Diffusion - Determination of Diffusion Constants using the Schlieren Method

Protocol for the PC 2 lab course by
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Abstract:

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1 Theory

Molecular diffusion describes the movement of particles from a region of high concentration to a region of low concentration, which leads to an even distribution of particles.[1] It was first discovered by Robert Brown, who observed the random movement of tiny particles suspended in a liquid, which is called Brownian motion. Then German physicist Adolf Fick lay the theoretical foundation for diffusion by formulating two laws.
The first Fick's law describes how the particle flux density $j(z)$ is connected to the Diffusion coefficient D and the concentration gradient $\frac{\partial c(z)}{\partial z}$, which is shown in Equation 1.

$$j(z) = -D \cdot \frac{\partial c}{\partial z} \tag{1}$$

The physical meaning of the first Fick's law is that a concentration gradient leads to mutual diffusion of particles, leading to an even distribution. The diffusion coefficient can also be defined by the „Einstein relation" in Equation 2.

$$D = \frac{1}{2}\nu \Delta z^2 \tag{2}$$

ν is the jump rate and Δz^2 the mean square jump distance of particles, assuming that they can only move along the z axis. Second Fick's law describes the temporal course of concentration as shown in Equation 3.

$$\frac{\partial c}{\partial t} = D \cdot \frac{\partial^2 c}{\partial z^2} \tag{3}$$

Equation 3 states that the concentration equalization proceeds faster the stronger the concentration gradient. By integrating seconds Fick's law from Equation 3 under certain boundary conditions and using the substition $\mu = \frac{x}{\sqrt{4Dt}}$, Equation 4 for the profile of concentration can be obtained.

$$c = \frac{c_2}{2} \cdot \left(1 - \frac{2}{\sqrt{\pi}} \int_0^{\mu} e^{-\mu^2}\, d\nu\right) \tag{4}$$

By deriving Equation 4, the function is simplified into Equation 5 which leads to the concentration gradient profile having the shape of a Gaussian bell curve, as is illustrated in Figure 1.

$$\frac{dc}{dz} = -\frac{c_2}{2\sqrt{\pi D t}} \cdot e^{-\mu^2} \tag{5}$$

Assuming that the refractive index n is proportional to c and knowing that the gradient has an extreme value at $z = 0$, as can be seen in Figure 1, Equation 6 describes the refractive index gradient at $z = 0$ with the indices of water n_1 and of the salt solution n_2.[1]

$$\left(\frac{dn}{dz}\right)_{z=0} = -\frac{n_1 - n_2}{2\sqrt{\pi D t}} \tag{6}$$

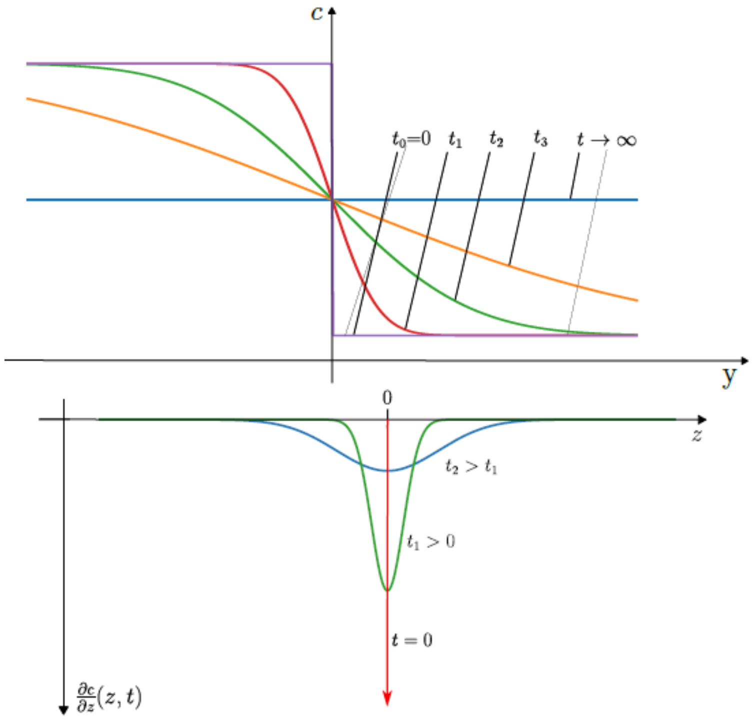

Figure 1: Profiles of the concentration and concentration gradient.[1]

