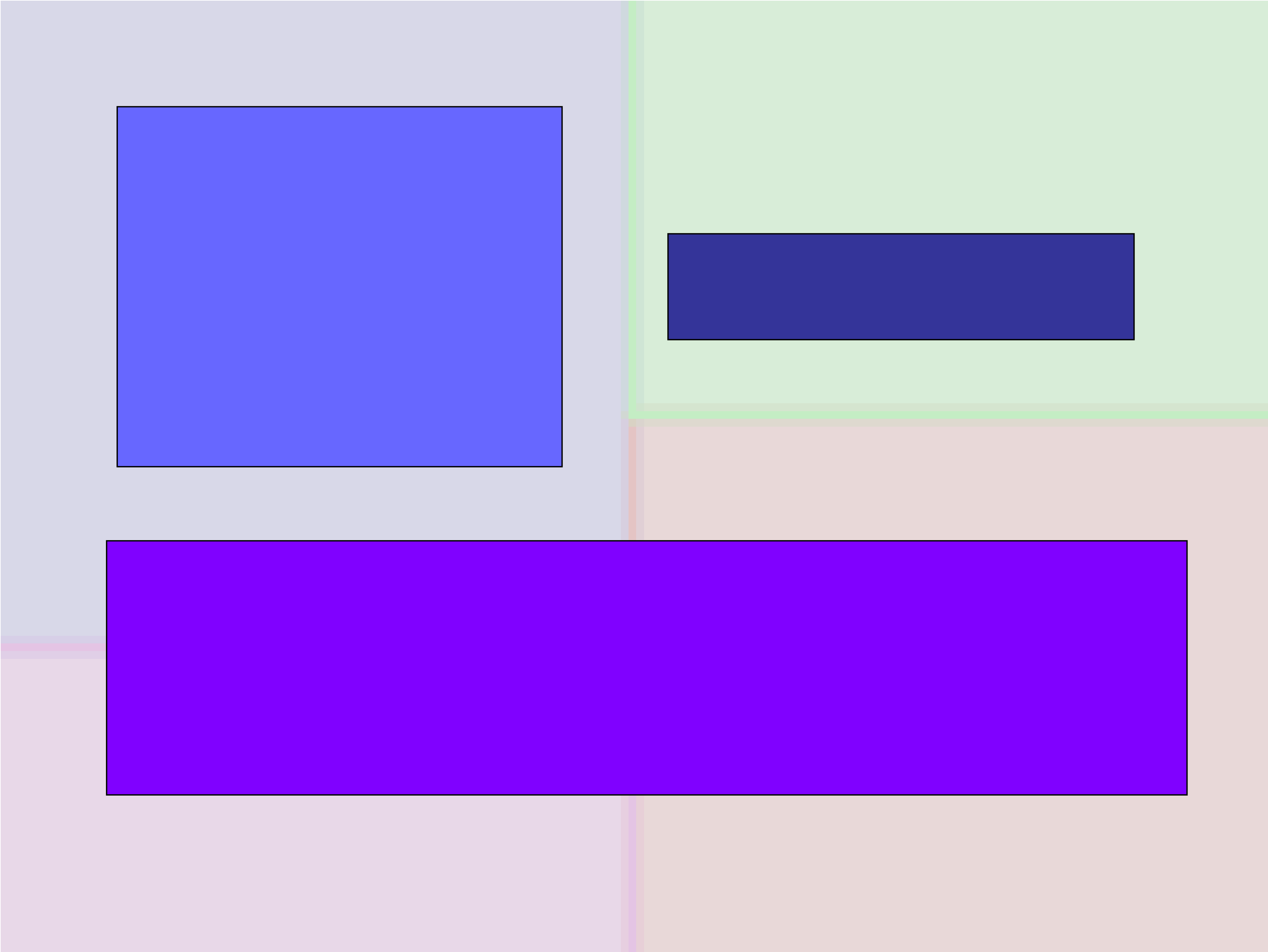
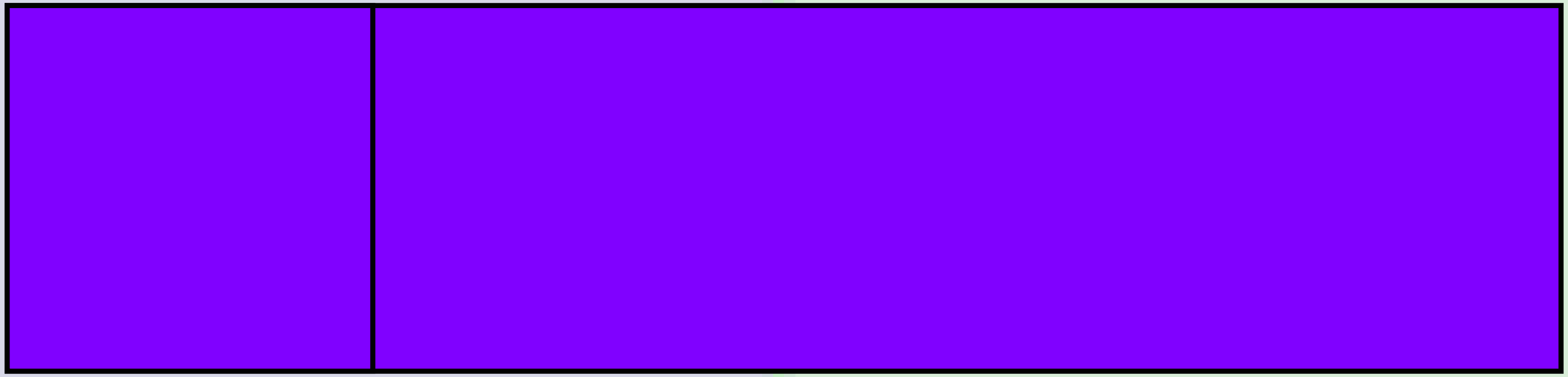
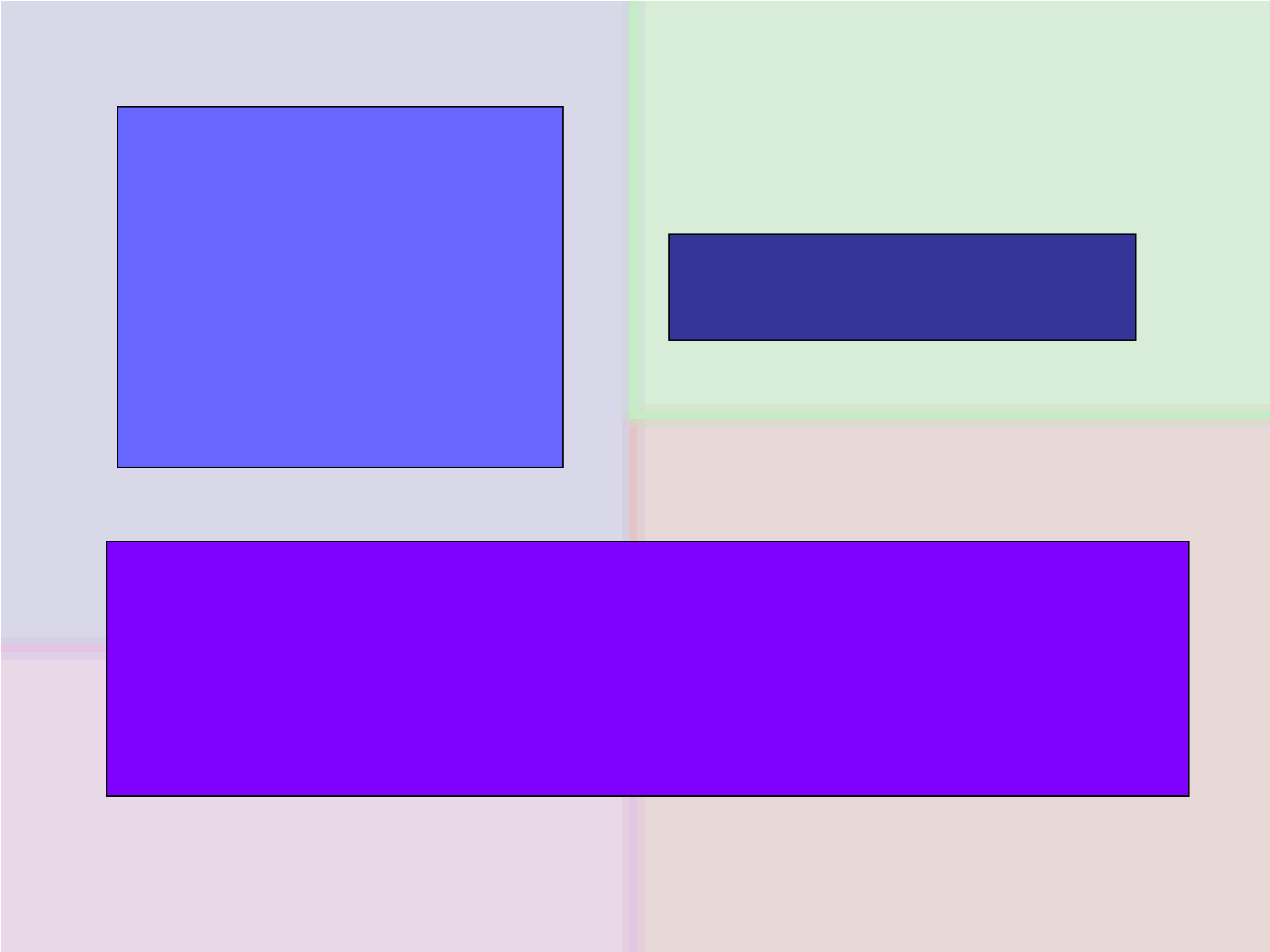




