Implementation approaches of

Fixed-point iteration method for Systems of

Nonlinear equations

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# Basic Concepts

Let we need to solve a system:

Let us write it as operator:

,

We shall rewrite it in the form:

*,*

where ,

– numeric parameter, – matrix.

Consider an iterative process:

, [1; Pt. 2, Ch. 5, p. 208]

when

For such a case:

* [2; Ch. 4, §24, p. 183]
* Such estimates of accuracy is fair

, [3; Ch. 1, §1.5.3, p. 44]

, [4; Ch. 7, §1, p. 326]

[2; Ch. 4, §24, p. 181]

* Sufficient convergence condition:

[2; Ch. 4, §24, p. 181]

* Necessary convergence condition:

[4; §5, p. 52]

* Priority estimate for accuracy :

[2; Ch. 4, §24, p. 181]

# Computer implementation questions

1. Norm selection

The most convenient norm is:

Because is searched as max value and most of math packages uses box searching borders, which corresponds norm.

1. selection

We should pick minimum possible to facilitate search of maximum:

Also we should substitute max appropriate , it equals 1-EPS.

1. selection

Eight systems were reviewed:

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3)

– does not work

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4)

– does not work

– does not work

– does not work

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5)

– does not work

– does not work

6)

– does not work

– does not work

7)

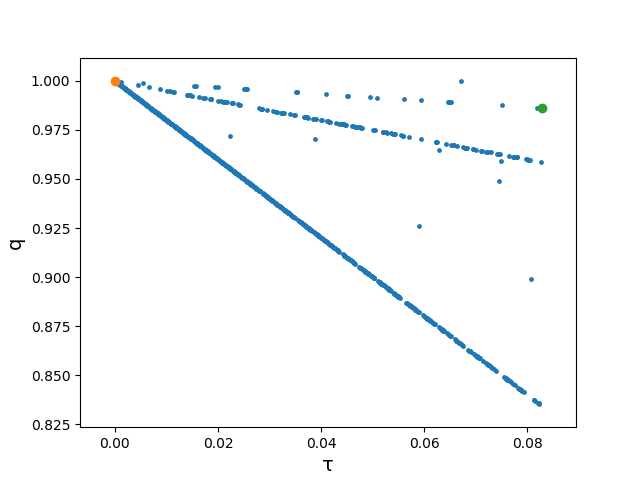
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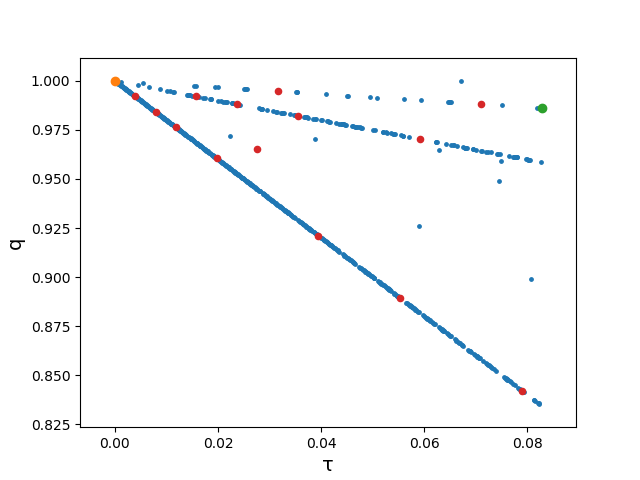
8)

