PHYC90045 Introduction to Quantum Computing

Lab Session 1

1.1 Introduction

Welcome to Lab-1 of PHYC90045 Introduction to Quantum Computing.

The purpose of this first lab session is to:
- learn to use the Quantum User Interface (QUI) to program/simulate quantum circuits
- use the QUI to analyse single qubit states and Bloch sphere representation
- use the QUI to understand single qubit operations.

1.2 The Quantum User Interface

The QUI is a web-based graphical user interface developed by the Hollenberg group at the University of Melbourne to program, simulate and analyse quantum circuits. The QUI allows the users to specify qubit number, build and simulate quantum circuits by easy drag-and-drop placement of quantum gates, and examine the quantum state at every time step in the circuit/program. The latter feature is critical to understanding QC, and distinguishes the QUI from other on-line QC programing/simulation tools.

1.3 Sign up and start the QUI

The QUI is accessed through a web-based interface (quispace.org – click on blue QUI logo). As per the subject announcement you should have signed up prior to the Lab-1 class – if you haven't please let a demonstrator know and follow these steps.

Step 1: Open a web browser (preferably Google Chrome or Firefox), go to quispace.org.

Step 2: You will need to create an account to access QUI for the first time.

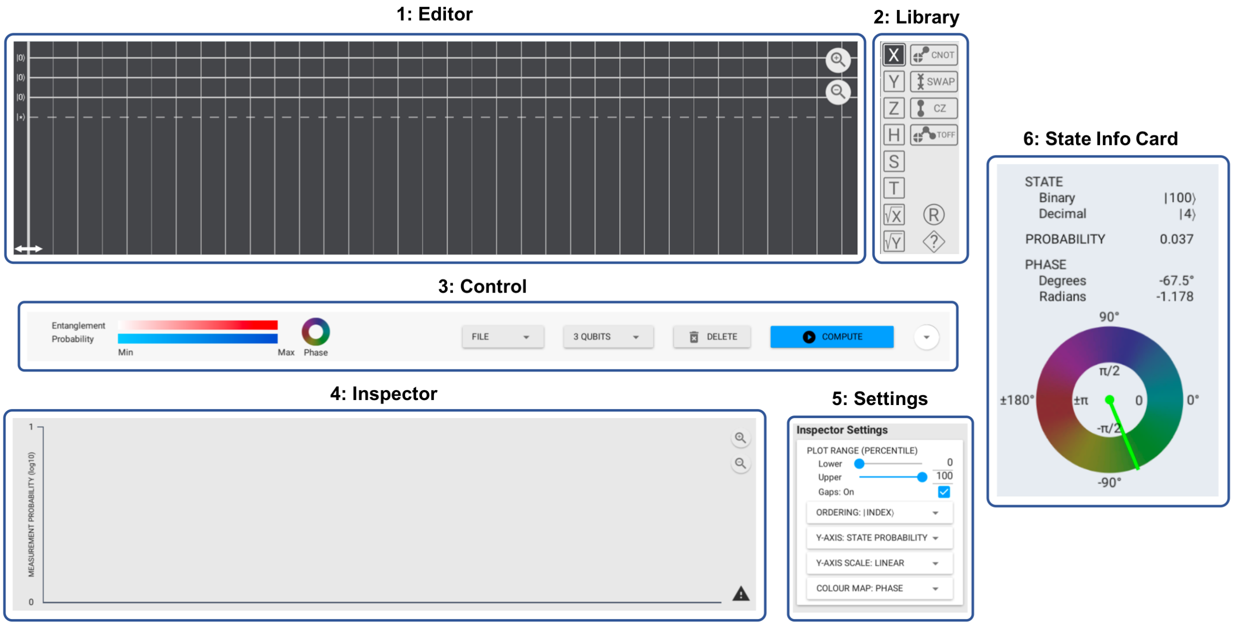
Note: you must use your University of Melbourne email address as your login name.

Follow the steps to create your account (including answering a few simple questions about use and education level).

Step 3: Once you have signed-up, start the QUI.

1.4 The QUI panels

Shown below are the various QUI panels.



Editor (Panel 1): This is where a quantum circuit is programed by dragging and dropping from the gate Library (Panel 2). On the left side, the initial states of three qubits (for this example) are shown in the default |0⟩ state. You can add more qubits using the |*> icon (or QUBITS in the Control panel). The vertical lines separate time steps left to right. The thicker vertical with the left-right arrow is the "time-slider", which can be moved to analyse the quantum state at different times in the quantum circuit. The time-slider will also display the degree of entanglement at various parts of the quantum circuit (Lab 2).