Create a square inside the rectangle by drawing only one line.

Continue this process without using the area of the previous square until you can no longer fit a square.



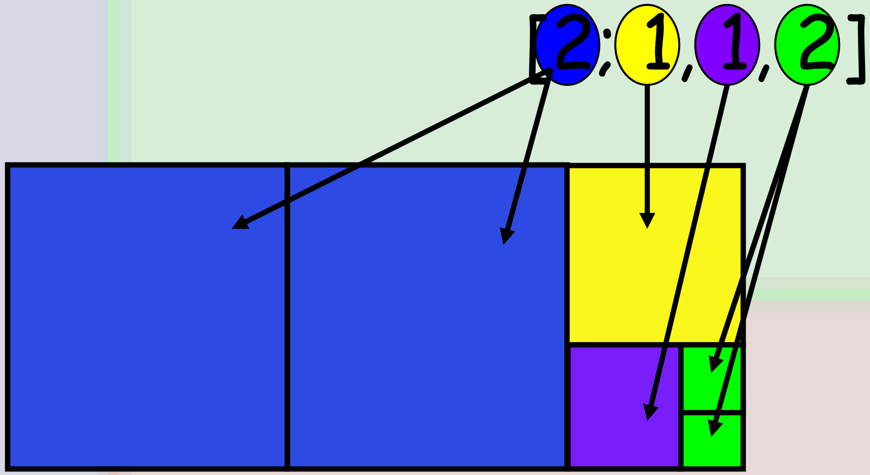
Let’s try more…

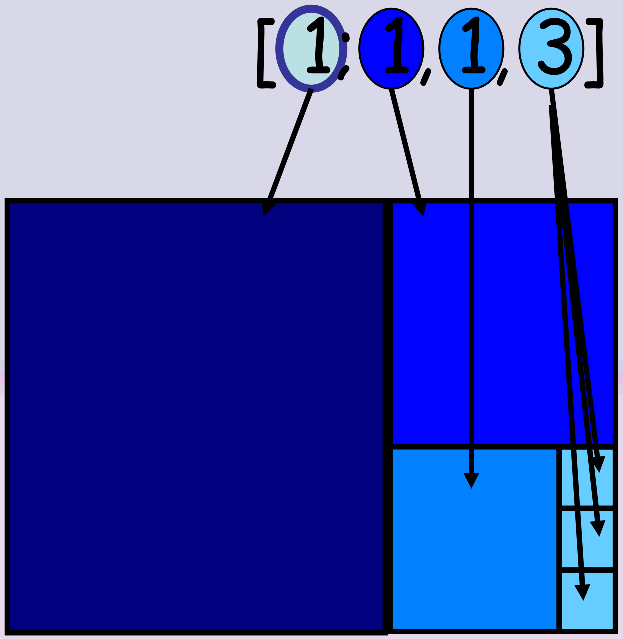
WHAT DO YOU THINK?

- Did all of your rectangles split up the same way?
- Were you always able to fill the rectangle completely?
- If we told you the squares configuration could you tell us what the rectangle will look like?

First let's look at your rectangles on paper…

We can show this square configuration
in short notation

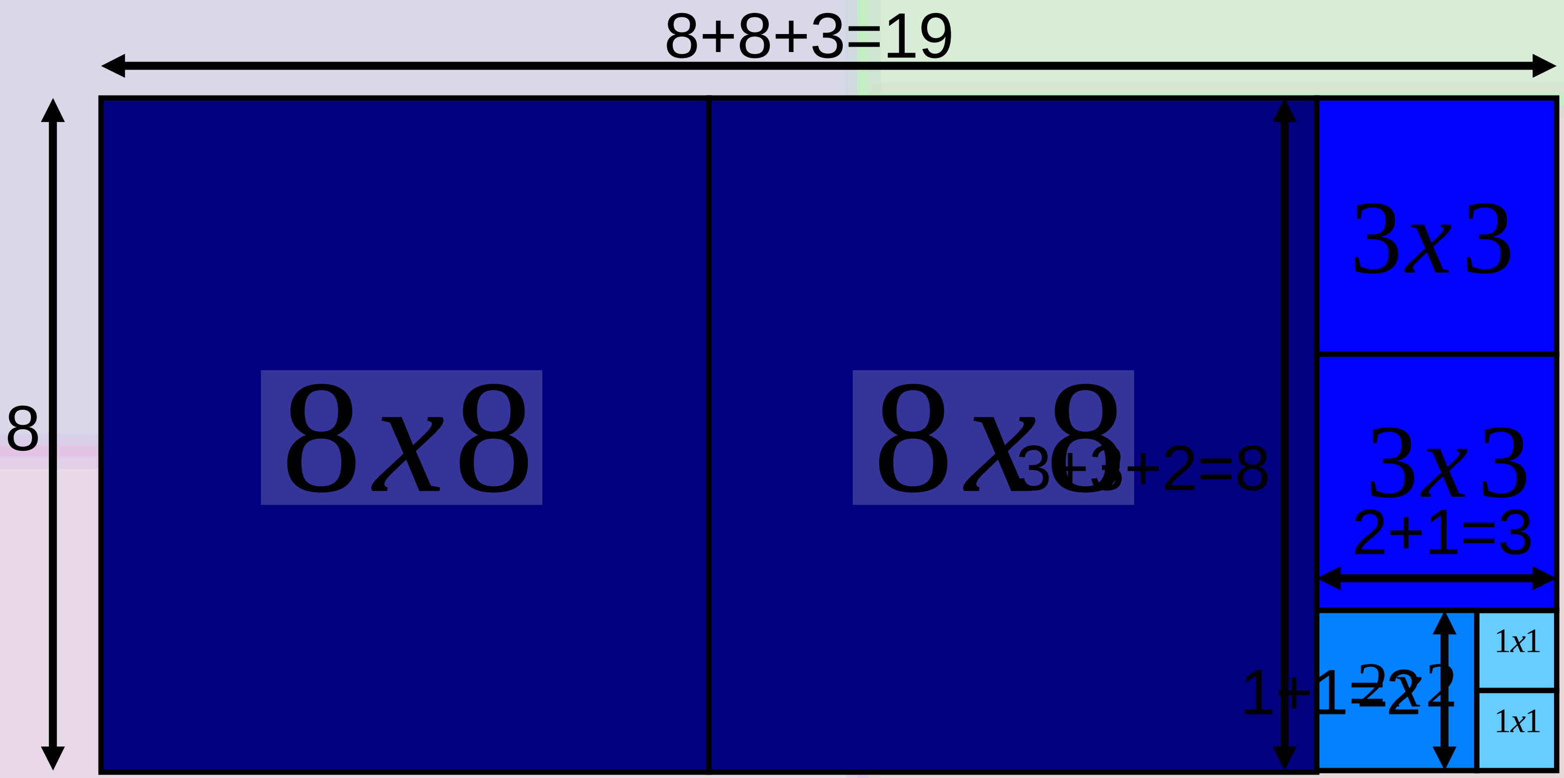




Now you try one...

$$\frac{19}{8}$$

Evaluate: [2; 2, 1, 2]



What were the ratios of the rectangles that you've tried so far?

$$\frac{13}{5} \qquad \frac{11}{7} \qquad \frac{26}{10} \qquad \frac{19}{8}$$

What kind of numbers are these?

