PROBLEM 302

A centrifugal pump has an efficiency of 80% at its design point specific speed of 1800. The impeller diameter is 220 mm. At design point flow conditions, the volume flow rate is 90 cu.m/h of water at 1200 rpm. To obtain higher flow rate, the pump is to be filled with new impeller diameter of 250 mm and 1800 rpm motor.

Use the Affinity Laws to find the design point performance characteristics of the pump at higher speed. Determine the volume flow rate, pump head and required power input at higher speed.

SOLUTION

a) Use pump specific speed

$$N_{sd} = \frac{n_1\sqrt{Q_1}}{h_1^{3/4}}$$

Calculating head at lower speed

$$h_1 = \left[\frac{n_1\sqrt{Q_1}}{N_{sd}}\right]^{4/3}$$

$$h_1 = \left[\frac{1200\,\frac{\text{rev}}{\text{min}}\sqrt{\left(90\ \text{m}^3/\text{h}\right)\left(1000\ \text{L}/1\ \text{m}^3\right)\left(1\ \text{gal}/3.785\ \text{L}\right)\left(1\ \text{h}/60\ \text{min}\right)}}{1800}\right]^{4/3}$$

$$h_1 = 31.4216\ \text{ft}$$

Calculating head at higher speed

$$\frac{gh_1}{n_1^2 D_1^2} = \frac{gh_2}{n_2^2 D_2^2}$$

$$h_2 = h_1\left[\frac{n_2 D_2}{n_1 D_1}\right]^2 = 31.4216\,\text{ft}\left[\frac{1800\times 0.25}{1200\times 0.22}\right]^2$$

$$h_2 = 91.2947\ \text{ft}$$

SOLUTION

b) Identifying Q_2

$$\frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3}$$

$$Q_2 = Q_1\left[\frac{n_2 D_2^3}{n_1 D_1^3}\right] = 90\frac{\text{m}^3}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\left[\frac{1800 \times 0.25^3}{1200 \times 0.22^3}\right]$$

$$Q_2 = 0.055\,\text{m}^3/\text{s}$$

c) Calculating the brake power at lower speed

$$\eta_p = \frac{WP}{BP_1}$$

$$BP_1 = \frac{WP}{\eta_p} = \frac{\gamma Q h}{\eta_p}$$

$$BP_1 = \frac{9.81\frac{\text{kN}}{\text{m}^3} \times 0.025\frac{\text{m}^3}{s} \times 31.4216\,\text{ft}}{0.80 \times \frac{3.28\,\text{ft}}{1\text{m}}}$$

$$BP_1 = 2.9368\,\text{kW}$$

SOLUTION

Use Pump Affinity Laws to determine the brake power at higher speed

$$\frac{BP_1}{\rho n_1^3 D_1^5} = \frac{BP_2}{\rho n_2^3 D_2^5}$$

$$BP_2 = BP_1 \left[\frac{n_2}{n_1}\right]^3 \left[\frac{D_2}{D_1}\right]^5$$

$$BP_2 = 2.9368\,\text{kW} \left[\frac{1800}{1200}\right]^3 \left[\frac{0.25}{0.22}\right]^5$$

$$BP_2 = 18.78\,\text{kW}$$

PROBLEM 303

At its optimum point of operation a given centrifugal pump with an impeller diameter of 50 cm delivers 3.2 cu.m/s of water against a head of 25 m when rotating at 1,450 rpm. (a) If its efficiency is 82 %, what is the brake power of the driving shaft? (b) If a homologous pump with an impeller diameter of 80 cm is rotating at 1,200 rpm, what would be the discharge, head and shaft power? Assume both pumps operate at the same efficiency. (c) Compute the specific speed of both pumps.

PROBLEM 303

At its optimum point of operation a given centrifugal pump with an impeller diameter of 50 cm delivers 3.2 cu.m/s of water against a head of 25 m when rotating at 1,450 rpm. (a) If its efficiency is 82 %, what is the brake power of the driving shaft? (b) If a homologous pump with an impeller diameter of 80 cm is rotating at 1,200 rpm, what would be the discharge, head and shaft power? Assume both pumps operate at the same efficiency. (c) Compute the specific speed of both pumps.