Library (Panel 2): This panel shows the gate library (quantum operations), including measurement. Hovering the cursor over any single-qubit gate will display the animation of the corresponding operation on the Bloch sphere. Select one of these gates and place it in the Editor panel at the desired location(s). There is even a paint feature.

Control (Panel 3): The colour scales on the left side indicate the min-max range of various output parameters (probability, entanglement, phase). Next is a drop-down FILE menu (New, Save, Load, Save As). QUBITS allows the users to select the number of qubits. <u>DELETE</u> removes selected quantum gates from the circuit. <u>COMPUTE</u> will run the current circuit and display the results in the Inspector panel (more accurately, the circuit is sent to our quantum simulator and the entire set of results (all time points) is sent back to your QUI session for analysing – please don't sit there and hit compute a million times).

Inspector (Panel 4): Here you can visualise the quantum state (after compute) at any time point corresponding to the position of the sliding time-slider. By default, the horizontal axis is ordered according to the state index in the computational basis (|11> = |3> etc), and the vertical axis by default gives the probability of each state. Detailed information about the state amplitudes (magnitude and phase) is given in the State Info Card that pops up when the mouse hovers over a state in the histogram. Note the hazard warning symbol bottom right – this indicates the current Inspector plot does not contain up-to-date data (i.e. with respect to changes in the circuit shown in the Editor).

Settings (Panel 5): Here the user has additional control over the various plotting options in the Inspector, such as PLOT RANGE, ORDERING, and AXIS (data) options.

State Info Card (Panel 6): This panel appears when the cursor is on a particular state in the Inspector panel and will display all information about the state's identity (binary and integer index), complex valued amplitude (magnitude and phase), and probability.

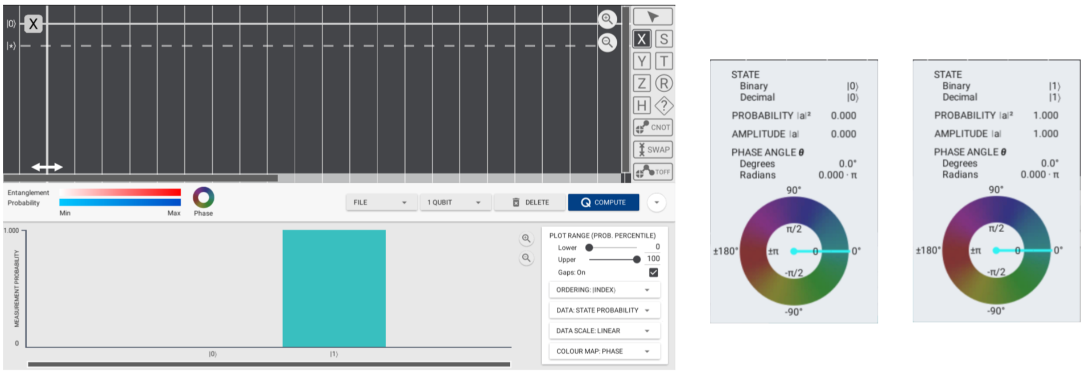
1.5 Quantum Gates, Amplitude, Probability and Phase

Now that you have started the QUI, the next step is to program some simple quantum gates to see how it works.

Exercise 1.5.1 Let's start with a simple X gate. Recall that an X gate flips the state of a qubit: for example: X|0⟩ → |1⟩. To program a X gate in the QUI do the following:

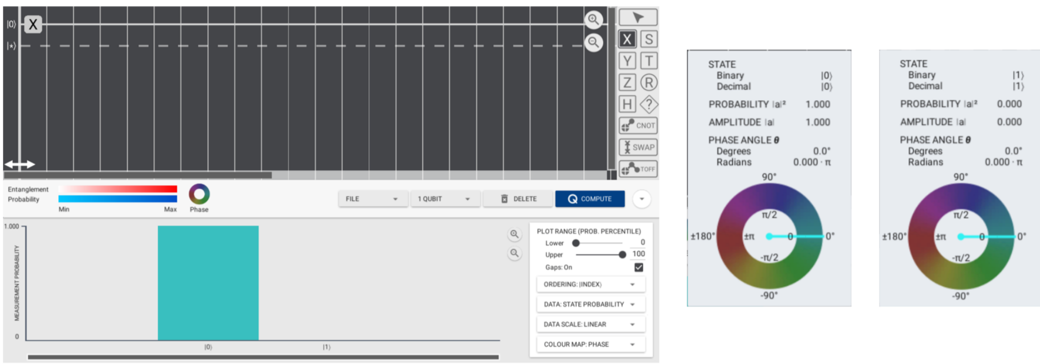
- select 1 QUBIT (Control Panel)
- click on an X gate (Library), place it in the first position on the qubit line (Editor).
- hit the COMPUTE button.