Whole numbers

Decimals

Fractions

Rational numbers

Integers

Irrational numbers

Let's try another rectangle…

NOW…
WHAT DO YOU THINK?

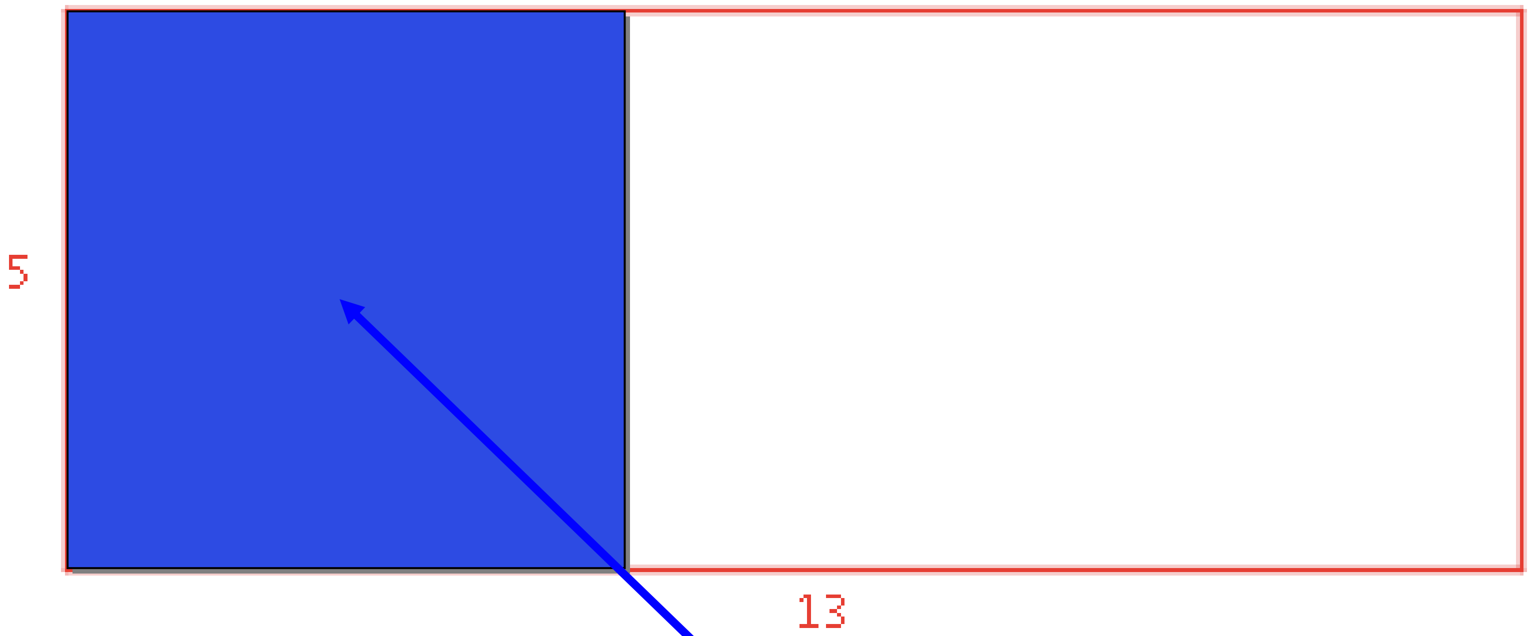
- Did this rectangle split up the same way?
- Were you able to fill the rectangle completely?
  - Why not?

Hmmmm…What was the ratio of the sides for that rectangle?