Solution:

a) The brake power of driving shaft can be expressed as

$$\eta_p = \frac{WP}{BP}$$

$$BP = \frac{WP}{\eta_p} = \frac{\gamma Q h}{\eta_p}$$

$$BP = \frac{9.81\,\frac{\text{kN}}{\text{m}^3} \times 3.2\,\frac{\text{m}^3}{s} \times 25\,\text{m}}{0.82} = 957\,\text{kW}$$

PROBLEM 303

b) Using Affinity Laws, the discharge flow rate and head yield

$$\frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3}$$

$$Q_2 = Q_1 \left[\frac{n_2}{n_1}\right]\left[\frac{D_2}{D_1}\right]^3$$

$$Q_2 = 3.2\frac{\text{m}^3}{s}\left[\frac{1200}{1450}\right]\left[\frac{0.80}{0.50}\right]^3 = 10.847\frac{\text{m}^3}{s}$$

$$\frac{gh_1}{n_1^2 D_1^2} = \frac{gh_2}{n_2^2 D_2^2}$$

$$h_2 = h_1\left[\frac{n_2 D_2}{n_1 D_1}\right]^2$$

$$h_2 = 25\text{ m}\left[\frac{1200\times 0.80}{1450\times 0.50}\right]^2 = 43.834\text{ m}$$

However, the brake power at lower speed gives

$$\frac{BP_1}{\rho_1 n_1^3 D_1^5} = \frac{BP_2}{\rho_2 n_2^3 D_2^5}$$

$$BP_2 = BP_1\left[\frac{\rho_2}{\rho_1}\right]\left[\frac{n_2}{n_1}\right]^3\left[\frac{D_2}{D_1}\right]^5$$

$$BP_2 = 957\text{ kW}\left[\frac{1200}{1450}\right]^3\left[\frac{0.8}{0.5}\right]^5$$

$$BP_2 = 5{,}687.90\text{ kW}$$

c) Calculating the pump specific speed for both conditions

$$N_{S_1}^* = \frac{1450\sqrt{3.2}}{25^{3/4}}$$

$$N_{S_1}^* = 232$$

$$N_{S_2}^* = \frac{1200\sqrt{10.85}}{43.8^{3/4}}$$

$$N_{S_2}^* = 232$$

EXERCISE 301

1. A group of researchers from NASA conducted an experiment to identify the pump performance characteristics. At its optimum point of operation a given centrifugal pump with an impeller diameter of 30 cm delivers 2.5 cu.m/s of water against a head of 20 m when mounted in a 4-pole and 50 Hz electric motor. If its efficiency is 75%, what is the brake power of the driving shaft? If a homologous pump with an impeller diameter of 60 cm is mounted in a 4-pole and 60 Hz electric motor which to be performed on MARS with a gravity of 3.711 m/s^2, what would be the discharge, head and shaft power? Assume both pumps operate at the same efficiency. Compute the specific speed of both pumps.

2. A model centrifugal pump was tested at 3,600 rpm and delivered 85 L/s at a head of 38 m with an efficiency of 90%. Assuming the prototype to have an efficiency of 91% and to develop the same head, what will be its speed, capacity, and power required? The liquid pumped is oil with a specific gravity of 0.80.

SUCTION SPECIFIC SPEED

With an analysis similar to that used to obtain the specific speed pi term, the *suction specific speed*, S_s, can be expressed as

$$S_s = \frac{\omega(\text{rad/s})\sqrt{Q(\text{m}^3/\text{s})}}{\left[g(\text{m/s}^2)\,NPSH_R(\text{m})\right]^{3/4}} \qquad \textbf{Eq. 3-18}$$

where the head in Equation 3-15 has been replaced by the required *net positive suction head*, $NPSH_R$. This dimensionless parameter is useful in determining the required operating conditions on the suction side of the pump.

As was true for the specific speed, N_s, the value for S_s commonly used is for peak efficiency. For a family of geometrically similar pumps, S_s should have a fixed value. If this value is known, then the $NPSH_R$ can be estimated for other pumps within the same family operating at different values of ω and *Q.*

SUCTION SPECIFIC SPEED

As noted for N_s, the suction specific speed is also dimensionless, and the value for S_s is independent of the system of units used. However, as was the case for specific speed, in the United States a modified dimensional form for the suction specific speed, designated as S_{sd}, is commonly used, where

$$S_{sd} = \frac{n(\text{rpm})\sqrt{Q(\text{gpm})}}{\left[NPSH_R(\text{ft})\right]^{3/4}} \qquad \textbf{Eq. 3-19}$$

For double-suction pumps the discharge, Q, is one-half the total discharge.

Typical values for S_{sd} fall in the range of 7000 to 12000. If S_{sd} is specified, Eq. 3-19 can be used to estimate the $NPSH_R$ for a given set of operating conditions.

However, this calculation would generally only provide an approximate value for the $NPSH_R$, and the actual determination of the $NPSH_R$ for a particular pump should be made through a direct measurement whenever possible.