The results will look like this:



The output state (time-slider after X) contains the |1⟩ state with probability 1.0 (and zero phase) and the |0⟩ state with zero probability (i.e. doesn't exist). Point to the relevant part of the plot to bring up the State Info Cards (SICs) – shown on right above.

Now drag the time-slider to the first time step to inspect the state before the application of the X gate. The results will look like this (SICs for each state shown on the right):



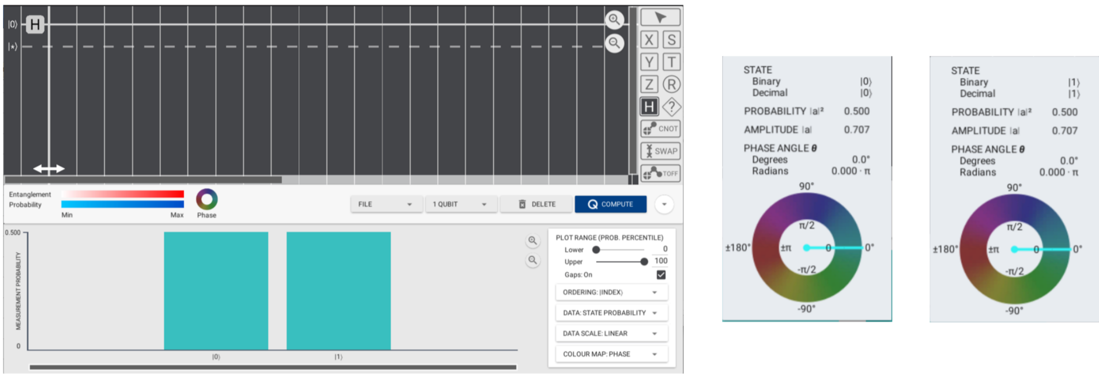
Before you go on further make sure you understand the information in the State Info Cards for this example (ask a tutor, or refer to lecture notes).

Exercise 1.5.2 Now let's instead use a Hadamard gate (denoted by H). Recall that this gate places the qubit into an equal superposition state:

$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

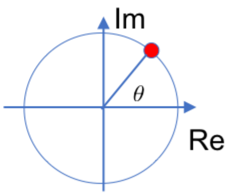
In order to test this gate in QUI:

- delete the X gate (using the DELETE button)
- replace it by a H gate from the Library.
- click on COMPUTE to find the outcome state after H operation:



Quantitative information on the SICs indicates that both $|0\rangle$ and $|1\rangle$ states have equal amplitudes of $\frac{1}{\sqrt{2}}$, zero phase, and probabilities of $[1/\sqrt{2}]^2 = 1/2$.

In the above cases the phase angle of the amplitude was zero, but we will soon encounter examples where this is not the case. You may need to recall the basic transformations between Cartesian and polar descriptions of complex numbers:



$e^{i\theta} = \cos\theta + i\sin\theta$

$$a = \mathrm{Re}[a] + i\,\mathrm{Im}[a] = |a|e^{i\theta} \rightarrow \begin{array}{l} |a| = \sqrt{\mathrm{Re}[a]^2 + \mathrm{Im}[a]^2} \\ \theta = \tan^{-1}(\mathrm{Im}[a]/\mathrm{Re}[a]) \end{array}$$

As per lecture notes, the State Info Card (SIC) in the QUI gives the value of the complex amplitude a_i of a given basis state component $|i\rangle = |0\rangle, |1\rangle$ is given by $a_i = |a_i|e^{i\theta_i}$ where $|a_i|$ is the magnitude, and θ_i is the phase angle (colour wheel scale). The probability of measuring the state $|i\rangle$ is $|a_i|^2$.

To get some practice, time to do some calculations by hand.

Exercise 1.5.3 Compute by hand the single gate operations H, X, Y, Z, S, and T on the state $|0\rangle$, and complete the table below. Compare with the QUI in each case.