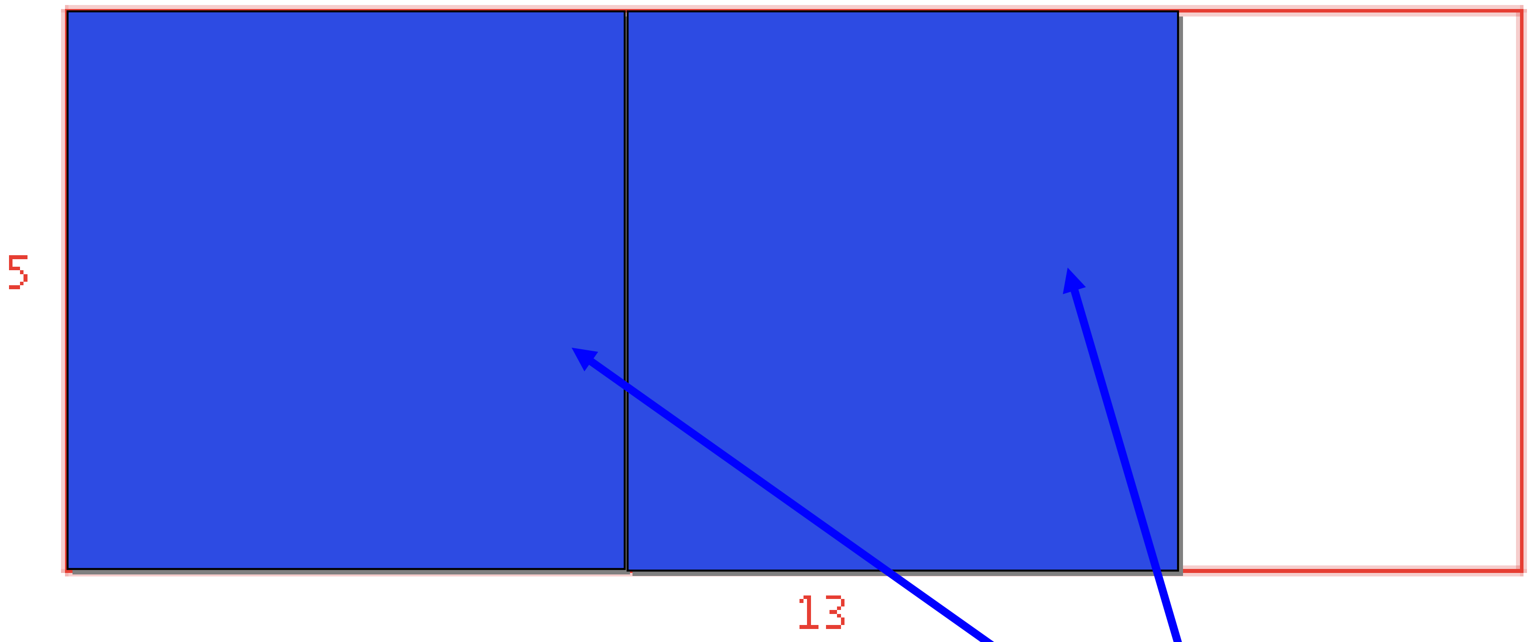
Do you think it is possible to figure out the decomposition of a rectangle without drawing a picture?

Is it possible that the math is hidden within the ratio itself?