PROBLEM 304:

Water is pumped with a centrifugal pump, and measurements made on the pump indicate that for a flow rate of 240 gpm the required input power is 6 hp. For a pump efficiency of 62%, what is the actual head rise of the water being pumped?

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Solution:

The actual head rise of water can be computed by using the pump efficiency given by

$$\eta_p = \frac{WP}{BP} = \frac{\gamma Q h}{BP}$$

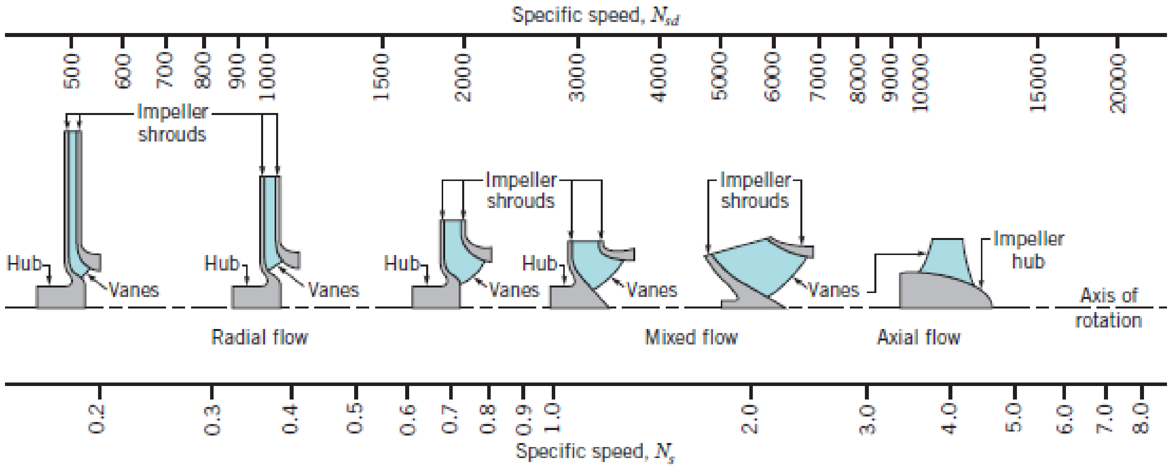
$$h = \frac{BP\,\eta_p}{\gamma Q}$$

$$h = \frac{6\text{ hp} \times 0.62 \times \dfrac{33{,}000\text{ ft - lb/min}}{1\text{ hp}}}{62.4\dfrac{\text{lb}}{\text{ft}^3} \times 240\dfrac{\text{gal}}{\text{min}} \times \dfrac{1\text{ ft}^3}{7.48\text{ gal}}}$$

$$h = 61.31\text{ ft}$$

PROBLEM 305:

In certain application a pump is required to deliver 5000 gpm against a 300-ft head when operating at 1200 rpm. What type of pump would you recommend?



PROBLEM 305:

In certain application a pump is required to deliver 5000 gpm against a 300-ft head when operating at 1200 rpm. What type of pump would you recommend?

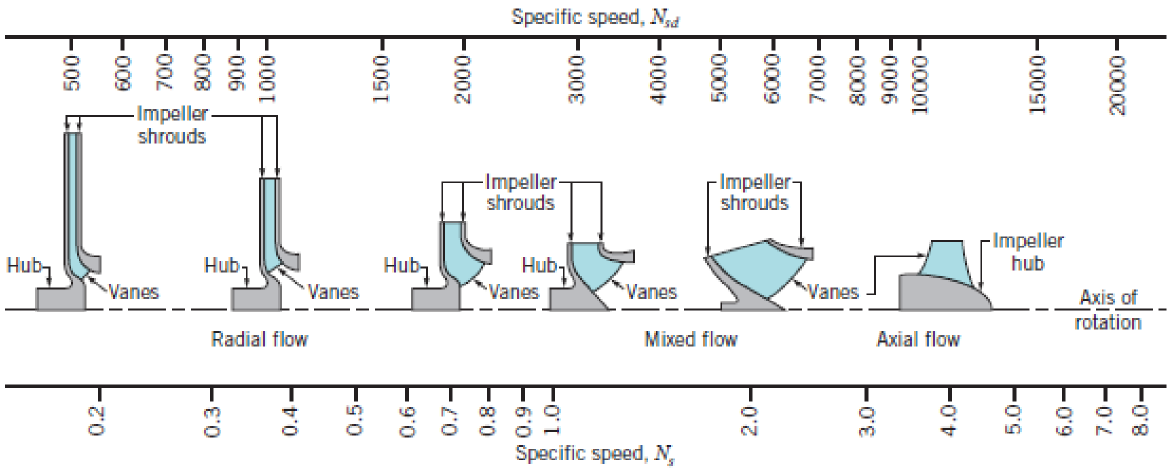
Solution:

Calculate the pump specific speed in English unit

$$N_{sd} = \frac{n\sqrt{Q}}{h^{3/4}} = \frac{1200\,(\text{rpm})\sqrt{5000(\text{gpm})}}{[300(\text{ft})]^{3/4}}$$

$$N_{sd} = 1177$$

Use radial flow type impeller or centrifugal pump



PROBLEM 306:

A centrifugal pump with a 12-in diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60-ft head. The impeller is changed to one with a 10-in diameter. Determine the expected flowrate, head and input power if the pump speed remains the same.

PROBLEM 306:

A centrifugal pump with a 12-in diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60-ft head. The impeller is changed to one with a 10-in diameter. Determine the expected flowrate, head and input power if the pump speed remains the same.

Solution:

a) Use Pump Affinity Laws to calculate the flowrate by the formula given as

$$\frac{Q_1}{D_1^3} = \frac{Q_2}{D_2^3}$$

$$Q_2 = Q_1\left[\frac{D_2^3}{D_1^3}\right]$$

$$Q_2 = 3200\text{ gpm}\left[\frac{10^3}{12^3}\right]$$

$$Q_2 = 1851.85\text{ gpm}$$

b) Calculating the head rise as a function of impeller diameter

$$\frac{h_1}{D_1^2} = \frac{h_2}{D_2^2}$$

$$h_2 = h_1\left[\frac{D_2}{D_1}\right]^2$$

$$h_2 = 60\text{ ft}\left[\frac{10}{12}\right]^2 = 41.67\text{ ft}$$

PROBLEM 306:

c) Calculating the brake power as a function of impeller diameter

$$\frac{BP_1}{\rho_1 n_1^3 D_1^5} = \frac{BP_2}{\rho_2 n_2^3 D_2^5}$$

$$\frac{BP_1}{D_1^5} = \frac{BP_2}{D_2^5}$$

$$BP_2 = BP_1\left[\frac{D_2}{D_1}\right]^5$$

$$BP_2 = 60\ \text{hp}\left[\frac{10}{12}\right]^5 = 24.113\ \text{hp}$$

PROBLEM 307:

(a) It is desired to deliver 100 L/s at a head of 270 m with a single-stage pump. What would be the minimum rotative speed that could be used? Assuming that the minimum practical specific speed is 10. (b) For the conditions of (a), how many stages must the pump have if a rotative speed of 600 rpm is to be used?.

PROBLEM 307:

(a) It is desired to deliver 100 L/s at a head of 270 m with a single-stage pump. What would be the minimum rotative speed that could be used? Assuming that the minimum practical specific speed is 10. (b) For the conditions of (a), how many stages must the pump have if a rotative speed of 600 rpm is to be used?.

Solution:

a) Use Pump Specific Speed by Daugherty, Franzini & Finnemore

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$n = \frac{N_s^* h^{3/4}}{\sqrt{Q}} = \frac{10(270\text{ m})^{3/4}}{\sqrt{(100\text{ L/s}/1000\text{ L})1\text{ m}^3}}$$

$$n = 2{,}106.31\text{ rpm}$$

b) Calculating the head if the rotating speed is 600 rpm

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$h = \left[\frac{n\sqrt{Q}}{N_s^*}\right]^{4/3}$$

$$h = \left[\frac{600\text{ rpm}\sqrt{0.1\text{ m}^3/s}}{10}\right]^{4/3}$$

$$h = 50.61\text{ m}$$

Calculating the number of stages yields

$$\text{Number of Stages} = \frac{270\text{ m}}{50.61\text{ m}} = 5.33\text{ say }6\text{ stages}$$

PROBLEM 308:

(a) Determine the specific speed of a pump that is to deliver 125 L/s against a head of 45 m with a rotative speed of 600 rpm. (b) If the rotative speed were doubled, what would be the flowrate and the head developed by this pump? Assume no change in efficiency. (c) Check the specific speed for the conditions given in (b). (d) Find the required operating speed of a two-stage pump to satisfy the requirements in (a).

PROBLEM 308:

(a) Determine the specific speed of a pump that is to deliver 125 L/s against a head of 45 m with a rotative speed of 600 rpm. (b) If the rotative speed were doubled, what would be the flowrate and the head developed by this pump? Assume no change in efficiency. (c) Check the specific speed for the conditions given in (b). (d) Find the required operating speed of a two-stage pump to satisfy the requirements in (a).