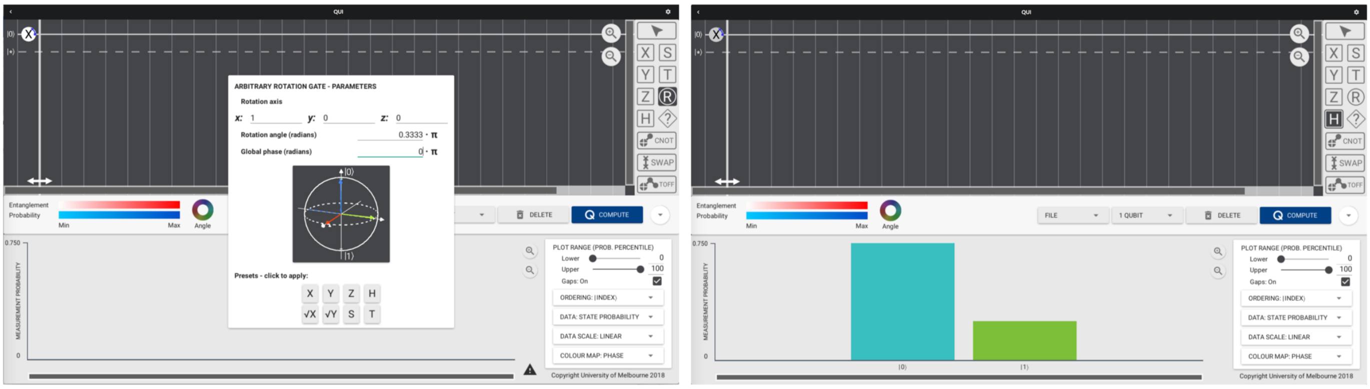
Gate	Operator (matrix rep)	Operation (matrix rep)	Operation (ket rep)	Final state $\|0\rangle$ amplitude	Final state $\|1\rangle$ amplitude	Final state probabilities
H	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$H\|0\rangle$ $= \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$ $= \|a_0\|e^{i\theta_0}\|0\rangle + \|a_1\|e^{i\theta_1}\|1\rangle$	$\|a_0\| = \frac{1}{\sqrt{2}}$ $\theta_0 = 0$	$\|a_1\| = \frac{1}{\sqrt{2}}$ $\theta_1 = 0$	$p_0 = 0.5$ $p_1 = 0.5$
X	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$X\|0\rangle = \|1\rangle$	$\|a_0\| = 0$ $\theta_0 = 0$	$\|a_1\| = 1$ $\theta_1 = 0$	$p_0 = 0$ $p_1 = 1$
Y	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$					
Z	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$					
S	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$					
T	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$					

Exercise 1.5.4 Compute by hand the single gate operations H, X, Y, Z, S, and T on the state $|1\rangle$, and complete the table below. Compare with the QUI in each case.

Gate	Operator (matrix rep)	Operation (matrix rep)	Operation (ket rep)	Final state $\|0\rangle$ amplitude	Final state $\|1\rangle$ amplitude	Final state probabilities
H	$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$ $=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$	$H\|1\rangle$ $=\frac{1}{\sqrt{2}}\|0\rangle-\frac{1}{\sqrt{2}}\|1\rangle$ $=\|a_0\|e^{i\theta_0}\|0\rangle+\ \|a_1\|e^{i\theta_1}\|1\rangle$	$\|a_0\|=\frac{1}{\sqrt{2}}$ $\theta_0=0$	$\|a_1\|=\frac{1}{\sqrt{2}}$ $\theta_1=\pi$	$p_0=\frac{1}{2}$ $p_1=\frac{1}{2}$
X	$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}$ $=\begin{pmatrix}1\\ 0\end{pmatrix}$	$X\|1\rangle=\|0\rangle$	$\|a_0\|=1$ $\theta_0=0$	$\|a_1\|=0$ $\theta_1=0$	$p_0=1$ $p_1=0$
Y	$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$					
Z	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$					
S	$\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$					
T	$\begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}$					

In the lecture notes summarising each gate (Lecture 2) we used a particular superposition example state which we will now construct and reproduce those instances.

Exercise 1.5.5 To construct our example state, start from a blank 1-qubit circuit and insert a R-gate. Once placed in the circuit right click on the R-gate in the circuit to bring up the rotation gate menu: Edit Parameters -> set the axis to X and rotation angle to $\theta_R = \frac{\pi}{3}$, and set global phase to zero (bottom left). The output should look like this (bottom right).



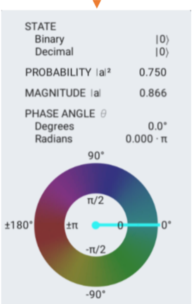
Don't worry how this gate works for now, we will just look at the final output state to further illustrate the QUI notation for the individual complex amplitudes, the Bloch representation of the overall state, and the gate operation examples given in the lectures.