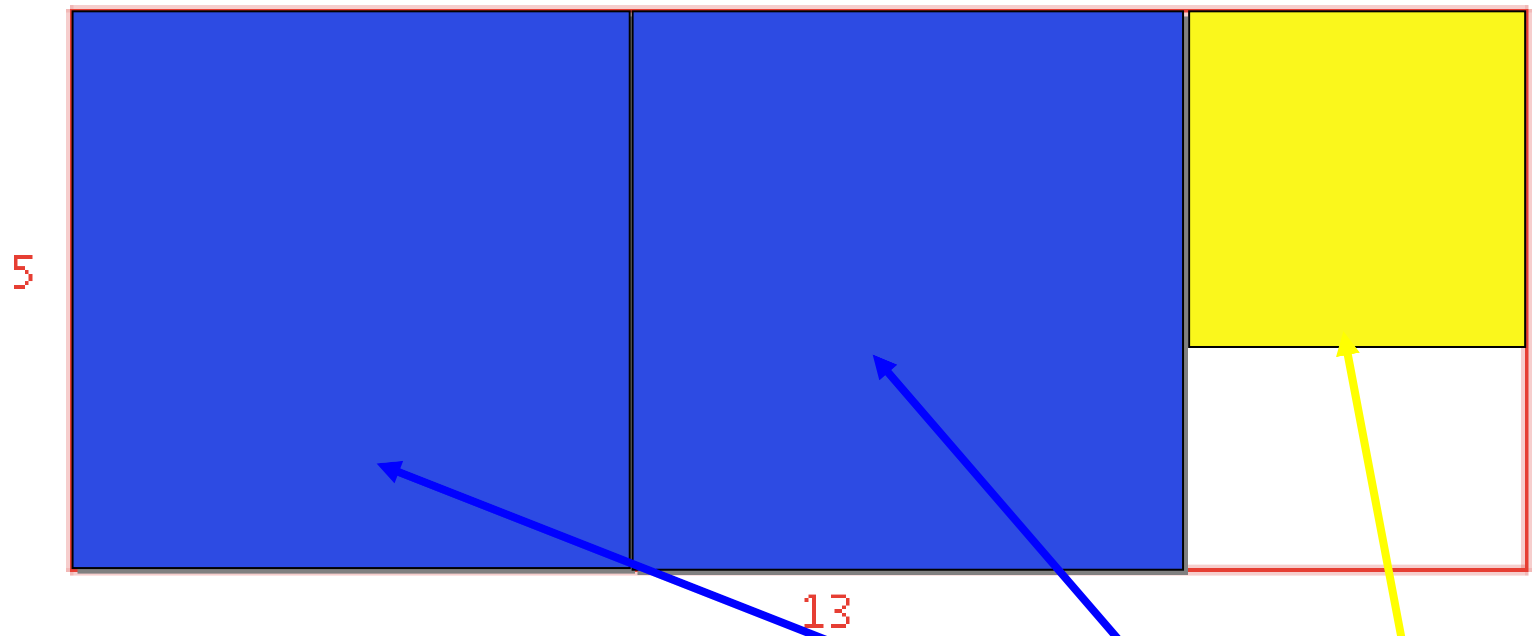
Let's find out…



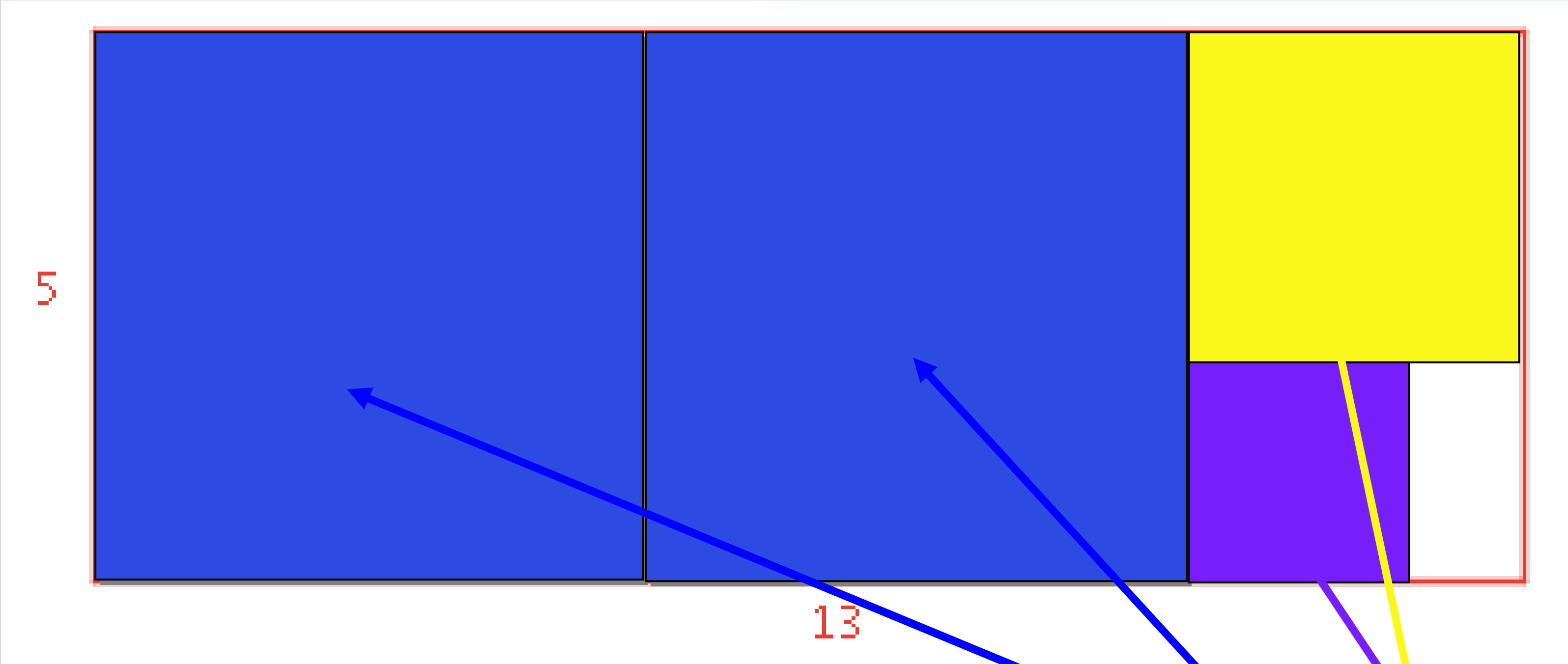
$$\frac{13}{5} = \frac{5}{5} + \frac{8}{5} = 