Unsteady-State Heat Conduction

Transient Heat Conduction

Important Equations

Lumped Heat Capacity (LHC) Model when Bi < 0.1

General Equation for Temperature

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = exp\left[-\frac{hA}{\rho CV} t\right]$$

but note

$$\frac{\theta}{\theta_i} = exp\left[\frac{t}{\tau}\right] \quad \text{where Tau, } \tau = \frac{\rho CV}{hA}$$

General Equation for Heat Transfer

$$Q = (\rho CV)\theta_i \left(1 - exp\left[-\frac{t}{\tau}\right]\right)$$

Biot Number, Bi and Consideration for LHC

Biot Number Equation and Consideration for LHC

$$Bi = \frac{hL_c}{k} < 0.1$$

where: L_c = Characteristic Length = $\frac{V}{A}$

Characteristic Length (for Common Geometries)

Plane Wall	Cylinder	Sphere
$L_c = L$	$L_c = \frac{r_o}{2}$	$L_c = \frac{r_o}{3}$

Fourier Number

$$Fo = \frac{\alpha t}{L^2}$$

Important Equations

note:

$$Q_o = \rho c V(T_i - T_\infty)$$

One-Term Approximation Model when Bi > 0.1

Plane Wall

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo)\cos(\zeta_1 x^*)$$

or

$$\theta^* = \theta_o^* \cos(\zeta_1 x^*)$$

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$$

$$\frac{Q}{Q_o} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*$$

Infinitely Long (L >>> D) Cylinder

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo) J_0(\zeta_1 r^*)$$

or

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$$

$$\theta^* = \theta_o^* J_0(\zeta_1 r^*)$$

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)$$

Sphere

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo)\frac{1}{\zeta_1 r^*}\sin(\zeta_1 r^*)$$

or

$$\theta^* = \theta_o^* \frac{1}{\zeta_1 r^*}\sin(\zeta_1 r^*)$$

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$$

$$\frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3}[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)]$$

The quantities $J1$ and $J0$ are Bessel functions of the first kind