The overall quantum state at the end of this circuit is a superposition $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$. We will now inspect the individual complex amplitudes in the SICs. In the following, convert from the QUI polar notation for the complex amplitudes to cartesian as indicated.

Overall quantum state: $|\psi\rangle = \boxed{a_0\,|0\rangle} + a_1\,|1\rangle$

QUI representation of complex amplitude a_0 in polar form:

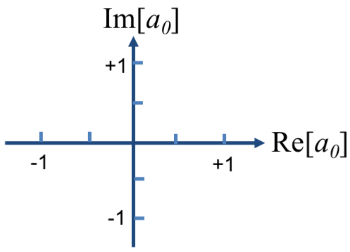
$|a_0|e^{i\theta_0}$



Convert amplitude a_0 to cartesian form and plot:

$$a_0 = |a_0|e^{i\theta_0} = \mathrm{Re}[a_0] + i\mathrm{Im}[a_0]$$

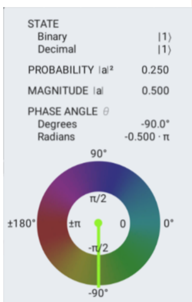
$a_0 =$ _______ $+\, i$ _______



Overall quantum state: $|\psi\rangle = a_0\,|0\rangle + \boxed{a_1\,|1\rangle}$

QUI representation of complex amplitude a_1 in polar form:

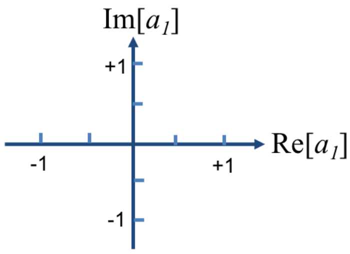
$|a_1|e^{i\theta_1}$



Convert amplitude a_1 to cartesian form and plot:

$$a_1 = |a_1|e^{i\theta_1} = \mathrm{Re}[a_1] + i\mathrm{Im}[a_1]$$

$a_1 =$ _______ $+\, i$ _______



Hence, verify that the state created is:

$$|0\rangle \rightarrow \frac{\sqrt{3}}{2}|0\rangle + \frac{-i}{2}|1\rangle = \qquad\quad |0\rangle + \qquad\quad |1\rangle$$

In above space provided write the complex amplitudes in polar form, $|a_0|e^{i\theta_0}|0\rangle + |a_1|e^{i\theta_1}|1\rangle$.

Exercise 1.5.6 Now we will look at how we represent this state on the Bloch sphere by converting from the QUI polar form.

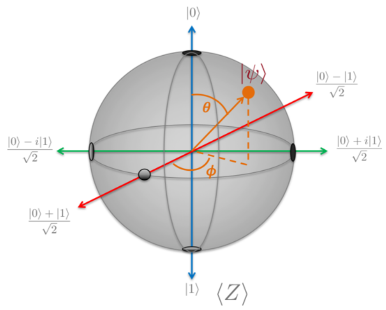
Specifying overall state on the Bloch sphere

Manipulate polar form:

$$|\psi\rangle = a_0\,|0\rangle + a_1\,|1\rangle = |a_0|e^{i\theta_0}\,|0\rangle + |a_1|e^{i\theta_1}\,|1\rangle$$

$$|\psi\rangle = e^{i\theta_0}\left(|a_0|\,|0\rangle + |a_1|e^{i(\theta_1-\theta_0)}\,|1\rangle\right) = e^{i\theta_{\text{global}}}\left(\cos\frac{\theta_{\text{B}}}{2}\,|0\rangle + \sin\frac{\theta_{\text{B}}}{2}e^{i\phi_{\text{B}}}\,|1\rangle\right)$$

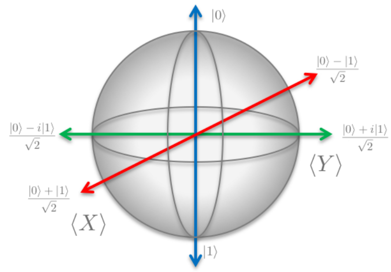
Ignore global phase: $\boxed{|\psi\rangle = a_0\,|0\rangle + a_1\,|a_1\rangle \rightarrow \cos\frac{\theta_{\text{B}}}{2}\,|0\rangle + \sin\frac{\theta_{\text{B}}}{2}e^{i\phi_{\text{B}}}\,|1\rangle}$