1 + \frac{8}{5}$$



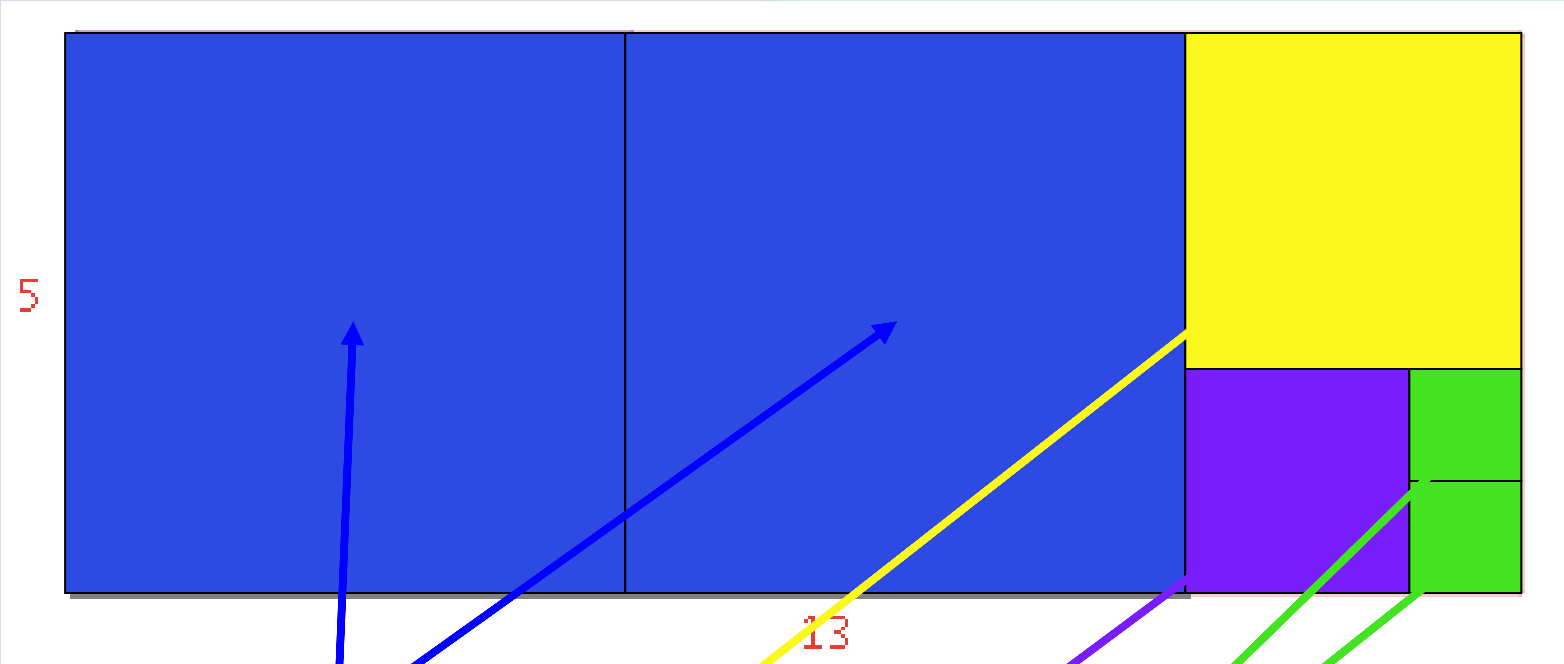
$$\frac{13}{5} = 1 + \frac{8}{5} = 1 + 1 + \frac{3}{5} = 2 + \frac{3}{5}$$



