation, then $Q = Q_1 = Q_2$

$$V_1 = \frac{4Q}{\pi\phi_1^2}; \quad V_2 = \frac{4Q}{\pi\phi_2^2}$$

Eq. 2

Combining Equations (1) & (2), eliminating the velocities yields

$$h_a = \left[\frac{p_2 - p_1}{SG\gamma_{water}}\right] + (z_2 - z_1) + \left[\frac{16Q^2}{2g\pi^2}\left(\frac{1}{\phi_2^4} - \frac{1}{\phi_1^4}\right)\right]$$

$$h_a = \left[\frac{p_2 - p_1}{SG\gamma_{water}}\right] + (z_2 - z_1) + \left[\frac{8Q^2}{g\pi^2}\left(\frac{1}{\phi_2^4} - \frac{1}{\phi_1^4}\right)\right]$$

Eq. 3

$$h_a = \left[\frac{80\text{ kPa} + 95\text{ mm Hg}\left(\frac{101.325\text{ kPa}}{760\text{ mm Hg}}\right)}{0.9\left(9.81\frac{\text{kN}}{\text{m}^3}\right)}\right] + 0.5\text{ m} + \left[\frac{8\left(120\frac{\text{gal}}{\text{min}} \times \frac{1\text{min}}{60\text{ s}} \times \frac{3.785\text{ L}}{1\text{ gal}} \times \frac{1\text{ m}^3}{1000\text{ L}}\right)^2}{\pi^2\left(9.81\frac{\text{m}}{\text{s}^2}\right)}\left(\frac{1}{(0.055\text{ m})^4} - \frac{1}{(0.110\text{ m})^4}\right)\right]$$

$$h_a = 10.4956\text{ m} + 0.5\text{ m} + 0.4851\text{ m} = 11.4807\text{ m}$$

b) If the pump and motor efficiencies are given, then we could use the following formula.

$$BP = \frac{WP}{\eta_p}; \quad EP = \frac{BP}{\eta_m}$$

Eq. 4 & Eq. 5

$$EP = \frac{WP}{\eta_p\eta_m}$$

Eq. 6