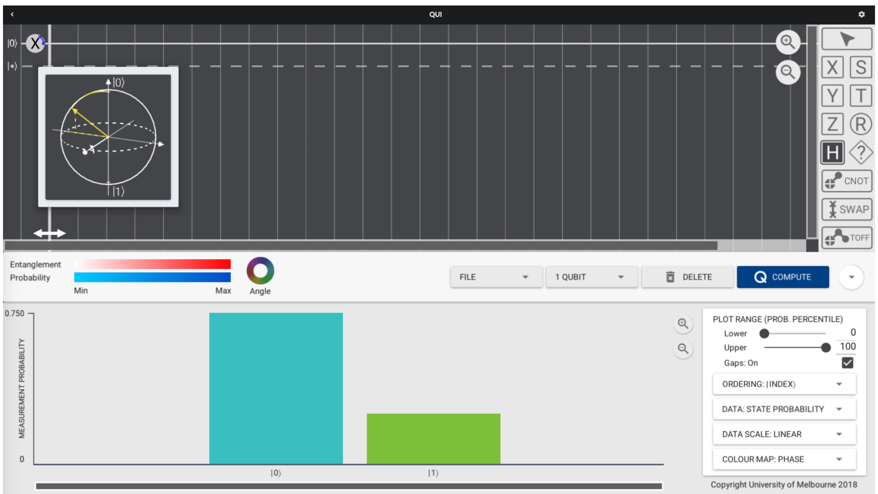
Find the position of the final state on the Bloch sphere and plot:

$\theta_{\text{B}} =$

$\phi_{\text{B}} =$



A useful QUI feature to note: in single qubit mode, hovering the mouse over each gate in the programmed circuit will display the Bloch sphere rotation corresponding to that operation on the state at that time point in the circuit. Hence, you can check your calculations in the above by inspecting the Bloch animation at the final H-gate.



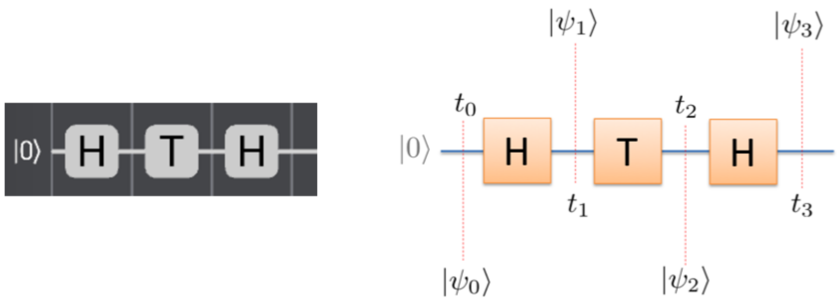
By following the Bloch sphere animations for each time step you can see how the quantum evolves over time.

Exercise 1.5.7 Using the state created in **Ex 1.5.5** above verify the examples of applying the various single qubit gates (X, Y, Z, H, S, T) on this state as given in Lecture 2.

1.6 Sequential gates in detail

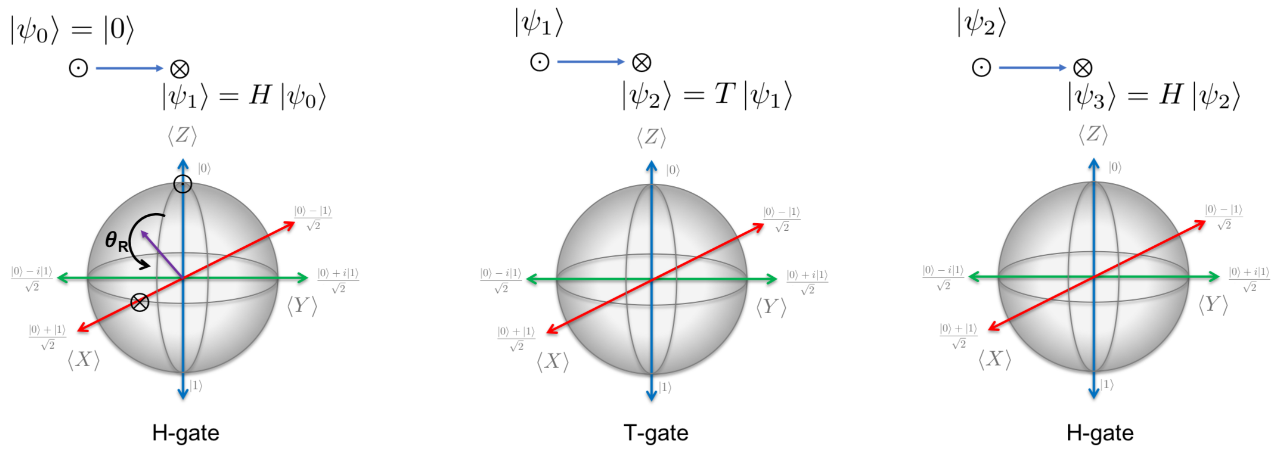
In the following exercises we will look at the mathematics of the quantum state evolution in more detail. In order to understand what is happening we will compute some examples by hand and compare with the QUI output.