– does not work

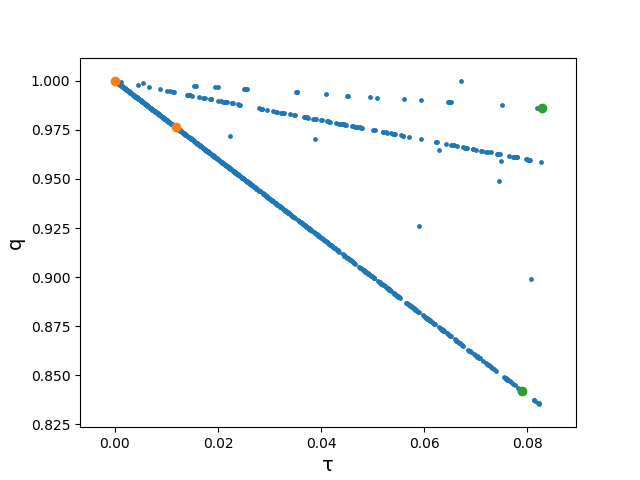
– does not work

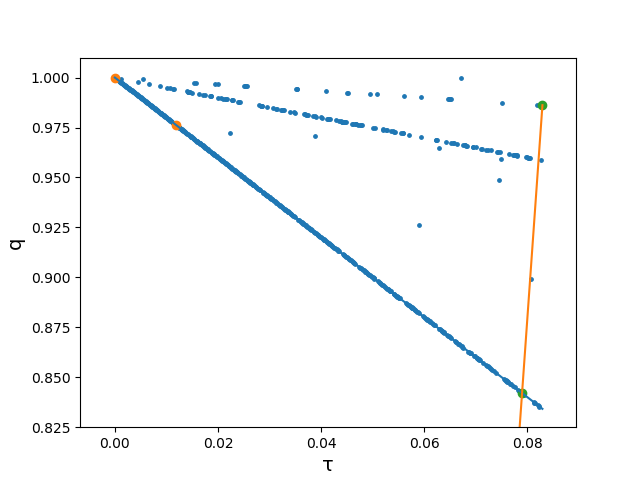
## Algorithm

1. Find first point from left and from right, with EPS step:
2. Divide interval by N equal distance points and find maximum angle from end points:

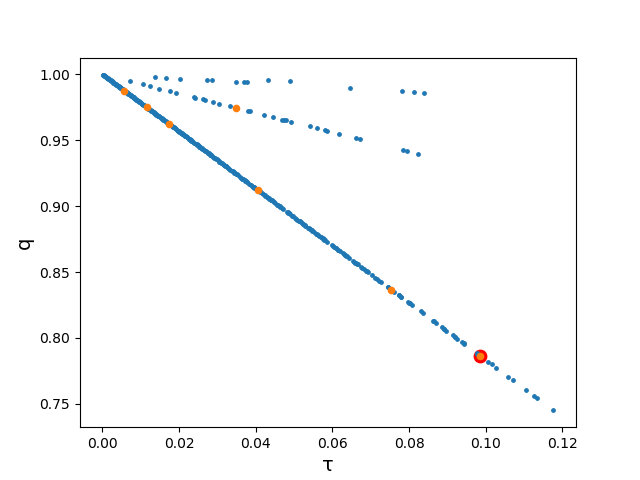


(gray – some angle, purple – optimal)



1. Find intersection of lines formed by this points:

In case when end point is under max-angle point we take the first or the last of N points founded before:



Afterword

The above method allows us to locate almost optimal value in minimal computations (22 in examples).

References

1. *A. A. Samarsky, A. V. Gulin “Chislennyye metody” (“Numerical methods”). “Nauka”, 1989.*
2. *E. A. Volkov “Chislennyye metody” (“Numerical methods”). Nauka", 1987.*
3. *Ya. S. Bugrov, S. M. Nikolsky "Vysshaya matematika. Differentsial'nyye uravneniya. Kratnyye integraly. Ryady. Funktsii kompleksnogo peremennogo" (“Higher Mathematics. Differential Equations, Multiple Integrals, Series, Theory of Functions of a Complex Variable”). “Drofa”, 2004. ISBN 5-7107-8450-8.*
4. *M. M. Moskalkov, A. I. Ryzhenko, S. O. Voitsekhovsky, A. V. Kuzmin, O. F. Kashpur etc. “Praktykum z metodiv obchyslenʹ” (“Workshop on calculation methods”). MAUP (IAPM—Interregional Academy of Personnel Management), 2006. ISBN 966-608-504-6.*