Solutions to Exercise Set 5.

10.3. (a) $\chi^2 = [(15-15)^2 + (21-15)^2 + (17-15)^2 + (7-15)^2]/15 = 104/15 = 6.93$. It has 3 degrees of freedom.

(b) $\lambda = [(12-15)^2 + (21-15)^2 + (18-15)^2 + (9-15)^2]/15 = 90/15 = 6.0$.

(c) At 3 degrees of freedom it requires a non-centrality parameter of 14.172 at 5% level of significance. This means $n(\lambda/60) = 14.172$, which reduces to $n = 142$.

11.3. Since $\sum |z_j| = \sum_{j=0}^{\infty} \beta^j = 1/(1-\beta) < \infty$ for $|\beta| < 1$, we have from Exercise 11.7 that $\sqrt{n}(\overline{Y}_n - \mu) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \sigma^2)$. Here $\mu = \xi \sum_0^{\infty} \beta^j = \xi/(1-\beta)$. Similarly we find $\sigma_{00} = \tau^2/(1-\beta^2)$, and $\sigma_{0k} = \beta^k \tau^2/(1-\beta^2)$. Hence, $\sigma^2 = (\tau^2/(1-\beta^2))[1 + 2\beta + 2\beta^2 + 2\beta^3 + \cdots] = \tau^2/(1-\beta)^2$.

11.6. (a) $\mathrm{E}(Y_n) = 2p^2 + (1-p)p = p + p^2$. $\mathrm{E}(Y_n^2) = 4p^2 + (1-p)p = p + 3p^2$, so $\mathrm{Var}(Y_n) = (p + 3p^2) - (p + p^2)^2 = p + 2p^2 - 2p^3 - p^4$,

(b) The sequence Y_n is 1-dependent stationary. Since $\mathrm{E}(Y_1 Y_2) = 4p^3 + 2(1-p)p^2 = 2p^2 + 2p^3$, we have $\mathrm{Cov}(Y_1, Y_2) = (2p^2 + 2p^3) - (p + p^2)^2 = p^2 - p^4$. Therefore,

$$\sqrt{n}\left(\frac{1}{n} S_n - (p + p^2)\right) \xrightarrow{\mathcal{L}} \mathcal{N}(0, \mathrm{Var}(Y_1) + 2\mathrm{Cov}(Y_1, Y_2)) = \mathcal{N}(0, p + 4p^2 - 2p^3 - 3p^4).$$