Exercise 1.6.1 Program the following sequence of single qubit gates H-T-H (shown below). Compute by hand the states at each time step in the matrix representation, covert to ket representation and fill out the table below. Now compare the amplitudes you obtained with the QUI output at each time step and check they agree.



Matrix representation	**Ket representation**	**Amplitudes**
$\lvert\psi_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\lvert\psi_0\rangle = \lvert 0\rangle$	$\lvert a_0\rvert = 1$ $\quad \lvert a_1\rvert = 0$ $\theta_0 = 0$ $\quad \theta_1 = 0$
$\lvert\psi_1\rangle = H\lvert\psi_0\rangle$		
$\lvert\psi_2\rangle = T\lvert\psi_1\rangle$		
$\lvert\psi_3\rangle = H\lvert\psi_2\rangle$		

Exercise 1.6.2 Examine the QUI Bloch sphere animations and complete the following representations of these gates in the sequence H-T-H as per below (i.e. plot $\odot$ and $\otimes$):

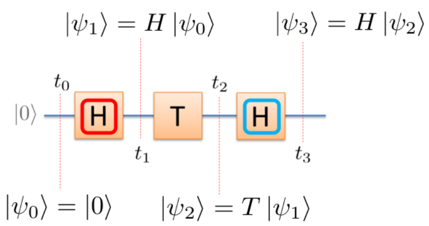


Exercise 1.6.3 Writing the above example as a string of operations on the initial state would look like the following:

$$|\psi_3\rangle = H\,|\psi_2\rangle = H\,T\,|\psi_1\rangle = H\,T\,H\,|\psi_0\rangle$$

This looks exactly like the circuit ordering, but that's because this example is palindromic (looks the same from either direction) – see below.

Circuit & time-ordering



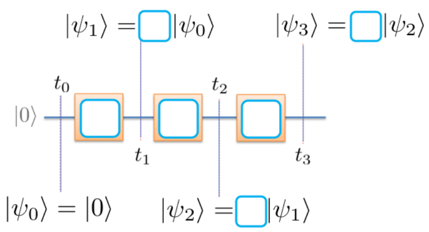
Operator-ordering

$$|\psi_3\rangle = \boxed{H}|\psi_2\rangle = \boxed{H}T\,|\psi_1\rangle = \boxed{H}T\boxed{H}|\psi_0\rangle$$

(reverse of circuit ordering!)

Complete the same analysis for the circuit comprising the combination X-Y-Z:

<u>Circuit & time-ordering</u>



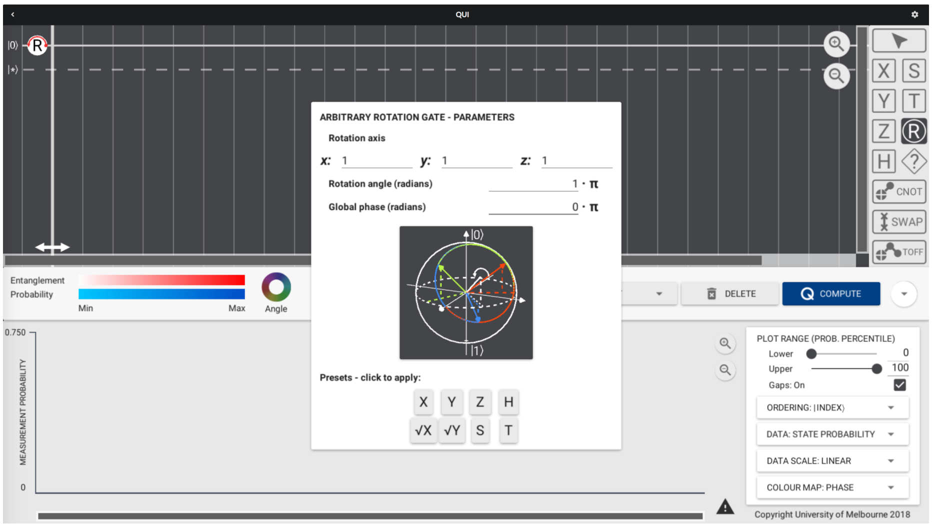
<u>Operator-ordering</u>

$$|\psi_3\rangle = \boxed{}\,|\psi_2\rangle = \boxed{}\boxed{}|\psi_1\rangle = \boxed{}\boxed{}\boxed{}|\psi_0\rangle$$

The reversal of operator and time orderings is something to keep in mind.

