

## D&D Deep Dive

November 2025

For this lab you will be focusing on functions of dice rolls to calculate probability distributions. You will then be introduced to expected values. Finally, this lab sets us up for sampling distributions on Wednesday.

Alright, so the following questions are going to deal with DnD dice which have varying number of sides (most common number of sides are 4,6,8,10,12, and 20). As a reminder, if you see 3d8 you read it as three 8-sided dice.

1. Write out the probability distributions for 1d4, and the sum of 2d4.
  2. Comment on the shape of the two distributions
  3. Find the probability distribution for 1d4 if we multiple the dice's value by 2. (Don't overthink the question, it's still uniform).

My DnD group once asked if for critical hits do we double the sum of the dice or do we roll twice as many dice. I replied that rolling twice as many dice is the safer choice.

Let  $x$  be the value of the dice roll and  $p_i$  be the probability of seeing outcome  $i$  ( $i = 1, 2, \dots, S$ ) on an  $S$ -sided dice. The **expected value** of the random variable  $X$  is the theoretical average of our dice rolls which for discrete variables can be defined as...

$$E(x) = \sum_{i=1}^S x_i p_i$$

which means each outcome is multiplied by its probability.

4. Find the expected value for the sum of 2d4.
5. Find the expected value for the probability distribution you found in question 3.
6. Graph, by hand, the distributions questions 4 and 5. HINT: A histogram or two is probably your best bet.
7. Given the three questions above, why did I claim that rolling twice the number of dice is safer than doubling the value of the dice? Assume larger dice totals are better.

8. *Roll with advantage* means to roll the dice twice and take the better of the two results. Find the probability distribution for rolling a 1d8 with advantage. HINT: Draw an 8x8 grid; each cell/intersection will be the larger of the two numbers. You can count the points for each outcome or you can try to see the pattern.
  9. *Roll with disadvantage* means that we take the worse (usually lower) of the two rolls. How do you think a 1d8 rolled with disadvantage will look like compared to the above question?
  10. Find the distribution of 1d8 rolled with disadvantage.
  11. Find the expected value for the 1d8 rolled with disadvantage  
dcm
  12. Take whatever die (singular of dice but feels weird to use...) was assigned to you and/or your lab group and begin to roll it.
    1. Roll the die 100 times
    2. Every 10 rolls calculate the mean of those ten rolls, put it in the chart on the next page in the middle column.
    3. Fill in the other column in the table which is the mean for all rolls up to that point. HINT: You can average the middle column to get the running average
    4. Add the values from the middle column onto the histogram on the board
    5. Add your final mean for all rolls on the board under your dice

Roll Number	Mean of Last 10 Rolls	Mean of All Rolls (ie running average)
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		

13. Find the expected value for the dice that was assigned to you
14. Comment on if there seems to be a relationship between the expected value from question 13 and the “Mean of All Rolls” column.
15. Bonus Point: What is the probability distribution for 2d20? HINT: Carry the pattern in question 3 outwards.