Solution:

a) Use Pump Specific Speed by Daugherty, Franzini & Finnemore

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$N_s^* = \frac{600\text{ rpm}\sqrt{0.125\text{ m}^3/s}}{(45\text{ m})^{3/4}}$$

$$N_s^* = 12.21$$

b) Calculating the flowrate and head if the rotating speed were doubled using Affinity Laws

$$\frac{Q_1}{n_1 D_1^3} = \frac{Q_2}{n_2 D_2^3}$$

$$Q_2 = Q_1\left[\frac{n_2}{n_1}\right]$$

$$Q_2 = 0.125\frac{\text{m}^3}{s}\left[\frac{1200}{600}\right] = 0.25\frac{\text{m}^3}{s}$$

$$\frac{gh_1}{n_1^2 D_1^2} = \frac{gh_2}{n_2^2 D_2^2}$$

$$h_2 = h_1\left[\frac{n_2}{n_1}\right]^2$$

$$h_2 = 45\text{ m}\left[\frac{1200}{600}\right]^2 = 180\text{ m}$$

PROBLEM 308:

c) Calculating the Pump Specific Speed for the conditions given in (b).

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$N_s^* = \frac{1200\ \text{rpm}\sqrt{0.25\ \text{m}^3/s}}{(180\ \text{m})^{3/4}}$$

$$N_s^* = 12.21$$

d) Identifying the operating speed of a two-stage pump but satisfying the requirements in (a).

$$h_{new} = \frac{45\ \text{m}}{2} = 22.5\ \text{m}$$

Also,

$$N_s^* = \frac{n\sqrt{Q}}{h^{3/4}}$$

$$n = \frac{N_s^* h^{3/4}}{\sqrt{Q}}$$

$$n = \frac{(12.21)(22.5\ \text{m})^{3/4}}{\sqrt{0.125\ \text{m}^3/s}} = 356.78\ \text{rpm}$$

RESTRICTION ON USE OF SIMILARITY LAWS

Similarity laws are of great practical value, but care must be exercised when applying them. Thus in comparing two machines of different sizes, the two must be homologous and the variation in the values of *h*, *D* and *n* should not be too large.

For example, a machine which operates satisfactorily at low speeds may cavitate at high speeds. The values of coefficients change somewhat as *h*, *D* and *n* are varied because the efficiencies of homologous machines are not identical.

Large machines are usually more efficient than smaller ones because their flow passages are larger. Also, efficiency usually increases with speed of rotation because power output varies with the cube of the speed while mechanical losses increase only as the square of the speed.

PERIPHERAL VELOCITY FACTOR

For a pump impeller, the ratio of its peripheral velocity to $\sqrt{2gh}$ is referred to as the *peripheral-velocity factor*, denoted by ϕ. Thus, for a centrifugal pump,

$$u_2 = \phi\sqrt{2gh} \qquad \textbf{Eq. 3-20}$$

But for practical engineering use,

$$n = \frac{60u}{\pi D} = \frac{60\phi\sqrt{2gh}}{\pi D} \qquad \textbf{Eq. 3-21}$$

which may be reduced to the convenient form

$$nD = 84.6\phi\sqrt{h} \qquad \textbf{Eq. 3-22}$$

For any machine its peripheral velocity might be any value from zero up to some maximum under a given head, and ϕ would consequently vary through as wide a range.

But the speed which is of most practical significance is that at which the efficiency is a maximum. The value of this dimensionless factor for this particular speed may be designated as ϕ_e.

EXERCISE 302

1. All dimensions of pump A are one-third as large as the corresponding dimensions of pump B. When operating at 300 rpm, B delivers 6 L/s of water against a head of 15 m. Assuming the same efficiency: (a) What will be the speed and capacity of A when it develops a head of 15 m: (b) what will be the speed and head of A when it delivers 6 L/s; (c) what will be the head and capacity of A when it operates at 300 rpm?

2. A 45 cm-diameter centrifugal-pump runner discharges 0.7 cu.m/s at a head of 30 m when running at 1,200 rpm. (a) If its efficiency is 85%, what is the brake power? (b) If the same pump were run at 1,800 rpm, what would be h, Q, and brake power for homologous conditions?

3. What head will the pump of Problem 2 develop if it is operating on MARS at 1200 rpm and delivering 0.7 cu.m/s?

4. For the pump in Problem 2, what are the values of flow, head and power coefficients? What is the specific speed?

THANK YOU!