$$\frac{13}{5} = 2 + \frac{3}{5} = 2 + \frac{1}{\frac{5}{3}} = 2 + \frac{1}{\frac{3}{3} + \frac{2}{3}} = 2 + \frac{1}{1 + \frac{2}{3}}$$



$$\frac{13}{5} = 2 + \cfrac{1}{1 + \cfrac{2}{3}} = 2 + \cfrac{1}{1 + \cfrac{1}{\cfrac{3}{2}}} = 2 + \cfrac{1}{1 + \cfrac{1}{\cfrac{2}{2} + \cfrac{1}{2}}} = 2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$$



$$\frac{13}{5} = 2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$$

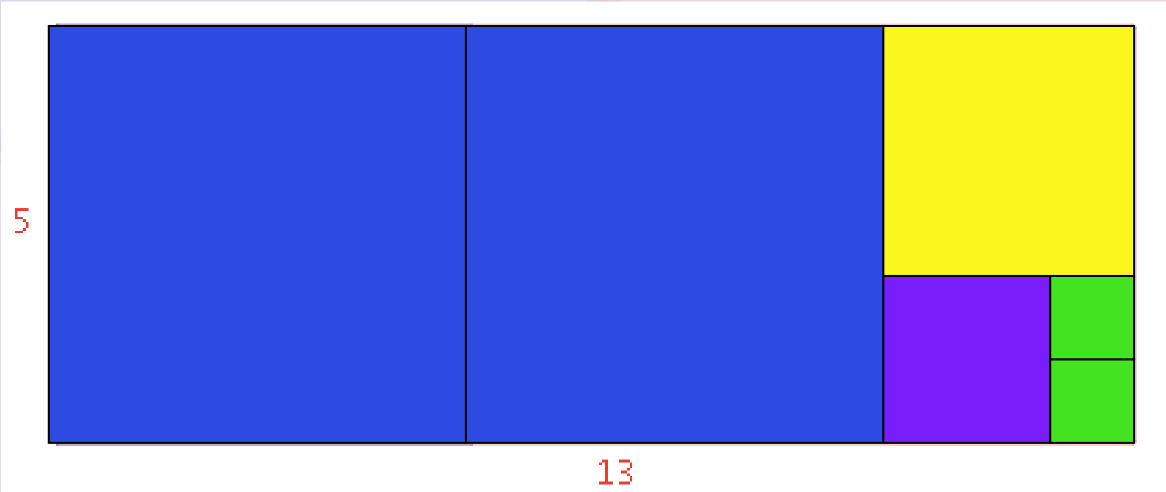
A Continued Fraction

Continued Fractions

$$\frac{13}{5} = 2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$$

Notation

[2;1,1,2]



Try the math again…Write the Continued Fraction

$$\frac{16}{9}=1+\frac{7}{9}=1+\frac{1}{\frac{9}{7}}=1+\frac{1}{1+\frac{2}{7}}=1+\frac{1}{1+\frac{1}{\frac{7}{2}}}=1+\frac{1}{1+\frac{1}{3+\frac{1}{2}}}$$

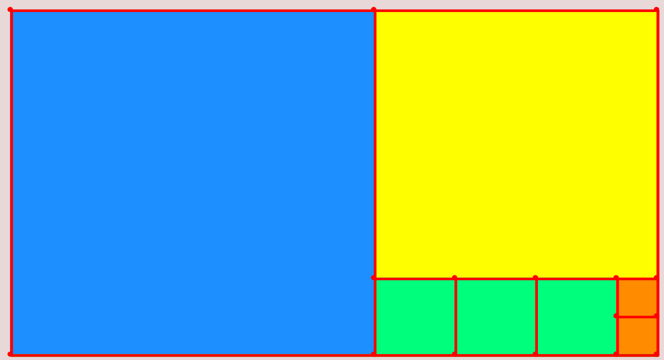
$$\frac{16}{9}=[1;1,3,2]$$

How can we check if this is correct?