Following this argumentation, diffusion leads to a continuously changing refractive index profile which causes a light beam directed to a cuvette of a salt solution with a concentration gradient to be refracted toward the optically denser medium. That is called Snell's law of refraction and shown in Figure 2. The exit angle β und the degree of curvature can be described with Equation 7 and 8, where α is the angle of incidence and r the radius of the curvature of the light beam.[1]

$$\beta = \frac{n}{n_0}\alpha \tag{7}$$

$$\frac{1}{r} = \frac{1}{n}\frac{dn}{dz} = \frac{d\ln(n)}{dz} \tag{8}$$

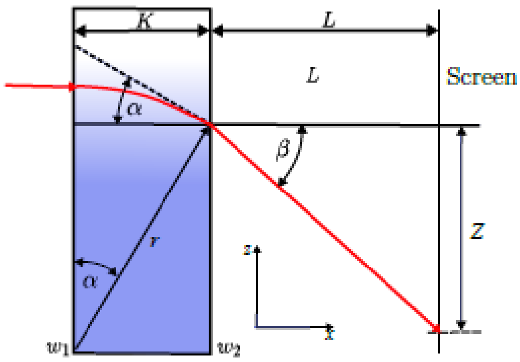

Figure 2: Scheme of the experimental setup geometry.[1]

The Huygen's principle states that every point of a wavefront can be seen a the origin of a spherical elementary wave. That's why the path of the light beam is curved in a medium with a spatially dependent refractive index $n(z)$ due to a lateral phase shift within the wavefront. The strongest deflection is experienced by the part of the beam that passes through the steepest concentration gradient region. With the small-angle approximation and Equation 7 and 8, the refractive index gradient can be described by Equation 9.

$$\frac{dn}{dz} \approx \frac{Z}{L} \cdot \frac{n_0}{K} \tag{9}$$

L is the distance between the cuvette and the screen, K the thinkness of the cuvette, n_0 the refractive index of air and Z the distance between the straight extension of the beam at the exit point and the impact point on the screen. By equating Equation 9 and Equation 6 and rearranging to the diffusion constant D, Equation 10 is derived to calculate D using the Schlieren Method.[1]

$$D = \frac{(n_1 - n_2)^2 L^2 K^2}{4\pi n_0^2 Z^2 t} \tag{10}$$

2 Procedure

The experimental setup was already built prior to the start of the measurements as shown in Figure 2. The laser beam was light path was tilted at about 45°.

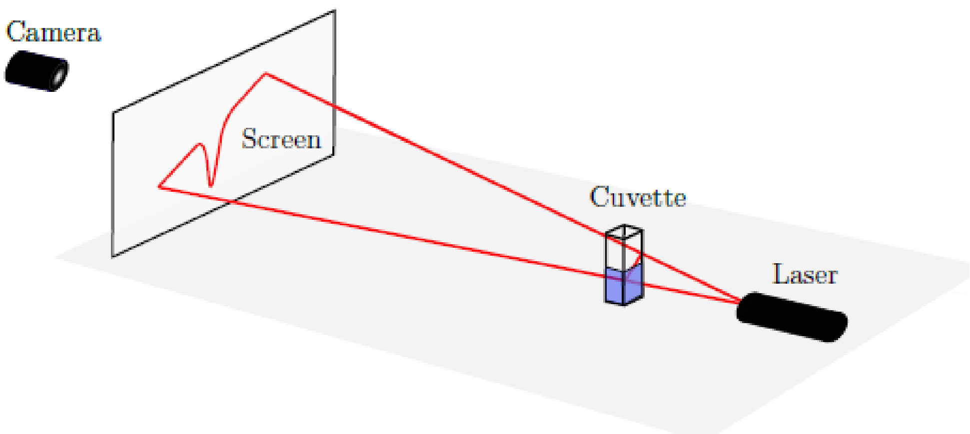

Figure 3: Scheme of the laser beam geometry.[1]

The experiment began with

3 Analysis

To evaluate **??**, Equation 10 is rearranged to Z^{-2} to get Equation 12, which shows the connection between the slope m and the diffusion coefficient.

$$Z^{-2} = \frac{4\pi D n_0^2}{(n_1 - n_2)^2 L^2 K^2} \cdot t \tag{11}$$

$$= m \cdot t \tag{12}$$

4 Error Discussion

5 Conclusion

6 References

[1] H. Dilger, *2025-pc2-script-en*, **2025**.