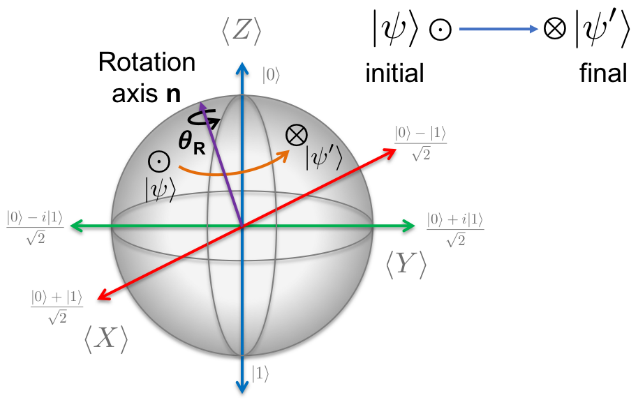
1.7 Arbitrary rotation and global phase

Again, to program the arbitrary rotation gate, R, place it on a single qubit in QUI and right click on the gate. Select "edit parameters" option and the pop-up window will appear:



In this window, you can select rotation axis (will self-normalise), angle, and global phase.

For reference, the R-gate is defined as (refer to Lecture 2):



$$|\psi'\rangle = R_{\hat{\mathbf{n}}}(\theta_{\mathrm{R}})\,|\psi\rangle$$

$$R_{\hat{\mathbf{n}}}(\theta_R) = e^{i\theta_g}\left(I\cos\frac{\theta_R}{2} - i\,\hat{\mathbf{n}}\cdot\mathbf{X}\sin\frac{\theta_R}{2}\right)$$

$$\mathbf{X} = (X, Y, Z) \qquad\qquad \hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|}$$

Exercise 1.7.1 Keeping the global phase zero for now, program in various combinations of rotation axis and angles. In QUI look at the animation to see where the overall state ends up on the Bloch sphere and the state created – can you understand the evolution from the initial state $|0\rangle$ in terms of the rotation operator programmed? You can repeat by starting from the $|1\rangle$ state and/or a non-trivial superposition initial state (e.g. similar to that created in **Ex 1.5.5**).

Exercise 1.7.2 Using the arbitrary rotation gate, R, program a rotation which is exactly equivalent to the Hadamard gate rotation (i.e. by setting the axis and global phase appropriately). Confirm that it produces the correct final states when acting on both computational states. Hint: you can program a H-gate and inspect the Bloch animation to understand the rotation performed (and refer to Lecture notes).

Answers: Axis = _____ ; Rotation angle = _____ ; Global phase = _____

Exercise 1.7.3 Program HTH using R-gates with global phase set to zero. Compare the final state with that of **Ex 1.6.1** and verify that the probabilities are unaffected by the choice of global phase. Find and set the global phase for each R-gate to exactly match the final phase of the HTH circuit.

Answer: Global phase (R = H-gate) = _____ ; Global phase (R = T-gate) = _____

Exercise 1.7.4 A product of two or more rotations is another rotation. Can you find a single R-gate which is exactly equivalent to HTH? Fill in your answers below.

Rotation axis:

X = _______ Y = _______ Z =_______

Rotation angle:

θ_R = _____

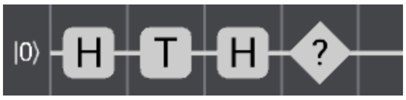
Global phase:

θ_g = _____

Check these parameters by ensuring that your rotation has the same effect on the computational basis states, $|0\rangle$ and $|1\rangle$, as the original HTH combination.

1.8 Measurements

Exercise 1.8.1 Add a measurement gate at the end of the HTH sequence. Hit the compute button many times (say N = 100) and record the number of 0 and 1 outcomes and fill in the table below. Compare the estimated probabilities with those expected.



$\|\psi\rangle = HTH\|0\rangle$ components	Exact probability	Measurement record	# outcomes, n	Estimated Prob = n/N
$\|0\rangle$	0.854	𝍸 𝍸 ⋯		
$\|1\rangle$	0.146	𝍸 ⋯		