Look at the decomposition of the rectangle

Algebraically, work from the bottom up

$$\frac{16}{9} = [1; 1, 3, 2] =$$



Algebraically…

$$1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2}}}$$

Add

$$1+\cfrac{1}{1+\cfrac{1}{\frac{7}{2}}}$$

Simplify

$$1+\cfrac{1}{1+\frac{2}{7}}$$

Add

$$1+\cfrac{1}{\frac{9}{7}}$$

Simplify

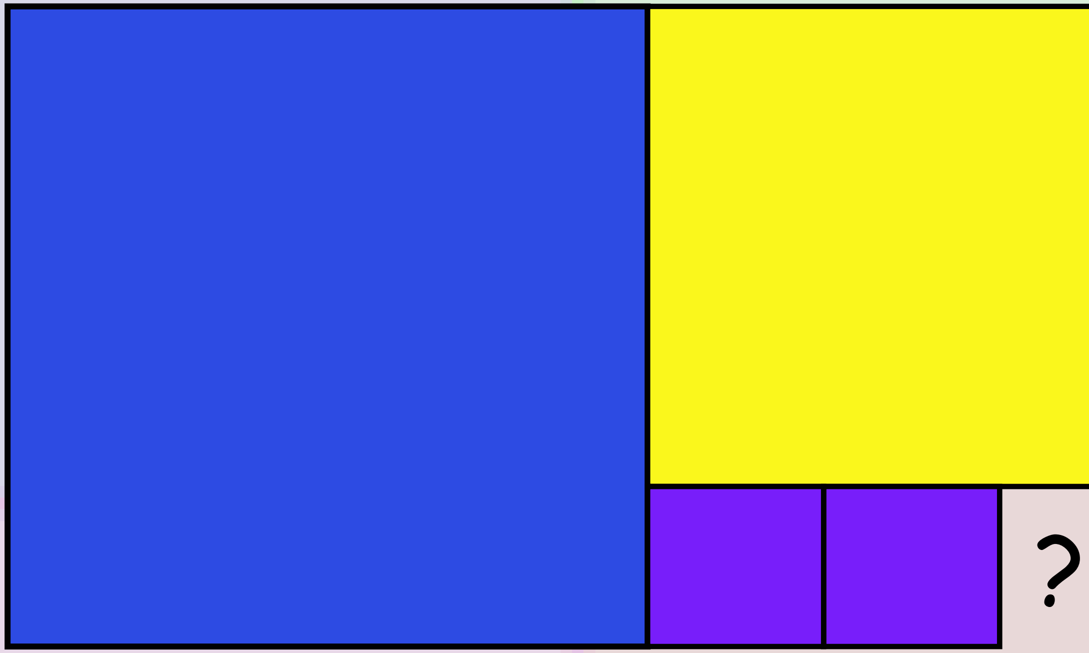
$$1+\frac{7}{9}$$

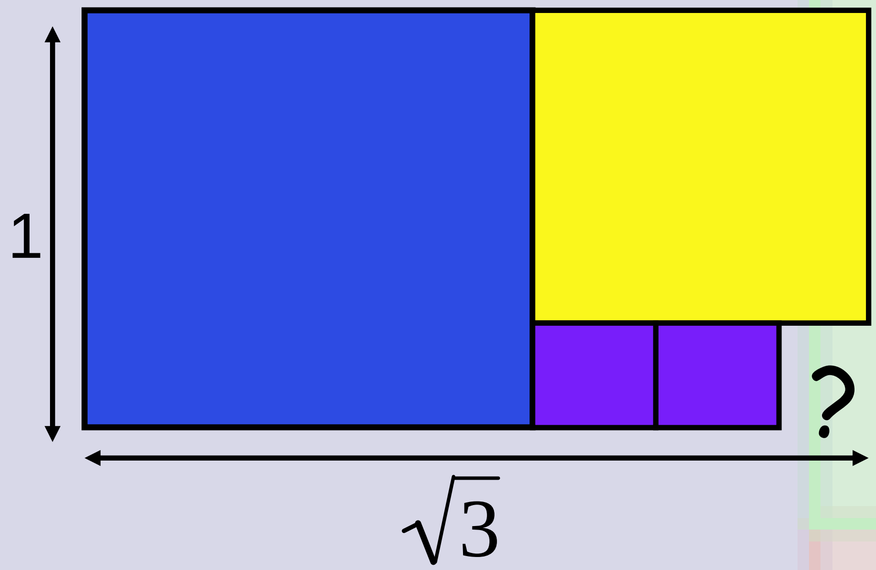
Add

$$=\frac{16}{9}$$

Why didn't this one work?

The ratio of the sides is irrational





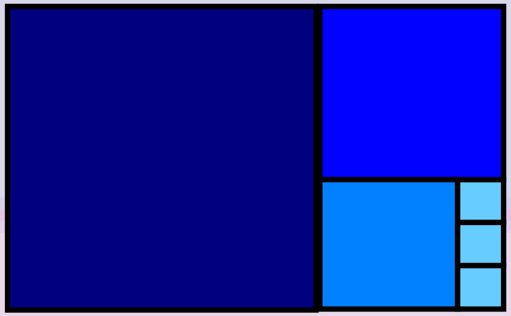
$$\frac{\sqrt{3}}{1}$$

This rectangle had side lengths of : $\sqrt{3}\ X\ 1$

Decimal Expansion begins: 1.73205080756887

Continued Fraction Notation for is: [1; 1, 2, 1, 2, 1, 2…]

A number is rational if and only if it can be expressed as a finite continued fraction



$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}$$

Continued Fractions

– Relates to the oldest algorithm known to Greek Mathematicians 300 BC

Euclid's Algorithm (gcf)

SUNSHINE STATE STANDARDS…

MA.A.1.3.1	MA.A.1.3.4
MA.A.3.3.1	MA.A.3.4.3
MA.C.2.3.1	MA.C.2.3.2